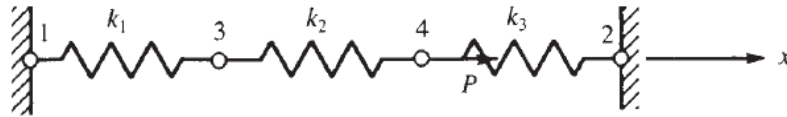


Chapter 2

2.1

(a)



$$[k^{(1)}] = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & 0 & 0 & 0 \\ -k_1 & 0 & k_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[k^{(2)}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k_2 & -k_2 \\ 0 & 0 & -k_2 & k_2 \end{bmatrix}$$

$$[k^{(3)}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_3 & 0 & -k_3 \\ 0 & 0 & 0 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix}$$

$$[K] = [k^{(1)}] + [k^{(2)}] + [k^{(3)}]$$

$$[K] = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_3 & 0 & -k_3 \\ -k_1 & 0 & k_1 + k_2 & -k_2 \\ 0 & -k_3 & -k_2 & k_2 + k_3 \end{bmatrix}$$

(b) Nodes 1 and 2 are fixed so $u_1 = 0$ and $u_2 = 0$ and $[K]$ becomes

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}$$

$$\{F\} = [K] \{d\}$$

$$\begin{Bmatrix} F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} 0 \\ P \end{Bmatrix} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$

$$\{F\} = [K] \{d\} \Rightarrow [K]^{-1} \{F\} = [K]^{-1} [K] \{d\}$$

$$\Rightarrow [K]^{-1} \{F\} = \{d\}$$

Using the adjoint method to find $[K^{-1}]$

$$C_{11} = k_2 + k_3 \quad C_{21} = (-1)^3 (-k_2)$$

$$C_{12} = (-1)^{1+2} (-k_2) = k_2 \quad C_{22} = k_1 + k_2$$

$$[C] = \begin{bmatrix} k_2 + k_3 & k_2 \\ k_2 & k_1 + k_2 \end{bmatrix} \text{ and } C^T = \begin{bmatrix} k_2 + k_3 & k_2 \\ k_2 & k_1 + k_2 \end{bmatrix}$$

$$\det [K] = |[K]| = (k_1 + k_2)(k_2 + k_3) - (-k_2)(-k_2)$$

$$\Rightarrow |[K]| = (k_1 + k_2)(k_2 + k_3) - k_2^2$$

$$[K^{-1}] = \frac{[C^T]}{\det K}$$

$$[K^{-1}] = \frac{\begin{bmatrix} k_2 + k_3 & k_2 \\ k_2 & k_1 + k_2 \end{bmatrix}}{(k_1 + k_2)(k_2 + k_3) - k_2^2} = \frac{\begin{bmatrix} k_2 + k_3 & k_2 \\ k_2 & k_1 + k_2 \end{bmatrix}}{k_1 k_2 + k_1 k_3 + k_2 k_3}$$

$$\begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \frac{\begin{bmatrix} k_2 + k_3 & k_2 \\ k_2 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} 0 \\ P \end{Bmatrix}}{k_1 k_2 + k_1 k_3 + k_2 k_3}$$

$$\Rightarrow u_3 = \frac{k_2 P}{k_1 k_2 + k_1 k_3 + k_2 k_3}$$

$$\Rightarrow u_4 = \frac{(k_1 + k_2) P}{k_1 k_2 + k_1 k_3 + k_2 k_3}$$

(c) In order to find the reaction forces we go back to the global matrix $F = [K] \{d\}$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_3 & 0 & -k_3 \\ -k_1 & 0 & k_1 + k_2 & -k_2 \\ 0 & -k_3 & -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

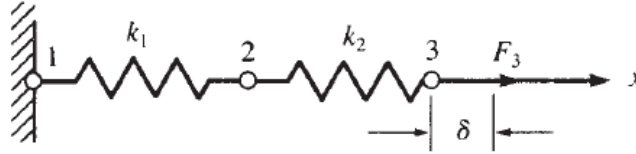
$$F_{1x} = -k_1 u_3 = -k_1 \frac{k_2 P}{k_1 k_2 + k_1 k_3 + k_2 k_3}$$

$$\Rightarrow F_{1x} = \frac{-k_1 k_2 P}{k_1 k_2 + k_1 k_3 + k_2 k_3}$$

$$F_{2x} = -k_3 u_4 = -k_3 \frac{(k_1 + k_2) P}{k_1 k_2 + k_1 k_3 + k_2 k_3}$$

$$\Rightarrow F_{2x} = \frac{-k_3 (k_1 + k_2) P}{k_1 k_2 + k_1 k_3 + k_2 k_3}$$

2.2



$$k_1 = k_2 = k_3 = 1000 \frac{\text{lb}}{\text{in.}}$$

$$[k^{(1)}] = \begin{matrix} & \begin{matrix} (1) & (2) \end{matrix} \\ \begin{matrix} (1) \\ (2) \end{matrix} & \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \end{matrix}; \quad [k^{(2)}] = \begin{matrix} & \begin{matrix} (2) & (3) \end{matrix} \\ \begin{matrix} (2) \\ (3) \end{matrix} & \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \end{matrix}$$

By the method of superposition the global stiffness matrix is constructed.

$$[K] = \begin{matrix} & \begin{matrix} (1) & (2) & (3) \end{matrix} \\ \begin{matrix} (1) \\ (2) \\ (3) \end{matrix} & \begin{bmatrix} k & -k & 0 \\ -k & k+k & -k \\ 0 & -k & k \end{bmatrix} \end{matrix} \Rightarrow [K] = \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}$$

Node 1 is fixed $\Rightarrow u_1 = 0$ and $u_3 = \delta$

$$\{F\} = [K] \{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 0 \\ F_{3x} = ? \end{Bmatrix} = \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = \delta \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} 0 \\ F_{3x} \end{Bmatrix} = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_2 \\ \delta \end{Bmatrix} \Rightarrow \begin{Bmatrix} 0 \\ F_{3x} \end{Bmatrix} = \begin{Bmatrix} 2ku_2 - k\delta \\ -ku_2 + k\delta \end{Bmatrix}$$

$$\Rightarrow u_2 = \frac{k\delta}{2k} = \frac{\delta}{2} = \frac{1 \text{ in.}}{2} \Rightarrow u_2 = 0.5''$$

$$F_{3x} = -k(0.5'') + k(1'')$$

$$F_{3x} = (-1000 \frac{\text{lb}}{\text{in.}})(0.5'') + (1000 \frac{\text{lb}}{\text{in.}})(1'')$$

$$F_{3x} = 500 \text{ lbs}$$

Internal forces

Element (1)

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(2)} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = 0.5'' \end{Bmatrix}$$

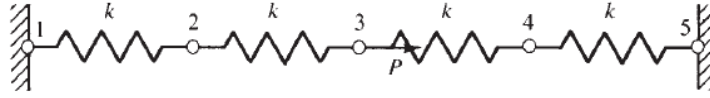
$$\Rightarrow f_{1x}^{(1)} = (-1000 \frac{\text{lb}}{\text{in.}})(0.5'') \Rightarrow f_{1x}^{(1)} = -500 \text{ lb}$$

$$f_{2x}^{(1)} = (1000 \frac{\text{lb}}{\text{in.}})(0.5'') \Rightarrow f_{2x}^{(1)} = 500 \text{ lb}$$

Element (2)

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_2 = 0.5'' \\ u_3 = 1'' \end{Bmatrix} \Rightarrow \begin{matrix} f_{2x}^{(2)} = -500 \text{ lb} \\ f_{3x}^{(2)} = 500 \text{ lb} \end{matrix}$$

2.3



$$(a) [k^{(1)}] = [k^{(2)}] = [k^{(3)}] = [k^{(4)}] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

By the method of superposition we construct the global $[K]$ and knowing $\{F\} = [K] \{d\}$ we have

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 0 \\ F_{3x} = P \\ F_{4x} = 0 \\ F_{5x} = ? \end{Bmatrix} = \begin{bmatrix} k & -k & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 \\ 0 & -k & 2k & -k & 0 \\ 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & -k & k \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 \\ u_4 \\ u_5 = 0 \end{Bmatrix}$$

$$(b) \begin{Bmatrix} 0 \\ P \\ 0 \end{Bmatrix} = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} \Rightarrow \begin{matrix} 0 = 2ku_2 - ku_3 \\ P = -ku_2 + 2ku_3 - ku_4 \\ 0 = -ku_3 + 2ku_4 \end{matrix} \quad \begin{matrix} (1) \\ (2) \\ (3) \end{matrix}$$

$$\Rightarrow u_2 = \frac{u_3}{2} ; u_4 = \frac{u_3}{2}$$

Substituting in the second equation above

$$P = -k u_2 + 2k u_3 - k u_4$$

$$\Rightarrow P = -k \left(\frac{u_3}{2} \right) + 2k u_3 - k \left(\frac{u_3}{2} \right)$$

$$\Rightarrow P = k u_3$$

$$\Rightarrow u_3 = \frac{P}{k}$$

$$u_2 = \frac{P}{2k} ; u_4 = \frac{P}{2k}$$

(c) In order to find the reactions at the fixed nodes 1 and 5 we go back to the global equation $\{F\} = [K] \{d\}$

$$F_{1x} = -k u_2 = -k \frac{P}{2k} \Rightarrow F_{1x} = -\frac{P}{2}$$

$$F_{5x} = -k u_4 = -k \frac{P}{2k} \Rightarrow F_{5x} = -\frac{P}{2}$$

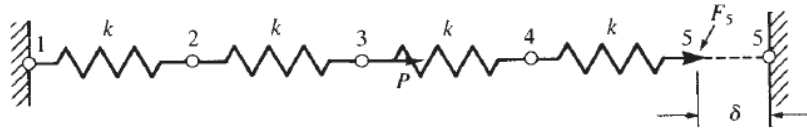
Check

$$\Sigma F_x = 0 \Rightarrow F_{1x} + F_{5x} + P = 0$$

$$\Rightarrow -\frac{P}{2} + \left(-\frac{P}{2}\right) + P = 0$$

$$\Rightarrow 0 = 0$$

2.4



(a) $[k^{(1)}] = [k^{(2)}] = [k^{(3)}] = [k^{(4)}] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$

By the method of superposition the global $[K]$ is constructed.

Also $\{F\} = [K] \{d\}$ and $u_1 = 0$ and $u_5 = \delta$

$$\begin{Bmatrix} F_{1x}=? \\ F_{2x}=0 \\ F_{3x}=0 \\ F_{4x}=0 \\ F_{5x}=? \end{Bmatrix} = \begin{bmatrix} k & -k & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 \\ 0 & -k & 2k & -k & 0 \\ 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & -k & k \end{bmatrix} \begin{Bmatrix} u_1=0 \\ u_2=? \\ u_3=? \\ u_4=? \\ u_5=\delta \end{Bmatrix}$$

(b) $0 = 2k u_2 - k u_3$ (1)

$0 = -k u_2 + 2k u_3 - k u_4$ (2)

$0 = -k u_3 + 2k u_4 - k \delta$ (3)

From (2)

$$u_3 = 2 u_2$$

From (3)

$$u_4 = \frac{\delta + 2 u_2}{2}$$

Substituting in Equation (2)

$$\Rightarrow -k(u_2) + 2k(2u_2) - k\left(\frac{\delta + 2u_2}{2}\right)$$

$$\Rightarrow -u_2 + 4u_2 - u_2 - \frac{\delta}{2} = 0 \Rightarrow u_2 = \frac{\delta}{4}$$

$$\Rightarrow u_3 = 2 \frac{\delta}{4} \Rightarrow u_3 = \frac{\delta}{2}$$

$$\Rightarrow u_4 = \frac{\delta + 2 \frac{\delta}{4}}{2} \Rightarrow u_4 = \frac{3\delta}{4}$$

(c) Going back to the global equation

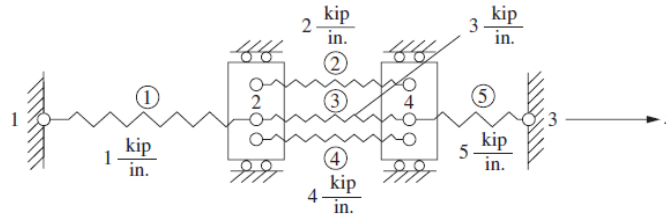
$$\{F\} = [K] \{d\}$$

$$F_{1x} = -k u_2 = -k \frac{\delta}{4} \Rightarrow F_{1x} = -\frac{k \delta}{4}$$

$$F_{5x} = -k u_4 + k \delta = -k \left(\frac{3\delta}{4} \right) + k \delta$$

$$\Rightarrow F_{5x} = \frac{k \delta}{4}$$

2.5



$$[k^{(1)}] = \begin{bmatrix} u_1 & u_2 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}; \quad [k^{(2)}] = \begin{bmatrix} u_2 & u_4 \\ 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$[k^{(3)}] = \begin{bmatrix} u_2 & u_4 \\ 3 & -3 \\ -3 & 3 \end{bmatrix}; \quad [k^{(4)}] = \begin{bmatrix} u_2 & u_4 \\ 4 & -4 \\ -4 & 4 \end{bmatrix}$$

$$[k^{(5)}] = \begin{bmatrix} u_4 & u_3 \\ 5 & -5 \\ -5 & 5 \end{bmatrix}$$

Assembling global $[K]$ using direct stiffness method

$$[K] = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1+2+3+4 & 0 & -2-3-4 \\ 0 & 0 & 5 & -5 \\ 0 & -2-3-4 & -5 & 2+3+4+5 \end{bmatrix}$$

Simplifying

$$[K] = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 10 & 0 & -9 \\ 0 & 0 & 5 & -5 \\ 0 & -9 & -5 & 14 \end{bmatrix} \frac{\text{kip}}{\text{in.}}$$

2.6 Now apply +3 kip at node 2 in spring assemblage of P 2.5.

$$\therefore F_{2x} = 3 \text{ kip}$$

$$[K]\{d\} = \{F\}$$

[K] from P 2.5

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 10 & 0 & -9 \\ 0 & 0 & 5 & -5 \\ 0 & -9 & -5 & 14 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 = 0 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 3 \\ F_3 \\ 0 \end{Bmatrix} \quad (A)$$

where $u_1 = 0, u_3 = 0$ as nodes 1 and 3 are fixed.

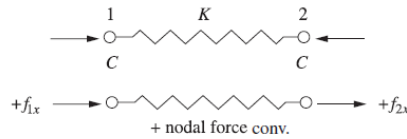
Using Equations (1) and (3) of (A)

$$\begin{bmatrix} 10 & -9 \\ -9 & 14 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 0 \end{Bmatrix}$$

Solving

$$u_2 = 0.712 \text{ in.}, \quad u_4 = 0.458 \text{ in.}$$

2.7



$$f_{1x} = C, \quad f_{2x} = -C$$

$$f = -k\delta = -k(u_2 - u_1)$$

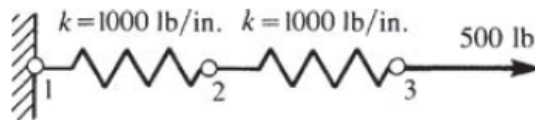
$$\therefore f_{1x} = -k(u_2 - u_1)$$

$$f_{2x} = -(-k)(u_2 - u_1)$$

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{Bmatrix} k & -k \\ -k & k \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\therefore [K] = \begin{Bmatrix} k & -k \\ -k & k \end{Bmatrix} \text{ same as for tensile element}$$

2.8



$$k_1 = 1000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}; \quad k_2 = 1000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

So

$$[K] = 1000 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\{F\} = [K] \{d\}$$

$$\Rightarrow \begin{bmatrix} F_1 = ? \\ F_2 = 0 \\ F_3 = 500 \end{bmatrix} = 1000 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = ? \end{bmatrix}$$

$$\Rightarrow 0 = 2000 u_2 - 1000 u_3 \quad (1)$$

$$500 = -1000 u_2 + 1000 u_3 \quad (2)$$

From (1)

$$u_2 = \frac{1000}{2000} u_3 \Rightarrow u_2 = 0.5 u_3 \quad (3)$$

Substituting (3) into (2)

$$\Rightarrow 500 = -1000 (0.5 u_3) + 1000 u_3$$

$$\Rightarrow 500 = 500 u_3$$

$$\Rightarrow u_3 = 1 \text{ in.}$$

$$\Rightarrow u_2 = (0.5) (1 \text{ in.}) \Rightarrow u_2 = 0.5 \text{ in.}$$

Element 1-2

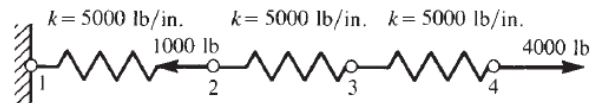
$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = 1000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \text{ in.} \\ 0.5 \text{ in.} \end{Bmatrix} \Rightarrow \begin{matrix} f_{1x}^{(1)} = -500 \text{ lb} \\ f_{2x}^{(1)} = 500 \text{ lb} \end{matrix}$$

Element 2-3

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = 1000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.5 \text{ in.} \\ 1 \text{ in.} \end{Bmatrix} \Rightarrow \begin{matrix} f_{2x}^{(2)} = -500 \text{ lb} \\ f_{3x}^{(2)} = 500 \text{ lb} \end{matrix}$$

$$F_{1x} = 500 [1 \ -1 \ 0] \begin{bmatrix} 0 \\ 0.5 \text{ in.} \\ 1 \text{ in.} \end{bmatrix} \Rightarrow F_{1x} = -500 \text{ lb}$$

2.9



$$\begin{matrix} (1) & (2) \\ [k^{(1)}] = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix} \\ (2) & (3) \end{matrix}$$

$$[k^{(2)}] = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix}$$

(3) (4)

$$[k^{(3)}] = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix}$$

(1) (2) (3) (4)

$$[K] = \begin{bmatrix} 5000 & -5000 & 0 & 0 \\ -5000 & 10000 & -5000 & 0 \\ 0 & -5000 & 10000 & -5000 \\ 0 & 0 & -5000 & 5000 \end{bmatrix}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = -1000 \\ F_{3x} = 0 \\ F_{4x} = 4000 \end{Bmatrix} = \begin{bmatrix} 5000 & -5000 & 0 & 0 \\ -5000 & 10000 & -5000 & 0 \\ 0 & -5000 & 10000 & -5000 \\ 0 & 0 & -5000 & 5000 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\Rightarrow \begin{aligned} u_1 &= 0 \text{ in.} \\ u_2 &= 0.6 \text{ in.} \\ u_3 &= 1.4 \text{ in.} \\ u_4 &= 2.2 \text{ in.} \end{aligned}$$

Reactions

$$F_{1x} = [5000 \quad -5000 \quad 0 \quad 0] \begin{Bmatrix} u_1 = 0 \\ u_2 = 0.6 \\ u_3 = 1.4 \\ u_4 = 2.2 \end{Bmatrix} \Rightarrow F_{1x} = -3000 \text{ lb}$$

Element forces

Element (1)

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.6 \end{Bmatrix} \Rightarrow \begin{aligned} f_{1x}^{(1)} &= -3000 \text{ lb} \\ f_{2x}^{(1)} &= 3000 \text{ lb} \end{aligned}$$

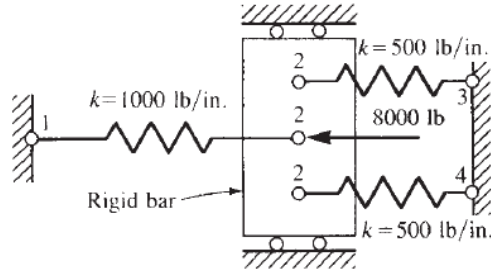
Element (2)

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix} \begin{Bmatrix} 0.6 \\ 1.4 \end{Bmatrix} \Rightarrow \begin{aligned} f_{2x}^{(2)} &= -4000 \text{ lb} \\ f_{3x}^{(2)} &= 4000 \text{ lb} \end{aligned}$$

Element (3)

$$\begin{Bmatrix} f_{3x}^{(3)} \\ f_{4x}^{(3)} \end{Bmatrix} = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix} \begin{Bmatrix} 1.4 \\ 2.2 \end{Bmatrix} \Rightarrow \begin{aligned} f_{3x}^{(3)} &= -4000 \text{ lb} \\ f_{4x}^{(3)} &= 4000 \text{ lb} \end{aligned}$$

2.10



$$[k^{(1)}] = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix}$$

$$[k^{(2)}] = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}$$

$$[k^{(3)}] = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}$$

$$\{F\} = [K] \{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = -8000 \\ F_{3x} = ? \\ F_{4x} = ? \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = 0 \\ u_4 = 0 \end{Bmatrix}$$

$$\Rightarrow u_2 = \frac{-8000}{2000} = -4 \text{ in.}$$

Reactions

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{Bmatrix} 4000 \\ -8000 \\ 2000 \\ 2000 \end{Bmatrix} \text{ lb}$$

Element (1)

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = \begin{Bmatrix} 4000 \\ -4000 \end{Bmatrix} \text{ lb}$$

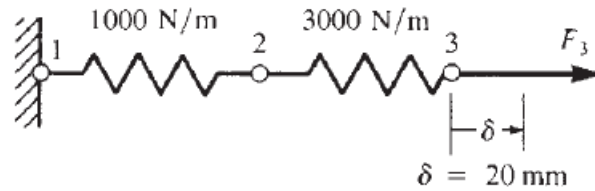
Element (2)

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -4 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{Bmatrix} -2000 \\ 2000 \end{Bmatrix} \text{ lb}$$

Element (3)

$$\begin{Bmatrix} f_{2x}^{(3)} \\ f_{4x}^{(3)} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -4 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x}^{(3)} \\ f_{4x}^{(3)} \end{Bmatrix} = \begin{Bmatrix} -2000 \\ 2000 \end{Bmatrix} \text{ lb}$$

2.11



$$[k^{(1)}] = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix}; \quad [k^{(2)}] = \begin{bmatrix} 3000 & -3000 \\ -3000 & 3000 \end{bmatrix}$$

$$\{F\} = [K] \{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 0 \\ F_{3x} = ? \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 \\ -1000 & 4000 & -3000 \\ 0 & -3000 & 3000 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = 0.02 \text{ m} \end{Bmatrix}$$

$$\Rightarrow u_2 = 0.015 \text{ m}$$

Reactions

$$F_{1x} = (-1000)(0.015) \Rightarrow F_{1x} = -15 \text{ N}$$

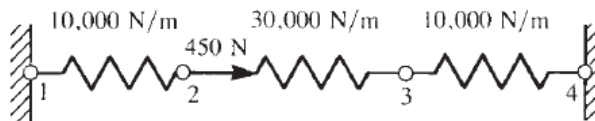
Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.015 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{Bmatrix} -15 \\ 15 \end{Bmatrix} \text{ N}$$

Element (2)

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 3000 & -3000 \\ -3000 & 3000 \end{bmatrix} \begin{Bmatrix} 0.015 \\ 0.02 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{Bmatrix} -15 \\ 15 \end{Bmatrix} \text{ N}$$

2.12



$$[k^{(1)}] = [k^{(3)}] = 10000 \begin{Bmatrix} 1 & -1 \\ -1 & 1 \end{Bmatrix}$$

$$[k^{(2)}] = 10000 \begin{Bmatrix} 3 & -3 \\ -3 & 3 \end{Bmatrix}$$

$$\{F\} = [K] \{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 450 \text{ N} \\ F_{3x} = 0 \\ F_{4x} = ? \end{Bmatrix} = 10000 \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 4 & -3 & 0 \\ 0 & -3 & 4 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = ? \\ u_4 = 0 \end{Bmatrix}$$

$$0 = -3 u_2 + 4 u_3 \Rightarrow u_2 = \frac{4}{3} u_3 \Rightarrow u_2 = 1.33 u_3$$

$$450 \text{ N} = 40000 (1.33 u_3) - 30000 u_3$$

$$\Rightarrow 450 \text{ N} = (23200 \frac{\text{N}}{\text{m}}) u_3 \Rightarrow u_3 = 1.93 \times 10^{-2} \text{ m}$$

$$\Rightarrow u_2 = 1.5 (1.94 \times 10^{-2}) \Rightarrow u_2 = 2.57 \times 10^{-2} \text{ m}$$

Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = 10000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 2.57 \times 10^{-2} \end{Bmatrix} \Rightarrow \begin{matrix} f_{1x}^{(1)} = -257 \text{ N} \\ f_{2x}^{(1)} = 257 \text{ N} \end{matrix}$$

Element (2)

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = 30000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 2.57 \times 10^{-2} \\ 1.93 \times 10^{-2} \end{Bmatrix} \Rightarrow \begin{matrix} f_{2x}^{(2)} = 193 \text{ N} \\ f_{3x}^{(2)} = -193 \text{ N} \end{matrix}$$

Element (3)

$$\begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = 10000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 1.93 \times 10^{-2} \\ 0 \end{Bmatrix} \Rightarrow \begin{matrix} f_{3x}^{(3)} = 193 \text{ N} \\ f_{4x}^{(3)} = -193 \text{ N} \end{matrix}$$

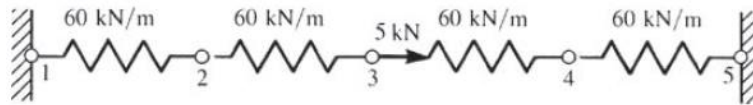
Reactions

$$\{F_{1x}\} = (10000 \frac{\text{N}}{\text{m}}) [1 \ -1] \begin{Bmatrix} 0 \\ 2.57 \times 10^{-2} \end{Bmatrix} \Rightarrow F_{1x} = -257 \text{ N}$$

$$\{F_{4x}\} = (10000 \frac{\text{N}}{\text{m}}) [-1 \ 1] \begin{Bmatrix} 1.93 \times 10^{-2} \\ 0 \end{Bmatrix}$$

$$\Rightarrow F_{4x} = -193 \text{ N}$$

2.13



$$[k^{(1)}] = [k^{(2)}] = [k^{(3)}] = [k^{(4)}] = 60 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\{F\} = [K] \{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 0 \\ F_{3x} = 5 \text{ kN} \\ F_{4x} = 0 \\ F_{5x} = ? \end{Bmatrix} = 60 \begin{Bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{Bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = ? \\ u_4 = ? \\ u_5 = 0 \end{Bmatrix}$$

$$\begin{cases} 0 = 2u_2 - u_3 \Rightarrow u_2 = 0.5u_3 \\ 0 = -u_3 + 2u_4 \Rightarrow u_4 = 0.5u_3 \end{cases} \Rightarrow u_2 = u_4$$

$$\Rightarrow 5 \text{ kN} = -60u_2 + 120(2u_2) - 60u_2$$

$$\Rightarrow 5 = 120u_2 \Rightarrow u_2 = 0.042 \text{ m}$$

$$\Rightarrow u_4 = 0.042 \text{ m}$$

$$\Rightarrow u_3 = 2(0.042) \Rightarrow u_3 = 0.084 \text{ m}$$

Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = 60 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.042 \end{Bmatrix} \Rightarrow \begin{cases} f_{1x}^{(1)} = -2.5 \text{ kN} \\ f_{2x}^{(1)} = 2.5 \text{ kN} \end{cases}$$

Element (2)

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = 60 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.042 \\ 0.084 \end{Bmatrix} \Rightarrow \begin{cases} f_{2x}^{(2)} = -2.5 \text{ kN} \\ f_{3x}^{(2)} = 2.5 \text{ kN} \end{cases}$$

Element (3)

$$\begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = 60 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.084 \\ 0.042 \end{Bmatrix} \Rightarrow \begin{cases} f_{3x}^{(3)} = 2.5 \text{ kN} \\ f_{4x}^{(3)} = -2.5 \text{ kN} \end{cases}$$

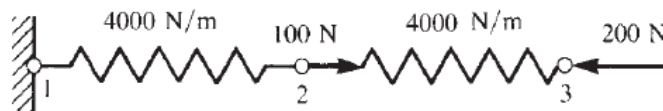
Element (4)

$$\begin{Bmatrix} f_{4x} \\ f_{5x} \end{Bmatrix} = 60 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.042 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} f_{4x}^{(4)} = 2.5 \text{ kN} \\ f_{5x}^{(4)} = -2.5 \text{ kN} \end{cases}$$

$$F_{1x} = 60 \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.042 \end{Bmatrix} \Rightarrow F_{1x} = -2.5 \text{ kN}$$

$$F_{5x} = 60 \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.042 \\ 0 \end{Bmatrix} \Rightarrow F_{5x} = -2.5 \text{ kN}$$

2.14



$$[k^{(1)}] = [k^{(2)}] = 4000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\{F\} = [K] \{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 100 \\ F_{3x} = -200 \end{Bmatrix} = 4000 \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = ? \end{Bmatrix}$$

$$100 = 8000 u_2 - 4000 u_3$$

$$-200 = -4000 u_2 + 4000 u_3$$

$$-100 = 4000 u_2 \Rightarrow u_2 = -0.025 \text{ m}$$

$$100 = 8000 (-0.025) - 4000 u_3 \Rightarrow u_3 = -0.075 \text{ m}$$

Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = 4000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ -0.025 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{1x}^{(1)} = 100 \text{ N} \\ f_{2x}^{(1)} = -100 \text{ N} \end{Bmatrix}$$

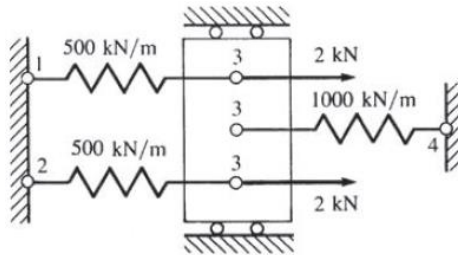
Element (2)

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = 4000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} -0.025 \\ -0.075 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x}^{(2)} = 200 \text{ N} \\ f_{3x}^{(2)} = -200 \text{ N} \end{Bmatrix}$$

Reaction

$$\{F_{1x}\} = 4000 [1 \quad -1] \begin{Bmatrix} 0 \\ -0.025 \end{Bmatrix} \Rightarrow F_{1x} = 100 \text{ N}$$

2.15



$$[k^{(1)}] = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}; [k^{(2)}] = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}; [k^{(3)}] = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = ? \\ F_{3x} = 4 \text{ kN} \\ F_{4x} = ? \end{Bmatrix} = \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = 0 \\ u_3 = ? \\ u_4 = 0 \end{Bmatrix}$$

$$\Rightarrow u_3 = 0.002 \text{ m}$$

Reactions

$$F_{1x} = (-500)(0.002) \Rightarrow F_{1x} = -1.0 \text{ kN}$$

$$F_{2x} = (-500)(0.002) \Rightarrow F_{2x} = -1.0 \text{ kN}$$

$$F_{4x} = (-1000)(0.002) \Rightarrow F_{4x} = -2.0 \text{ kN}$$

Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.002 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{1x} \\ f_{3x} \end{Bmatrix} = \begin{Bmatrix} -1.0 \text{ kN} \\ 1.0 \text{ kN} \end{Bmatrix}$$

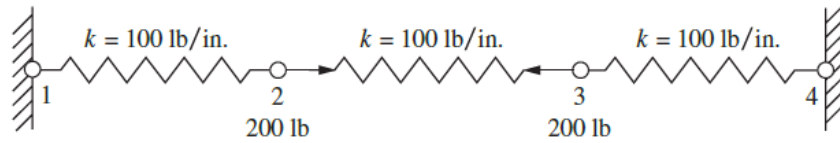
Element (2)

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.002 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{Bmatrix} -1.0 \text{ kN} \\ 1.0 \text{ kN} \end{Bmatrix}$$

Element (3)

$$\begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0.002 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = \begin{Bmatrix} 2.0 \text{ kN} \\ -2.0 \text{ kN} \end{Bmatrix}$$

2.16



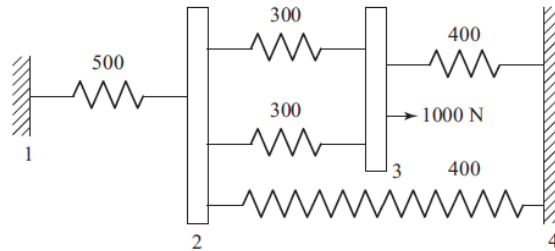
$$\begin{Bmatrix} F_{1x} \\ 200 \\ -200 \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 100 + 100 & -100 & 0 \\ 0 & -100 & 100 + 100 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 200 \\ -200 \end{Bmatrix} = \begin{Bmatrix} 200 & -100 \\ -100 & 200 \end{Bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$u_2 = \frac{2}{3} \text{ in.}$$

$$u_3 = -\frac{2}{3} \text{ in.}$$

2.17



$$\begin{Bmatrix} F_{1x} = ? \\ 0 \\ 1000 \text{ N} \\ F_{4x} = ? \end{Bmatrix} = \begin{bmatrix} -500 & -500 & 0 & 0 \\ -500 & (400 + 300) & -300 - 300 & -400 \\ 0 & -300 - 300 & (300 + 300 + 400) & -400 \\ 0 & -400 & -400 & 400 + 400 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 \\ u_4 = 0 \end{Bmatrix}$$

$$0 = 1500 u_2 - 600 u_3$$

$$1000 = -600 u_2 + 1000 u_3$$

$$u_3 = \frac{15 \cancel{\theta} \cancel{\theta}}{6 \cancel{\theta} \cancel{\theta}} u_2 = 2.5 u_2$$

$$1000 = -600 u_2 + 1000 (2.5 u_2)$$

$$1000 = 1900 u_2$$

$$u_2 = \frac{1000}{1900} = \frac{1}{1.9} \text{ mm} = 0.526 \text{ mm}$$

$$u_3 = 2.5 \left(\frac{1}{1.9} \right) \text{ mm} = 1.316 \text{ mm}$$

$$F_{1x} = -500 \left(\frac{1}{1.9} \right) = -263.16 \text{ N}$$

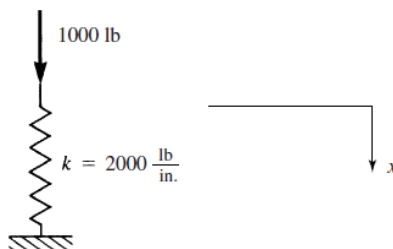
$$F_{4x} = -400 \left(\frac{1}{1.9} \right) - 400 \left(2.5 \left(\frac{1}{1.9} \right) \right)$$

$$= -400 \left(\frac{1}{1.9} + \frac{2.5}{1.9} \right) = -736.84 \text{ N}$$

$$\Sigma F_x = -263.16 + 1000 - 736.84 = 0$$

2.18

(a)



As in Example 2.4

$$\pi_p = U + \Omega$$

$$U = \frac{1}{2} k x^2, \Omega = -Fx$$

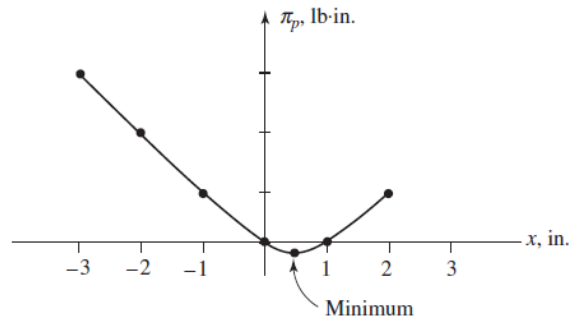
Set up table

$$\pi_p = \frac{1}{2} (2000) x^2 - 1000 x = 1000 x^2 - 1000 x$$

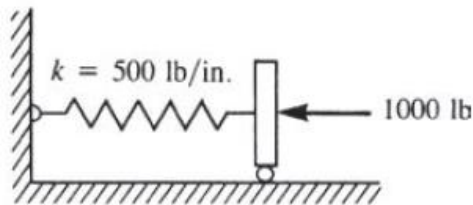
Deformation x , in.	π_p , lb·in.
-3.0	6000
-2.0	3000
-1.0	1000
0.0	0
0.5	-125
1.0	0

2.0	1000
-----	------

$$\frac{\partial \pi_p}{\partial x} = 2000x - 1000 = 0 \Rightarrow x = 0.5 \text{ in. yields minimum } \pi_p \text{ as table verifies.}$$



(b)



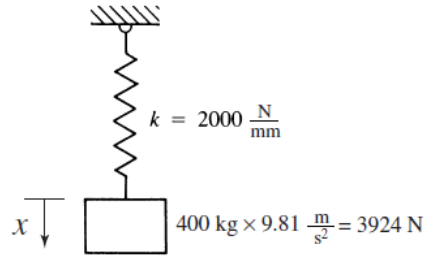
$$\pi_p = \frac{1}{2} kx^2 - F_x = 250x^2 - 1000x$$

$x, \text{ in.}$	$\pi_p, \text{ lb}\cdot\text{in.}$
-3.0	11250
-2.0	3000
-1.0	1250
0	0
1.0	-750
2.0	-1000
3.0	-750

$$\frac{\partial \pi_p}{\partial x} = 500x - 1000 = 0$$

$$\Rightarrow x = 2.0 \text{ in. yields } \pi_p \text{ minimum}$$

(c)



$$\pi_p = \frac{1}{2} (2000) x^2 - 3924 x = 1000 x^2 - 3924 x$$

$$\frac{\partial \pi_p}{\partial x} = 2000 x - 3924 = 0$$

$$\Rightarrow x = 1.962 \text{ mm yields } \pi_p \text{ minimum}$$

$$\pi_{p \text{ min}} = \frac{1}{2} (2000) (1.962)^2 - 3924 (1.962)$$

$$\Rightarrow \pi_{p \text{ min}} = -3849.45 \text{ N}\cdot\text{mm}$$

$$(d) \quad \pi_p = \frac{1}{2} (400) x^2 - 981 x$$

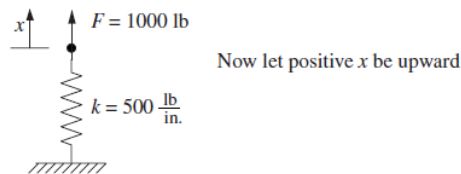
$$\frac{\partial \pi_p}{\partial x} = 400 x - 981 = 0$$

$$\Rightarrow x = 2.4525 \text{ mm yields } \pi_p \text{ minimum}$$

$$\pi_{p \text{ min}} = \frac{1}{2} (400) (2.4525)^2 - 981 (2.4525)$$

$$\Rightarrow \pi_{p \text{ min}} = -1202.95 \text{ N}\cdot\text{mm}$$

2.19



$$\pi_p = \frac{1}{2} kx^2 - Fx$$

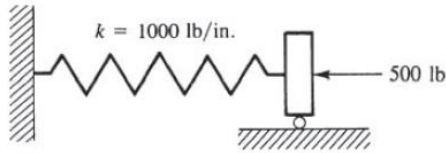
$$\pi_p = \frac{1}{2} (500) x^2 - 1000 x$$

$$\pi_p = 250 x^2 - 1000 x$$

$$\frac{\partial \pi_p}{\partial x} = 500x - 1000 = 0$$

$$\Rightarrow x = 2.0 \text{ in. } \uparrow$$

2.20



$$F = k\delta^2 \quad (x = \delta)$$

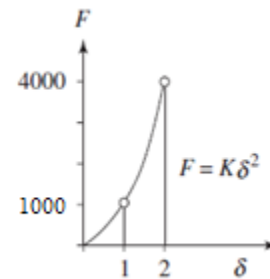
$$dU = F dx$$

$$U = \int_0^x (kx^2) dx$$

$$U = \frac{kx^3}{3}$$

$$\Omega = -Fx$$

$$\pi_p = \frac{1}{3} kx^3 - 500x$$



$$\frac{\partial \pi_p}{\partial x} = 0 = kx^2 - 500$$

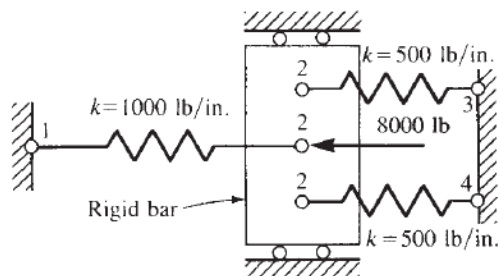
$$0 = 1000x^2 - 500$$

$$\Rightarrow x = 0.707 \text{ in. (equilibrium value of displacement)}$$

$$\pi_{p \text{ min}} = \frac{1}{3} (1000) (0.707)^3 - 500 (0.707)$$

$$\pi_{p \text{ min}} = -235.7 \text{ lb}\cdot\text{in.}$$

2.21 Solve Problem 2.10 using P.E. approach



$$\pi_p = \sum_{e=1}^3 \pi_p^{(e)} = \frac{1}{2} k_1 (u_2 - u_1)^2 + \frac{1}{2} k_2 (u_3 - u_2)^2 + \frac{1}{2} k_3 (u_4 - u_2)^2$$

$$- f_{1x}^{(1)} u_1 - f_{2x}^{(1)} u_2 - f_{2x}^{(2)} u_2$$

$$- f_{3x}^{(2)} u_3 - f_{2x}^{(3)} u_2 - f_{4x}^{(3)} u_4$$

$$\frac{\partial \pi_p}{\partial u_1} = -k_1 u_2 + k_1 u_1 - f_{1x}^{(1)} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial \pi_p}{\partial u_2} &= k_1 u_2 - k_1 u_1 - k_2 u_3 + k_2 u_2 - k_3 u_4 \\ &+ k_3 u_2 - f_{2x}^{(1)} - f_{2x}^{(2)} - f_{2x}^{(3)} = 0 \end{aligned} \quad (2)$$

$$\frac{\partial \pi_p}{\partial u_3} = k_2 u_3 - k_2 u_2 - f_{3x}^{(2)} = 0 \quad (3)$$

$$\frac{\partial \pi_p}{\partial u_4} = k_3 u_4 - k_3 u_2 - f_{4x}^{(3)} = 0 \quad (4)$$

In matrix form (1) through (4) become

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 + k_2 + k_3 & -k_2 & -k_3 \\ 0 & -k_2 & k_2 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} + f_{2x}^{(2)} + f_{2x}^{(3)} \\ f_{3x}^{(2)} \\ f_{4x}^{(3)} \end{Bmatrix} \quad (5)$$

or using numerical values

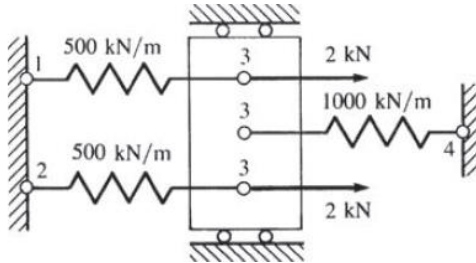
$$\begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 = 0 \\ u_4 = 0 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ -8000 \\ F_{3x} \\ F_{4x} \end{Bmatrix} \quad (6)$$

Solution now follows as in Problem 2.10

Solve 2nd of Equations (6) for $u_2 = -4$ in.

For reactions and element forces, see solution to Problem 2.10

2.22 Solve Problem 2.15 by P.E. approach



$$\begin{aligned}\pi_p = \sum_{e=1}^3 \pi_p^{(e)} &= \frac{1}{2} k_1 (u_3 - u_1)^2 + \frac{1}{2} k_2 (u_3 - u_2)^2 \\ &+ \frac{1}{2} k_3 (u_4 - u_3)^2 - f_{1x}^{(1)} u_1 \\ &- f_{3x}^{(1)} u_3 - f_{2x}^{(2)} u_2 - f_{3x}^{(2)} u_3 \\ &- f_{3x}^{(3)} u_3 - f_{3x}^{(4)} u_4\end{aligned}$$

$$\frac{\partial \pi_p}{\partial u_1} = 0 = -k_1 u_3 + k_1 u_1 - f_{1x}^{(1)}$$

$$\frac{\partial \pi_p}{\partial u_2} = 0 = -k_2 u_3 + k_2 u_2 - f_{2x}^{(2)}$$

$$\frac{\partial \pi_p}{\partial u_3} = 0 = k_1 u_3 + k_2 u_3 - k_2 u_2 - k_3 u_4 + k_3 u_3 - f_{3x}^{(2)} - f_{3x}^{(3)} - f_{3x}^{(1)} - k_1 u_1$$

$$\frac{\partial \pi_p}{\partial u_4} = 0 = k_3 u_4 - k_3 u_3 - f_{3x}^{(4)}$$

In matrix form

$$\begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & -k_2 & 0 \\ -k_1 & -k_2 & k_1 + k_2 + k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} = 4 \text{ kN} \\ F_{4x} \end{Bmatrix}$$

For rest of solution, see solution of Problem 2.15.

2.23

$$I = a_1 + a_2 x$$

$$I(0) = a_1 = I_1$$

$$I(L) = a_1 + a_2 L = I_2$$

$$a_2 = \frac{I_2 - I_1}{L}$$

$$\therefore I = I_1 + \frac{I_2 - I_1}{L} x$$

Now $V = IR$

$$V = -V_1 = R(I_2 - I_1)$$

$$V = V_2 = R(I_2 - I_1)$$

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = R \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$$

