

THE FIRST LAW OF THERMODYNAMICS

Problem 1.1 (a) $W_{i \rightarrow f} = \int_i^f P dV = P_1 V_2$. Next, to calculate T_f we note that from (i) to (f) we have

$$\frac{dM}{dt} = \dot{m}$$

$$\frac{dU}{dt} = -\dot{W} + \dot{m} h_1$$

where $\dot{m}$ is the instantaneous flowrate into the cylinder, and M and U are the mass and energy inventories of the system (the "system" is the cylinder volume). Integrating in time,

$$M_f - M_i = \int_i^f \dot{m} dt$$

$$U_f - U_i = -P_1 V_2 + h_1 (M_f - M_i) \quad (1)$$

and recognizing that $U_i = 0$ and $M_i = 0$, the first law reduces to

$$U_f = M_f h_1 - P_1 V_2 \quad (1')$$

For the "ideal gas" working fluid we write

$$U_f = M_f c_v (T_f - T_0)$$

$$h_1 = c_v (T_1 - T_0) + P v_1$$

hence, eq. (1') becomes

$$M_f c_v (T - T_0) = M_f [c_v (T_1 - T_0) + P v_1] - P_1 V_2$$

Noting that $V_2 = M_f v_f$ and dividing everything by M_f yields

$$c_v T_f + P_1 v_f = c_v T_1 + P v_1$$

or

$$c_v T_f + R T_f = c_v T_1 + R T_1$$

in other words, $T_f = T_1$. The final ideal-gas mass admitted is

$$m_f = M_f = \frac{P_1 V_2}{R T_1}$$

hence the goodness ratio

$$\frac{W_{i-f}}{m_f} = \frac{P_1 V_2}{P_1 V_2 / (RT_1)} = RT_1$$

(b) $m_1 = P_1 V_1 / RT_1$, based on the solution for m_f given in part (a), and

$$\begin{aligned} W_{i-f} &= \int_0^{V_1} P dV + \int_{V_1}^{V_2} P dV \\ &= P_1 V_1 + \frac{c_v}{R} (P_1 V_1 - P_2 V_2) \end{aligned}$$

The second group of terms on the right-hand-side is the work output during the reversible & adiabatic expansion (path: $PV^k = \text{constant}$). Finally, the goodness ratio is

$$\frac{W_{i-f}}{m_1} = \frac{P_1 V_1 + \frac{c_v}{R} (P_1 V_1 - P_2 V_2)}{P_1 V_1 / (RT_1)} = RT_1 \left[1 + \frac{c_v}{R} \left(1 - \frac{P_2 V_2}{P_1 V_1} \right) \right]$$

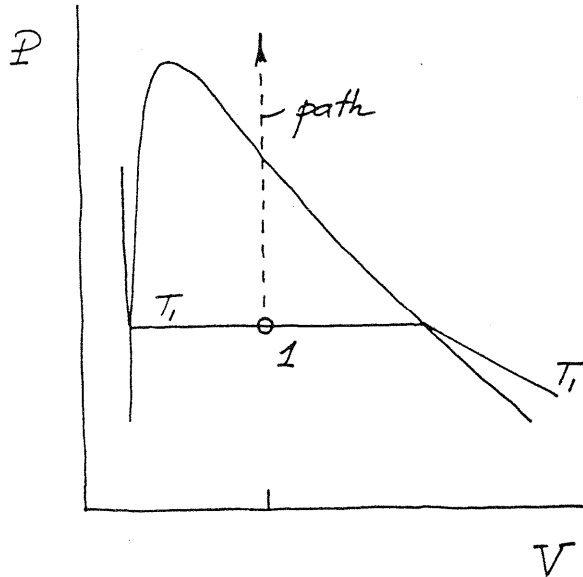
(c) The relative goodness is

$$\begin{aligned} \frac{(W_{i-f}/m_1)_{\text{part (b)}}}{(W_{i-f}/m_f)_{\text{part (a)}}} &= 1 + \frac{c_v}{R} \left(1 - \frac{P_2 V_2}{P_1 V_1} \right) \\ &= 1 + \frac{c_v}{R} \left[1 - \left(\frac{V_1}{V_2} \right)^{k-1} \right] \end{aligned}$$

The quantity in the square brackets is positive because $k > 1$ and $V_1 < V_2$, therefore

$$\left(\frac{W_{i-f}}{m_1} \right)_{\text{part (b)}} > \left(\frac{W_{i-f}}{m_f} \right)_{\text{part (a)}}$$

Problem 1.2 (a) Given are $m = 1$ kg, $T_1 = 100^\circ\text{C}$, and $x_1 = 0.5$. The path is constant volume.



(b) To pinpoint state (2) we must determine two properties at the final state. The first one is the volume

$$\begin{aligned} v_2 &= v_1 = v_{f,T_1} + x_1 v_{fg,T_1} = \\ &= 0.001044 + 0.5 (1.6729 - 0.001044) \\ &= 0.837 \text{ m}^3 / \text{kg} \end{aligned}$$

The second property is the internal energy: this comes from the first law

$$Q_{1-2} - W_{1-2} = m(u_2 - u_1) \quad (1)$$

where $W_{1-2} = 0$ and

$$u_1 = u_{f,T_1} + x_1 u_{fg,T_1} = 418.94 + (0.5) (2087.6) = 1462.74 \text{ kJ/kg}$$

Equation (1) yields

$$u_2 = u_1 + \frac{1}{m} Q_{1-2} = 3662 \text{ kJ / kg}$$

(c) To find T_2 and P_2 we must first locate state (2) on the $P(v, t)$ surface (or tables). At state (2) we know u_2 and v_2 , therefore, one way to proceed is to look at the table of superheated steam properties and find the u values of order 3662 kJ / kg. This is the equivalent of traveling along the $u = u_2$ line and looking for the v value that comes closest to v_2 . This search leads to this portion of the table:

T	P = 0.5 MPa		P = 0.6 MPa	
	v	u	v	u
...	...	...	...	...
800°C	0.9896	3662.1	0.8245	3661.8
...	...	...	...	...

Fitting v_2 between 0.9896 and 0.8245, we interpolate linearly for pressure and find

$$P_2 \cong 0.592 \text{ MPa}$$

The final temperature is clearly $T_2 \cong 800^\circ\text{C}$.

(d) At state (2) the system is superheated steam. This particular fluid approaches ideal gas behavior if near state (2) the following *two conditions* are met:

$$(i) \quad u = u(T)$$

$$(ii) \quad Pv = RT, \text{ i.e. } Pv / T = \text{constant.}$$

Condition (i) is satisfied, as shown by the u values listed in the preceding table (u depends on T , while being practically independent of P). As a way of testing condition (ii), we calculate the group (Pv/T) for the states immediately to the left and right of state (2):

$$\left(\frac{Pv}{T}\right)_{\text{left}} = \frac{(0.5) 10^6 (0.9896)}{273.15 + 800} = 461.1 \frac{\text{Pa m}^3 / \text{kg}}{\text{K}}$$

$$\left(\frac{Pv}{T}\right)_{\text{right}} = \frac{(0.5) 10^6 (0.8245)}{273.15 + 800} = 461.0 \frac{\text{Pa m}^3 / \text{kg}}{\text{K}}$$

Condition (ii) is also satisfied (approximately, or course), therefore, the ideal gas model could be used to describe the behavior of the system at states that are sufficiently close to state (2).

Observation: note the use of absolute temperature in the denominators of the (Pv/T) calculations presented above.

Problem 1.3 Taking the m gas as "system", we write the first law for the process (1) - (2),

$$Q_{1-2} - W_{1-2} = U_2 - U_1$$

which means

$$0 - P_{1A} (V_2 - V_1) = \frac{m}{3} c_v (T_2 - T_1) + \frac{m}{3} c_v (T_2 - T_1) + \frac{m}{3} c_v (T_2 - T_1)$$

or

$$-P_{1A} \left(\frac{mRT_2}{P_2} - \frac{\frac{m}{3} RT_1}{P_{1A}} - \frac{\frac{m}{3} RT_1}{P_{1B}} - \frac{\frac{m}{3} RT_1}{P_{1C}} \right) = mc_v (T_2 - T_1)$$

Noting that $P_2 = P_{1A}$, the above statement can be written as

$$\frac{T_2}{T_1} = \frac{1}{1 + R/c_v} + \frac{1 + (P_{1A}/P_{1B}) + (P_{1A}/P_{1C})}{3(1 + R/c_v)} \cdot \frac{R}{c_v}$$

$$= \frac{c_v}{c_p} + \frac{R}{3c_p} \left(1 + \frac{P_{1A}}{P_{1B}} + \frac{P_{1A}}{P_{1C}} \right)$$

Problem 1.4 The process is one of heating at constant volume. Let m_f and m_g represent the instantaneous liquid and vapor inventories in the system,

$$m_f + m_g = m, \quad (\text{constant})$$

Furthermore, the constant-volume constraint reads

$$m_f v_f + m_g v_g = V, \quad (\text{constant})$$

The first law of thermodynamics requires on a per unit time basis that

$$\dot{Q} - \dot{W} = \frac{dU}{dt}$$

or, since $\dot{W} = 0$,

$$\begin{aligned} \dot{Q} &= \frac{d}{dt} (m_f u_f + m_g u_g) \\ &= u_f \frac{dm_f}{dt} + m_f \frac{du_f}{dt} + u_g \frac{dm_g}{dt} + m_g \frac{du_g}{dt} \end{aligned} \quad (3)$$

$$\underbrace{\frac{du_f}{dP} \frac{dP}{dt}} \quad \underbrace{\frac{du_g}{dP} \frac{dP}{dt}}$$

The time derivatives dm_f/dt and dm_g/dt follow from solving the system of two equations

$$\frac{d}{dt} \quad (1) \quad \text{and} \quad \frac{d}{dt} \quad (2)$$

The solution is

$$\frac{dm_f}{dt} = -\frac{A}{v_{fg}} \quad \text{and} \quad \frac{dm_g}{dt} = \frac{A}{v_{fg}} \quad (4)$$

where

$$A = -m_f \frac{dv_f}{dt} - m_g \frac{dv_g}{dt} = -m_f \frac{dv_f}{dP} \frac{dP}{dt} - m_g \frac{dv_g}{dP} \frac{dP}{dt}$$

Combining (3) and (4) we obtain after a few manipulations

$$\frac{dP}{dt} = \frac{\dot{Q}/m}{\frac{du_f}{dP} - \frac{u_{fg}}{v_{fg}} \frac{dv_f}{dP} + x \left(\frac{du_{fg}}{dP} - \frac{u_{fg}}{v_{fg}} \frac{dv_{fg}}{dP} \right)}$$

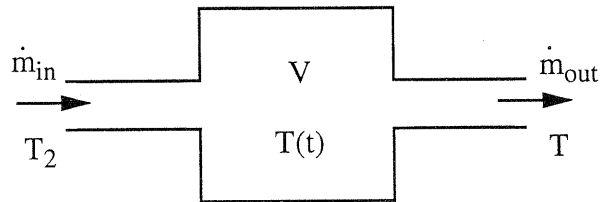
Problem 1.5 After the seal is broken the atmospheric air rushes into the glass tube and the pressure inside the tube becomes atmospheric. If we wait long enough, the temperature inside the tube becomes equal to the atmospheric temperature also (this scenario is described in Example 1.2 in the text). In that example we learned that en route to thermal equilibrium the bottle air rejects heat to the ambient. This means that immediately after we break the seal the air that occupies the glass tube is warmer than the ambient. If we dip the open end of the tube into the pool of water immediately after breaking the seal, the water will rise into the tube as the air mass that is trapped inside the glass tube cools down (and *shrinks*) to atmospheric temperature.

Problem 1.6 (a) Applying the first law to the water container as an open system, we have

$$\frac{d}{dt} (mu) = (\dot{m}h)_{in} - (\dot{m}h)_{out} \quad (1)$$

where

$$m = \frac{V}{v_w} = \text{constant}$$



Mass conservation dictates

$$\frac{d}{dt} (m) = \dot{m}_{in} - \dot{m}_{out} = 0$$

hence $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$. The first law (1) reads finally

$$\frac{V}{v_w} \frac{du}{dt} = \dot{m} (h_{in} - h_{out})$$

For an incompressible fluid we also have

$$du = cdT \quad \text{and} \quad dh = cdT + v_w dP$$

In the present case $P_{in} = P_{out}$, therefore

$$h_{in} - h_{out} = c(T_{in} - T_{out}) = c(T_2 - T)$$

Equation (1) becomes

$$\frac{V}{v_w} c \frac{dT}{dt} = \dot{m} c (T_2 - T)$$

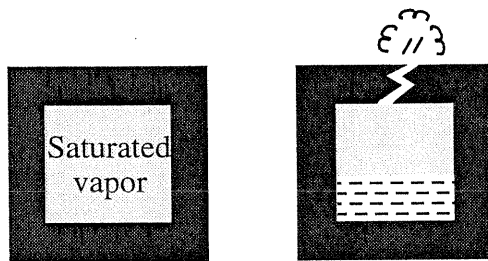
which, integrated from 0 to t, means

$$\ln \frac{T_2 - T}{T_2 - T_1} = - \frac{\dot{m} v_w}{V} t$$

(b) The mass of hot water that raises the container water temperature from 10°C to 20°C is

$$\begin{aligned} \dot{m} t &= - \frac{V}{v_w} \ln \frac{T_2 - T}{T_2 - T_1} \\ &= - \frac{m^3}{10^{-3} \text{ m}^3/\text{kg}} \ln \frac{40 - 20}{40 - 10} = 405.5 \text{ kg} \end{aligned}$$

Problem 1.7 (a) We treat the instantaneous water inventory of the container as an open system operating unsteadily, so that the first law and the principle of mass conservation read



State 1

State 2

$$\frac{d}{dt} (mu) = \dot{Q} - \dot{W} + (\dot{m}h)_{\text{in}} - (\dot{m}h)_{\text{out}} \quad (1)$$

$$\frac{d}{dt} (m) = \dot{m}_{\text{in}} - \dot{m}_{\text{out}} \quad (2)$$

where $\dot{Q} = \dot{W} = 0$ and $\dot{m}_{\text{in}} = 0$. Integrating (2) and writing m_1 and m_2 for the initial and final mass inventories of the system yields

$$\int_1^2 \dot{m}_{\text{out}} dt = m_1 - m_2$$

The objective is to calculate x_2 , therefore, we focus on pinpointing state (2). We already know one property at that state (namely, P_2). To obtain the second property we use the first law (1) in integral form

$$\begin{aligned} m_2 u_2 - m_1 u_1 &= -h_{\text{out}} \int_1^2 \dot{m}_{\text{out}} dt \\ &= h_{\text{out}} (m_2 - m_1) \end{aligned}$$

or the dimensionless form

$$\frac{u_2}{u_1} - \frac{m_1}{m_2} = \frac{h_{\text{out}}}{u_1} \left(1 - \frac{m_1}{m_2} \right) \quad (3)$$

Next, we note that the volume of the container is fixed,

$$V = m_1 v_1 = m_2 v_2 \quad (4)$$

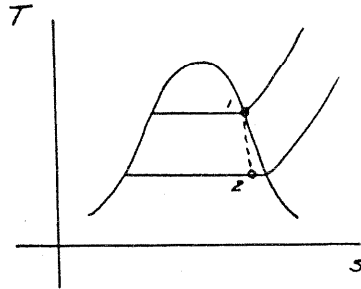
where $v_2 = v_{f,2} + x_2 v_{fg,2}$. What follows from eq. (4) is a relationship between m_1/m_2 and x_2 :

$$\begin{aligned} \frac{m_1}{m_2} &= \frac{v_2}{v_1} = \frac{v_{f,2}}{v_1} + x_2 \frac{v_{fg,2}}{v_1} \\ &= \frac{0.001154}{0.03944} + x_2 \frac{0.13064}{0.03944} \\ &= 0.02926 + x_2 (3.312) \end{aligned} \quad (5)$$

A similar relation exists between u_2/u_1 and x_2 ,

$$\begin{aligned} \frac{u_1}{u_2} &= \frac{u_{f,2}}{u_1} + x_2 \frac{u_{fg,2}}{u_1} \\ &= \frac{843.16}{2597.1} + x_2 \frac{1751.3}{2597.1} \\ &= 0.3246 + x_2 (0.6743) \end{aligned} \quad (6)$$

Substituting eqs. (5, 6) into eq. (3) yields an equation for x_2 alone, whose solution is $x_2 = 0.805$.



(b) The final vapor / liquid volume ratio is

$$\begin{aligned} \left(\frac{V_g}{V_f} \right)_2 &= \left(\frac{m_g v_g}{m_f v_f} \right)_2 = \frac{x_2}{1 - x_2} \left(\frac{v_g}{v_f} \right)_2 \\ &= \frac{0.805}{1 - 0.805} \left(\frac{0.13177}{0.001154} \right) = 471.4 \end{aligned}$$

Problem 1.8 Selected for analysis is the system that contains the two masses (m_1, m_2). In the initial state (a) the velocities of the two masses are different (V_1, V_2), while in the final state (b) mutual friction brings the velocities to the same level (V_∞). Since there are no forces between the system and its environment, the total momentum of the ensemble is conserved,

$$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V_\infty \quad (1)$$

The initial and final kinetic energy inventories of the ensemble are

$$KE_a = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 \quad (2)$$

$$KE_b = \frac{1}{2} (m_1 + m_2) V_\infty^2 \quad (3)$$

The evolution of the total kinetic energy during the process (a) - (b) is described by the "efficiency" ratio

$$\eta = \frac{KE_b}{KE_a} \quad (4)$$

Eliminating V_∞ between Eqs. (1) and (3), the efficiency can be expressed in terms of the initial mass and velocity ratios m_2 / m_1 and V_2 / V_1 ,

$$\eta = \frac{\left(1 + \frac{m_2}{m_1} \frac{V_2}{V_1}\right)^2}{\left(1 + \frac{m_2}{m_1}\right) \left[1 + \frac{m_2}{m_1} \left(\frac{V_2}{V_1}\right)^2\right]} < 1 \quad (5)$$

It can be shown analytically that η is less than 1 as soon as V_2 is different than V_1 , for any value of the ratio m_2 / m_1 . Two limits of Eq. (5) are worth noting,

$$\eta = \frac{1}{\frac{m_2}{m_1} + 1} \quad \left(\frac{V_2}{V_1} \rightarrow 0\right) \quad (6)$$

$$\eta = \frac{1}{\frac{m_1}{m_2} + 1} \quad \left(\frac{V_2}{V_1} \rightarrow \infty\right) \quad (7)$$

with the special case $\eta = 1$ when $V_1 = V_2$ for any m_2/m_1 . Equations (5) - (7) show that the order of magnitude of η is 1 when m_2 / m_1 is a number of order 1.

In conclusion, the kinetic energy of the system decreases from state (a) to state (b). According to the first law of thermodynamics, this decrease is balanced by the other energy interactions and energy changes of the system,

$$Q_{a-b} - W_{a-b} = U_b - U_a + KE_b - KE_a \quad (8)$$

where $W_{a-b} = 0$. If the process is adiabatic, $Q_{a-b} = 0$, then the KE decrease is balanced by an increase in U ,

$$U_b - U_a = KE_a - KE_b \quad (9)$$

If the system boundary is diathermal, and (a) and (b) are states of thermal equilibrium with the ambient temperature reservoir (T_0), then

$$Q_{a-b} = U_b - U_a + KE_b - KE_a \quad (10)$$

If m_1 and m_2 are two incompressible substances then $U = U(T)$, and at thermal equilibrium (T_0) the energy change $U_b - U_a$ is zero,

$$Q_{a-b} = KE_b - KE_a < 0 \quad (11)$$
