C H A P T E R 1
Equations, Inequalities, and Mathematical Modeling

Section 1.1 Graphs of Equations **50**

Section 1.2 Linear Equations in One Variable **59**

Section 1.3 Modeling with Linear Equations **68**

Section 1.4 Quadratic Equations and Applications **78**

Section 1.5 Complex Numbers **94**

Section 1.6 Other Types of Equations **100**

Section 1.7 Linear Inequalities in One Variable **119**

Section 1.8 Other Types of Inequalities **127**

Review Exercises **144**

Problem Solving **159**

Practice Test **163**

CHAPTER 1
Equations, Inequalities, and Mathematical Modeling

Section 1.1 Graphs of Equations

1. solution or solution point

2. graph

3. intercepts

4. y-axis

5. circle; (h, k); r

6. numerical

7. (a) $(0, 2)$: $2 \stackrel{?}{=} \sqrt{0 + 4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 3)$: $3 \stackrel{?}{=} \sqrt{5 + 4}$

$3 \stackrel{?}{=} \sqrt{9}$

$3 = 3$

Yes, the point *is* on the graph.

8. (a) $(1, 2)$: $2 \stackrel{?}{=} \sqrt{5 - 1}$

$2 \stackrel{?}{=} \sqrt{4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 0)$: $0 \stackrel{?}{=} \sqrt{5 - 5}$

$0 = 0$

Yes, the point *is* on the graph.

9. (a) $(2, 0)$: $(2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$4 - 6 + 2 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

(b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$4 + 6 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

No, the point *is not* on the graph.

10. (a) $(-1, 1)$: $1 \stackrel{?}{=} 3 - 2(-1)^2$

$1 \stackrel{?}{=} 3 - 2(1)$

$1 = 1$

Yes, the point *is* on the graph.

(b) $(-2, 11)$: $11 \stackrel{?}{=} 3 - 2(-2)^2$

$11 \stackrel{?}{=} 3 - 2(4)$

$11 \neq -5$

No, the point *is not* on the graph.

11. (a) $(1, 5)$: $5 \stackrel{?}{=} 4 - |1 - 2|$

$5 \stackrel{?}{=} 4 - 1$

$5 \neq 3$

No, the point *is not* on the graph.

(b) $(6, 0)$: $0 \stackrel{?}{=} 4 - |6 - 2|$

$0 \stackrel{?}{=} 4 - 4$

$0 = 0$

Yes, the point *is* on the graph.

12. (a) $(2, 3)$: $3 \stackrel{?}{=} |2 - 1| + 2$

$3 \stackrel{?}{=} 1 + 2$

$3 = 3$

Yes, the point *is* on the graph.

(b) $(-1, 0)$: $0 \stackrel{?}{=} |-1 - 1| + 2$

$0 \stackrel{?}{=} 2 + 2$

$0 \neq 4$

No, the point *is not* on the graph.

13. (a) $(3, -2)$: $(3)^2 + (-2)^2 \stackrel{?}{=} 20$

$9 + 4 \stackrel{?}{=} 20$

$13 \neq 20$

No, the point *is not* on the graph.

(b) $(-4, 2)$: $(-4)^2 + (2)^2 \stackrel{?}{=} 20$

$16 + 4 \stackrel{?}{=} 20$

$20 = 20$

Yes, the point *is* on the graph.

14. (a) $(6, 0): 2(6)^2 + 5(0)^2 \stackrel{?}{=} 8$

$$2(36) + 5(0) \stackrel{?}{=} 8$$

$$72 + 0 \stackrel{?}{=} 8$$

$$72 \neq 8$$

No, the point *is not* on the graph.

(b) $(0, 4): 2(0)^2 + 5(4)^2 \stackrel{?}{=} 8$

$$2(0) + 5(16) \stackrel{?}{=} 8$$

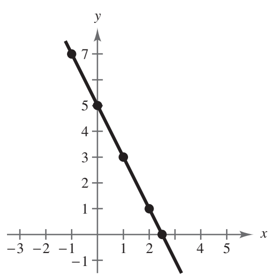
$$0 + 80 \stackrel{?}{=} 8$$

$$80 \neq 8$$

No, the point *is not* on the graph.

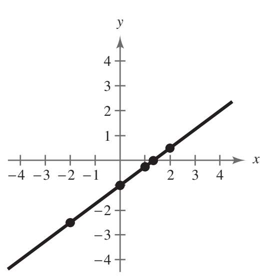
15. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



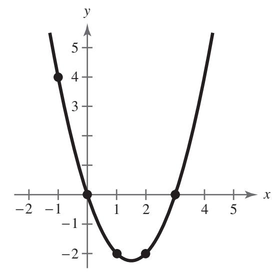
16. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



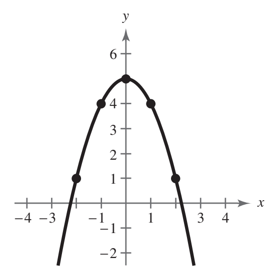
17. $y = x^2 - 3x$

x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



18. $y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
x, y	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



19. x-intercept: $(3, 0)$

y-intercept: $(0, 9)$

20. x-intercepts: $(\pm 2, 0)$

y-intercept: $(0, 16)$

21. x-intercept: $(-2, 0)$

y-intercept: $(0, 2)$

22. x-intercept: $(4, 0)$

y-intercepts: $(0, \pm 2)$

23. x-intercept: $(1, 0)$

y-intercept: $(0, 2)$

24. x-intercepts: $(0, 0), (0, \pm 2)$

y-intercept: $(0, 0)$

25. $x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

26. $x - y^2 = 0$

$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No y-axis symmetry

$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow$ x-axis symmetry

$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No origin symmetry

27. $y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

28. $y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

29. $y = \dfrac{x}{x^2 + 1}$

$y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{x}{x^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow -y = \dfrac{-x}{x^2 + 1} \Rightarrow y = \dfrac{x}{x^2 + 1} \Rightarrow$ Origin symmetry

30. $y = \dfrac{1}{1 + x^2}$

$y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{1}{1 + x^2} \Rightarrow$ y-axis symmetry

$-y = \dfrac{1}{1 + x^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No origin symmetry

31. $xy^2 + 10 = 0$

$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No y-axis symmetry

$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow$ x-axis symmetry

$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No origin symmetry

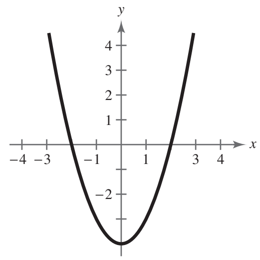
32. $xy = 4$

$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow$ No y-axis symmetry

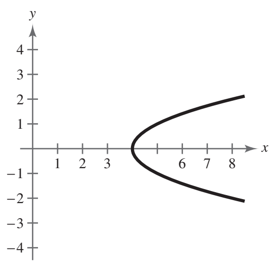
$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow$ No x-axis symmetry

$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow$ Origin symmetry

33.



34.



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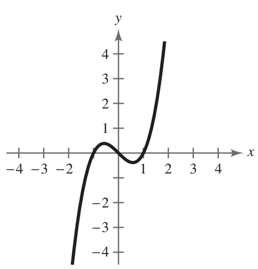
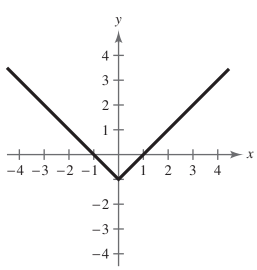


36.

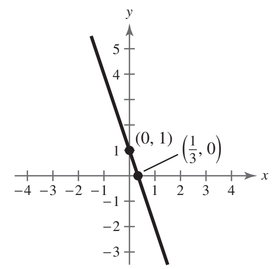


37. $y = -3x + 1$

x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

No symmetry

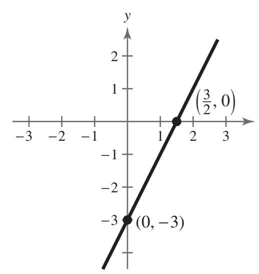


38. $y = 2x - 3$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

No symmetry



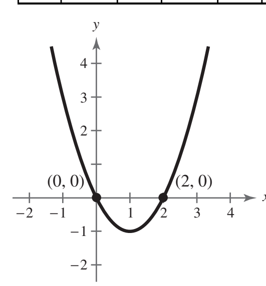
39. $y = x^2 - 2x$

x-intercepts: $(0, 0), (2, 0)$

y-intercept: $(0, 0)$

No symmetry

x	-1	0	1	2	3
y	3	0	-1	0	3

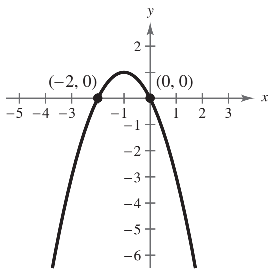


40. $y = -x^2 - 2x$

x-intercepts: $(-2, 0), (0, 0)$

y-intercept: $(0, 0)$

No symmetry



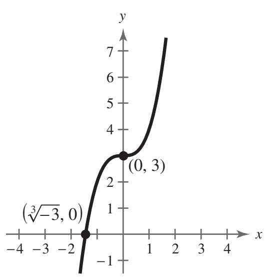
41. $y = x^3 + 3$

x-intercept: $\left(\sqrt[3]{-3}, 0\right)$

y-intercept: $(0, 3)$

No symmetry

x	-2	-1	0	1	2
y	-5	2	3	4	11

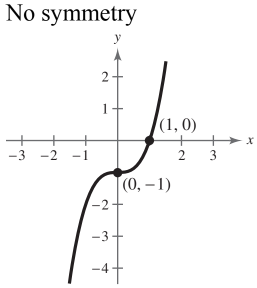


42. $y = x^3 - 1$

x-intercept: $(1, 0)$

y-intercept: $(0, -1)$

No symmetry



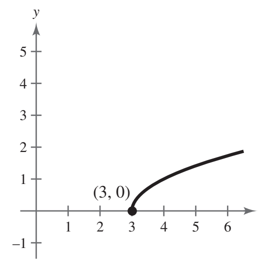
43. $y = \sqrt{x - 3}$

x-intercept: $(3, 0)$

y-intercept: none

No symmetry

x	3	4	7	12
y	0	1	2	3

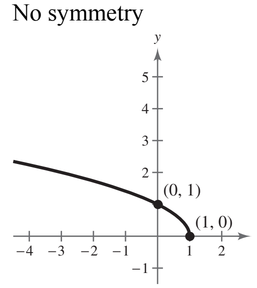


44. $y = \sqrt{1 - x}$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

No symmetry



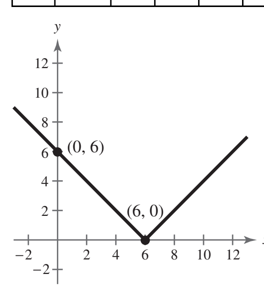
45. $y = |x - 6|$

x-intercept: $(6, 0)$

y-intercept: $(0, 6)$

No symmetry

x	-2	0	2	4	6	8	10
y	8	6	4	2	0	2	4

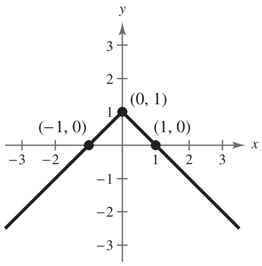


46. $y = 1 - |x|$

x-intercepts: $(1, 0), (-1, 0)$

y-intercept: $(0, 1)$

y-axis symmetry



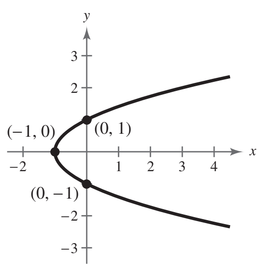
47. $x = y^2 - 1$

x-intercept: $(-1, 0)$

y-intercepts: $(0, -1), (0, 1)$

x-axis symmetry

x	-1	0	3
y	0	± 1	± 2

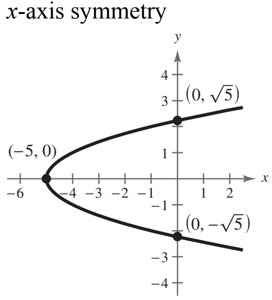


48. $x = y^2 - 5$

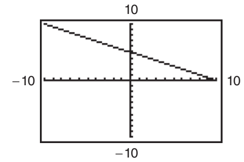
x-intercept: $(-5, 0)$

y-intercepts: $(0, \sqrt{5}), (0, -\sqrt{5})$

x-axis symmetry

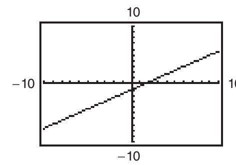


49. $y = 5 - \frac{1}{2}x$



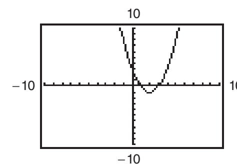
Intercepts: $(10, 0), (0, 5)$

50. $y = \frac{2}{3}x - 1$



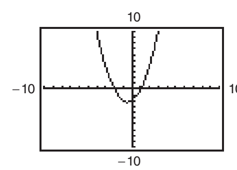
Intercepts: $(0, -1), \left(\frac{3}{2}, 0\right)$

51. $y = x^2 - 4x + 3$



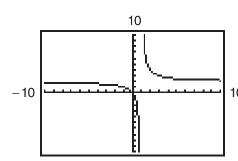
Intercepts: $(3, 0), (1, 0), (0, 3)$

52. $y = x^2 + x - 2$



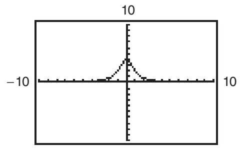
Intercepts: $(-2, 0), (1, 0), (0, -2)$

53. $y = \dfrac{2x}{x - 1}$



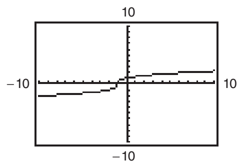
Intercept: $(0, 0)$

54. $y = \dfrac{4}{x^2 + 1}$



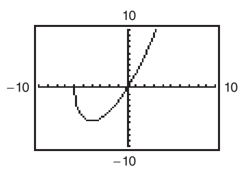
Intercept: $(0, 4)$

55. $y = \sqrt[3]{x + 1}$



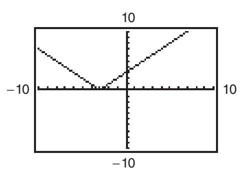
Intercepts: $(-1, 0), (0, 1)$

56. $y = x\sqrt{x + 6}$



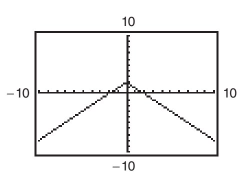
Intercepts: $(0, 0), (-6, 0)$

57. $y = |x + 3|$



Intercepts: $(-3, 0), (0, 3)$

58. $y = 2 - |x|$



Intercepts: $(\pm 2, 0), (0, 2)$

59. Center: $(0, 0)$; Radius: 3

$$\begin{aligned}(x - 0)^2 + (y - 0)^2 &= 3^2\\ x^2 + y^2 &= 9\end{aligned}$$

60. Center: $(0, 0)$; Radius: 7

$$\begin{aligned}(x - 0)^2 + (y - 0)^2 &= 7^2\\ x^2 + y^2 &= 49\end{aligned}$$

61. Center: $(-4, 5)$; Radius: 2

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ [x - (-4)]^2 + [y - 5]^2 &= 2^2\\ (x + 4)^2 + (y - 5)^2 &= 4\end{aligned}$$

62. Center: $(1, -3)$; Radius: $\sqrt{11}$

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ (x - 1)^2 + [y - (-3)]^2 &= \sqrt{11}^2\\ (x - 1)^2 + (y + 3)^2 &= 11\end{aligned}$$

63. Center: $(3, 8)$; Solution point: $(-9, 13)$

$$\begin{aligned}r &= \sqrt{(x - h)^2 + (y - k)^2}\\ &= \sqrt{(-9 - 3)^2 + (13 - 8)^2}\\ &= \sqrt{(-12)^2 + (5)^2}\\ &= \sqrt{144 + 25}\\ &= \sqrt{169}\\ &= 13\end{aligned}$$

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ (x - 3)^2 + (y - 8)^2 &= 13^2\\ (x - 3)^2 + (y - 8)^2 &= 169\end{aligned}$$

64. Center: $(-2, -6)$; Solution point: $(1, -10)$

$$\begin{aligned}r &= \sqrt{(x - h)^2 + (y - k)^2}\\ &= \sqrt{[1 - (-2)]^2 + [-10 - (-6)]^2}\\ &= \sqrt{(3)^2 + (-4)^2}\\ &= \sqrt{9 + 16}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ [x - (-2)]^2 + [y - (-6)]^2 &= 5^2\\ (x + 2)^2 + (y + 6)^2 &= 25\end{aligned}$$

65. Endpoints of a diameter: $(3, 2), (-9, -8)$

$$r = \frac{1}{2}\sqrt{(-9-3)^2 + (-8-2)^2}$$
$$= \frac{1}{2}\sqrt{(-12)^2 + (-10)^2}$$
$$= \frac{1}{2}\sqrt{144 + 100}$$
$$= \frac{1}{2}\sqrt{244} = \frac{1}{2}(2)\sqrt{61} = \sqrt{61}$$

$$(h, k): \left(\frac{3 + (-9)}{2} \cdot \frac{2 + (-8)}{2}\right) = \left(\frac{-6}{2} \cdot \frac{-6}{2}\right)$$
$$= (-3, -3)$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$\left[x - (-3)\right]^2 + \left[y - (-3)\right]^2 = \left(\sqrt{61}\right)^2$$
$$(x + 3)^2 + (y + 3)^2 = 61$$

66. Endpoints of a diameter: $(11, -5), (3, 15)$

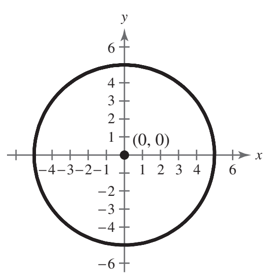
$$r = \frac{1}{2}\sqrt{(3-11)^2 + \left[15 - (-5)\right]^2}$$
$$= \frac{1}{2}\sqrt{(-8)^2 + (20)^2}$$
$$= \frac{1}{2}\sqrt{64 + 400}$$
$$= \frac{1}{2}\sqrt{464} = \frac{1}{2}(4)\sqrt{29} = 2\sqrt{29}$$

$$(h, k): \left(\frac{11 + 3}{2} \cdot \frac{-5 + 15}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 7)^2 + (y - 5)^2 = \left(2\sqrt{29}\right)^2$$
$$(x - 7)^2 + (y - 5)^2 = 116$$

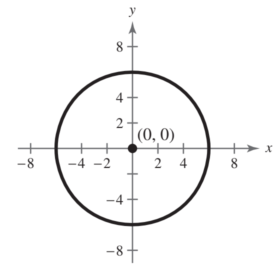
67. $x^2 + y^2 = 25$

Center: $(0, 0)$, Radius: 5



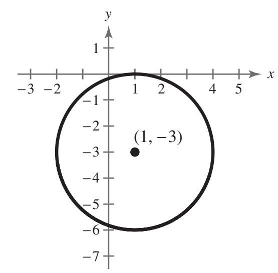
68. $x^2 + y^2 = 36$

Center: $(0, 0)$, Radius: 6



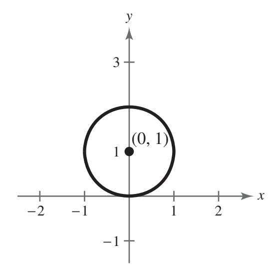
69. $(x - 1)^2 + (y + 3)^2 = 9$

Center: $(1, -3)$, Radius: 3



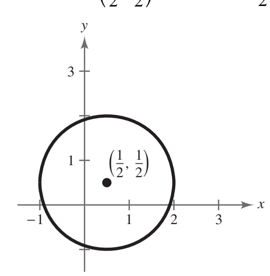
70. $x^2 + (y - 1)^2 = 1$

Center: $(0, 1)$, Radius: 1



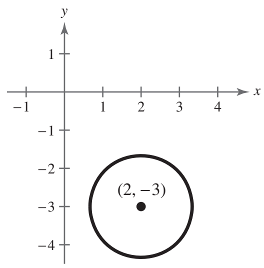
71. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$, Radius: $\frac{3}{2}$

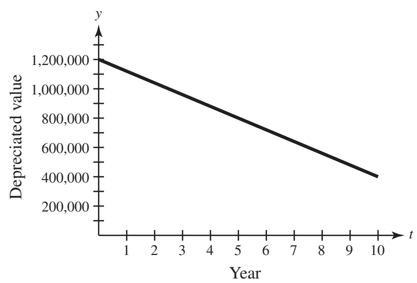


72. $(x-2)^2+(y+3)^2=\frac{16}{9}$

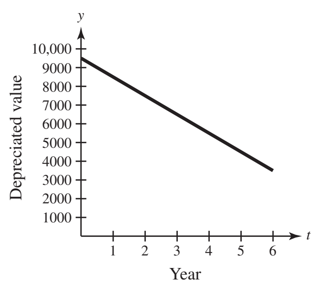
Center: $(2,-3)$, Radius: $\frac{4}{3}$



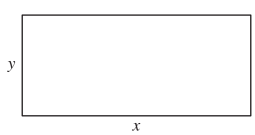
73. $y=1{,}200{,}000-80{,}000t,\ 0\le t\le 10$



74. $y=9500-1000t,\ 0\le t\le 6$

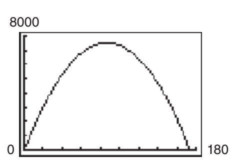


75. (a)



(b) $$2x+2y=\frac{1040}{3}$$
$$2y=\frac{1040}{3}-2x$$
$$y=\frac{520}{3}-x$$
$$A=xy=x\left(\frac{520}{3}-x\right)$$

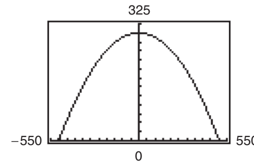
(c)



(d) When $x=y=86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

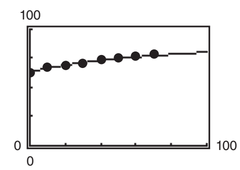
(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

76. (a)



(b) *Sample answer:* $(500, 0)$. The graph is symmetric with respect to the y-axis, so the other x-intercept is $(-500, 0)$ and the width is 1000 feet.

77. (a)



The model fits the data well.

(b) Graphically: The point $(50, 74.7)$ represents a life expectancy of 74.7 years in 1990.

Algebraically: $$y=\frac{63.6+0.97(50)}{1+0.01(50)}=\frac{112.1}{1.5}=74.7$$

So, the life expectancy in 1990 was about 74.7 years.

(c) Graphically: The point $(24.2, 70.1)$ represents a life expectancy of 70.1 years during the year 1964.

Algebraically:
$$y=\frac{63.6+0.97t}{1+0.01t}$$
$$70.1=\frac{63.6+0.97t}{1+0.01t}$$
$$70.1(1+0.01t)=63.6+0.97t$$
$$70.1+0.701t=63.6+0.97t$$
$$6.5=0.269t$$
$$t=24.2$$

When $y=70.1$, $t=24.2$ which represents the year 1964.

(d) $$y=\frac{63.6+0.97(0)}{1+0.01(0)}=\frac{63.6}{1}=63.6$$

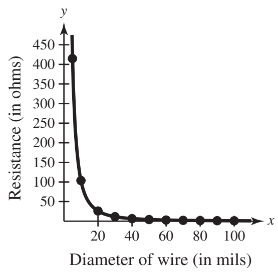
The y-intercept is $(0, 63.6)$. In 1940, the life expectancy of a child (at birth) was 63.6 years.

(e) Answers will vary.

78. (a)

x	5	10	20	30	40	50	60	70	80	90	100
y	414.8	103.7	25.93	11.52	6.48	4.15	2.88	2.12	1.62	1.28	1.04

(b)



When $x = 85.5$, the resistance is about 1.4 ohms.

(c) When $x = 85.5$,

$$y = \frac{10{,}370}{(85.5)^2} = 1.42 \text{ ohms.}$$

(d) As the diameter of the copper wire increases, the resistance decreases.

79. False. The line $y = x$ is symmetric with respect to the origin.

80. False. The line $y = 0$ has infinitely many x-intercepts.

81. True. Depending upon the center and radius, the graph of a circle could intersect one, both, or neither axis.

82. x-axis symmetry:

$$\begin{aligned} x^2 + y^2 &= 1 \\ x^2 + (-y)^2 &= 1 \\ x^2 + y^2 &= 1 \end{aligned}$$

y-axis symmetry:

$$\begin{aligned} x^2 + y^2 &= 1 \\ (-x)^2 + y^2 &= 1 \\ x^2 + y^2 &= 1 \end{aligned}$$

Origin symmetry:

$$\begin{aligned} x^2 + y^2 &= 1 \\ (-x)^2 + (-y)^2 &= 1 \\ x^2 + y^2 &= 1 \end{aligned}$$

So, the graph of the equation is symmetric with respect to x-axis, y-axis, and origin.

83. $y = ax^2 + bx^3$

(a) $$\begin{aligned} y &= a(-x)^2 + b(-x)^3 \\ &= ax^2 - bx^3 \end{aligned}$$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

Sample answer: $a = 1, b = 0$

(b) $$\begin{aligned} -y &= a(-x)^2 + b(-x)^3 \\ -y &= ax^2 - bx^3 \\ y &= -ax^2 + bx^3 \end{aligned}$$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

Sample answer: $a = 0, b = 1$

Section 1.2 Linear Equations in One Variable

1. equation

2. identities; conditional; contradictions

3. $ax + b = 0$

4. equivalent

5. rational

6. extraneous

7. The equation $3(x - 1) = 3x - 3$ is an *identity* by the Distributive Property. The equation is true for all real values of x.

8. The equation $2(x + 1) = 2x - 1$ is a *contradiction*. There are no real values of x for which the equation is true.

9. The equation $2(x - 1) = 3x + 1$ is a *conditional equation*. The only value in the domain that satisfies the equation is $x = -3$.

10. The equation $4(x + 2) = 2x + 2$ is a *conditional equation*. The only value in the domain that satisfies the equation is $x = -3$.

11. The equation $3(x + 2) = 3x + 2$ is a *contradiction*. There are no real values of x for which the equation is true.

12. The equation $5(x + 2) = 5x + 10$ is an *identity* by the Distributive Property. The equation is true for all real values of x.

13. The equation $2(x + 3) - 5 = 2x + 1$ is an *identity* by simplification. The equation is true for all real values of x.

14. The equation $3(x - 1) + 2 = 4x - 2$ is a *conditional equation*. The only value in the domain that satisfies the equation is $x = 1$.

15.
$$\begin{aligned} x + 11 &= 15 \\ x + 11 - 11 &= 15 - 11 \\ x &= 4 \end{aligned}$$

16.
$$\begin{aligned} 7 - x &= 19 \\ 7 - x + x &= 19 + x \\ 7 &= 19 + x \\ 7 - 19 &= 19 + x - 19 \\ -12 &= x \end{aligned}$$

17.
$$\begin{aligned} 7 - 2x &= 25 \\ 7 - 7 - 2x &= 25 - 7 \\ -2x &= 18 \\ \frac{-2x}{-2} &= \frac{18}{-2} \\ x &= -9 \end{aligned}$$

18.
$$\begin{aligned} 7x + 2 &= 23 \\ 7x + 2 - 2 &= 23 - 2 \\ 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$

19.
$$\begin{aligned} 3x - 5 &= 2x + 7 \\ 3x - 2x - 5 &= 2x - 2x + 7 \\ x - 5 &= 7 \\ x - 5 + 5 &= 7 + 5 \\ x &= 12 \end{aligned}$$

20.
$$\begin{aligned} 5x + 3 &= 6 - 2x \\ 5x + 2x + 3 &= 6 - 2x + 2x \\ 7x + 3 &= 6 \\ 7x + 3 - 3 &= 6 - 3 \\ 7x &= 3 \\ \frac{7x}{7} &= \frac{3}{7} \\ x &= \frac{3}{7} \end{aligned}$$

21.
$$\begin{aligned} 4y + 2 - 5y &= 7 - 6y \\ 4y - 5y + 2 &= 7 - 6y \\ -y + 2 &= 7 - 6y \\ -y + 6y + 2 &= 7 - 6y + 6y \\ 5y + 2 &= 7 \\ 5y + 2 - 2 &= 7 - 2 \\ 5y &= 5 \\ \frac{5y}{5} &= \frac{5}{5} \\ y &= 1 \end{aligned}$$

22.
$$\begin{aligned} 5y + 1 &= 8y - 5 + 6y \\ 5y + 1 &= 8y + 6y - 5 \\ 5y + 1 &= 14y - 5 \\ 5y - 5y + 1 &= 14y - 5y - 5 \\ 1 &= 9y - 5 \\ 1 + 5 &= 9y - 5 + 5 \\ 6 &= 9y \\ \frac{6}{9} &= \frac{9y}{9} \\ \frac{2}{3} &= y \end{aligned}$$

23.
$$\begin{aligned} x - 3(2x + 3) &= 8 - 5x \\ x - 6x - 9 &= 8 - 5x \\ -5x - 9 &= 8 - 5x \\ -5x + 5x - 9 &= 8 - 5x + 5x \\ -9 &\neq 8 \end{aligned}$$

Because $-9 = 8$ is a contradiction, the equation has no solution.

24. $9x - 10 = 5x + 2(2x - 5)$

$9x - 10 = 5x + 4x - 10$

$9x - 10 = 9x - 10$

Because the equation is an identity, the solution is the set of all real numbers.

25. $0.25x + 0.75(10 - x) = 3$

$0.25x + 7.5 - 0.75x = 3$

$-0.50x + 7.5 = 3$

$-0.50x = -4.5$

$x = 9$

26. $0.60x + 0.40(100 - x) = 50$

$0.60x + 40 - 0.40x = 50$

$0.20x = 10$

$x = 50$

27. $\frac{3x}{8} - \frac{4x}{3} = 4$

$(24)\frac{3x}{8} - (24)\frac{4x}{3} = (24)4$

$9x - 32x = 96$

$-23x = 96$

$x = -\frac{96}{23}$

28. $\frac{2x}{5} + 5x = \frac{4}{3}$

$(15)\frac{2x}{5} + (15)5x = (15)\frac{4}{3}$

$6x + 75x = 20$

$81x = 20$

$x = \frac{20}{81}$

29. $\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$

$(4)\frac{5x}{4} + (4)\frac{1}{2} = (4)x - (4)\frac{1}{2}$

$5x + 2 = 4x - 2$

$x = -4$

30. $\frac{x}{5} - \frac{x}{2} = 3 + \frac{3x}{10}$

$(10)\frac{x}{5} - (10)\frac{x}{2} = (10)3 + (10)\frac{3x}{10}$

$2x - 5x = 30 + 3x$

$-6x = 30$

$x = -5$

31. $\frac{5x - 4}{5x + 4} = \frac{2}{3}$

$3(5x - 4) = 2(5x + 4)$

$15x - 12 = 10x + 8$

$5x = 20$

$x = 4$

32. $\frac{10x + 3}{5x + 6} = \frac{1}{2}$

$2(10x + 3) = 1(5x + 6)$

$20x + 6 = 5x + 6$

$15x = 0$

$x = 0$

33. $10 - \frac{13}{x} = 4 + \frac{5}{x}$

$\frac{10x - 13}{x} = \frac{4x + 5}{x}$

$10x - 13 = 4x + 5$

$6x = 18$

$x = 3$

34. $\frac{15}{x} - 4 = \frac{6}{x} + 3$

$\frac{15}{x} - \frac{6}{x} = 7$

$\frac{9}{x} = 7$

$9 = 7x$

$\frac{9}{7} = x$

35. $3 = 2 + \frac{2}{z + 2}$

$3(z + 2) = \left(2 + \frac{2}{z + 2}\right)(z + 2)$

$3z + 6 = 2z + 4 + 2$

$z = 0$

36. $\frac{1}{x} + \frac{2}{x - 5} = 0$

$x(x - 5)\frac{1}{x} + x(x - 5)\frac{2}{x - 5} = x(x - 5)0$

$x - 5 + 2x = 0$

$3x - 5 = 0$

$3x = 5$

$x = \frac{5}{3}$

37. $$\frac{x}{x+4} + \frac{4}{x+4} + 2 = 0$$
$$\frac{x+4}{x+4} + 2 = 0$$
$$1 + 2 = 0$$
$$3 \neq 0$$

Because $3 = 0$ is a contradiction, the equation has no solution.

38. $$\frac{7}{2x+1} - \frac{8x}{2x-1} = -4$$ Multiply each term by $(2x + 1)(2x - 1)$.
$$7(2x-1) - 8x(2x+1) = -4(2x+1)(2x-1)$$
$$14x - 7 - 16x^2 - 8x = -16x^2 + 4$$
$$6x = 11$$
$$x = \frac{11}{6}$$

39. $$\frac{2}{(x-4)(x-2)} = \frac{1}{x-4} + \frac{2}{x-2}$$ Multiply each term by $(x - 4)(x - 2)$.
$$2 = 1(x-2) + 2(x-4)$$
$$2 = x - 2 + 2x - 8$$
$$2 = 3x - 10$$
$$12 = 3x$$
$$4 = x$$

A check reveals that $x = 4$ yields a denominator of zero. So, $x = 4$ is an extraneous solution, and the original equation has no real solution.

40. $$\frac{12}{(x-1)(x+3)} = \frac{3}{x-1} + \frac{2}{x+3}$$ Multiply each term by $(x - 1)(x + 3)$.
$$(x-1)(x+3)\frac{12}{(x-1)(x+3)} = (x-1)(x+3)\frac{3}{x-1} + (x-1)(x+3)\frac{2}{x+3}$$
$$12 = 3(x+3) + 2(x-1)$$
$$12 = 3x + 9 + 2x - 2$$
$$12 = 5x + 7$$
$$5 = 5x$$
$$x = 1$$

A check reveals that $x = 1$ yields a denominator of zero. So, $x = 1$ is an extraneous solution, and the original equation has no real solution.

41. $$\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9}$$ Multiply each term by $(x + 3)(x - 3)$.
$$\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{(x+3)(x-3)}$$
$$1(x+3) + 1(x-3) = 10$$
$$2x = 10$$
$$x = 5$$

42. $\dfrac{1}{x-2}+\dfrac{3}{x+3}=\dfrac{4}{x^2+x-6}$

$$\begin{aligned}
\frac{1}{x-2}+\frac{3}{x+3} &= \frac{4}{(x+3)(x-2)} \quad \text{Multiply each term by } (x+3)(x-2).\\
(x+3)+3(x-2) &= 4\\
x+3+3x-6 &= 4\\
4x-3 &= 4\\
4x &= 7\\
x &= \frac{7}{4}
\end{aligned}$$

43. $\dfrac{3}{x^2-3x}+\dfrac{4}{x}=\dfrac{1}{x-3}$

$$\begin{aligned}
\frac{3}{x(x-3)}+\frac{4}{x} &= \frac{1}{x-3} \quad \text{Multiply each term by } x(x-3).\\
3+4(x-3) &= x\\
3+4x-12 &= x\\
3x &= 9\\
x &= 3
\end{aligned}$$

A check reveals that $x = 3$ yields a denominator of zero. So, $x = 3$ is an extraneous solution, and the original equation has no solution.

44. $\dfrac{6}{x}-\dfrac{2}{x+3}=\dfrac{3(x+5)}{x(x+3)}$ Multiply each term by $x(x+3)$.

$$\begin{aligned}
6(x+3)-2x &= 3(x+5)\\
6x+18-2x &= 3x+15\\
4x+18 &= 3x+15\\
x &= -3
\end{aligned}$$

Check: $\dfrac{6}{-3}-\dfrac{2}{-3+3}=\dfrac{3(-3+5)}{-3(-3+3)}$

$$-2-\frac{2}{0}=\frac{-6}{-3(0)}$$

Division by zero is undefined. So, $x = -3$ is an extraneous solution, and the original equation has no solution.

45.

$$\begin{aligned}
y &= 12-5x & y &= 12-5x\\
0 &= 12-5x & y &= 12-5(0)\\
5x &= 12 & y &= 12\\
x &= \tfrac{12}{5}
\end{aligned}$$

The x-intercept is $\left(\frac{12}{5}, 0\right)$ and the y-intercept is $(0, 12)$.

46.

$$\begin{aligned}
y &= 16-3x & y &= 16-3x\\
0 &= 16-3x & y &= 16-3(0)\\
-16 &= -3x & y &= 16\\
\tfrac{16}{3} &= x
\end{aligned}$$

The x-intercept is $\left(\frac{16}{3}, 0\right)$ and the y-intercept is $(0, 16)$.

47.

$$\begin{aligned}
y &= -3(2x+1) & y &= -3(2x+1)\\
0 &= -3(2x+1) & y &= -3(2(0)+1)\\
0 &= 2x+1 & y &= -3\\
x &= -\tfrac{1}{2}
\end{aligned}$$

The x-intercept is $\left(-\frac{1}{2}, 0\right)$ and the y-intercept is $(0, -3)$.

48.

$$\begin{aligned}
y &= 5-(6-x) & y &= 5-(6-x)\\
0 &= 5-(6-x) & y &= 5-(6-0)\\
0 &= -1+x & y &= -1\\
1 &= x
\end{aligned}$$

The x-intercept is $(1, 0)$ and the y-intercept is $(0, -1)$.

49.

$$\begin{aligned} 2x + 3y &= 10 \\ 2x + 3(0) &= 10 \\ 2x &= 10 \\ x &= 5 \end{aligned} \qquad \begin{aligned} 2x + 3y &= 10 \\ 2(0) + 3y &= 10 \\ 3y &= 10 \\ y &= \tfrac{10}{3} \end{aligned}$$

The x-intercept is $(5, 0)$ and the y-intercept is $\left(0, \frac{10}{3}\right)$.

50.

$$\begin{aligned} 4x - 5y &= 12 \\ 4x - 5(0) &= 12 \\ 4x &= 12 \\ x &= 3 \end{aligned} \qquad \begin{aligned} 4x - 5y &= 12 \\ 4(0) - 5y &= 12 \\ -5y &= 12 \\ y &= -\tfrac{12}{5} \end{aligned}$$

The x-intercept is $(3, 0)$ and the y-intercept is $\left(0, -\frac{12}{5}\right)$.

51.

$$\begin{aligned} 4y - 0.75x + 1.2 &= 0 \\ 4(0) - 0.75x + 1.2 &= 0 \\ -0.75 + 1.2 &= 0 \\ x = \frac{1.2}{0.75} &= 1.6 \end{aligned} \qquad \begin{aligned} 4y - 0.75x + 1.2 &= 0 \\ 4y - 0.75(0) + 1.2 &= 0 \\ 4y + 1.2 &= 0 \\ y = \frac{-1.2}{4} &= -0.3 \end{aligned}$$

The x-intercept is $(1.6, 0)$ and the y-intercept is $(0, -0.3)$.

52.

$$\begin{aligned} 3y + 2.5x - 3.4 &= 0 \\ 3(0)2.5x - 3.4 &= 0 \\ 2.5x &= 3.4 \\ x &= \frac{3.4}{2.5} \\ x &= 1.36 \end{aligned} \qquad \begin{aligned} 3y + 2.5x - 3.4 &= 0 \\ 3y + 2.5(0) - 3.4 &= 0 \\ 3y &= 3.4 \\ y &= \frac{3.4}{3} \\ y &= 1.1\overline{3} \end{aligned}$$

The x-intercept is $(1.36, 0)$ and the y-intercept is $\left(0, 1.1\overline{3}\right)$.

53.

$$\begin{aligned} \frac{2x}{5} + 8 - 3y &= 0 \\ 2x + 40 - 15y &= 0 \\ 2x + 40 - 15(0) &= 0 \\ 2x + 40 &= 0 \\ x &= -20 \end{aligned} \qquad \begin{aligned} 2x + 40 - 15y &= 0 \\ 2(0) + 40 - 15y &= 0 \\ 40 - 15y &= 0 \\ y = \frac{40}{15} &= \frac{8}{3} \end{aligned}$$

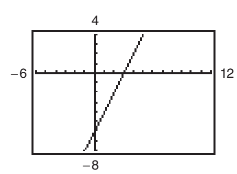
The x-intercept is $(-20, 0)$ and the y-intercept is $\left(0, \frac{8}{3}\right)$.

54.

$$\begin{aligned} \frac{8x}{3} + 5 - 2y &= 0 \\ \frac{8x}{3} + 5 - 2(0) &= 0 \\ \frac{8x}{3} &= -5 \\ \frac{3}{8} \cdot \frac{8x}{3} &= \frac{3}{8} \cdot (-5) \\ x &= -\frac{15}{8} \end{aligned} \qquad \begin{aligned} \frac{8x}{3} + 5 - 2y &= 0 \\ \frac{8(0)}{3} + 5 - 2y &= 0 \\ -2y &= -5 \\ y &= \frac{5}{2} \end{aligned}$$

The x-intercept is $\left(-\frac{15}{8}, 0\right)$ and the y-intercept is $\left(0, \frac{5}{2}\right)$.

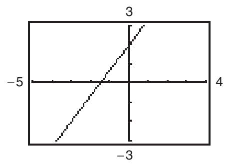
55. $y = 2(x - 1) - 4$



$$\begin{aligned} 0 &= 2(x - 1 - 4) \\ 0 &= 2x - 2 - 4 \\ 0 &= 2x - 6 \\ 6 &= 2x \\ 3 &= x \\ x &= 3 \end{aligned}$$

The x-intercept is $x = 3$. The solution of $0 = 2(x - 1) - 4$ and the x-intercept of $y = 2(x - 1) - 4$ are the same. They are both $x = 3$. The x-intercept is $(3, 0)$.

56. $y = \frac{4}{3}x + 2$

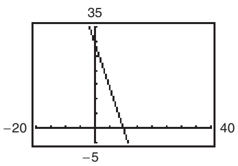


$$\begin{aligned} y &= \tfrac{4}{3}x + 2 \\ -\tfrac{4}{3}x &= 2 \\ \left(-\tfrac{3}{4}\right)\left(-\tfrac{4}{3}x\right) &= \left(-\tfrac{3}{4}\right)(2) \\ x &= -\tfrac{3}{2} \end{aligned}$$

Intercept: $\left(-\frac{3}{2}, 0\right)$

The solution to $0 = \frac{4}{3}x + 2$ is the same as the x-intercept of $y = \frac{4}{3}x + 2$. They are both $x = -\frac{3}{2}$.

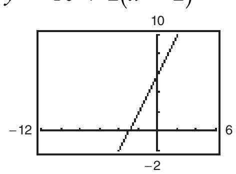
57. $y = 20 - (3x - 10)$



$$\begin{aligned} 0 &= 20 - (3x - 10) \\ 0 &= 20 - 3x + 10 \\ 0 &= 30 - 3x \\ 3x &= 30 \\ x &= 10 \end{aligned}$$

The x-intercept is $x = 10$. The solution of $0 = 20 - (3x - 10)$ and the x-intercept of $y = 20 - (3x - 10)$ are the same. They are both $x = 10$. The x-intercept is $(10, 0)$.

58. $y = 10 + 2(x - 2)$

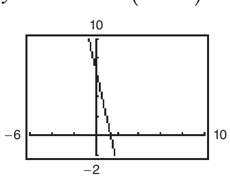


$$\begin{aligned} 0 &= 10 + 2(x - 2) \\ 0 &= 10 + 2x - 4 \\ 0 &= 6 + 2x \\ -2x &= 6 \\ x &= -3 \end{aligned}$$

Intercept: $(-3, 0)$

The solution to $0 = 10 + 2(x - 2)$ is the same as the x-intercept of $y = 10 + 2(x - 2)$. They are both $x = -3$.

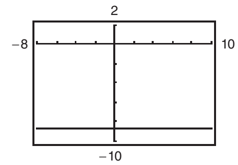
59. $y = -38 + 5(9 - x)$



$$\begin{aligned} 0 &= -38 + 5(9 - x) \\ 0 &= -38 + 45 - 5x \\ 0 &= 7 - 5x \\ 5x &= 7 \\ x &= \tfrac{7}{5} \end{aligned}$$

The x-intercept is at $x = \frac{7}{5}$. The solution of $0 = -38 + 5(9 - x)$ and the x-intercept of $y = -38 + 5(9 - x)$ are the same. They are both $x = \frac{7}{5}$. The x-intercept is $\left(\frac{7}{5}, 0\right)$.

60. $y = 6x - 6\left(\frac{6}{11} + x\right)$



$$\begin{aligned} 0 &= 6x - 6\left(\tfrac{16}{11} + x\right) \\ 0 &= 6x - \tfrac{96}{11} - 6x \\ 0 &\neq -\tfrac{96}{11} \end{aligned}$$

There is no x-intercept.

61.
$$\begin{aligned} 0.275x + 0.725(500 - x) &= 300 \\ 0.275x + 362.5 - 0.725x &= 300 \\ -0.45x &= -62.5 \\ x &= \frac{62.5}{0.45} \approx 138.889 \end{aligned}$$

62.
$$\begin{aligned} 2.763 - 4.5(2.1x - 5.1432) &= 6.32x + 5 \\ 2.763 - 9.45x + 23.1444 &= 6.32x + 5 \\ 20.9074 &= 15.77x \\ 1.326 &\approx x \end{aligned}$$

63. $\dfrac{2}{7.398} - \dfrac{4.405}{x} = \dfrac{1}{x}$ Multiply both sides by $7.398x$.
$$\begin{aligned} 2x - (4.405)(7.398) &= 7.398 \\ 2x &= (4.405)(7.398) + 7.398 \\ 2x &= (5.405)(7.398) \\ x &= \frac{(5.405)(7.398)}{2} \approx 19.993 \end{aligned}$$

64. $\dfrac{3}{6.350} - \dfrac{6}{x} = 18$ Multiply both sides by $6.350x$.
$$\begin{aligned} 3x - 6(6.350) &= 18(6.350)x \\ 3x - 38.1 &= 114.3x \\ -38.1 &= 111.3x \\ -0.342 &\approx x \end{aligned}$$

65.
$$\begin{aligned} 471 &= 2\pi(25) + 2\pi(5h) \\ 471 &= 50\pi + 10\pi h \\ 471 - 50\pi &= 10\pi h \\ h &= \frac{471 - 50\pi}{10\pi} = \frac{471 - 50(3.14)}{10(3.14)} = 10 \\ h &= 10 \text{ feet} \end{aligned}$$

66.
$$\begin{aligned} 248 &= 2(24) + 2(4x) + 2(6x) \\ 248 &= 48 + 8x + 12x \\ 200 &= 20x \\ x &= 10 \text{ centimeters} \end{aligned}$$

67. Let $y = 18$.

$$\begin{aligned} y &= 0.514x - 14.75 \\ 18 &= 0.514x - 14.75 \\ 32.75 &= 0.514x \\ \frac{32.75}{0.514} &= x \\ 63.7 &= x \end{aligned}$$

So, the height of the female is about 63.7 inches.

68. Let $y = 23$.

$$\begin{aligned} y &= 0.532x - 17.03 \\ 23 &= 0.532x - 17.03 \\ 40.03 &= 0.532x \\ \frac{40.03}{0.532} &= x \\ 75.2 &= x \end{aligned}$$

The height of the missing man is about 75.2 inches.

Because 75.2 in. $\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 6.27$ ft is about 6 feet 3 inches, it is possible the femur belongs to the missing man.

69. (a) The y-intercept is about $(0, 295)$.

(b) Let $t = 0$.

$$\begin{aligned} y &= 11.09t + 293.4 \\ &= 11.09(0) + 293.4 \\ &= 293.4 \end{aligned}$$

The y-intercept is $(0, 293.4)$.

In 2000, the population of Raleigh was about 293.4 thousand, or 293,400.

(c) Let $y = 538$.

$$\begin{aligned} y &= 11.09t + 293.4 \\ 538 &= 11.09t + 293.4 \\ 244.6 &= 11.09t \\ \frac{244.6}{11.09} &= t \\ 22.1 &\approx t \end{aligned}$$

In 2022, the population is expected to reach 538,000. Answers will vary. *Sample answer:* If the population continues to increase at a constant (linear) rate, the answer seems reasonable.

70. (a) The y-intercept is about $(0, 292)$.

(b) Let $t = 0$.

$$\begin{aligned} y &= -2.60t + 291.7 \\ &= -2.60(0) + 291.7 \\ &= 291.7 \end{aligned}$$

The y-intercept is $(0, 291.7)$.

In 2000, the population of Buffalo was about 291.7 thousand, or 291,700.

(c) Let $y = 239$.

$$\begin{aligned} y &= -2.60t + 291.7 \\ 239 &= -2.60t + 291.7 \\ -52.7 &= -2.60t \\ \frac{-52.7}{-2.60} &= t \\ 20.3 &\approx t \end{aligned}$$

In 2020, the population is expected to decrease to 239,000.
Answers will vary. *Sample answer:* If the population continues to decrease at a constant (linear) rate, the answer seems reasonable.

71. Let $c = 10{,}000$.

$$\begin{aligned} c &= 0.37m + 2600 \\ 10{,}000 &= 0.37m + 2600 \\ 7400 &= 0.37m \\ \frac{7400}{0.37} &= m \\ m &= 20{,}000 \end{aligned}$$

So, the number of miles is 20,000.

72. Let $y = 1$.

$$\begin{aligned} y &= -0.25t + 8 \\ 1 &= -0.25t + 8 \\ 0.25t &= 7 \\ t &= 28 \text{ hours} \end{aligned}$$

73. $x(3 - x) = 10$

$3x - x^2 = 10$

False. This is a quadratic equation. The equation cannot be written in the form $ax + b = 0$.

74. $2x + 3 = x$

$x + 3 = 0$

True. The equation is a linear equation because it can be written in the form $ax + b = 0$.

75. $2(x - 3) + 1 = 2x - 5$

$2x - 6 + 1 = 2x - 5$

$2x - 5 = 2x - 5$

False. The equation is an identity, so every real number is a solution.

76. $2(x + 3) = 3x + 3$

$2x + 6 = 3x + 3$

$3 = x$

False. $x = 3$ is a solution.

77. $3(x - 1) - 2 = 3x - 6$

$3x - 3 - 2 = 3x - 6$

$3x - 5 = 3x - 6$

$-5 \neq -6$

False. The equation $-5 = -6$ is a contradiction, so the original equation has no solution.

78. $$2 - \frac{1}{x - 2} = \frac{3}{x - 2}$$

$$(x - 2)\left(2 - \frac{1}{x - 2}\right) = (x - 2)\left(\frac{3}{x - 2}\right)$$

$2(x - 2) - 1 = 3$

$2x - 4 - 1 = 3$

$2x - 5 = 3$

$2x = 8$

$x = 4$

False. $x = 4$ is a solution.

79. (a)

x	-1	0	1	2	3	4
$3.2x - 5.8$	-9	-5.8	-2.6	0.6	3.8	7

(b) Since the sign changes from negative at 1 to positive at 2, the root is somewhere between 1 and 2.

$1 < x < 2$

(c)

x	1.5	1.6	1.7	1.8	1.9	2
$3.2x - 5.8$	-1	-0.68	-0.36	-0.04	0.28	0.6

(d) Since the sign changes from negative at 1.8 to positive at 1.9, the root is somewhere between 1.8 and 1.9.

$1.8 < x < 1.9$

To improve accuracy, evaluate the expression at subintervals within this interval and determine where the sign changes.

(e) $0.3(x - 1.5) - 2 = 0$

x	6	7	8	9	10
$0.3(x - 1.5) - 2$	-0.65	-0.35	-0.05	0.25	0.55

The solution of $0.3(x - 1.5) - 2 = 0$ is in the interval $8 < x < 9$.

x	8.1	8.2	8.3	8.4
$0.3(x - 1.5) - 2$	-0.02	0.01	0.04	0.07

The solution is in the interval $8.1 < x < 8.2$.

x	8.14	8.15	8.16	8.17	8.18
$0.3(x - 1.5) - 2$	-0.008	-0.005	-0.002	0.001	0.004

The solution is in the interval $8.16 < x < 8.17$.

80. $\dfrac{3x+2}{5} = 7$

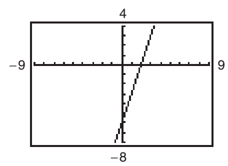
$3x + 2 = 35$ and $x + 9 = 20$

$3x = 33$ $\quad x = 11$

$x = 11$

Yes, they are equivalent equations. They both have the solution $x = 11$.

81. (a)



(b) x-intercept: $(2, 0)$

(c) The x-intercept is the solution of the equation $3x - 6 = 0$.

82. (a) The x-intercept is $(20{,}000, 0)$ and the y-intercept is $(0, 10{,}000)$. The subsidy is \$0 for an earned income of \$20,000. The subsidy is \$10,000 for an earned income of \$0.

(b) Set one of S or E equal to 0 and solve for the other.

(c) The earned income is \$8000.

(d) Set T equal to 14,000, substitute $10{,}000 - \frac{1}{2}E$ for S in the equation $T = E + S$, and solve for E.

83. (a) To find the x-intercept, let $y = 0$ and solve for x.

$$0 = ax + b$$
$$-b = ax$$
$$\frac{-b}{a} = x$$

The x-intercept is $\left(-\dfrac{b}{a}, 0\right)$.

(b) To find the y-intercept, let $x = 0$, and solve for y.

$$y = a(0) + b$$
$$y = b$$

The y-intercept is $(0, b)$.

(c) x-intercept:

$$x = \frac{-b}{a} = \frac{-10}{5} = -2$$

The x-intercept is $(-2, 0)$.

y-intercept:

$$y = b = 10$$

The y-intercept is $(0, 10)$.

84. (a) To find the x-intercept, let $y = 0$, and solve for x.

$$ax + by = c$$
$$ax + b(0) = c$$
$$ax = c$$
$$x = \frac{c}{a}$$

The x-intercept is $\left(\dfrac{c}{a}, 0\right)$.

(b) To find the y-intercept, let $x = 0$, and solve for y.

$$a(0) + by = c$$
$$by = c$$
$$y = \frac{c}{b}$$

The y-intercept is $\left(0, \dfrac{c}{b}\right)$.

(c) x-intercept:

$$x = \frac{c}{a} = \frac{11}{2}$$

The x-intercept is $\left(\dfrac{11}{2}, 0\right)$.

y-intercept:

$$y = \frac{c}{b} = \frac{11}{7}$$

The y-intercept is $\left(0, \dfrac{11}{7}\right)$.

Section 1.3 Modeling with Linear Equations

1. mathematical modeling

2. verbal model; algebraic equation

3. $y + 2$

The sum of a number and 2

A number increased by 2

4. $x - 8$

The difference of a number and 8

A number decreased by 8

5. $\dfrac{t}{6}$

A number divided by 6

6. $\dfrac{1}{3}u$

The product of $\dfrac{1}{3}$ and a number

7. $\dfrac{z - 2}{3}$

A number decreased by 2, then divided by 3

8. $\dfrac{x + 9}{5}$

A number increased by 9, then divided by 5

9. $-2(d + 5)$

The product of -2 and a number increased by 5

10. $10y(y - 3)$

The product of 10, a number, and 3 less than the same number

11. $\dfrac{3(x - 2)}{x}$

The product of 2 less than a number and 3, then divided by the same number

12. $\dfrac{(r + 3)(2 - r)}{3r}$

The product of 3 more than a number and 2 decreased by the same number, then divided by 3 times the same number

13. *Verbal Model*: (Sum) = (first number) + (second number)

Labels: Sum $= S$, first number $= n$, second number $= n + 1$

Equation: $S = n + (n + 1) = 2n + 1$

14. *Verbal Model*: Product = (first number) · (second number)

Labels: Product $= P$, first number $= n$, second number $= n + 1$

Equation: $P = n(n + 1) = n^2 + n$

15. *Verbal Model:* Product = (first odd integer) · (second odd integer)

Labels: Product $= P$, first odd integer $= 2n - 1$, second odd integer $= 2n - 1 + 2 = 2n + 1$

Equation: $P = (2n - 1)(2n + 1) = 4n^2 - 1$

16. *Verbal Model:* $(\text{Sum}) = (\text{first even number})^2 + (\text{second even number})^2$

Labels: Sum $= S$, first even number $= 2n$, second even number $= 2n + 2$

Equation: $S = (2n)^2 + (2n + 2)^2 = 4n^2 + 4n^2 + 8n + 4 = 8n^2 + 8n + 4$

17. *Verbal Model:* (Distance) = (rate) · (time)

Labels: Distance $= d$, rate $= 55$ mph, time $= t$

Equation: $d = 55t$

18. *Verbal Model:* (time) = (distance) ÷ (rate)

Labels: time $= t$, distance $= 900$ km, rate $= r$

Equation: $t = \dfrac{900}{r}$

19. *Verbal Model:* (Amount of acid) = 20% · (amount of solution)

Labels: Amount of acid (in gallons) $= A$, amount of solution (in gallons) $= x$

Equation: $A = 0.20x$

20. *Verbal Model:* $(\text{Sale price}) = (\text{list price}) - (\text{discount})$

Labels: Sale price $= S$, list price $= L$, discount $= 0.33L$

Equation: $S = L - 033L = 0.67L$

21. *Verbal Model:* Perimeter $= 2(\text{width}) + 2(\text{length})$

Labels: Perimeter $= P$, width $= x$, length $= 2(\text{width}) = 2x$

Equation: $P = 2x + 2(2x) = 6x$

22. *Verbal Model:* $(\text{Area}) = \frac{1}{2}(\text{base})(\text{height})$

Labels: Area $= A$, base $= 16$ in., height $= h$

Equation: $A = \frac{1}{2}(16)h = 8h$

23. *Verbal Model:* $(\text{Total cost}) = (\text{unit cost})(\text{number of units}) + (\text{fixed cost})$

Labels: Total cost $= C$, unit cost $= \$40$, number of units $= x$, fixed cost $= \$2500$

Equation: $C = 2500 + 40x$

24. *Verbal Model:* $(\text{Revenue}) - (\text{price})(\text{number of units})$

Labels: Revenue $= R$, price $= \$12.99$, number of units $= x$

Equation: $R = 12.99x$

25. *Verbal Model:* $(\text{Discount}) = (\text{percent}) \cdot (\text{list price})$

Equation: $d = 0.30L$

26. *Labels:* $A =$ amount of water, $q =$ number of quarts

Verbal Model: $(\text{Amount of water}) = \dfrac{(\text{percent})}{100} \cdot (\text{number of quarts})$

Equation: $A = 0.72q$

27. *Labels:* $N =$ the number, $p =$ percent % of the number

Verbal Model: $(\text{The number}) = \dfrac{(\text{percent})}{100} \cdot 672$

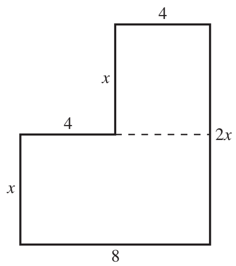
Equation: $N = \dfrac{p}{100} \cdot 672$

28. *Labels:* $S_2 =$ sales for this month, $S_1 =$ sales from last month

Verbal Model: $(\text{Sales for this month}) = (\text{Sales from last month}) + \dfrac{(\text{percent})}{100} \cdot (\text{Sales from last month})$

Equation: $S_2 = S_1 + 0.2S_1$

29.



Area = Area of top rectangle + Area of bottom rectangle

$A = 4x + 8x = 12x$

30. Area $= \frac{1}{2}$(base)(height)

$A = \frac{1}{2}\left(\frac{2}{3}b + 1\right) = \frac{1}{3}b^2 + \frac{1}{2}b$

31. *Verbal Model:* Sum = (first number) + (second number)

Labels: Sum = 525, first number = n, second number = $n + 1$

Equation:
$525 = n + (n + 1)$
$525 = 2n + 1$
$524 = 2n$
$n = 262$

Answer: First number $= n = 262$, second number $= n + 1 = 263$

32. *Verbal Model:* Sum = (first number) + (second number) + (third number)

Labels: Sum = 804, first number = n, second number = $n + 1$, third number = $n + 2$

Equation:
$804 = n + n + 1 + n + 2$
$804 = 3n + 3$
$801 = 3n$
$267 = n$

Answer: $n = 267$, $n + 1 = 268$ (second number), and $n + 2 = 269$ (third number)

33. *Verbal Model:* Difference = (one number) − (another number)

Labels: Difference = 148, one number = $5x$, another number = x

Equation:
$148 = 5x - x$
$148 = 4x$
$x = 37$
$5x = 185$

Answer: The two numbers are 37 and 185.

34. *Verbal Model:* Difference = (number) − (one-fifth of number)

Labels: Difference = 76, number = n, one-fifth of number = $\frac{1}{5}n$

Equation:
$76 = n - \frac{1}{5}n$
$76 = \frac{4}{5}n$
$95 = n$

Answer: The numbers are 95 and $\frac{1}{5} \cdot 95 = 19$.

35. *Verbal Model:* Product $=$ (smaller number) $\cdot$ (larger number) $=$ (smaller number)$^2 - 5$

Labels: Smaller number $= n$, larger number $= n + 1$

Equation:
$$n(n+1) = n^2 - 5$$
$$n^2 + n = n^2 - 5$$
$$n = -5$$

Answer: Smaller number $= n = -5$, larger number $= n + 1 = -4$

36. *Verbal Model:* Difference $=$ (reciprocal of smaller number) $-$ (reciprocal of larger number)

$= \frac{1}{4}$ (reciprocal of smaller number)

Labels: Smaller number $= n$, larger number $= n + 1$, difference $= \dfrac{1}{4n}$

Equation:
$$\frac{1}{4n} = \frac{1}{n} - \frac{1}{n+1}$$ Multiply both sides by $4n(n+1)$.
$$4n(n+1)\frac{1}{4n} = 4n(n+1)\frac{1}{n} - 4n(n+1)\frac{1}{n+1}$$
$$n + 1 = 4(n+1) - 4n$$
$$n + 1 = 4n + 4 - 4n$$
$$n = 3$$

Answer: The numbers are 3 and $n + 1 = 4$.

37. *Verbal Model:* (first paycheck) $+$ (second paycheck) $=$ total

Labels: second paycheck $= x$, first paycheck $= 0.85x$, total $= \$1125$

Equation:
$$0.85x + x = 1125$$
$$1.85x = 1125$$
$$x \approx 608.11$$
$$0.85x \approx 516.89$$

Answer: The first salesperson's weekly paycheck is \$516.89 and the second salesperson's weekly paycheck is \$608.11.

38. *Verbal Model:* (Sale price) $=$ (list price) $-$ (discount)

Labels: Sale price $-$ \$1210.75, list price $= L$, discount $= 0.165L$

Equation:
$$1210.75 = L - 0.165L$$
$$1210.75 = 0.835L$$
$$1450 = L$$

Answer: The list price of the pool is \$1450.

39. *Verbal Model:* (Loan payments) $=$ (Percent) $\cdot$ (Annual Income)

Labels: Loan payments $= 15{,}680$ (dollars)

Percent $= 0.32$

Annual income $= I$ (dollars)

Equation:
$$15{,}680 = 0.32I$$
$$\frac{15{,}680}{0.32} = \frac{0.32I}{0.32}$$
$$49{,}000 = I$$

Answer: The family's annual income is \$49,000.

40. *Verbal Model:* (Mortgage payment) $=$ (Percent) $\cdot$ (Monthly income)

Labels: Mortgage payment $= 760$ (dollars)

Percent $= 0.16$

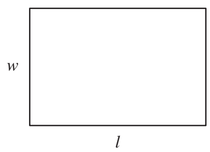
Monthly income $= I$ (dollars)

Equation:

$$760 = 0.16I$$
$$\frac{760}{0.16} = \frac{0.16I}{0.16}$$
$$4750 = I$$

Answer: The family's monthly income is \$4750.

41. (a)



(b) $l = 1.5w$

$$\begin{aligned} P &= 2l + 2w \\ &= 2(1.5w) + 2w \\ &= 5w \end{aligned}$$

(c) $25 = 5w$

$5 = w$

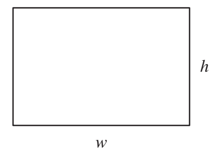
Width: $w = 5$ meters

Length: $l = 1.5w = 7.5$ meters

Dimensions: 7.5 meters $\times$ 5 meters

42. The perimeter is 3 meters and the height is two thirds of the width, so $h = \frac{2}{3}w$.

First,
$$\begin{aligned} P &= 2h + 2w \\ &= 2\left(\tfrac{2}{3}w\right) + 2w \\ &= \tfrac{10}{3}w \end{aligned}$$



Then let $P = 3$

$$3 = \tfrac{10}{3}w$$
$$\tfrac{9}{10} = w$$

Since $h = \frac{2}{3}w = \frac{2}{3}\left(\frac{9}{10}\right) = \frac{3}{5}$

So, the dimensions of the frame are $\frac{9}{10}$ meter by $\frac{3}{5}$ meter.

43. *Verbal Model:* $\text{Average} = \dfrac{(\text{test \#1}) + (\text{test \#2}) + (\text{test \#3}) + (\text{test \#4})}{4}$

Labels: Average $= 90$, test #1 $= 87$, test #2 $= 92$, test #3 $= 84$, test #4 $= x$

Equation: $90 = \dfrac{87 + 92 + 84 + x}{4}$

Answer: You must score 97 or better on test #4 to earn an A for the course.

44. *Verbal Model:* $\text{Average} = \dfrac{(\text{test \#1}) + (\text{test \#2}) + (\text{test \#3}) + (\text{test \#4})}{5}$

Labels: Average $= 90$, test #1 $= 87$, test #2 $= 92$, test #3 $= 84$, test #4 $= x$

Equation:

$$90 = \frac{87 + 92 + 84 + x}{5}$$
$$450 = 87 + 92 + 84 + x$$
$$450 = 263 + x$$
$$187 = x$$

Answer: You must score 187 out of 200 on the last test to get an A in the course.

45. $\text{Rate} = \dfrac{\text{distance}}{\text{time}} = \dfrac{50 \text{ kilometers}}{\frac{1}{2} \text{ hour}} = 100 \text{ kilometers/hour}$

$\text{Total time} = \dfrac{\text{total distance}}{\text{rate}} = \dfrac{500 \text{ kilometers}}{100 \text{ kilometers/hour}} = 5 \text{ hours}$

The entire trip takes 5 hours.

46. *Verbal Model:* $(\text{Distance}) = (\text{rate})(\text{time}_1 + \text{time}_2)$

Labels: Distance $= 2 \cdot 200 = 400$ miles, rate $= 2$,

$$\text{time}_1 = \frac{\text{distance}}{\text{rate}_1} = \frac{200}{55} \text{ hours,}$$

$$\text{time}_2 = \frac{\text{distance}}{\text{rate}_2} = \frac{200}{40} \text{ hours}$$

Equation:

$$400 = r\left(\frac{200}{55} + \frac{200}{40}\right)$$

$$400 = r\left(\frac{1600}{440} + \frac{2200}{440}\right) = \frac{3800}{440}r$$

$$46.3 \approx r$$

The average speed for the round trip was approximately 46.3 miles per hour.

47. *Verbal Model:* $(\text{Distance}) = (\text{rate})(\text{time})$

Labels: Distance $= 1.5 \times 10^{11}$ (meters)

Rate $= 3.0 \times 10^{8}$ (meters per second)

Time $= t$

Equation:

$$1.5 \times 10^{11} = \left(3.0 \times 10^{8}\right)t$$

$$500 = t$$

Light from the sun travels to the Earth in 500 seconds or approximately 8.33 minutes.

48. *Verbal Model:* $\text{time} = \dfrac{\text{distance}}{\text{rate}}$

Equation:

$$t = \frac{3.84 \times 10^{8} \text{ meters}}{3.0 \times 10^{8} \text{ meters per second}}$$

$$t = 1.28 \text{ seconds}$$

The radio wave travels from Mission Control to the moon in 1.28 seconds.

49. *Verbal Model:* $\dfrac{(\text{Height of building})}{(\text{Length of building's shadow})} = \dfrac{(\text{Height of post})}{(\text{Length of post's shadow})}$

Labels: Height of building $= x$ (feet)

Length of building's shadow $= 105$ (feet)

Height of post $= 3 \cdot 12 = 36$ (inches)

Length of post's shadow $= 4$ (inches)

Equation:

$$\frac{x}{105} = \frac{36}{4}$$

$$x = 945$$

One Liberty Place is 945 feet tall.

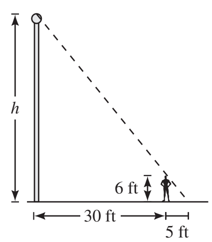
50. *Verbal Model:* $\dfrac{(\text{height of tree})}{(\text{length of tree's shadow})} = \dfrac{(\text{height of lamppost})}{(\text{length of lamppost's shadow})}$

Labels: height of tree $= h$, height of tree's shadow $= 8$ meters, height of lamppost $= 2$ meters, height of lamppost's shadow $= 0.75$ meter

Equation:
$$\frac{h}{8} = \frac{2}{0.75}$$
$$h = \frac{8(2)}{0.75} = 21\frac{1}{3}$$

The tree is $21\frac{1}{3}$ meters tall.

51. (a)



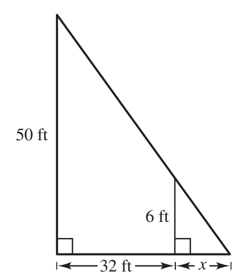
(b) *Verbal Model:* $\dfrac{(\text{height of pole})}{(\text{height of pole's shadow})} = \dfrac{(\text{height of person})}{(\text{height of person's shadow})}$

Labels: Height of pole $= h$, height of pole's shadow $= 30 + 5 = 35$ feet, height of person $= 6$ feet, height of person's shadow $= 5$ feet

Equation:
$$\frac{h}{35} = \frac{6}{5}$$
$$h = \frac{6}{5} \cdot 35 = 42$$

The pole is 42 feet tall.

52. *Verbal Model:* $\dfrac{(\text{height of tower})}{(\text{height of tower's shadow})} = \dfrac{(\text{height of person})}{(\text{height of person's shadow})}$



Labels: Let $x =$ length of person's shadow.

Equation:
$$\frac{50}{32 + x} = \frac{6}{x}$$
$$50x = 6(32 + x)$$
$$50x = 192 + 6x$$
$$44x = 192$$
$$x \approx 4.36 \text{ feet}$$

53. *Verbal Model:* $\boxed{\text{Interest from } 4\tfrac{1}{2}\%} + \boxed{\text{Interest from } 5\%} = \boxed{\text{Total interest}}$

Labels:
Amount invested at $4\frac{1}{2}\% = x$ dollars
Amount invested at $5\% = 12{,}000 - x$ dollars
Interest from $4\frac{1}{2}\% = x(0.045)$ dollars
Interest from $5\% = (12{,}000 - x)(0.05)$ dollars
Total annual interest $= 580$ dollars

Equation:
$$0.045x + 0.05(12{,}000 - x) = 580$$
$$0.045x + 600 - 0.05x = 580$$
$$-0.005x = -20$$
$$x = 4000$$

So, \$4000 was invested at $4\frac{1}{2}\%$ and $\$12{,}000 - \$4000 = \$8000$ was invested at 5%.

54. *Verbal Model:* $\boxed{\text{Interest from } 3\%} + \boxed{\text{Interest from } 4\tfrac{1}{2}\%} = \boxed{\text{Total interest}}$

Labels: Amount invested at 3% $= x$

Amount invested at $4\frac{1}{2}\% = 25{,}000 - x$

Equation:

$$\begin{aligned} 0.03x + 0.045(25{,}000 - x) &= 900 \\ 0.03x + 1125 - 0.045x &= 900 \\ -0.015x &= -225 \\ x &= 15{,}000 \end{aligned}$$

So, \$15,000 was invested at 3% and $\$25{,}000 - \$15{,}000 = \$10{,}000$ was invested at $4\frac{1}{2}\%$.

55. *Verbal Model:* (Profit from dogwood trees) + (profit from red maple trees) = (total profit)

Labels: Inventory of dogwood trees $= x$, inventory of red maple trees $= 40{,}000 - x$,
profit from dogwood trees $= 0.25x$, profit from red maple trees $= 0.17(40{,}000 - x)$,
total profit $= 0.20(40{,}000) = 8000$

Equation:

$$\begin{aligned} 0.25x + 0.17(40{,}000) &= 8000 \\ 0.25x + 6800 - 0.17x &= 8000 \\ 0.08x &= 1200 \\ x &= 15{,}000 \end{aligned}$$

The amount invested in dogwood trees was \$15,000 and the amount invested in red maple trees was $\$40{,}000 - \$15{,}000 = \$25{,}000$.

56. *Verbal Model:* (Profit from minivans) + (profit from alternative-fueled vehicles) = (total profit)

Labels: Inventory of minivans $= x$, inventory of alternative-fueled vehicles $= 600{,}000 - x$,
profit from minivans $= 0.24x$, profit from alternative-fueled vehicles $= 0.28(600{,}000 - x)$,
total profit $= 0.25(600{,}000) = 150{,}000$

Equation:

$$\begin{aligned} 0.24x + 0.28(600{,}000 - x) &= 150{,}000 \\ 0.24x + 168{,}000 - 0.28x &= 150{,}000 \\ -0.04x &= -18{,}000 \\ x &= 450{,}000 \end{aligned}$$

The amount invested in minivans was \$450,000 and the amount invested in alternative-fueled vehicles was. $\$600{,}000 - \$450{,}000 = \$150{,}000$.

57. *Verbal Model:* $\boxed{\begin{array}{l}\text{Amount of gasoline}\\ \text{in mixture}\end{array}} + \boxed{\begin{array}{l}\text{Amount of gasoline}\\ \text{to add}\end{array}} = \boxed{\begin{array}{l}\text{Amount of gasoline}\\ \text{in final mixture}\end{array}}$

Labels: Amount of gasoline in mixture $= \frac{32}{33}(2)$ (gallons)

Amount of gasoline to add $= x$ (gallons)

Amount of gasoline in final mixture $= \frac{50}{51}(2 + x)$ (gallons)

Equation:

$$\begin{aligned} \tfrac{64}{33} + x &= \tfrac{50}{51}(2 + x) \\ \tfrac{64}{33} + x &= \tfrac{100}{51} + \tfrac{50}{51}x \\ 3264 + 1683x &= 3300 + 1650x \\ 33x &= 36 \\ x &\approx 1.09 \end{aligned}$$

The forester should add about 1.09 gallons of gasoline to the mixture.

58. *Verbal Model:* $\begin{pmatrix}\text{Price per pound}\\ \text{of peanuts}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\ \text{peanuts}\end{pmatrix} + \begin{pmatrix}\text{price per pound}\\ \text{of walnuts}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\ \text{walnuts}\end{pmatrix} = \begin{pmatrix}\text{price per pound}\\ \text{of nut mixture}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\ \text{nut mixture}\end{pmatrix}$

Labels: Price per pound of peanuts $= \$1.49$, pounds of peanuts $= x$, price per pound of peanuts $= \$2.69$, pounds of walnuts $= 100 - x$, price per pound of nut mixture $= \$2.21$, pounds of nut mixture $= 100$

Equation:
$$\begin{aligned} 1.49x + 2.69(100 - x) &= 2.21(100)\\ 1.49x + 2.69 - 2.69x &= 2.21\\ -1.2x &= -48\\ x &= 40 \end{aligned}$$

There were 40 pounds of peanuts and $100 - 40 = 60$ pounds of walnuts in the mixture.

59.
$$\begin{aligned} A &= \frac{1}{2}bh\\ 2A &= bh\\ \frac{2A}{b} &= h \end{aligned}$$

60.
$$\begin{aligned} V &= lwh\\ \frac{V}{wh} &= l \end{aligned}$$

61.
$$\begin{aligned} S &= C + RC\\ S &= C(1 + R)\\ \frac{S}{1 + R} &= C \end{aligned}$$

62.
$$\begin{aligned} S &= L - RL\\ S &= L(1 - R)\\ \frac{S}{1 - R} &= L \end{aligned}$$

63.
$$\begin{aligned} A &= P + Prt\\ A - P &= Prt\\ \frac{A - P}{Pt} &= r \end{aligned}$$

64.
$$\begin{aligned} A &= \frac{1}{2}(a + b)h\\ 2A &= (a + b)h\\ \frac{2A}{h} &= a + b\\ b &= \frac{2A}{h} - a \end{aligned}$$

65.
$$\begin{aligned} C &= \tfrac{5}{9}(F - 32)\\ &= \tfrac{5}{9}(98.6 - 32)\\ &= \tfrac{5}{9}(66.6)\\ &= 37^\circ \end{aligned}$$

The temperature is 37°C.

66.
$$\begin{aligned} C &= \tfrac{5}{9}(F - 32)\\ &= \tfrac{5}{9}(101.3 - 32)\\ &= \tfrac{5}{9}(69.3)\\ &= 38.5^\circ \end{aligned}$$

The temperature is 38.5°C.

67.
$$\begin{aligned} F &= \tfrac{9}{5}C + 32\\ &= \tfrac{9}{5}(27) + 32\\ &= 48.6 + 32\\ &= 80.6^\circ F \end{aligned}$$

The temperature is 80.6°F.

68.
$$\begin{aligned} F &= \tfrac{9}{5}C + 32\\ &= \tfrac{9}{5}(58.8) + 32\\ &= 105.84 + 32\\ &= 137.84^\circ F \end{aligned}$$

The temperature is 137.84°F.

69.
$$\begin{aligned} V &= \frac{4}{3}\pi r^3\\ 5.96 &= \frac{4}{3}\pi r^3\\ 17.88 &= 4\pi r^3\\ \frac{17.88}{4\pi} &= r^3\\ r &= \sqrt[3]{\frac{4.47}{\pi}} \approx 1.12 \text{ inches} \end{aligned}$$

70.
$$\begin{aligned} V &= \pi r^2 h\\ h &= \frac{V}{\pi r^2} = \frac{603.2}{\pi(2)^2} \approx 48 \text{ feet} \end{aligned}$$

71. (a)
$$\begin{aligned} W_1x &= W_2(L - x) \\ 50x &= 75(10 - x) \\ 50x &= 750 - 75x \\ 125x &= 750 \\ x &= 6 \text{ feet from 50-pound child} \end{aligned}$$

(b)
$$\begin{aligned} W_1x &= W_2(L - x) \\ W_1 &= 200 \text{ pounds} \\ W_2 &= 550 \text{ pounds} \\ L &= 5 \text{ feet} \\ 200x &= 550(5 - x) \\ 200x &= 2750 - 550x \\ 750x &= 2750 \\ x &= 3\tfrac{2}{3} \text{ feet from the person} \end{aligned}$$

72. (a) *Verbal Model:* $\dfrac{\text{Height of building}}{\text{Length of building's shadow}} = \dfrac{\text{Height of post}}{\text{Length of post's shadow}}$

(b) *Equation:* $\dfrac{x}{30} = \dfrac{4}{3}$

73. False, it should be written as $\dfrac{z^3 - 8}{z^2 - 9}$.

74. True. The expression $\dfrac{x^3}{(x - 4)^2}$ can be described as x cubed divided by the square of the difference of x and 4.

75. Area of circle: $A = \pi r^2 = \pi(2)^2 = 4\pi \approx 12.56 \text{ in.}^2$

Area of square: $A = s^2 = (4)^2 = 16 \text{ in.}^2$

True. $12.56 \text{ in.}^2 < 16 \text{ in.}^2$, so the area of the circle is less than the area of the square.

76. Cube: $V = s^3 = 9.5^3 = 857.375 \text{ in.}^3$

Sphere: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5.9)^3 \approx 860.290 \text{ in.}^3$

False. $857.375 < 860.290$, so the volume of the cube is not greater than the volume of the sphere.

Section 1.4 Quadratic Equations and Applications

1. quadratic equation

2. second-degree polynomial

3. factoring; square roots; completing; square; Quadratic Formula

4. discriminant

5. position equation

6. Pythagorean Theorem

7.
$$\begin{aligned} 6x^2 + 3x &= 0 \\ 3x(2x + 1) &= 0 \\ 3x = 0 \quad &\text{or} \quad 2x + 1 = 0 \\ x = 0 \quad &\text{or} \quad x = -\tfrac{1}{2} \end{aligned}$$

8.
$$\begin{aligned} 8x^2 - 2x &= 0 \\ 2x(4x - 1) &= 0 \\ 8x = 0 \quad &\text{or} \quad 4x - 1 = 0 \\ x = 0 \quad &\text{or} \quad x = \tfrac{1}{4} \end{aligned}$$

9.
$$\begin{aligned} 3 + 5x - 2x^2 &= 0 \\ (3 - x)(1 + 2x) &= 0 \\ 3 - x = 0 \quad &\text{or} \quad 1 + 2x = 0 \\ x = 3 \quad &\text{or} \quad x = -\tfrac{1}{2} \end{aligned}$$

10.
$$\begin{aligned} x^2 + 6x + 9 &= 0 \\ (x + 3)(x + 3) &= 0 \\ x + 3 &= 0 \\ x &= -3 \end{aligned}$$

11. $$\begin{aligned} x^2 + 10x + 25 &= 0 \\ (x + 5)(x + 5) &= 0 \\ x + 5 &= 0 \\ x &= -5 \end{aligned}$$

12. $$\begin{aligned} 4x^2 + 12x + 9 &= 0 \\ (2x + 3)(2x + 3) &= 0 \\ 2x + 3 = 0 \Rightarrow x &= -\tfrac{3}{2} \end{aligned}$$

13. $$\begin{aligned} 16x^2 - 9 &= 0 \\ (4x + 3)(4x + 3) &= 0 \\ 4x + 3 = 0 \Rightarrow x &= -\tfrac{3}{4} \\ 4x - 3 = 0 \Rightarrow x &= \tfrac{3}{4} \end{aligned}$$

14. $$\begin{aligned} x^2 - 2x - 8 &= 0 \\ (x - 4)(x + 2) &= 0 \\ x - 4 = 0 \quad \text{or} \quad x + 2 &= 0 \\ x = 4 \quad \text{or} \quad x &= -2 \end{aligned}$$

15. $$\begin{aligned} 2x^2 &= 19x + 33 \\ 2x^2 - 19x - 33 &= 0 \\ (2x + 3)(x - 11) &= 0 \\ 2x + 3 = 0 \Rightarrow x &= -\tfrac{3}{2} \\ x - 11 = 0 \Rightarrow x &= 11 \end{aligned}$$

16. $$\begin{aligned} -x^2 + 4x &= 3 \\ -x^2 + 4x - 3 &= 0 \\ (-1)(-x^2 + 4x - 3) &= (-1)(0) \\ x^2 - 4x + 3 &= 0 \\ (x - 3)(x - 1) &= 0 \\ x - 3 = 0 \Rightarrow x &= 3 \\ x - 1 = 0 \Rightarrow x &= 1 \end{aligned}$$

17. $$\begin{aligned} \tfrac{3}{4}x^2 + 8x + 20 &= 0 \\ 4\left(\tfrac{3}{4}x^2 + 8x + 20\right) &= 4(0) \\ 3x^2 + 32x + 80 &= 0 \\ (3x + 20)(x + 4) &= 0 \\ 3x + 20 = 0 \quad \text{or} \quad x + 4 &= 0 \\ x = -\tfrac{20}{3} \quad \text{or} \quad x &= -4 \end{aligned}$$

18. $$\begin{aligned} \tfrac{1}{8}x^2 - x - 16 &= 0 \\ x^2 - 8x - 128 &= 0 \\ (x - 16)(x + 8) &= 0 \\ x - 16 = 0 \Rightarrow x &= 16 \\ x + 8 = 0 \Rightarrow x &= -8 \end{aligned}$$

19. $$\begin{aligned} x^2 &= 49 \\ x &= \pm 7 \end{aligned}$$

20. $$\begin{aligned} x^2 &= 144 \\ x &= \pm 12 \end{aligned}$$

21. $$\begin{aligned} x^2 &= 19 \\ x &= \pm\sqrt{19} \\ x &\approx \pm 4.36 \end{aligned}$$

22. $$\begin{aligned} x^2 &= 43 \\ x &= \pm\sqrt{43} \\ x &\approx 6.56 \end{aligned}$$

23. $$\begin{aligned} 3x^2 &= 81 \\ x^2 &= 27 \\ x &= \pm 3\sqrt{3} \\ &\approx \pm 5.20 \end{aligned}$$

24. $$\begin{aligned} 9x^2 &= 36 \\ x^2 &= 4 \\ x &= \pm\sqrt{4} = \pm 2 \end{aligned}$$

25. $$\begin{aligned} (x - 4)^2 &= 49 \\ x - 4 &= \pm 7 \\ x &= 4 \pm 7 \\ x &= 11 \text{ or } x = -3 \end{aligned}$$

26. $$\begin{aligned} (x - 5)^2 &= 25 \\ x - 5 &= \pm 5 \\ x &= 5 \pm 5 \\ x &= 0 \text{ or } x = 10 \end{aligned}$$

27. $$\begin{aligned} (x + 2)^2 &= 14 \\ x + 2 &= \pm\sqrt{14} \\ x &= -2 \pm \sqrt{14} \\ &\approx 1.74,\ -5.74 \end{aligned}$$

28. $(x+9)^2 = 24$

$x + 9 = \pm\sqrt{24}$

$x = -9 \pm 2\sqrt{6}$

$\approx -4.10, \ -13.90$

29. $(2x-1)^2 = 18$

$2x - 1 = \pm\sqrt{18}$

$2x = 1 \pm 3\sqrt{2}$

$x = \dfrac{1 \pm 3\sqrt{2}}{2}$

$\approx 2.62, \ -1.62$

30. $(4x+7)^2 = 44$

$4x + 7 = \pm\sqrt{44}$

$4x = -7 \pm 2\sqrt{11}$

$x = \dfrac{-7 \pm 2\sqrt{11}}{4} = -\dfrac{7}{4} \pm \dfrac{\sqrt{11}}{2}$

$\approx -0.09, \ -3.41$

31. $(x-7)^2 = (x+3)^2$

$x - 7 = \pm(x+3)$

$x - 7 = x + 3$ or $x - 7 = -x - 3$

$-7 \neq 3$ or $2x = 4$

$x = 2$

The only solution of the equation is $x = 2$.

32. $(x+5)^2 = (x+4)^2$

$x + 5 = \pm(x+4)$

$x + 5 = +(x+4)$ or $x + 5 = -(x+4)$

$5 \neq 4$ or $x + 5 = -x - 4$

$2x = -9$

$x = -\frac{9}{2}$

The only solution of the equation is $x = -\frac{9}{2}$.

33. $x^2 + 4x - 32 = 0$

$x^2 + 4x = 32$

$x^2 + 4x + 2^2 = 32 + 2^2$

$(x+2)^2 = 36$

$x + 2 = \pm 6$

$x = -2 \pm 6$

$x = 4$ or $x = -8$

34. $x^2 - 2x - 3 = 0$

$x^2 - 2x = 3$

$x^2 - 2x + (-1)^2 = 3 + (1)^2$

$(x-1)^2 = 4$

$x - 1 = \pm\sqrt{4}$

$x = 1 \pm 2$

$x = 3$ or $x = -1$

35. $x^2 + 4x + 2 = 0$

$x^2 + 4x = -2$

$x^2 + 4x + 2^2 = -2 + 2^2$

$(x+2)^2 = 2$

$x + 2 = \pm\sqrt{2}$

$x = -2 \pm \sqrt{2}$

36. $x^2 + 8x + 14 = 0$

$x^2 + 8x = -14$

$x^2 + 8x + 4^2 = -14 + 16$

$(x+4)^2 = 2$

$x + 4 = \pm\sqrt{2}$

$x = -4 \pm \sqrt{2}$

37. $6x^2 - 12x = -3$

$x^2 - 2x = -\dfrac{1}{2}$

$x^2 - 2x + 1^2 = -\dfrac{1}{2} + 1^2$

$(x-1)^2 = \dfrac{1}{2}$

$x - 1 = \pm\sqrt{\dfrac{1}{2}}$

$x = 1 \pm \sqrt{\dfrac{1}{2}}$

$x = 1 \pm \dfrac{\sqrt{2}}{2}$

38.
$$\begin{aligned}4x^2 - 4x &= 1\\ x^2 - x &= \frac{1}{4}\\ x^2 - x + \left(-\frac{1}{2}\right)^2 &= \frac{1}{4} + \left(-\frac{1}{2}\right)^2\\ \left(x - \frac{1}{2}\right)^2 &= \frac{1}{2}\\ x - \frac{1}{2} &= \pm\frac{\sqrt{2}}{2}\\ x &= \frac{1}{2} \pm \frac{\sqrt{2}}{2}\\ x &= \frac{1 \pm \sqrt{2}}{2}\end{aligned}$$

39.
$$\begin{aligned}7 + 2x - x^2 &= 0\\ -x^2 + 2x + 7 &= 0\\ x^2 - 2x - 7 &= 0\\ x^2 - 2x &= 7\\ x^2 - 2x + (-1)^2 &= 7 + (-1)^2\\ (x - 1)^2 &= 8\\ x - 1 &= \pm 2\sqrt{2}\\ x &= 1 \pm 2\sqrt{2}\end{aligned}$$

40.
$$\begin{aligned}-x^2 + x - 1 &= 0\\ x^2 - x + 1 &= 0\\ x^2 - x + \tfrac{1}{4} &= -1 + \tfrac{1}{4}\\ \left(x - \tfrac{1}{2}\right)^2 &= -\tfrac{3}{4}\end{aligned}$$

No real solution

41.
$$\begin{aligned}2x^2 + 5x - 8 &= 0\\ 2x^2 + 5x &= 8\\ x^2 + \frac{5}{2}x &= 4\\ x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 &= 4 + \left(\frac{5}{4}\right)^2\\ \left(x + \frac{5}{4}\right)^2 &= \frac{89}{16}\\ x + \frac{5}{4} &= \pm\frac{\sqrt{89}}{4}\\ x &= -\frac{5}{4} \pm \frac{\sqrt{89}}{4}\\ x &= \frac{-5 \pm \sqrt{89}}{4}\end{aligned}$$

42.
$$\begin{aligned}3x^2 - 4x - 7 &= 0\\ 3x^2 - 4x &= 7\\ x^2 - \frac{4}{3}x &= \frac{7}{3}\\ x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 &= \frac{7}{3} + \left(-\frac{2}{3}\right)^2\\ \left(x - \frac{2}{3}\right)^2 &= \frac{25}{9}\\ x - \frac{2}{3} &= \pm\frac{5}{3}\\ x &= \frac{2}{3} \pm \frac{5}{3}\\ x &= -1 \text{ or } x = \frac{7}{3}\end{aligned}$$

43.
$$\begin{aligned}\frac{1}{x^2 - 2x + 5} &= \frac{1}{x^2 - 2x + 1^2 + 5}\\ &= \frac{1}{(x - 1)^2 + 4}\end{aligned}$$

44.
$$\begin{aligned}\frac{1}{x^2 + 6x + 10} &= \frac{1}{x^2 + 6x + (3)^2 - (3)^2 + 10}\\ &= \frac{1}{x^2 + 6x + 9 + 1}\\ &= \frac{1}{(x + 3)^2 + 1}\end{aligned}$$

45.
$$\begin{aligned}\frac{4}{x^2 + 10x + 74} &= \frac{4}{x^2 + 10x + (5)^2 - (5)^2 + 74}\\ &= \frac{4}{x^2 + 10x + 25 + 49}\\ &= \frac{4}{(x + 5)^2 + 49}\end{aligned}$$

46.
$$\begin{aligned}\frac{5}{x^2 - 18x + 162} &= \frac{5}{x^2 - 18x + (9)^2 - (9)^2 + 162}\\ &= \frac{5}{x^2 - 18x + 81 + 81}\\ &= \frac{5}{(x - 9)^2 + 81}\end{aligned}$$

47. $\dfrac{1}{\sqrt{3 + 2x - x^2}} = \dfrac{1}{\sqrt{-1(x^2 - 2x - 3)}}$

$$= \frac{1}{\sqrt{-1\left[x^2 - 2x + (1)^2 - (1)^2 - 3\right]}}$$

$$= \frac{1}{\sqrt{-1(x^2 - 2x + 1) + 4}}$$

$$= \frac{1}{\sqrt{4 - (x - 1)^2}}$$

48. $\dfrac{1}{\sqrt{9 + 8x - x^2}} = \dfrac{1}{\sqrt{-1(x^2 - 8x - 9)}}$

$$= \frac{1}{\sqrt{-1\left[x^2 - 8x + (4)^2 - (4)^2 - 9\right]}}$$

$$= \frac{1}{\sqrt{-1\left[(x^2 - 8x + 16) - 25\right]}}$$

$$= \frac{1}{\sqrt{25 - (x - 4)^2}}$$

49. $\dfrac{1}{\sqrt{12 + 4x - x^2}} = \dfrac{1}{\sqrt{-1(x^2 - 4x - 12)}} = \dfrac{1}{\sqrt{-1\left[x^2 - 4x + (2)^2 - (2)^2 - 12\right]}}$

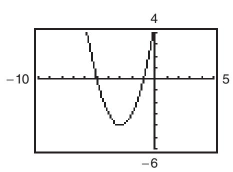
$$= \frac{1}{\sqrt{-1\left[(x^2 - 4x + 4) - 16\right]}} = \frac{1}{\sqrt{16 - (x - 2)^2}}$$

50. $\dfrac{1}{\sqrt{16 - 6x - x^2}} = \dfrac{1}{\sqrt{16 - 1(x^2 + 6x)}}$

$$= \frac{1}{\sqrt{16 - (x^2 + 6x + 3^2) + 9}}$$

$$= \frac{1}{\sqrt{25 - (x + 3)^2}}$$

51. (a) $y = (x + 3)^2 - 4$

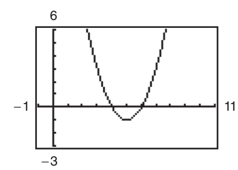


(b) The x-intercepts are $(-1, 0)$ and $(-5, 0)$.

(c)
$$\begin{aligned} 0 &= (x + 3)^2 - 4 \\ 4 &= (x + 3)^2 \\ \pm\sqrt{4} &= x + 3 \\ -3 \pm 2 &= x \\ x &= -1 \quad \text{or} \quad x = -5 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = (x + 3)^2 - 4$.

52. (a) $y = (x - 5)^2 - 1$

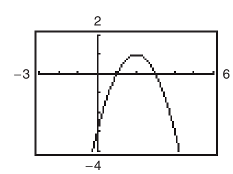


(b) The x-intercepts are $(4, 0)$ and $(6, 0)$.

(c)
$$\begin{aligned} 0 &= (x - 5)^2 - 1 \\ (x - 5)^2 &= 1 \\ x - 5 &= \pm\sqrt{1} \\ x &= 5 \pm 1 = 6, 4 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = (x - 5)^2 - 1$.

53. (a) $y = 1 - (x - 2)^2$

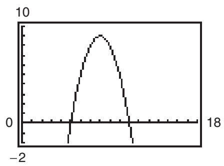


(b) The x-intercepts are $(1, 0)$ and $(3, 0)$.

(c)
$$\begin{aligned} 0 &= 1 - (x - 2)^2 \\ (x - 2)^2 &= 1 \\ x - 2 &= \pm 1 \\ x &= 2 \pm 1 \\ x &= 3 \quad \text{or} \quad x = 1 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 1 - (x - 2)^2$.

54. (a) $y = 9 - (x - 8)^2$

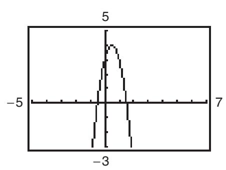


(b) The x-intercepts are $(5, 0)$ and $(11, 0)$.

(c)
$$0 = 9 - (x - 8)^2$$
$$(x - 8)^2 = 9$$
$$x - 8 = \pm\sqrt{9}$$
$$x = 8 \pm 3 = 11, 5$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 9 - (x - 8)^2$.

55. (a) $y = -4x^2 + 4x + 3$

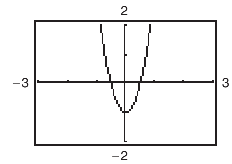


(b) The x-intercepts are $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{3}{2}, 0\right)$.

(c)
$$0 = -4x^2 + 4x + 3$$
$$4x^2 - 4x = 3$$
$$4(x^2 - x) = 3$$
$$x^2 - x = \frac{3}{4}$$
$$x^2 - x + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \left(\frac{1}{2}\right)^2$$
$$\left(x - \frac{1}{2}\right)^2 = 1$$
$$x - \frac{1}{2} = \pm\sqrt{1}$$
$$x = \frac{1}{2} \pm 1$$
$$x = \frac{3}{2} \quad \text{or} \quad x = -\frac{1}{2}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = -4x^2 + 4x + 3$.

56. (a) $y = 4x^2 - 1$

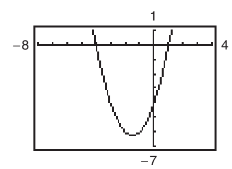


(b) The x-intercepts are $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{2}, 0\right)$.

(c)
$$0 = 4x^2 - 1$$
$$4x^2 = 1$$
$$x^2 = \frac{1}{4}$$
$$x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 4x^2 - 1$.

57. (a) $y = x^2 + 3x - 4$

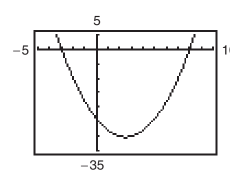


(b) The x-intercepts are $(-4, 0)$ and $(1, 0)$.

(c)
$$0 = x^2 + 3x - 4$$
$$0 = (x + 4)(x - 1)$$
$$x + 4 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -4 \quad \text{or} \quad x = 1$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = x^2 + 3x - 4$.

58. (a) $y = x^2 - 5x - 24$



(b) The x-intercepts are $(8, 0)$ and $(-3, 0)$.

(c)
$$0 = x^2 - 5x - 24$$
$$(x - 8)(x + 3) = 0$$
$$x - 8 = 0 \Rightarrow x = 8$$
$$x + 3 = 0 \Rightarrow x = -3$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = x^2 - 5x - 24$.

59. $9x^2 + 12x + 4 = 0$

$b^2 - 4ac = (12)^2 - 4(9)(4) = 0$

One repeated real solution

60. $x^2 + 3x + 4 = 0$

$b^2 - 4ac = (2)^2 - 4(1)(4) = 4 - 16 = -12 < 0$

No real solution

61. $2x^2 - 5x + 5 = 0$

$b^2 - 4ac = (-5)^2 - 4(2)(5) = -15 < 0$

No real solution

62. $-5x^2 - 4x + 1 = 0$

$b^2 - 4ac = (-4)^2 - 4(-5)(1) = 16 + 20 = 36 > 0$

Two real solutions

63. $2x^2 - z - 1 = 0$

$b^2 - 4ac = (-1)^2 - 4(2)(-1) = 9 > 0$

Two real solutions

64. $x^2 - 4x + 4 = 0$

$b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$

One repeated solution

65. $\frac{1}{3}x^2 - 5x + 25 = 0$

$b^2 - 4ac = (-5)^2 - 4\left(\frac{1}{3}\right)(25) = -\frac{25}{3} < 0$

No real solution

66. $\frac{4}{7}x^2 - 8x + 280$

$b^2 - 4ac = (-8)^2 - 4\left(\frac{4}{7}\right)(28) = 64 - 64 = 0$

One repeated solution

67. $0.2x^2 + 1.2x - 8 = 0$

$b^2 - 4ac = (1.2)^2 - 4(0.2)(-8) = 7.84 > 0$

Two real solutions

68. $9 + 2.4x - 8.3x^2 = 0$

$$\begin{aligned} b^2 - 4ac &= (2.4)^2 - 4(-8.3)(9) \\ &= 5.76 + 298.8 = 304.56 > 0 \end{aligned}$$

Two real solutions

69. $2x^2 + x - 1 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{-1 \pm 3}{4} = \frac{1}{2}, -1 \end{aligned}$$

70. $2x^2 - x - 1 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{1 \pm \sqrt{1 + 8}}{4} \\ &= \frac{1 \pm 3}{4} = 1, -\frac{1}{2} \end{aligned}$$

71. $16x^2 + 8x - 3 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{8^2 - 4(16)(-3)}}{2(16)} \\ &= \frac{-8 \pm 16}{32} = \frac{1}{4}, -\frac{3}{4} \end{aligned}$$

72. $25x^2 - 20x + 3 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-20) \pm \sqrt{(-20)^2 - (25)(3)}}{2(25)} \\ &= \frac{20 \pm \sqrt{400 - 300}}{50} \\ &= \frac{20 \pm 10}{50} = \frac{3}{5}, \frac{1}{5} \end{aligned}$$

73. $x^2 + 8x - 4 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)} \\ &= \frac{-8 \pm 4\sqrt{5}}{2} = -4 \pm 2\sqrt{5} \end{aligned}$$

74. $9x^2 + 30x + 25 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-30 \pm \sqrt{30^2 - 4(9)(25)}}{2(9)}$$
$$= \frac{-30 \pm 0}{18} = -\frac{5}{3}$$

75. $2x^2 - 7x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)}$$
$$= \frac{7 \pm \sqrt{49 - 8}}{2(2)}$$
$$= \frac{7 \pm \sqrt{41}}{4}$$
$$= \frac{7}{4} \pm \frac{\sqrt{41}}{4}$$

76. $36x^2 + 24x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-24 \pm \sqrt{24^2 - 4(36)(-7)}}{2(36)}$$
$$= \frac{-24 \pm \sqrt{576 + 1008}}{72}$$
$$= \frac{-24 \pm \sqrt{(144)(11)}}{72}$$
$$= -\frac{1}{3} \pm \frac{\sqrt{11}}{6}$$

77. $2 + 2x - x^2 = 0$

$$-x^2 + 2x + 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)}$$
$$= \frac{-2 \pm 2\sqrt{3}}{-2}$$
$$= 1 \pm \sqrt{3}$$

78. $x + 10 + 8x = 0$

$$x + 8x + 10 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(10)}}{2(1)}$$
$$= \frac{-8 \pm \sqrt{64 - 40}}{2(1)}$$
$$= \frac{-8 \pm \sqrt{24}}{2}$$
$$= \frac{-8 \pm 2\sqrt{6}}{2}$$
$$= \frac{2\left(-4 \pm \sqrt{6}\right)}{2}$$
$$= -4 \pm \sqrt{6}$$

79. $x^2 + 16 = -12x$

$$x^2 + 12x + 16 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(12) \pm \sqrt{(12)^2 - 4(1)(16)}}{2(1)}$$
$$= \frac{-12 \pm \sqrt{144 - 64}}{2(1)}$$
$$= \frac{-12 \pm \sqrt{80}}{2}$$
$$= \frac{-12 \pm 4\sqrt{5}}{2}$$
$$= \frac{4\left(-3 \pm \sqrt{5}\right)}{2}$$
$$= 2\left(-3 \pm \sqrt{5}\right) = -6 \pm 2\sqrt{5}$$

80. $4x = 8 - x^2$

$$x^2 + 4x - 8 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)}$$
$$= \frac{-4 \pm 4\sqrt{3}}{2} = -2 \pm 2\sqrt{3}$$

81. $4x^2 + 6x = 8$

$$4x^2 + 6x - 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(6) \pm \sqrt{(6)^2 - 4(4)(-8)}}{2(4)}$$

$$= \frac{-6 \pm \sqrt{36 + 128}}{2(4)}$$

$$= \frac{-6 \pm \sqrt{164}}{8}$$

$$= \frac{-6 \pm 2\sqrt{41}}{8}$$

$$= \frac{2\left(-3 \pm \sqrt{41}\right)}{8}$$

$$= \frac{-3 \pm \sqrt{41}}{4}$$

$$= \frac{-3}{4} \pm \frac{\sqrt{41}}{4}$$

82. $16x^2 + 5 = 40x$

$$16x^2 - 40x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-40) \pm \sqrt{(-40)^2 - 4(16)(5)}}{2(16)}$$

$$= \frac{40 \pm \sqrt{1600 - 320}}{32}$$

$$= \frac{40 \pm \sqrt{1280}}{32}$$

$$= \frac{40 \pm 16\sqrt{5}}{32}$$

$$= \frac{5}{4} \pm \frac{\sqrt{5}}{2}$$

83. $28x - 49x^2 = 4$

$$-49x^2 + 28x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-28 \pm \sqrt{28^2 - 4(-49)(-4)}}{2(-49)}$$

$$= \frac{-28 \pm 0}{-98} = \frac{2}{7}$$

84. $3x + x^2 - 1 = 0$

$$x^2 + 3x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{13}}{2} = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

85. $8t = 5 + 2t^2$

$$-2t^2 + 8t - 5 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4(-2)(-5)}}{2(-2)}$$

$$= \frac{-8 \pm 2\sqrt{6}}{-4} = 2 \pm \frac{\sqrt{6}}{2}$$

86. $25h^2 + 80h + 61 = 0$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)}$$

$$= \frac{-80 \pm \sqrt{6400 - 6100}}{50}$$

$$= -\frac{8}{5} \pm \frac{10\sqrt{3}}{50}$$

$$= -\frac{8}{5} \pm \frac{\sqrt{3}}{5}$$

87. $(y - 5)^2 = 2y$

$$y^2 - 12y + 25 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{12 \pm 2\sqrt{11}}{2} = 6 \pm \sqrt{11}$$

88. $(z+6)^2 = -2z$

$$z^2 + 12z + 36 = -2z$$
$$z^2 + 14z + 36 = 0$$
$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-14 \pm \sqrt{14^2 - 4(1)(36)}}{2(1)}$$
$$= \frac{-14 \pm \sqrt{52}}{2} = -7 \pm \sqrt{13}$$

89. $\frac{1}{2}x^2 + \frac{3}{8}x = 2$

$$4x^2 + 3x = 16$$
$$4x^2 + 3x - 16 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(4)(-16)}}{2(4)}$$
$$= \frac{-3 \pm \sqrt{265}}{8} = -\frac{3}{8} \pm \frac{\sqrt{265}}{8}$$

90. $\left(\frac{5}{7}x - 14\right)^2 = 8x$

$$\frac{25}{49}x^2 - 20x + 196 = 8x$$
$$\frac{25}{49}x^2 - 28x + 196 = 0$$
$$25x^2 - 1372x + 9604 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1372) \pm \sqrt{(-1372)^2 - 4(25)(9604)}}{2(25)}$$
$$= \frac{1372 \pm \sqrt{931{,}984}}{50}$$
$$= \frac{686 \pm 196\sqrt{6}}{25}$$

91. $5.1x^2 - 1.7x - 3.2 = 0$

$$x = \frac{1.7 \pm \sqrt{(-1.7)^2 - 4(5.1)(-3.2)}}{2(5.1)}$$
$$\approx 0.976, -0.643$$

92. $2x^2 - 2.50x - 0.42 = 0$

$$x = \frac{-(-2.50) \pm \sqrt{(-2.50)^2 - 4(2)(-0.42)}}{2(2)}$$
$$= \frac{2.50 \pm \sqrt{9.61}}{4} = 1.400, -0.150$$

93. $-0.67x^2 + 0.5x + 1.375 = 0$

$$x = \frac{0.5 \pm \sqrt{(0.5)^2 - 4(-0.67)(1.375)}}{2(5.1)}$$
$$\approx -1.107, 1.853$$

94. $-0.005x^2 + 0.101x - 0.193 = 0$

$$x = \frac{-0.101 \pm \sqrt{(0.101)^2 - 4(-0.005)(-0.193)}}{2(-0.005)}$$
$$= \frac{-0.101 \pm \sqrt{0.006341}}{-0.01}$$
$$\approx 2.137, 18.063$$

95. $12.67x^2 + 31.55x + 8.09 = 0$

$$x = \frac{-31.55 \pm \sqrt{(31.55)^2 - 4(12.67)(8.09)}}{2(12.67)}$$
$$\approx -2.200, -0.290$$

96. $-3.22x^2 - 0.08x + 28.651 = 0$

$$x = \frac{-(-0.08) \pm \sqrt{(-0.08)^2 - 4(-3.22)(28.651)}}{2(-3.22)}$$
$$= \frac{0.08 \pm \sqrt{369.031}}{-6.44} \approx -2.995, 2.971$$

97. $x^2 - 2x - 1 = 0$ Complete the square.

$$x^2 - 2x = 1$$
$$x^2 - 2x + 1^2 = 1 + 1^2$$
$$(x-1)^2 = 2$$
$$x - 1 = \pm\sqrt{2}$$
$$x = 1 \pm \sqrt{2}$$

98. $14x^2 + 42x = 0$ Factor.

$$14x(x + 3x) = 0$$
$$x(x+3) = 0$$
$$x = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = -3$$

99. $(x + 2)^2 = 64$ Extract square roots.

$$x + 2 = \pm 8$$

$$x + 2 = 8 \quad \text{or} \quad x + 2 = -8$$

$$x = 6 \quad \text{or} \quad x = -10$$

100. $x^2 - 14x + 49 = 0$ Extract square roots.

$$(x - 7)^2 = 0$$

$$x - 7 = 0$$

$$x = 7$$

101. $x^2 - x - \frac{11}{4} = 0$ Complete the square.

$$x^2 - x = \tfrac{11}{4}$$

$$x^2 - x + \left(\tfrac{1}{2}\right)^2 = \tfrac{11}{4} + \left(\tfrac{1}{2}\right)^2$$

$$\left(x - \tfrac{1}{2}\right)^2 = \tfrac{12}{4}$$

$$x - \tfrac{1}{2} = \pm\sqrt{\tfrac{12}{4}}$$

$$x = \tfrac{1}{2} \pm \sqrt{3}$$

102. $x^2 + 3x - \frac{3}{4} = 0$ Complete the square.

$$x^2 + 3x + \left(\tfrac{3}{2}\right)^2 = \tfrac{3}{4} + \tfrac{9}{4}$$

$$\left(x + \tfrac{3}{2}\right)^2 = 3$$

$$x + \tfrac{3}{2} = \pm\sqrt{3}$$

$$x = -\tfrac{3}{2} \pm \sqrt{3}$$

103. $3x + 4 = 2x^2 - 7$ Quadratic Formula

$$0 = 2x^2 - 3x - 11$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{97}}{4}$$

$$= \frac{3}{4} \pm \frac{\sqrt{97}}{4}$$

104. $(x + 1)^2 = x^2$ Extract square roots.

$$x^2 = (x + 1)^2$$

$$x = \pm(x + 1)$$

For $x = +(x + 1)$:

$0 \neq 1$ No solution

For $x = -(x + 1)$:

$$2x = -1$$

$$x = -\tfrac{1}{2}$$

105. (a) $w(w + 14) = 1632$

(b) $w^2 + 14w - 1632 = 0$

$$(w + 48)(w - 34) = 0$$

$$w = -48 \quad \text{or} \quad w = 34$$

Because the width must be greater than zero, $w = 34$ feet and the length is $w + 14 = 48$ feet.

106. Total fencing: $4x + 3y = 100$

Total area: $2xy = 350$

$$x = \frac{100 - 3y}{4}$$

$$2\left(\frac{100 - 3y}{4}\right)y = 350$$

$$\frac{1}{2}\left(100y - 3y^2\right) - 350 = 0$$

$$100y - 3y^2 - 700 = 0$$

$$-3y^2 + 100y - 700 = 0$$

$$(3y - 70)(-y + 10) = 0$$

$$3y - 70 = 0 \Rightarrow y = \frac{70}{3}$$

$$-y + 10 = 0 \Rightarrow y = 10$$

For $y = \dfrac{70}{3}$:

$$2x\left(\frac{70}{3}\right) = 350$$

$$x = 7.5$$

For $y = 10$:

$$2x(10) = 350$$

$$x = 17.5$$

There are two solutions: $x = 7.5$ meters and $y = 23\frac{1}{3}$ meters or $x = 17.5$ meters and $y = 10$ meters.

107. $S = x^2 + 4xh$

$$108 = x^2 + 4x(3)$$

$$0 = x^2 + 12x - 108$$

$$0 = (x + 18)(x - 6)$$

$$x = -18 \quad \text{or} \quad x = 6$$

Because x must be positive, $x = 6$ inches. The dimensions of the box are 6 inches $\times$ 6 inches $\times$ 3 inches.

108. Volume: $4x^2 = 576$

$$x^2 = 144$$

$$x = \pm 12$$

Because x must be positive, $x = 12$ centimeters and the side length of the original material is $x + 8 = 20$ centimeters. The dimensions of the original material are 20 centimeters $\times$ 20 centimeters.

109. (a) Volume is 1024 cubic feet.

$$V = l \cdot w \cdot h = x(x+1)(4) = 4x^2 + 4x$$

So, $4x^2 + 4x = 1024$

$$4x^2 + 4x - 1024 = 0$$
$$4(x^2 + x - 256) = 0$$
$$x^2 + x - 256 = 0$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4(1) - 256}}{2(1)}$$
$$x \approx 15.51$$
$$x + 1 \approx 16.51$$

So the base of the pool is approximately 15.51 feet × 16.51 feet.

(b) Because 1 cubic foot of water weighs approximately 62.4 pounds,

$$1024 \text{ cubic feet} \cdot \frac{62.4 \text{ pounds}}{1 \text{ cubic foot}} = 63{,}897.6 \text{ pounds.}$$

110. Original arrangement: x rows, y seats per row,

$$xy = 72, y = \frac{72}{x}$$

New arrangement: $(x - 2)$ rows, $(y + 3)$ seats per row

$$(x-2)(y+3) = 72$$
$$(x-2)\left(\frac{72}{x} + 3\right) = 72$$
$$x(x-2)\left(\frac{72}{x} + 3\right) = 72x$$
$$(x-2)(72+3x) = 72x$$
$$72x + 3x^2 - 144 - 6x = 72x$$
$$3x^2 - 6x - 144 = 0$$
$$x^2 - 2x - 48 = 0$$
$$(x-8)(x+6) = 0$$

$x - 8 = 0 \Rightarrow x = 8$

$x + 6 = 0 \Rightarrow x = -6$

Originally, there were 8 rows of seats with $\frac{72}{9} = 9$ seats per row.

111. (a) $s = -16t^2 + v_0t + s_0$

Since the object was dropped, $v_0 = 0$, and the initial height is $s_0 = 984$. Thus, $s = -16t^2 + 984$.

(b) $s = -16(4)^2 + 984 = 728$ feet

(c)
$$0 = -16t^2 + 984$$
$$16t^2 = 984$$
$$t^2 = \frac{984}{16}$$
$$t = \sqrt{\frac{984}{16}} = \frac{\sqrt{246}}{2} \approx 7.84$$

It will take the coin about 7.84 seconds to strike the ground.

112. (a) $s = -16t^2 + v_0t + s_0$

Since the object was dropped, $v_0 = 0$, and the initial height is $s_0 = 1815$. Thus, $s = -16t^2 + 1815$.

(b)

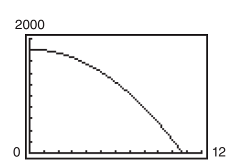
Time, t	0	2	4	6	8	10	12
Height, s	1815	1751	1559	1239	791	215	−489

(c) The object reaches the ground between $t = 10$ seconds and $t = 12$ seconds. Numerical approximation will vary, though 10.7 seconds is a reasonable estimate.

(d)
$$0 = -16t^2 + 1815$$
$$16t^2 = 1815$$
$$t^2 = \frac{1815}{16}$$
$$t = \sqrt{\frac{1815}{16}} = \frac{11\sqrt{15}}{4} \approx 10.65$$

It will take the object about 10.65 seconds to reach the ground.

(e)



The zero of the graph is at $t \approx 10.65$ seconds.

113. (a) $s = -16t^2 + v_0t + s_0$

$s = -16t^2 + 550$

Let $s = 0$ and solve for t.

$$0 = -16t^2 + 550$$

$$16t^2 = 550$$

$$t^2 = \frac{550}{16}$$

$$t = \sqrt{\frac{550}{16}}$$

$$t \approx 5.86$$

The supply package will take about 5.86 seconds to reach the ground.

(b) *Verbal Model:* $(\text{Distance}) = (\text{Rate}) \cdot (\text{Time})$

Labels: Distance $= d$

Rate = 138 miles per hour

Time $= \dfrac{5.86 \text{ seconds}}{3600 \text{ seconds per hour}} \approx 0.0016$ hour

Equation: $d = (138)(0.0016) \approx 0.22$ mile

The supply package will travel about 0.2 mile.

114. (a) $s = -16t^2 + v_0t + s_0$

$$v_0 = 100 \text{ mph} = \frac{(100)(5280)}{3600} = 146\frac{2}{3} \text{ ft/sec}$$

$$s_0 = 6 \text{ feet } 3 \text{ inches} = 6\frac{1}{4} \text{ feet}$$

$$s = -16t^2 + 146\frac{2}{3}t + 6\frac{1}{4}$$

$$s = -16t^2 + 146\frac{2}{3}t + 6.25$$

(b) When $t = 4$: $s(4) \approx 336.92$ feet

When $t = 5$: $s(5) \approx 339.58$ feet

When $t = 6$: $s(6) \approx 310.25$ feet

During the interval $4 \le t \le 6$, the baseball reached its maximum height.

(c) The ball hits the ground when $s = 0$.

$$-16t^2 + 146\tfrac{2}{3}t + 6\tfrac{1}{4} = 0$$

Using the Quadratic Formula, $t = \dfrac{-146\frac{2}{3} \pm \sqrt{\left(146\frac{2}{3}\right)^2 - 4(-16)\left(6\frac{1}{4}\right)}}{2(-16)} \approx \dfrac{-146\frac{2}{3} \pm 148.02}{-32}$,

$t \approx -0.042$ or $t \approx 9.209$. Time is always positive, so the ball will be in the air for approximately 9.209 seconds.

115. (a)

t	8	9	10	11	12	13	14
D	8.98	10.71	12.30	13.75	15.06	16.22	17.24

Sometime during the year 2010, the total public debt reached \$13 trillion.

(b) *Algebraically:*

$$D = -0.071t^2 + 2.94t - 10.0$$
$$13 = -0.071t^2 + 2.94t - 10.0$$
$$0 = -0.071t^2 + 2.94t - 23.0$$

Let $a = -0.071$, $b = 2.94$, and $c = 23.0$.

Using the Quadratic Formula,

$$t = \frac{-(2.94) \pm \sqrt{(2.94)^2 - 4 -(0.071)(23.0)}}{2(0.157)}$$

$$= \frac{-2.94 \pm \sqrt{2.1116}}{-0.142}$$

$t \approx 10.47$ and $t \approx 30.94$

Because the domain of the model is $8 \leq t \leq 14$, $t \approx 10.47$ is the only solution. So, the total public debt reached \$13 trillion during 2010.

Graphically: Use a graphing utility to graph $y_1 = -0.071t^2 - 2.94t + 10.0$ and $y_2 = 13$ in the same viewing window. Then use the intersect feature to find that the graphs intersect when $t \approx 10.47$ and $t \approx 30.94$. Choose $t \approx 10.47$ because it is in the domain.

So, the total public debt reached \$13 trillion during 2010.

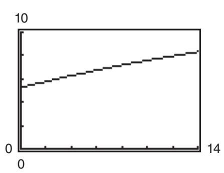
(c) For 2025, let $t = 25$

$$D = -0.071t^2 + 2.94t - 10.0 = -0.071(25)^2 + 2.94(25) - 10.0 = \$19.125 \text{ trillion}$$

Using the model, in 2025, the total public debt will be \$19.125 trillion.

Answers will vary. *Sample answer*: Yes. In the short time period beyond the interval, $8 \leq t \leq 14$, the public can be modeled by the equation.

116. (a)



During 2007, the average ticket price reached \$7.00.

(b)

$$P = -0.0038t^2 + 0.272t + 5.25$$
$$7.00 = -0.0038t^2 + 0.272t + 5.25$$
$$0 = -0.0038t^2 + 0.272t - 1.75$$

Using the Quadratic Formula,

$$t = \frac{-0.272 \pm \sqrt{(0.272)^2 - 4(-0.0038)(-1.75)}}{2(0.0133)} = \frac{-0.272 \pm \sqrt{0.047384}}{2(0.0133)}$$

$t \approx 7.15$ and $t \approx 64.43$

Because the domain of the model is $1 \leq t \leq 14$, $t \approx 5.50$ is the only solution. So, the average ticket price reached \$7.00 during 2007.

(c) For 2025, let $t = 25$.

$$P = -0.0038t^2 + 0.272t + 5.25$$

$$P = -0.0038(25)^2 + 0.272(25) + 5.25 \approx \$9.68$$

Using the model, the average ticket price in the year 2025 will be about \$9.68.

Answers will vary. *Sample answer*: No, it may be more likely the average ticket price will be higher than that predicted using the model.

117. $L = -0.270t^2 + 3.59t + 83.1$

$93 = -0.270t^2 + 3.59t + 83.1$

$0 = -0.270t^2 + 3.59t - 9.9$

$0 = 0.270t^2 - 3.59t + 9.9$

Using the Quadratic Formula,

$$t = \frac{-(-3.59) \pm \sqrt{(-3.59)^2 - 4(0.270)(9.9)}}{2(0.270)} = \frac{3.59 \pm \sqrt{2.1961}}{0.54}$$

$t \approx 3.9$ and $t \approx 9.4$

Because the domain of the model is $2 \le t \le 7$, $t \approx 3.9$ is the only solution. The patient's blood oxygen level was 93% at approximately 4:00 P.M.

118. (a) $150 = 0.45x^2 - 1.65x + 50.75\angle$

$0 = 0.45x^2 - 1.65x - 99.25$

$$x = \frac{1.65 \pm \sqrt{(-1.65)^2 - 4(0.45)(-99.25)}}{2(0.45)}$$

$\approx -13.1, 16.8$

Because $10 \le x \le 25$, choose $16.8°$C.

$x = 10$: $0.45(10)^2 - 1.65(10) + 50.75 = 79.25$

(b) $x = 20$: $0.45(20)^2 - 1.65(20) + 50.75 = 197.75$

$197.75 \div 79.25 \approx 2.5$

Oxygen consumption is increased by a factor of approximately 2.5.

119. (a) *Model:* $(\text{winch})^2 + (\text{distance to dock})^2 = (\text{length of rope})^2$

Labels: winch $= 15$, distance to dock $= x$, length of rope $= l$

Equation: $15^2 + x^2 = l^2$

(b) When $l = 75$: $15^2 + x^2 = 75^2$

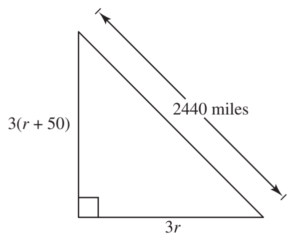
$$x^2 = 5625 - 225 = 5400$$

$$x = \sqrt{5400} = 30\sqrt{6} \approx 73.5$$

The boat is approximately 73.5 feet from the dock when there is 75 feet of rope out.

120.

$$d_N = (3 \text{ hours})(r + 50 \text{ mph})$$
$$d_E = (3 \text{ hours})(r \text{ mph})$$
$$d_N^2 + d_E^2 = 2440^2$$
$$9(r + 50)^2 + 9r^2 = 2440^2$$
$$18r^2 + 900r - 5{,}931{,}100 = 0$$
$$r = \frac{-900 \pm \sqrt{900^2 - 4(18)(-5{,}931{,}100)}}{2(18)} = \frac{-900 \pm \sqrt{118{,}847}}{36}$$



Using the positive value for r, we have one plane moving northbound at $r + 50 \approx 600$ miles per hour and one plane moving eastbound at $r \approx 550$ miles per hour.

121. $-3x^2 + x = -5$

$-3x^2 + x + 5 = 0$

$b^2 - 4ac = (1)^2 - 4(-3)(5) = 1 + 60 = 61 > 0.$

True. The quadratic equation has two real solutions.

122. False. The product must equal zero for the Zero Factor Property to be used.

123. *Sample answer:* $(x - 0)(x - 4) = 0$

$x(x - 5) = 0$

$x^2 - 4x = 0$

124. *Sample answer:* $(x - (-2))(x - (-8)) = 0$

$(x + 2)(x + 8) = 0$

$x^2 + 10x + 16 = 0$

125. One possible equation is:

$(x - 8)(x - 14) = 0$

$x^2 - 22x + 112 = 0$

Any non-zero multiple of this equation would also have these solutions.

126. $x = \frac{1}{6} \Rightarrow 6x = 1 \Rightarrow 6x - 1$ is a factor.

$x = -\frac{2}{5} \Rightarrow 5x = -2 \Rightarrow 5x + 2$ is a factor.

$(6x - 1)(5x + 2) = 0$

$30x^2 + 7x - 2 = 0$

127. One possible equation is:

$$\left[x - \left(1 + \sqrt{2}\right)\right]\left[x - \left(1 - \sqrt{2}\right)\right] = 0$$
$$\left[(x - 1) - \sqrt{2}\right]\left[(x - 1) + \sqrt{2}\right] = 0$$
$$(x - 1)^2 - \left(\sqrt{2}\right)^2 = 0$$
$$x^2 - 2x + 1 - 2 = 0$$
$$x^2 - 2x - 1 = 0$$

Any non-zero multiple of this equation would also have these solutions.

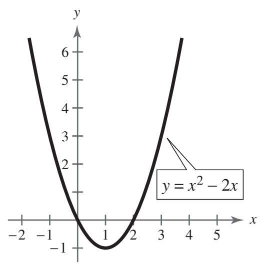
128. $x = -3 + \sqrt{5}, x = -3 - \sqrt{5}$, so:

$$\left(x - \left(-3 + \sqrt{5}\right)\right)\left(x - \left(-3 - \sqrt{5}\right)\right) = 0$$
$$\left(x + 3 - \sqrt{5}\right)\left(x + 3 + \sqrt{5}\right) = 0$$
$$x^2 + 6x + 4 = 0$$

129. Yes, the vertex of the parabola would be on the x-axis.

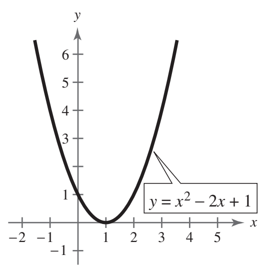
130. (a) The discriminant is positive because the graph has two x-intercepts. $y = x^2 - 2x$

$$b^2 = 4ac = (-2)^2 - 4(1)(0) = 4$$



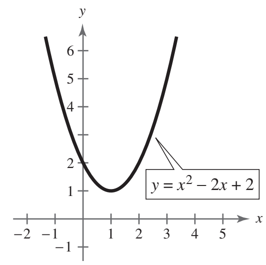
(b) The discriminant is zero because the graph has one x-intercept. $y = x^2 - 2x + 1$

$$b^2 = 4ac = (-2)^2 - 4(1)(1) = 0$$



(c) The discriminant is negative because the graph has no x-intercepts. $y = x^2 - 2x + 2$

$$b = 4ac = (-2)^2 - 4(1)(2) = -4$$



Section 1.5 Complex Numbers

1. real

2. imaginary

3. pure imaginary

4. $\sqrt{-1}$; -1

5. principal square

6. complex conjugates

7. $a + bi = 9 + 8i$
$a = 9$
$b = 8$

8. $a + bi = 10 - 5i$
$a = 10$
$b = -5$

9. $(a - 2) + (b + 1)i = 6 + 5i$
$a - 2 = 6 \Rightarrow a = 8$
$b + 1 = 5 \Rightarrow b = 4$

10. $(a + 2) + (b - 3)i = 4 + 7i$
$a + 2 = 4 \Rightarrow a = 2$
$b - 3 = 7 \Rightarrow b = 10$

11. $2 + \sqrt{-25} = 2 + 5i$

12. $4 + \sqrt{-49} = 4 + 7i$

13. $1 - \sqrt{-12} = 1 - 2\sqrt{3}i$

14. $2 - \sqrt{-18} = 2 - 3\sqrt{2}i$

15. $\sqrt{-40} = 2\sqrt{10}\,i$

16. $\sqrt{-27} = 3\sqrt{3}\,i$

17. 23

18. 50

19. $-6i + i^2 = -6i + (-1)$
$= -1 - 6i$

20. $-2i^2 + 4i = -2(-1) + 4\,i$
$= 2 + 4\,i$

21. $\sqrt{-0.04} = \sqrt{0.04}\,i$
$= 0.2i$

22. $\sqrt{-0.0025} = \sqrt{0.0025}\,i$
$= 0.05i$

23. $(5 + i) + (2 + 3i) = 5 + i + 2 + 3i$
$= 7 + 4i$

24. $(13 - 2i) + (-5 + 6i) = 8 + 4i$

25. $(9 - i) - (8 - i) = 1$

26. $(3 + 2i) - (6 + 13i) = 3 + 2i - 6 - 13i$
$= -3 - 11i$

27. $\left(-2 + \sqrt{-8}\right) + \left(5 - \sqrt{-50}\right) = -2 + 2\sqrt{2}i + 5 - 5\sqrt{2}i = 3 - 3\sqrt{2}i$

28. $\left(8 + \sqrt{-18}\right) - \left(4 + 3\sqrt{2}i\right) = 8 + 3\sqrt{2}i - 4 - 3\sqrt{2}i = 4$

29. $13i - (14 - 7i) = 13i - 14 + 7i$
$= -14 + 20i$

30. $25 + (-10 + 11i) + 15i = 15 + 26i$

31. $(1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2$
$= 3 + i + 2 = 5 + i$

32. $(7 - 2i)(3 - 5i) = 21 - 35i - 6i + 10i^2$
$= 21 - 41i - 10$
$= 11 - 41i$

33. $12i(1 - 9i) = 12i - 108i^2$
$= 12i + 108$
$= 108 + 12i$

34. $-8i(9 + 4i) = -72i - 32i^2$
$= 32 - 72i$

35. $\left(\sqrt{2} + 3i\right)\left(\sqrt{2} - 3i\right) = 2 - 9t^2$
$= 2 + 9 = 11$

36. $\left(4 + \sqrt{7}i\right)\left(4 - \sqrt{7}i\right) = 16 - 7i^2$
$= 16 + 7 = 23$

37. $(6 + 7i)^2 = 36 + 84i + 49i^2$
$= 36 + 84i - 49$
$= -13 + 84i$

38. $(5 - 4i)^2 = 25 - 40i + 16i^2$
$= 25 - 40i - 16$
$= 9 - 40i$

39. The complex conjugate of $9 + 2i$ is $9 - 2i$.
$(9 + 2i)(9 - 2i) = 81 - 4i^2$
$= 81 + 4$
$= 85$

40. The complex conjugate of $8 - 10i$ is $8 + 10i$.
$(8 - 10i)(8 + 10i) = 64 - 100i^2$
$= 64 + 100$
$= 164$

41. The complex conjugate of $-1 - \sqrt{5}i$ is $-1 + \sqrt{5}i$.
$\left(-1 - \sqrt{5}i\right)\left(-1 + \sqrt{5}i\right) = 1 - 5i^2$
$= 1 + 5 = 6$

42. The complex conjugate of $-3 + \sqrt{2}i$ is $-3 - \sqrt{2}i$.
$\left(-3 + \sqrt{2}i\right)\left(-3 - \sqrt{2}i\right) = 9 - 2i^2$
$= 9 + 2$
$= 11$

43. The complex conjugate of $\sqrt{-20} = 2\sqrt{5}i$ is $-2\sqrt{5}i$.
$\left(2\sqrt{5}i\right)\left(-2\sqrt{5}i\right) = -20i^2 = 20$

44. The complex conjugate of $\sqrt{-15} = \sqrt{15}i$ is $-\sqrt{15}i$.
$\left(\sqrt{15}i\right)\left(-\sqrt{15}i\right) = -15i^2 = 15$

45. The complex conjugate of $\sqrt{6}$ is $\sqrt{6}$.

$$\left(\sqrt{6}\right)\left(\sqrt{6}\right) = 6$$

46. The complex conjugate of $1 + \sqrt{8}$ is $1 + \sqrt{8}$.

$$\left(1 + \sqrt{8}\right)\left(1 + \sqrt{8}\right) = 1 + 2\sqrt{8} + 8$$
$$= 9 + 4\sqrt{2}$$

47. $$\frac{2}{4 - 5i} = \frac{2}{4 - 5i} \cdot \frac{4 + 5i}{4 + 5i}$$
$$= \frac{2(4 + 5i)}{16 + 25} = \frac{8 + 10i}{41} = \frac{8}{41} + \frac{10}{41}i$$

48. $$\frac{13}{1 - i} \cdot \frac{(1 + i)}{(1 + i)} = \frac{13 + 13i}{1 - i^2} = \frac{13 + 13i}{2} = \frac{13}{2} + \frac{13}{2}i$$

49. $$\frac{5 + i}{5 - i} \cdot \frac{(5 + i)}{(5 + i)} = \frac{25 + 10i + i^2}{25 - i^2}$$
$$= \frac{24 + 10i}{26} = \frac{12}{13} + \frac{5}{13}i$$

50. $$\frac{6 - 7i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} = \frac{6 + 12i - 7i - 14i^2}{1 - 4i^2}$$
$$= \frac{20 + 5i}{5} = 4 + i$$

51. $$\frac{9 - 4i}{i} \cdot \frac{-i}{-i} = \frac{-9i + 4i^2}{-i^2} = -4 - 9i$$

52. $$\frac{8 + 16i}{2i} \cdot \frac{-2i}{-2i} = \frac{-16i - 32i^2}{-4i^2} = 8 - 4i$$

53. $$\frac{3i}{(4 - 5i)^2} = \frac{3i}{16 - 40i + 25i^2} = \frac{3i}{-9 - 40i} \cdot \frac{-9 + 40i}{-9 + 40i}$$
$$= \frac{-27i + 120i^2}{81 + 1600} = \frac{-120 - 27i}{1681}$$
$$= -\frac{120}{1681} - \frac{27}{1681}i$$

54. $$\frac{5i}{(2 + 3i)^2} = \frac{5i}{4 + 12i + 9i^2}$$
$$= \frac{5i}{-5 + 12i} \cdot \frac{-5 - 12i}{-5 - 12i}$$
$$= \frac{-25i - 60i^2}{25 - 144i^2}$$
$$= \frac{60 - 25i}{169} = \frac{60}{169} - \frac{25}{169}i$$

55. $$\frac{2}{1 + i} - \frac{3}{1 - i} = \frac{2(1 - i) - 3(1 + i)}{(1 + i)(1 - i)}$$
$$= \frac{2 - 2i - 3 - 3i}{1 + 1}$$
$$= \frac{-1 - 5i}{2}$$
$$= -\frac{1}{2} - \frac{5}{2}i$$

56. $$\frac{2i}{2 + i} + \frac{5}{2 - i} = \frac{2i(2 - i) + 5(2 + i)}{(2 + i)(2 - i)}$$
$$= \frac{4i - 2i^2 + 10 + 5i}{4 - i^2}$$
$$= \frac{12 + 9i}{5}$$
$$= \frac{12}{5} + \frac{9}{5}i$$

57. $$\frac{i}{3 - 2i} + \frac{2i}{3 + 8i} = \frac{i(3 + 8i) + 2i(3 - 2i)}{(3 - 2i)(3 + 8i)}$$
$$= \frac{3i + 8i^2 + 6i - 4i^2}{9 + 24i - 6i - 16i^2}$$
$$= \frac{4i^2 + 9i}{9 + 18i + 16}$$
$$= \frac{-4 + 9i}{25 + 18i} \cdot \frac{25 - 18i}{25 - 18i}$$
$$= \frac{-100 + 72i + 225i - 162i^2}{625 + 324}$$
$$= \frac{62 + 297i}{949} = \frac{62}{949} + \frac{297}{949}i$$

58. $$\frac{1 + i}{i} - \frac{3}{4 - i} = \frac{(1 + i)(4 - i) - 3i}{i(4 - i)}$$
$$= \frac{4 - i + 4i - i^2 - 3i}{4i - i^2}$$
$$= \frac{5}{1 + 4i} \cdot \frac{1 - 4i}{1 - 4i}$$
$$= \frac{5 - 20i}{1 - 16i^2}$$
$$= \frac{5}{17} - \frac{20}{17}i$$

59. $$\sqrt{-6} \cdot \sqrt{-2} = \left(\sqrt{6}i\right)\left(\sqrt{2}i\right) = \sqrt{12}i^2 = \left(2\sqrt{3}\right)(-1)$$
$$= -2\sqrt{3}$$

60. $\sqrt{-5} \cdot \sqrt{-10} = \left(\sqrt{5}i\right)\left(\sqrt{10}i\right)$
$= \sqrt{50}i^2 = 5\sqrt{2}(-1) = -5\sqrt{2}$

61. $\left(\sqrt{-15}\right)^2 = \left(\sqrt{15}i\right)^2 = 15i^2 = -15$

62. $\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$

63. $\sqrt{-8} + \sqrt{-50} = \sqrt{8}i + \sqrt{50}i$
$= 2\sqrt{2}i + 5\sqrt{2}i$
$= 7\sqrt{2}i$

64. $\sqrt{-45} - \sqrt{-5} = \sqrt{45}i - \sqrt{5}i$
$= 3\sqrt{5}i - \sqrt{5}i$
$= 2\sqrt{5}i$

65. $\left(3 + \sqrt{-5}\right)\left(7 - \sqrt{-10}\right) = \left(3 + \sqrt{5}i\right)\left(7 - \sqrt{10}i\right)$
$= 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2$
$= \left(21 + \sqrt{50}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i$
$= \left(21 + 5\sqrt{2}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i$

66. $\left(2 - \sqrt{-6}\right)^2 = \left(2 - \sqrt{6}i\right)\left(2 - \sqrt{6}i\right)$
$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6i^2$
$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6(-1)$
$= 4 - 6 - 4\sqrt{6}i$
$= -2 - 4\sqrt{6}i$

67. $x^2 - 2x + 2 = 0;\ a = 1,\ b = -2,\ c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

68. $x^2 + 6x + 10 = 0;\ a = 1,\ b = 6,\ c = 10$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 + 2i}{2} = -3 \pm i$$

69. $4x^2 + 16x + 17 = 0;\ a = 4,\ b = 16,\ c = 17$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)} = \frac{-16 \pm \sqrt{-16}}{8} = \frac{-16 \pm 4i}{8} = -2 \pm \frac{1}{2}i$$

70. $9x^2 - 6x + 37 = 0;\ a = 9,\ b = -6,\ c = 37$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)} = \frac{6 \pm \sqrt{-1296}}{18} = \frac{6 \pm 36i}{18} = \frac{1}{3} \pm 2i$$

71. $4x^2 + 16x + 21 = 0;\ a = 4,\ b = 16,\ c = 21$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(21)}}{2(4)} = \frac{-16 \pm \sqrt{-80}}{8} = \frac{-16 \pm \sqrt{80}\,i}{8} = \frac{-16 \pm 4\sqrt{5}\,i}{8} = -2 \pm \frac{\sqrt{5}}{2}i$$

72. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$\begin{aligned} t &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)} \\ &= \frac{4 \pm \sqrt{-176}}{32} \\ &= \frac{4 \pm 4\sqrt{11}i}{32} \\ &= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i \end{aligned}$$

73. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3,\ b = -12,\ c = 18$

$$\begin{aligned} x &= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)} \\ &= \frac{12 \pm \sqrt{-72}}{6} \\ &= \frac{12 \pm 6\sqrt{2}i}{6} \\ &= 2 \pm \sqrt{2}i \end{aligned}$$

74. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$ Multiply both sides by 16.

$14x^2 - 12x + 5 = 0;\ a = 14,\ b = -12,\ c = 5$

$$\begin{aligned} x &= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)} \\ &= \frac{12 \pm \sqrt{-136}}{28} \\ &= \frac{12 \pm 2\sqrt{34}i}{28} \\ &= \frac{3}{7} \pm \frac{\sqrt{34}}{14}i \end{aligned}$$

75. $1.4x^2 - 2x + 10 = 0 \Rightarrow 14x^2 - 20x + 100 = 0;$

$a = 14,\ b = -20,\ c = 100$

$$\begin{aligned} x &= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(14)(100)}}{2(14)} \\ &= \frac{20 \pm \sqrt{-5200}}{28} \\ &= \frac{20 \pm 20\sqrt{13}\,i}{28} \\ &= \frac{20}{28} \pm \frac{20\sqrt{13}\,i}{28} \\ &= \frac{5}{7} \pm \frac{5\sqrt{13}}{7}i \end{aligned}$$

76. $4.5x^2 - 3x + 12 = 0;\ a = 4.5,\ b = -3,\ c = 12$

$$\begin{aligned} x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)} \\ &= \frac{3 \pm \sqrt{-207}}{9} \\ &= \frac{3 \pm 3\sqrt{23}i}{9} \\ &= \frac{1}{3} \pm \frac{\sqrt{23}}{3}i \end{aligned}$$

77. $$\begin{aligned} -6i^3 + i^2 &= -6i^2i + i^2 \\ &= -6(-1)i + (-1) \\ &= 6i - 1 \\ &= -1 + 6i \end{aligned}$$

78. $4i^2 - 2i^3 = 4i^2 - 2i^2i = 4(-1) - 2(-1)i = -4 + 2i$

79. $-14i^5 = -14i^2i^2i = -14(-1)(-1)(i) = -14i$

80. $(-i)^3 = (-1)(i^3) = (-1)i^2i = (-1)(-1)i = i$

81. $$\begin{aligned} \left(\sqrt{-72}\right)^3 &= \left(6\sqrt{2}i\right)^3 \\ &= 6^3\left(\sqrt{2}\right)^3 i^3 \\ &= 216\left(2\sqrt{2}\right)i^2i \\ &= 432\sqrt{2}(-1)i \\ &= -432\sqrt{2}i \end{aligned}$$

82. $\left(\sqrt{-2}\right)^6 = \left(\sqrt{2}i\right)^6 = 8i^6 = 8i^2i^2i^2 = 8(-1)(-1)(-1) = -8$

83. $\frac{1}{i^3} = \frac{1}{i^2i} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = i$

84. $\dfrac{1}{(2i)^3} = \dfrac{1}{8i^3} = \dfrac{1}{8i^2 i} = \dfrac{1}{-8i} = \dfrac{1}{-8i}\cdot\dfrac{8i}{8i} = \dfrac{8i}{-64i^2} = \dfrac{1}{8}i$

85. $(3i)^4 = 81i^4 = 81i^2i^2 = 81(-1)(-1) = 81$

86. $(-i)^6 = i^6 = i^2i^2i^2 = (-1)(-1)(-1) = -1$

87. (a) $z_1 = 9 + 16i,\ z_2 = 20 - 10i$

(b) $\dfrac{1}{z} = \dfrac{1}{z_1} + \dfrac{1}{z_2} = \dfrac{1}{9 + 16i} + \dfrac{1}{20 - 10i} = \dfrac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \dfrac{29 + 6i}{340 + 230i}$

$$z = \left(\frac{340 + 230i}{29 + 6i}\right)\left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11{,}240 + 4630i}{877} = \frac{11{,}240}{877} + \frac{4630}{877}i$$

88. (a)
$$\begin{aligned}\left(-1 + \sqrt{3}i\right)^3 &= (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3\\ &= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3\\ &= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^2 i\\ &= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i\\ &= 8\end{aligned}$$

(b)
$$\begin{aligned}\left(-1 - \sqrt{3}i\right)^3 &= (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3\\ &= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3\\ &= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^2 i\\ &= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i\\ &= 8\end{aligned}$$

89. False.

Sample answer: $(1 + i) + (3 + i) = 4 + 2i$ which is not a real number.

90. False.

If $b = 0$ then $a + bi = a - bi = a$.

That is, if the complex number is real, the number equals its conjugate.

91. True.

$$\begin{aligned}x^4 - x^2 + 14 &= 56\\ \left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 &\stackrel{?}{=} 56\\ 36 + 6 + 14 &\stackrel{?}{=} 56\\ 56 &= 56\end{aligned}$$

92. False.

$$\begin{aligned}i^{44} + i^{150} - i^{74} - i^{109} + i^{61} &= \left(i^2\right)^{22} + \left(i^2\right)^{75} - \left(i^2\right)^{37} - \left(i^2\right)^{54} i + \left(i^2\right)^{30} i\\ &= (-1)^{22} + (-1)^{75} - (-1)^{37} - (-1)^{54} i + (-1)^{30} i\\ &= 1 - 1 + 1 - i + i = 1\end{aligned}$$

93. $i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i^4 i = i$

$i^6 = i^4 i^2 = -1$

$i^7 = i^4 i^3 = -i$

$i^8 = i^4 i^4 = 1$

$i^9 = i^4 i^4 i = i$

$i^{10} = i^4 i^4 i^2 = -1$

$i^{11} = i^4 i^4 i^3 = -i$

$i^{12} = i^4 i^4 i^4 = 1$

The pattern $i, -1, -i, 1$ repeats. Divide the exponent by 4.

If the remainder is 1, the result is i.

If the remainder is 2, the result is -1.

If the remainder is 3, the result is $-i$.

If the remainder is 0, the result is 1.

94. (i) D

(ii) F

(iii) B

(iv) E

(v) A

(vi) C

95. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$

96. $$\begin{aligned}(a_1 + b_1 i)(a_2 + b_2 i) &= a_1a_2 + a_1b_2i + a_2b_1i + b_1b_2i^2 \\ &= (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i\end{aligned}$$

The complex conjugate of this product is

$(a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i.$

The product of the complex conjugates is

$$\begin{aligned}(a_1 - b_1 i)(a_2 - b_2 i) &= a_1a_2 - a_1b_2i - a_2b_1i - b_1b_2i^2 \\ &= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i.\end{aligned}$$

So, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

97. $(a_1 + b_1 i) + (a_2 + b_2 i) = (a_1 + a_2) + (b_1 + b_2)i$

The complex conjugate of this sum is $(a_1 + a_2) - (b_1 + b_2)i.$

The sum of the complex conjugates is $(a_1 - b_1 i) + (a_2 - b_2 i) = (a_1 + a_2) - (b_1 + b_2)i.$

So, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

Section 1.6 Other Types of Equations

1. polynomial

2. quadratic type

3. radical

4. absolute value

5. $$\begin{aligned}6x^4 - 54x^2 &= 0 \\ 6x^2(x^2 - 9) &= 0 \\ 6x^2 = 0 &\Rightarrow x = 0 \\ x^2 - 9 = 0 &\Rightarrow x = \pm 3\end{aligned}$$

6. $$\begin{aligned}36x^3 - 100x &= 0 \\ 4x(9x^2 - 25) &= 0 \\ 4x(3x + 5)(3x - 5) &= 0 \\ 4x = 0 &\Rightarrow x = 0 \\ 3x + 5 = 0 &\Rightarrow x = -\tfrac{5}{3} \\ 3x - 5 = 0 &\Rightarrow x = \tfrac{5}{3}\end{aligned}$$

7. $$\begin{aligned}5x^3 + 3 - x^2 + 45x &= 0 \\ 5x(x^2 + 6x + 9) &= 0 \\ 5x(x + 3)^2 &= 0 \\ 5x = 0 &\Rightarrow x = 0 \\ x + 3 = 0 &\Rightarrow x = -3\end{aligned}$$

8.
$$\begin{aligned} 9x^4 - 24x^3 + 16x^2 &= 0 \\ x^2(9x^2 - 24x + 16) &= 0 \\ x^2(3x - 4)^2 &= 0 \\ x^2 &= 0 \Rightarrow x = 0 \\ 3x - 4 &= 0 \Rightarrow x = \tfrac{4}{3} \end{aligned}$$

9.
$$\begin{aligned} x^4 - 81 &= 0 \\ (x^2 + 9)(x + 3)(x - 3) &= 0 \\ x^2 + 9 &= 0 \Rightarrow x = \pm 3i \\ x + 3 &= 0 \Rightarrow x = -3 \\ x - 3 &= 0 \Rightarrow x = 3 \end{aligned}$$

10.
$$\begin{aligned} x^6 - 64 &= 0 \\ (x^3 - 8)(x^3 + 8) &= 0 \\ (x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4) &= 0 \\ x - 2 &= 0 \Rightarrow x = 2 \\ x^2 + 2x + 4 &= 0 \Rightarrow x = -1 \pm \sqrt{3}i \\ x + 2 &= 0 \Rightarrow x = -2 \\ x^2 - 2x + 4 &= 0 \Rightarrow x = 1 \pm \sqrt{3}i \end{aligned}$$

11.
$$\begin{aligned} x^3 + 512 &= 0 \\ x^3 + 8^3 &= 0 \\ (x + 8)(x^2 - 8x + 64) &= 0 \\ x + 8 &= 0 \Rightarrow x = -8 \\ x^2 - 8x + 64 &= 0 \Rightarrow x = 4 \pm 4\sqrt{3}i \end{aligned}$$

12.
$$\begin{aligned} 27x^3 - 343 &= 0 \\ (3x)^3 - 7^3 &= 0 \\ (3x - 7)(9x^2 + 21x + 49) &= 0 \\ 3x - 7 &= 0 \Rightarrow x = \frac{7}{3} \\ 9x^2 + 21x + 49 &= 0 \Rightarrow -\frac{7}{6} \pm \frac{7\sqrt{3}}{6}i \end{aligned}$$

13.
$$\begin{aligned} x^3 - 3x^2 - x + 3 &= 0 \\ x^2(x - 3) - (x - 3) &= 0 \\ (x - 3)(x^2 - 1) &= 0 \\ (x - 3)(x + 3)(x - 1) &= 0 \\ x - 3 &= 0 \Rightarrow x = 3 \\ x + 1 &= 0 \Rightarrow x = -1 \\ x - 1 &= 0 \Rightarrow x = 1 \end{aligned}$$

14.
$$\begin{aligned} x^3 + 2x^2 + 3x + 6 &= 0 \\ x^2(x + 2) + 3(x + 2) &= 0 \\ (x + 2)(x^2 + 3) &= 0 \\ x + 2 &= 0 \Rightarrow x = -2 \\ x^2 + 3 &= 0 \Rightarrow x = \pm\sqrt{3}i \end{aligned}$$

15.
$$\begin{aligned} x^4 - x^3 + x - 1 &= 0 \\ x^3(x - 1) + (x - 1) &= 0 \\ (x - 1)(x^3 + 1) &= 0 \\ (x - 1)(x + 1)(x^2 - x + 1) &= 0 \\ x - 1 &= 0 \Rightarrow x = 1 \\ x + 1 &= 0 \Rightarrow x = -1 \\ x^2 - x + 1 &= 0 \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i \end{aligned}$$

16.
$$\begin{aligned} x^4 + 2x^3 - 8x - 16 &= 0 \\ x^3(x + 2) - 8(x + 2) &= 0 \\ (x^3 - 8)(x + 2) &= 0 \\ (x - 2)(x^2 + 2x + 4)(x + 2) &= 0 \\ x - 2 &= 0 \Rightarrow x = 2 \\ x^2 + 2x + 4 &= 0 \Rightarrow x = -1 \pm \sqrt{3}i \\ x + 2 &= 0 \Rightarrow x = -2 \end{aligned}$$

17.
$$\begin{aligned} x^4 - 4x^2 + 3 &= 0 \\ (x^2)^2 - 4(x^2) + 3 &= 0 \end{aligned}$$

Let $u = x^2$.

$$\begin{aligned} u^2 - 4u + 3 &= 0 \\ (u - 3)(u - 1) &= 0 \\ u - 3 &= 0 \Rightarrow u = 3 \\ u - 1 &= 0 \Rightarrow u = 1 \end{aligned}$$

$$\begin{aligned} u &= 1 & u &= 3 \\ x^2 &= 1 & x^2 &= 3 \\ x &= \pm 1 & x &= \pm\sqrt{3} \end{aligned}$$

18. $x^4 - 13x^2 + 36 = 0$

$(x^2)^2 - 13(x^2) + 36 = 0$

Let $u = x^2$.

$u^2 - 13u + 36 = 0$

$(u - 9)(u - 4) = 0$

$u - 9 = 0 \Rightarrow u = 9$

$u - 4 = 0 \Rightarrow u = 4$

$u = 9 \qquad u = 4$

$x^2 = 9 \qquad x^2 = 4$

$x = \pm 3 \qquad x = \pm 2$

19. $4x^4 - 65x^2 + 16 = 0$

$4(x^2) - 65(x^2) + 16 = 0$

Let $u = x^2$.

$4u - 65u + 16 = 0$

$(4u - 1)(u - 16) = 0$

$4u - 1 = 0 \Rightarrow u = \frac{1}{4}$

$u - 16 = 0 \Rightarrow u = 16$

$u = \frac{1}{4} \qquad u = 16$

$x^2 = \frac{1}{4} \qquad x^2 = 16$

$x = \pm\frac{1}{2} \qquad x = \pm 4$

20. $36t^4 + 29t^2 - 7 = 0$

$36(t^2)^2 + 29(t^2) - 7 = 0$

Let $u = t^2$.

$36u^2 + 29u - 7 = 0$

$(36u - 7)(u + 1) = 0$

$36u - 7 = 0 \Rightarrow u = \frac{7}{36}$

$u + 1 = 0 \Rightarrow u = -1$

$u = \frac{7}{36} \qquad u = -1$

$x^2 = \frac{7}{36} \qquad x^2 = -1$

$x = \pm\frac{\sqrt{7}}{6} \qquad x = \pm i$

21. $x^6 + 7x^3 - 8 = 0$

$(x^3)^2 + 7(x^3) - 8 = 0$

Let. $u = x^3$

$u^2 + 7u - 8 = 0$

$(u + 8)(u - 1) = 0$

$u + 8 = 0$

$x^3 + 8 = 0$

$(x + 2)(x^2 - 2x + 4) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x^2 - 2x + 4 = 0 \Rightarrow x = 1 \pm \sqrt{3}i$

$u - 1 = 0$

$x^3 - 1 = 0$

$(x - 1)(x^2 + x + 1) = 0$

$x - 1 = 0$

$x = 1$

$x^2 + x + 1 = 0$

$x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

22. $x^6 + 3x^3 + 2 = 0$

$(x^3)^2 + 3(x^3) + 2 = 0$

Let $u = x^3$.

$u^2 + 3u + 2 = 0$

$(u + 2)(u + 1) = 0$

$u + 2 = 0$

$x^3 + 2 = 0$

$(x + \sqrt[3]{2})\left[x^2 - \sqrt[3]{2x} + (\sqrt[3]{2})^2\right] = 0$

$x + \sqrt[3]{2} = 0 \Rightarrow x = -\sqrt[3]{2}$

$x^2 - \sqrt[3]{2x} + (\sqrt[3]{2})^2 = 0 \Rightarrow x = \frac{1 \pm \sqrt{3}i}{2^{2/3}}$

$u + 1 = 0$

$x^3 + 1 = 0$

$(x + 1)(x^2 - x + 1) = 0$

$x + 1 = 0 \Rightarrow x = -1$

$x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{3}i}{2}$

23.
$$\frac{1}{x^2} + \frac{8}{x} + 15 = 0$$
$$\left(\frac{1}{x}\right)^2 + 8\left(\frac{1}{x}\right) + 15 = 0$$
Let $u = \frac{1}{x}$.
$$u^2 + 8u + 15 = 0$$
$$(u + 5)(u + 3) = 0$$
$$u + 5 = 0 \Rightarrow u = -5$$
$$u + 3 = 0 \Rightarrow u = -3$$

$u = -5$	$u = -3$
$\frac{1}{x} = -5$	$\frac{1}{x} = -3$
$x = -\frac{1}{5}$	$x = -\frac{1}{3}$

24.
$$1 + \frac{3}{x} = -\frac{2}{x^2}$$
$$\frac{2}{x^2} + \frac{3}{x} + 1 = 0$$
$$2\left(\frac{1}{x}\right)^2 + 3\left(\frac{1}{x}\right) + 1 = 0$$
Let $u = \frac{1}{x}$.
$$2u^2 + 3u + 1 = 0$$
$$(2u + 1)(u + 1) = 0$$
$$2u + 1 = 0 \Rightarrow u = -\frac{1}{2}$$
$$u + 1 = 0 \Rightarrow u = -1$$

$u = -\frac{1}{2}$	$u = -1$
$\frac{1}{x} = -\frac{1}{2}$	$\frac{1}{x} = -1$
$x = -2$	$x = -1$

25. $2\left(\frac{x}{x + 2}\right)^2 - 3\left(\frac{x}{x + 2}\right) - 2 = 0$

Let $u = \frac{x}{x + 2}$.
$$2u^2 - 3u - 2 = 0$$
$$(2u + 1)(u - 2) = 0$$
$$2u + 1 = 0 \Rightarrow u = -\frac{1}{2}$$
$$u - 2 = 0 \Rightarrow u = 2$$

$u = -\frac{1}{2}$	$u = 2$
$\frac{x}{x + 2} = -\frac{1}{2}$	$\frac{x}{x + 2} = 2$
$x = -\frac{2}{3}$	$x = -4$

26. $6\left(\frac{x}{x + 1}\right)^2 + 5\left(\frac{x}{x + 1}\right) - 6 = 0$

Let $u = \frac{x}{x + 1}$.
$$6u^2 + 5u - 6 = 0$$
$$(3u - 2)(2u + 3) = 0$$
$$3u - 2 = 0 \Rightarrow u = \frac{2}{3}$$
$$2u + 3 = 0 \Rightarrow u = -\frac{3}{2}$$

$u = \frac{2}{3}$	$u = -\frac{3}{2}$
$\frac{x}{x + 1} = \frac{2}{3}$	$\frac{x}{x + 1} = -\frac{3}{2}$
$x = 2$	$x = -\frac{3}{5}$

27. $2x + 9\sqrt{x} = 5$

$2x + 9\sqrt{x} - 5 = 0$

$2\left(\sqrt{x}\right)^2 + 9\left(\sqrt{x}\right) - 5 = 0$

Let $u = \sqrt{x}$.

$2u^2 + 9u - 5 = 0$

$(2u - 1)(u + 5) = 0$

$2u - 1 = 0 \Rightarrow u = \frac{1}{2}$

$u + 5 = 0 \Rightarrow u = -5$

$u = \frac{1}{2}$ $\qquad u = -5 \Rightarrow \sqrt{x} \neq -5$

$\sqrt{x} = \frac{1}{2}$ $\qquad \left(\sqrt{x} = -5 \text{ is not a solution.}\right)$

$x = \frac{1}{4}$

28. $6x - 7\sqrt{x} - 3 = 0$

$6\left(\sqrt{x}\right)^2 - 7\left(\sqrt{x}\right) - 3 = 0$

Let $u = \sqrt{x}$.

$6u^2 - 7u - 3 = 0$

$(3u + 1)(2u - 2) = 0$

$3u + 1 = 0 \Rightarrow u = -\frac{1}{3}$

$2u - 3 = 0 \Rightarrow u = \frac{3}{2}$

$u = -\frac{1}{3}$ $\qquad u = \frac{3}{2}$

$\sqrt{x} \neq -\frac{1}{3}$ $\qquad \sqrt{x} = \frac{3}{2}$

$\left(\sqrt{x} = -\frac{1}{3} \text{ is not a solution.}\right)$

29. $9t^{2/3} + 24^{1/3} + 16 = 0$

$9\left(t^{1/3}\right)^2 + 24\left(t^{1/3}\right) + 16 = 0$

Let $u = t^{1/3}$.

$9u^2 + 24u + 16 = 0$

$(3u + 4)^2 = 0$

$3u + 4 = 0 \Rightarrow u = -\frac{4}{3}$

$u = -\frac{4}{3}$

$t^{1/3} = -\frac{4}{3}$

$t = -\frac{64}{27}$

30. $3x^{1/3} + 2x^{2/3} = 5$

$2x^{2/3} + 3x^{1/3} - 5 = 0$

$2\left(x^{1/3}\right)^2 + 3\left(x^{1/3}\right) - 5 = 0$

Let $u = x^{1/3}$.

$(2u + 5)(u - 1) = 0$

$2u + 5 = 0 \Rightarrow u = -\frac{5}{2}$

$u - 1 = 0 \Rightarrow u = 1$

$u = -\frac{5}{2}$ $\qquad u = 1$

$x^{1/3} = -\frac{5}{2}$ $\qquad x^{1/3} = 1$

$x = -\frac{125}{2}$ $\qquad x = 1$

31. $\sqrt{5x} - 10 = 0$

$\sqrt{5x} = 10$

$\left(\sqrt{5x}\right)^2 = (10)^2$

$5x = 100$

$x = 20$

32. $6 - 2\sqrt{x} = 0$

$6 = 2\sqrt{x}$

$(6)^2 = \left(2\sqrt{x}\right)^2$

$36 = 4x$

$x = 9$

33. $\sqrt{x + 8} - 5 = 0$

$\sqrt{x + 8} = 5$

$\left(\sqrt{x + 8}\right)^2 = (5)^2$

$x + 8 = 25$

$x = 17$

34. $\sqrt{3x + 1} = 7$

$\left(\sqrt{3x + 1}\right)^2 = (7)^2$

$3x + 1 = 49$

$3x = 48$

$x = 16$

35. $$\begin{aligned} 4 + \sqrt[3]{2x - 9} &= 0 \\ \sqrt[3]{2x - 9} &= -4 \\ \left(\sqrt[3]{2x - 9}\right)^3 &= (-4)^3 \\ 2x - 9 &= -64 \\ 2x &= -55 \\ x &= -\frac{55}{2} \end{aligned}$$

36. $$\begin{aligned} \sqrt[3]{12 - x} - 3 &= 0 \\ \sqrt[3]{12 - x} &= 3 \\ \left(\sqrt[3]{12 - x}\right)^3 &= (3)^3 \\ 12 - x &= 27 \\ -x &= 15 \\ x &= -15 \end{aligned}$$

37. $$\begin{aligned} \sqrt{x + 8} &= 2 + x \\ \left(\sqrt{x + 8}\right)^2 &= (2 + x)^2 \\ x + 8 &= x^2 + 4x + 4 \\ 0 &= x^2 + 3x - 4 \\ x^2 + 3x - 4 &= 0 \\ (x + 4)(x - 1) &= 0 \\ x + 4 &= 0 \Rightarrow x = -4, \text{ extraneous} \\ x - 1 &= 0 \Rightarrow x = 1 \end{aligned}$$

38. $$\begin{aligned} 2x &= \sqrt{-5x + 24} - 3 \\ 2x + 3 &= \sqrt{-5x + 24} \\ (2x + 3)^2 &= \left(\sqrt{-5x + 24}\right)^2 \\ 4x^2 + 12x + 9 &= -5x + 24 \\ 4x^2 + 17x - 15 &= 0 \\ (4x - 3)(x + 5) &= 0 \\ 4x - 3 &= 0 \Rightarrow x = \tfrac{3}{4} \\ x + 5 &\Rightarrow x = -5, \text{ extraneous} \end{aligned}$$

39. $$\begin{aligned} \sqrt{x - 3} + 1 &= \sqrt{x} \\ \sqrt{x - 3} &= \sqrt{x} - 1 \\ \left(\sqrt{x - 3}\right)^2 &= \left(\sqrt{x} - 1\right)^2 \\ x - 3 &= x - 2\sqrt{x} + 1 \\ -4 &= -2\sqrt{x} \\ 2 &= \sqrt{x} \\ (2)^2 &= \left(\sqrt{x}\right)^2 \\ 4 &= x \end{aligned}$$

40. $$\begin{aligned} \sqrt{x} + \sqrt{x - 24} &= 2 \\ \sqrt{x} &= 2 - \sqrt{x - 24} \\ \left(\sqrt{x}\right)^2 &= \left(2 - \sqrt{x - 24}\right)^2 \\ x &= 4 - 4\sqrt{x - 24} + x - 24 \\ 20 &= -4\sqrt{x - 24} \\ 5 &= -\sqrt{x - 24} \\ 5^2 &= \left(-\sqrt{x - 24}\right)^2 \\ 25 &= x - 24 \\ 49 &= x \end{aligned}$$

$x = 49$ is an extraneous solution, so the equation has no solution.

41. $$\begin{aligned} 2\sqrt{x + 1} - \sqrt{2x + 3} &= 1 \\ 2\sqrt{x + 1} &= 1 + \sqrt{2x + 3} \\ \left(2\sqrt{x + 1}\right)^2 &= \left(1 + \sqrt{2x + 3}\right)^2 \\ 4(x + 1) &= 1 + 2\sqrt{2x + 3} + 2x + 3 \\ 2x &= 2\sqrt{2x + 3} \\ x &= \sqrt{2x + 3} \\ x^2 &= 2x + 3 \\ x^2 - 2x - 3 &= 0 \\ (x - 3)(x + 1) &= 0 \\ x - 3 &= 0 \Rightarrow x = 3 \\ x + 1 &= 0 \Rightarrow x = -1, \text{ extraneous} \end{aligned}$$

42.
$$\begin{aligned}
4\sqrt{x-3} - \sqrt{6x-17} &= 3\\
4\sqrt{x-3} &= 3 + \sqrt{6x-17}\\
\left(4\sqrt{x-3}\right)^2 &= \left(3+\sqrt{6x-17}\right)^2\\
16(x-3) &= 9 + 6\sqrt{6x-17} + 6x - 17\\
16x - 48 &= 6\sqrt{6x-17} + 6x - 8\\
10x - 40 &= 6\sqrt{6x-17}\\
5x - 20 &= 3\sqrt{6x-17}\\
(5x-20)^2 &= \left(3\sqrt{6x-17}\right)^2\\
25x^2 - 200x + 400 &= 9(6x-17)\\
25x^2 - 200x + 400 &= 54x - 153\\
25x^2 - 254x + 553 &= 0\\
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a}\\
x &= \frac{-(-254) \pm \sqrt{(-254)^2 - 4(25)(553)}}{2(25)}\\
x &= \frac{254 \pm \sqrt{9261}}{50}\\
x &= \frac{254+96}{50} = \frac{350}{50} = 7\\
x &= \frac{254-96}{50} = \frac{158}{50} = \frac{79}{25}\text{, extraneous}
\end{aligned}$$

43.
$$\begin{aligned}
\sqrt{4\sqrt{4x+9}} &= \sqrt{8x+2}\\
\left(\sqrt{4\sqrt{4x+9}}\right)^2 &= \left(\sqrt{8x+2}\right)^2\\
4\sqrt{4x+9} &= 8x+2\\
2\sqrt{4x+9} &= 4x+1\\
\left(2\sqrt{4x+9}\right)^2 &= (4x+1)^2\\
4(4x+9) &= 16x^2 + 8x + 1\\
16x + 36 &= 16x^2 + 8x + 1\\
0 &= 16x^2 - 8x - 35\\
0 &= (4x+5)(4x-7)\\
4x + 5 = 0 \Rightarrow x &= -\frac{5}{4}\text{, extraneous}\\
4x - 7 = 0 \Rightarrow x &= \frac{7}{4}
\end{aligned}$$

44.
$$\begin{aligned}
\sqrt{16+9\sqrt{x}} &= 4 + \sqrt{x}\\
\left(\sqrt{16+9\sqrt{x}}\right)^2 &= \left(4+\sqrt{x}\right)^2\\
16 + 9\sqrt{x} &= 16 + 8\sqrt{x} + x\\
\sqrt{x} &= x\\
\left(\sqrt{x}\right)^2 &= (x)^2\\
x &= x^2\\
x^2 - x &= 0\\
x(x-1) &= 0\\
x &= 0\\
x - 1 = 0 \Rightarrow x &= 1
\end{aligned}$$

45.
$$\begin{aligned}
(x-5)^{3/2} &= 8\\
(x-5)^3 &= 8^2\\
x - 5 &= \sqrt[3]{64}\\
x &= 5 + 4 = 9
\end{aligned}$$

46.
$$\begin{aligned}(x+2)^{2/3} &= 9\\ (x+2)^2 &= 9^3\\ x+2 &= \pm\sqrt{729}\\ x &= -2 \pm 27 = -29,\ 25\end{aligned}$$

47.
$$\begin{aligned}(x^2-5)^{3/2} &= 27\\ (x^2-5)^3 &= 27^2\\ x^2-5 &= \sqrt[3]{27^2}\\ x^2 &= 5+9\\ x^2 &= 14\\ x &= \pm\sqrt{14}\end{aligned}$$

48.
$$\begin{aligned}(x^2-x-22)^{3/2} &= 27\\ x^2-x-22 &= 27^{2/3}\\ x^2-x-22 &= 9\\ x^2-x-31 &= 0\end{aligned}$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-31)}}{2(1)} = \frac{1 \pm \sqrt{125}}{2} = \frac{1 \pm 5\sqrt{5}}{2}$$

49.
$$\begin{aligned}3x(x-1)^{1/2} + 2(x-1)^{3/2} &= 0\\ (x-1)^{1/2}\left[3x + 2(x-1)\right] &= 0\\ (x-1)^{1/2}(5x-2) &= 0\\ (x-1)^{1/2} &= 0 \Rightarrow x-1=0 \Rightarrow x = 1\\ 5x-2 &= 0 \Rightarrow x = \tfrac{2}{5},\ \text{extraneous}\end{aligned}$$

50.
$$\begin{aligned}4x^2(x-1)^{1/3} + 6x(x-1)^{4/3} &= 0\\ 2x\left[2x(x-1)^{1/3} + 3(x-1)^{4/3}\right] &= 0\\ 2x(x-1)^{1/3}\left[2x + 3(x-1)\right] &= 0\\ 2x(x-1)^{1/3}(5x-3) &= 0\\ 2x &= 0 \Rightarrow x = 0\\ x-1 &= 0 \Rightarrow x = 1\\ 5x-3 &= 0 \Rightarrow x = \tfrac{3}{5}\end{aligned}$$

51.
$$\begin{aligned}x &= \frac{3}{x} + \frac{1}{2}\\ (2x)(x) &= (2x)\left(\frac{3}{x}\right) + (2x)\left(\frac{1}{2}\right)\\ 2x^2 &= 6 + x\\ 2x^2 - x - 6 &= 0\\ (2x+3)(x-2) &= 0\\ 2x+3 &= 0 \Rightarrow x = -\frac{3}{2}\\ x-2 &= 0 \Rightarrow x = 2\end{aligned}$$

52.
$$\begin{aligned}\frac{4}{x} - \frac{5}{3} &= \frac{x}{6}\\ (6x)\frac{4}{x} - (6x)\frac{5}{3} &= (6x)\frac{x}{6}\\ 24 - 10x &= x^2\\ x^2 + 10x - 24 &= 0\\ (x+12)(x-2) &= 0\\ x+12 &= 0 \Rightarrow x = -12\\ x-2 &= 0 \Rightarrow x = 2\end{aligned}$$

53.
$$\begin{aligned}\frac{1}{x} - \frac{1}{x+1} &= 3\\ x(x+1)\frac{1}{x} - x(x+1)\frac{1}{x+1} &= x(x+1)(3)\\ x+1-x &= 3x(x+1)\\ 1 &= 3x^2 + 3x\\ 0 &= 3x^2 + 3x - 1\end{aligned}$$
$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(3)(-1)}}{2(3)} = \frac{-3 \pm \sqrt{21}}{6}$$

54. $\dfrac{4}{x+1} - \dfrac{3}{x+2} = 1$

$4(x+2) - 3(x+1) = (x+1)(x+2)$

$4x + 8 - 3x = x^2 + 3x + 2$

$x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$

$x - 1 = 0 \Rightarrow x = 1$

$x + 3 = 0 \Rightarrow x = -3$

55. $3 - \dfrac{14}{x} - \dfrac{5}{x^2} = 0$

$\dfrac{5}{x^2} + \dfrac{14}{x} - 3 = 0$

$5\left(\dfrac{1}{x}\right)^2 + 14\left(\dfrac{1}{x}\right) - 3 = 0$

Let $u = \dfrac{1}{x}$.

$5u^2 + 14u - 3 = 0$

$(5u-1)(u+3) = 0$

$5u - 1 = 0 \Rightarrow u = \dfrac{1}{5}$

$u + 3 = 0 \Rightarrow u = -3$

$u = \dfrac{1}{5}$ $\qquad$ $u = -3$

$\dfrac{1}{x} = \dfrac{1}{5}$ $\qquad$ $\dfrac{1}{x} = -3$

$x = 5$ $\qquad$ $x = -\dfrac{1}{3}$

56. $5 = \dfrac{18}{x} + \dfrac{8}{x^2}$

$\dfrac{8}{x^2} + \dfrac{18}{x} - 5 = 0$

$8\left(\dfrac{1}{x}\right)^2 + 18\left(\dfrac{1}{x}\right) - 5 = 0$

Let $u = \dfrac{1}{x}$.

$8u^2 + 18u - 5 = 0$

$(4u-1)(2u+5) = 0$

$4u - 1 = 0 \Rightarrow u = \dfrac{1}{4}$

$2u + 5 = 0 \Rightarrow u = -\dfrac{5}{2}$

$u = \dfrac{1}{4}$ $\qquad$ $u = -\dfrac{5}{2}$

$\dfrac{1}{x} = \dfrac{1}{4}$ $\qquad$ $\dfrac{1}{x} = -\dfrac{5}{2}$

$x = 4$ $\qquad$ $x = -\dfrac{2}{5}$

57. $\dfrac{x}{x^2-4} + \dfrac{1}{x+2} = 3$

$(x+2)(x-2)\dfrac{x}{x^2-4} + (x+2)(x-2)\dfrac{1}{x+2} = 3(x+2)(x-2)$

$x + x - 2 = 3x^2 - 12$

$3x^2 - 2x - 10 = 0$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-10)}}{2(3)}$

$= \dfrac{2 \pm \sqrt{124}}{6} = \dfrac{2 \pm 2\sqrt{31}}{6} = \dfrac{1 \pm \sqrt{31}}{3}$

58.
$$\frac{x+1}{3} - \frac{x+1}{x+2} = 0$$
$$3(x+2)\frac{x+1}{3} - 3(x+2)\frac{x+1}{x+2} = 0$$
$$(x+2)(x+1) - 3(x+1) = 0$$
$$x^2 + 3x + 2 - 3x - 3 = 0$$
$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x - 1 = 0 \Rightarrow x = 1$$

59. $|2x - 5| = 11$
$$2x - 5 = 11 \Rightarrow x = 8$$
$$-(2x - 5) = 11 \Rightarrow x = -3$$

60. $|3x + 2| = 7$
$$3x + 2 = 7 \Rightarrow x = \tfrac{5}{3}$$
$$-(3x + 2) = 7$$
$$-3x - 2 = 7 \Rightarrow x = -3$$

61. $|x| = x^2 + x - 24$

First equation:
$$x = x^2 + x - 24$$
$$x^2 - 24 = 0$$
$$x^2 = 24$$
$$x = \pm 2\sqrt{6}$$

Second equation:
$$-x = x^2 + x - 24$$
$$x^2 + 2x - 24 = 0$$
$$(x+6)(x-4) = 0$$
$$x + 6 = 0 \Rightarrow x = -6$$
$$x - 4 = 0 \Rightarrow x = 4$$

Only $x = 2\sqrt{6}$ and $x = -6$ are solutions of the original equation. $x = -2\sqrt{6}$ and $x = 4$ are extraneous.

62. $|x^2 + 6x| = 3x + 18$

First equation:
$$x^2 + 6x = 3x + 18$$
$$x^2 + 3x - 18 = 0$$
$$(x-3)(x+6) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 6 = 0 \Rightarrow x = -6$$

Second equation:
$$-(x^2 + 6x) = 3x + 18$$
$$0 = x^2 + 9x + 18$$
$$0 = (x+3)(x+6)$$
$$0 = x + 3 \Rightarrow x = -3$$
$$x = x + 6 \Rightarrow x = -6$$

The solutions of the original equation are $x = \pm 3$ and $x = -6$.

63. $|x + 1| = x^2 - 5$

First equation:
$$x + 1 = x^2 - 5$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 2 = 0 \Rightarrow x = -2$$

Second equation:
$$-(x+1) = x^2 - 5$$
$$-x - 1 = x^2 - 5$$
$$x^2 + x - 4 = 0$$
$$x = \frac{-1 + \sqrt{17}}{2}$$

Only $x = 3$ and $x = \dfrac{-1 - \sqrt{17}}{2}$ are solutions of the original equation. $x = -2$ and $x = \dfrac{-1 + \sqrt{17}}{2}$ are extraneous.

64. $|x - 5| = x^2 - 15x$

First equation:

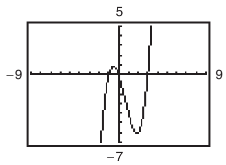
$$\begin{aligned} x - 15 &= x^2 - 15x \\ x^2 - 16x + 15 &= 0 \\ (x - 1)(x - 15) &= 0 \\ x - 1 = 0 &\Rightarrow x = 1 \\ x - 15 = 0 &\Rightarrow x = 15 \end{aligned}$$

Second equation:

$$\begin{aligned} -(x - 15) &= x^2 - 15x \\ x^2 - 14x - 15 &= 0 \\ (x + 1)(x - 15) &= 0 \\ x + 1 = 0 &\Rightarrow x = -1 \\ x - 15 = 0 &\Rightarrow x = 15 \end{aligned}$$

Only $x = 15$ and $x = -1$ are solutions of the original equation. $x = 1$ is extraneous.

65. (a)

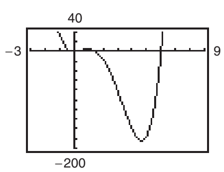


(b) x-intercepts: $(-1, 0), (0, 0), (3, 0)$

(c)
$$\begin{aligned} 0 &= x^3 - 2x^2 - 3x \\ 0 &= x(x + 1)(x - 3) \\ x &= 0 \\ x + 1 = 0 &\Rightarrow x = -1 \\ x - 3 = 0 &\Rightarrow x = 3 \end{aligned}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

66. (a)

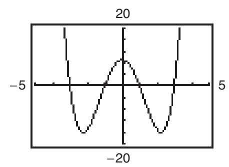


(b) x-intercepts: $(0, 0), \left(\frac{3}{2}, 0\right), (6, 0)$

(c)
$$\begin{aligned} y &= 2x^4 - 15x^3 + 18x^2 \\ 0 &= 2x^4 - 15x^3 + 18x^2 \\ &= x^2(2x^2 - 15x + 18) \\ &= x^2(2x - 3)(x - 6) \\ 0 = x^2 &\Rightarrow x = 0 \\ 0 = 2x - 3 &\Rightarrow x = \tfrac{3}{2} \\ 0 = x - 6 &\Rightarrow x = 6 \end{aligned}$$

(d) The x-intercepts and the solutions are the same.

67. (a)



(b) x-intercepts: $(\pm1, 0), (\pm3, 0)$

(c)
$$\begin{aligned} 0 &= x^4 - 10x^2 + 9 \\ 0 &= (x^2)^2 - 10(x^2) + 9 \end{aligned}$$

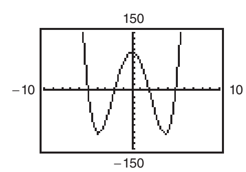
Let $u = x^2$.

$$0 = (u - 1)(u - 9)$$

$$\begin{aligned} u - 1 &= 0 & u - 9 &= 0 \\ u &= 1 & u &= 9 \\ x^2 &= 1 & x^2 &= 9 \\ x &= \pm1 & x &= \pm3 \end{aligned}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

68. (a)



(b) x-intercepts: $(-2, 0), (2, 0), (-5, 0), (5, 0)$

(c)
$$\begin{aligned} y &= x^4 - 29x^2 + 100 \\ 0 &= x^4 - 29x^2 + 100 \\ 0 &= (x^2)^2 - 29(x^2) + 100 \end{aligned}$$

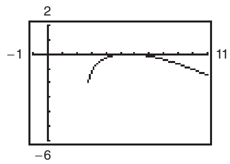
Let $u = x^2$.

$$\begin{aligned} 0 &= u^2 - 29u + 100 \\ 0 &= (u - 4)(u - 25) \end{aligned}$$

$$\begin{aligned} u - 4 &= 0 & u - 25 &= 0 \\ u &= 4 & u &= 25 \\ x^2 &= 4 & x^2 &= 25 \\ x &= \pm2 & x &= \pm5 \end{aligned}$$

(d) The x-intercepts and the solutions are the same.

69. (a)

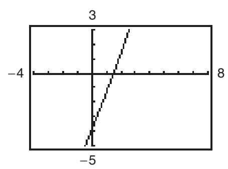


(b) x-intercepts: $(5, 0), (6, 0)$

(c)
$$\begin{aligned}
0 &= \sqrt{11x - 30} - x\\
x &= \sqrt{11x - 30}\\
x^2 &= 11x - 30\\
x^2 - 11x + 30 &= 0\\
(x - 5)(x - 6) &= 0\\
x - 5 = 0 &\Rightarrow x = 5\\
x - 6 = 0 &\Rightarrow x = 6
\end{aligned}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

70. (a)

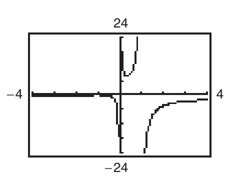


(b) x-intercept: $\left(\frac{3}{2}, 0\right)$

(c)
$$\begin{aligned}
y &= 2x - \sqrt{15 - 4x}\\
0 &= 2x - \sqrt{15 - 4x}\\
\sqrt{15 - 4x} &= 2x\\
15 - 4x &= 4x^2\\
0 &= 4x^2 + 4x - 15\\
0 &= (2x + 5)(2x - 3)\\
0 &= 2x + 5 \Rightarrow x = -\tfrac{5}{2}, \text{ extraneous}\\
0 &= 2x - 3 \Rightarrow x = \tfrac{3}{2}\\
x &= \tfrac{3}{2}
\end{aligned}$$

(d) The x-intercept and the solution are the same.

71. (a)

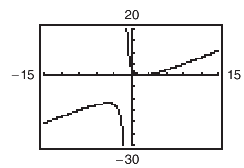


(b) x-intercept: $(-1, 0)$

(c)
$$\begin{aligned}
0 &= \frac{1}{x} - \frac{4}{x - 1} - 1\\
0 &= (x - 1) - 4x - x(x - 1)\\
0 &= x - 1 - 4x - x^2 + x\\
0 &= -x^2 - 2x - 1\\
0 &= x^2 + 2x + 1\\
0 &= (x + 1)^2\\
x + 1 = 0 &\Rightarrow x = -1
\end{aligned}$$

(d) The x-intercept of the graph is the same as the solution of the equation.

72. (a)

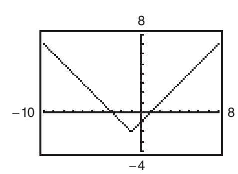


(b) x-intercept: $(2, 0)$

(c)
$$\begin{aligned}
0 &= x + \frac{9}{x + 1} - 5\\
0 &= x + \frac{9}{x + 1} - 5\\
0 &= x(x + 1) + (x + 1)\frac{9}{x + 1} - 5(x + 1)\\
0 &= x^2 + x + 9 - 5x - 5\\
0 &= x^2 - 4x + 4\\
0 &= (x - 2)(x - 2)\\
0 &= x - 2 \Rightarrow x = 2\\
x &= 2
\end{aligned}$$

(d) The x-intercept and the solution are the same.

73. (a)

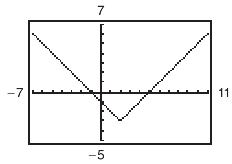


(b) x-intercepts: $(1, 0), (-3, 0)$

(c)
$$\begin{aligned}
0 &= |x + 1| - 2\\
2 &= |x + 1|
\end{aligned}$$
$$\begin{aligned}
x + 1 &= 2 \quad &\text{or}\quad -(x + 1) &= 2\\
x &= 1 \quad &\text{or}\quad -x - 1 &= 2\\
& & -x &= 3\\
& & x &= -3
\end{aligned}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

74. (a)



(b) x-intercepts: $(5, 0), (-1, 0)$

(c) $0 = |x - 2| - 3$

$3 = |x - 2|$

$x - 2 = 3 \Rightarrow x = 5$ or $-(x - 2) = 3$

$-x + 2 = 3 \Rightarrow x = -1$

$x = 5, -1$

(d) The x-intercepts and the solutions are the same.

75. $3.2x^4 - 1.5x^2 - 2.1 = 0$

$$x^2 = \frac{1.5 \pm \sqrt{1.5^2 - 4(3.2)(-2.1)}}{2(3.2)}$$

Using the positive value for x^2, we have

$$x = \pm\sqrt{\frac{1.5 + \sqrt{29.13}}{6.4}} \approx \pm 1.038.$$

76. $0.1x^4 - 2.4x^2 - 3.6 = 0$

$$x^2 = \frac{2.4 \pm \sqrt{(-2.4)^2 - 4(0.1)(-3.6)}}{2(0.1)} = \frac{2.4 \pm 7.2}{0.2}$$

Using the positive values for x^2,

$$x = \pm\sqrt{\frac{2.4 + \sqrt{7.2}}{0.2}} \approx \pm 5.041.$$

77. $7.08x^6 + 4.15x^3 - 9.6 = 0$

$a = 7.8, b = 4.15, c = -9.6$

$$x^3 = \frac{-4.15 \pm \sqrt{(4.15)^2 - 4(7.08)(-9.6)}}{2(7.08)}$$

$$= \frac{-4.15 \pm \sqrt{2.89.0945}}{14.16}$$

$$x = \sqrt[3]{\frac{-4.15 + \sqrt{289.0945}}{14.16}} \approx 0.968$$

$$x = \sqrt[3]{\frac{-4.15 - \sqrt{289.0945}}{14.16}} \approx -1.143$$

78. $5.25x^6 - 0.2x^3 - 1.55 = 0$

$$x^3 = \frac{0.2 \pm \sqrt{(-0.2)^2 - 4(5.25)(-1.55)}}{2(5.25)}$$

$$= \frac{0.2 \pm \sqrt{32.59}}{10.5}$$

$$x = \sqrt[3]{\frac{0.2 + \sqrt{32.59}}{10.5}} \approx 0.826$$

$$x = \sqrt[3]{\frac{0.2 - \sqrt{32.59}}{10.5}} \approx -0.807$$

79. $1.8x - 6\sqrt{x} - 5.6 = 0$ Given equation

$1.8\left(\sqrt{x}\right)^2 - 6\sqrt{x} - 5.6 = 0$

Use the Quadratic Formula with $a = 1.8, b = -6,$ and $c = -5.6.$

$$\sqrt{x} = \frac{6 \pm \sqrt{36 - 4(1.8)(-5.6)}}{2(1.8)} \approx \frac{6 \pm 8.7361}{3.6}$$

Considering only the positive value for $\sqrt{x}$, we have

$\sqrt{x} \approx 4.0934$

$x \approx 16.756.$

80. $5.3x + 3.1 = 9.8\sqrt{x}$

$(5.3x + 3.1)^2 = \left(9.8\sqrt{x}\right)^2$

$28.09x^2 + 32.86x + 9.61 = 96.04x$

$28.09x^2 - 63.18x + 9.61 = 0$

$$x = \frac{-(-63.18) \pm \sqrt{(-63.18)^2 - 4(28.09)(9.61)}}{2(28.09)}$$

$$= \frac{63.18 \pm \sqrt{2911.9328}}{56.18}$$

$\approx 2.085, 0.164$

81. $4x^{2/3} + 8x^{1/3} + 3.6 = 0$

$a = 4, b = 8, c = 3.6$

$$x^{1/3} = \frac{-8 \pm \sqrt{8^2 - 4(4)(3.6)}}{2(4)}$$

$$x = \left[\frac{-8 + \sqrt{6.4}}{8}\right]^3 \approx -0.320$$

$$x = \left[\frac{-8 - \sqrt{6.4}}{8}\right]^3 \approx -2.280$$

82. $8.4x^{2/3} - 1.2x^{1/3} - 24 = 0$

$$x^{1/3} = \frac{1.2 \pm \sqrt{(-1.2)^2 - 4(8.4)(-24)}}{2(8.4)}$$

$$= \frac{1.2 \pm \sqrt{807.84}}{16.8}$$

$$x = \left(\frac{1.2 + \sqrt{807.84}}{16.8}\right)^3 \approx 5.482$$

$$x = \left(\frac{1.2 - \sqrt{807.84}}{16.8}\right)^3 \approx -4.255$$

83. $-4, 7$

Sample answer: $(x - (-4))(x - 7) = 0$

$$(x + 4)(x - 7) = 0$$

$$x^2 - 3x - 28 = 0$$

84. $0, 2, 9$

Sample answer: $(x - 0)(x - 2)(x - 9) = 0$

$$x(x - 2)(x - 9) = 0$$

$$x(x^2 - 11x + 18) = 0$$

$$x^3 - 11x^2 + 18x = 0$$

85. $-\frac{7}{3}, \frac{6}{7}$

One possible equation is:

$x = -\frac{7}{3} \Rightarrow 3x = -7 \Rightarrow 3x + 7$ is a factor.

$x = \frac{6}{7} \Rightarrow 7x = 6 \Rightarrow 7x - 6$ is a factor.

$$(3x + 7)(7x - 6) = 0$$

$$21x^2 + 31x - 42 = 0$$

Any non-zero multiple of this equation would also have these solutions.

86. $-\frac{1}{8}, -\frac{4}{5}$

$$\left(x - \left(-\tfrac{1}{8}\right)\right)\left(x - \left(-\tfrac{4}{5}\right)\right) = 0$$

$$\left(x + \tfrac{1}{8}\right)\left(x + \tfrac{4}{5}\right) = 0$$

$$x^2 + \tfrac{4}{5}x + \tfrac{1}{8}x + \tfrac{4}{40} = 0$$

$$40x^2 + 32x + 5x + 4 = 0$$

$$40x^2 + 37x + 4 = 0$$

Any non-zero multiple of this equation would also have these solutions.

87. $\sqrt{3}, -\sqrt{3}$, and 4

One possible equation is:

$$\left(x - \sqrt{3}\right)\left(x - \left(-\sqrt{3}\right)\right)(x - 4) = 0$$

$$\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x - 4) = 0$$

$$(x^2 - 3)(x - 4) = 0$$

$$x^3 - 4x^2 - 3x + 12 = 0$$

Any non-zero multiple of this equation would also have these solutions.

88. $2\sqrt{7}, -\sqrt{7}$

$$\left(x - 2\sqrt{7}\right)\left(x + \sqrt{7}\right) = 0$$

$$x^2 + x\sqrt{7} - 2x\sqrt{7} - 2(7) = 0$$

$$x^2 - x\sqrt{7} - 14 = 0$$

Any non-zero multiple of this equation would also have these solutions.

89. $i, -i$

Sample answer: $(x - i)(x - (-i)) = 0$

$$(x - i)(x + i) = 0$$

$$x^2 - i^2 = 0$$

$$x^2 + 1 = 0$$

90. $2i, -2i$

Sample answer: $(x - 2i)(x - (-2i)) = 0$

$$(x - 2i)(x + 2i) = 0$$

$$x^2 - 4i^2 = 0$$

$$x^2 + 4 = 0$$

91. $-1, 1, i$, and $-i$

One possible equation is:

$$(x - (-1))(x - 1)(x - i)(x - (-i)) = 0$$

$$(x + 1)(x - 1)(x - i)(x + i) = 0$$

$$(x^2 - 1)(x^2 + 1) = 0$$

$$x^4 - 1 = 0$$

Any non-zero multiple of this equation would also have these solutions.

92. $4i, -4i, 6, -6$

Sample answer: $(x - 4i)(x + 4i)(x - 6)(x + 6) = 0$

$$(x^2 + 16)(x^2 - 36) = 0$$

$$x^4 - 20x^2 - 576 = 0$$

93. *Labels*: Let $x =$ the number of students in the original group. Then, $\dfrac{1700}{x} =$ the original cost per student.

When six more students join the group, the cost per student becomes $\dfrac{1700}{x} - 7.50$.

Model: $(\text{Cost per student}) \cdot (\text{Number of students}) = (\text{Total cost})$

Equation: $\left(\dfrac{1700}{x} - 7.5\right)(x + 6) = 1700$

$(3400 - 15x)(x + 6) = 3400x$ Multiply both sides by $2x$ to clear fraction.

$-15x^2 - 90x + 20{,}400 = 0$

$$x = \frac{90 \pm \sqrt{(-90)^2 - 4(-15)(20{,}400)}}{2(-15)} = \frac{90 \pm 1110}{-30}$$

Using the positive value for x, we conclude that the original number was $x = 34$ students.

94. *Model:* $\begin{pmatrix}\text{Cost per}\\ \text{student}\end{pmatrix} \cdot \begin{pmatrix}\text{Number of}\\ \text{students}\end{pmatrix} = \begin{pmatrix}\text{Monthly}\\ \text{rent}\end{pmatrix}$

Labels: Monthly rent $= x$

Number of students $= 4$

Original cost per student $= \dfrac{x}{3}$

Cost per student $= \dfrac{x}{3} - 150$

Equation: $\left(\dfrac{x}{3} - 150\right)(4) = x$

$\dfrac{4x}{3} - 600 = x$

$\dfrac{4x}{3} - x = 600$

$\dfrac{x}{3} = 600$

$x = 1800$

The monthly rent is \$1800.

95. *Model:* $\text{Time} = \dfrac{\text{Distance}}{\text{Rate}}$

Labels: Let $x =$ average speed of the plane. Then we have a travel time of $t = 145/x$. If the average speed is increased by 40 mph, then

$t - \dfrac{12}{60} = \dfrac{145}{x + 40}$

$t = \dfrac{145}{x + 40} + \dfrac{1}{5}.$

Now, we equate these two equations and solve for x.

Equation: $\dfrac{145}{x} = \dfrac{145}{x + 40} + \dfrac{1}{5}$

$145(5)(x + 40) = 145(5)x + x(x + 40)$

$725x + 29{,}000 = 725x + x^2 + 40x$

$0 = x^2 + 40x - 29{,}000$

Using the positive value for x found by the Quadratic Formula, we have $x \approx 151.5$ mph and $x + 40 = 191.5$ mph. The airspeed required to obtain the decrease in travel time is 191.5 miles per hour.

96. *Model:* $(\text{Rate}) \cdot (\text{time}) = (\text{distance})$

Labels: Distance $= 1080$

Original time $= t$

Original rate $= \dfrac{1080}{t}$

Return time $= t + 2.5$

Return rate $= \dfrac{1080}{t} - 6$

Equation: $\left(\dfrac{1080}{t} - 6\right)(t + 2.5) = 1080$

$$1080 + \frac{2700}{t} - 6t - 15 = 1080$$
$$\frac{2700}{t} - 6t - 15 = 0$$
$$270 - 0 - 6t^2 - 15t = 0$$
$$2t^2 + 5t - 900 = 0$$
$$(2t + 45)(t - 20) = 0$$
$$2t + 45 = 0 \Rightarrow t = -22.5$$
$$t - 20 = 0 \Rightarrow t = 20$$

The average speed was $\frac{1080}{20} = 54$ miles per hour.

97.
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$2694.58 = 2500\left(1 + \frac{r}{12}\right)^{(12)(5)}$$
$$\frac{2694.58}{2500} = \left(1 + \frac{r}{12}\right)^{60}$$
$$1.077832 = \left(1 + \frac{r}{12}\right)^{60}$$
$$(1.077832)^{1/60} = 1 + \frac{r}{12}$$
$$\left[(1.0.77832)^{1/60} - 1\right](12) = r$$
$$r \approx 0.015 = 1.5\%$$

98.
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$7734.27 = 6000\left(1 + \frac{r}{4}\right)^{(4)(5)}$$
$$\frac{7734.27}{6000} = \left(1 + \frac{r}{4}\right)^{20}$$
$$1.289045 = \left(1 + \frac{r}{4}\right)^{20}$$
$$(1.289045)^{1/20} = 1 + \frac{r}{4}$$
$$\left[(1.289045)^{1/20}\right](4) = r$$
$$r \approx 0.0511 = 5.1\%$$

99. When $C = 2.5$ we have:

$$2.5 = \sqrt{0.2x + 1}$$
$$6.25 = 0.2x + 1$$
$$5.25 = 0.2x$$
$$x = 26.25 = 26{,}250 \text{ passengers}$$

100. (a)
$$N = \sqrt{734.024 + 1839.666t}$$
$$135 = \sqrt{734.024 + 1839.666t}$$
$$(135)^2 = \left(\sqrt{734.024 + 1839.666t}\right)^2$$
$$18{,}225 = 734.024 + 1839.666t$$
$$17{,}490.976 = 1839.666t$$
$$9.508 \approx t$$

So, the number of nurses reached 135,000 in the year 2009.

(b)
$$N = \sqrt{734.024 + 1839.666t}$$
$$200 = \sqrt{734.024 + 1839.666t}$$
$$(200)^2 = \left(\sqrt{734.024 + 1839.666t}\right)^2$$
$$40{,}000 = 734.024 + 1839.666t$$
$$39{,}265.976 = 1839.666t$$
$$21.34 \approx t$$

Using the model, the number of nurses will reach 200,000 in the year 2021. Answers will vary. *Sample answer*: Yes, it is reasonable that the number of registered nurses continues to increase at a rate in which the number reaches 200,000 in the year 2021.

101. $T = 75.82 - 2.11x + 43.51\sqrt{x},\ 5 \le x \le 40$

(a) $212 = 75.82 - 2.11x + 43.51\sqrt{x}$

$0 = -2.11x + 43.51\sqrt{x} - 136.18$

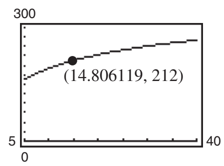
By the Quadratic Formula, we have

$\sqrt{x} \approx 16.77928 \Rightarrow x \approx 281.333$

$\sqrt{x} \approx 3.84787 \Rightarrow x \approx 14.806.$

Since x is restricted to $5 \le x \le 40$, let $x = 14.806$ pounds per square inch.

(b)



102. (a)

$$P = \frac{184.64 + 0.7524t^2}{1 + 0.0028t^2}$$

$$\frac{184.64 + 0.7524t^2}{1 + 0.0028t^2} = 210$$

$$184.64 + 0.7524t^2 = 210\left(1 + 0.0028t^2\right)$$

$$184.64 + 0.7524t^2 = 210 + 0.588t^2$$

$$0.1644t^2 = 25.36$$

$$t^2 = \frac{25.36}{0.1644}$$

$$t \approx 12.42 \rightarrow \text{during } 2002$$

(b) To find the year when the total voting age population is expected to reach 260 million, let $P = 260$ and solve for t.

$$P = \frac{184.64 + 0.7524t^2}{1 + 0.0028t^2}$$

$$\frac{184.64 + 0.7524t^2}{1 + 0.0028t^2} = 260$$

$$184.64 + 0.7524t^2 = 260\left(1 + 0.0028t^2\right)$$

$$184.64 + 0.7524t^2 = 260 + 0.728t^2$$

$$0.0244t^2 = 75.36$$

$$t^2 = \frac{75.36}{0.0244}$$

$$t \approx 55.57 \rightarrow \text{during } 2045$$

This prediction is reasonable if population of the United States in general continues to increase at a similar rate as it had during the years 1990 through 2014. You could assume that the voting age population would increase at a rate similar to that for which the model was created.

103.

$$37.55 = 40 - \sqrt{0.01x + 1}$$

$$\sqrt{0.01x + 1} = 2.45$$

$$0.01x + 1 = 6.0025$$

$$0.01x = 5.0025$$

$$x = 500.25$$

Rounding x to the nearest whole unit yields $x \approx 500$ units.

104. *Verbal Model:* Total cost $=$ Cost underwater $\cdot$ Distance underwater $+$ Cost overland $\cdot$ Distance overland

Labels: Total cost : \$1,098,662.40

Cost overland: \$24 per foot

Distance overland in feet: $5280(8 - x)$

Cost underwater: \$30 per foot

Distance underwater in feet: $5280\sqrt{x^2 + (3/4)^2} = 5280\sqrt{\dfrac{16x^2 + 9}{16}} = 1320\sqrt{16x^2 + 9}$

Equation:

$$\begin{aligned}
1{,}098{,}662.40 &= 30\left(1320\sqrt{16x^2 + 9}\right) + 24\left[5280(8 - x)\right] \\
1{,}098{,}662.40 &= 39{,}600\sqrt{16x^2 + 9} + 126{,}720(8 - x) \\
1{,}098{,}662.40 &= 7920\left[5\sqrt{16x^2 + 9} + 16(8 - x)\right] \\
138.72 &= 5\sqrt{16x^2 + 9} + 16(8 - x) \\
138.72 &= 5\sqrt{16x^2 + 9} + 128 - 16x \\
16x + 10.72 &= 5\sqrt{16x^2 + 9} \\
(16x + 10.72)^2 &= \left(5\sqrt{16x^2 + 9}\right)^2 \\
256x^2 + 343.04x + 114.9184 &= 25\left(16x^2 + 9\right) \\
256x^2 + 343.04x + 114.9184 &= 400x^2 + 225 \\
0 &= 144x^2 - 343.04x + 110.0816
\end{aligned}$$

By the Quadratic Formula, $x \approx 2$ or $x \approx 0.382$.

So, the length of x could either be 0.382 mile or 2 miles.

105.

$$\begin{aligned}
\frac{1}{t} + \frac{1}{t + 3} &= \frac{1}{y} \\
\frac{1}{t} + \frac{1}{t + 3} &= \frac{1}{2} \\
2t(t + 3)\frac{1}{t} + 2t(t + 3)\frac{1}{t + 3} &= 2t(t + 3)\frac{1}{2} \\
2(t + 3) + 2t &= t(t + 3) \\
2t + 6 + 2t &= t^2 + 3t \\
0 &= t^2 - t - 6 \\
0 &= (t - 3)(t + 2) \\
t - 3 = 0 &\Rightarrow t = 3 \\
t + 2 = 0 &\Rightarrow t = -2
\end{aligned}$$

Since t represents time, $t = 3$ is the only solution. It takes 3 hours for you working alone to tile the floor.

106.

$$\frac{1}{t} + \frac{1}{t+2} = \frac{1}{y}$$

$$\frac{1}{t} + \frac{1}{t+2} = \frac{1}{3}$$

$$3t(t+2)\left(\frac{1}{t}\right) + 3t(t+2)\left(\frac{1}{t+2}\right) = 3t(t+2)\left(\frac{1}{3}\right)$$

$$3(t+2) + 3t = t(t+2)$$

$$3t + 6 + 3t = t^2 + 2t$$

$$0 = t^2 - 4t - 6$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{40}}{2}$$

$$= \frac{4 \pm 2\sqrt{10}}{2}$$

$$= \frac{2\left(2 \pm \sqrt{10}\right)}{2}$$

$$= 2 \pm \sqrt{10}$$

$$t \approx 5.2 \text{ or } t \approx -1.2$$

Since t represents time, $t \approx 5.2$ is the only solution. It takes approximately 5.2 hours for you working alone to paint the fence.

107.

$$d = \sqrt{\frac{2U}{k}}$$

$$d^2 = \frac{2U}{k}$$

$$d^2k = 2U$$

$$\frac{kd^2}{2} = U$$

108.

$$C = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$$

$$C^2 = \left(2\pi\sqrt{\frac{a^2 + b^2}{2}}\right)^2$$

$$\frac{C^2}{4\pi^2} = \frac{a^2 + b^2}{2}$$

$$\frac{2C^2}{4\pi^2} = a^2 + b^2$$

$$\frac{C^2}{2\pi^2} - b^2 = a^2$$

$$a = \pm\sqrt{\frac{C^2}{2\pi^2} - b^2}$$

109. False. See Example 7 on page 125.

110.

$$\sqrt{x+10} - \sqrt{x-10} = 0$$

$$\sqrt{x+10} = \sqrt{x-10}$$

$$x + 10 = x - 10$$

$$10 \neq -10$$

True. There is no value to satisfy this equation.

111. The distance between $(3, -5)$ and $(x, 7)$ is 13.

$$\sqrt{(x-3)^2 + (-5-7)^2} = 13$$

$$(x-3)^2 + (-12)^2 = 13^2$$

$$x^2 - 6x + 9 + 144 = 169$$

$$x^2 - 6x - 16 = 0$$

$$(x-8)(x+2) = 0$$

$$x - 8 = 0 \Rightarrow x = 8$$

$$x + 2 = 0 \Rightarrow x = -2$$

Both $(8, 7)$ and $(-2, 7)$ are a distance of 13 from $(3, -5)$.

112. The distance between $(10, y)$ and $(4, -3)$ is 10.

$$\begin{aligned}\sqrt{(10-4)^2 + (y-(-3))^2} &= 10\\ (6)^2 + (y+3)^2 &= 10^2\\ 36 + (y+3)^2 &= 100\\ (y+3)^2 &= 64\\ y+3 &= \pm 8\\ y &= -3 \pm 8 = -11, 5\end{aligned}$$

Both $(10, -11)$ and $(10, 5)$ are a distance of 10 from $(4, -3)$.

113. The quadratic equation was not written in general form before the values of *a*, *b*, and *c* were substituted in the Quadratic Formula. As a result, the substitutions in the Quadratic Formula are incorrect.

114. (a) The formula for volume of the glass cube is V = Length × Width × Height. The volume of water in the cube is the length × width × height of the water. So, the volume is $x \cdot x \cdot (x-3) = x^2(x-3)$.

(b) Given the equation $x^2(x-3) = 320$. The dimensions of the glass cube can be found by solving for *x*. Then, substitute that value into the expression x^3 to find the volume of the cube.

115. $9 + |9-a| = b$

$|9-a| = b - 9$

$$\begin{aligned}9-a &= b-9 &\quad\text{or}\quad 9-a &= -(b-9)\\ -a &= b-18 & 9-a &= -b+9\\ a &= 18-b & -a &= -b\\ & & a &= b\end{aligned}$$

Thus, $a = 18 - b$ *or* $a = b$. From the original equation we know that $b \ge 9$.

Some possibilities are: $b = 9, a = 9$

$b = 10, a = 8$ *or* $a = 10$

$b = 11, a = 7$ *or* $a = 11$

$b = 12, a = 6$ *or* $a = 12$

$b = 13, a = 5$ *or* $a = 13$

$b = 14, a = 4$ *or* $a = 14$

116. Isolate the absolute value by subtracting *x* from both sides of the equations. The expression inside the absolute value signs can be positive or negative, so two separate equations must be solved. Each solution must be checked since extraneous solutions may be included.

117.
$$\begin{aligned}20 + \sqrt{20-1} &= b\\ \sqrt{20-a} &= b-20\\ 20-a &= b^2 - 40b + 400\\ -a &= b^2 - 40b + 380\\ a &= -b^2 + 40b - 380\end{aligned}$$

This formula gives the relationship between *a* and *b*. From the original equation we know that $a \le 20$ and $b \ge 20$. Choose a *b* value, where $b \ge 20$ and then solve for *a*, keeping in mind that $a \le 20$.

Some possibilities are: $b = 20, a = 20$

$b = 21, a = 19$

$b = 22, a = 16$

$b = 23, a = 11$

$b = 24, a = 4$

$b = 25, a = -5$

118. First isolate the radical by subtracting *x* from both sides of the equation. Then, square both sides and solve the resulting equation using the Quadratic Formula. Each solution must be checked since extraneous solutions may be included.

Section 1.7 Linear Inequalities in One Variable

1. solution set

2. graph

3. double

4. union

5. Interval: $[-2, 6)$

   Inequality: $-2 \le x < 6$; The interval is bounded.

6. Interval: $(-7, 4)$

Inequality: $-7 \le x \le 4$; The interval is bounded.

7. Interval: $[-1, 5]$

Inequality: $-1 \le x \le 5$; The interval is bounded.

8. Interval: $(2, 10]$

Inequality: $2 < x \le 10$; The interval is bounded.

9. Interval: $(11, \infty)$

Inequality: $x > 11$; The interval is unbounded.

10. Interval: $[-5, \infty)$

Inequality: $-5 \le x < \infty$ or $x \ge -5$; The interval is unbounded.

11. Interval: $(-\infty, -2)$

Inequality: $x < -2$; The interval is unbounded.

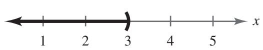
12. Interval: $(-\infty, 7]$

Inequality: $-\infty < x \le 7$ or $x \le 7$; The interval is unbounded.

13. $4x < 12$

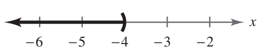
$\frac{1}{4}(4x) < \frac{1}{4}(12)$

$x < 3$



14. $10x < -40$

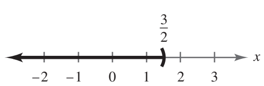
$x < -4$



15. $-2x > -3$

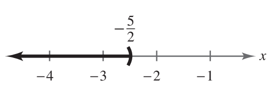
$-\frac{1}{2}(-2x) < \left(-\frac{1}{2}\right)(-3)$

$x < \frac{3}{2}$



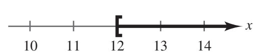
16. $-6x > 15$

$x < -\frac{15}{6}$ or $x < -\frac{5}{2}$



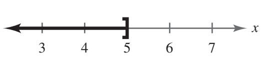
17. $x - 5 \ge 7$

$x \ge 12$



18. $x + 7 \le 12$

$x \le 5$



19. $2x + 7 < 3 + 4x$

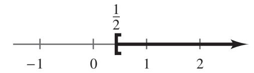
$-2x < -4$

$x > 2$

20. $3x + 1 \ge 2 + x$

$2x \ge 1$

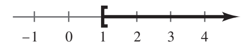
$x \ge \frac{1}{2}$



21. $3x - 4 \ge 4 - 5x$

$8x \ge 8$

$x \ge 1$



22. $6x - 4 \le 2 + 8x$

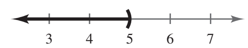
$-2x \le 6$

$x \ge -3$

23. $4 - 2x < 3(3 - x)$

$4 - 2x < 9 - 3x$

$x < 5$

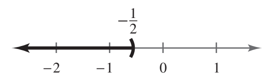


24. $4(x + 1) < 2x + 3$

$4x + 4 < 2x + 3$

$2x < -1$

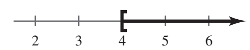
$x < -\frac{1}{2}$



25. $\frac{3}{4}x - 6 \le x - 7$

$-\frac{1}{4}x \le -1$

$x \ge 4$

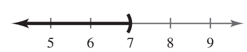


26. $3 + \frac{2}{7}x > x - 2$

$21 + 2x > 7x - 14$

$-5x > -35$

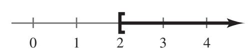
$x < 7$



27. $\frac{1}{2}(8x + 1) \ge 3x + \frac{5}{2}$

$4x + \frac{1}{2} \ge 3x + \frac{5}{2}$

$x \ge 2$

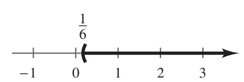


28. $9x - 1 < \frac{3}{4}(16x - 2)$

$36x - 4 < 48x - 6$

$-12x < -2$

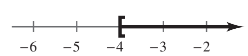
$x > \frac{1}{6}$



29. $3.6x + 11 \ge -3.4$

$3.6x \ge 14.4$

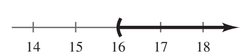
$x \ge -4$



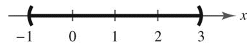
30. $15.6 - 1.3x < -5.2$

$-1.3x < -20.8$

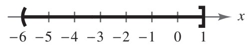
$x > 16$



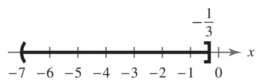
31. $1 < 2x + 3 < 9$
$-2 < 2x < 6$
$-1 < x < 3$



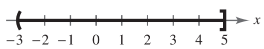
32. $-9 \le -2x - 7 < 5$
$-2 \le -2x < 12$
$1 \ge x > -6$
$-6 < x \le 1$



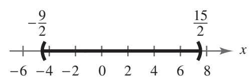
33. $0 < 3(x + 7) \le 20$
$0 < x + 7 \le \frac{20}{3}$
$-7 < x \le -\frac{1}{3}$



34. $-1 \le -(x - 4) < 7$
$1 \ge x - 4 > -7$
$5 \ge x > -3$
$-3 < x \le 5$

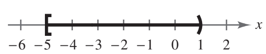


35. $-4 < \dfrac{2x - 3}{3} < 4$
$-12 < 2x - 3 < 12$
$-9 < 2x < 15$
$-\dfrac{9}{2} < x < \dfrac{15}{2}$

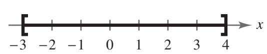


36. $0 \le \dfrac{x + 3}{2} < 5$
$0 \le x + 3 < 10$
$-3 \le x < 7$

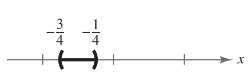
37. $-1 < \dfrac{-x - 2}{3} \le 1$
$-3 < -x - 2 \le 3$
$-1 < -x \le 5$
$1 > x \ge -5$
$-5 \le x < 1$



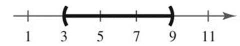
38. $-1 \le \dfrac{-3x + 5}{7} \le 2$
$-7 \le -3x + 5 \le 14$
$-12 \le -3x \le 9$
$4 \ge x \ge -3$
$-3 \le x \le 4$



39. $\frac{3}{4} > x + 1 > \frac{1}{4}$
$-\frac{1}{4} > x > -\frac{3}{4}$
$-\frac{3}{4} < x < -\frac{1}{4}$

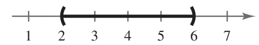


40. $-1 < 2 - \dfrac{x}{3} < 1$
$-3 < 6 - x < 3$
$-9 < -x < -3$
$3 < x < 9$



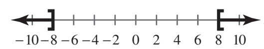
41. $3.2 \le 0.4x - 1 \le 4.4$
$4.2 \le 0.4x \le 5.4$
$10.5 \le x \le 13.5$

42. $1.6 < 0.3x + 1 < 2.8$
$0.6 < 0.3x < 1.8$
$2 < x < 6$

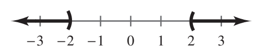


43. $|x| < 5$
$-5 < x < 5$

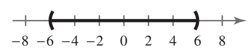
44. $|x| \ge 8$
$x \ge 8$ or $x \le -8$



45. $\left|\dfrac{x}{2}\right| > 1$
$\dfrac{x}{2} < -1$ or $\dfrac{x}{2} > 1$
$x < -2 \qquad x > 2$



46. $\left|\dfrac{x}{3}\right| < 2$
$-2 < \dfrac{x}{3} < 2$
$-6 < x < 6$



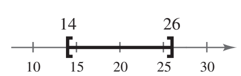
47. $|x - 5| < -1$

No solution. The absolute value of a number cannot be less than a negative number.

48. $|x - 7| < -5$

No solution. The absolute value of a number cannot be less than a negative number.

49. $|x - 20| \le 6$
$-6 \le x - 20 \le 6$
$14 \le x \le 26$



50. $|x - 8| \geq 0$

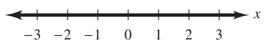
$x - 8 \geq 0$ or $-(x - 8) \geq 0$

$x \geq 8 \qquad -x + 8 \geq 0$

$-x \geq -8$

$x \leq 8$

All real numbers x



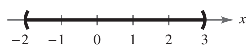
51. $|7 - 2x| \geq 9$

$7 - 2x \leq -9$ or $7 - 2x \geq 9$

$-2x \leq -16 \qquad -2x \geq 2$

$x \geq 8 \qquad x \leq -1$

52. $|1 - 2x| < 5$



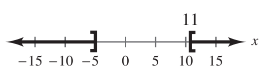
$-5 < 1 - 2x < 5$

$-6 < -2x < 4$

$3 > x > -2$

$-2 < x < 3$

53. $\left|\dfrac{x - 3}{2}\right| \geq 4$



$\dfrac{x - 3}{2} \leq -4$ or $\dfrac{x - 3}{2} \geq 4$

$x - 3 \leq -8 \qquad x - 3 \geq 8$

$x \leq -5 \qquad x \geq 11$

54. $\left|1 - \dfrac{2x}{3}\right| < 1$

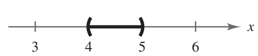
$-1 < 1 - \dfrac{2x}{3} < 1$

$-2 < -\dfrac{2x}{3} < 0$

$3 > x > 0$

$0 < x < 3$

55. $|9 - 2x| - 2 < -1$



$|9 - 2x| < 1$

$-1 < 9 - 2x < 1$

$-10 < -2x < -8$

$5 > x > 4$

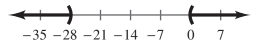
$4 < x < 5$

56. $|x + 14| + 3 > 17$

$|x + 14| > 14$

$x + 14 < -14$ or $x + 14 > 14$

$x < -28 \qquad x > 0$

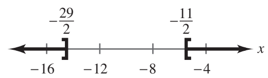


57. $2|x + 10| \geq 9$

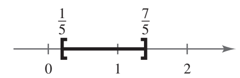
$|x + 10| \geq \frac{9}{2}$

$x + 10 \leq -\frac{9}{2}$ or $x + 10 \geq \frac{9}{2}$

$x \leq -\frac{29}{2} \qquad x \geq -\frac{11}{2}$



58. $3|4 - 5x| \leq 9$



$|4 - 5x| \leq 3$

$-3 \leq 4 - 5x \leq 3$

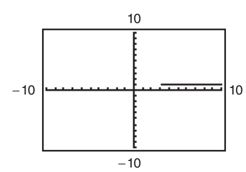
$-7 \leq -5x \leq -1$

$\frac{7}{5} \geq x \geq \frac{1}{5}$

$\frac{1}{5} \leq x \leq \frac{7}{5}$

59. $7x > 21$

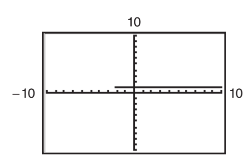
$x > 3$



60. $-4x \leq 9$

$x \geq -\dfrac{9}{4}$

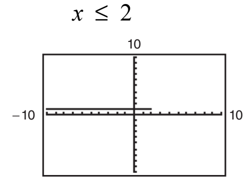
$x \geq -2.25$



61. $8 - 3x \geq 2$

$-3x \geq -6$

$x \leq 2$

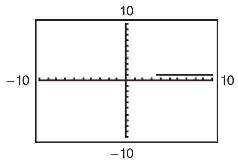


62. $20 < 6x - 1$

$21 < 6x$

$\frac{7}{2} < x$

$x > 3.5$

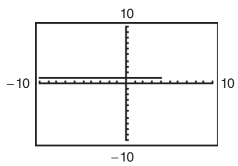


63. $4(x - 3) \le 8 - x$

$4x - 12 \le 8 - x$

$5x \le 20$

$x \le 4$

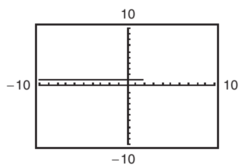


64. $3(x + 1) < x + 7$

$3x + 3 < x + 7$

$2x < 4$

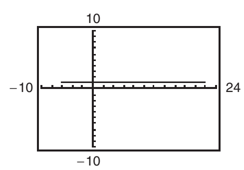
$x < 2$



65. $|x - 8| \le 14$

$-14 \le x - 8 \le 14$

$-6 \le x \le 22$

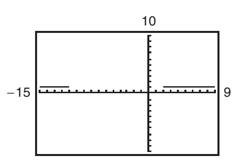


66. $|2x + 9| > 13$

$2x + 9 < -13 \quad \text{or} \quad 2x + 9 > 13$

$2x < -22 \qquad 2x > 4$

$x < -11 \qquad x > 2$

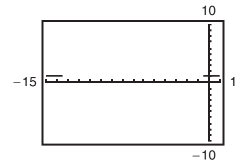


67. $2|x + 7| \ge 13$

$|x + 7| \ge \frac{13}{2}$

$x + 7 \le -\frac{13}{2} \quad \text{or} \quad x + 7 \ge \frac{13}{2}$

$x \le -\frac{27}{2} \qquad x \ge -\frac{1}{2}$

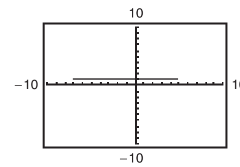


68. $\frac{1}{2}|x + 1| \le 3$

$|x + 1| \le 6$

$-6 \le x + 1 \le 6$

$-7 \le x \le 5$



69. $y = 3x - 1$

(a) $y \ge 2$

$3x - 1 \ge 2$

$3x \ge 3$

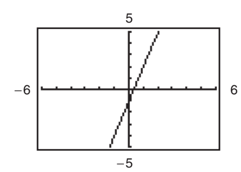
$x \ge 1$

(b) $y \le 0$

$3x - 1 \le 0$

$3x \le 1$

$x \le \frac{1}{3}$



70. $y = \frac{2}{3}x + 1$

(a) $y \le 5$

$\frac{2}{3}x + 1 \le 5$

$\frac{2}{3}x \le 4$

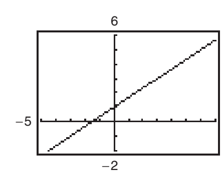
$x \le 6$

(b) $y \ge 0$

$\frac{2}{3}x + 1 \ge 0$

$\frac{2}{3}x \ge -1$

$x \ge -\frac{3}{2}$



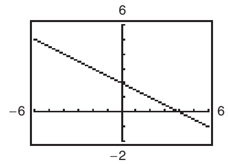
71. $y = -\frac{1}{2}x + 2$

(a) $0 \le y \le 3$

$0 \le -\frac{1}{2}x + 2 \le 3$

$-2 \le -\frac{1}{2}x \le 1$

$4 \ge x \ge -2$



(b) $y \ge 0$

$-\frac{1}{2}x + 2 \ge 0$

$-\frac{1}{2}x \ge -2$

$x \le 4$

72. $y = -3x + 8$

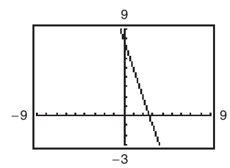
(a) $-1 \le y \le 3$

$-1 \le -3x + 8 \le 3$

$-9 \le -3x \le -5$

$3 \ge x \ge \frac{5}{3}$

$\frac{5}{3} \le x \le 3$



(b) $y \le 0$

$-3x + 8 \le 0$

$-3x \le -8$

$x \ge \frac{8}{3}$

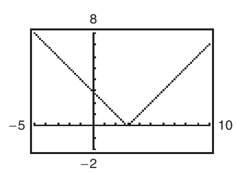
73. $y = |x - 3|$

(a) $y \le 2$

$|x - 3| \le 2$

$-2 \le x - 3 \le 2$

$1 \le x \le 5$



(b) $y \ge 4$

$|x - 3| \ge 4$

$x - 3 \le -4$ or $x - 3 \ge 4$

$x \le -1$ or $x \ge 7$

74. $y = \left|\frac{1}{2}x + 1\right|$

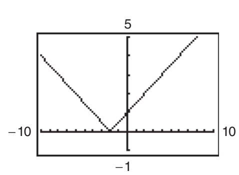
(a) $y \le 4$

$\left|\frac{1}{2}x + 1\right| \le 4$

$-4 \le \frac{1}{2}x + 1 \le 4$

$-5 \le \frac{1}{2}x \le 3$

$-10 \le x \le 6$



(b) $y \ge 1$

$\left|\frac{1}{2}x + 1\right| \ge 1$

$\frac{1}{2}x + 1 \le -1$ or $\frac{1}{2}x + 1 \ge 1$

$\frac{1}{2}x \le -2$ $\qquad$ $\frac{1}{2}x \ge 0$

$x \le -4$ $\qquad$ $x \ge 0$

75. All real numbers less than 8 units from 10.

76. $|x - 8| > 4$

All real numbers more than 4 units from 8

77. The midpoint of the interval $[-3, 3]$ is 0. The interval represents all real numbers x no more than 3 units from 0.

$|x - 0| \le 3$

$|x| \le 3$

78. The graph shows all real numbers more than 3 units from 0.

$|x - 0| > 3$

$|x| > 3$

79. The graph shows all real numbers at least 3 units from 7.

$|x - 7| \ge 3$

80. The graph shows all real numbers no more than 4 units from -1.

$|x + 1| \le 4$

81. All real numbers less than 3 units from 7

$|x - 7| \ge 3$

82. All real numbers at least 5 units from 8

$|x - 8| \ge 5$

83. All real numbers less than 4 units from -3

$|x - (-3)| < 4$

$|x + 3| < 4$

84. All real numbers no more than 7 units from -6

$|x + 6| \le 7$

85. $\$7.25 \le P \le \$7.75$

86. $180 < w < 185.5$

87. $r \le 0.08$

88. $I \ge \$239{,}000{,}000$

89. $r = 220 - A = 220 - 20 = 200$ beats per minute

$$0.50(200) \le r \le 0.85(200)$$
$$100 \le r \le 170$$

The target heart rate is at least 100 beats per minute and at most 170 beats per minute.

90. $r = 220 - A = 220 - 40 = 180$ beats per minute

$$0.50(180) \le r \le 0.85(180)$$
$$90 \le r \le 153$$

The target heartrate is at least 90 beats per minute and at most 153 beats per minute.

91. $9.00 + 0.75x > 13.50$

$$0.75x > 4.50$$
$$x > 6$$

You must produce at least 6 units each hour in order to yield a greater hourly wage at the second job.

92. $10.00 + 1.25x > 13.75$

$$1.25x > 3.75$$
$$x > 3$$

You must produce more than 3 units each hour in order to yield a greater hourly wage at the second job.

93. $1000(1 + r(10)) > 2000.00$

$$1 + 10r > 2$$
$$10r > 1$$
$$r > 0.1$$

The rate must be greater than 10%.

94. $750 < 500(1 + r(5))$

$$1.5 < 1 + 5r$$
$$0.5 < 5r$$
$$0.1 < r$$

The rate must be more than 10%.

95. $R > C$

$$115.95x > 95x + 750$$
$$20.95x > 750$$
$$x \ge 35.7995$$
$$x \ge 36 \text{ units}$$

96. $24.55x > 15.4x + 150{,}000$

$$9.15 > 150{,}000$$
$$x > 16{,}393.44262$$

Because the number of units x must be an integer, the product will return a profit when at least 16,394 units are sold.

97. Let $x =$ number of dozen doughnuts sold per day.

Revenue: $R = 7.95x$

Cost: $C = 1.45x + 165$

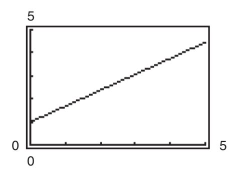
$$P = R - C$$
$$= 7.95x - (1.45x + 165)$$
$$= 6.50x - 165$$
$$400 \le P \le 1200$$
$$400 \le 6.50x - 165 \le 1200$$
$$565 \le 6.50x \le 1365$$
$$86.9 \le x \le 210$$

The daily sales vary between 87 and 210 dozen doughnuts per day.

98. The goal is to lose $164 - 128 = 36$ pounds. At $1\frac{1}{2}$ pounds per week, it will take 24 weeks.

$$36 \div 1\tfrac{1}{2} = 36 \times \tfrac{2}{3}$$
$$= 12 \times 2$$
$$= 24$$

99. (a)



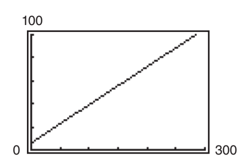
(b) From the graph you see that $y \ge 3$ when $x \ge 2.9$.

(c) Algebraically:

$$3 \le 0.692x + 0.988$$
$$2.012 \le 0.692x$$
$$2.91 \le x$$
$$x \ge 2.91$$

(d) IQ scores are not a good predictor of GPAs. Other factors include study habits, class attendance, and attitudes.

100. (a)



(b) One estimate is $x \le 224$ pounds.

(c) $0.33x + 6.20 \le 80$

$$0.33x \le 73.8$$
$$x \le 223.636$$

(d) An athlete's bench press load weight may not be a particularly good indicator of the athlete's barbell curl weight. Other factors, such as if the athlete focuses solely on his/her bench press maximum, other exercise habits, the athlete's hand strength, etc.

101. (a) $W = 0.903t + 26.08$

$30 \le 0.903t + 26.08 \le 32$

$3.92 \le 0.903t \le 6.56$

$4.34 \le t \le 5.92$

Between the years 2004 and 2006, the mean hourly wage was between \$30 and \$32.

(b) $0.903t + 26.08 \ge 45$

$0.903t \ge 18.92$

$t \ge 20.95$

The mean hourly wage will exceed \$45 sometime during the year 2020.

102. (a) $M = 3.00t + 163.3$

$180 < 3.00t + 163.3 \le 190$

$16.7 < 3.00t \le 26.7$

$5.6 < t \le 8.9$

Milk production was between 180 billion pounds and 190 billion pounds between the years 2005 and 2008.

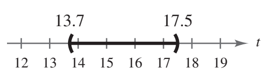
(b) $3.00t + 163.3 > 230$

$3.00t > 66.7$

$t > 22.2$

Milk production will exceed 230 billion pounds sometime during the year 2022.

103. $\left|\dfrac{t - 15.6}{1.9}\right| < 1$



$-1 < \dfrac{t - 15.6}{1.9} < 1$

$-1.9 < t - 15.6 < 1.9$

$13.7 < t < 17.5$

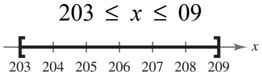
Two-thirds of the workers could perform the task in the time interval between 13.7 minutes and 17.5 minutes.

104. (a) $|x - 206| \le 3$

$-3 \le x - 206 \le 3$

$203 \le x \le 09$

(b)



105. $1 \text{ oz} = \dfrac{1}{16} \text{ lb, so } \dfrac{1}{2} \text{ oz} = \dfrac{1}{32} \text{ lb.}$

Because $8.99 \cdot \dfrac{1}{32} = 0.2809375$, you may be undercharged or overcharged by \$0.28.

106. $\dfrac{1}{10}(2.22) = \$0.22$ per gallon

You might have been undercharged or overcharged by \$0.22.

107. $|s - 10.4| \le \frac{1}{16}$

$-\frac{1}{16} \le s - 10.4 \le \frac{1}{16}$

$-0.0625 \le s - 10.4 \le 0.0625$

$10.3375 \le s \le 10.4625$

Because $A = s^2$,

$(10.3375)^2 \le \text{area} \le (10.4625)^2$

$106.864 \text{ in.}^2 \le \text{area} \le 109.464 \text{ in.}^2.$

108. $24.2 - 0.25 \le s \le 24.2 + 0.25$

$23.95 \le s \le 24.45$

The interval containing the possible side lengths s in centimeters of the square is $[23.95, 24.45]$, so the interval containing the possible areas in square centimeters is $\left[23.95^2, 24.45^2\right]$, or $[573.6025, 597.8025]$.

109. True. This is the Addition of a Constant Property of Inequalities.

110. False. If c is negative, then $ac \ge bc$.

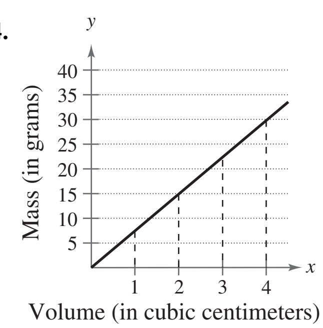
111. False. If $-10 \le x \le 8$, then $10 \ge -x$ and $-x \ge -8$.

112. True. This is the Multiplication by a Constant Property of Inequalities,

for $c < 0, a < b \Rightarrow ac > bc$

113. Answer not unique. Sample answer: $x < x + 1$

114.



(a) When the volume is 2 cubic centimeters, the mass is approximately 15 grams.

(b) When the volume is greater than or equal to 0 cubic centimeters, and less than 4 cubic centimeters, $0 \le x < 4$ the mass is greater than or equal to 0 grams and less than 30 grams, $0 \le y < 30$.

115. Answer not unique. *Sample answer:* $|ax - b| \le c$, if $a = 1, b = 5$, and $c = 5$, then

$|x - 5| \le 5$

$-5 \le x - 5 \le 5$

$0 \le x \le 10.$

Section 1.8 Other Types of Inequalities

1. positive; negative

2. key; test intervals

3. zeros; undefined values

4. $P = R - C$

5. $x^2 - 3 < 0$

(a) $x = 3$

$(3)^2 - 3 \overset{?}{<} 0$

$6 \not< 0$

No, $x = 3$ *is not* a solution.

(b) $x = 0$

$(0)^2 - 3 \overset{?}{<} 0$

$-3 < 0$

Yes, $x = 0$ *is* a solution.

(c) $x = \frac{3}{2}$

$\left(\frac{3}{2}\right)^2 - 3 \overset{?}{<} 0$

$-\frac{3}{4} < 0$

Yes, $x = \frac{3}{2}$ *is* a solution.

(d) $x = -5$

$(-5)^2 - 3 \overset{?}{<} 0$

$22 \not< 0$

No, $x = -5$ *is not* a solution

6. $x^2 - 2x - 8 \geq 0$

(a) $x = 5$

$(5)^2 - 2(5) - 8 \overset{?}{\geq} 0$

$7 \geq 0$

Yes, $x = 5$ *is* a solution.

(b) $x = 0$

$(0)^2 - 2(0) - 8 \overset{?}{\geq} 0$

$-8 \not\geq 0$

No, $x = 0$ *is not* a solution.

(c) $x = -4$

$(-4)^2 - 2(-4) - 8 \overset{?}{\geq} 0$

$16 + 8 - 8 \overset{?}{\geq} 0$

$16 \geq 0$

Yes, $x = -4$ *is* a solution.

(d) $x = 1$

$(1)^2 - 2(1) - 8 \overset{?}{\geq} 0$

$1 - 2 - 8 \overset{?}{\geq} 0$

$-9 \not\geq 0$

No, $x = 1$ *is not* a solution

7. $\dfrac{x + 2}{x - 4} \geq 3$

(a) $x = 5$

$\dfrac{5 + 2}{5 - 4} \overset{?}{\geq} 3$

$7 \geq 3$

Yes, $x = 5$ *is* a solution.

(b) $x = 4$

$\dfrac{4 + 2}{4 - 4} \overset{?}{\geq} 3$

$\dfrac{6}{0}$ is undefined.

No, $x = 4$ *is not* a solution.

(c) $x = -\dfrac{9}{2}$

$\dfrac{-\frac{9}{2} + 2}{-\frac{9}{2} - 4} \overset{?}{\geq} 3$

$\dfrac{5}{17} \not\geq 3$

No, $x = -\dfrac{9}{2}$ *is not* a solution.

(d) $x = \dfrac{9}{2}$

$\dfrac{\frac{9}{2} + 2}{\frac{9}{2} - 4} \overset{?}{\geq} 3$

$13 \geq 3$

Yes, $x = \dfrac{9}{2}$ *is* a solution.

8. $\dfrac{3x^2}{x^2 + 4} < 1$

(a) $x = -2$

$\dfrac{3(-2)^2}{(-2)^2 + 4} \overset{?}{<} 1$

$\dfrac{12}{8} \not< 1$

No, $x = -2$ *is not* a solution.

(b) $x = -1$

$\dfrac{3(-1)^2}{(-1)^2 + 4} \overset{?}{<} 1$

$\dfrac{3}{5} < 1$

Yes, $x = -1$ *is* a solution.

(c) $x = 0$

$\dfrac{3(0)^2}{(0)^2 + 4} \overset{?}{<} 1$

$0 < 1$

Yes, $x = 0$ *is* a solution.

(d) $x = 3$

$\dfrac{3(3)^2}{(3)^2 + 4} \overset{?}{<} 1$

$\dfrac{27}{13} \not< 1$

No, $x = 3$ *is not* a solution.

9. $x^2 - 3x - 18 = (x + 3)(x - 6)$

$x + 3 = 0 \Rightarrow x = -3$

$x - 6 = 0 \Rightarrow x = 6$

The key numbers are -3 and 6.

10. $9x^3 - 25x^2 = 0$

$x^2(9x - 25) = 0$

$x^2 = 0 \Rightarrow x = 0$

$9x - 25 = 0 \Rightarrow x = \frac{25}{9}$

The key numbers are 0 and $\frac{25}{9}$.

11. $\dfrac{1}{x - 5} + 1 = \dfrac{1 + 1(x - 5)}{x - 5}$

$= \dfrac{x - 4}{x - 5}$

$x - 4 = 0 \Rightarrow x = 4$

$x - 5 = 0 \Rightarrow x = 5$

The key numbers are 4 and 5.

12. $\dfrac{x}{x + 2} - \dfrac{2}{x - 1} = \dfrac{x(x - 1) - 2(x + 2)}{(x + 2)(x - 1)}$

$= \dfrac{x^2 - x - 2x - 4}{(x + 2)(x - 1)}$

$= \dfrac{(x - 4)(x + 1)}{(x + 2)(x - 1)}$

$(x - 4)(x + 1) = 0$

$x - 4 = 0 \Rightarrow x = 4$

$x + 1 = 0 \Rightarrow x = -1$

$(x + 2)(x - 1) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x - 1 = 0 \Rightarrow x = 1$

The key numbers are $-2, -1, 1,$ and 4.

13. $2x^2 + 4x < 0$

$2x(x + 2) < 0$

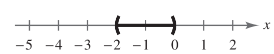
Key numbers: $x = 0, \ -2$

Test intervals: $(-\infty, -2), (-2, 0), (0, \infty)$

Test: Is $2x(x + 2) < 0$?

Interval	x-Value	Value of $2x(x + 2)$	Conclusion
$(-\infty, -2)$	-3	6	Positive
$(-2, 0)$	-1	-2	Negative
$(0, \infty)$	1	3	Positive

Solution set: $(-2, 0)$



14. $3x^2 - 9x \geq 0$

$3x(x - 3) \geq 0$

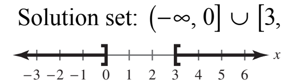
Key numbers: $x = 0, 3$

Test intervals: $(-\infty, 0), (0, 3), (3, \infty)$

Test: Is $3x(x - 3) > 0$?

Interval	x-Value	Value of $3x(x - 3)$	Conclusion
$(-\infty, 0)$	-1	12	Positive
$(0, 3)$	1	-6	Negative
$(3, \infty)$	4	12	Positive

Solution set: $(-\infty, 0] \cup [3, \infty)$



15.
$$x^2 < 9$$
$$x^2 - 9 < 0$$
$$(x+3)(x-3) < 0$$

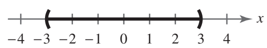
Key numbers: $x = \pm 3$

Test intervals: $(-\infty, -3), (-3, 3), (3, \infty)$

Test: Is $(x+3)(x-3) < 0$?

Interval	x-Value	Value of $x^2 - 9$	Conclusion
$(-\infty, -3)$	-4	7	Positive
$(-3, 3)$	0	-9	Negative
$(3, \infty)$	4	7	Positive

Solution set: $(-3, 3)$



16.
$$x^2 \le 25$$
$$x^2 - 25 \le 0$$
$$(x+5)(x-5) \le 0$$

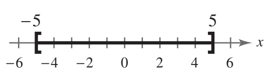
Key numbers: $x = \pm 5$

Test intervals: $(-\infty, -5), (-5, 5), (5, \infty)$

Test: Is $(x+5)(x-5) \le 0$?

Interval	x-Value	Value of $x^2 - 25$	Conclusion
$(-\infty, -5)$	-6	11	Positive
$(-5, 5)$	0	-25	Negative
$(5, \infty)$	6	11	Positive

Solution set: $[-5, 5]$



17.
$$(x+2)^2 \le 25$$
$$x^2 + 4x + 4 \le 25$$
$$x^2 + 4x - 21 \le 0$$
$$(x+7)(x-3) \le 0$$

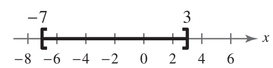
Key numbers: $x = -7, x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

Test: Is $(x+7)(x-3) \le 0$?

Interval	x-Value	Value of $(x+7)(x-3)$	Conclusion
$(-\infty, -7)$	-8	$(-1)(-11) = 11$	Positive
$(-7, 3)$	0	$(7)(-3) = -21$	Negative
$(3, \infty)$	4	$(11)(1) = 11$	Positive

Solution set: $[-7, 3]$



18.
$$(x-3)^2 \ge 1$$
$$x^2 - 6x + 8 \ge 0$$
$$(x-2)(x-4) \ge 0$$

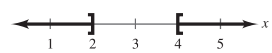
Key numbers: $x = 2, x = 4$

Test intervals: $(-\infty, 2) \Rightarrow (x-2)(x-4) > 0$

$(2, 4) \Rightarrow (x-2)(x-4) < 0$

$(4, \infty) \Rightarrow (x-2)(x-4) > 0$

Solution set: $(-\infty, 2] \cup [4, \infty)$



19.
$$x^2 + 6x + 1 \ge -7$$
$$x^2 + 6x + 8 \ge 0$$
$$(x+2)(x+4) \ge 0$$

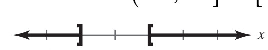
Key numbers: $x = -2, x = -4$

Test Intervals: $(-\infty, -4), (-4, -2), (-2, \infty)$

Test: Is $(x+2)(x+4) > 0$?

Interval	x-Value	Value of $(x+2)(x+4)$	Conclusion
$(-\infty, -4)$	-6	8	Positive
$(-4, -2)$	-3	-1	Negative
$(-2, \infty)$	0	8	Positive

Solution set: $(-\infty, -4] \cup [-2, \infty)$



20. $x^2 - 8x + 2 < 11$

$x^2 - 8x - 9 < 0$

$(x - 9)(x + 1) < 0$

Key numbers: $x = -1, x = 9$

Test intervals: $(-\infty, -1) \Rightarrow (x - 9)(x + 1) > 0$

$(-1, 9) \Rightarrow (x - 9)(x + 1) < 0$

$(9, \infty) \Rightarrow (x - 9)(x + 1) > 0$

Solution set: $(-1, 9)$

21. $x^2 + x < 6$

$x^2 + x - 6 < 0$

$(x + 3)(x - 2) < 0$

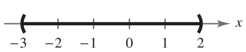
Key numbers: $x = -3, x = 2$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Test: Is $(x + 3)(x - 2) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 2)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-6) = 6$	Positive
$(-3, 2)$	0	$(3)(-2) = -6$	Negative
$(2, \infty)$	3	$(6)(1) = 6$	Positive

Solution set: $(-3, 2)$



22. $x^2 + 2x > 3$

$x^2 + 2x - 3 > 0$

$(x + 3)(x - 1) > 0$

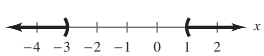
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 1) > 0$

$(-3, 1) \Rightarrow (x + 3)(x - 1) < 0$

$(1, \infty) \Rightarrow (x + 3)(x - 1) > 0$

Solution set: $(-\infty, -3) \cup (1, \infty)$



23. $x^2 < 3 - 2x$

$x^2 + 2x - 3 < 0$

$(x + 3)(x - 1) < 0$

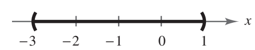
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3), (-3, 1), (1, \infty)$

Test: Is $(x + 3)(x - 1) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 1)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-5) = 5$	Positive
$(-3, 1)$	0	$(3)(-1) = -3$	Negative
$(1, \infty)$	2	$(5)(1) = 5$	Positive

Solution set: $(-3, 1)$



24. $x^2 > 2x + 8$

$x^2 - 2x - 8 > 0$

$(x - 4)(x + 2) > 0$

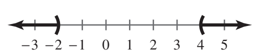
Key numbers: $x = -2, x = 4$

Test intervals: $(-\infty, -2), (-2, 4), (4, \infty)$

Test: Is $(x - 4)(x + 2) > 0$?

Interval	x-Value	Value of $(x - 4)(x + 2)$	Conclusion
$(-\infty, -2)$	-3	$(-7)(-1) = 7$	Positive
$(-2, 4)$	0	$(-4)(2) = -8$	Negative
$(4, \infty)$	5	$(1)(7) = 7$	Positive

Solution set: $(-\infty, -2) \cup (4, \infty)$



25. $$3x^2 - 11x > 20$$
$$3x^2 - 11x - 20 > 0$$
$$(3x + 4)(x - 5) > 0$$

Key numbers: $x = 5, x = -\frac{4}{3}$

Test intervals: $\left(-\infty, -\frac{4}{5}\right), \left(-\frac{4}{3}, 5\right), (5, \infty)$

Test: Is $(3x + 4)(x - 5) > 0$?

Interval	x-Value	Value of $(3x + 4)(x - 5)$	Conclusion
$\left(-\infty, -\frac{4}{3}\right)$	-3	$(-5)(-8) = 40$	Positive
$\left(-\frac{4}{3}, 5\right)$	0	$(4)(-5) = -20$	Negative
$(5, \infty)$	6	$(22)(1) = 22$	Positive

Solution set: $\left(-\infty, -\frac{4}{3}\right) \cup (5, \infty)$

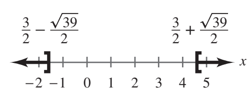
26. $$-2x^2 + 6x + 15 \le 0$$
$$2x^2 - 6x - 15 \ge 0$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)}$$
$$= \frac{6 \pm \sqrt{156}}{4}$$
$$= \frac{6 \pm 2\sqrt{39}}{4}$$
$$= \frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

Key numbers: $x = \frac{3}{2} - \frac{\sqrt{39}}{2}, x = \frac{3}{2} + \frac{\sqrt{39}}{2}$

Test intervals:

$$\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 < 0$$
$$\left(\frac{3}{2} - \frac{\sqrt{39}}{2}, \frac{3}{2} + \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 > 0$$
$$\left(\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right) \Rightarrow -2x^2 + 6x + 15 < 0$$

Solution set: $\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right] \cup \left[\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right)$



27. $$x^3 - 3x^2 - x + 3 > 0$$
$$x^2(x - 3) - (x - 3) > 0$$
$$(x - 3)(x^2 - 1) > 0$$
$$(x - 3)(x + 1)(x - 1) > 0$$

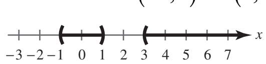
Key numbers: $x = -1, x = 1, x = 3$

Test intervals: $(-\infty\ -1), (-1, 1), (1, 3), (3, \infty)$

Test: Is $(x - 3)(x + 1)(x - 1) > 0$?

Interval	x-Value	Value of $(x - 3)(x + 1)(x - 1)$	Conclusion
$(-\infty, -1)$	-2	$(-5)(-1)(-3) = -15$	Negative
$(-1, 1)$	0	$(-3)(1)(-1) = 3$	Positive
$(1, 3)$	2	$(-1)(3)(1) = -3$	Negative
$(3, \infty)$	4	$(1)(5)(3) = 15$	Positive

Solution set: $(-1, 1) \cup (3, \infty)$



28. $x^3 + 2x^2 - 4x - 8 \le 0$

$x^2(x + 2) - 4(x + 2) \le 0$

$(x + 2)(x^2 - 4) \le 0$

$(x + 2)^2(x - 2) \le 0$

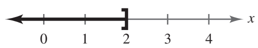
Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(-2, 2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(2, \infty) \Rightarrow x^3 + 2x^2 - 4x - 8 > 0$

Solution set: $(-\infty, 2]$



29. $-x^3 + 7x^2 + 9x > 63$

$x^3 - 7x^2 - 9x < -63$

$x^3 - 7x^2 - 9x + 63 < 0$

$x^2(x - 7) - 9(x - 7) < 0$

$(x - 7)(x^2 - 9) < 0$

$(x - 7)(x + 3)(x - 3) < 0$

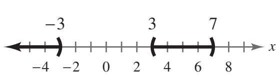
Key numbers: $x = -3, x = 3, x = 7$

Test intervals: $(-\infty, -3), (-3, 3), (3, 7), (7, \infty)$

Test: Is $(x - 7)(x + 3)(x - 3) < 0$?

Interval	x-Value	Value of $(x - 7)(x + 3)(x - 3)$	Conclusion
$(-\infty, -3)$	-4	$(-11)(-1)(-7) = -77$	Negative
$(-3, 3)$	0	$(-7)(3)(-3) = 63$	Positive
$(3, 7)$	4	$(-3)(7)(1) = -21$	Negative
$(7, \infty)$	8	$(1)(11)(5) = 55$	Positive

Solution set: $(-\infty, -3) \cup (3, 7)$



30. $$\begin{aligned} 2x^3 + 13x^2 - 8x - 46 &\ge 6 \\ 2x^3 + 13x^2 - 8x - 52 &\ge 0 \\ x^2(2x + 13) - 4(2x + 13) &\ge 0 \\ (2x + 13)(x^2 - 4) &\ge 0 \\ (2x + 13)(x + 2)(x - 2) &\ge 0 \end{aligned}$$

Key numbers: $x = -\frac{13}{2}, x = -2, x = 2$

Test intervals:

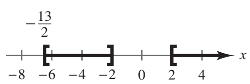
$\left(-\infty, -\frac{13}{2}\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$\left(-\frac{13}{2}, -2\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

$(-2, 2) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$(2, \infty) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

Solution set: $\left[-\frac{13}{2}, -2\right], [2, \infty)$



31. $$\begin{aligned} 4x^3 - 6x^2 &< 0 \\ 2x^2(2x - 3) &< 0 \end{aligned}$$

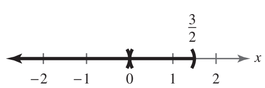
Key numbers: $x = 0, x = \frac{3}{2}$

Test intervals: $(-\infty, 0) \Rightarrow 2x^2(2x - 3) < 0$

$\left(0, \frac{3}{2}\right) \Rightarrow 2 \Rightarrow 2x^2(2x - 3) < 0$

$\left(\frac{3}{2}, \infty\right) \Rightarrow 2x^2(2x - 3) > 0$

Solution set: $(-\infty, 0) \cup \left(0, \frac{3}{2}\right)$



32. $$\begin{aligned} 4x^3 - 12x^2 &> 0 \\ 4x^2(x - 3) &> 0 \end{aligned}$$

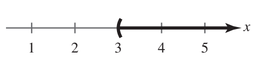
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow 4x^2(x - 3) < 0$

$(0, 3) \Rightarrow 4x^2(x - 3) < 0$

$(3, \infty) \Rightarrow 4x^2(x - 3) > 0$

Solution set: $(3, \infty)$



33. $$\begin{aligned} x^3 - 4x &\ge 0 \\ x(x + 2)(x - 2) &\ge 0 \end{aligned}$$

Key numbers: $x = 0, x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow x(x + 2)(x - 2) < 0$

$(-2, 0) \Rightarrow x(x + 2)(x - 2) > 0$

$(0, 2) \Rightarrow x(x + 2)(x - 2) < 0$

$(2, \infty) \Rightarrow x(x + 2)(x - 2) > 0$

Solution set: $[-2, 0] \cup [2, \infty)$

34. $$\begin{aligned} 2x^3 - x^4 &\le 0 \\ x^3(2 - x) &\le 0 \end{aligned}$$

Key numbers: $x = 0, x = 2$

Test intervals: $(-\infty, 0) \Rightarrow x^3(2 - x) < 0$

$(0, 2) \Rightarrow x^3(2 - x) > 0$

$(2, \infty) \Rightarrow x^3(2 - x) < 0$

Solution set: $(-\infty, 0] \cup [2, \infty)$

35. $(x - 1)^2(x + 2)^3 \ge 0$

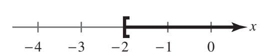
Key numbers: $x = 1, x = -2$

Test intervals: $(-\infty, -2) \Rightarrow (x - 1)^2(x + 2)^3 < 0$

$(-2, 1) \Rightarrow (x - 1)^2(x + 2)^3 > 0$

$(1, \infty) \Rightarrow (x - 1)^2(x + 2)^3 > 0$

Solution set: $[-2, \infty)$



36. $x^4(x - 3) \le 0$

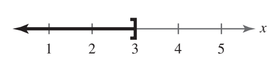
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow x^4(x - 3) < 0$

$(0, 3) \Rightarrow x^4(x - 3) < 0$

$(3, \infty) \Rightarrow x^4(x - 3) > 0$

Solution set: $(-\infty, 3]$



37. $4x^2 - 4x + 1 \le 0$

$(2x - 1)^2 \le 0$

Key number: $x = \frac{1}{2}$

Test Interval	x-Value	Polynomial Value	Conclusion
$\left(-\infty, \frac{1}{2}\right)$	$x = 0$	$[2(0) - 1]^2 = 1$	Positive
$\left(\frac{1}{2}, \infty\right)$	$x = 1$	$[2(1) - 1]^2 = 1$	Positive

The solution set consists of the single real number $\frac{1}{2}$.

38. $x^2 + 3x + 8 > 0$

Using the Quadratic Formula you can determine the key numbers are $x = -\dfrac{3}{2} \pm \dfrac{\sqrt{23}}{2}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 + 3(0) + 8 = 8$	Positive

The solution set is the set of all real numbers.

39. $x^2 - 6x + 12 \le 0$

Using the Quadratic Formula, you can determine that the key numbers are $x = 3 \pm \sqrt{3i}$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 - 6(0) + 12 = 12$	Positive

The solution set is empty, that is there are no real solutions.

40. $x^2 - 8x + 16 > 0$

$(x - 4)^2 > 0$

Key number: $x = 4$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, 4)$	$x = 0$	$(0 - 4)^2 = 16$	Positive
$(4, \infty)$	$x = 5$	$(5 - 4)^2 = 1$	Positive

The solution set consists of all real numbers except $x = 4$, or $(-\infty, 4) \cup (4, \infty)$.

41. $\dfrac{4x-1}{x} > 0$

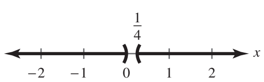
Key numbers: $x = 0, x = \dfrac{1}{4}$

Test intervals: $(-\infty, 0), \left(0, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $\dfrac{4x-1}{x} > 0$?

Interval	x-Value	Value of $\dfrac{4x-1}{x}$	Conclusion
$(-\infty, 0)$	-1	$\dfrac{-5}{-1} = 5$	Positive
$\left(0, \dfrac{1}{4}\right)$	$\dfrac{1}{8}$	$\dfrac{-\frac{1}{2}}{\frac{1}{8}} = -4$	Negative
$\left(\dfrac{1}{4}, \infty\right)$	1	$\dfrac{3}{1} = 3$	Positive

Solution set: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$



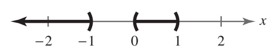
42.
$$\frac{x^2-1}{x} < 0$$
$$\frac{(x-1)(x+1)}{x} < 0$$

Key numbers: $x = -1, x = 0, x = 1$

Test intervals: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$

Interval	x-Value	Value of $\dfrac{(x-1)(x+1)}{x}$	Conclusion
$(-\infty, -1)$	-2	$\dfrac{(-3)(-1)}{-2} = -\dfrac{3}{2}$	Negative
$(-1, 0)$	$-\dfrac{1}{2}$	$\dfrac{\left(-\frac{3}{2}\right)\left(\frac{1}{2}\right)}{-\frac{1}{2}} = \dfrac{3}{2}$	Positive
$(0, 1)$	$\dfrac{1}{2}$	$\dfrac{\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right)}{\frac{1}{2}} = -\dfrac{3}{2}$	Negative
$(1, \infty)$	2	$\dfrac{(1)(3)}{2} = \dfrac{3}{2}$	Positive

Solution set: $(-\infty, -1) \cup (0, 1)$



43.
$$\frac{3x+5}{x-1} < 2$$
$$\frac{3x+5}{x-1} - 2 < 0$$
$$\frac{3x+5-2(x-1)}{x-1} < 0$$
$$\frac{x+7}{x-1} < 0$$

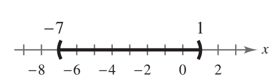
Key numbers: $x = -7, x = 1$

Test intervals: $(-\infty, -7), (-7, 1), (1, \infty)$

Test: Is $\dfrac{x+7}{x-1} < 0$?

Interval	x-Value	Value of $\dfrac{x+7}{x-1}$	Conclusion
$(-\infty, -7)$	-8	$\dfrac{-1}{-9} = \dfrac{1}{9}$	Positive
$(-7, 1)$	0	$\dfrac{0+7}{0-1} = -7$	Negative
$(1, \infty)$	2	$\dfrac{2+9}{2-1} = 11$	Positive

Solution set: $(-7, 1)$



44.

$$\frac{x+12}{x+2} \geq 3$$
$$\frac{x+12}{x+2} - 3 \geq 0$$
$$\frac{x+12-3(x+2)}{x+2} \geq 0$$
$$\frac{6-2x}{x+2} \geq 0$$

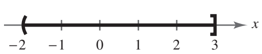
Key numbers: $x = -2, x = 3$

Test intervals: $(-\infty, -2), (-2, 3), (-3, \infty)$

Test: Is $\frac{6-2x}{x+2} > 0$?

Interval	x-Value	Value of $\frac{6-2x}{x+2}$	Conclusion
$(-\infty, -2)$	-3	$\frac{6-2(-3)}{(-3)+2} = -12$	Negative
$(-2, 3)$	0	$\frac{6-0}{0+2} = 3$	Positive
$(3, \infty)$	4	$\frac{6-8}{4+2} = \frac{1}{3}$	Negative

Solution set: $(-2, 3]$



45.

$$\frac{2}{x+5} > \frac{1}{x-3}$$
$$\frac{2}{x+5} - \frac{1}{x-3} > 0$$
$$\frac{2(x-3)-1(x+5)}{(x+5)(x-3)} > 0$$
$$\frac{x-11}{(x+5)(x-3)} > 0$$

Key numbers: $x = -5, x = 3, x = 11$

Test intervals: $(-\infty, -5) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(-5, 3) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

$(3, 11) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(11, \infty) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

Solution set: $(-5, 3) \cup (11, \infty)$

46.

$$\frac{5}{x-6} > \frac{3}{x+2}$$
$$\frac{5(x+2)-3(x-6)}{(x-6)(x+2)} > 0$$
$$\frac{2x+28}{(x-6)(x+2)} > 0$$

Key numbers: $x = -14, x = -2, x = 6$

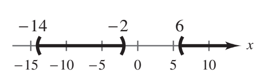
Test intervals: $(-\infty, -14) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(-14, -2) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

$(-2, 6) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(6, \infty) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

Solution intervals: $(-14, -2) \cup (6, \infty)$



47.
$$\frac{1}{x-3} \le \frac{9}{4x+3}$$
$$\frac{1}{x-3} - \frac{9}{4x+3} \le 0$$
$$\frac{4x+3-9(x-3)}{(x-3)(4x+3)} \le 0$$
$$\frac{30-5x}{(x-3)(4x+3)} \le 0$$

Key numbers: $x = 3, x = -\frac{3}{4}, x = 6$

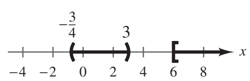
Test intervals: $\left(-\infty, -\frac{3}{4}\right) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} > 0$

$\left(-\frac{3}{4}, 3\right) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} < 0$

$(3, 6) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} > 0$

$(6, \infty) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} < 0$

Solution set: $\left(-\frac{3}{4}, 3\right) \cup [6, \infty)$



48.
$$\frac{1}{x} \ge \frac{1}{x+3}$$
$$\frac{1(x+3)-1(x)}{x(x+3)} \ge 0$$
$$\frac{3}{x(x+3)} \ge 0$$

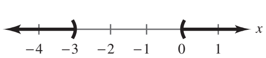
Key numbers: $x = -3, x = 0$

Test intervals: $(-\infty, -3) \Rightarrow \frac{3}{x(x+3)} > 0$

$(-3, 0) \Rightarrow \frac{3}{x(x+3)} < 0$

$(0, \infty) \Rightarrow \frac{3}{x(x+3)} > 0$

Solution intervals: $(-\infty, -3) \cup (0, \infty)$



49.
$$\frac{x^2+2x}{x^2-9} \le 0$$
$$\frac{x(x+2)}{(x+3)(x-3)} \le 0$$

Key numbers: $x = 0, x = -2, x = \pm 3$

Test intervals: $(-\infty, -3) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

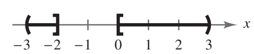
$(-3, -2) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} < 0$

$(-2, 0) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} < 0$

$(3, \infty) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

Solution set: $(-3, -2] \cup [0, 3)$



50.
$$\frac{x^2+x-6}{x} \ge 0$$
$$\frac{(x+3)(x-2)}{x} \ge 0$$

Key numbers: $x = -3, x = 0, x = 2$

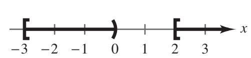
Test intervals: $(-\infty, -3) \Rightarrow \frac{(x+3)(x-2)}{x} < 0$

$(-3, 0) \Rightarrow \frac{(x+3)(x-2)}{x} > 0$

$(0, 2) \Rightarrow \frac{(x+3)(x-2)}{x} < 0$

$(2, \infty) \Rightarrow \frac{(x+3)(x-2)}{x} > 0$

Solution set: $[-3, 0) \cup [2, \infty)$



51.
$$\frac{3}{x-1} + \frac{2x}{x+1} > -1$$
$$\frac{3(x+1) + 2x(x-1) + 1(x+1)(x-1)}{(x-1)(x+1)} > 0$$
$$\frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$$

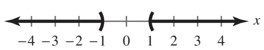
Key numbers: $x = -1, x = 1$

Test intervals: $(-\infty, -1) \Rightarrow \dfrac{3x^2 + x + 2}{(x-1)(x+1)} > 0$

$(-1, 1) \Rightarrow \dfrac{3x^2 + x + 2}{(x-1)(x+1)} < 0$

$(1, \infty) \Rightarrow \dfrac{3x^2 + x + 2}{(x-1)(x+1)} > 0$

Solution set: $(-\infty, -1) \cup (1, \infty)$



52.
$$\frac{3x}{x-1} \le \frac{x}{x+4} + 3$$
$$\frac{3x(x+4) - x(x-1) - 3(x+4)(x-1)}{(x-1)(x+4)} \le 0$$
$$\frac{-x^2 + 4x + 12}{(x-1)(x+4)} \le 0$$
$$\frac{-(x-6)(x+2)}{(x-1)(x+4)} \le 0$$

Key numbers: $x = -4, x = -2, x = 1, x = 6$

Test intervals: $(-\infty, -4) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

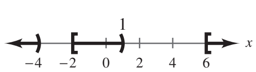
$(-4, -2) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(-2, 1) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

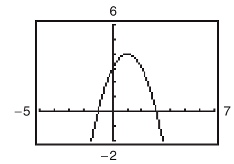
$(1, 6) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(6, \infty) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

Solution set: $(-\infty, -4) \cup [-2, 1) \cup [6, \infty)$



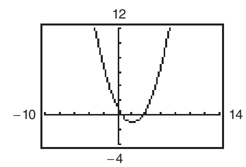
53. $y = -x^2 + 2x + 3$



(a) $y \le 0$ when $x \le -1$ or $x \ge 3$.

(b) $y \ge 3$ when $0 \le x \le 2$.

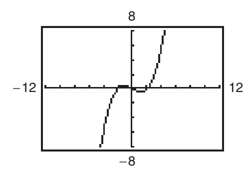
54. $y = \frac{1}{2}x^2 - 2x + 1$



(a) $y \le 0$ when $2 - \sqrt{2} \le x \le 2 + \sqrt{2}$.

(b) $y \ge 7$ when $x \le -2$ or $x \ge 6$.

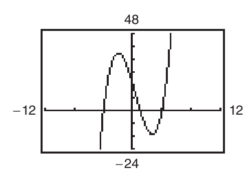
55. $y = \frac{1}{8}x^3 - \frac{1}{2}x$



(a) $y \ge 0$ when $-2 \le x \le 0$ or $2 \le x < \infty$.

(b) $y \le 6$ when $x \le 4$.

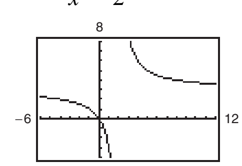
56. $y = x^3 - x^2 - 16x + 16$



(a) $y \le 0$ when $-\infty < x \le -4$ or $1 \le x \le 4$.

(b) $y \ge 36$ when $x = -2$ or $5 \le x < \infty$.

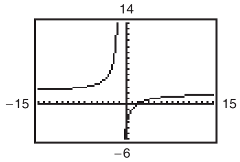
57. $y = \dfrac{3x}{x-2}$



(a) $y \le 0$ when $0 \le x < 2$.

(b) $y \ge 6$ when $2 < x \le 4$.

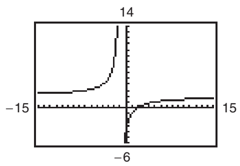
58. $y = \dfrac{2(x-2)}{x+1}$



(a) $y \le 0$ when $-1 < x \le 2$.

(b) $y \ge 8$ when $-2 \le x < -1$.

59. $y = \dfrac{2x^2}{x^2+4}$



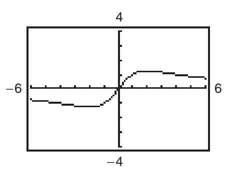
(a) $y \ge 1$ when $x \le -2$ or $x \ge 2$.

This can also be expressed as $|x| \ge 2$.

(b) $y \le 2$ for all real numbers x.

This can also be expressed as $-\infty < x < \infty$.

60. $y = \dfrac{5x}{x^2+4}$



(a) $y \ge 1$ when $1 \le x \le 4$.

(b) $y \le 0$ when $-\infty < x \le 0$.

61. $0.3x^2 + 6.26 < 10.8$

$0.3x^2 + 4.54 < 0$

Key numbers: $x \approx \pm 3.89$

Test intervals: $(-\infty, -3.89), (-3.89, 3.89), (3.89, \infty)$

Solution set: $(-3.89, 3.89)$

62. $-1.3x^2 + 3.78 > 2.12$

$-1.3x^2 + 1.66 > 0$

Key numbers: $x \approx \pm 1.13$

Test intervals: $(-\infty, -1.13), (-1.13, 1.13), (1.13, \infty)$

Solution set: $(-1.13, 1.13)$

63. $-0.5x^2 + 12.5x + 1.6 > 0$

Key numbers: $x \approx -0.13, x \approx 25.13$

Test intervals: $(-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)$

Solution set: $(-0.13, 25.13)$

64. $1.2x^2 + 4.8x + 3.1 < 5.3$

$1.2x^2 + 4.8x - 2.2 < 0$

Key numbers: $x \approx -4.42, x \approx 0.42$

Test intervals: $(-\infty, -4.42), (-4.42, 0.42), (0.42, \infty)$

Solution set: $(-4.42, 0.42)$

65.
$$\frac{1}{2.3x - 5.2} > 3.4$$
$$\frac{1}{2.3x - 5.2} - 3.4 > 0$$
$$\frac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0$$
$$\frac{-7.82x + 18.68}{2.3x - 5.2} > 0$$

Key numbers: $x \approx 2.39, x \approx 2.26$

Test intervals: $(-\infty, 2.26), (2.26, 2.39), (2.39, \infty)$

Solution set: $(2.26, 2.39)$

66.
$$\frac{2}{3.1x - 3.7} > 5.8$$
$$\frac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0$$
$$\frac{23.46 - 17.98x}{3.1x - 3.7} > 0$$

Key numbers: $x \approx 1.19, x \approx 1.30$

Test intervals: $(-\infty, 1.19) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

$(1.19, 1.30) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

$(1.30, \infty) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

Solution set: $(1.19, 1.30)$

67. $s = -16t^2 + v_0t + s_0 = -16t^2 + 160t$

(a) $-16t^2 + 160t = 0$

$-16t(t - 10) = 0$

$t = 0, t = 10$

It will be back on the ground in 10 seconds.

(b) $-16t^2 + 160t > 384$

$-16t^2 + 160t - 384 > 0$

$-16(t^2 - 10t + 24) > 0$

$t^2 - 10t + 24 < 0$

$(t - 4)(t - 6) < 0$

Key numbers: $t = 4, t = 6$

Test intervals: $(-\infty, 4), (4, 6), (6, \infty)$

Solution set: 4 seconds $< t <$ 6 seconds

68. $s = -16t^2 + v_0t + s_0 = -16t^2 + 128t$

(a) $-16t^2 + 128t = 0$

$-16t(t - 8) = 0$

$-16t = 0 \Rightarrow t = 0$

$t - 8 = 0 \Rightarrow t = 8$

It will be back on the ground in 8 seconds.

(b) $-16t^2 + 128t < 128$

$-16t^2 + 128t - 128 < 0$

$-16(t^2 - 8t + 8) < 0$

$t^2 - 8t + 8 > 0$

Key numbers: $t = 4 - 2\sqrt{2}, t = 4 + 2\sqrt{2}$

Test intervals:
$(-\infty, 4 - 2\sqrt{2}), (4 - 2\sqrt{2}, 4 + 2\sqrt{2}),$
$(4 + 2\sqrt{2}, \infty)$

Solution set: 0 seconds $\le t < 4 - 2\sqrt{2}$ seconds
and
$4 + 2\sqrt{2}$ seconds $< t \le 8$ seconds

69. $R = x(75 - 0.0005x)$ and $C = 30x + 250{,}000$

$$\begin{aligned} P &= R - C \\ &= (75x - 0.0005x^2) - (30x + 250{,}000) \\ &= -0.0005x^2 + 45x - 250{,}000 \end{aligned}$$

$$P \ge 750{,}000$$
$$-0.0005x^2 + 45x - 250{,}000 \ge 750{,}000$$
$$-0.0005x^2 + 45x - 1{,}000{,}000 \ge 0$$

Key numbers: $x = 40{,}000, x = 50{,}000$
(These were obtained by using the Quadratic Formula.)

Test intervals:
$(0, 40{,}000), (40{,}000, 50{,}000), (50{,}000, \infty)$

The solution set is $[40{,}000, 50{,}000]$ or $40{,}000 \le x \le 50{,}000$. The price per unit is

$$p = \frac{R}{x} = 75 - 0.0005x.$$

For $x = 40{,}000$, $p = \$55$. For $x = 50{,}000$, $p = \$50$. So, for $40{,}000 \le x \le 50{,}000$, $\$50.00 \le p \le \$55.00$.

70. $R = x(50 - 0.0002x)$ and $C = 12x + 150{,}000$

$$\begin{aligned} P &= R - C \\ &= (50x - 0.0002x^2) - (12x + 150{,}000) \\ &= -0.0002x^2 + 38x - 150{,}000 \end{aligned}$$

$$P \ge 1{,}650{,}000$$
$$-0.0002x^2 + 38x - 150{,}000 \ge 1{,}650{,}000$$
$$-0.0002x^2 + 38x - 1{,}800{,}000 \ge 0$$

Key numbers: $x = 90{,}000$ and $x = 100{,}000$

Test intervals:
$(0, 90{,}000), (90{,}000, 100{,}000), (100{,}000, \infty)$

The solution set is $[90{,}000, 100{,}000]$ or $90{,}000 \le x \le 100{,}000$. The price per unit is

$$p = \frac{R}{x} = 50 - 0.0002x.$$

For $x = 90{,}000$, $p = \$32$. For $x = 100{,}000$, $p = \$30$. So, for $90{,}000 \le x \le 100{,}000$, $\$30 \le p \le \$32$.

71. $4 - x^2 \ge 0$

$(2 + x)(2 - x) \ge 0$

Key numbers: $x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow 4 - x^2 < 0$

$(-2, 2) \Rightarrow 4 - x^2 > 0$

$(2, \infty) \Rightarrow 4 - x^2 < 0$

Domain: $[-2, 2]$

72. The domain of $\sqrt{x^2 - 9}$ can be found by solving the inequality:

$x^2 - 9 \ge 0$

$(x + 3)(x - 3) \ge 0$

Key numbers: $x = -3, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 3) > 0$

$(-3, 3) \Rightarrow (x + 3)(x - 3) < 0$

$(3, \infty) \Rightarrow (x + 3)(x - 3) > 0$

Domain: $(-\infty, -3] \cup [3, \infty)$

73. $x^2 - 9x + 20 \ge 0$

$(x - 4)(x - 5) \ge 0$

Key numbers: $x = 4, x = 5$

Test intervals: $(-\infty, 4), (4, 5), (5, \infty)$

Interval	x-Value	Value of $(x - 4)(x - 5)$	Conclusion
$(-\infty, 4)$	0	$(-4)(-5) = 20$	Positive
$(4, 5)$	$\frac{9}{2}$	$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4}$	Negative
$(5, \infty)$	6	$(2)(1) = 2$	Positive

Domain: $(-\infty, 4] \cup [5, \infty)$

74. The domain of $\sqrt{49 - x^2}$ can be found by solving the inequality:

$49 - x^2 \ge 0$

$x^2 - 49 \le 0$

$(x + 7)(x - 7) \le 0$

Key numbers: $x = -7, x = 7$

Test intervals: $(-\infty, -7) \Rightarrow (x + 7)(x - 7) > 0$

$(-7, 7) \Rightarrow (x + 7)(x - 7) < 0$

$(7, \infty) \Rightarrow (x + 7)(x - 7) > 0$

Domain: $[-7, 7]$

75. $\dfrac{x}{x^2 - 2x - 35} \ge 0$

$\dfrac{x}{(x + 5)(x - 7)} \ge 0$

Key numbers: $x = 0, x = -5, x = 7$

Test intervals: $(-\infty, -5) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} < 0$

$(-5, 0) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} > 0$

$(0, 7) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} < 0$

$(7, \infty) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} > 0$

Domain: $(-5, 0] \cup (7, \infty)$

76. $\dfrac{x}{x^2 - 9} \ge 0$

$\dfrac{x}{(x + 3)(x - 3)} \ge 0$

Key numbers: $x = -3, x = 0, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

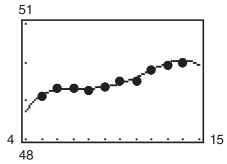
$(-3, 0) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

$(0, 3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

Domain: $(-3, 0] \cup (3, \infty)$

77. (a) and (c)



(b) $N = -0.001231t^4 + 0.04723t^3 - 0.6452t^2 + 3.783t + 41.21$

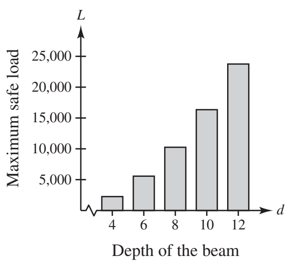
The model fits the data well.

(d) Using the zoom and trace features, the number of students enrolled in elementary and secondary schools fell below 48 million in the year 2017.

(e) No. The model can be used to predict enrollments for years close to those in its domain, $5 \le t \le 14$, but when you project too far into the future, the numbers predicted by the model decrease too rapidly to be considered reasonable.

78. (a)

d	4	6	8	10	12
Load	2223.9	5593.9	10,312	16,378	23,792



(b)
$$\begin{aligned} 2000 &\le 168.5d^2 - 472.1 \\ 2472.1 &\le 168.5d^2 \\ 14.67 &\le d^2 \\ 3.83 &\le d \end{aligned}$$

The minimum depth is 3.83 inches.

79. $2L + 2W = 100 \Rightarrow W = 50 - L$

$$\begin{aligned} LW &\ge 500 \\ L(50 - L) &\ge 500 \\ -L^2 + 50L - 500 &\ge 0 \end{aligned}$$

By the Quadratic Formula you have:

Key numbers: $L = 25 \pm 5\sqrt{5}$

Test: Is $-L^2 + 50L - 500 \ge 0$?

Solution set: $25 - 5\sqrt{5} \le L \le 25 + 5\sqrt{5}$

13.8 meters $\le L \le$ 36.2 meters

80. $2L + 2W = 440 \Rightarrow W = 220 - L$

$$\begin{aligned} LW &\ge 8000 \\ L(220 - L) &\ge 8000 \\ -L^2 + 220L - 8000 &\ge 0 \end{aligned}$$

By the Quadratic Formula we have:

Key numbers: $L = 110 \pm 10\sqrt{41}$

Test: Is $-L^2 + 220L - 8000 \ge 0$?

Solution set: $110 - 10\sqrt{41} \le L \le 110 + 10\sqrt{41}$

45.97 feet $\le L \le$ 174.03 feet

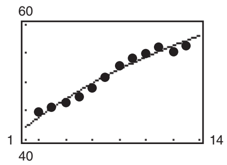
81.
$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{2} \\ 2R_1 &= 2R + RR_1 \\ 2R_1 &= R(2 + R_1) \\ \frac{2R_1}{2 + R_1} &= R \end{aligned}$$

Because $R \ge 1$,

$$\begin{aligned} \frac{2R_1}{2 + R_1} &\ge 1 \\ \frac{2R_1}{2 + R_1} - 1 &\ge 0 \\ \frac{R_1 - 2}{2 + R_1} &\ge 0. \end{aligned}$$

Because $R_1 > 0$, the only key number is $R_1 = 2$. The inequality is satisfied when $R_1 \ge 2$ ohms.

82. (a)



(b) The model fits the data well; each data value is close to the graph of the model.

(c)
$$S = \frac{40.32 + 3.53t}{1 + 0.039t}$$
$$65 \le \frac{40.32 + 3.53t}{1 + 0.039t}$$
$$0 \le \frac{40.32 + 3.53t}{1 + 0.039t} - 65$$
$$0 \le \frac{-24.68 + 0.995t}{1 + 0.039t}$$

Key numbers: $t \approx -25.6$ and $t \approx 24.8$. Use the domain of the model to create test intervals.

Test Intervals	t-Value	Expression Value	Conclusion
$(2, 24.8)$	$t = 5$	< 0	Negative
$(24.8, \infty)$	$t = 30$	> 0	Positive

So, the mean salary for classroom teachers will exceed \$65,000 during the year 2024.

(d) Yes. The model yields a steady gradual increase in salaries for values of $t \ge 13$.

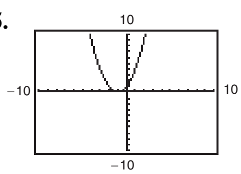
83. False.

There are four test intervals. The test intervals are $(-\infty, -3), (-3, 1), (1, 4),$ and $(4, \infty)$.

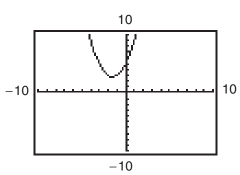
84. True.

The y-values are greater than zero for all values of x.

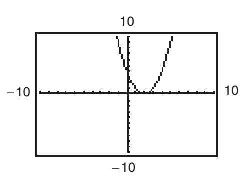
85.



For part (b), the y-values that are less than or equal to 0 occur only at $x = -1$.



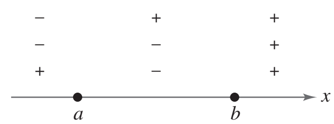
For part (c), there are no y-values that are less than 0.



For part (d), the y-values that are greater than 0 occur for all values of x except 2.

86. (a) $x = a, x = b$

(b)



(c) The real zeros of the polynomial.

87. $x^2 + bx + 9 = 0$

(a) To have at least one real solution, $b^2 - 4ac \ge 0$.

$$b^2 - 4(1)(9) \ge 0$$
$$b^2 - 36 \ge 0$$

Key numbers: $b = -6, b = 6$

Test intervals: $(-\infty, -6) \Rightarrow b^2 - 36 > 0$

$(-6, 6) \Rightarrow b^2 - 36 < 0$

$(6, \infty) \Rightarrow b^2 - 36 > 0$

Solution set: $(-\infty, -6] \cup [6, \infty)$

(b) $b^2 - 4ac \ge 0$

Key numbers: $b = -2\sqrt{ac}, b = 2\sqrt{ac}$

Similar to part (a), if $a > 0$ and $c > 0$, $b \le -2\sqrt{ac}$ or $b \ge 2\sqrt{ac}$.

88. $x^2 + bx - 4 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(1)(-4) \geq 0$

$b^2 + 16 \geq 0$

Key numbers: none

Test intervals: $(-\infty, \infty) \Rightarrow b^2 + 16 > 0$

Solution set: $(-\infty, \infty)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c < 0$, b can be any real number.

89. $3x^2 + bx + 10 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(3)(10) \geq 0$

$b^2 - 120 \geq 0$

Key numbers: $b = -2\sqrt{30}, b = 2\sqrt{30}$

Test intervals: $\left(-\infty, -2\sqrt{30}\right) \Rightarrow b^2 - 120 > 0$

$\left(-2\sqrt{30}, 2\sqrt{30}\right) \Rightarrow b^2 - 120 < 0$

$\left(2\sqrt{30}, \infty\right) \Rightarrow b^2 - 120 > 0$

Solution set: $\left(-\infty, -2\sqrt{30}\right] \cup \left[2\sqrt{30}, \infty\right)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

90. $2x^2 + bx + 5 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(2)(5) \geq 0$

$b^2 - 40 \geq 0$

Key numbers: $b = -2\sqrt{10}, b = 2\sqrt{10}$

Test intervals: $\left(-\infty, -2\sqrt{10}\right) \Rightarrow b^2 - 40 > 0$

$\left(-2\sqrt{10}, 2\sqrt{10}\right) \Rightarrow b^2 - 40 < 0$

$\left(2\sqrt{10}, \infty\right) \Rightarrow b^2 - 40 > 0$

Solution set: $\left(-\infty, -2\sqrt{10}\right] \cup \left[2\sqrt{10}, \infty\right)$

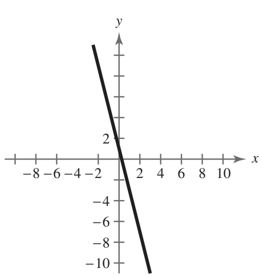
(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

Review Exercises for Chapter 1

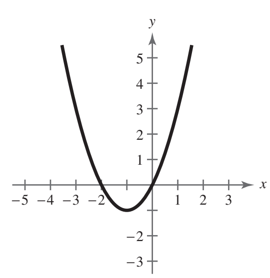
1. $y = -4x + 1$

x	-2	-1	0	1	2
y	9	5	1	-3	-7



2. $y = x^2 + 2x$

x	-3	-2	-1	0	1
y	3	0	-1	0	3



3. x-intercepts: $(1, 0), (5, 0)$

y-intercept: $(0, 5)$

4. x-intercepts: $(-4, 0), (2, 0)$

y-intercept: $(0, -2)$

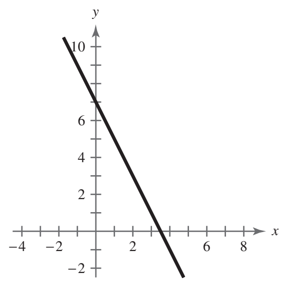
5. $y = -3x + 7$

Intercepts: $\left(\frac{7}{3}, 0\right), (0, 7)$

$y = -3(-x) + 7 \Rightarrow y = 3x + 7 \Rightarrow$ No y-axis symmetry

$-y = -3x + 7 \Rightarrow y = 3x - 7 \Rightarrow$ No x-axis symmetry

$-y = -3(-x) + 7 \Rightarrow y = -3x - 7 \Rightarrow$ No origin symmetry



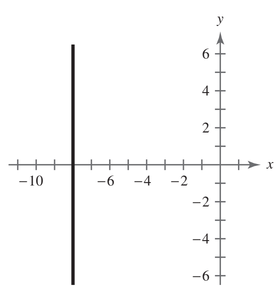
6. $x = -8$

Intercept: $(-8, 0)$, No y-intercept.

$-x = -8 \Rightarrow x = 8 \Rightarrow$ No y-axis symmetry

$x = -8 \Rightarrow x$-axis symmetry

$-x = -8 \Rightarrow x = 8 \Rightarrow$ No origin symmetry



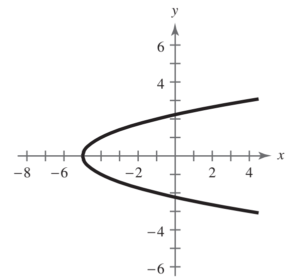
7. $x = y^2 - 5$

Intercepts: $(-5, 0), \left(0, \pm\sqrt{5}\right)$

$-x = y^2 - 5 \Rightarrow x = -y^2 + 5 \Rightarrow$ No y-axis symmetry

$x = \left(-y^2\right) - 5 \Rightarrow x = y^2 - 5 \Rightarrow x$-axis symmetry

$-x = (-y)^2 - 5 \Rightarrow x = -y^2 + 5 \Rightarrow$ No origin symmetry



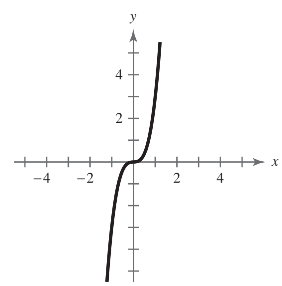
8. $y = 3x^3$

Intercepts: $(0, 0)$

$y = 3(-x)^3 \Rightarrow y = -3x^3 \Rightarrow$ No y-axis symmetry

$-y = 3x^3 \Rightarrow y = -3x^3 \Rightarrow$ No x-axis symmetry

$-y = 3(-x)^3 \Rightarrow y = 3x^3 \Rightarrow$ Original symmetry



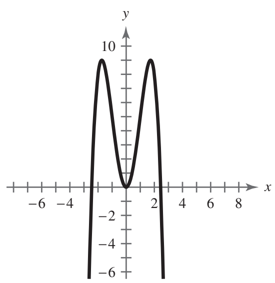
9. $y = -x^4 + 6x^2$

Intercept: $(0, 0)$

$y = -(-x)^4 + 6(-x)^2 \Rightarrow y = -x^4 + 6x^2 \Rightarrow y$-axis symmetry

$-y = -x^4 + 6x^2 \Rightarrow y = x^4 - 6x^2 \Rightarrow$ No x-axis symmetry

$-y = -(-x)^4 + 6(-x)^2 \Rightarrow y = x^4 - 6x^2 \Rightarrow$ No origin symmetry



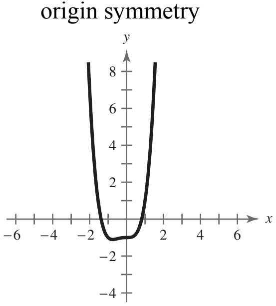
10. $y = x^4 + x^3 - 1$

Intercept: $(0, -1)$

$y = (-x)^4 + (-x)^3 - 1 \Rightarrow y = x^4 - x^3 - 1 \Rightarrow y$-axis symmetry

$-y = x^4 + x^3 - 1 \Rightarrow y = -x^4 - x^3 + 1 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 + (-x)^3 - 1 \Rightarrow y = x^4 - x^3 - 1 \Rightarrow$ No origin symmetry



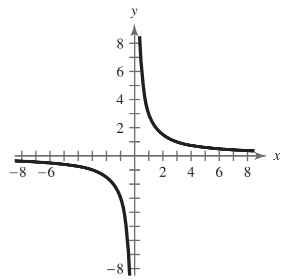
11. $y = \dfrac{3}{x}$

Intercept: None

$y = \dfrac{3}{-x} \Rightarrow y = -\dfrac{3}{x} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{3}{x} \Rightarrow y = -\dfrac{3}{x} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{3}{-x} \Rightarrow y = \dfrac{3}{x} \Rightarrow$ origin symmetry



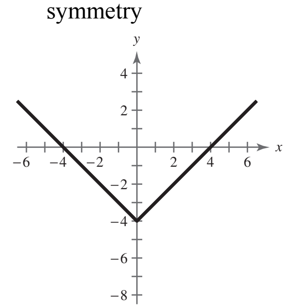
12. $y = |x| - 4$

Intercepts: $(\pm 4, 0), (0, -4)$

$y = |-x| - 4 \Rightarrow y = |x| - 4 \Rightarrow y$-axis symmetry

$-y = |x| - 4 \Rightarrow y = -|x| + 4 \Rightarrow$ No x-axis symmetry

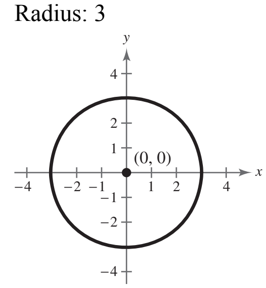
$-y = |-x| - 4 \Rightarrow y = -|x| + 4 \Rightarrow$ No origin symmetry



13. $x^2 + y^2 = 9$

Center: $(0, 0)$

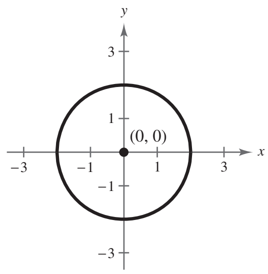
Radius: 3



14. $x^2 + y^2 = 4$

Center: $(0, 0)$

Radius: 2

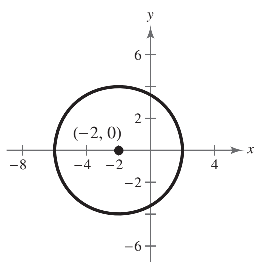


15. $(x + 2)^2 + y^2 = 16$

$(x - (-2))^2 + (y - 0)^2 = 4^2$

Center: $(-2, 0)$

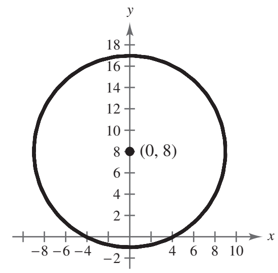
Radius: 4



16. $x^2 + (y - 8)^2 = 81$

Center: $(0, 8)$

Radius: 9



17. Endpoints of a diameter: $(0, 0)$ and $(4, -6)$

Center: $\left(\dfrac{0 + 4}{2}, \dfrac{0 + (-6)}{2}\right) = (2, -3)$

Radius: $r = \sqrt{(2 - 0)^2 + (-3 - 0)^2} = \sqrt{4 + 9} = \sqrt{13}$

Standard form: $(x - 2)^2 + (y - (-3))^2 = \left(\sqrt{13}\right)^2$

$$(x - 2)^2 + (y + 3)^2 = 13$$

18. Endpoints of a diameter: $(-2, -3)$ and $(4, -10)$

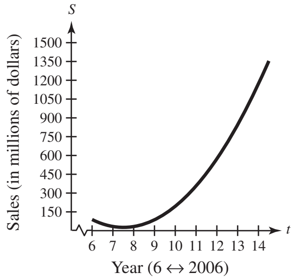
Center: $\left(\dfrac{-2 + 4}{2}, \dfrac{-3 + (-10)}{2}\right) = \left(1, -\dfrac{13}{2}\right)$

Radius: $r = \sqrt{(1 - (-2))^2 + \left(-\dfrac{13}{2} - (-3)\right)^2} = \sqrt{9 + \dfrac{49}{4}} = \sqrt{\dfrac{85}{4}}$

Standard form: $(x - 1)^2 + \left(y - \left(-\dfrac{13}{2}\right)\right)^2 = \left(\sqrt{\dfrac{85}{4}}\right)^2$

$$(x - 1)^2 + \left(y + \frac{13}{2}\right)^2 = \frac{85}{4}$$

19. (a)



(b) $27.215t^2 - 409.06t + 1563.6 = 200$

$27.215t^2 - 409.06t + 1363.6 = 0$

$$t = \frac{-(-409.06) \pm \sqrt{(-409.06)^2 - 4(27.215)(1363.6)}}{2(27.215)}$$

$$t = 10.04, 4.99$$

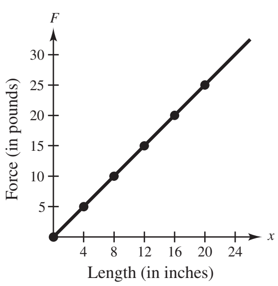
Using the zoom and trace features, sales were $200 million during the year 2010.

20. $F = \frac{5}{4}x, 0 \le x \le 20$

(a)

x	0	4	8	12	16	20
F	0	5	10	15	20	25

(b)



(c) When $x = 10, F = \frac{50}{4} = 12.5$ pounds.

21. $2(x - 2) = 2x - 4$

$2x - 4 = 2x - 4$

$0 = 0$ Identity

All real numbers are solutions.

22. $2(x + 3) = 2x - 2$

$2x + 6 = 2x - 2$

$6 = -2$ Contradiction

No solution

23. $3(x - 2) + 2x = 2(x + 3)$

$3x - 6 + 2x = 2x + 6$

$3x = 12$

$x = 4$

Conditional equation

24. $5(x - 1) - 2x = 3x - 5$

$5x - 5 - 2x = 3x - 5$

$3x - 5 = 3x - 5$

$0 = 0$ Identity

All real numbers are solutions.

25. $8x - 5 = 3x + 20$

$5x = 25$

$x = 5$

26. $7x + 3 = 3x - 17$

$4x = -20$

$x = -5$

27. $2(x + 5) - 7 = x + 9$

$2x + 10 - 7 = x + 9$

$2x + 3 = x + 9$

$x = 6$

28. $7(x - 4) = 1 - (x + 9)$

$7x - 28 = 1 - x - 9$

$7x - 28 = -x - 8$

$8x = 20$

$x = \frac{5}{2}$

29. $\frac{x}{5} - 3 = \frac{x}{3} + 1$

$15\left(\frac{x}{5} - 3\right) = \left(\frac{x}{3} + 1\right)15$

$3x - 45 = 5x + 15$

$-2x = 60$

$x = -30$

30.
$$\begin{aligned}\frac{4x-3}{6}+\frac{x}{4}&=x-2\\2(4x-3)+3x&=12x-24\\8x-6+3x&=12x-24\\-x&=-18\\x&=18\end{aligned}$$

31.
$$\begin{aligned}3+\frac{2}{x-5}&=\frac{2x}{x-5}\\\frac{3(x-5)+2}{x-5}&=\frac{2x}{x-5}\\\frac{3x-15+2}{x-5}&=\frac{2x}{x-5}\\\frac{3x-13}{x-5}&=\frac{2x}{x-5}\\(x-5)\frac{3x-13}{x-5}&=\frac{2x}{x-5}(x-5)\\3x-13&=2x\\x&=3\end{aligned}$$

32.
$$\begin{aligned}\frac{1}{x^2+3x-18}-\frac{3}{x+6}&=\frac{4}{x-3}\\\frac{1}{(x+6)(x-3)}-\frac{3(x-3)}{(x+6)(x-3)}&=\frac{4(x+6)}{(x+6)(x-3)}\\(x+6)(x-3)\left[\frac{1}{(x+6)(x-3)}-\frac{3(x-3)}{(x+6)(x-3)}\right]&=\frac{4(x+6)}{(x+6)(x-3)}(x+6)(x-3)\\1-3(x-3)&=4(x+6)\\1-3x+9&=4x+24\\-7x&=14\\x&=-2\end{aligned}$$

33. $y=3x-1$

x-intercept: $0=3x-1\Rightarrow x=\frac{1}{3}$

y-intercept: $y=3(0)-1\Rightarrow y=-1$

The x-intercept is $\left(\frac{1}{3},0\right)$ and the y-intercept is $(0,-1)$.

34.
$$\begin{aligned}y&=-5x+6 & y&=-5x+6\\0&=-5x+6 & y&=-5(0)+6\\-6&=-5x & y&=6\\\tfrac{6}{5}&=x\end{aligned}$$

The x-intercept is $\left(\frac{6}{5},0\right)$ and the y-intercept is $(0,6)$.

35. $y=2(x-4)$

x-intercept: $0=2(x-4)\Rightarrow x=4$

y-intercept: $y=2(0-4)\Rightarrow y=-8$

The x-intercept is $(4,0)$ and the y-intercept is $(0,-8)$.

36.
$$\begin{aligned}y&=4(7x+1) & y&=4(7x+1)\\0&=4(7x+1) & y&=4[7(0)+1]\\0&=28x+4 & y&=4\\-4&=28x\\-\tfrac{1}{7}&=x\end{aligned}$$

The x-intercept is $\left(-\frac{1}{7},0\right)$ and the y-intercept is $(0,4)$.

37. $y=-\dfrac{1}{2}x+\dfrac{2}{3}$

x-intercept: $0=-\dfrac{1}{2}x+\dfrac{2}{3}\Rightarrow x=\dfrac{2/3}{1/2}=\dfrac{4}{3}$

y-intercept: $y=-\dfrac{1}{2}(0)+\dfrac{2}{3}\Rightarrow y=\dfrac{2}{3}$

The x-intercept is $\left(\dfrac{4}{3},0\right)$ and the y-intercept is $\left(0,\dfrac{2}{3}\right)$.

38.
$$\begin{aligned}y&=\tfrac{3}{4}x-\tfrac{1}{4} & y&=\tfrac{3}{4}x-\tfrac{1}{4}\\0&=\tfrac{3}{4}x-\tfrac{1}{4} & y&=\tfrac{3}{4}(0)-\tfrac{1}{4}\\\tfrac{4}{3}\cdot\tfrac{1}{4}&=\tfrac{4}{3}\cdot\tfrac{3}{4}x & y&=-\tfrac{1}{4}\\\tfrac{1}{3}&=x\end{aligned}$$

The x-intercept is $\left(\frac{1}{3},0\right)$ and the y-intercept is $\left(0,-\frac{1}{4}\right)$.

39. $244.92 = 2(3.14)(3)^2 + 2(3.14)(3)h$

$244.92 = 56.52 + 18.84h$

$188.40 = 18.84h$

$10 = h$

The height is 10 inches.

40. $C = \frac{5}{9}F - \frac{160}{9}$

$\frac{5}{9}F = C + \frac{160}{9}$

$F = \frac{9}{5}\left(C + \frac{160}{9}\right)$

For $C = 100°$, $F = \frac{9}{5}\left(100 + \frac{160}{9}\right) = 212°\text{F}$.

41. *Verbal Model:* Revenue in 2013 + Revenue in 2014 = Total Revenue

Labels: Let x = Revenue in 2014. Then $x = 0.452x$ = Revenue in 2013.

Equation:

$$x + (x + 0.452x) = 3.75$$
$$2.452x = 3.75$$
$$x \approx 1.53$$
$$x + 0.452x \approx 2.22$$

So, Revenue was \$2.22 billion in 2013 and was \$1.53 billion in 2014.

42. *Model:* $(\text{Original price}) = \dfrac{(\text{sale price})}{(1 - \text{discount rate})}$

Labels: Original price $= x$

Discount rate $= 0.2$

Sale price $= 340$

Equation:

$$x = \frac{340}{1 - 0.2}$$
$$x = 425$$

The original price was \$425.

43. Let x = the total investment required.

Each person's share is $\dfrac{x}{9}$. If three more people invest, each person's share is $\dfrac{x}{12} + 2500$.

Since this is \$2500 less than the original cost, we have:

$$\frac{x}{9} = \frac{x}{12} + 2500$$
$$\frac{x}{9} - \frac{x}{12} = 2500$$
$$\frac{8x - 6x}{72} = 2500$$
$$\frac{2x}{72} = 2500$$
$$x = 90{,}000$$

The total investment to start the business is \$90,000.

44.

	Rate	Time	Distance
To work	r	$\dfrac{56}{r}$	56
From work	$r + 8$	$\dfrac{56}{r+8}$	56

$\text{Time} = \dfrac{\text{distance}}{\text{rate}}$

Time to work = time from work + 10 minutes

$$\frac{56}{r} = \frac{56}{r+8} + \frac{1}{6} \quad \text{Convert minutes to portion of an hour.}$$

$$\begin{aligned}
6(r + 8)(56) &= 6r(56) + r(r + 8)\\
336r + 2688 &= 336r + r^2 + 8r\\
0 &= r^2 + 8r - 2688\\
0 &= (r - 48)(r + 56)
\end{aligned}$$

Using the positive value for r, we have $r = 48$ miles per hour. The average speed on the trip home was $r + 8 = 56$ miles per hour.

45. Let x = the number of liters of pure antifreeze.

$$\begin{aligned}
30\% \text{ of } (10 - x) + 100\% \text{ of } x &= 50\% \text{ of } 10\\
0.30(10 - x) + 1.00x &= 0.50(10)\\
3 - 0.30x + 1.00x &= 5\\
0.70x &= 2\\
x &= \frac{2}{0.70} = \frac{20}{7} \approx 2.857 \text{ liters}
\end{aligned}$$

46. *Model:* $(\text{Interest from } 4\tfrac{1}{2}\%) + (\text{Interest from } 5\tfrac{1}{2}\%) = (\text{total interest})$

Labels: Amount invested at $4\tfrac{1}{2}\% = x$, amount invested at $5\tfrac{1}{2}\% = 6000 - x$

Interest from $4\tfrac{1}{2}\% = x(0.045)(1)$, interest from $5\tfrac{1}{2}\% = (6000 - x)(0.055)(1)$, total interest = \$3.05

Equation:

$$\begin{aligned}
0.045x + 0.055(6000 - x) &= 305\\
0.045x + 330 - 0.055x &= 305\\
-0.01x &= -25\\
x &= 2500
\end{aligned}$$

The amount invested at $4\tfrac{1}{2}\%$ was \$2500 and the amount invested at $5\tfrac{1}{2}\%$ was $6000 - 2500 = \$3500$.

47.

$$\begin{aligned}
V &= \frac{1}{3}\pi r^2 h\\
3V &= \pi r^2 h\\
\frac{3V}{\pi r^2} &= h
\end{aligned}$$

48.

$$\begin{aligned}
E &= \frac{1}{2}mv^2\\
mv^2 &= 2E\\
m &= \frac{2E}{v^2}
\end{aligned}$$

49.

$$\begin{aligned}
15 + x - 2x^2 &= 0\\
0 &= 2x^2 - x - 15\\
0 &= (2x + 5)(x - 3)\\
2x + 5 = 0 &\Rightarrow x = -\tfrac{5}{2}\\
x - 3 = 0 &\Rightarrow x = 3
\end{aligned}$$

50. $2x^2 - x - 28 = 0$

$(2x + 7)(x - 4) = 0$

$2x + 7 = 0 \Rightarrow x = -\frac{7}{2}$

$x - 4 = 0 \Rightarrow x = 4$

51. $6 = 3x^2$

$2 = x^2$

$\pm\sqrt{2} = x$

52. $16x^2 = 25$

$16x^2 - 25 = 0$

$(4x - 5)(4x + 5) = 0$

$4x - 5 = 0 \Rightarrow x = \frac{5}{4}$

$4x + 5 = 0 \Rightarrow x = -\frac{5}{4}$

53. $(x + 13)^2 = 25$

$x + 13 = \pm 5$

$x = -13 \pm 5$

$x = -18$ or $x = -8$

54. $(x - 5)^2 = 30$

$x - 5 = \pm\sqrt{30}$

$x = 5 \pm \sqrt{30}$

55. $x^2 + 12x + 250 = 0$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{-12 \pm 2\sqrt{11}}{2}$$

$$= -6 \pm \sqrt{11}$$

56. $9x^2 - 12x = 14$

$9x^2 - 12x - 14 = 0$

$$x = \frac{-12 \pm \sqrt{(-12)^2 - 4(9)(-14)}}{2(9)}$$

$$= \frac{-12 \pm 18\sqrt{2}}{18}$$

$$= \frac{2}{3} \pm \sqrt{2}$$

57. $-2x^2 - 5x + 27 = 0$

$2x^2 + 5x - 27 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-27)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{241}}{4}$$

58. $-20 - 3x + 3x^2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

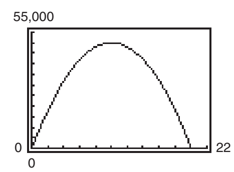
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)}$$

$$= \frac{3 \pm \sqrt{249}}{6} = \frac{1}{2} \pm \frac{\sqrt{249}}{6}$$

59. $M = 500x(20 - x)$

(a) $500x(20 - x) = 0$ when $x = 0$ feet and $x = 20$ feet.

(b)



(c) The bending moment is greatest when $x = 10$ feet.

60. (a) $h(t) = -16t^2 + 30t + 5.8$

(b) $h(1) = -16 \cdot 1^2 + 30 \cdot 1 + 5.8 = 19.8$ feet

(c) $-16t^2 + 30t + 5.8 = 6.2$

$-16t^2 + 30t - 0.4 - 0$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(-0.4)}}{2(-16)}$$

$$= \frac{-30 \pm \sqrt{874.4}}{-32}$$

≈ 1.86157 or 0.01343

The ball will hit the ground in about 1.86 seconds.

61. $4 + \sqrt{-9} = 4 + 3i$

62. $3 + \sqrt{-16} = 3 + 4i$

63. $i^2 + 3i = -1 + 3i$

64. $-5i + i^2 = -1 - 5i$

65. $(6 - 4i) + (-9 + i) = (6 + (-9)) + (-4i + i) = -3 - 3i$

66. $(7 - 2i) - (3 - 8i) = (7 - 3) + (-2i + 8i) = 4 + 6i$

67. $$\begin{aligned} -3i(-2 + 5i) &= 6i - 15i^2 \\ &= 6i - 15(-1) \\ &= 15 + 6i \end{aligned}$$

68. $$\begin{aligned} (4 + i)(3 - 10i) &= 12 - 40i + 3i - 10i^2 \\ &= 12 - 37i - 10(-1) \\ &= 22 - 37i \end{aligned}$$

69. $$\begin{aligned} (1 + 7i)(1 - 7i) &= 1 - 49i^2 \\ &= 1 - 49(-1) \\ &= 1 + 49 \\ &= 50 \end{aligned}$$

70. $$\begin{aligned} (5 - 9i)^2 &= 25 - 90i + 81i^2 \\ &= 25 - 90i + 81(-1) \\ &= 25 - 81 - 90i \\ &= -56 - 90i \end{aligned}$$

71. $$\begin{aligned} \frac{4}{1 - 2i} &= \frac{4}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} \\ &= \frac{4 + 8i}{1 - 4i^2} \\ &= \frac{4 + 8i}{5} \\ &= \frac{4}{5} + \frac{8}{5}i \end{aligned}$$

72. $$\begin{aligned} \frac{6 - 5i}{i} &= \frac{6 - 5i}{i} \cdot \frac{-i}{-i} \\ &= \frac{-6i + 5i^2}{-i^2} \\ &= -5 - 6i \end{aligned}$$

73. $$\begin{aligned} \frac{3 + 2i}{5 + i} &= \frac{3 + 2i}{5 + i} \cdot \frac{5 - i}{5 - i} \\ &= \frac{15 - 3i + 10i - 2i^2}{25 - i^2} \\ &= \frac{17 + 7i}{26} \\ &= \frac{17}{26} + \frac{7i}{26} \end{aligned}$$

74. $$\begin{aligned} \frac{7i}{(3 + 2i)^2} &= \frac{7i}{9 + 12i + 4i^2} \\ &= \frac{7i}{9 + 12i + 4(-1)} \\ &= \frac{7i}{5 + 12i} \\ &= \frac{7i}{5 + 12i} \cdot \frac{5 - 12i}{5 - 12i} \\ &= \frac{35i - 84i^2}{25 - 144i^2} \\ &= \frac{35i - 84(-1)}{25 + 144} \\ &= \frac{84 + 35i}{169} \\ &= \frac{84}{169} + \frac{35}{169}i \end{aligned}$$

75. $$\begin{aligned} \frac{4}{2 - 3i} + \frac{2}{1 + i} &= \frac{4}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} + \frac{2}{1 + i} \cdot \frac{1 - i}{1 - i} \\ &= \frac{8 + 12i}{4 + 9} + \frac{2 - 2i}{1 + 1} \\ &= \frac{8}{13} + \frac{12}{13}i + 1 - i \\ &= \left(\frac{8}{13} + 1\right) + \left(\frac{12}{13}i - i\right) \\ &= \frac{21}{13} - \frac{1}{13}i \end{aligned}$$

76. $$\begin{aligned} \frac{1}{2 + i} - \frac{5}{1 + 4i} &= \frac{(1 + 4i) - 5(2 + i)}{(2 + i)(1 + 4i)} \\ &= \frac{1 + 4i - 10 - 5i}{2 + 81 + i + 4i^2} \\ &= \frac{-9 - i}{-2 + 9i} \cdot \frac{(-2 - 9i)}{(-2 - 9i)} \\ &= \frac{18 + 81i + 2i + 9i^2}{4 - 81i^2} \\ &= \frac{9 + 83i}{85} = \frac{9}{85} + \frac{83i}{85} \end{aligned}$$

77. $x^2 - 2x + 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{-36}}{2}$$
$$= \frac{2 \pm 6i}{2}$$
$$= 1 \pm 3i$$

78. $x^2 + 6x + 34 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6 \pm \sqrt{(6)^2 - 4(1)(34)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{-100}}{2}$$
$$= \frac{-6 \pm 10i}{2}$$
$$= -3 \pm 5i$$

79. $4x^2 + 4x + 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{(4)^2 - 4(4)(7)}}{2(4)}$$
$$= \frac{-4 \pm \sqrt{-96}}{8}$$
$$= \frac{-4 \pm 4\sqrt{6}i}{8}$$
$$= -\frac{1}{2} \pm \frac{\sqrt{6}}{2}i$$

80. $6x^2 + 3x + 27 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(6)(27)}}{2(6)}$$
$$= \frac{-3 \pm \sqrt{-639}}{12}$$
$$= \frac{-3 \pm 3i\sqrt{71}}{12} = -\frac{1}{4} \pm \frac{\sqrt{71}}{4}i$$

81. $5x^4 - 12x^3 = 0$

$$x^3(5x - 12) = 0$$
$$x^3 = 0 \text{ or } 5x - 12 = 0$$
$$x = 0 \text{ or } \qquad x = \tfrac{12}{5}$$

82. $4x^3 - 6x^2 = 0$

$$x^2(4x - 6) = 0$$
$$x^2 = 0 \Rightarrow x = 0$$
$$4x - 6 = 0 \Rightarrow x = \tfrac{3}{2}$$

83. $x^3 - 7x^2 + 4x - 28 = 0$

$$x^2(x - 7) + 4(x - 7) = 0$$
$$(x - 7)(x^2 + 4) = 0$$
$$x - 7 = 0 \Rightarrow x = 7$$
$$x^2 + 4 = 0 \Rightarrow x^2 = -4$$
$$x = \pm\sqrt{-4} = \pm 2i$$

84. $9x^4 + 27x^3 - 4x^2 - 12x = 0$

$$9x^3(x + 3) - 4x(x + 3) = 0$$
$$(x + 3)(9x^3 - 4x) = 0$$
$$(x + 3)(x)(9x^2 - 4) = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$
$$x = 0$$
$$9x^2 - 4 = 0 \Rightarrow x = \pm\tfrac{2}{3}$$

85.

$$x^6 - 7x^3 - 8 = 0$$
$$(x^3)^2 - 7(x^3) - 8 = 0$$
$$u^2 - 7u - 8 = 0$$
$$(u - 8)(u + 1) = 0$$
$$u - 8 = 0$$
$$x^3 - 8 = 0$$
$$(x - 2)(x^2 + 2x + 4) = 0$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x^2 + 2x + 4 = 0 \Rightarrow x = -1 \pm \sqrt{3}i$$
$$u + 1 = 0$$
$$x^3 + 1 = 0$$
$$(x + 1)(x^2 - x + 1) = 0$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x^2 - x + 1 = 0 \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

86.
$$\begin{aligned} x^4 - 13x^2 - 48 &= 0 \\ (x^2)^2 - 13(x^2) - 48 &= 0 \\ \text{Let } u &= x^2. \\ u^2 - 13u - 48 &= 0 \\ (u - 16)(u + 3) &= 0 \\ u - 16 = 0 &\Rightarrow u = 16 \\ u + 3 = 0 &\Rightarrow u = -3 \end{aligned}$$

$u = 16 \quad u = -3$

$x^2 = 16 \quad x^2 = -3$

$x = \pm 4 \quad x = \pm\sqrt{3}i$

87.
$$\begin{aligned} \sqrt{2x + 3} + \sqrt{x - 2} &= 2 \\ \left(\sqrt{2x + 3}\right)^2 &= \left(2 - \sqrt{x - 2}\right)^2 \\ 2x + 3 &= 4 - 4\sqrt{x - 2} + x - 2 \\ x + 1 &= -4\sqrt{x - 2} \\ (x + 1)^2 &= \left(-4\sqrt{x - 2}\right)^2 \\ x^2 + 2x + 1 &= 16(x - 2) \\ x^2 - 14x + 33 &= 0 \\ (x - 3)(x - 11) &= 0 \end{aligned}$$

$x = 3$, extraneous or $x = 11$, extraneous

No solution

88.
$$\begin{aligned} 5\sqrt{x} - \sqrt{x - 1} &= 6 \\ 5\sqrt{x} &= 6 + \sqrt{x - 1} \\ 25x &= 36 + 12\sqrt{x - 1} + x - 1 \\ 24x - 35 &= 12\sqrt{x - 1} \\ 576x^2 - 1680x + 1225 &= 144(x - 1) \\ 576x^2 - 1824x + 1369 &= 0 \end{aligned}$$

$$x = \frac{-(-1824) \pm \sqrt{(-1824)^2 - 4(576)(1369)}}{2(576)} = \frac{1824 \pm \sqrt{172{,}800}}{1152} = \frac{1824 \pm 240\sqrt{3}}{1152}$$

$$x = \frac{38 + 5\sqrt{3}}{24}$$

$$x = \frac{38 - 5\sqrt{3}}{24}, \text{ extraneous}$$

89.
$$\begin{aligned} (x - 1)^{2/3} - 25 &= 0 \\ (x - 1)^{2/3} &= 25 \\ (x - 1)^2 &= 25^3 \\ x - 1 &= \pm\sqrt{25^3} \\ x &= 1 \pm 125 \\ x &= 126 \text{ or } x = -124 \end{aligned}$$

90.
$$\begin{aligned} (x + 2)^{3/4} &= 27 \\ x + 2 &= 27^{4/3} \\ x + 2 &= 81 \\ x &= 79 \end{aligned}$$

91.
$$\begin{aligned} \frac{5}{x} &= 1 + \frac{3}{x + 2} \\ 5(x + 2) &= 1(x)(x + 2) + 3x \\ 5x + 10 &= x^2 + 2x + 3x \\ 10 &= x^2 \\ \pm\sqrt{10} &= x \end{aligned}$$

92.
$$\begin{aligned} \frac{6}{x} + \frac{8}{x + 5} &= 3 \\ x(x + 5)\frac{6}{x} + x(x + 5)\frac{8}{x + 5} &= 3x(x + 5) \\ 6(x + 5) + 8x &= 3x(x + 5) \\ 14x + 30 &= 3x^2 + 15x \\ 0 &= 3x^2 + x - 30 \\ 0 &= (3x + 10)(x - 3) \\ 0 &= 3x + 10 \Rightarrow x = -\frac{10}{3} \\ 0 &= x - 3 \Rightarrow x = 3 \end{aligned}$$

93. $|x - 5| = 10$

$x - 5 = -10$ or $x - 5 = 10$

$x = -5$ $\quad$ $x = 15$

94. $|2x + 3| = 7$

$|2x + 3| = 7$ or $2x + 3 = -7$

$2x = 4$ $\quad$ $2x = -10$

$x = 2$ $\quad$ $x = -5$

95. $|x^2 - 3| = 2x$

$$\begin{aligned} x^2 - 3 &= 2x && \text{or} & x^2 - 3 &= -2x \\ x^2 - 2x - 3 &= 0 && & x^2 + 2x - 3 &= 0 \\ (x - 3)(x + 1) &= 0 && & (x + 3)(x - 1) &= 0 \\ x = 3 \quad \text{or} \quad x &= -1 && & x = -3 \quad \text{or} \quad x &= 1 \end{aligned}$$

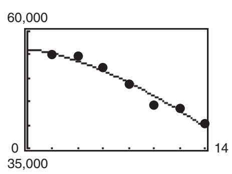
The only solutions of the original equation are $x = 3$ or $x = 1$. ($x = 3$ and $x = -1$ are extraneous.)

96. $|x^2 - 6| = x$

$$\begin{aligned} x^2 - 6 &= x && \text{or} & -(x^2 - 6) &= x \\ x^2 - x - 6 &= 0 && & x^2 + x - 6 &= 0 \\ (x - 3)(x + 2) &= 0 && & (x + 3)(x - 2) &= 0 \\ x - 3 = 0 &\Rightarrow x = 3 && & x - 2 = 0 &\Rightarrow x = 2 \\ x + 2 = 0 &\Rightarrow x = -2, \text{ extraneous} && & x + 3 = 0 &\Rightarrow x = -3, \text{ extraneous} \end{aligned}$$

97.
$$\begin{aligned} 29.95 &= 42 - \sqrt{0.001x + 2} \\ -12.05 &= -\sqrt{0.001x + 2} \\ \sqrt{0.001x + 2} &= 12.05 \\ 0.001x + 2 &= 145.2025 \\ 0.001x &= 143.2025 \\ x &= 143{,}202.5 \\ &\approx 143{,}203 \text{ units} \end{aligned}$$

98. (a)



The model fits the data well.

(b) Using trace and zoom features, there was an average of 50,000,000 daily newspapers in 2007.

(c)
$$\begin{aligned} 50{,}000 &= 56{,}146 - 314{,}980t^{3/2} \\ -6146 &= -314.980t^{3/2} \\ \frac{6146}{314.980} &= t^{3/2} \\ t^{3/2} &\approx 19.5123 \\ t &\approx 19.5123^{2/3} \\ t &\approx 7.24 \Rightarrow 2007 \end{aligned}$$

99. Interval: $(-7, 2]$

Inequality: $-7 < x \le 2$

The interval is bounded.

100. Interval: $(4, \infty)$

Inequality: $4 < x$

The interval is unbounded.

101. Interval: $(-\infty, -10]$

Inequality: $x \le -10$

The interval is unbounded.

102. Interval: $[-2, 2]$

Inequality: $-2 \le x \le 2$

The interval is bounded.

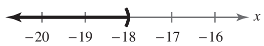
103. $3(x + 2) + 7 < 2x - 5$

$3x + 6 + 7 < 2x - 5$

$3x + 13 < 2x - 5$

$x < -18$

$(-\infty\ -18)$



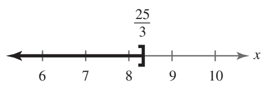
104. $2(x + 7) - 4 \ge 5(x - 3)$

$2x + 10 \ge 5x - 15$

$-3x \ge -25$

$x \le \frac{25}{3}$

$\left(-\infty, \frac{25}{3}\right]$



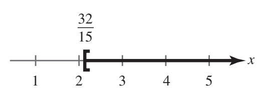
105. $4(5 - 2x) \le \frac{1}{2}(8 - x)$

$20 - 8x \le 4 - \frac{1}{2}x$

$-\frac{15}{2}x \le -16$

$x \ge \frac{32}{15}$

$\left[\frac{32}{15}, \infty\right)$



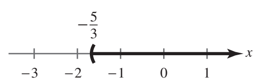
106. $\frac{1}{2}(3 - x) > \frac{1}{3}(2 - 3x)$

$9 - 3x > 4 - 6x$

$3x > -5$

$x > -\frac{5}{3}$

$\left(-\frac{5}{3}, \infty\right)$

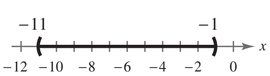


107. $|x + 6| < 5$

$-5 < x + 6 < 5$

$-11 < x < -1$

$(-11, -1)$



108. $\frac{2}{3}|3 - x| \ge 4$

$|3 - x| \ge 6$

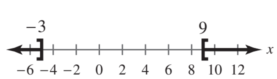
$3 - x \le -6$ or $3 - x \ge 6$

$-x \le -9$ or $-x \ge 3$

$x \ge 9$ or $x \le -3$

$x \le -3$ or $x \ge 9$

$(-\infty, -3] \cup [9, \infty)$



109. $125.33x > 92x + 1200$

$33.33x > 1200$

$x > 36$ units

So, the smallest value of x for which the product returns a profit is 37 units.

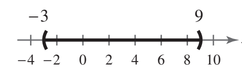
110. If the side is 19.3 cm, then with the possible error of 0.5 cm we have:

$18.8 \le \text{side} \le 19.8$

$353.44\text{ cm}^2 \le \text{area} \le 392.04\text{ cm}^2$

111. $x^2 - 6x - 27 < 0$

$(x + 3)(x - 9) < 0$



Key numbers: $x = -3, x = 9$

Test intervals: $(-\infty, -3), (-3, 9), (9, \infty)$

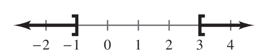
Test: Is $(x + 3)(x - 9) < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-3, 9)$.

112. $x^2 - 2x \ge 3$

$x^2 - 2x - 3 \ge 0$

$(x - 3)(x + 1) \ge 0$



Key numbers: $x = -1, x = 3$

Test intervals: $(-\infty, -1), (-1, 3), (3, \infty)$

Test: Is $(x - 3)(x + 1) \ge 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, -1] \cup [3, \infty)$.

113. $5x^3 - 45x < 0$

$5x(x^2 - 9) < 0$

$5x(x + 3)(x - 3) < 0$

Key numbers: $x = \pm 3, x = 0$

Test intervals: $(-\infty, -3), (-3, 0), (0, 3), (3, \infty)$

Test: Is $5x(x + 3)(x - 3) < 0$?

By testing an x-value in each test interval in the inequality, the solution set is $(-\infty, -3) \cup (0, 3)$.

114. $2x^3 - 5x^2 - 3x \ge 0$

$x(2x^2 - 5x - 3) \ge 0$

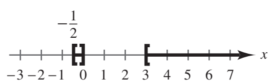
$x(2x + 1)(x - 3) \ge 0$

Key numbers: $x = 0 - \frac{1}{2}, 3$

Test intervals: $\left(-\infty, -\frac{1}{2}\right), \left(-\frac{1}{2}, 0\right), (0, 3), (3, \infty)$

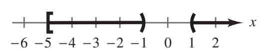
Test: Is $x(2x + 1)(x - 3) \ge 0$?

By testing an x-value in each test interval in the inequality, the solution set is $\left[-\frac{1}{2}, 0\right] \cup [3, \infty)$.



115.

$$\frac{2}{x+1} \le \frac{3}{x-1}$$



$$\frac{2(x-1) - 3(x+1)}{(x+1)(x-1)} \le 0$$

$$\frac{2x - 2 - 3x - 3}{(x+1)(x-1)} \le 0$$

$$\frac{-(x+5)}{(x+1)(x-1)} \le 0$$

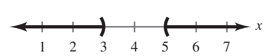
Key numbers: $x = -5, x = -1, x = 1$

Test intervals: $(-5, -1), (-1, 1), (1, \infty)$

Test: Is $\dfrac{-(x+5)}{(x+1)(x-1)} \le 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $[-5, -1) \cup (1, \infty)$.

116. $\dfrac{x-5}{3-x} < 0$



Key numbers: $x = 5, x = 3$

Test intervals: $(-\infty, 3), (3, 5), (5, \infty)$

Test: Is $\dfrac{x-5}{3-x} < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, 3) \cup (5, \infty)$.

117.

$$\begin{aligned} 5000(1 + r)^2 &> 5500 \\ (1 + r)^2 &> 1.1 \\ 1 + r &> 1.0488 \\ r &> 0.0488 \\ r &> 4.9\% \end{aligned}$$

118.

$$\begin{aligned} P &= \frac{1000(1 + 3t)}{5 + t} \\ 2000 &\le \frac{1000(1 + 3t)}{5 + t} \\ 2000(5 + t) &\le 1000(1 + 3t) \\ 10{,}000 + 2000t &\le 1000 + 3000t \\ -1000t &\le -9000 \\ t &\ge 9 \text{ days} \end{aligned}$$

At least 9 days are required.

119. False.

$$\sqrt{-18}\sqrt{-2} = \left(\sqrt{18}i\right)\left(\sqrt{2}i\right) = \sqrt{36}i^2 = -6$$

$$\sqrt{(-8)(-2)} = \sqrt{36} = 6$$

120. False. The equation has no real solution.

The solutions are

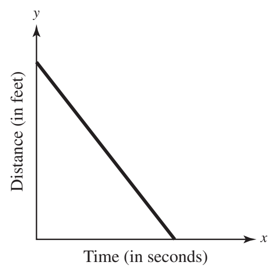
$$\frac{717}{650} \pm \frac{\sqrt{3311}i}{650}.$$

121. Rational equations, equations involving radicals, and absolute value equations, may have "solutions" that are extraneous. So checking solutions, in the original equations, is crucial to eliminate these extraneous values.

122. $\left[-\infty, -\frac{36}{11}\right]$ or $[2, \infty)$. *Sample answer:* The first equivalent inequality written is incorrect. It should be $11x + 4 \le -26$. This leads to the solution $\left(-\infty, -\frac{30}{11}\right]$ or $[2, \infty)$.

Problem Solving for Chapter 1

1.



2. (a) $1 + 2 + 3 + 4 + 5 = 15$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

(b) $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$

When $n = 5$: $\frac{1}{2}(5)(6) = 15$

When $n = 8$: $\frac{1}{2}(8)(9) = 36$

When $n = 10$: $\frac{1}{2}(10)(11) = 55$

(c)
$$\begin{aligned} \tfrac{1}{2}n(n + 1) &= 210 \\ n(n + 1) &= 420 \\ n^2 + n - 420 &= 0 \\ (n + 21)(n - 20) &= 0 \\ n = -21 \text{ or } n &= 20 \end{aligned}$$

Since n is a natural number, choose $n = 20$.

3. (a) $A = \pi ab$

$a + b = 20 \Rightarrow b = 20 - a$, thus:

$A = \pi a(20 - a)$

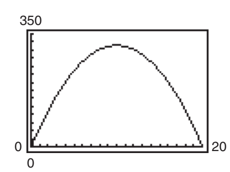
(b)

a	4	7	10	13	16
A	64π	91π	100π	91π	64π

(c)
$$\begin{aligned} 300 &= \pi a(20 - a) \\ 300 &= 20\pi a - \pi a^2 \\ \pi a^2 - 20\pi a + 300 &= 0 \\ a &= \frac{20\pi \pm \sqrt{(-20\pi)^2 - 4\pi(300)}}{2\pi} \\ &= \frac{20\pi \pm \sqrt{400\pi^2 - 1200\pi}}{2\pi} \\ &= \frac{20\pi \pm 20\sqrt{\pi(\pi - 3)}}{2\pi} \\ &= 10 \pm \frac{10}{\pi}\sqrt{\pi(\pi - 3)} \end{aligned}$$

$a \approx 12.12$ or $a \approx 7.88$

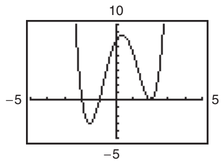
(d)



(e) The a-intercepts occur at $a = 0$ and $a = 20$. Both yield an area of 0. When $a = 0, b = 20$ and you have a vertical line of length 40. Likewise when $a = 20, b = 0$ and you have a horizontal line of length 40. They represent the minimum and maximum values of a.

(f) The maximum value of A is $100\pi \approx 314.159$. This occurs when $a = b = 10$ and the ellipse is actually a circle.

4. $y = x^4 - x^3 - 6x^2 + 4x + 8 = (x-2)^2(x+1)(x+2)$



From the graph you see that $x^4 - x^3 - 6x^2 + 4x + 8 > 0$ on the intervals $(-\infty, -2) \cup (-1, 2) \cup (2, \infty)$.

5. $P = 0.00256s^2$

(a) $0.00256s^2 = 20$

$$s^2 = 7812.5$$
$$s \approx 88.4 \text{ miles per hour}$$

(b) $0.00256s^2 = 40$

$$s^2 = 15625$$
$$s = 125 \text{ miles per hour}$$

No, actually it can survive wind blowing at $\sqrt{2}$ times the speed found in part (a).

(c) The wind speed in the formula is squared, so a small increase in wind speed could have potentially serious effects on a building.

6. $h = \left(\sqrt{h_0} - \dfrac{2\pi d^2\sqrt{3}}{lw}t\right)^2$

$l = 60''$, $w = 30''$, $h_0 = 25''$, $d = 2''$

$$h = \left(5 - \frac{8\pi\sqrt{3}}{1800}t\right)^2 = \left(5 - \frac{\pi\sqrt{3}}{225}t\right)^2$$

(a) $$12.5 = \left(5 - \frac{\pi\sqrt{3}}{225}t\right)^2$$

$$\sqrt{12.5} = 5 - \frac{\pi\sqrt{3}}{225}t$$

$$t = \frac{225}{\pi\sqrt{3}}\left(5 - \sqrt{12.5}\right) \approx 60.6 \text{ seconds}$$

(b) $$0 = \left(\sqrt{12.5} - \frac{\pi\sqrt{3}}{225}t\right)^2$$

$$t = \frac{225\sqrt{12.5}}{\pi\sqrt{3}} \approx 146.2 \text{ seconds}$$

(c) The speed at which the water drains decreases as the amount of the water in the bathtub decreases.

7. (a) If $x^2 + 9 = (x+m)(x+n)$ then

$mn = 9$ and $m + n = 0$.

(b) $m + n = 0 \Rightarrow n = -m$

$m(-m) = 9 \Rightarrow -m^2 = 9 \Rightarrow m^2 = -9$

There is no **integer** m such that m^2 equals a negative number. $x^2 + 9$ cannot be factored over the integers.

8.

$$4\sqrt{x} = 2x + k$$

$$2x - 4\sqrt{x} + k = 0 \quad \text{Complete the square.}$$

$$x - 2\sqrt{x} = -\frac{k}{2}$$

$$x - 2\sqrt{x} + 1 = 1 - \frac{k}{2}$$

$$\left(\sqrt{x} - 1\right)^2 = 1 - \frac{k}{2}$$

Number of solutions (real)	Some k-values
2	$-1, 0, 1$
1	2 only
0	3, 4, 5

This equation will have two solutions when $1 - \frac{k}{2} > 0$ or when $k < 2$.

This equation will have one solution when $1 - \frac{k}{2} = 0$ or when $k = 2$.

This equation will have no solutions when $1 - \frac{k}{2} < 0$ or when $k > 2$.

9. (a) 5, 12, and 13; 8, 15, and 17

7, 24, and 25

(b) $5 \cdot 12 \cdot 13 = 780$ which is divisible by 3, 4, and 5.

$8 \cdot 15 \cdot 17 = 2040$ which is divisible by 3, 4, and 5.

$7 \cdot 24 \cdot 25 = 4200$ which is also divisible by 3, 4, and 5.

(c) Conjecture: If $a^2 + b^2 = c^2$ where a, b, and c are positive integers, then abc is divisible by 60.

10.

Equation	x_1, x_2	$x_1 + x_2$	$x_1 \cdot x_2$
(a) $x^2 - x - 6 = 0$ $(x + 2)(x - 3) = 0$	$-2, 3$	1	-6
(b) $2x^2 + 5x - 3 = 0$ $(2x - 1)(x + 3) = 0$	$\frac{1}{2}, -3$	$-\frac{5}{2}$	$-\frac{3}{2}$
(c) $4x^2 - 9 = 0$ $(2x + 3)(2x - 3) = 0$	$-\frac{3}{2}, \frac{3}{2}$	0	$-\frac{9}{4}$
(d) $x^2 - 10x + 34 = 0$ $x = 5 \pm 3i$	$5 + 3i, 5 - 3i$	10	34

11. (a) $S = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} + \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-2a}{2a}$

$= -\dfrac{b}{a}$

(b) $P = \left(\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$

$= \dfrac{b^2 - (b^2 - 4ac)}{4a^2}$

$= \dfrac{4ac}{4a^2}$

$= \dfrac{c}{a}$

12. (a) (i) $\left(\dfrac{-5 + 5\sqrt{3}i}{2}\right)^3 = 125$

(ii) $\left(\dfrac{-5 - 5\sqrt{3}i}{2}\right)^3 = 125$

(b) (i) $\left(\dfrac{-3 + 3\sqrt{3}i}{2}\right)^3 = 27$

(ii) $\left(\dfrac{-3 - 3\sqrt{3}i}{2}\right)^3 = 27$

(c) (i) The cube roots of 1 are: $1, \dfrac{-1 \pm \sqrt{3}i}{2}$

(ii) The cube roots of 8 are: $2, -1 \pm \sqrt{3}i$

(iii) The cube roots of 64 are: $4, -2 \pm 2\sqrt{3}i$

13. (a) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{1 + i} = \dfrac{1}{1 + i} \cdot \dfrac{1 - i}{1 - i}$

$= \dfrac{1 - i}{2} = \dfrac{1}{2} - \dfrac{1}{2}i$

(b) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{3 - i} = \dfrac{1}{3 - i} \cdot \dfrac{3 + i}{3 + i}$

$= \dfrac{3 + i}{10} = \dfrac{3}{10} + \dfrac{1}{10}i$

(c) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{-2 + 8i}$

$= \dfrac{1}{-2 + 8i} \cdot \dfrac{-2 - 8i}{-2 - 8i}$

$= \dfrac{-2 - 8i}{68} = -\dfrac{1}{34} - \dfrac{2}{17}i$

14. $(a + bi)(a - bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$

Since *a* and *b* are real numbers, $a^2 + b^2$ is also a real number.

15. (a) $c = 1$

The terms are: $i, -1 + i, -i, -1 + i, -i, -1 + i, -i, -1 + i, -i, \ldots$

The sequence is bounded so $c = i$ *is* in the Mandelbrot Set.

(b) $c = -2$

The terms are: $1 + i, 1 + 3i, -7 + 7i, 1 - 97i, -9407 - 1931i, \ldots$

The sequence is unbounded so $c = 1 + i$ *is not* in the Mandelbrot Set.

(c) $c = -2$

The terms are: $-2, 2, 2, 2, 2, \ldots$

The sequence is bounded so $c = -2$ *is* in the Mandelbrot Set.

Practice Test for Chapter 1

1. Graph $3x - 5y = 15$.

2. Graph $y = \sqrt{9 - x}$.

3. Solve $5x + 4 = 7x - 8$.

4. Solve $\frac{x}{3} - 5 = \frac{x}{5} + 1$.

5. Solve $\frac{3x + 1}{6x - 7} = \frac{2}{5}$.

6. Solve $(x - 3)^2 + 4 = (x + 1)^2$.

7. Solve $A = \frac{1}{2}(a + b)h$ for a.

8. 301 is what percent of 4300?

9. Cindy has $6.05 in quarter and nickels. How many of each coin does she have if there are 53 coins in all?

10. Ed has $15,000 invested in two fund paying $9\frac{1}{2}\%$ and 11% simple interest, respectively. How much is invested in each if the yearly interest is $1582.50?

11. Solve $28 + 5x - 3x^2 = 0$ by factoring.

12. Solve $(x - 2)^2 = 24$ by taking the square root of both sides.

13. Solve $x^2 - 4x - 9 = 0$ by completing the square.

14. Solve $x^2 + 5x - 1 = 0$ by the Quadratic Formula.

15. Solve $3x^2 - 2x + 4 = 0$ by the Quadratic Formula.

16. The perimeter of a rectangle is 1100 feet. Find the dimensions so that the enclosed area will be 60,000 square feet.

17. Find two consecutive even positive integers whose product is 624.

18. Solve $x^3 - 10x^2 + 24x = 0$ by factoring.

19. Solve $\sqrt[3]{6 - x} = 4$.

20. Solve $(x^2 - 8)^{2/5} = 4$.

21. Solve $x^4 - x^2 - 12 = 0$.

22. Solve $4 - 3x > 16$.

23. Solve $\left|\frac{x - 3}{2}\right| < 5$.

24. Solve $\frac{x + 1}{x - 3} < 2$.

25. Solve $|3x - 4| \geq 9$.