An Introduction to Econometric Theory: Solutions Manual for the Exercises

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Chapter 1

Elementary Data Analysis: Exercises

1. *Are the following statements True or False?*

   (a) *The correlation of a variable with itself is always 1.*
   TRUE. Put $y_t = x_t$ in (1.11)

   (b) *Chebyshev's rule says that at least a proportion* $1/m^2$ *of any sample lies beyond* m *standard deviations from the mean.*
   FALSE. "At most", not "At least".

   (c) *The least absolute values fitting method is more influenced by outlying observations than least squares.*
   FALSE. Less not more.

   (d) *The sample mean is the least squares measure of location.*
   TRUE, see (1.29).

   (e) *The slope coefficient in the simple regression is the tangent of the angle of the regression line with the horizontal axis.*
   TRUE, see Figure 1.3

   (f) *The least squares estimator of the slope coefficient* (y on x) *is the sum of the products of* y *with the mean deviations of* x, *divided by the sum of squares of the mean deviations of* x.
   TRUE, see (1.27).

   (g) *Run a regression in both directions* (y on x *and* x on y), *and the product of the two slope coefficients is equal to the squared correlation coefficient.*
   TRUE, compare (1.11) with (1.27).

2. *Here are 12 observations on a variable* x:

   86,109,81,53,86,−14,65,−39,65,34,79,−27

(a) *Compute the mean.*

$$\bar{x} = 48.1667$$

(b) *Compute the sequence of mean deviations.*

$$60.83, 32.83, 4.83, 37.83, -62.17, 16.83, -87.17, 16.83, -14.17, 30.83, -75.17$$

(c) *Compute the standard deviation.*

$$s = 49.0785$$

(d) *How many of these data points lie more than (i) one, (ii) two, (iii) three standard deviations from the mean?*

(i) 1 SD bounds: $[-0.91, 97.24]$ and 4/12 lie outside.

(ii) 2SD bounds: $[-49.99, 146.32]$ and none lie outside.

(e) *Include the following observations in the set, and obtain the mean and standard deviation for this case.*

$$209,\ 475,\ -114,\ 46$$

$$\bar{x} = 74.625, \quad s = 129.009$$

(f) *Repeat exercise (d) for the enlarged data set.*

(i) 1 SD bounds: $[-54.38, 203.63]$ and 3/12 lie outside.

(ii) 2SD bounds: $[-183.99, 332.64]$ and none lie outside.

3. *Here are 12 observations on a variable y.*

43,62,26,46,48,3,52,-10,37,5,42,4

(a) *Compute the mean.*

$$\bar{x} = 29.833$$

(b) *Compute the standard deviation.*

$$s = 23.54$$

(c) *Compute the correlation coefficient of y with x in Question 1.*

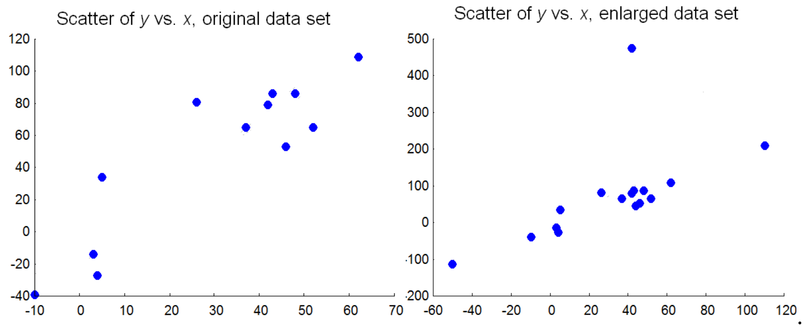
$$r = 0.896$$

(d) *Include the following data points in the set and compute the correlation coefficient with the enlarged data set of Question 1.*

110,42,-50,44

$$r = 0.602$$

(e) *Draw scatter plots of the two cases*



4. *Compute the regression of y on x (original sample).*

(a) *Report the fitted slope and intercept coefficients and the residuals, $\hat{u}$.*

$$\hat{y} = 9.11 + 0.43x$$

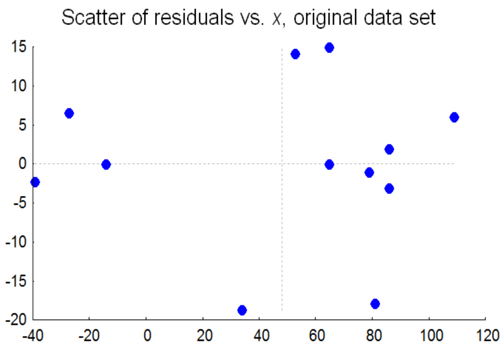
Residuals:

$-3.1, 5.99, -17.96, 14.09, 1.89, -0.091, 14.93, -2.34, -0.074, -18.7, -1.09, 6.5$

Fitted values:

$46.11, 56, 43.96, 31.91, 46.11, 3.09, 37.07, -7.66, 37.07, 23.74, 43.1, -2.5$

(b) *Verify that $\hat{u}$ has zero correlation with x. Draw the scatter plot of the two variables.*



Correlation coefficient is zero.

(c) *Consider the prediction equation*

$$\breve{y} = 5 + 0.6x$$

Show that the prediction errors in this case are correlated with x, and also have a larger mean squared error than the regression predictions.

The predictions are

$$56.6, 70.4, 53.6, 36.8, 56.6, -3.4, 44, -18.4, 44, 25.4, 52.4, -11.2$$

Correlation with x is -0.62. MSE is 194.47. MSE of the least squares residuals is 108.45

(d) *Compute the regression of y on x (extended sample). Report the fitted coefficients and comment*

$$\hat{y} = 19.037 + 0.167x.$$

Comment: large shift in the coefficients shows that extreme observations are influential in the least squares fit. See scatter plot - a single observation makes the difference.

5.

(a) *Solve the following equation system for $\hat{\beta}_1$.*

$$\hat{\beta}_1 \sum_{t=1}^{T} x_{1t}^2 + \hat{\beta}_2 \sum_{t=1}^{T} x_{1t}x_{2t} = \sum_{t=1}^{T} x_{1t}y_t \tag{1}$$

$$\hat{\beta}_1 \sum_{t=1}^{T} x_{2t}x_{1t} + \hat{\beta}_2 \sum_{t=1}^{T} x_{2t}^2 = \sum_{t=1}^{T} x_{2t}y_t \tag{2}$$

First step: get $\hat{\beta}_2$ a function of $\hat{\beta}_1$ in equation (2)

$$\hat{\beta}_2 = \frac{\sum_{t=1}^{T} x_{2t}y_t - \hat{\beta}_1 \sum_{t=1}^{T} x_{2t}x_{1t}}{\sum_{t=1}^{T} x_{2t}^2}$$

Second step. Substitute into equation (1)

$$\hat{\beta}_1 \sum_{t=1}^{T} x_{1t}^2 \sum_{t=1}^{T} x_{2t}^2 + \sum_{t=1}^{T} x_{2t}y_t \sum_{t=1}^{T} x_{1t}x_{2t} - \hat{\beta}_1 \sum_{t=1}^{T} x_{2t}x_{1t} \sum_{t=1}^{T} x_{1t}x_{2t} = \sum_{t=1}^{T} x_{1t}y_t \sum_{t=1}^{T} x_{2t}^2$$

Third step. Solve for $\hat{\beta}_1$

$$\hat{\beta}_1 = \frac{\sum_{t=1}^{T} x_{1t}y_t \sum_{t=1}^{T} x_{2t}^2 - \sum_{t=1}^{T} x_{2t}y_t \sum_{t=1}^{T} x_{1t}x_{2t}}{\sum_{t=1}^{T} x_{1t}^2 \sum_{t=1}^{T} x_{2t}^2 - \left(\sum_{t=1}^{T} x_{2t}x_{1t}\right)^2}.$$

(b) *Define $\hat{\alpha} = \bar{y} - \hat{\beta}_1\bar{x}_1 - \hat{\beta}_2\bar{x}_2$. Show that if $T\hat{\alpha}\bar{x}_1$ is subtracted from the right-hand side of equation (1), and $T\hat{\alpha}\bar{x}_2$ is subtracted from the right-hand side of equation (2), the resulting equations are modified by having the variables expressed in mean deviation form.*

$$\hat{\beta}_1 \sum_{t=1}^{T} x_{1t}^2 + \hat{\beta}_2 \sum_{t=1}^{T} x_{1t}x_{2t} = \sum_{t=1}^{T} x_{1t}y_t - T(\bar{y} - \hat{\beta}_1\bar{x}_1 - \hat{\beta}_2\bar{x}_2)\bar{x}_1$$

$$\hat{\beta}_1 \sum_{t=1}^{T} x_{2t}x_{1t} + \hat{\beta}_2 \sum_{t=1}^{T} x_{2t}^2 = \sum_{t=1}^{T} x_{2t}y_t - T(\bar{y} - \hat{\beta}_1\bar{x}_1 - \hat{\beta}_2\bar{x}_2)\bar{x}_2$$

are the same as

$$\hat{\beta}_1\left(\sum_{t=1}^{T} x_{1t}^2 - T\bar{x}_1^2\right) + \hat{\beta}_2\left(\sum_{t=1}^{T} x_{1t}x_{2t} - T\bar{x}_1\bar{x}_2\right) = \left(\sum_{t=1}^{T} x_{1t}y_t - T\bar{y}\bar{x}_1\right)$$
$$\hat{\beta}_1\left(\sum_{t=1}^{T} x_{2t}x_{1t} - T\bar{x}_1\bar{x}_2\right) + \hat{\beta}_2\left(\sum_{t=1}^{T} x_{2t}^2 - T\bar{x}_2^2\right) = \left(\sum_{t=1}^{T} x_{2t}y_t - T\bar{y}\bar{x}_2\right)$$

(c) *What is* $\hat{\alpha}$*?*
Write the equations as

$$\hat{\beta}_1\sum_{t=1}^{T} x_{1t}^2 + \hat{\beta}_2\sum_{t=1}^{T} x_{1t}x_{2t} + \hat{\alpha}\sum_{t=1}^{T} x_{1t} = \sum_{t=1}^{T} x_{1t}y_t$$
$$\hat{\beta}_1\sum_{t=1}^{T} x_{2t}x_{1t} + \hat{\beta}_2\sum_{t=1}^{T} x_{2t}^2 + \hat{\alpha}\sum_{t=1}^{T} x_{2t} = \sum_{t=1}^{T} x_{2t}y_t$$
$$\hat{\beta}_1\sum_{t=1}^{T} x_{1t} + \hat{\beta}_2\sum_{t=1}^{T} x_{2t} + T\hat{\alpha} = \sum_{t=1}^{T} y_t$$

and these are the normal equations with intercept, with the solution for $\hat{\alpha}$ indicated.

6. *Show that the* α *that minimizes* $\sum_{t=1}^{T}(y_t-\alpha)^2$ *is the sample mean of* $y_1,\ldots,y_T$.

Write

$$\begin{aligned}\sum_{t=1}^{T}(y_t-\alpha)^2 &= \sum_{t=1}^{T}(y_t-\bar{y}+\bar{y}-\alpha)^2\\ &= \sum_{t=1}^{T}(y_t-\bar{y})^2 + T(\bar{y}-\alpha)^2 + 2(\bar{y}-\alpha)\sum_{t=1}^{T}(y_t-\bar{y})\\ &\geq \sum_{t=1}^{T}(y_t-\bar{y})^2\end{aligned}$$

because

$$\sum_{t=1}^{T}(y_t-\bar{y}) = 0.$$

Chapter 2

Matrix Representation: Exercises

1. *Are the following statements True or False?*

   (a) *If for matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ both of the products $\boldsymbol{AB}$ and $\boldsymbol{BA}$ exist, then $\boldsymbol{A}$ and $\boldsymbol{B}$ must be square.*
   FALSE. Need $\boldsymbol{A}$ $m \times n$ and $\boldsymbol{B}$ $n \times m$.

   (b) *Symmetric matrices must be square.*
   TRUE, by definition

   (c) *If $\boldsymbol{A}$ and $\boldsymbol{B}$ are square matrices conformable for multiplication, then*

   $$\boldsymbol{AB} = \boldsymbol{BA}$$

   FALSE, by definition of matrix multiplication.

   (d) *If $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are matrices having the same dimensions $m \times n$, then*

   $$(\boldsymbol{A}'(\boldsymbol{B} + \boldsymbol{C}))' = \boldsymbol{B}'\boldsymbol{A} + \boldsymbol{C}'\boldsymbol{A}$$

   TRUE.

   $$\begin{aligned} (\boldsymbol{A}'(\boldsymbol{B} + \boldsymbol{C}))' &= (\boldsymbol{B} + \boldsymbol{C})'(\boldsymbol{A}')' \\ &= (\boldsymbol{B}' + \boldsymbol{C}')\boldsymbol{A} \\ &= \boldsymbol{B}'\boldsymbol{A} + \boldsymbol{C}'\boldsymbol{A} \end{aligned}$$

   (e) T*he inner product is defined for pairs of conformable vectors, while the outer product is defined for any pair of vectors.*
   TRUE, by definition.

   (f) *The scalar product obeys the commutative rule.*
   TRUE.

   $$\boldsymbol{B}(a\boldsymbol{C}) = a\boldsymbol{BC}.$$

(g) *If $\boldsymbol{A}$ and $\boldsymbol{B}$ are matrices conformable for multiplication such that AB exists, and are also partitioned conformably, then*

$$\begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1\boldsymbol{B}_1 & \boldsymbol{A}_2\boldsymbol{B}_1 \\ \boldsymbol{A}_1\boldsymbol{B}_2 & \boldsymbol{A}_2\boldsymbol{B}_2 \end{bmatrix}$$

FALSE. See (2.9).

2. *Let*

$$\boldsymbol{A} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}.$$

Compute

(a) $\boldsymbol{A} + \boldsymbol{B}'$

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 4 & 7 \\ 6 & 9 \end{bmatrix}$$

(b) $5\boldsymbol{B}$

$$\mathbf{5} \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 10 & 10 \\ 15 & 15 \end{bmatrix}$$

(c) $\boldsymbol{AB}$

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ 7 & 14 & 21 \\ 9 & 18 & 27 \end{bmatrix}$$

(d) $\boldsymbol{BA}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 14 & 32 \\ 14 & 32 \end{bmatrix}$$

(e) $\boldsymbol{A}'\boldsymbol{A}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}$$

(f) $\boldsymbol{BB}'$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 14 & 14 \\ 14 & 14 \end{bmatrix}$$

3. *Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be as in Question 2 and*

$$\boldsymbol{c} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Compute

(a) $\boldsymbol{B}\boldsymbol{c}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

(b) $\boldsymbol{B}'\boldsymbol{d}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

(c) $\boldsymbol{A} + \boldsymbol{c}\boldsymbol{d}'$

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 2 & 5 \\ 4 & 7 \end{bmatrix}$$

(d) $\boldsymbol{c}'\boldsymbol{c}$

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 2$$

(e) $\boldsymbol{c}\boldsymbol{c}'$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(f) $\|\boldsymbol{c}\|$

$$\sqrt{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} = \sqrt{2} = 1.414$$

(g) $\boldsymbol{d}'\boldsymbol{B}\boldsymbol{c}$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 8.$$

(h) $\boldsymbol{c}'\boldsymbol{A}\boldsymbol{A}'\boldsymbol{c}$

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & 10 \end{bmatrix} \begin{bmatrix} 4 \\ 10 \end{bmatrix} = 116.$$

4. *If* $\boldsymbol{A}$*,* $\boldsymbol{B}$*,* $\boldsymbol{c}$ *and* $\boldsymbol{d}$ *are defined as in Questions 2 and 3, write down ten matrix expressions involving one or more of these components that do not exist (violate conformability).*

Among others, $\boldsymbol{A}+\boldsymbol{B}$, $\boldsymbol{A}\boldsymbol{B}'$, $\boldsymbol{A}\boldsymbol{A}$, $\boldsymbol{B}+\boldsymbol{B}'$, $\boldsymbol{c}\boldsymbol{d}$, $\boldsymbol{c}'\boldsymbol{d}$, $\boldsymbol{B}'\boldsymbol{c}$, $\boldsymbol{c}+\boldsymbol{d}$, $\boldsymbol{d}\boldsymbol{d}$, $\boldsymbol{A}-\boldsymbol{d}\boldsymbol{c}'$.