

ANTENNA THEORY

SOLUTIONS MANUAL TO ACCOMPANY

ANTENNA THEORY ANALYSIS AND DESIGN

FOURTH EDITION

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WILEY

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Preface

This Solutions Manual consists of solutions for all the problems found in *Antenna Theory: Analysis and Design* (4th edition, 2016) at the end of Chapters 2–16. There are 699 (103 new) problems, most of them with multiple parts. The degree of difficulty and length varies. While certain solutions need special functions, found in graphical form in the appendices, others require the use of the computer program. These computer programs are placed on a password protected website. All of the computer programs, especially those at the end of Chapters 6, 9, 11, 13 and 14 have been developed to design, respectively, uniform and nonuniform arrays, impedance transformers, log-periodic arrays, horns and microstrip patch antennas. In some cases, the computer programs also perform analysis on the designs. The programs at the end of Chapters 2, 4, 5, 7, 8, 10, 12, 15 and 16 are primarily developed for analysis. The problems have been designed to test the student's grasp of this text's material and to apply the concepts to the analysis and design of many practical radiators. In this fourth edition, more emphasis has been placed on design. To accomplish this, equations, procedures, examples, graphs, end-of-the-chapter problems, and computer programs have been developed.

This manual has been prepared to assist the instructor in making homework and test assignments, and to provide one set of solutions for all of the problems. There maybe undoubtedly errors which have been overlooked. In addition, the solutions contained in this manual are not necessarily the simplest and/or the best. The author will, therefore, appreciate having errors brought to his attention and solicits alternate solutions to the problems.

This Solutions Manual for the fourth edition has been prepared from the manuals of the first, second and third editions and many other new problems provided by the author.

CHAPTER 2



Solution Manual

Exact

2.1. (a) $d\Omega = \sin \theta \, d\theta \, d\phi$

$$\Omega_A = \int_{45^\circ}^{60^\circ} \int_{\pi/4}^{\pi/3} d\Omega = \int_{\pi/4}^{\pi/3} \int_{\pi/6}^{\pi/3} \sin \theta \, d\theta \, d\phi$$

$$= (\phi) \Big|_{\pi/4}^{\pi/3} (-\cos \theta) \Big|_{\pi/6}^{\pi/3}$$

$$= \left(\frac{\pi}{3} - \frac{\pi}{4} \right) (-0.5 + 0.866)$$

$$\Omega_A = \left(\frac{\pi}{12} \right) (0.366) = 0.09582 \text{ sterads}$$

$$\Omega_A = \begin{cases} 0.09582 \text{ sterads} \\ 0.09582 \left(\frac{180}{\pi} \right) \left(\frac{180}{\pi} \right) = 314.5585 \text{ (degrees)}^2 \end{cases}$$

Approximate

$$\Omega_A \simeq \left(\frac{\pi}{3} - \frac{\pi}{4} \right) \left(\frac{\pi}{3} - \frac{\pi}{6} \right)$$

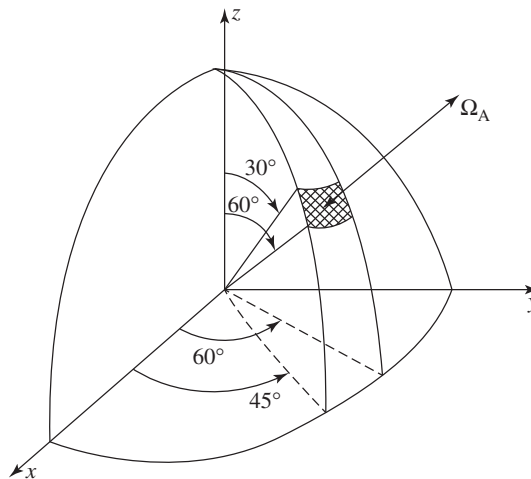
$$\simeq \left(\frac{\pi}{12} \right) \left(\frac{\pi}{6} \right) = \frac{\pi^2}{72}$$

$$\Omega_A \simeq 0.13708 \text{ sterads}$$

$$\Omega_A \simeq (60 - 45)(60 - 30)$$

$$\simeq 450 \text{ (degrees)}^2 \text{ or error of}$$

$$\left(\frac{450 - 314.5585}{314.5585} \right) \times 100 = 43.06\%$$



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$$\begin{aligned} \text{(b) } D_0 &= \frac{4\pi}{\Omega_A(\text{sterads})} = \frac{4\pi}{0.09582} = 131.1456 \text{ (dimensionless)} \\ &= 10 \log_{10}(131.1456) = 21.1775 \text{ dB} \end{aligned}$$

or

$$\begin{aligned} D_0 &= \frac{4\pi \left(\frac{180}{\pi}\right) \left(\frac{180}{\pi}\right)}{\Omega_A(\text{degrees})^2} = 131.1456 \text{ (dimensionless)} = 21.1775 \text{ dB} \\ D_0 &= \begin{cases} 131.1456 \text{ (dimensionless)} \\ 21.1775 \text{ (dB)} \end{cases} \end{aligned}$$

2.2. $\underline{\mathcal{W}} = \underline{\mathcal{E}} \times \underline{\mathcal{H}} = \text{Re} [\underline{E}e^{j\omega t}] \times \text{Re} [\underline{H}e^{j\omega t}]$

Using the identity $\text{Re} [\underline{A}e^{j\omega t}] = \frac{1}{2} [\underline{A}e^{j\omega t} + \underline{A}^*e^{-j\omega t}]$

The instant Poynting vector can be written as

$$\begin{aligned} \underline{\mathcal{W}} &= \left\{ \frac{1}{2} [\underline{E}e^{j\omega t} + \underline{E}^*e^{-j\omega t}] \right\} \times \left\{ \frac{1}{2} [\underline{H}e^{j\omega t} + \underline{H}^*e^{-j\omega t}] \right\} \\ &= \frac{1}{2} \left\{ \frac{1}{2} [\underline{E} \times \underline{H}^* + \underline{E}^* \times \underline{H}] + \frac{1}{2} [\underline{E} \times \underline{H}e^{j2\omega t} + \underline{E}^* \times \underline{H}^*e^{-j2\omega t}] \right\} \\ &= \frac{1}{2} \left\{ \frac{1}{2} [\underline{E} \times \underline{H}^* + (\underline{E} \times \underline{H}^*)^*] + \frac{1}{2} [\underline{E} \times \underline{H}e^{j2\omega t} + (\underline{E} \times \underline{H}e^{j2\omega t})^*] \right\} \end{aligned}$$

Using the above identity again, but this time in reverse order, we can write that

$$\underline{\mathcal{W}} = \frac{1}{2} [\text{Re}(\underline{E} \times \underline{H}^*)] + \frac{1}{2} [\text{Re}(\underline{E} \times \underline{H}e^{j2\omega t})]$$

2.3. (a) $\underline{W}_{\text{rad}} = \frac{1}{2} \text{Re}[\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta} \hat{a}_r = \frac{5^2}{2(120\pi)} \hat{a}_r = 0.03315 \hat{a}_r \text{ Watts/m}^2$

$$\begin{aligned} \text{(b) } P_{\text{rad}} &= \oint_s \underline{W}_{\text{rad}} \cdot d\mathbf{s} = \int_0^{2\pi} \int_0^\pi (0.03315)(r^2 \sin \theta \, d\theta \, d\phi) \\ &= \int_0^{2\pi} \int_0^\pi (0.03315)(100)^2 \sin \theta \, d\theta \, d\phi \\ &= 2\pi(0.03315)(100)^2 \int_0^\pi \sin \theta \, d\theta = 2\pi(0.03315)(100)^2 \cdot (2) \\ &= 4165.75 \text{ Watts} \end{aligned}$$

2.4. (a) $U(\theta) = \cos \theta$

$$U(\theta_h) = 0.5 = \cos \theta_h \Rightarrow \theta_h = \cos^{-1}(0.5) = 60^\circ$$

$$\Rightarrow \Theta_h = 2(60^\circ) = 120^\circ = \frac{2\pi}{3} \text{ rads.}$$

$$U(\theta_n) = 0 = \cos \theta_n \Rightarrow \theta_n = \cos^{-1}(0) = 90^\circ$$

$$\Rightarrow \Theta_n = 2(90^\circ) = 180^\circ = \pi \text{ rads.}$$

(b) $U(\theta) = \cos^2 \theta$

$$U(\theta_h) = 0.5 = \cos^2 \theta_h \Rightarrow \theta_h = \cos^{-1}(0.5)^{1/2} = 45^\circ$$

$$\Rightarrow \Theta_h = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

$$U(\theta_n) = 0 = \cos^2 \theta_n \Rightarrow \theta_n = \cos^{-1}(0) = 90^\circ$$

$$\Rightarrow \Theta_n = 2(90^\circ) = 180^\circ = \pi \text{ rads}$$

(c) $U(\theta) = \cos(2\theta)$

$$U(\theta_h) = 0.5 = \cos(2\theta_h) \Rightarrow \theta_h = \frac{1}{2} \cos^{-1}(0.5) = 30^\circ$$

$$\Rightarrow \Theta_h = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$

$$U(\theta_n) = 0 = \cos(2\theta_n) \Rightarrow \theta_n = \frac{1}{2} \cos^{-1}(0) = 45^\circ$$

$$\Rightarrow \Theta_n = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

(d) $U(\theta) = \cos^2(2\theta)$

$$U(\theta_h) = 0.5 = \cos^2(2\theta_h) \Rightarrow \theta_h = \frac{1}{2} \cos^{-1}(0.5)^{1/2} = 22.5^\circ$$

$$\Rightarrow \Theta_h = 2(22.5^\circ) = 45^\circ = \frac{\pi}{4} \text{ rads}$$

$$U(\theta_n) = 0 = \cos^2(2\theta_n) \Rightarrow \theta_n = \frac{1}{2} \cos^{-1}(0) = 45^\circ$$

$$\Rightarrow \Theta_n = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

(e) $U(\theta) = \cos(3\theta)$

$$U(\theta_h) = \cos(3\theta_h) = 0.5 \Rightarrow \theta_h = \frac{1}{3} \cos^{-1}(0.5) = 20^\circ$$

$$\Rightarrow \Theta_h = 2(20^\circ) = 40^\circ = 0.698 \text{ rads}$$

$$U(\theta_n) = \cos(3\theta_n) = 0 \Rightarrow \theta_n = \frac{1}{3} \cos^{-1}(0) = 30^\circ$$

$$\Rightarrow \Theta_n = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$

(f) $U(\theta) = \cos^2(3\theta)$

$$U(\theta_h) = 0.5 = \cos^2(3\theta_h) \Rightarrow \theta_h = \frac{1}{3} \cos^{-1}(0.5)^{1/2} = 15^\circ$$

$$\Rightarrow \Theta_h = 2(15^\circ) = 30^\circ = \pi/6 \text{ rads}$$

$$U(\theta_n) = 0 = \cos^2(3\theta_n) \Rightarrow \theta_n = \frac{1}{3} \cos^{-1}(0) = 30^\circ$$

$$\Rightarrow \Theta_n = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$

2.5. Using the results of Problem 2.4 and a nonlinear solver to find the half power beamwidth of the radiation intensity represented by the transcendental functions, we have that:

(a) $U(\theta) = \cos \theta \cos(2\theta) \Rightarrow \begin{cases} \text{HPBW} = 55.584^\circ \\ \text{FNBW} = 90^\circ \end{cases}$

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$$(b) U(\theta) = \cos^2 \theta \cos^2(2\theta) \Rightarrow \begin{cases} \text{HPBW} = 40.985^\circ \\ \text{FNBW} = 90^\circ \end{cases}$$

$$(c) U = \cos \theta \cos(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 38.668^\circ \\ \text{FNBW} = 60^\circ \end{cases}$$

$$(d) U = \cos^2 \theta \cos^2(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 28.745^\circ \\ \text{FNBW} = 60^\circ \end{cases}$$

$$(e) U = \cos(2\theta) \cos(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 34.942^\circ \\ \text{FNBW} = 60^\circ \end{cases}$$

$$(f) U = \cos^2(2\theta) \cos^2(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 25.583^\circ \\ \text{FNBW} = 60^\circ \end{cases}$$

$$2.6. (a) D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(200 \times 10^{-3})}{0.9(125.66 \times 10^{-3})} = 22.22 = 13.47 \text{ dB}$$

$$G_0 = \epsilon_{cd} D_0 = 0.9(22.22) = 20 = 13.01 \text{ dB}$$

$$(b) D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(200 \times 10^{-3})}{(125.66 \times 10^{-3})} = 20 = 13.01 \text{ dB}$$

$$G_0 = \epsilon_{cd} D_0 = 0.9 \cdot (20) = 18 = 12.55 \text{ dB}$$

$$2.7. \boxed{U = B_0 \cos^2 \theta}$$

$$(a) P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} U \sin \theta d\theta = 2\pi B_0 \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta$$

$$= 2\pi B_0 \int_0^{\pi/2} \cos^2 \theta d(-\cos \theta)$$

$$P_{\text{rad}} = -2\pi B_0 \left. \frac{\cos^3 \theta}{3} \right|_0^{\pi/2} = -2\pi B_0 \left[\frac{-1}{3} \right] = \frac{2\pi}{3} B_0 = 10 \Rightarrow B_0 = \frac{15}{\pi}$$

$$U = \frac{15}{\pi} \cos^2 \theta \Rightarrow W_{\text{rad}} \Big|_{\max} = \frac{U}{r^2} \Big|_{\max} = \frac{15}{\pi} \frac{\cos^2 \theta}{r^2} \Big|_{\max}$$

$$= \frac{15}{\pi(10^3)^2} = 4.7746 \times 10^{-6} \text{ Watts/m}^2 @ \theta = 0^\circ$$

$$W_{\text{rad}} \Big|_{\max} = 4.7746 \times 10^{-6} \text{ Watts/m}^2 @ \theta = 0^\circ$$

$$(b) \Omega_A (\text{exact}) = \int_0^{2\pi} \int_0^\pi U_n \cos^2 \theta \sin \theta d\theta d\phi$$

$$\Omega_A (\text{exact}) = \frac{2\pi}{3} \text{ steradians} = 2.0944 \text{ sterads} = 6,875.51 (\text{degrees})^2$$

$$U = 0.5 = \cos^2 \theta_h \Rightarrow \theta_h = \cos^{-1}(0.5)^{1/2} = 45^\circ$$

$$\Rightarrow \Theta_h = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

$$\Omega_A \left(\frac{\text{Kraus}'}{\text{approx}} \right) = \Theta_h^2 = (\pi/2)^2 = \frac{\pi^2}{4} = 2.4674 \text{ sterads} = 8,099.997 (\text{degrees})^2$$

$$(c) \quad D_0 \text{ (exact)} = \frac{4\pi}{\Omega_A \text{ (exact)}} = \frac{4\pi}{2\pi/3} = 6 = 7.782 \text{ dB}$$

$$D_0 \text{ (approx/Kraus')} = \frac{4\pi}{\Omega_A \text{ (approx)}} = \frac{4\pi}{\pi^2/4} = \frac{16}{\pi} = 5.093 = 7.0697$$

(d) G_0 Assuming lossless antenna ($P_{in} = P_{rad}$)

$$G_0 \text{ (exact)} = D_0 \text{ (exact)} = 6 = 7.782 \text{ dB}$$

$$G_0 \text{ (approx)} = D_0 \text{ (approx)} = 5.093 = 7.0697 \text{ dB}$$

$$\boxed{U = B_0 \cos^3 \theta}$$

$$(a) \quad P_{rad} = -2\pi B_0 \left(-\frac{1}{4}\right) = \frac{\pi}{2} B_0 = 10 \Rightarrow B_0 = 20/\pi$$

$$W_{rad} \Big|_{\max} = \frac{20}{\pi} \frac{1}{\pi^2} = \frac{20}{\pi} \times 10^{-6} = 6.366 \times 10^{-6} \text{ Watts/m}^2$$

(b) $\Omega_A \text{ (exact)} = (\pi/2) = 1.5708 \text{ sterads}$

$$U = 0.5 = \cos^3 \theta_h \Rightarrow \theta_h \cos^{-1}(0.5)^{1/3} = 37.467^\circ$$

$$\Rightarrow \Theta_h = 2(37.467^\circ) = 74.934^\circ = 1.30785 \text{ rads}$$

$$\Omega_A \text{ (approx)} = (1.30785)^2 = 1.71 \text{ sterads}$$

(c) $D_0 \text{ (exact)} = 4\pi/(\pi/2) = 8 = 9.031 \text{ dB}$

$$D_0 \text{ (approx)} = \frac{4\pi}{1.71} = 7.347 = 8.66 \text{ dB}$$

(d) Assuming lossless antenna $\Rightarrow$ Gain = Directivity (see part c)

$$\mathbf{2.8.} \quad \underline{E}_a = \hat{a}_\theta E_a \sin^{1.5} \theta \frac{e^{-jkr}}{r} \Rightarrow U_n = (\sin^{1.5} \theta)^2 = \sin^3 \theta \quad \mathbf{Normalized} \quad U_n$$

$$(a) \quad D_0 = \frac{4\pi U_{\max}}{P_{rad}}, U_{\max} = U_n \Big|_{\max} = \sin^3 \theta \Big|_{\theta=\theta_{\max}} = 1, \theta_{\max} = 90^\circ$$

$$P_{rad} = \int_0^{2\pi} \int_0^\pi U_n \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \int_0^\pi \sin^3 \theta \sin \theta \, d\theta \, d\phi = 2\pi \int_0^\pi \sin^4 \theta \, d\theta$$

$$= 2\pi \left[-\frac{\sin^3 \theta \cos \theta}{4} \Big|_0^\pi + \frac{3}{4} \int_0^\pi \sin^2 \theta \, d\theta \right] = 2\pi \left[\frac{3}{4} \left(\frac{1}{2} - \frac{1}{4} \sin(2\theta) \right) \Big|_0^\pi \right]$$

$$= 2\pi \left[\frac{3}{4} \left(\frac{\pi}{2} \right) \right] = \frac{3\pi^2}{4}$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi(1)}{3\pi^2/4} = \frac{16}{3\pi} = \boxed{1.698 = 2.298 \text{ dB}}$$

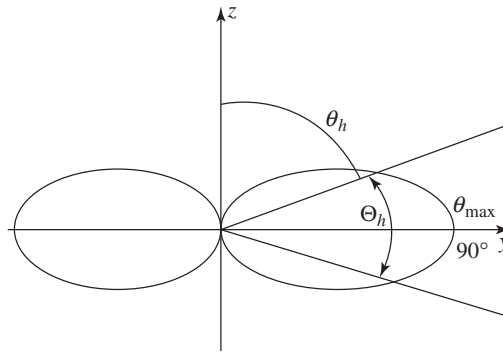
(b) $U_n = \sin^3 \theta, U_{n\max} = 1, \theta_{\max} = 90^\circ$

$$U_n \Big|_{\theta=\theta_h} = 0.5 = \sin^3 \theta_h \Rightarrow \theta_h = [\sin^{-1}(0.5^4)^3] = \sin^{-1}(0.794) = 52.533^\circ$$

$$\text{HPBW} = \Theta_h = 2(\theta_{\max} - \theta_h) = 2(90 - 52.533)$$

$$\boxed{\Theta_h = 2(37.467) = 74.934^\circ}$$

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(c) Because pattern is **omnidirectional**:

$$D_0(\text{McDonald}) = \frac{101}{\text{HPBW} - 0.0027(\text{HPBW})^2} = \frac{101}{74.934 - 0.0027(74.934)^2}$$

$$D_0(\text{McDonald}) = \frac{101}{74.934 - 15.161} = \frac{101}{59.773} = \boxed{1.690 = 10 \log_{10}(1.690) = 2.278}$$

(d) Because pattern is **omnidirectional**:

$$D_0(\text{Pozar}) = -172.4 + 191 \sqrt{0.818 + \frac{1}{\text{HPBW}}} = -172.4 + 191 \sqrt{0.818 + \frac{1}{74.934}}$$

$$= -172.4 + 191(0.912) = -172.4 + 174.150 = 1.750$$

$$P_0(\text{Pozar}) = \boxed{1.750 = 10 \log_{10}(1.750) = 2.431 \text{ dB}}$$

(e) Computer Program Directivity: $\boxed{D_0 = 1.693 = 2.2864 \text{ dB}}$

Input parameters:

The lower bound of theta in degrees = 0
 The upper bound of theta in degrees = 180
 The lower bound of phi in degrees = 0
 The upper bound of phi in degrees = 360

Output parameters:

Radiated power (watts) = 7.4228
 Directivity (dimensionless) = 1.6930
 Directivity (dB) = 2.2864

2.9. $U(\theta, \phi) = \cos^n(\theta) \quad 0 \leq \theta \leq \pi/2, 0 \leq \phi \leq 2\pi$

(a) $U_n(\theta_n, \phi) = 0.5 = \cos^n(5^\circ) = [\cos(5^\circ)]^n = (0.99619)^n$

$$0.5 = (0.99619)^n$$

$$\log_{10}(0.5) = \log[(0.99619)^n] = n \log_{10}(0.99619) = n(-0.00166)$$

$$-0.30103 = -0.00166n$$

$$\boxed{n = 181.34}$$

(b) $U(\theta, \phi) = \cos^{181.34}(\theta); U_{\max} = 1, \theta = 0^\circ$

$$\begin{aligned} P_{\text{rad}} &= \int_0^{2\pi} \int_0^{\pi/2} U(\theta, \phi) \sin \theta \, d\theta \, d\phi = 2\pi \int_0^{\pi/2} \cos^{181.34}(\theta) \sin \theta \, d\theta \\ &= 2\pi \left[-\frac{\cos^{182.34}(\theta)}{182.34} \right]_0^{\pi/2} = \left[-0 + \frac{1}{182.34} \right] 2\pi = \frac{2\pi}{182.34} = 0.03446 \\ D_0 &= \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(1)}{2\pi} (182.34) = 2(182.34) = 364.68 \end{aligned}$$

$$D_0 = 364.68 = 25.62 \text{ dB}$$

(c) *Kraus' Approximation (2.27):*

$$D_0 \simeq \frac{41,253}{\Theta_{1d}\Theta_{2d}} = \frac{41,253}{(10)(10)} = 412.53 = 26.15 \text{ dB}$$

$$D_0 \simeq 412.53 = 26.15 \text{ dB}$$

(d) *Tai & Pereira (2.30b):*

$$D_0 \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{2(10)^2} = \frac{72,815}{200} = 364.075 = 25.61 \text{ dB}$$

$$D_0 \simeq 364.075 = 25.61 \text{ dB}$$

2.10.

$$U(\theta, \phi) = \begin{cases} 1 & 0^\circ \leq \theta \leq 20^\circ \\ 0.342 \csc(\theta) & 20^\circ \leq \theta \leq 60^\circ \\ 0 & 60^\circ \leq \theta \leq 180^\circ \end{cases} \quad 0^\circ \leq \phi \leq 360^\circ$$

$$\begin{aligned} P_{\text{rad}} &= \int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta \, d\theta \, d\phi \\ &= 2\pi \left[\int_0^{20^\circ} \sin \theta \, d\theta + \int_{20^\circ}^{60^\circ} 0.342 \csc(\theta) \times \sin \theta \, d\theta \right] \\ &= 2\pi \left\{ -\cos \theta \Big|_0^{\pi/9} + 0.342 \cdot \theta \Big|_{\pi/9}^{\pi/3} \right\} \\ &= 2\pi \left\{ \left[-\cos \left(\frac{\pi}{9} \right) + 1 \right] + 0.342 \left(\frac{\pi}{3} - \frac{\pi}{9} \right) \right\} \\ &= 2\pi \left\{ [-0.93969 + 1] + 0.342\pi \left(\frac{2}{9} \right) \right\} \\ &= 2\pi \{0.06031 + 0.23876\} = 1.87912 \\ D_0 &= \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(1)}{1.87912} = 6.68737 = 8.25255 \text{ dB} \end{aligned}$$

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$$2.11. \quad (a) \quad D_0 \simeq \frac{41,253}{\Theta_{1d}\Theta_{2d}} = \frac{41,253}{30(35)} = 39.29 = 15.94 \text{ dB}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

$$(b) \quad D_0 \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{(30)^2 + (35)^2} = 34.27 = 15.35 \text{ dB}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

$$2.12. \quad D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}}$$

$$(a) \quad U = \sin \theta \sin \phi \quad \text{for} \quad 0 \leq \theta \leq \pi, 0 \leq \phi \leq \pi$$

$$U|_{\max} = 1 \text{ and it occurs when } \theta = \phi = \pi/2.$$

$$P_{\text{rad}} = \int_0^\pi \int_0^\pi U \sin \theta \, d\theta \, d\phi = \int_0^\pi \sin \phi \, d\phi \int_0^\pi \sin^2 \theta \, d\theta = 2 \left(\frac{\pi}{2} \right) = \pi.$$

$$\text{Thus } D_0 = \frac{4\pi(1)}{\pi} = 4 = 6.02 \text{ dB}$$

The half-power beamwidths are equal to

$$\text{HPBW (az.)} = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

$$\text{HPBW (el.)} = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

In a similar manner, it can be shown that for the following:

$$(b) \quad U = \sin \theta \sin^2 \phi \Rightarrow D_0 = 5.09 = 7.07 \text{ dB}$$

$$\text{HPBW (el.)} = 120^\circ, \text{HPBW (az.)} = 90^\circ$$

$$(c) \quad U = \sin \theta \sin^3 \phi \Rightarrow D_0 = 6 = 7.78 \text{ dB}$$

$$\text{HPBW (el.)} = 120^\circ, \text{HPBW (az.)} = 74.93^\circ$$

$$(d) \quad U = \sin^2 \theta \sin \phi \Rightarrow D_0 = 12\pi/8 = 4.71 = 6.73 \text{ dB}$$

$$\text{HPBW (el.)} = 90^\circ, \text{HPBW (az.)} = 120^\circ$$

$$(e) \quad U = \sin^2 \theta \sin^2 \phi \Rightarrow D_0 = 6 = 7.78 \text{ dB}$$

$$\text{HPBW (az.)} = \text{HPBW (el.)} = 90^\circ$$

$$(f) \quad U = \sin^2 \theta \sin^3 \phi \Rightarrow D_0 = 9\pi/4 = 7.07 = 8.49 \text{ dB}$$

$$\text{HPBW (el.)} = 90^\circ, \text{HPBW (az.)} = 74.93^\circ$$

$$2.13. \quad U = \sin \theta \cos^2 \phi, \quad 0 \leq \theta \leq 180^\circ, \quad 90^\circ \leq \phi \leq 270^\circ$$

$$(a) \quad D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}}, \quad U_{\max} = \sin \theta \cos^2 \phi \Big|_{\substack{\theta=90^\circ \\ \phi=180^\circ}} = 1$$

$$\begin{aligned}
 P_{\text{rad}} &= \int_{\pi/2}^{3\pi/2} \int_0^\pi U(\theta, \phi) \sin \theta \, d\theta \, d\phi = \int_{\pi/2}^{3\pi/2} \int_0^\pi \sin \theta \cos^2 \phi \sin \theta \, d\theta \, d\phi \\
 &= \int_{\pi/2}^{3\pi/2} \cos^2 \phi \, d\phi \int_0^\pi \sin^2 \theta \, d\theta \\
 P_{\text{rad}} &= \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} \\
 D_0 &= \frac{4\pi(1)}{\pi^2/4} = \frac{16}{\pi} = 5.09296 = 10 \log_{10}(5.09296) = 7.0697 \text{ dB}
 \end{aligned}$$

$$D_0(\text{exact}) = 5.09296(\text{dim}) = 7.0697 \text{ dB}$$

(b) Azimuth (Horizontal) Principal Plane ($\theta = 90^\circ$):

$$\begin{aligned}
 U(\theta = 90^\circ) &= \sin \theta \cos^2 \phi|_{\theta=90^\circ} = \cos^2 \phi \\
 U_h &= \cos^2 \phi|_{\phi=\phi_h} = 0.5 \Rightarrow \phi_h = \cos^{-1}(\pm\sqrt{0.5}) = \cos^{-1}(\pm 0.707) = 135^\circ \\
 \Phi_h(az) &= 2(180 - 135) = 2(45^\circ) = 90^\circ \\
 \Phi_h(az) &= 90^\circ
 \end{aligned}$$

(c) Elevation (vertical) Principal plane ($\phi = 180^\circ$):

$$\begin{aligned}
 U(\phi = 180^\circ) &= \sin \theta \cos^2 \phi|_{\phi=180^\circ} = \sin \theta \\
 U_h &= \sin \theta|_{\theta=\theta_h} = 0.5 \Rightarrow \theta_h = \sin^{-1}(0.5) = 30^\circ \\
 \Theta_h &= 2(90^\circ - 30^\circ) = 2(60) = 120^\circ \\
 \Theta_h(\text{elev}) &= 120^\circ
 \end{aligned}$$

(d) **Either:** $D_0(\text{Kraus}) = \frac{41,253}{\Phi_h \Theta_h} = \frac{41,253}{90^\circ(120^\circ)} = 3.8197 = 5.82 \text{ dB}$

$$D_0(\text{Kraus}) = 3.8197 \text{ dim} = 5.82 \text{ dB}$$

or:

$$D_0(\text{Tai \& Pereira}) = \frac{72,815}{\Phi_h^2 + \Theta_h^2} = \frac{72,815}{(90^\circ)^2 + (120^\circ)^2} = \frac{72,815}{25,500} = 3.236$$

$$D_0(\text{T\&P}) = 3.236 \text{ dim} = 5.1 \text{ dB}$$

2.14. Using the half-power beamwidths found in Problem 2.12, the directivity for each intensity using Kraus' and Tai & Pereira's formulas is given by

$$\begin{aligned}
 U &= \sin \theta \sin \phi; \\
 (a) \ D_0 &\simeq \frac{41253}{\Theta_{1d} \Theta_{2d}} = \frac{41253}{120(120)} = 2.86 = 4.57 \text{ dB}
 \end{aligned}$$

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$$(b) D_0 \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{(120)^2 + (120)^2} = 2.53 = 4.03 \text{ dB}$$

$$U = \sin \theta \sin^2 \phi;$$

$$(a) D_0 \simeq 3.82 = 5.82 \text{ dB}$$

$$(b) D_0 \simeq 3.24 = 5.10 \text{ dB}$$

$$U = \sin \theta \sin^3 \phi;$$

$$(a) D_0 \simeq 4.59 = 6.62 \text{ dB}$$

$$(b) D_0 \simeq 3.64 = 5.61 \text{ dB}$$

$$U = \sin^2 \theta \sin \phi;$$

$$(a) D_0 \simeq 3.82 = 5.82 \text{ dB}$$

$$(b) D_0 \simeq 3.24 = 5.10 \text{ dB}$$

$$U = \sin^2 \theta \sin^2 \phi;$$

$$(a) D_0 \simeq 5.09 = 7.07 \text{ dB}$$

$$(b) D_0 \simeq 4.49 = 6.53 \text{ dB}$$

$$U = \sin^2 \theta \sin^3 \phi;$$

$$(a) D_0 \simeq 6.12 = 7.87 \text{ dB}$$

$$(b) D_0 \simeq 5.31 = 7.25 \text{ dB}$$

$$\mathbf{2.15.} \quad (a) D_0 = \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi}{(1.5064)^2} = 5.5377 = 7.433 \text{ dB}$$

$$(b) D_0 = \frac{32 \ln(2)}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{32 \ln(2)}{(1.5064)^2 + (1.5064)^2} = 4.88725 = 6.8906 \text{ dB}$$

$$\mathbf{2.16.} \quad (a) D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{U_{\max}}{U_0}$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta \, d\theta \, d\phi = 2\pi \int_0^\pi U \sin \theta \, d\theta = 2\pi \left\{ \int_0^{30^\circ} \sin \theta \, d\theta \right. \\ \left. + \int_{30^\circ}^{60^\circ} (0.5) \sin \theta \, d\theta + \int_{60^\circ}^{90^\circ} (0.1) \sin \theta \, d\theta \right\}$$

$$= 2\pi \left\{ (-\cos \theta) \Big|_0^{30^\circ} + \left(-\frac{\cos \theta}{2} \right) \Big|_{30^\circ}^{60^\circ} + (-0.1 \cos \theta) \Big|_{60^\circ}^{90^\circ} \right\}$$

$$= 2\pi \left\{ (-0.866 + 1) + \left(\frac{-0.5 + 0.866}{2} \right) + \left(\frac{-0 + 0.5}{10} \right) \right\}$$

$$P_{\text{rad}} = 2\pi \{-0.866 + 1 - 0.25 + 0.433 + 0.05\} = 2\pi(0.367)$$

$$= 0.734\pi = 2.3059$$

$$D_0 = \frac{1(4\pi)}{2.3059} = 5.4496 = 7.3636 \text{ dB}$$

$$(b) D_0(\text{dipole}) = 1.5 = 1.761 \text{ dB}$$

$$D_0(\text{above dipole}) = (7.3636 - 1.761) \text{ dB} = 5.6026 \text{ dB}$$

$$D_0(\text{above dipole}) = \frac{5.45}{1.5} = 3.633 = 5.603 \text{ dB}$$

$$\begin{aligned}
 \mathbf{2.17.} \quad (a) \quad P_{\text{rad}} &= \int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \sin^2 \phi \, d\phi \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta \\
 &= (\pi) \left(\frac{1}{5} \right) = \frac{\pi}{5} \\
 U_{\text{max}} &= U(\theta = 0^\circ, \phi = \pi/2) = 1 \\
 D_0 &= \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi}{(\pi/5)} = 20 = 13.0 \text{ dB}
 \end{aligned}$$

(b) Elevation Plane: θ varies, ϕ fixed

$\Rightarrow$ Choose $\phi = \pi/2$.

$$U(\theta, \phi = \pi/2) = \cos^4 \theta, \quad 0 \leq \theta \leq \pi/2.$$

$$\cos^4 \left[\frac{\text{HPBW}(\text{el.})}{2} \right] = \frac{1}{2}$$

$$\text{HPBW}(\text{el.}) = 2 \cos^{-1} \{ \sqrt{0.5} \}^{1/2} = 65.5^\circ$$

$$\begin{aligned}
 \mathbf{2.18.} \quad (a) \quad P_{\text{rad}} &= \int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta \, d\theta \, d\phi = 2\pi \\
 &\cdot \left\{ \int_0^{30^\circ} \sin \theta \, d\theta + \int_{30^\circ}^{90^\circ} \frac{\cos \theta \sin \theta}{0.866} d\theta \right\} \\
 &= 2\pi \left\{ \int_0^{\pi/6} \sin \theta \, d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{0.866} \cos \theta \sin \theta \, d\theta \right\} \\
 &= 2\pi \left\{ -\cos \theta \Big|_0^{\pi/6} + \frac{1}{0.866} \left(-\frac{\cos^2 \theta}{2} \right) \Big|_{\pi/6}^{\pi/2} \right\} = 2\pi[-0.866 + 1 + 0.433] \\
 P_{\text{rad}} &= 3.5626 \\
 D_0 &= \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi(1)}{3.5626} = 3.5273 = 5.4745 \text{ dB}
 \end{aligned}$$

$$(b) \quad U = \frac{\cos(\theta)}{0.866} = 0.5 \Rightarrow \cos \theta = 0.5(0.866) = 0.433, \theta = \cos^{-1}(0.433) = 64.34^\circ$$

$$\Theta_{1r} = 2(64.34) = 128.68^\circ = 2.246 \text{ rad} = \Theta_{2r}$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi}{(2.246)^2} = 2.4912 = 3.9641 \text{ dB}$$

2.19. (a) 35 dB

$$(b) \quad 20 \log_{10} \left| \frac{E_{\text{max}}}{E_s} \right| = 35, \log_{10} \left| \frac{E_{\text{max}}}{E_s} \right| = \frac{35}{20} = 1.75$$

$$\left| \frac{E_{\text{max}}}{E_s} \right| = 10^{1.75} = 56.234$$

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2.20. (a) $U = \sin \theta, U_{\max} = 1, P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta d\theta d\phi$

$$= \int_0^{2\pi} \int_0^\pi \sin^2 \theta d\theta d\phi = \pi^2$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.2732$$

(b) HPBW = $120^\circ, 2\pi/3$

The directivity based on (2-33a) is equal to,

$$D_0 = \frac{101}{120^\circ - 0.0027(120^\circ)^2} = 1.2451$$

while that based on (2-33b) is equal to,

$$D_0 = -172.4 + 191 \sqrt{0.818 + \frac{1}{120^\circ}} = 1.2245$$

(c) Computer Program: $D_0 = 1.2732$

2.21. (a) $U = \sin^3 \theta, U_{\max} = 1, P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi \sin^4 \theta d\theta d\phi = \frac{3}{4}\pi^2$

$$D_0 = \frac{4\pi}{\frac{3}{4}\pi^2} = \frac{16}{3\pi} = 1.6976$$

(b) HPBW = 74.93°

From (2-33a), $D_0 = \frac{101}{(74.93^\circ) - 0.0027(74.93^\circ)^2} = 1.68971$

From (2-33b), $D_0 = -172.4 + 191 \sqrt{0.818 + \frac{1}{74.93^\circ}} = 1.75029$

(c) Computer program: $D_0 = 1.693$

The value of $D_0 = 1.693$ is similar to that of (4-91) or 1.643

2.22. (a) $U = J_1^2(ka \sin \theta),$

$$a = \lambda/10, ka \sin \theta = \frac{\pi}{5} \sin \theta. \text{ HPBW} = 93.10^\circ$$

From (2-33a): $D_0 = 101 / [(93.10) - 0.0027(93.10)^2] = 1.449120$

From (2-33b): $D_0 = -172.4 + 191 \sqrt{0.818 + \frac{1}{93.10}} = 1.477271$

$$a = \lambda/20, ka \sin \theta = \frac{\pi}{10} \sin \theta, \text{ HPBW} = 91.10^\circ$$

From (2-33a), $D_0 = 1.47033$; From (2-33b), $D_0 = 1.502$

(b) $a = \frac{\lambda}{10}: P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi J_1^2(ka \sin \theta) \cdot \sin \theta d\theta d\phi = 0.7638045$

$$U_{\max} = 0.0893, D_0 = \frac{4\pi(0.0893)}{0.7638045} = 1.469193$$

$$a = \frac{\lambda}{20}: P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi J_1^2(\pi/10 \sin \theta) \sin \theta d\theta d\phi = 0.202604$$

$$U_{\text{max}} = 0.0240714, D_0 = \frac{4\pi(0.0240714)}{0.202604} = 1.49257$$

If the radius of loop is smaller than $\lambda/20$, the directivity approaches 1.5.

2.23. Using the numerical techniques, the directivity for each intensity of Prob. 2.12, with 10° uniform divisions is equal to for $U = \sin \theta \sin \phi$:

$$\begin{aligned} \text{(a) Midpoint: } D_0 &= \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} \\ U_{\text{max}} = 1: P_{\text{rad}} &= \frac{\pi}{18} \left(\frac{\pi}{18} \right) \sum_{j=1}^{18} \sin \phi_j \sum_{i=1}^{18} \sin^2 \theta_i \\ \theta_i &= \frac{\pi}{36} + (i-1)\frac{\pi}{18}, i = 1, 2, 3, \dots, 18 \\ \phi_j &= \frac{\pi}{36} + (j-1)\frac{\pi}{18}, j = 1, 2, 3, \dots, 18 \\ P_{\text{rad}} &= \left(\frac{\pi}{18} \right)^2 (11.38656)(8.9924) = 3.119 \\ D_0 &= \frac{4\pi(1)}{3.119} = 4.03 = 6.05 \text{ dB} \end{aligned}$$

(b) Trailing edge of each division:

$$\begin{aligned} \text{Trailing edge: } \theta_i &= i(\pi/18), i = 1, 2, 3, \dots, 18 \\ \phi_j &= j(\pi/18), j = 1, 2, 3, \dots, 18 \\ P_{\text{rad}} &= \left(\frac{\pi}{18} \right)^2 (11.25640)(8.96985) = 3.076 \\ D_0 &= \frac{4\pi(1)}{3.119} = 4.09 = 6.11 \text{ dB} \end{aligned}$$

In a similar manner:

$$\begin{aligned} U &= \sin \theta \sin^2 \phi; \\ \text{(a) } P_{\text{rad}} &= 2.463 \Rightarrow D_0 = 5.10 = 7.07 \text{ dB} \\ \text{(b) } P_{\text{rad}} &= 2.451 \Rightarrow D_0 = 5.13 = 7.10 \text{ dB} \\ U &= \sin \theta \sin^3 \phi; \\ \text{(a) } P_{\text{rad}} &= 2.092 \Rightarrow D_0 = 6.01 = 7.79 \text{ dB} \\ \text{(b) } P_{\text{rad}} &= 2.086 \Rightarrow D_0 = 6.02 = 7.80 \text{ dB} \\ U &= \sin^2 \theta \sin \phi; \\ \text{(a) } P_{\text{rad}} &= 2.469 \Rightarrow D_0 = 4.74 = 6.76 \text{ dB} \\ \text{(b) } P_{\text{rad}} &= 2.618 \Rightarrow D_0 = 4.80 = 6.81 \text{ dB} \\ U &= \sin^2 \theta \sin^2 \phi; \\ \text{(a) } P_{\text{rad}} &= 2.092 \Rightarrow D_0 = 6.01 = 7.79 \text{ dB} \\ \text{(b) } P_{\text{rad}} &= 2.086 \Rightarrow D_0 = 6.02 = 7.80 \text{ dB} \\ U &= \sin^2 \theta \sin^3 \phi; \\ \text{(a) } P_{\text{rad}} &= 1.777 \Rightarrow D_0 = 7.07 = 8.49 \text{ dB} \\ \text{(b) } P_{\text{rad}} &= 1.775 \Rightarrow D_0 = 7.08 = 8.50 \text{ dB} \end{aligned}$$

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2.24. Using the computer program **Directivity** of Chapter 2, the directivities for each radiation intensity of Problem 2.12 are equal to:

(a) $U = \sin \theta \sin \phi; P_{\text{rad}} = 3.1318$

$$U_{\text{max}} = 1; \quad D_0 = \frac{4\pi \cdot U_{\text{max}}}{3.1318} = 4.0125 \Rightarrow 6.034 \text{ dB}$$

(b) $U = \sin \theta \sin^2 \phi; P_{\text{rad}} = 2.4590$

$$U_{\text{max}} = 1; \quad D_0 = \frac{4\pi \cdot 1}{2.4590} = 5.110358 \Rightarrow 7.0845 \text{ dB}$$

(c) $U = \sin \theta \sin^3 \phi; P_{\text{rad}} = 2.0870$

$$U_{\text{max}} = 1; \quad D_0 = \frac{4\pi \cdot 1}{2.0870} = 6.02124 \Rightarrow 7.80 \text{ dB}$$

(d) $U = \sin^2 \theta \sin \phi; P_{\text{rad}} = 2.6579$

$$U_{\text{max}} = 1; \quad D_0 = \frac{4\pi \cdot 1}{2.6579} = 4.72793 \Rightarrow 6.746 \text{ dB}$$

(e) $U = \sin^2 \theta \sin^2 \phi; P_{\text{rad}} = 2.0870$

$$U_{\text{max}} = 1; \quad D_0 = \frac{4\pi \cdot 1}{2.0870} = 6.02126 \Rightarrow 7.7968 \text{ dB}$$

(f) $U = \sin^2 \theta \sin^3 \phi; P_{\text{rad}} = 1.7714$

$$U_{\text{max}} = 1; \quad D_0 = \frac{4\pi \cdot 1}{1.7714} = 7.09403 \Rightarrow 8.5089 \text{ dB}$$

2.25. (a) $E|_{\text{max}} = \cos \left[\frac{\pi}{4}(\cos \theta - 1) \right] |_{\text{max}} = 1 \text{ at } \theta = 0^\circ.$

$$0.707E_{\text{max}} = 0.707 \cdot (1) = \cos \left[\frac{\pi}{4}(\cos \theta_1 - 1) \right]$$

$$\frac{\pi}{4}(\cos \theta_1 - 1) = \pm \frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(2) = \text{does not exist} \\ \cos^{-1}(0) = 90^\circ = \frac{\pi}{2} \text{ rad.} \end{cases}$$

$$\Theta_{1r} = \Theta_{2r} = 2 \left(\frac{\pi}{2} \right) = \pi$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.049 \text{ dB}$$

(b) Using the computer program **Directivity** of Chapter 2

$$D_0 = 2.00789 = 3.027 \text{ dB}$$

Since the pattern is not very narrow, the answer obtained using Kraus' approximate formula is not as accurate.

2.26. (a) $E|_{\text{max}} = \cos \left[\frac{\pi}{4}(\cos \theta + 1) \right] |_{\text{max}} = 1 \text{ at } \theta = \pi.$

$$0.707 = \cos \left[\frac{\pi}{4}(\cos \theta_1 + 1) \right]$$

$$\frac{\pi}{4}(\cos \theta_1 + 1) = \pm \frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(-2) \rightarrow \text{does not exist.} \\ \cos^{-1}(0) \rightarrow 90^\circ \rightarrow \frac{\pi}{2} \text{ rad} \end{cases}$$

$$\Theta_{1r} = \Theta_{2r} = 2 \left(\frac{\pi}{2} \right) = \pi$$

$$D_0 \simeq \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.049 \text{ dB}$$

(b) Computer Program **Directivity**:

$$D_0 = 2.00789 = 3.027 \text{ dB}$$

$$\begin{aligned} 2.27. \quad (a) \quad P_{\text{rad}} &= \int_0^{2\pi} \int_0^{\pi/2} U_0 \sin(\pi \sin \theta) \sin \theta \, d\theta \, d\phi = 2\pi U_0 \frac{\pi}{2} J_1(\pi) = U_0 \pi^2 J_1(\pi) \\ D_0 &= \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi U_0}{U_0 \pi^2 J_1(\pi)} = \frac{4}{\pi} \frac{1}{J_1(\pi)} = 4.4735 \\ \frac{\pi}{2} J_1(\pi) &= 0.447 \end{aligned}$$

(b) Computer program **Directivity**:

$$\begin{aligned} P_{\text{rad}} &= \int_0^{2\pi} \int_0^{\pi/2} U_0 \sin(\pi \sin \theta) \sin \theta \, d\theta \, d\phi = 2\pi(0.447) \\ D_0 &= 4.4735 \end{aligned}$$

$$2.28. \quad E_\phi = C_0 \sin^{1.5} \theta \frac{e^{-jkr}}{r}$$

$$(a) \quad U_n = |E_\phi|^2 = C_0^2 \sin^3 \theta, \Rightarrow U_n|_{\text{max}} = C_0^2$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta \, d\theta \, d\phi = 2\pi \int_0^\pi C_0^2 \sin^3 \theta \sin \theta \, d\theta = C_0^2 (2\pi) \int_0^\pi \sin^4 \theta \, d\theta$$

$$\begin{aligned} \int_0^\pi \sin^4 \theta \, d\theta &= -\frac{\sin^3 \theta \cos \theta}{4} \Big|_0^\pi + \frac{4-3}{4} \int_0^\pi \sin^2 \theta \, d\theta = \frac{3}{4} \int_0^\pi \sin^2 \theta \, d\theta \\ &= \frac{3}{4} \left[\frac{\theta}{2} - \frac{1}{4} \sin(2\theta) \right]_0^\pi = \frac{3}{4} \left(\frac{\pi}{2} \right) = \frac{3\pi}{8} \end{aligned}$$

$$P_{\text{rad}} = 2\pi C_0^2 \left(\frac{3\pi}{8} \right) = \frac{3\pi^2}{4} C_0^2$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi C_0^2}{\frac{3\pi^2}{4} C_0^2} = \frac{16}{3\pi} = 1.69765 = 2.298 \text{ dB}$$

$$\boxed{D_0 = 1.69765 = 2.298 \text{ dB}}$$

$$(b) \quad U_n = C_0 \sin^3 \theta, \quad U_n|_{\text{max}} = C_0^2 \text{ at } \theta = 90^\circ, \quad U_n|_{\theta=\theta_h} = 0.5 C_0^2 = \sin^3 \theta_h C_0^2$$

$$\sin^3 \theta_h = 0.5, \theta_h = \sin^{-1}(0.5)^{1/3} = \sin^{-1}(0.7937) = 52.5327^\circ$$

$$\Theta_h = 2(90^\circ - 52.5327^\circ) = 74.935^\circ$$

$$D_0(\text{McDonald}) = \frac{101}{74.935 - 0.0027(74.935)^2} = \frac{101}{59.7738} = \boxed{1.6897 = 2.278 \text{ dB}}$$

$$D_0(\text{McDonald}) = \boxed{1.6897 \text{ dimensionless} = 2.278 \text{ dB}}$$

$$D_0(\text{Pozar}) = -172.4 + 191 \sqrt{0.818 + \frac{1}{74.935^\circ}} = -172.4 + 191(0.91178)$$

$$D_0(\text{Pozar}) = -172.4 + 174.1502 = \boxed{1.7502 \text{ dimensionless} = 2.431 \text{ dB}}$$

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2.29. (a) Using the computer program **Directivity** of Chapter 2.

$$D_0 = 14.0707 \text{ dimensionless} = 11.48 \text{ dB}$$

$$(b) U|_{\max} = \left[\frac{\sin(\pi \sin \theta)}{\pi \sin \theta} \right]_{\max}^2 = 1 \quad \text{when } \theta = 0^\circ.$$

$$U = \frac{1}{2} U_{\max} = \frac{1}{2}(1) = \left[\frac{\sin(\pi \sin \theta_1)}{\pi \sin \theta_1} \right]^2$$

Iteratively we obtain $\theta_1 = 26.3^\circ$. Therefore

$$\Theta_{1d} = \Theta_{2d} = 2(26.3^\circ) = 52.6^\circ.$$

$$\text{and } D_0 \simeq \frac{41,253}{(52.6)^2} = 14.91 \text{ dimensionless} = 11.73 \text{ dB using the Kraus' formula}$$

(c) For Tai and Pereira's formula

$$D_0 = \frac{72,815}{2 \cdot \Theta_{1d}^2} = \frac{72,815}{2(52.6)^2} = 13.16 \text{ dimensionless} = 11.19 \text{ dB}$$

$$2.30. U = \frac{1}{2\eta} |E|^2 = \frac{1}{2\eta} \sin^2 \theta \cos^2 \phi \Rightarrow U_{\max} = \frac{1}{2\eta}$$

$$(a) P_{\text{rad}} = 2 \cdot \int_0^{\pi/2} \int_0^\pi \frac{1}{2\eta} \sin^2 \theta \cos^2 \phi \, d\theta \, d\phi = \frac{1}{\eta} \left(\frac{\pi}{4} \right) \left(\frac{\pi}{2} \right) = \frac{\pi^2}{8\eta}$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi \left(\frac{1}{2\eta} \right)}{\frac{\pi^2}{8\eta}} = \frac{16}{\pi} = 5.09 = 7.07 \text{ dB}$$

$$(b) U_{\max} = \frac{1}{2\eta} \text{ at } \theta = \pi/2, \phi = 0$$

In the elevation plane through the maximum $\phi = 0$ and $U = \frac{1}{2\eta} \sin^2 \theta$, the 3-dB point occurs when

$$U = 0.5U_{\max} = 0.5 \left(\frac{1}{2\eta} \right) = \frac{1}{2\eta} \sin^2 \theta_1 \Rightarrow \theta_1 = \sin^{-1}(0.5) = 30^\circ$$

Therefore $\Theta_{1d} = 2(90 - 30) = 120^\circ$

In the azimuth plane through the maximum $\theta = \pi/2$ and $U = \frac{1}{2\eta} \cos^2 \phi$, the 3-dB point

occurs when $U = 0.5U_{\max} = 0.5 \left(\frac{1}{2\eta} \right) = \frac{1}{2\eta} \cos^2 \theta_1 \Rightarrow \phi_1 = \cos^{-1}(0.707) = 45^\circ$

$$\Theta_{2d} = 2(90^\circ - 45^\circ) = 90^\circ$$

Therefore using Kraus' formula: $D_0 \simeq \frac{41,253}{120(90)} = 3.82 \text{ dimensionless} = 5.82 \text{ dB}$

(c) Using Tai and Pereira's formula:

$$D_0 \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{(120)^2 + (90)^2} = 3.24 \text{ dimensionless} = 5.10 \text{ dB}$$

(d) Using the computer program **Directivity** of Chapter 2.

$$D_0 = 5.16425 = 7.13 \text{ dB}$$

$$2.31. \quad U = \left[\frac{J_1(ka \sin \theta)}{\sin \theta} \right]^2 = (ka)^2 \left[\frac{J_1(ka \sin \theta)}{ka \sin \theta} \right]^2 = U_0 \left[\frac{J_1(ka \sin \theta)}{ka \sin \theta} \right]^2$$

(a) $U_{\max} = U_0 \left(\frac{1}{2} \right)^2 = \frac{U_0}{4}$ and it occurs when $ka \sin \theta = 0 \Rightarrow \theta = 0^\circ$. The 3-dB point is obtained using

$$U = \frac{1}{2} U_{\max} = \frac{U_0}{8} = U_0 \left[\frac{J_1(ka \sin \theta)}{ka \sin \theta} \right]^2 \Rightarrow \frac{J_1(ka \sin \theta)}{ka \sin \theta} = 0.3535$$

with the aid of the $J_1(x)/x$ of Appendix V.

$$x = ka \sin \theta_1 = 1.61 \Rightarrow \theta_1 = \sin^{-1}(1.61/2\pi) = 14.847^\circ \\ \Rightarrow \Theta_{1r} = 29.694^\circ$$

(b) Since $\Theta_{1r} = \Theta_{2r} = 29.694^\circ$, the directivity using Kraus' formula is equal to

$$D_0 \simeq \frac{41,253}{(29.694)^2} = 46.79 \text{ dimensionless} = 16.70 \text{ dB}$$

$$2.32. \quad G_0 = 16 \text{ dB} \Rightarrow 16 = 10 \log_{10} G_0 \text{ (dimensionless)} \Rightarrow G_0(\text{dimensionless}) = 10^{1.6} = 39.81$$

$$r = 100 \text{ meters} = 10,000 \text{ cm} = 10^4 \text{ cm}$$

$$P_{\text{rad}} = e_{cd} P_{in} = (1) P_{in} = 8 \text{ watts}$$

$$f = 1,900 \text{ MHz} \Rightarrow \lambda = 30 \times 10^9 / 1.9 \times 10^9 = 15.789 \text{ cm}$$

$$(a) \quad W_0 = \frac{P_{\text{rad}}}{4\pi r^2} = \frac{8}{4\pi(10^4)^2} = \frac{8}{4\pi \times 10^8} \\ = \frac{2}{\pi} \times 10^{-8} = 0.6366 \times 10^{-8} \text{ Watts/cm}^2$$

$$W_0 = 0.6366 \times 10^{-8} = 6.366 \times 10^{-9} \text{ Watts/cm}^2$$

$$W_{\max} = W_0 G_0(\text{dim}) = 6.366 \times 10^{-9} (39.81) = 253.438 \times 10^{-9}.$$

$$\boxed{W_{\max} = 253.438 \times 10^{-9} \text{ Watts/cm}^2}$$

$$(b) \quad D_0(\lambda/4 \text{ monopole}) = 1.643$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi} (1.643) = \frac{1.643(15.789)^2}{4\pi} = 32.5938 \text{ cm}^2$$

$$A_{em} = 32.5938 \text{ cm}^2$$

$$P(\text{received}) = W_{\max} A_{em} = (253.438 \times 10^{-9})(32.5938)$$

$$\boxed{P(\text{received}) = 8.2606 \times 10^{-6} \text{ Watts}}$$

2.33. (a) Linear because $\Delta\phi = 0$.

(b) Linear because $\Delta\phi = 0$.

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(c) Circular because

1. $E_x = E_y$

2. $\Delta\phi = \pi/2$.

CCW because E_y leads E_x . AR = 1, $\tau = 90^\circ$

(d) Circular because

1. $E_x = E_y$

2. $\Delta\phi = -\pi/2$

CW because E_y lags E_x . AR = 1, $\tau = 90^\circ$

(e) Elliptical because $\Delta\phi$ is not odd multiples of $\pi/2$. CCW because E_y leads E_x .
 AR = OA/OB

Letting $E_x = E_y = E_0$

$$\left. \begin{aligned} \text{OA} &= E_0[0.5(1 + 1 + \sqrt{2})]^{1/2} = 1.30656E_0 \\ \text{OB} &= E_0[0.5(1 + 1 + \sqrt{2})]^{1/2} = 0.541196E_0 \end{aligned} \right\} \Rightarrow \text{AR} = \frac{1.30656}{0.541196} = 2.414$$

$$\begin{aligned} \tau &= 90^\circ - \frac{1}{2} \tan^{-1} \left[\frac{2(1) \cos(45^\circ)}{1 - 1} \right] = 90^\circ - \frac{1}{2} \tan^{-1} \left(\frac{1.414}{0} \right) \\ &= 90^\circ - \frac{1}{2}(90^\circ) = 45^\circ \end{aligned}$$

(f) Elliptical because $\Delta\phi$ is not odd multiples of $\pi/2$. CW because E_y lags E_x .

$$\left. \begin{aligned} \text{From above } \text{OA} &= 1.30656E_0 \\ \text{OB} &= 0.541196E_0 \end{aligned} \right\} \Rightarrow \text{AR} = \frac{1.30656}{0.541196} = 2.414$$

$$\text{From above } \tau = 90^\circ - \frac{1}{2}(90^\circ) = 45^\circ$$

(g) Elliptical because

1. $E_x \neq E_y$

2. $\Delta\phi$ is not zero or multiples of π .

CCW because E_y leads E_x .

$$\left. \begin{aligned} \text{OA} &= E_y \left\{ \frac{1}{2} [0.25 + 1 + 0.75] \right\}^{1/2} = E_y \\ \text{OB} &= E_y \left\{ \frac{1}{2} [0.25 + 1 - 0.75] \right\}^{1/2} = 0.5E_y \end{aligned} \right\} \Rightarrow \text{AR} = \frac{1}{0.5} = 2$$

$$\tau = 90^\circ - \frac{1}{2} \tan^{-1} \left(\frac{0}{-0.75} \right) = 90^\circ - \frac{1}{2}(180^\circ) = 0^\circ$$

(h) Elliptical because

1. $E_x \neq E_y$

2. $\Delta\phi$ is not zero or multiples of π .

CCW because E_y lags E_x .

$$\left. \begin{aligned} \text{From above } \text{OA} &= E_y \\ \text{OB} &= 0.5E_y \end{aligned} \right\} \Rightarrow \text{AR} = \frac{1}{0.5} = 2$$

$$\tau = 90^\circ - \frac{1}{2}(180^\circ) = 0^\circ$$

2.34. $\mathcal{E}_x(z, t) = \text{Re}[E_x e^{j(\omega t + kz + \phi_x)}] = E_x \cos(\omega t + kz + \phi_x)$

$$\mathcal{E}_y(z, t) = \text{Re}[E_y e^{j(\omega t + kz + \phi_y)}] = E_y \cos(\omega t + kz + \phi_y)$$

where E_x and E_y are real positive constants.

Choosing $z = 0$ and letting $\Delta\phi = \phi_y - \phi_x = \phi_y - 0 = \phi$

$$\mathcal{E}_x(t) = E_x \cos(\omega t) \quad (1)$$

$$\mathcal{E}_y(t) = E_y \cos(\omega t + \phi)$$

and

$$\mathcal{E}(t) = \sqrt{\mathcal{E}_x^2 + \mathcal{E}_y^2} = \sqrt{E_x^2 \cos^2(\omega t) + E_y^2 \cos^2(\omega t + \phi)} \quad (2)$$

The maximum and minimum values of (2) are the major and minor axes of the polarization ellipse. Squaring (2) and using the half-angle identity, (2) can be written as

$$\mathcal{E}^2(t) = \frac{1}{2} \{E_x^2 + E_y^2 + E_x^2 \cos(2\omega t) + E_y^2 \cos^2[2(\omega t + \phi)]\} \quad (3)$$

Since E_x and E_y are constants, the maximum and minimum values of (3) occur when $f(t) = E_x^2 \cos(2\omega t) + E_y^2 \cos^2[2(\omega t + \phi)]$ is maximum or minimum. These are found by differentiating (4) and setting it equal to zero. Thus

$$\frac{df}{d(2\omega t)} = -E_x^2 \sin(2\omega t) - E_y^2 \sin[2(\omega t + \phi)] = 0 \quad (4)$$

or

$$\begin{aligned} E_x^2 \sin(2\omega t) &= -E_y^2 \sin[2(\omega t + \phi)] \\ &= -E_y^2 \{ \sin 2\omega t \cos 2\phi + \cos 2\omega t \sin 2\phi \} \end{aligned} \quad (5)$$

Dividing (5) by $\cos(2\omega t)$ yields

$$E_x^2 \tan(2\omega t) = -E_y^2 [\tan(2\omega t) \cos(2\phi) + \sin(2\phi)]$$

or

$$\tan(2\omega t) = \frac{-E_y^2 \sin(2\phi)}{E_x^2 + E_y^2 \cos(2\phi)}$$

from which we obtain that

$$\cos(2\omega t) = \frac{E_x^2 + E_y^2 \cos(2\phi)}{\pm \rho} \quad (6)$$

$$\cos(2\omega t + 2\phi) = \frac{E_y^2 + E_x^2 \cos(2\phi)}{\pm \rho} \quad (7)$$

where

$$\rho = \sqrt{E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos(2\phi)} \quad (8)$$

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Substituting (6)–(8) into (3) yields

$$\mathcal{E}^2 = \frac{1}{2} \left[E_x^2 + E_y^2 \pm \frac{1}{\rho} (\rho^2) \right]$$

whose maximum value is

$$\begin{aligned} \mathcal{E}_{\max} = \text{OA} &= \left\{ \frac{1}{2} [E_x^2 + E_y^2 + (E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos 2\phi)^{1/2}] \right\}^{1/2} \\ \mathcal{E}_{\max} = \text{OB} &= \left\{ \frac{1}{2} [E_x^2 + E_y^2 - (E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos 2\phi)^{1/2}] \right\}^{1/2} \end{aligned}$$

The tilt angle τ can be obtained by expanding (1) and writing the two as

$$\frac{\mathcal{E}_x^2}{E_x^2} - \frac{2\mathcal{E}_x \mathcal{E}_y \cos \phi}{E_x E_y} + \frac{\mathcal{E}_y^2}{E_y^2} = \sin^2 \phi \quad (9)$$

which is the equation of a tilted ellipse. Choosing a coordinate system whose principal axes coincide with the major and minor axes of the tilted ellipse, we can write that

$$\begin{aligned} \mathcal{E}_x &= \mathcal{E}'_x \sin(z) - \mathcal{E}'_y \cos(z) \\ \mathcal{E}_y &= \mathcal{E}'_x \cos(z) + \mathcal{E}'_y \sin(z) \end{aligned} \quad (10)$$

where $\mathcal{E}'_x$ and $\mathcal{E}'_y$ are the new field values along the new principal axes x', y', z' . Substituting (10) into (9) yields

$$\frac{2\mathcal{E}'_x \mathcal{E}'_y \cos(z) \sin(z)}{E_x^2} - \frac{2\mathcal{E}'_x \mathcal{E}'_y \cos(z) \sin(z)}{E_y^2} - \frac{2\mathcal{E}'_x \mathcal{E}'_y \cos \phi}{E_x E_y} (\sin^2 z - \cos^2 z) = 0$$

which when solved for the tilt angle τ reduces to

$$\tan \left[2 \left(\frac{\pi}{2} - \tau \right) \right] = \frac{2E_x E_y \cos \phi}{E_x^2 - E_y^2}$$

or

$$\tau = \frac{\pi}{2} - \frac{1}{2} \tan^{-1} \left(\frac{2E_x E_y \cos \phi}{E_x^2 - E_y^2} \right)$$

For more details on the tilt angle derivation, see J.D. Kraus, *Antennas*, McGraw-Hill, 1950, pp. 464–476.

2.35. (a) $\hat{\rho}_w = \hat{a}_x \cos \phi_1 + \hat{a}_y \sin \phi_1$

$$\hat{\rho}_a = \hat{a}_x \cos \phi_2 + \hat{a}_y \sin \phi_2$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |(\hat{a}_x \cos \phi_1 + \hat{a}_y \sin \phi_1) \cdot (\hat{a}_x \cos \phi_2 + \hat{a}_y \sin \phi_2)|^2$$

$$= |\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2|^2 = |\cos(\phi_1 - \phi_2)|^2$$

$$\begin{aligned}
 (b) \quad \hat{\rho}_w &= \hat{a}_x \sin \theta_1 \cos \phi_1 + \hat{a}_y \sin \theta_1 \sin \phi_1 + \hat{a}_z \cos \theta_1 \\
 \hat{\rho}_a &= \hat{a}_x \sin \theta_2 \cos \phi_2 + \hat{a}_y \sin \theta_2 \sin \phi_2 + \hat{a}_z \cos \theta_2 \\
 \text{PLF} &= |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\sin \theta_1 \cos \phi_1 \sin \theta_2 \cos \phi_2 + \sin \theta_1 \sin \phi_1 \sin \theta_2 \sin \phi_2 \\
 &\quad + \cos \theta_1 \cos \theta_2|^2 \\
 \text{PLF} &= |\sin \theta_1 \sin \theta_2 (\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2) + \cos \theta_1 \cos \theta_2|^2 \\
 \text{PLF} &= |\sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2) + \cos \theta_1 \cos \theta_2|^2
 \end{aligned}$$

2.36. Assuming electric field is x -polarized

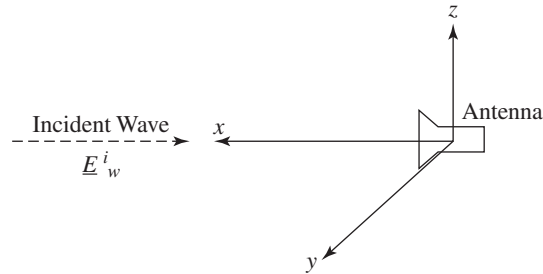
$$\begin{aligned}
 (a) \quad \underline{E}_w &= \hat{a}_x E_1 e^{-jkz} \Rightarrow \hat{\rho}_w = \hat{a}_x \\
 \underline{E}_a &= (\hat{a}_\theta - j\hat{a}_\phi) E_0 f(r, \theta, \phi) \Rightarrow \hat{\rho}_a = \left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}} \right) \\
 \text{PLF} &= |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \frac{1}{2} |\hat{a}_x \cdot \hat{a}_\theta - j\hat{a}_x \cdot \hat{a}_\phi|^2 \\
 \text{since } \hat{a}_\theta &= \hat{a}_x \cos \theta \cos \phi + \hat{a}_y \cos \theta \sin \phi - \hat{a}_z \sin \theta \\
 \hat{a}_\phi &= -\hat{a}_x \sin \phi + \hat{a}_y \cos \phi \\
 \text{PLF} &= \frac{1}{2} (\cos^2 \theta \cos^2 \phi + \sin^2 \phi)
 \end{aligned}$$

(b) when $\underline{E}_a = (\hat{a}_\theta + j\hat{a}_\phi) E_0 f(r, \theta, \phi)$, PLF is also

$$\text{PLF} = \frac{1}{2} (\cos^2 \theta \cos^2 \phi + \sin^2 \phi)$$

A more general, but also more complex, expression can be derived when the incident electric field is of the form $\underline{E}_w = (a\hat{a}_x + b\hat{a}_y)e^{-jkz}$ where a, b are real constants. It can be shown (using the same procedure) that

$$\text{PLF} = \frac{1}{\sqrt{2(a^2 + b^2)}} [(a \cos \theta \cos \phi + b \sin \theta \sin \phi)^2 + (a \sin \phi - b \cos \phi)^2]^{1/2}$$



2.37. (a) $\underline{E}_w = E_0(j\hat{a}_y + 3\hat{a}_z)e^{+jkx}$

1. **Elliptical polarization; AR = $\frac{3}{1} = 3$; Left Hand (CCW)**

- 2 components orthogonal to direction of propagation
- Not of same magnitude
- 90° phase difference between them
- y component is leading the z component *or* z component is lagging the y component

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$$(b) \underline{E}_a = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx}$$

1. **Linear polarization; AR = ∞; No rotation**

- 2 components orthogonal to direction of propagation.
- Not of same magnitude
- 0° phase difference between them,

$$(c) \text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$\underline{E}_w = E_0(j\hat{a}_y + 3\hat{a}_z)e^{+jkx} = E_0 \underbrace{\left(\frac{j\hat{a}_y + 3\hat{a}_z}{\sqrt{10}} \right)}_{\hat{\rho}_w} \sqrt{10}e^{+jkx}$$

$$\hat{\rho}_w = \left(\frac{j\hat{a}_y + 3\hat{a}_z}{\sqrt{10}} \right)$$

$$\underline{E}_a = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx} = E_0 \underbrace{\left(\frac{\hat{a}_y + 2\hat{a}_z}{\sqrt{5}} \right)}_{\hat{\rho}_a} \sqrt{5}e^{-jkx}$$

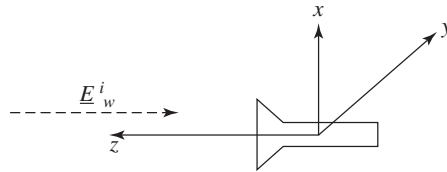
$$\hat{\rho}_a = \left(\frac{\hat{a}_y + 2\hat{a}_z}{\sqrt{5}} \right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{(j\hat{a}_y + 3\hat{a}_z)}{\sqrt{10}} \cdot \frac{(\hat{a}_y + 2\hat{a}_z)}{\sqrt{5}} \right|^2 = \frac{|j + 6|^2}{50} = \frac{37}{50}$$

$$\text{PLF} = \frac{37}{50} = \boxed{0.740 = -1.31 \text{ dB}}$$

2.38. $\underline{E}_w^i = (\hat{a}_x + j\hat{a}_y)E_0e^{+jkz}$

$$\underline{E}_a = (\hat{a}_x + 2\hat{a}_y)E_1 \frac{e^{-jkr}}{r} \Big|_{\substack{\theta = 0^\circ \\ z \text{ axis}}} = (\hat{a}_x + 2\hat{a}_y)E_1 \frac{e^{-jkz}}{z}$$



$$(a) \underline{E}_w^i = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2}E_0e^{+jkz}$$

Circular: 2 components, same amplitude, 90° phase difference

(b) **Clockwise** (y component is leading the x component)

$$(c) \underline{E}_a = \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}} \right) \sqrt{5}E_1 \frac{e^{-jkz}}{z}$$

Linear: 2 components, 0° phase difference

(d) No rotation

$$(e) \quad \hat{\rho}_w = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right), \quad \hat{\rho}_a = \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}} \right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left[\left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \cdot \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}} \right) \right]^2 = \frac{|1 + j2|^2}{10} = \frac{5}{10}$$

$$\text{PLF} = \frac{5}{10} = 0.5 = 10 \log_{10}(0.5) = -3 \text{ dB}$$

$$2.39. (a) \quad \underline{E}_w = (4\hat{a}_z + j2\hat{a}_x)E_w \frac{e^{+jky}}{y} = \underbrace{\left(\frac{4\hat{a}_z + j2\hat{a}_x}{\sqrt{20}} \right)}_{\hat{\rho}_w} \sqrt{20}E_w \frac{e^{+jky}}{y}$$

• **Elliptical** (2 components, not of same magnitude, 90° phase difference)

(b) **CW**; x -components leads z -component by 90°; rotate x into z while looking (observing) in the $-y$ direction (from behind the wave).

$$(c) \quad \mathbf{AR} = \frac{4}{2} = 2$$

$$(d) \quad \hat{\rho}_w = \left(\frac{4\hat{a}_z + j2\hat{a}_x}{\sqrt{20}} \right); \quad \hat{\rho}_a = \hat{a}_z$$

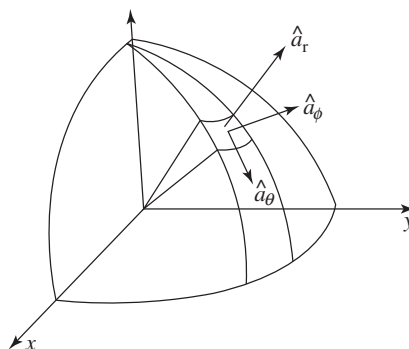
$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \left(\frac{4\hat{a}_z + j2\hat{a}_x}{\sqrt{20}} \right) \cdot \hat{a}_z \right|^2 = \frac{16}{20} = 0.8 = 10 \log_{10}(0.8)$$

$$\text{PLF} = 0.8 = -0.969 \text{ dB}$$

$$2.40. (a) \quad \underline{E}_a = E_0(j\hat{a}_\theta + 2\hat{a}_\phi)f_0(\theta_0, \phi_0) \frac{e^{-jkr}}{r} = E_0 \underbrace{\left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}} \right)}_{\hat{\rho}_a} \sqrt{5}f_0(\theta_0, \phi_0) \frac{e^{-jkr}}{r}$$

$$\hat{\rho}_a = \left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}} \right)$$

Elliptical, CW



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$$\begin{aligned}
 \text{(b)} \quad \underline{E}_w &= E_1(2\hat{a}_\theta + j\hat{a}_\phi)f_1(\theta_0, \phi_0)\frac{e^{+jkr}}{r} \\
 &= E_1 \underbrace{\left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right)}_{\hat{\rho}_w} \sqrt{5}f_1(\theta_0, \phi_0)\frac{e^{+jkr}}{r} \\
 \hat{\rho}_w &= \left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right)
 \end{aligned}$$

Elliptical, CW

$$\text{(c)} \quad \text{PLF} = |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \left| \left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}}\right) \cdot \left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right) \right|^2 = \left| \frac{2j + j2}{\sqrt{25}} \right|^2 = \left| \frac{4j}{\sqrt{25}} \right|^2$$

$$\boxed{\text{PLF} = \frac{16}{25} = 0.64 = -1.938 \text{ dB}}$$

$$\text{2.41. (a)} \quad \underline{E}_w = E_0(\hat{a}_x \pm j\hat{a}_y)e^{-jkz} \Rightarrow \hat{\rho}_w = \frac{1}{\sqrt{2}}(\hat{a}_x \pm j\hat{a}_y)$$

$$\underline{E}_a \simeq E_1(\hat{a}_\theta - j\hat{a}_\phi)f(r, \theta, \phi) \Rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_\theta - j\hat{a}_\phi)$$

$$\text{PLF} = \frac{1}{2} \left| (\hat{a}_x \pm j\hat{a}_y) \cdot (\hat{a}_\theta - j\hat{a}_\phi) \right|^2 = \frac{1}{2} \left| (\hat{a}_x \cdot \hat{a}_\theta \pm \hat{a}_y \cdot \hat{a}_\phi) - j(\hat{a}_x \hat{a}_\phi \mp \hat{a}_y \hat{a}_\theta) \right|^2$$

Converting the spherical unit vectors to rectangular, as it was done in Problem 2.35, leads to

$$\text{PLF} = \frac{1}{2}(\cos \theta \pm 1)^2$$

(b) When

$$\underline{E}_w = E_0(\hat{a}_x \pm j\hat{a}_y)e^{-jkz}$$

$$\underline{E}_a \simeq E_1(\hat{a}_\theta + j\hat{a}_\phi)f(r, \theta, \phi)$$

the PLF is equal to

$$\text{PLF} = \frac{1}{2}(\cos \theta \mp 1)^2$$

$$\text{2.42.} \quad \underline{E}_w = (\hat{a}_\theta \cos \phi - \hat{a}_\phi \sin \phi \cos \theta)f(r, \theta, \phi) \text{ or}$$

$$\underline{E}_w = \left[\frac{\hat{a}_\theta \cos \phi - \hat{a}_\phi \sin \phi \cos \theta}{\sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 \theta}} \right] \sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 \theta} \cdot f(r, \theta, \phi)$$

$$\text{Thus } \hat{\rho}_w = \frac{\hat{a}_\theta \cos \phi - \hat{a}_\phi \sin \phi \cos \theta}{\sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 \theta}}$$

and

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \left(\frac{\hat{a}_\theta \cos \phi - \hat{a}_\phi \sin \phi \cos \theta}{\sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 \theta}} \right) \cdot \hat{a}_x \right|^2$$

Transforming the rectangular unit vector to spherical using

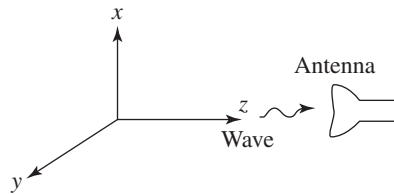
$$\hat{a}_x = \hat{a}_r \sin \theta \cos \phi + \hat{a}_\theta \cos \theta \cos \phi - \hat{a}_\phi \sin \phi$$

the PLF reduces to

$$\text{PLF} = \frac{\cos^2 \theta}{\cos^2 \phi + \sin^2 \phi \cos^2 \theta}$$

The same answer is obtained by transforming the spherical unit vectors to rectangular, as was done in Prob. 2.35.

2.43. $\underline{E}_a \simeq (2\hat{a}_x \pm j\hat{a}_y)f(r, \theta, \phi) = \left(\frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}} \right) \sqrt{5}f(r, \theta, \phi)$



(a) $\hat{\rho}_w = \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \Rightarrow \text{Wave is Right Hand (RH)}$

$$\hat{\rho}_a = \left(\frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}} \right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$= \begin{cases} \frac{9}{10} = -0.4576 \text{ dB using the } + \text{ sign} & \text{(Antenna is LH in receiving mode and RH in transmitting)} \\ \frac{1}{10} = -10 \text{ dB using the } - \text{ sign} & \text{(Antenna is RH in receiving mode and LH in transmitting)} \end{cases}$$

(b) $\hat{\rho}_w = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \Rightarrow \text{Wave is Left Hand (LH)}$

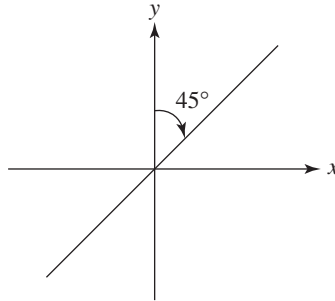
$$\hat{\rho}_a = \left(\frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}} \right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$= \begin{cases} \frac{1}{10} = -10 \text{ dB using the } + \text{ sign} & \text{(Antenna is LH in receiving mode and RH in transmitting)} \\ \frac{9}{10} = -0.4576 \text{ dB using the } - \text{ sign} & \text{(Antenna is RH in receiving mode and LH in transmitting)} \end{cases}$$

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2.44. For $\hat{\rho}_w$



$$\hat{\rho}_w = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}; \text{ PLF} = \left| \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} \cdot \frac{4\hat{a}_x + j\hat{a}_y}{\sqrt{17}} \right|^2$$

$$\text{PLF} = \frac{1}{34} |(\hat{a}_x \cdot 4\hat{a}_x) + (\hat{a}_y \cdot j\hat{a}_y)|^2 = \frac{1}{34} |4 + j|^2 = 0.5 \text{ dimensionless} = -3 \text{ dB}$$

2.45. (a) RHCP; $\hat{\rho}_a = \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}$

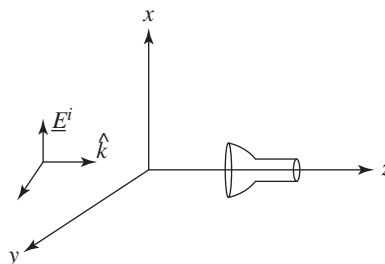
$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}} \cdot \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right|^2 = 0.9 \text{ dimensionless} = -0.46 \text{ dB}$$

(b) LHCP; $\hat{\rho}_a = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}} \cdot \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right|^2 = 0.1 \text{ dimensionless} = -10.0 \text{ dB}$$

2.46. $\underline{E}^i = (\hat{a}_x - j\hat{a}_y)E_0 e^{-jkz} = \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2}E_0 e^{-jkz}$

$$\hat{\rho}_w = \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \quad \text{CW}$$



$$\begin{aligned} \text{(a)} \quad \underline{E}^a &= (\hat{a}_x + j\hat{a}_y)E_1 e^{+jkz} \\ &= \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2}E_1 e^{+jkz} \\ \hat{\rho}_a &= \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \quad \text{CW} \end{aligned}$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \cdot \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \right|^2 = \left(\frac{1-j^2}{2} \right)^2 = 1$$

$$\text{PLF} = 1 = 0 \text{ dB}$$

$$\begin{aligned} \text{(b)} \quad \underline{E}^a &= \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2}E_1 e^{+jkz} \\ \hat{\rho}_a &= \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \end{aligned}$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \cdot \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \right|^2 = \left| \frac{1+j^2}{2} \right|^2 = 0$$

$$\text{PLF} = 0 = -\infty \text{ dB}$$

$$\text{2.47.} \quad \underline{E}_a = (2\hat{a}_\theta + j4\hat{a}_\phi)E_a \frac{e^{-jkr}}{r} = \underbrace{\left(\frac{2\hat{a}_\theta + j4\hat{a}_\phi}{\sqrt{20}} \right)}_{\hat{\rho}_a} 20E_a \frac{e^{-jkr}}{r}$$

$$\underline{E}_w = (j4\hat{a}_\theta + 2\hat{a}_\phi)E_w \frac{e^{+jkr}}{r} = \underbrace{\left(\frac{j4\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{20}} \right)}_{\hat{\rho}_w} 20E_w \frac{e^{+jkr}}{r}$$

Antenna

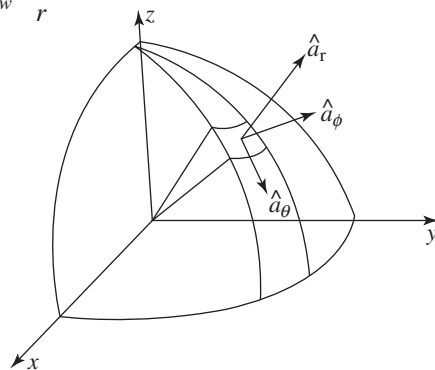
- a. Elliptical
- b. CCW
- c. $\mathbf{AR} = \frac{4}{2} = 2$

Wave

- d. Elliptical
- e. CCW
- f. $\mathbf{AR} = \frac{4}{2} = 2$

$$\begin{aligned} \text{g.} \quad \text{PLF} &= |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \left| \left(\frac{2\hat{a}_\theta + j4\hat{a}_\phi}{\sqrt{20}} \right) \cdot \left(\frac{j4\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{20}} \right) \right|^2 \\ &= \left| \frac{j8 + j8}{20} \right| = \left| \frac{16}{20} \right|^2 = (0.8)^2 = 0.64 \end{aligned}$$

$$\text{PLF} = 0.64 \text{ dimensionless} = -1.9382 \text{ dB}$$



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2.48. $\underline{E}^i = \hat{a}_x E_0 e^{-jkz}, \hat{\rho}_w = \hat{a}_x$

$$\underline{E}^a = (\hat{a}_x + j\hat{a}_y) E_1 e^{+jkz} = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2} E_1 e^{+jkz}$$

$$\hat{\rho}_a = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right)$$

$$(a) A_{em} = \frac{\lambda^2}{4\pi} e_o D_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \frac{\lambda^2}{4\pi} G_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2$$

$$(e_o D_0 = G_0)$$

$$\text{At } 10 \text{ GHz} \Rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = \frac{3 \times 10^8}{10^{10}} = 3 \times 10^{-2}$$

$$G_0 = 10 = 10 \log_{10} G_0(\text{dim}) \Rightarrow G_0(\text{dim}) = 10^1 = 10$$

$$A_{em} = \frac{\lambda^2}{4\pi} G_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \frac{(3 \times 10^{-2})^2}{4\pi} (10) \left| \hat{a}_x \cdot \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \right|^2$$

$$= \frac{9 \times 10^{-4}}{4\pi} (10) \left(\frac{1}{2} \right) = \frac{9 \times 10^{-3}}{4\pi} \left(\frac{1}{2} \right) = (0.7162 \times 10^{-3}) \left(\frac{1}{2} \right)$$

$$A_{em} = 0.3581 \times 10^{-3} \text{ m}^2$$

$$(b) P_T = A_{em} W^i = (0.3581 \times 10^{-3})(10 \times 10^{-3}) = 3.581 \times 10^{-6} \text{ Watts}$$

$$P_T = 3.581 \times 10^{-6} \text{ Watts}$$

2.49. $\underline{E}_w = \hat{a}_z E_w \frac{e^{+jky}}{y}, \hat{\rho}_w = \hat{a}_z, E_a = -\hat{a}_z E_a \frac{e^{-jky}}{y}, \hat{\rho}_a = -\hat{a}_z, W_{\text{inc}} = 100 \times 10^{-3} \frac{W}{\text{cm}^2}$

$$(a) \text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |-\hat{a}_z \cdot \hat{a}_z|^2 = 1 = 0 \text{ dB}$$

(b) For the $\lambda/2$ dipole ($Z_a = 73 + j42.5$) with a loss resistance R_L of 5 ohms:

$$U_n = (E_{\theta n})^2 = (\sin^{1.3} \theta)^2 = \sin^3 \theta \Rightarrow (U_n)_{\text{max}} = 1$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}}$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta d\theta d\phi = \int_0^{2\pi} \left(\int_0^\pi \sin^3 \theta \sin \theta d\theta \right) d\phi = 2\pi \int_0^\pi \sin^4 \theta d\theta$$

$$\int_0^\pi \sin^4 \theta d\theta = -\frac{\sin^3 \theta \cos \theta}{4} \Big|_0^\pi + \frac{4-1}{4} \int_0^\pi \sin^2 \theta d\theta = \frac{3}{4} \int_0^\pi \sin^2 \theta d\theta$$

$$\int_0^\pi \sin^4 \theta d\theta = \frac{3}{4} \int_0^\pi \sin^2 \theta d\theta = \frac{3}{4} \left[\frac{\theta}{2} - \frac{1}{4} \sin(2\theta) \right] = \frac{3\pi}{8}$$

$$P_{\text{rad}} = 2\pi \int_0^\pi \sin^4 \theta d\theta = 2\pi \left(\frac{3\pi}{8} \right) = \frac{3\pi^2}{4}$$

$$\therefore D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = D_0 = \frac{4\pi(1)}{3\pi^2/4} = \frac{16}{3\pi} = 1.69765 \text{ (dimensionless)} = 2.298 \text{ dB}$$

Using the equivalent circuit of Figure 1.2 with $R_r = 73$ and $R_L = 5$

$$e_{cd} = \frac{R_r}{R_r + R_L} = \frac{73}{73 + 5} = 0.9359$$

$$\therefore G_0 = e_{cd} D_0 = 0.9359(1.69765) = 1.5888 = 2.011 \text{ dB}$$

$$|\Gamma| = \left| \frac{Z_{in} - Z_c}{Z_{in} + Z_c} \right| = \left| \frac{(Z_a + R_L) - Z_c}{(Z_a + R_L) + Z_c} \right| = \left| \frac{(73 + j42.5 + 5) - 50}{(73 + j42.5 + 5) + 50} \right| = \frac{50.8945}{134.8712} = 0.3774$$

$$|\Gamma|^2 = (0.3774)^2 = 0.1424 \Rightarrow (1 - |\Gamma|^2) = (1 - 0.1424) = 0.8576$$

$$G_{re0} = e_r G_0 = (1 - |\Gamma|^2) G_0 = (0.8576) 1.5888 = 1.3626 \text{ (dim)} = 1.344 \text{ dB}$$

$$P_{\text{received}} = A_{em}(e_{cd})(1 - |\Gamma|^2)\text{PLF} = \left(\frac{\lambda^2}{4\pi} D_0 \right) e_{cd}(1 - |\Gamma|^2)\text{PLF}$$

$$= \frac{\pi^2}{4\pi} \underbrace{\left[D_0 e_{cd} (1 - |\Gamma|^2) \right]}_{G_0} (\text{PLF}) W_{inc}$$

$$P_{\text{received}} = (W_{inc}) \frac{\lambda^2}{4\pi} G_{re} (\text{PLF}), \quad \lambda = \frac{30 \times 10^9}{10 \times 10^9} = 3 \text{ cm}$$

$$= \underbrace{(100 \times 10^{-3})}_{W_{inc}} \underbrace{\left(\frac{(3)^2}{4\pi} \right)}_{3^2/4\pi} \underbrace{(1.3562)}_{G_{re0}} \underbrace{(1)}_{\text{PLF}} = \frac{10^{-1}(9)(1.3562)}{4\pi}$$

$$P_{\text{received}} = 0.0981 \text{ Watts} = 98.1 \text{ mWatts} = 98.1 \times 10^{-3} \text{ Watts}$$

$$P_{\text{received}} = \underbrace{(100 \times 10^{-3})}_{W_{inc}} \underbrace{\left[\frac{(3)^2}{4\pi} \right]}_{3^2/4\pi} \underbrace{(1.3626)}_{G_{re0}} \underbrace{(1)}_{\text{PLF}} = 97.59 \text{ mW} = 97.59 \times 10^{-3} \text{ W}$$

2.50. $\underline{E}_a = (2\hat{a}_x \pm j\hat{a}_y)Ee^{-jkz}$

$$\hat{\rho}_a = \frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}}$$

(a) $\underline{E}_w = \hat{a}_x E_w \Rightarrow \hat{\rho}_w = \hat{a}_x$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{2}{\sqrt{5}} \right|^2 = \frac{4}{5} = 0.8 \text{ dimensionless} = -0.9691 \text{ dB}$$

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$$(b) \underline{E}_w = \hat{a}_y E_w \Rightarrow \hat{\rho}_w = \hat{a}_y$$

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{1}{\sqrt{5}} \right|^2 = \frac{1}{5} = 0.2 \text{ dimensionless} = -6.9897 \text{ dB}$$

$$2.51. (a) E_y = E'_y + E''_y = 3 \cos \omega t + 2 \cos \omega t = 5 \cos \omega t$$

$$E_x = E'_x + E''_x = 7 \cos \left(\omega t + \frac{\pi}{2} \right) + 3 \cos \left(\omega t - \frac{\pi}{2} \right)$$

$$= -7 \sin \omega t + 3 \sin \omega t = -4 \sin \omega t$$

$$AR = \frac{5}{4} = 1.25$$

$$(b) \text{ At } \omega t = 0, \underline{E} = 5\hat{a}_y$$

$$\text{At } \omega t = \pi/2 \Rightarrow \underline{E} = -4\hat{a}_x \Rightarrow \text{Rotation in CCW}$$

$$2.52. (a) PLF = \frac{1}{2} \text{ independent of } \psi \rightarrow \text{must have CP}$$

$$\therefore AR = 1.$$

$$(b) \text{ Polarization will be elliptical with major axis aligned with x-axis.}$$

$$\text{Guess: } AR = 2$$

$$\text{Verify: } \hat{\rho}_w = (2\hat{a}_x + ja_y)/\sqrt{5}$$

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{2 \cos \psi + j \sin \psi}{\sqrt{5}} \right|^2 = \frac{4 \cos^2 \psi + \sin^2 \psi}{5}$$

$$\psi = 0 : PLF = 0.8$$

$$\psi = 90^\circ : PLF = 0.2$$

$$(c) PLF = 1 \text{ at } \psi = 45^\circ \text{ and } 225^\circ$$

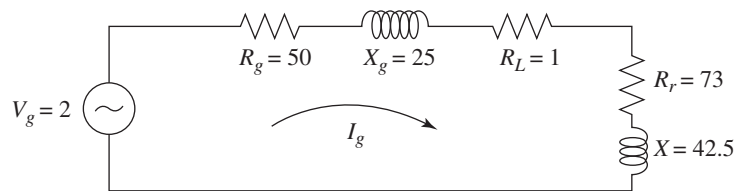
$$PLF = 0 \text{ at } \psi = 135^\circ \text{ and } 315^\circ$$

$$\text{Polarization must be linear at an angle of } 45^\circ$$

$$\therefore AR = \infty$$

$$2.53. I_g = \frac{2}{(50 + 1 + 73) + j(25 + 42.5)} = \frac{2}{124 + j67.5}$$

$$= (12.442 - j6.7724) \times 10^{-3} = 14.166 \times 10^{-3} \angle -28.56^\circ$$



$$(a) P_s = \frac{1}{2} \text{Re}(V_g \cdot I_g^*) = \text{Re}(12.442 + j6.7724) \times 10^{-3} = 12.442 \times 10^{-3} \text{ W}$$

$$(b) P_r = \frac{1}{2} |I_g|^2 R_r = 7.325 \times 10^{-3} \text{ W}$$

$$(c) P_L = \frac{1}{2} |I_g|^2 R_L = 0.1003 \times 10^{-3} \text{ W}$$

The remaining supplied power is dissipated as heat in the internal resistor of the generator, or

$$P_g = \frac{1}{2} |I_g|^2 R_g = 5.0169 \times 10^{-3} \text{ W}$$

Thus

$$P_r + P_L + P_g = (7.325 + 0.1003 + 5.0169) \times 10^{-3} = 12.4422 \times 10^{-3} = P_s$$

2.54. The impedance transfer equation of

$$Z_{in} = Z_c \left[\frac{Z_L + jZ_c \tan(kl)}{Z_c + jZ_L \tan(kl)} \right]$$

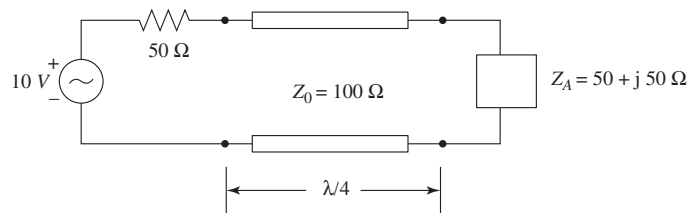
reduces for $l = \lambda/2$ to $Z_{in} = Z_L$.

Therefore the equivalent load impedance at the terminals of the generator is the same as that for Problem 2.53.

Thus the supplied, radiated, and dissipated powers are the same as those of Problem 2.53.

$$\mathbf{2.55.} \quad (a) \quad Z_{in} = \frac{(100)^2}{50 + j50} = \frac{10000}{5000}(50 - j50)2 = 100 - j100 \, \Omega$$

$$I_g = \frac{10}{150 - j100} = \frac{10}{180.3 \angle -33.7^\circ} = 0.05546 \angle 33.7^\circ \text{ A}$$



$$(b) \quad P_s = \frac{1}{2} \text{Re}\{V_g I_g^*\} = \frac{1}{2} \times 10 \times 0.05546 \times \cos(33.7^\circ) = 0.231 \text{ W}$$

$$(c) \quad P_A = \frac{1}{2} |I_g|^2 \text{Re}\{Z_{in}\} = \frac{1}{2} \times (0.05546)^2 \times 100 = 0.1538 \text{ W}$$

$$P_{\text{rad}} = e_{cd} P_A = 0.96 \times 0.1538 = 0.148 \text{ W}$$

$$\mathbf{2.56.} \quad \text{Gain} = \frac{P_{\text{rad}}}{P_{\text{accepted}}} (\text{Directivity})$$

$$\text{Realized Gain} = \frac{P_{\text{rad}}}{P_{\text{available}}} (\text{Directivity})$$

$$\frac{\text{Gain}}{\text{Realized Gain}} = \frac{P_{\text{available}}}{P_{\text{accepted}}}$$

$$P_{\text{available}} = \frac{1}{2} \frac{\left(\frac{V_s}{\sqrt{2}}\right)^2}{Z_0} = \frac{V_s^2}{4Z_0}$$

$$V(x) = A[e^{-jkx} + \Gamma(0)e^{jkx}]$$

$$I(x) = \frac{A}{Z_0}[e^{-jkx} - \Gamma(0)e^{jkx}]$$

$$V(0) = A[1 + \Gamma(0)]$$

$$I(0) = \frac{A}{Z_0}[1 - \Gamma(0)]$$

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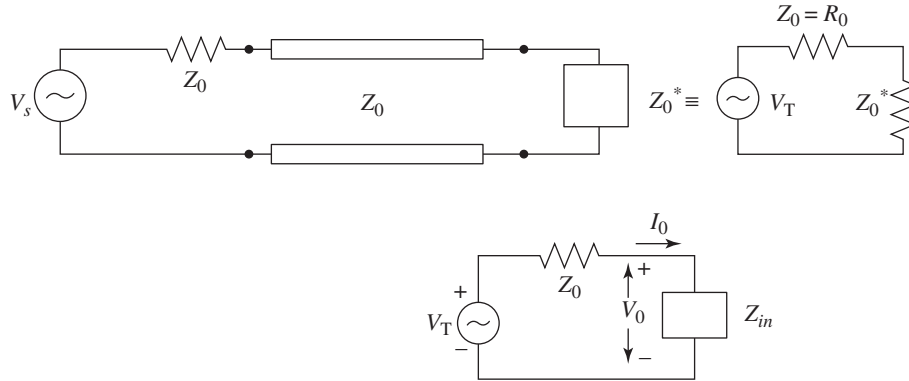


Fig. 1

From Fig. 1:

$$-V_s + I(0)Z_0 + V(0) = 0$$

$$-V_s + \frac{A}{Z_0}[1 - \Gamma(0)]Z_0 + A[1 + \Gamma(0)] = 0$$

$$-V_s + A - A\Gamma(0) + A + A\Gamma(0) = 0$$

$$2A = V_s \rightarrow A = \frac{V_s}{2}$$

$$P_{\text{accepted}} = \text{Re}[V(0)I^*(0)]$$

$$V(0) = \frac{V_s}{2}[1 + \Gamma(0)]$$

$$I(0) = \frac{V_s}{2Z_0}[1 - \Gamma(0)]$$

$$\Gamma(0) = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\Rightarrow V(0) = \frac{V_s}{2} \left(1 + \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right)$$

$$= \frac{V_s}{2} \left(1 + \frac{R_{in} + jX_{in} - Z_0}{R_{in} + jX_{in} + Z_0} \right)$$

$$= \frac{V_s}{2} \left(\frac{R_{in} + jX_{in} + Z_0 + R_{in} + jX_{in} - Z_0}{R_{in} + jX_{in} + Z_0} \right)$$

$$V(0) = \frac{V_s(R_{in} + jX_{in})}{R_{in} + jX_{in} + Z_0}$$

$$I(0) = \frac{V_s}{2Z_0} \left(1 - \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right) = \frac{V_s}{2Z_0} \left(\frac{Z_{in} + Z_0 - Z_{in} + Z_0}{Z_{in} + Z_0} \right)$$

$$I(0) = \frac{V_s}{Z_{in} + Z_0} = \frac{V_s}{R_{in} + jX_{in} + Z_0}$$

$$\text{Re}[V(0)I(0)^*] = \text{Re} \left[\frac{V_s R_{in} + jV_s X_{in}}{R_{in} + Z_0 + jX_{in}} \times \frac{V_s}{R_{in} + Z_0 - jX_{in}} \right]$$