**CHAPTER 2 RESERVE PROBLEMS**

The following problems have been reserved for your use in assignments and testing and do not appear in student versions of the text.

**Reserve Problems Chapter 2 Section 1 Problem 1**

Determine the sample space for the following random experiment.  
  
It is known that the delivery time is within 10 to 46 hours. Consider an experiment that records the time from an order until the shipment arrives.

**SOLUTION**  
  
*S* is the sample space for delivery time when measured in hours.

**Reserve Problems Chapter 2 Section 1 Problem 2**

Consider an experiment that records the number of bushels (and fractions of bushels) of corn produced in an acre. The yield is assumed to be between 130 and 230 bushels.  
  
Determine the corresponding sample space.

**SOLUTION**  
  
*S* is the sample space for the possible number of bushels of corn produced in an acre.

**Reserve Problems Chapter 2 Section 1 Problem 3**

Consider an experiment that records the error (difference) between the actual and estimated quarterly revenue of a corporation.   
  
Determine the corresponding sample space.

**SOLUTION**  
  
*S* is the sample space for the error between the actual and estimated quarterly revenue of a corporation.

**Reserve Problems Chapter 2 Section 1 Problem 4**

A conceptual smartphone design uses a parachute technique to avoid severe damage to the screen and phone-case. Each of four nozzles located on the case might be either functional or defective after a year.

(a) Describe the sample space in terms of the condition (functional or defective) of each nozzle after a year. Let "F" denote a functional nozzle after a year and "D" denote a defective one.

(b) How many outcomes are in the event defined by two defective nozzles?

**SOLUTION**  
  
Let "F" denote a functional nozzle after a year and "D" denote a defective one.  
  
The outcome could be represented as {XXXX}, where X denotes the condition of the nozzle and takes one of the two possible values: "F" if the nozzle is functional and "D" if it is defective. For example, {FFDD} denote that the first two nozzles of the smartphone are functional and the last two nozzles are defective.  
  
(a) The sample space in terms of the condition of each nozzle after a year is   
  
     {FDDD, DFDD, DDFD, DDDF,   
       FFDD, FDFD, FDDF, DFDF,   
       DFFD, DDFF, DFFF, FDFF,   
       FFDF, FFFD, FFFF, DDDD}  
  
  
(b) There are 6 suitable outcomes: FFDD, FDFD, FDDF, DFFD, DFDF, DDFF.

**Reserve Problems Chapter 2 Section 1 Problem 5**

Let *x* denote the grams of gold obtained in a ton of ore. Consider the two events  and . Determine the following events.

a) 

b) 

c) 

d) 

**SOLUTION**  
  
Because *x* denotes the grams of gold obtained in a ton of ore, all the possible values for *x* lie within the interval. ( 1 ton = 907184.74 grams)  
  
a)   
  
b)   
  
c)   
  
d) , thus

**Reserve Problems Chapter 2 Section 1 Problem 6**

Each of three machined parts is classified as either above or below the target specification for the part. Let *a* and *b* denote a part above and below the specification, respectively.  
  
Provide a reasonable description of the sample space for this random experiment.

**SOLUTION**



**Reserve Problems Chapter 2 Section 1 Problem 7**

Each of 24 Web sites is classified as containing or not containing banner ads.

Provide a reasonable description of the sample space for this random experiment.

**SOLUTION**

Let *y* and *n* denote a web site that contains and does not contain banner ads, respectively. The sample space is the set of all possible sequences of *y* and *n* of length 24. An example outcome in the sample space is .

**Reserve Problems Chapter 2 Section 1 Problem 8**

A scale that displays two decimal places is used to measure material feeds in a chemical plant in tons. Choose a reasonable description of the sample space for this random experiment.

**SOLUTION**

*S* is the sample space of 100 integer numbers from 0 to 99, including 0 and 99.

**Reserve Problems Chapter 2 Section 1 Problem 9**

Provide a reasonable description of the sample space for a measurement of the concentration of ozone to the nearest part per billion.

**SOLUTION**

 in ppb.

**Reserve Problems Chapter 2 Section 1 Problem 10**

The time of a chemical reaction is recorded to the nearest millisecond.   
  
Provide a reasonable description of the sample space for this experiment.

**SOLUTION**

 in milliseconds.

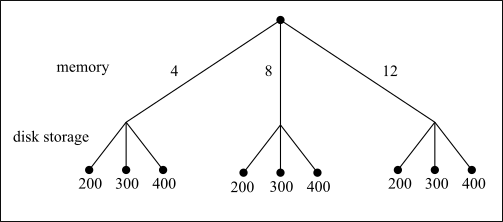
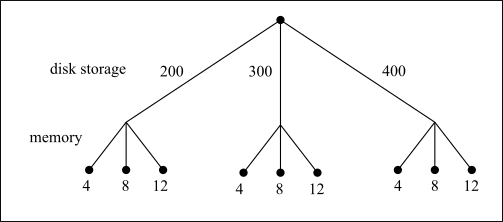
**Reserve Problems Chapter 2 Section 1 Problem 11**

An order for a computer system can specify memory of 4, 8, or 12 gigabytes and disk storage of 200, 300, or 400 gigabytes.  
  
Choose the diagrams that describe the set of possible orders.

|  |  |
| --- | --- |
| A | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\EAT_14939106254440_5401601254045579.png |
| B | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\EAT_14939106254460_2631990323029971.png |
| C | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\EAT_14968229880470_6285856719223288.png |
| D | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\EAT_14968229880470_4693655022129243.png |

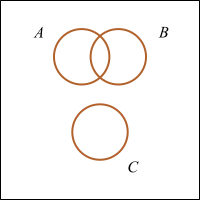
**SOLUTION**

The set of possible orders can be described by following diagrams:

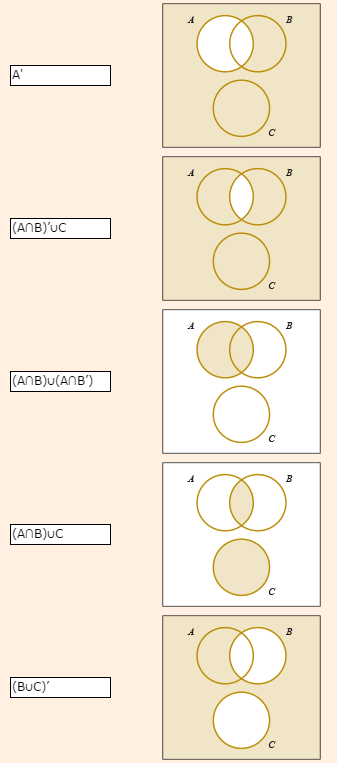
**Reserve Problems Chapter 2 Section 1 Problem 12**

Three events are shown on the Venn diagram in the following figure:



Match the figures and the corresponding events.

**SOLUTION**



**Reserve Problems Chapter 2 Section 1 Problem 13**

Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarized as follows:

|  |  |  |  |
| --- | --- | --- | --- |
|  | | Edge Finish | |
|  | | Excellent | Good |
| Surface Finish | Excellent | 70 | 1 |
| Good | 12 | 17 |

Let *A* denote the event that a sample has excellent surface finish, and let *B* denote the event that a sample has excellent edge finish. Determine the number of samples in each of the following events.

(a) 

(b) 

(c) 

**SOLUTION**

(a) 70

(b)  17 + 1 = 18

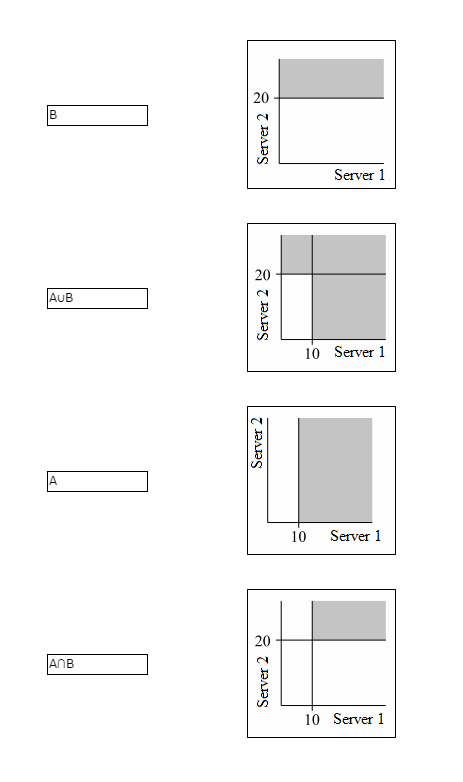
(c)  70 + 12 +1 = 99

**Reserve Problems Chapter 2 Section 1 Problem 14**

Counts of the Web pages provided by each of two computer servers in a selected hour of the day are recorded. Let *A* denote the event that at least 10 pages are provided by server 1, and let *B* denote the event that at least 20 pages are provided by server 2.

For each of the following events choose the corresponding sample space graph:

a) A  
  
b) B’  
  
c)   
  
d) 

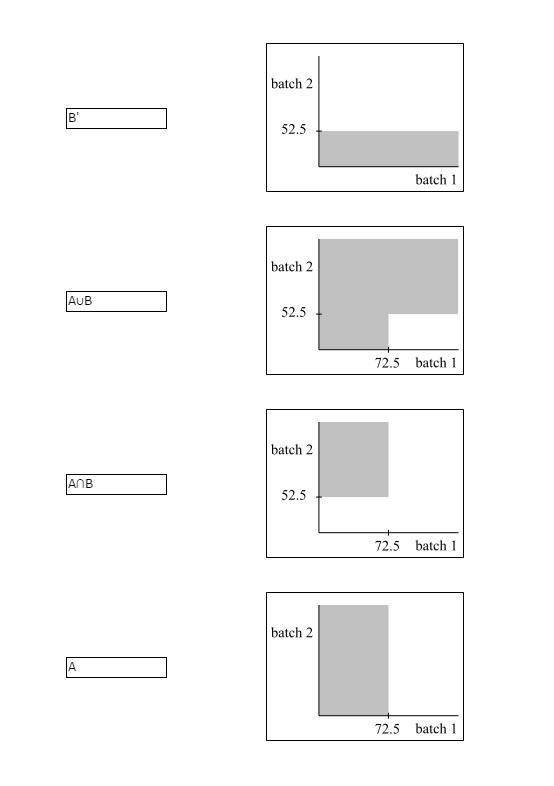
**SOLUTION**  


**Reserve Problems Chapter 2 Section 1 Problem 15**

A reactor's rise time is measured in minutes (and fractions of minutes). Let the sample space for the rise time of each batch be positive, real numbers. Consider the rise times of two batches. Let *A* denote the event that the rise time of batch 1 is less than 72.5 minutes, and let *B* denote the event that the rise time of batch 2 is greater than 52.5 minutes.  
  
For each of the following events choose the corresponding sample space graph:

a) A  
  
b) B’  
  
c)   
  
d) 

**SOLUTION**



**Reserve Problems Chapter 2 Section 1 Problem 16**

A *byte* is a sequence of eight bits and each bit is either 0 or 1.

(a) How many different bytes are possible?

(b) If the first bit of a byte is a parity check, that is, the first byte is determined from the other seven bits, how many different bytes are possible?

**SOLUTION**

(a)

From the multiplication rule,  bytes are possible.

(b)

From the multiplication rule,  bytes are possible.

**Reserve Problems Chapter 2 Section 1 Problem 17**

In a chemical plant, 24 holding tanks are used for final product storage. Four tanks are selected at random and without replacement. Suppose that four of the tanks contain material in which the viscosity exceeds the customer requirements.

(a) What is the probability that exactly one tank in the sample contains high-viscosity material?

(b) What is the probability that at least one tank in the sample contains high-viscosity material?

(c) In addition to the four tanks with high-viscosity levels, four different tanks contain material with high impurities. What is the probability that exactly one tank in the sample contains high-viscosity material and exactly one tank in the sample contains material with high impurities?

**SOLUTION**

(a)

The total number of samples possible is . The number of samples in which exactly one tank has high viscosity is. Therefore, the probability is.

(b)

The number of samples that contain no tank with high viscosity is. Therefore, the requested probability is.

(c)

The number of samples that meet the requirements is. Therefore, the probability is.

**Reserve Problems Chapter 2 Section 1 Problem 18**

An article in *The Journal of Data Science* ["A Statistical Analysis of Well Failures in Baltimore County" (2009, Vol. 7, pp. 111-127)] provided the following table of well failures for different geological formation groups in Baltimore County.

|  |  |  |
| --- | --- | --- |
| Geological Formation Group | Wells | |
| Failed | Total |
| Gneiss | 170 | 1685 |
| Granite | 2 | 28 |
| Loch raven schist | 443 | 3733 |
| Mafic | 14 | 363 |
| Marble | 29 | 309 |
| Prettyboy schist | 60 | 1403 |
| Other schists | 46 | 933 |
| Serpentine | 3 | 39 |

Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed. Determine the number of wells in each of the following events.

(a) 

(b) 

(c) 

(d) 

(e) 

**SOLUTION**

(a) 

(b) 

(c) 

(d) 

(e) 

**Reserve Problems Chapter 2 Section 2 Problem 1**

Customers who are registered on a corporate Web site are summarized by the type of shipping contract they use and the number of orders in the previous month. The number of customers in each category are shown in the following table. Customers are to be selected without replacement.

|  |  |  |  |
| --- | --- | --- | --- |
| Shipping Contract | Express | Standard | Total |
| No orders | 25 | 15 | 40 |
| One order | 65 | 45 | 110 |
| More than one order | 40 | 20 | 60 |

(a) How many different samples of eight express customers are possible?

(b) How many different samples of ten standard customers are possible?

(c) How many different samples of eight express and ten standard customers are possible?

**SOLUTION**  
  
(a) The total number of express customers is , and the number of different samples of eight express customers is  
  
  
  
(b) The total number of standard customers is , and the number of different samples of ten standard customers is  
  
  
  
(c) By the multiplication rule, the number of different samples of eight express and ten standard customers is  
  


**Reserve Problems Chapter 2 Section 2 Problem 2**

Customers who are registered on a corporate Web site are summarized by the type of shipping contract they use and the number of orders in the previous month. The number of customers in each category are shown in the following table. Customers are to be selected without replacement.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Shipping Contract |  | Express | Standard | Total |
| No orders |  | 25 | 15 | 40 |
| One order |  | 65 | 45 | 110 |
| More than one order |  | 40 | 20 | 60 |

(a) Suppose that 10 express customers are selected without replacement. How many samples contain exactly one customer with more than one order last month?

(b) Suppose that 10 express customers are selected without replacement. How many samples contain at least one customer with more than one order last month?

(c) Suppose that 10 express and 15 standard customers are selected without replacement. How many samples contain exactly one express customer and exactly one standard customer with no orders last month?

**SOLUTION**  
  
(a) The possible number of samples that contain exactly one customer with more than one order last month is  
  
  
  
  
(b) The possible number of samples that contain at least one customer with more than one order last month is   
  
  
  
  
(c) The number of samples that contain exactly one express customer and exactly one standard customer with no orders last month is  
  


**Reserve Problems Chapter 2 Section 2 Problem 3**

A committee will be formed with 4 managers and 3 engineers selected without replacement from 10 managers and 20 engineers.

How many different committees are possible?

**SOLUTION**  
The number of ways to choose 4 managers out of 10 is the number of ways to choose 3 engineers out of 20 is   
  
The total number of ways to choose 4 managers and 3 engineers is

.

**Reserve Problems Chapter 2 Section 2 Problem 4**

Code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). One character is held back as a start and stop delimiter.  
  
For each of the following cases, how many characters can be encoded?

(a) The constraint of exactly two wide bars is replaced with one that requires exactly one wide bar.

(b) The constraint of exactly two wide bars is replaced with one that allows either one or two wide bars.

(c) The constraint of exactly two wide bars is dropped.

(d) The constraints of exactly two wide bars and one wide space are dropped.

**SOLUTION**

(a)

Focus first on the bars. There are  permutations of the bars with one wide bar and four narrow bars. A code has three narrow spaces and one wide space so there are four possible locations for the wide space. Therefore, the possible number of codes , and if one is held back as a delimiter, 19 characters can be coded.

(b)

As in part (a), the number of codes with exactly two wide bars . From part (a), the number of codes with exactly one wide bar . Therefore, the possible number of codes is , and if one is held back as a delimiter, 59 characters can be coded.

(c)

There are two choices for each bar (wide or narrow) and 5 bars are used in total. Therefore, the number of possibilities for the bars . As in part (a), there are 4 possibilities for the spaces. Therefore, the number of codes is , and if one is held back as a delimiter, 127 characters can be coded.

(d)

As in part (c), there are 32 possibilities for the bars, and there are also  possibilities for the spaces. Therefore,  codes are possible, and if one is held back as a delimiter, 511 characters can be coded.

**Reserve Problems Chapter 2 Section 3 Problem 1**

In a car racing competition, four car brands are competing, each having two cars representing them. Cars from the same company have the same probability of winning. Each of the cars from Acura, Alfa Romeo and Audi has the same probability of winning the match, while each of Aston Martin’s cars has the probability of winning 1.57 times the probability of winning for each of the other brands’ cars.

(a) What is the chance of winning for each of the Aston Martin's cars? What is the chance of winning for each other cars?

(b) What is the probability of winning for Aston Martin brand? What is the probability of winning for each other brand?

**SOLUTION**

1. Let *P*(*A*) denote the probability that each car from Acura, Alfa Romeo, and Audi wins the competition, and *P*(*B*) denote the probability that each of Aston Martin’s car wins the competition. Because *P*(*S*) = 1, we have

Solving for P(A) and P(B) using any number of methods for example method of linear combinations we can multiply the second equation by two then add the two equations together.

Solving for we find that

and

1 or 11%, and the chance of winning for each of cars from Aston Martin is 0.17 or 17%.

(b) Because each brand has two cars to take part in the competition, the probability of winning for Acura, Alfa Romeo and Audi is   
  
 or 22%.  
  
and the probability of winning for Aston Martin is   
  
 or 34%

**Reserve Problems Chapter 2 Section 3 Problem 2**

Customers who are registered on a corporate Web site are summarized by the type of shipping contract they use and the number of orders in the previous month. The number of customers in each category are shown in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Shipping Contract |  | Express | Standard | Total |
| No orders |  | 25 | 15 | 40 |
| One order |  | 65 | 44 | 109 |
| More than one order |  | 44 | 20 | 64 |
| Total |  | 134 | 79 | 213 |

Suppose that three customers are selected randomly, without replacement, for a survey.

(a) What is the probability that at least two selected customers use express shipping?

(b) What is the probability that customers from all three order categories are selected?

**SOLUTION**  
  
(a) The sample space consists of all possible (unordered) subsets of three customers selected without replacement. The total number of subsets of size 3 is and the numbers of subsets with exactly two and exactly three express customers are and, respectively. So, the probability that at least two selected customers use express shipping is .

(b) The number of subsets with customers from all three order categories is . So, the probability that customers from all three order categories are selected is .

**Reserve Problems Chapter 2 Section 3 Problem 3**

A committee will be formed with 2 managers and 4 engineers selected randomly without replacement from 9 managers and 18 engineers.

(a) What is the probability that the specific engineer Jane and the specific manager Mary are on the committee?  
  
(b) Each of the engineers differ in years of experience. What is the probability that the most experienced and least experienced engineers are on the committee?

**SOLUTION**

(a) The probability that the specific engineer Jane is on the committee is

and the probability that the specific manager Mary is on the committee is.   
  
Therefore, the probability that both of them are on the committee is .

(b) The number of samples which contain the most and least experienced engineers is.  
  
And the probability that both of them are on the committee is

**Reserve Problems Chapter 2 Section 3 Problem 4**

Each of the possible five outcomes of a random experiment is equally likely. The sample space is [*a, b, c, d, e*]. Let *A* denote the event [*a, b*], and let *B* denote the event [*c, d, e*].   
Determine the following:

(a) 

(b) 

(c) 

(d) 

(e) 

**SOLUTION**

(a) 

(b) 

(c) 

(d) 

(e) 

**Reserve Problems Chapter 2 Section 3 Problem 5**

The following table summarizes 186 endothermic reactions involving sodium bicarbonate.

|  |  |  |
| --- | --- | --- |
| Final Temperature Conditions | Heat absorbed (cal) | |
| Below Target | Above Target |
| 266 K | 12 | 32 |
| 271 K | 44 | 16 |
| 274 K | 46 | 36 |

Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is above target. Determine the following probabilities.

(a) 

(b) 

(c) 

(d) 

(e) 

**SOLUTION**

(a) 

(b) 

(c) 

(d) 

(e) 

**Reserve Problems Chapter 2 Section 3 Problem 6**

An article in *The Journal of Data Science* ["A Statistical Analysis of Well Failures in Baltimore County" (2009, Vol. 7, pp. 111-127)] provided the following table of well failures for different geological formation groups in Baltimore County.

|  |  |  |
| --- | --- | --- |
| Geological Formation Group | Wells | |
| Failed | Total |
| Gneiss | 170 | 1685 |
| Granite | 2 | 28 |
| Loch raven schist | 443 | 3733 |
| Mafic | 14 | 363 |
| Marble | 29 | 309 |
| Prettyboy schist | 60 | 1403 |
| Other schists | 46 | 933 |
| Serpentine | 3 | 39 |

Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed. Determine the following probabilities.

(a) 

(b) 

(c) 

(d) 

(e) 

**SOLUTION**

(a) 

(b) 

(c) 

(d) 



(e) 



**Reserve Problems Chapter 2 Section 3 Problem 7**

Code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). Suppose that all 40 codes are equally likely (none is held back as a delimiter).

Determine the probability for each of the following:

(a) A wide space occurs before a narrow space.

(b) Two wide bars occur consecutively.

(c) Two consecutive wide bars are at the start or end.

(d) The middle bar is wide.

**SOLUTION**  
(a)

There are four spaces and exactly one is wide.  
Number of permutations of the spaces where the wide appears first is 1.  
Number of permutations of the bars is .  
Total number of permutations where a wide space occurs before a narrow space .  
*P*(wide space occurs before a narrow space)

(b)

There are 5 bars and 2 are wide.  
The spaces are handled as in part (a).  
Number of permutations of the bars where 2 wide bars are consecutive is 4.  
Therefore, the probability is .

(c)

The spaces are handled as in part (a).  
Number of permutations of the bars where 2 consecutive wide bars are at the start or end is 2. Therefore, the probability is 

(d)

The spaces are handled as in part (a).  
Number of permutations of the bars where a wide bar is at the center is 4 because there are 4 remaining positions for the second wide bar. Therefore, the probability is .

**\ Reserve Problems Chapter 2 Section 4 Problem 1**

Customers who are registered on a corporate Web site are summarized by the type of shipping contract they use and the number of orders in the previous month. The number of customers in each category are shown in the following table. Customers are to be selected without replacement.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Shipping Contract |  | Express | Standard | Total |
| No orders |  | 22 | 17 | 39 |
| One order |  | 64 | 44 | 108 |
| More than one order |  | 44 | 21 | 65 |

(a) What is the probability that a randomly selected customer had at least one order in the previous month or uses express shipping?  
  
(b) What is the probability that a randomly selected customer uses standard shipping or had no orders?

**SOLUTION**  
  
(a) The probability that the selected customer had at least one order (denote by A) is   
  
  
The probability that the selected customer uses express shipping (denote by B) is  
  
The probability that a customer had at least one order and also used the express shipping () is  
  
So 

(b) The probability that a selected customer uses a standard shipping (denote by C) is  
  
The probability that a selected customer had no order (denote by D) is  
  
The probability that a selected customer uses a standard shipping and also had no order is  
  
So 

**Reserve Problems Chapter 2 Section 4 Problem 2**

A committee will be formed with 4 managers and 4 engineers selected randomly without replacement from 11 managers and 16 engineers.

What is the probability that engineer Jane or manager Mary is on the committee?

**SOLUTION**

Let A denote that engineer Jane is on the committee, B denote that the manager Mary is on the committee.  
So  is that both Jane and Mary are on the committee,  is that Jane or Mary is on the committee.  
Using the additional rule,  


**Reserve Problems Chapter 2 Section 4 Problem 3**

Consider the hospital emergency department visits shown in table below. People may leave without being seen by a physician, and those visits are denoted as LWBS. The remaining visits are serviced at the emergency department, and the visitor may or may not be admitted for a stay in the hospital.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Hospital |  | 1 | 2 | 3 | 4 | Total |
| Total |  | 5337 | 7048 | 5754 | 4405 | 22544 |
| LWBS |  | 190 | 267 | 247 | 248 | 952 |
| Admitted |  | 1267 | 1538 | 669 | 964 | 4438 |
| Not admitted |  | 3880 | 5243 | 4838 | 3193 | 17154 |

Assume that the record is reviewed from a visit selected randomly from the table.

(a) What is the probability that the visit is LWBS or that the visit is from hospital 1?

(b) What is the probability that the visit is not LWBS and the hospital is 1 or 2?

**SOLUTION**

(a) The probability that the visit is LWBS is . The probability that the visit is from hospital 1 is , and the probability that that the visit is LWBS and from hospital 1 is. Using the addition rule:

(b) Let C denote the visit is from hospital 2. Because :

**Reserve Problems Chapter 2 Section 4 Problem 4**

Consider the endothermic reactions in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| Final Temperature Conditions |  | Heat Absorbed (cal) |  |
|  |  | Below Target | Above Target |
| 266 K |  | 21 | 40 |
| 271 K |  | 44 | 27 |
| 274 K |  | 56 | 36 |

Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is above target. Use the addition rules to calculate the following probabilities.

|  |
| --- |
| (a) |
| (b) |
| (c) |

**SOLUTION**

, ,

(a)

(b)

(c)

**Reserve Problems Chapter 2 Section 4 Problem 5**

Consider the well failure data in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | Wells |  |
| Geological Formation Group |  | Failed |  | Total |
| Gneiss |  | 110 |  | 1485 |
| Granite |  | 2 |  | 28 |
| Loch raven schist |  | 443 |  | 3733 |
| Mafic |  | 14 |  | 363 |
| Marble |  | 29 |  | 309 |
| Prettyboy schist |  | 60 |  | 1403 |
| Other schists |  | 46 |  | 933 |
| Serpentine |  | 3 |  | 39 |

Let *A* denote the event that the geological formation of a well has more than 1000 wells, and let *B* denote the event that a well failed. Use the addition rules to calculate the following probabilities.

|  |
| --- |
| (a) |
| (b) |
| (c) |

**SOLUTION**

,

,

,

|  |  |  |  |
| --- | --- | --- | --- |
| (a) |  | = |  |
| (b) |  | = |  |
|  |  | = |  |
| (c) |  | = |  |

**Reserve Problems Chapter 2 Section 4 Problem 6**

Code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6).

Suppose that all 40 codes are equally likely (none is held back as a delimiter). Determine the probability for each of the following:

(a) The first bar is wide or the second bar is wide.

(b) Neither the first nor the second bar is wide.

(c) The first bar is wide or the second bar is not wide.

(d) The first bar is wide or the first space is wide.

**SOLUTION**  
(a)

Number of permutations of the bars with the first bar wide is 4.  
Number of permutations of the bars with the second bar wide is 4.  
Number of permutations of the bars with both the first & second bar wide is 1.  
Number of permutations of the bars with either the first bar wide or the second bar wide = 4 + 4 − 1 = 7.  
Number of codes is multiplied this by the number of permutations for the spaces = 4.  
The probability that first bar is wide equals 16/40 = 0.4, the probability that second bar is wide equals 16/40 = 0.4, the probability that first & second bar is wide equals 4/40 = 0.1  
The probability that first or second bar is wide equals 4/10 + 4/10 − 1/10 = 7/10 = 0.7

(b)

Neither the first or second bar wide implies the two wide bars occur in the last 3 positions.  
Number of permutations of the bars with the wide bars in the last 3 positions is 3!/(2!1!) = 3  
The probability that neither first nor second bar is wide equals 12/40 = 0.3

(c)

The spaces are handled as in part (a).  
The probability that first bar is wide equals 16/40 = 0.4  
Number of permutations of the bars with the second bar narrow is 4!/(2!2!) = 6  
The probability that second bar is narrow equals 24/40 = 0.6  
Number of permutations with the first bar wide and the second bar narrow is 3!/(1!2!) = 3  
The probability that first bar is wide and the second bar is narrow equals 12/40 = 0.3  
The probability that first bar is wide or the second bar is narrow equals 0.4 + 0.6 − 0.3 = 0.7

(d)

The spaces are handled as in part (a).  
Number of permutations of the bars with the first bar wide is 4. Therefore, the probability that first bar is wide equals 16/40 = 0.4  
The number of permutations of the bars = 10. Number of permutations of the spaces with the first space wide is 1.  
Therefore, the probability that first space is wide equals 1(10)/40 = 0.25  
Number codes with the first bar wide and the first space wide is 4(1) = 4  
The probability that first bar is wide & the first space is wide equals 4/40 = 0.1  
The probability that first bar is wide or the first space is wide equals 0.4 + 0.25 −0.1 = 0.55

**Reserve Problems Chapter 2 Section 5 Problem 1**

Customers who are registered on a corporate Web site are summarized by the type of shipping contract they use and the number of orders in the previous month. The number of customers in each category are shown in the following table. Customers are to be selected randomly.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Shipping Contract |  | Express | Standard | Total |
| No orders |  | 22 | 17 | 39 |
| One order |  | 62 | 44 | 106 |
| More than one order |  | 40 | 23 | 63 |

(a) What is the conditional probability that the customer had one order last month given that the customer uses express shipping?

(b) What is the conditional probability that the customer uses express shipping given that the customer had at least one order last month?

**SOLUTION**  
(a)   
  
  
Let *A* denote that the customer had one order, *B* denote that the customer uses express shipping. Then

(b) Let *C* denote that the customer had at least one order.

**Reserve Problems Chapter 2 Section 5 Problem 2**

Consider the hospital emergency department visits shown in table below. People may leave without being seen by a physician, and those visits are denoted as LWBS. The remaining visits are serviced at the emergency department, and the visitor may or may not be admitted for a stay in the hospital.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Hospital |  | 1 | 2 | 3 | 4 | Total |
| Total |  | 5295 | 7172 | 5748 | 4445 | 22660 |
| LWBS |  | 198 | 271 | 241 | 238 | 948 |
| Admitted |  | 1277 | 1578 | 679 | 1014 | 4548 |
| Not admitted |  | 3820 | 5323 | 4828 | 3193 | 17164 |

Assume that the records are reviewed from two visits selected randomly without replacement from the table.

(a) What is the probability that the second visit selected is LWBS given that the first visit selected is LWBS?

(b) What is the probability that both visits are LWBS from hospital 4?

**SOLUTION**

(a) If the first visit selected is LWBS, then there are  LWBS visits. One of them should be the second chosen. Let A denote that the second visit selected is LWBS, and B denote that the first visit selected is LWBS. Thus

(b) The probability that the first visit is LWBS from hospital 4 is , the probability that the second visit is LWBS from hospital 4 given that the first visit is LWBS from hospital 4 is

. Thus, we get:

**Reserve Problems Chapter 2 Section 5 Problem 3**

A committee will be formed with 4 managers and 3 engineers selected randomly without replacement from 11managers and 19 engineers.

What is the conditional probability that engineer Al is on the committee given that engineer Jane is on the committee?

**SOLUTION**

Let A denote that the engineer Al is on the committee, B denote that the engineer Jane is on the committee. One of four vacancies already belongs to Jane, thus Al has 3 opportunities to get one of them. Thus,

**Reserve Problems Chapter 2 Section 5 Problem 4**

Consider the data on wafers classified by contamination and location in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| Number of Contamination Particles | Center | Edge | Totals |
| 0 | 0.28 | 0.12 | 0.40 |
| 1 | 0.17 | 0.03 | 0.20 |
| 2 | 0.1 | 0.05 | 0.15 |
| 3 | 0.06 | 0.04 | 0.10 |
| 4 | 0.04 | 0.01 | 0.05 |
| 5 or more | 0.07 | 0.03 | 0.10 |
| Totals | 0.72 | 0.28 | 1.00 |

Assume that one wafer is selected at random from this set. Let *A* denote the event that a wafer contains four or more particles, and let *B* denote the event that a wafer is from the center of the sputtering tool. Determine the following probabilities.

|  |  |  |  |
| --- | --- | --- | --- |
| (a) |  | = |  |
| (b) |  | = |  |
| (c) |  | = |  |
| (d) |  | = |  |
| (e) |  | = |  |
| (f) |  | = |  |

**SOLUTION**

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

**Reserve Problems Chapter 2 Section 5 Problem 5**

A batch of 370 samples of rejuvenated mitochondria contains 6 that are mutated (or defective). Two are selected from the batch, at random, without replacement.

(a) What is the probability that the second one selected is defective given that the first one was defective?

(b) What is the probability that both are defective?

(c) What is the probability that both are acceptable?

**SOLUTION**

(a) 

(b) 

(c) 

**Reserve Problems Chapter 2 Section 5 Problem 6**

A computer system uses passwords that are exactly 5 characters and each character is one of the 26 letters (*a*–*z*) or 10 integers (0–9). You maintain a password for this computer system. Let *A* denote the subset of passwords that begin witha vowel (either *a*, *e*, *i*, *o*, or *u*) and let *B* denote the subset of passwords that end with some predetermined number (either 0, 1, 2, 3, 4, 6, 8 or 9).

(a) Suppose a hacker selects a password at random. What is the probability that your password is selected?

(b) Suppose a hacker knows that your password is in event *A* and selects a password at random from this subset. What is the probability that your password is selected?

(c) Suppose a hacker knows that your password is in *A* and *B* and selects a password at random from this subset. What is the probability that your password is selected?

**SOLUTION**

(a) 

(b) 

(c) 

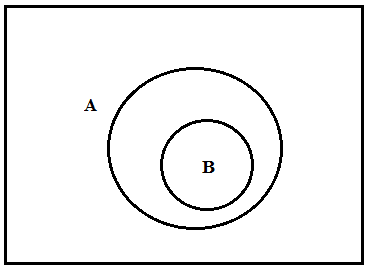
**Reserve Problems Chapter 2 Section 5 Problem 7**

If , must ? Choose a correct Venn diagram to explain your answer.

|  |  |
| --- | --- |
| A | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\img1496837108375_01981653128444738.png |
| B | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\img1496837108435_8469133366737387.png |
| C | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\img1496837108445_03865336381926043.png |
| D | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\img1496837108445_9932651646969254.png |
| E | D:\work\07. july\3. Montgomery docx (1d)\doc\art\qu\ch0\img1496837108455_14750709149567243.png |

**SOLUTION**

No, if , then 



**Reserve Problems Chapter 2 Section 5 Problem 8**

Consider the well failure data in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | Wells |  |
| Geological Formation Group |  | Failed |  | Total |
| Gneiss |  | 130 |  | 1685 |
| Granite |  | 2 |  | 28 |
| Loch raven schist |  | 443 |  | 3733 |
| Mafic |  | 14 |  | 363 |
| Marble |  | 32 |  | 309 |
| Prettyboy schist |  | 60 |  | 1403 |
| Other schists |  | 46 |  | 933 |
| Serpentine |  | 3 |  | 39 |

Let *A* denote the event that the geological formation of a well has more than 1000 wells, let *B* denote the event that a well failed, and let *C* denote the event that the geological formation of a well has fewer than 500 wells.

(a) What is the probability of a failure given there are more than 1,000 wells in a geological formation?

(b) What is the probability of a failure given there are fewer than 500 wells in a geological formation?

**SOLUTION**

(a) 

(b) 

**Reserve Problems Chapter 2 Section 5 Problem 9**

Code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6).

Suppose that all 40 codes are equally likely (none is held back as a delimiter). Determine the probability for each of the following.

Enter your answers as proper fractions.

(a) The second bar is wide given that the first bar is wide.

(b) The third bar is wide given that the first two bars are not wide.

(c) The first bar is wide given that the last bar is wide.

**SOLUTION**  
  
(a)

*A* = permutations with first bar wide, *B* = permutations with second bar wide



There are 5 bars and 2 are wide. Number of permutations of the bars with 2 wide and 3 narrow bars is 

Number of permutations of the 4 spaces is 

Number of permutations of the bars with the first bar wide is . Spaces are handled as previously. Therefore, 

Number of permutations of the bars with the first and second bar wide is 1. Spaces are handled as previously. Therefore, 

Therefore, 

Or one can use the fact that if the first bar is wide there are 4 other equally likely positions for the wide bar. Therefore, 

(b)

*A* = first two bars not wide, *B* = third bar wide



Number of permutations of the bars with the first two bars not wide is . Spaces are handled as in part (a).

Therefore, 

Number of permutations of the bars with the first two bars not wide and the third bar wide is 2. Spaces are handled as in part (a). Therefore, 

Therefore, 

Or one can use the fact that if the first two bars are not wide, there are only 3 equally likely positions for the 2 wide bars and 2 of these positions result in a wide bar in the third position. Therefore, 

(c)

*A* = first bar wide, *B* = last bar wide



Number of permutations of the bars with last bar wide is . Spaces are handled as in part (a). Therefore, 

Number of permutations of the bar with the first and last bar wide is 1. Spaces are handled as in part (a). Therefore,  and 

Or one can use the fact that if the last bar is wide there are 4 other equally likely positions for the wide bar. Therefore, 

**Reserve Problems Chapter 2 Section 5 Problem 10**

During a clinical trial the effect of two treatments and a control for treatment of hepatitis C were considered. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Complete Response** | **Total** |
| Ribavirin plus interferon alfa |  | 13 | 24 |
| Interferon alfa |  | 9 | 16 |
| Untreated controls |  | 0 | 20 |

Suppose that a patient is selected at random from this set. Let *A* denote the event that the patient is treated with ribavirin plus interferon alfa, and let *B* denote the event that the response is complete. Determine the following probabilities.

|  |  |  |  |
| --- | --- | --- | --- |
| (a) |  | = |  |
| (b) |  | = |  |
| (c) |  | = |  |
| (d) |  | = |  |

**SOLUTION**  
  
, , 

(a) 

(b) 

(c) 

(d) 

**Reserve Problems Chapter 2 Section 6 Problem 1**

Accidents on highways are one of the main causes of death or injury in developing countries and the weather conditions have an impact on the rates of death and injury. In foggy, rainy, and sunny conditions, 1/6, 1/10, and 1/25 of the accidents result in death, respectively. Sunny conditions occur 62% of the time, while rainy and foggy conditions each occur 19% of the time.

What is the probability that an accident results in a death?

**SOLUTION**  
  
Let *D* denote that accident result in a death, *S* denote sunny conditions, *R* denote rainy conditions, and *F* denote foggy conditions. *S*, *R*, *F* are mutually exclusive and exhaustive sets:  
  
  
  
With the total probability rule,

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

**Reserve Problems Chapter 2 Section 6 Problem 2**

A committee will be formed with 2 managers and 4 engineers selected randomly without replacement from 10 managers and 24 engineers.

What is the probability that engineer Al and engineer Jane are on the committee?

**SOLUTION**  
Let *A* and *B* denote the events that engineer Al and engineer Jane are on the committee, respectively.  
  


**Reserve Problems Chapter 2 Section 6 Problem 3**

An article in the *Transportation Research Part E Journal* ["Arc Routing Problems to Restore Connectivity of a Road Network" (2016)] considered ways of re-establishing the connectivity of road networks after a natural disaster — earthquake. Estimates of the probabilities of a randomly chosen road being under light debris, moderate debris, and heavy debris conditions after different disaster magnitudes are shown in the following table. Disaster magnitude is equally likely to be low, moderate or high.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Disaster  Magnitude |  | Light  Debris | Moderate  Debris | Heavy  Debris | Total |
| Low |  | 80 | 15 | 5 | 100 |
| Moderate |  | 50 | 40 | 10 | 100 |
| High |  | 30 | 50 | 20 | 100 |
| Total |  | 160 | 105 | 35 | 300 |

(a) What is the probability that a randomly selected road is under heavy debris after an earthquake?

(b) What is the probability that a randomly selected road is under light or moderate debris after an earthquake?

**SOLUTION**  
  
(a) Let *L* denote low-magnitude disaster, *M* denote moderate-magnitude disaster and *H* denote high-magnitude disaster.  
  
Let *LD* denote light debris, *MD* denote moderate debris and *HD* denote heavy debris.  
  
Because *L*, *M*, *H* are mutually exclusive and exhaustive sets,  
  
  
  
  
  


(b) Because ,  
  
  
  
  
  


**Reserve Problems Chapter 2 Section 6 Problem 4**

Consider the endothermic reactions in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| Final Temperature Conditions |  | Heat Absorbed (cal) |  |
|  |  | Below Target | Above Target |
| 266 K |  | 19 | 40 |
| 271 K |  | 44 | 11 |
| 274 K |  | 56 | 36 |

Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is above target. Determine the following probabilities.

(a) 

(b) 

(c)    
(d) Use the total probability rule to determine .

**SOLUTION**  
  
, 

(a) 

(b) 

(c) 

(d) 

**Reserve Problems Chapter 2 Section 6 Problem 5**

The following table summarizes visits to emergency departments at four hospitals in Arizona. People may leave without being seen by a physician, and those visits are denoted as LWBS. The remaining visits are serviced at the emergency department, and the visitor may or may not be admitted for a stay in the hospital.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | Hospital |  |  |  |
|  |  | **1** | **2** | **3** | **4** | **Total** |
|  |  |  |  |  |  |  |
| Total |  | 5292 | 6991 | 5640 | 4329 | 22,252 |
| LWBS |  | 235 | 230 | 226 | 262 | 953 |
| Admitted |  | 1277 | 1558 | 666 | 984 | 4485 |
| Not admitted |  | 3820 | 5163 | 4728 | 3103 | 16,814 |

Suppose that three visits that resulted in LWBS are selected randomly (without replacement) for a follow-up interview.

(a) What is the probability that all three are selected from hospital 2?

(b) What is the probability that all three are from the same hospital?

**SOLUTION**  
  
(a)



(b)



**Reserve Problems Chapter 2 Section 6 Problem 6**

Consider the well failure data in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | Wells |  |
| Geological Formation Group |  | Failed |  | Total |
| Gneiss |  | 170 |  | 2885 |
| Granite |  | 2 |  | 28 |
| Loch raven schist |  | 383 |  | 3733 |
| Mafic |  | 14 |  | 363 |
| Marble |  | 29 |  | 309 |
| Prettyboy schist |  | 60 |  | 1403 |
| Other schists |  | 46 |  | 933 |
| Serpentine |  | 3 |  | 39 |

Let *A* denote the event that the geological formation of a well has more than 1000 wells, and let *B* denote the event that a well failed. Determine the following probabilities.

(a) 

(b) 

(c) 

(d) Use the total probability rule to determine .

**SOLUTION**  
  
, 

(a) 

(b) 

(c) 

(d) 

**Reserve Problems Chapter 2 Section 6 Problem 7**

Consider the well failure data in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | Wells |  |
| Geological Formation Group |  | Failed |  | Total |
| Gneiss |  | 160 |  | 1685 |
| Granite |  | 2 |  | 28 |
| Loch raven schist |  | 453 |  | 3733 |
| Mafic |  | 14 |  | 363 |
| Marble |  | 29 |  | 309 |
| Prettyboy schist |  | 55 |  | 1403 |
| Other schists |  | 51 |  | 933 |
| Serpentine |  | 3 |  | 39 |

Suppose that two failed wells are selected randomly (without replacement) for a follow-up review.

(a) What is the probability that both are from the gneiss geological formation group?

(b) What is the probability that both are from the same geological formation group?

**SOLUTION**  
(a)



(b)

|  |  |  |
| --- | --- | --- |
| *P* | = |  |
|  | = | 0.4029 |

**Reserve Problems Chapter 2 Section 6 Problem 8**

Code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6).

Suppose that all 40 codes are equally likely (none is held back as a delimiter). Determine the probability for each of the following:

(a) The code starts and ends with a wide bar.

(b) Two wide bars occur consecutively.

(c) Two consecutive wide bars occur at the start or end.

(d) The middle bar is wide.

**SOLUTION**  
  
(a)   
Number of permutations of the bars that start and end with a wide bar is 1. Number of permutations of the spaces is 4!/(1!3!) = 4.

Number of codes that start and end with a wide bar is 4. The probability that code starts and ends with a wide bar is 4/40 = 0.1

(b)

Number of permutations of the bars where two wide bars are consecutive is 4. Spaces are handled as in part (a). The probability that two wide bars occur consecutively is 16/40 = 0.4

(c)

Number of permutations of the bars with two consecutive wide bars at the start or end is 2. Spaces are handled as in part (a). The probability that two consecutive wide bars occur at the start or end is 8/40 = 0.2

(d)

Number of permutations of the bars with the middle bar wide is 4. Spaces are handled as in part (a). The probability that the middle bar is wide equals 16/40 = 0.4

**Reserve Problems Chapter 2 Section 6 Problem 9**

A computer system uses passwords that contain exactly5 characters, and each character is one of the 26 lowercase letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let **Ω** denote the set of all possible passwords, and let *A* and *B* denote the events that consist of passwords with only letters or only integers, respectively.

Suppose that all passwords in **Ω** are equally likely. Determine the following probabilities.

(a) 

(b)    
(c) The probability that password contains exactly 2 integers given that it contains at least 1 integer. 

**SOLUTION**  
  
(a) 

, 



(b) 

From part (a),  and 

Therefore, 

This can also be solved as  because *A* and *B* are mutually exclusive.

(c) Let *C* denote the event that consists of passwords with exactly 2 integers, and let *D* denote the event that consists of passwords with at least one integer.



Number of positions for the integers is 

Number of ordering of the two integers is 

Number of orderings of the letters for passwords with exactly 2 integers is 

Total number of orderings is 

Therefore, .





**Reserve Problems Chapter 2 Section 7 Problem 1**

A cell phone user selects apps to download. Each of 10 apps is independently selected with probability 0.21  
  
(a) If each of the first 9 apps are downloaded, what is the probability that the last app is downloaded?   
  
(b) What is the probability that the cell phone user downloads at least 3 apps?

(c) What is the probability that app 1 or 2 is downloaded?

**SOLUTION**  
  
(a) Let *A* denote that the first 9 apps are downloaded, *B* denote that the last app is downloaded. Because event *B* and event *A* are independent,  
  
  
  
(b) The complementary event for this question is that the cell phone user downloads no apps, or 1 app, or 2 apps. Let *X* denote the number of apps that are downloaded.  
  
Since , , ,  
  
  
  
  
  
  
  
(c) Let *C* denote that app 1 is downloaded and *D* denote that app 2 is downloaded. Because *C* and *D* are independent,   
  
  


**Reserve Problems Chapter 2 Section 7 Problem 2**

An article in the *Transportation Research Part E Journal* ["Arc Routing Problems to Restore Connectivity of a Road Network" (2016)] considered ways of re-establishing the connectivity of road networks after a natural disaster — earthquake. Estimates of the probabilities of a randomly chosen road being under light debris, moderate debris, and heavy debris conditions after different disaster magnitudes are shown in the following table. Disaster magnitude is equally likely to be low, moderate or high.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Disaster  Magnitude |  | Light  Debris | Moderate  Debris | Heavy  Debris | Total |
| Low |  | 80 | 15 | 5 | 100 |
| Moderate |  | 50 | 40 | 10 | 100 |
| High |  | 30 | 50 | 20 | 100 |
| Total |  | 160 | 105 | 35 | 300 |

Let *A* and *B* denote the events that the earthquake magnitude is low and the road debris is heavy-. Are these events independent?

**SOLUTION**  
  
Events *A* and *B* are independent if .  
  
Check for the given events:  
  
  
  
  
  
, hence, events *A* and *B* are not independent.

**Reserve Problems Chapter 2 Section 7 Problem 3**

Suppose that a 9% discount is independently applied to Web orders for clothing with probability 0.150.

(a) What is the probability that the first discount is applied to the fifth order?

(b) What is the probability that at least one order in the next five receives the discount?

**SOLUTION**

(a) Let *N* denote that the discount is not applied to Web order, and *Y* denote that the discount is applied to Web order,  
  
  
  
Because the discount is independently applied to Web order,  
  
  
  
(b) The complementary event is that no order receives discount (denote by *C*),   
  


**Reserve Problems Chapter 2 Section 7 Problem 4**

Samples of emissions from three suppliers are classified for conformance to air-quality specifications.  
The results from 100 samples are summarized as follows:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Conforms | |
|  |  | Yes | No |
| Supplier | 1 | 21 | 9 |
| 2 | 25 | 5 |
| 3 | 30 | 10 |

Let *A* denote the event that a sample is from supplier 1, and let *B* denote the event that a sample conforms to specifications.

(a) Are *A* and *B* independent events?

(b) Determine *.*

**SOLUTION**  
(a)

,  , , then  Therefore, *A* and *B* are not independent.

(b)



**Reserve Problems Chapter 2 Section 7 Problem 5**

Six tissues are extracted from an ivy plant infested by spider mites. The plant in infested in 22% of its area. Each tissue is chosen from a randomly selected area on the ivy plant.

(a) What is the probability that there are exact four samples showing the signs of infestation and they are successive?

(b) What is the probability that there are exact three samples showing the signs of infestation and they are three out of four successive?

**SOLUTION**

(a)

Let *I* and *G* denote an infested and good sample. There are 3 ways to obtain four consecutive samples showing the signs of the infestation: *IIIIGG*, *GIIIIG*, *GGIIII*. Therefore, the probability is 

(b)

There are 10 ways to obtain three out of four consecutive samples showing the signs of infestation. The probability is 

**Reserve Problems Chapter 2 Section 7 Problem 6**

In an acid-base titration, a base or acid is gradually added to the other until they have completely neutralized each other. Because acids and bases are usually colorless (as are the water and salt produced in the neutralization reaction), pH is measured to monitor the reaction. Suppose that the equivalence point is reached after approximately 100 mL of an NaOH solution has been added (enough to react with all the acetic acid present) but that replicates are equally likely to indicate from 95 to 104 mL, measured to the nearest mL. Assume that two technicians each conduct titrations independently.  
  
(a) What is the probability that both technicians obtain equivalence at 101 mL?

(b) What is the probability that both technicians obtain equivalence between 98 and 102 mL (inclusive)?

(c) What is the probability that the average volume at equivalence from the technicians is 100 mL?

**SOLUTION**  
(a)

The probability that one technician obtains equivalence at 101 mL is 0.1010. So the probability that both technicians obtain equivalence at 100 mL is .

(b)

The probability that one technician obtains equivalence between 98 and 102 mL is 0.5. So the probability that both technicians obtain equivalence between 98 and 102 mL is .

(c)

The probability that the average volume at equivalence from the technician is 100 mL is .

**Reserve Problems Chapter 2 Section 7 Problem 7**

Consider the endothermic reactions given below. Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is above target.

|  |  |  |
| --- | --- | --- |
| Final Temperature Conditions | Heat Absorbed (cal) | |
|  | Below Target | Above Target |
| 266K | 11 | 41 |
| 271K | 44 | 16 |
| 274K | 56 | 36 |

Determine   
  
Determine   
  
Determine 

Are *A* and *B* independent events?

**SOLUTION**

Determine 



Determine 



Determine 



Are *A* and *B* independent events?

Because , *A* and *B* are not independent.

**Reserve Problems Chapter 2 Section 7 Problem 8**

Consider the well failure data given below. Let *A* denote the event that the geological formation of a well has more than 1000 wells, and let *B* denote the event that a well failed.

|  |  |  |
| --- | --- | --- |
|  | Wells | |
| Geological Formation Group | Failed | Total |
| Gneiss | 180 | 1985 |
| Granite | 2 | 28 |
| Loch raven schist | 443 | 3733 |
| Mafic | 14 | 363 |
| Marble | 29 | 309 |
| Prettyboy schist | 60 | 1403 |
| Other schists | 46 | 933 |
| Serpentine | 3 | 39 |

Determine   
  
Determine   
  
Determine 

Are *A* and *B* independent events?

**SOLUTION**

Determine 



Determine 



Determine 



Are *A* and *B* independent events?

Because , *A* and *B* are not independent.

**Reserve Problems Chapter 2 Section 7 Problem 9**

The code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). Suppose that all 40 codes are equally likely (none is held back as a delimiter).   
  
Let *A* denote the event that the first bar is wide and *B* denote the event that the second bar is wide.

Determine the following.  
  
Determine   
  
Determine    
  
Determine 

Are *A* and *B* independent events?

**SOLUTION**

(a) 

The total number of permutations of 2 wide and 3 narrow bars is . The number of permutations that begin with a wide bar is . Therefore, .

(b) 

A similar approach to that used in part (a) implies 

(c) 

Because a code contains exactly 2 wide bars, there is only 1 permutation with wide bars in the first and second positions. Therefore, .

(d) Are *A* and *B* independent events?

Because , the events are not independent.

**Reserve Problems Chapter 2 Section 7 Problem 10**

Consider the wafers categorized by location and contamination levels, so that the location is in the center or on the edge, and the contamination is low or high. Let the number of wafers with *low* contamination from the *center* and *edge* locations be denoted as and , respectively. Similarly, let and denote the number of wafers with *high* contamination from the *center* and *edge* locations, respectively. Suppose that and . That is, there are 12 times as many *low* contamination wafers as *high* ones from each location. Let *A* denote the event that contamination is low, and let *B* denote the event that the location is *center.*

Are *A* and *B* independent?

Does your conclusion change if the multiplier of 12(between *low* and *high* contamination wafers) is changed from 12 to another positive integer?

**SOLUTION**

Are *A* and *B* independent?

nlc=10nhc; nle=10nhe

|  |  |  |
| --- | --- | --- |
|  | center | edge |
| low | 10hc | 10nhe |
| high | nhc | nhe |

.





Because  the events are independent.

Does your conclusion change if the multiplier of 12 (between *low* and *high* contamination wafers) is changed from 12 to another positive integer?

The conclusion does not change. Even though the multiplier is changed, this relation does not change.

**Reserve Problems Chapter 2 Section 8 Problem 1**

An article in the *Transportation Research Part E Journal* ["Arc Routing Problems to Restore Connectivity of a Road Network" (2016)] considered ways of re-establishing the connectivity of road networks after a natural disaster — earthquake. Estimates of the probabilities of a randomly chosen road being under light debris, moderate debris, and heavy debris conditions after different disaster magnitudes are shown in the following table. Disaster magnitude is equally likely to be low, moderate or high.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Disaster  Magnitude |  | Light  Debris | Moderate  Debris | Heavy  Debris | Total |
| Low |  | 80 | 15 | 5 | 100 |
| Moderate |  | 50 | 40 | 10 | 100 |
| High |  | 30 | 50 | 20 | 100 |
| Total |  | 160 | 105 | 35 | 300 |

Given that a road had heavy debris after an earthquake, what is the conditional probability that the disaster magnitude was high?

**SOLUTION**  
  
Let H denote high-magnitude disaster, and let  denote moderate debris. Because , based on Bayes' Theorem,  
  


**Reserve Problems Chapter 2 Section 8 Problem 2**

Accidents on highways are one of the main causes of death or injury in developing countries and the weather conditions have an impact on the rates of death and injury. In foggy, rainy, and sunny conditions, 1/4, 1/8, and 1/21 of the accidents result in death, respectively. Sunny conditions occur 60% of the time, while rainy and foggy conditions each occur 20% of the time.

Given that an accident without deaths occurred, what is the conditional probability that it was foggy at the time?

**SOLUTION**

Let *D* denote that accident result in a death, *ND* denote that accident does not result in a death, and let *F* denote the foggy conditions.  
  
,  
  
,   
  
  
  
Based on Bayes' Theorem,  
  


**Question Title: Reserve Problems Chapter 2 Section 8 Problem 3**

Consider the endothermic reactions given below.

|  |  |  |
| --- | --- | --- |
|  | Heat Absorbed (cal) | |
| Final Temperature Conditions | Below Target | Above Target |
| 266K | 13 | 39 |
| 271K | 44 | 16 |
| 274K | 56 | 36 |

Use Bayes’ theorem to calculate the probability that a reaction's final temperature is 271 K or less given that the heat absorbed is above target.

**SOLUTION**





**Reserve Problems Chapter 2 Section 8 Problem 4**

Consider the well failure data given below.

|  |  |  |
| --- | --- | --- |
|  | Wells | |
| Geological Formation Group | Failed | Total |
| Gneiss | 170 | 1485 |
| Granite | 2 | 28 |
| Loch raven schist | 443 | 3733 |
| Mafic | 14 | 363 |
| Marble | 29 | 309 |
| Prettyboy schist | 60 | 1403 |
| Other schists | 46 | 933 |
| Serpentine | 3 | 39 |

Use Bayes’ theorem to calculate the probability that a randomly selected well is in the gneiss group given that the well has failed.

**SOLUTION**

Let *A* denote the event that a well is failed   
Let  denote the event that a well is in Gneiss   
Let denote the event that a well is in Granite   
Let  denote the event that a well is in Loch raven schist   
Let  denote the event that a well is in Mafic   
Let  denote the event that a well is in Marble   
Let  denote the event that a well is in Prettyboy schist   
Let  denote the event that a well is in Other schist   
Let  denote the event that a well is in Serpentine  


****

**Reserve Problems Chapter 2 Section 8 Problem 5**

The probabilities of poor print quality given no printer problem, misaligned paper, high ink viscosity, or printer-head debris are 0, 0.3, 0.4, and 0.6, respectively. The probabilities of no printer problem, misaligned paper, high ink viscosity, or printer-head debris are 0.8, 0.02, 0.08, and 0.1, respectively.

Determine the probability of high ink viscosity given poor print quality.  
  
Given poor print quality, what problem is most likely?

**SOLUTION**

(a)

*NP* = no problem; *PP* = poor print; *MP* = misaligned paper; *HV* = high ink viscosity; *HD* = print head debris  
  
*P*(*MP*) = 0.02; *P*(*HV*) = 0.08; *P*(*HD*) = 0.1; *P*(*NP*) = 0.8; *P*(*PP*|*NP*) = 0; *P*(*PP*|*MP*) = 0.3; *P*(*PP*|*HV*) = 0.4; *P*(*PP*|*HD*) = 0.6;   
  
  
  
  
Therefore, 

(b)

   
   
   
   
The problem most likely given poor print quality is head debris.

**Reserve Problems Chapter 2 Section 9 Problem 1**

Decide whether a discrete or continuous random variable is the best model for each of the following variables:

a) The time a customer spends at a grocery store is \_\_\_\_\_.

b) The number of arrivals to a grocery store on a given day is \_\_\_\_\_.

c) The number of items each customer purchases at a grocery store is \_\_\_\_\_.

d) The weekly revenue of a grocery store is \_\_\_\_\_.

**SOLUTION**

a) Continuous.  
b) Discrete.  
c) Discrete.  
d) Continuous.

**Question Title: Reserve Supplemental Exercises Chapter 2 Problem 1**

Decide whether a discrete or continuous random variable is the best model for each of the following variables:

a) The duration of a call at a mobile phone customer care service is \_\_\_\_\_.

b) The number of busy representatives at a mobile phone customer care service at a time *t* is \_\_\_\_\_.

c) The number of calls made to mobile phone customer care service on a given day is \_\_\_\_\_.

**SOLUTION**

a) Continuous.  
b) Discrete.  
c) Discrete.

**Reserve Supplemental Exercises Chapter 2 Problem 2**

An article in the *Transportation Research Part E Journal* ["Arc Routing Problems to Restore Connectivity of a Road Network" (2016)] considered ways of re-establishing the connectivity of road networks after a natural disaster — earthquake. Estimates of the probabilities of a randomly chosen road being under light debris, moderate debris, and heavy debris conditions after different disaster magnitudes are shown in the following table. Disaster magnitude is equally likely to be low, moderate or high.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Disaster  Magnitude |  | Light  Debris | Moderate  Debris | Heavy  Debris | Total |
| Low |  | 80 | 15 | 5 | 100 |
| Moderate |  | 50 | 40 | 10 | 100 |
| High |  | 30 | 50 | 20 | 100 |
| Total |  | 160 | 105 | 35 | 300 |

What is the probability that a randomly selected road is under heavy debris given that the earthquake is either moderate or high?

**SOLUTION**  
  
Let *M* denote moderate-magnitude disaster and *H* denote high-magnitude disaster, let  denote light debris.  
  
  
  
  
  
We can conclude that . Thus,  and .  
  
  
  


**Reserve Supplemental Exercises Chapter 2 Problem 3**

Customers who are registered on a corporate Web site are summarized by the type of shipping contract they use and the number of orders in the previous month. The number of customers in each category are shown in the following table. Suppose that four customers are selected randomly, without replacement, for a survey.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Shipping Contract |  | Express | Standard | Total |
| No orders |  | 30 | 18 | 48 |
| One order |  | 73 | 50 | 123 |
| More than one order |  | 40 | 28 | 68 |

a) What is the probability that three selected customers use express shipping and the other one uses standard shipping?

b) What is the conditional probability that no selected customers ordered in the previous month given that all selected customers use express shipping?

**SOLUTION**  
a) Let *C* denote that three selected customers use express shipping and the other one uses standard shipping.  
  


b) Let *A* denote that no selected customers ordered in the previous month and let *B* denote that all selected customers use express shipping.  
  


**Reserve Supplemental Exercises Chapter 2 Problem 4**

A conceptual smartphone design uses a parachute technique to avoid severe damage to the screen and phone-case. Each of four nozzles located on the case might be either functional or defective after a year. The probability that a year-old nozzle is functional is 0.66and assume that the nozzles fail independently. If at least two nozzles are functional, the phone will not be damaged in a drop. Otherwise, it will necessarily be damaged in a drop.

a) What is the probability that a year-old phone is not damaged in a drop?

b) What is the probability that all four nozzles of a year-old phone are functional?

c) What is the conditional probability that all four nozzles are functional given that a year-old phone is not damaged in a drop?

**SOLUTION**  
Let *X* denote the number of functional nozzles after a year.

a) The probability that a year-old phone is not damaged in a drop is the probability that at least two nozzles are functional after a year. Therefore,  
  
  
  
  
  

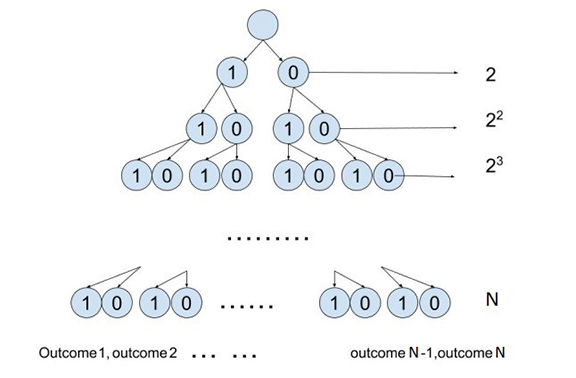

b) 

c) Let *A* denote that all nozzles are functional and let *B* denote that a year-old phone is not damaged in a drop.  
  
Since   
  


**Reserve Supplemental Exercises Chapter 2 Problem 5**

A cell phone user selects apps to download. Each of 5 apps is independently selected with probability 0.2. Describe the sample space for the app downloads and determine the number of outcomes in the sample space.

**SOLUTION**

Let 1 denote that an app is downloaded, and let 0 denote that an app is not downloaded. For each app, the sample space of whether an app is downloaded or not is .  
  
  
  
As shown above, the number of possible outcomes is , and .  
  
If we use binary vectors to represent the sample space, that is  
  


**Reserve Supplemental Exercises Chapter 2 Problem 6**

A committee will be formed with4 managers and 6 engineers selected randomly without replacement from 13 managers and 20 engineers.   
  
Consider the two events that engineer Jane and manager Mary are selected for the committee. Are these events independent?

**SOLUTION**

Let *A* and *B* denote the events that engineer Jane and manager Mary, respectively, are selected for the committee.  
  
The number of all possible committees is . The number of committees with Mary is , the number of committees with Jane is , and the number of committees with both Mary and Jane is . Therefore, we can see that  
  
  
  
  
  
  
  
Therefore, , so these two events are independent.

**Reserve Supplemental Exercises Chapter 2 Problem 7**

Customers specify delivery dates for orders. Suppose that 14 customers each independently, randomly select delivery days over the next year (365 days).

(a) What is the probability that none select the same day?

(b) What is the probability that none select the same week? Assume exactly 52 weeks in a year.

**SOLUTION**

(a) We will ignore leap years and assume all days are equally likely. The first customer will definitely have a unique day, so the probability that there are no same days is 1. The probability that the second customer’s day will not be the same as the first customer’s day is 364/365. The third customer will have a probability 363/365 and so on. Multiplying these together to get the overall probability, we have  
  


(b) The probability that none select the same week is   
  


**Reserve Supplemental Exercises Chapter 2 Problem 8**

You remove four fuses of 10, 20, 20, and 30 amperes each, but you do not mark the corresponding circuits.   
If you insert the fuses so that each sequence is equally likely, what is the probability that the appropriate amperage fuse is assigned to all the circuits?

**SOLUTION**

Consider four spots that need 10 amps, 20 amps, 20 amps, 30 amps fuses, respectively.   
  
If fuses are randomly assigned to each spot, there are 4×3×2×1 = 24 ways to do so.   
If we required that appropriate amperage fuse is assigned to all circuits, there are 1×2×1 = 2 ways to do so.  
  
Therefore, the probability that the appropriate amperage fuse is assigned to all circuits is  
  


**Reserve Supplemental Exercises Chapter 2 Problem 9**

Consider the hospital emergency room data from the table. Let *A* denote the event that a visit is to hospital 1 and let *B* denote the event that a patient is admitted to hospital 1.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Hospital |  | 1 | 2 | 3 | 4 | Total |
| Total |  | 5250 | 6877 | 5658 | 4301 | 22086 |
| LWBS |  | 187 | 255 | 249 | 246 | 937 |
| Admitted |  | 1239 | 1513 | 682 | 940 | 4374 |
| Not admitted |  | 3824 | 5109 | 4727 | 3115 | 16775 |

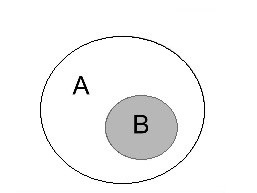
Determine the following probabilities.

a) 

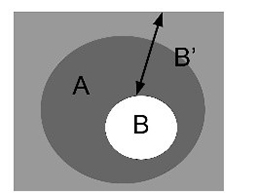
b) 

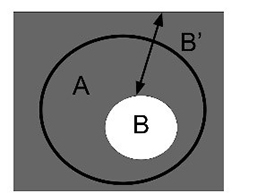
c) 

d) 

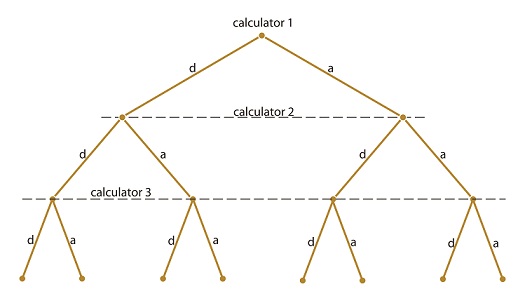
**SOLUTION**  
  
a) Because ,   
  
  
  


b) 

c) Because ,   
  
  
  


d) Because ,   
  


**Reserve Supplemental Exercises Chapter 2 Problem 10**

A sample of three calculators is selected from a manufacturing line, and each calculator is classified as either defective or acceptable. Let *A*, *B*, and *C* denote the events that the first, second, and third calculators, respectively, are defective. Sample space for this experiment could be described with a tree diagram. Let "d" denote a defective calculator and let "a" denote an acceptable calculator.  
  


(a) Use the tree diagram to describe event *A*

(b) Use the tree diagram to describe event *B*

(c) Use the tree diagram to describe event 

(d) Use the tree diagram to describe event 

**SOLUTION**  
(a) Use the tree diagram to describe event *A:*



(b) Use the tree diagram to describe event *B:*



(c) Use the tree diagram to describe event *:*



(d) Use the tree diagram to describe event :



**Reserve Supplemental Exercises Chapter 2 Problem 11**

Shafts are classified in terms of the machine tool that was used for manufacturing the shaft and conformance to surface finish and roundness.

|  |  |  |  |
| --- | --- | --- | --- |
| Tool 1 |  | Roundness Conforms | |
|  |  | Yes | No |
| Surface Finish | Yes | 197 | 4 |
| Conforms | No | 4 | 2 |

|  |  |  |  |
| --- | --- | --- | --- |
| Tool 2 |  | Roundness Conforms | |
|  |  | Yes | No |
| Surface Finish | Yes | 145 | 4 |
| Conforms | No | 10 | 4 |

(a) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or to roundness requirements or is from tool 1?

(b) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or does not conform to roundness requirements or is from tool 2?

(c) If a shaft is selected at random, what is the probability that the shaft conforms to both surface finish and roundness requirements or the shaft is from tool 2?

(d) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or the shaft is from tool 2?

**SOLUTION**

(a)   
(197 + 4 + 4 + 2 + 145 + 4 + 10)/370 = (370 - 4)/370 = 0.9892  
  
(b)   
366/370 = 0.989   
  
(c)   
(370—4-4-2)/370 = 0.9730   
  
(d)   
364/370 = 0.984

**Reserve Supplemental Exercises Chapter 2 Problem 12**

The data from 200 machined parts are summarized as follows:

|  |  |  |
| --- | --- | --- |
|  | Depth of Bore | |
| Edge Condition | Above Target | Below Target | |
| Coarse | 15 | 10 | |
| Moderate | 24 | 21 | |
| Smooth | 48 | 82 | |

(a) What is the probability that a part selected has a moderate edge condition and a below-target bore depth?

(b) What is the probability that a part selected has a moderate edge condition or a below-target bore depth?

(c) What is the probability that a part selected does not have a moderate edge condition or does not have a below-target bore depth?

**SOLUTION**

(a)



(b)



(c)



**Reserve Supplemental Exercises Chapter 2 Problem 13**

An e-mail message can travel through one of two server routes. The probability of transmission error in each of the servers and the proportion of messages that travel each route are shown in the following table. Assume that the servers are independent.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | **Probability of Error** | | | |
|  | Percentage of Messages | Server 1 | Server 2 | Server 3 | Server 4 |
| Route 1 | 55 | 0.01 | 0.015 | — | — |
| Route 2 | 45 | — | — | 0.02 | 0.003 |

(a) What is the probability that a message will arrive without error?

(b) If a message arrives in error, what is the probability it was sent through route 1?

**SOLUTION**

(a)

(0.55)(0.99)(0.985) + (0.45)(0.98)(0.997) = 0.9760

(b)



**Reserve Supplemental Exercises Chapter 2 Problem 14**

A lot of 50 spacing washers contains 30 washers that are thicker than the target dimension. Washers are selected from the lot at random without replacement.

(a) What is the minimum number of washers that need to be selected so that the probability that all the washers are thinner than the target is less than 0.10?

(b) What is the minimum number of washers that need to be selected so that the probability that 1 or more washers are thicker than the target is at least 0.90?

**SOLUTION**

(a)

If n washers are selected, then the probability they are all less than the target is 

|  |  |
| --- | --- |
| n | probability all selected washers are less than target |
| 1 | 20/50=0.4 |
| 2 | (20/50)(19/49) = 0.155 |
| 3 | (20/50)(19/49)(18/48)=0.058 |

Therefore, the answer is n = 3.

(b)

Then event *E* that one or more washers is thicker than target is the complement of the event that all are less than target. Therefore, *P*(*E*) equals one minus the probability in part a. Therefore, n = 3.

**Reserve Supplemental Exercises Chapter 2 Problem 15**

Natural red hair consists of two genes. People with red hair have two dominant genes, two regressive genes, or one dominant and one regressive gene. A group of 1000 people was categorized as follows:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Gene 2** | | |
| Gene 1 | Dominant | Regressive | Other |
| Dominant | 5 | 25 | 30 |
| Regressive | 7 | 63 | 35 |
| Other | 20 | 15 | 800 |

Let *A* denote the event that a person has a dominant red hair gene, and let *B* denote the event that a person has a regressive red hair gene. If a person is selected at random from this group, compute the following:

(a) 

(b) 

(c) 

(d) 

(e) 

(f) Probability that the selected person has red hair.

**SOLUTION**

(a) 

(b) 

(c) 

(d) 

(e) 

(f) Probability that the selected person has red hair.  
  
