Chapter 1

EXERCISE 1.1

1. a) $S = 2500\left[1 + (0.12)\left(\frac{18}{12}\right)\right] = \$2950$

   b) $S = 1200\left[1 + (0.085)\left(\frac{120}{360}\right)\right] = \$1234$

   c) $S = 10\ \ 000\left[1 + (0.07)\left(\frac{64}{365}\right)\right] = \$10\ \ 122.74$

   d) $S = 500\left[1 + (0.06)\left(\frac{175}{365}\right)\right] = \$5143.84$

2. a) $r = 420/(1000 \times 2.5) = 16.8\%$

   b) $r = 1/(1 \times 7)\ = 14.29\%$

   c) $r = 10/\left(500 \times \frac{2}{12}\right) = 12\%$

   d) $r = 50/\left(800 \times \frac{214}{265}\right) = 10.66\%$

3. a) $t = \frac{200}{1000(0.055)} = 3.636\ \ years\ \doteq 1328\ \ days$

   b) $t = \frac{30}{1600(0.035)} = 0.535714\ \ years\ \doteq 196\ \ days$

   c) $t = \frac{100}{5000(0.09)} = 0.222222\ \ years\ \doteq 82\ \ days$

4. $Ordinary\ \ interest = 5000\ \times 0.105\ \times \frac{90}{360} = \$131.25$

   $Exact\ \ interest = 5000\ \times 0.105 \times \frac{90}{365} = \$129.45$

5. $r = 2.50/\left(50 \times \frac{1}{12}\right)\ = 0.60 = 60\%$

6. $P = \ 5100\left[1 + (0.09)\left(\frac{6}{12}\right)\right]^{-1} = \$4880.38$

7. $P = 580[1 + (0.18)\left(\frac{120}{365}\right)]^{-1} = \$547.59$

8. *At ordinary interest* $S = 1000[1 + (0.065)\left(\frac{65}{360}\right)] = \$1011.74$

At exact interest $S = 1000[1 + (0.065)\left(\frac{65}{365}\right)] = \$1011.58$

9. $S = 1000\left[1 + (0.075)\left(\frac{212}{365}\right)\right] = \$1042.11$

Number of Days = *Sept*.16(259) - *Feb*.16(47) = 212 *days*

7 *months after Feb*. 16 *is Sept*.16

10. Exact time = (365 – 138) + 98 = 325 days

Exact interest $= 2000 \times 0.045 \times \frac{325}{365} = \$80.14$

11. $r = 300 / \left(4000 \times \frac{264}{365}\right) = 10.37\%$

12.

Dates	# of days	Balance	Interest
January 1 –March 1	59	1000	$1000(0.12)\left(\frac{59}{365}\right) = 19.40$
March 1 – March 31	30	900	$900(0.12)\left(\frac{30}{365}\right) = 8.88$
			March 31 pay $28.28
March 31 –April 17	17	900	$900(0.12)\left(\frac{17}{365}\right) = 5.03$
April 17 – June 30	74	600	$600(0.12)\left(\frac{74}{365}\right) = 14.60$
			June 30 pay $19.63
June 30 – July 12	12	600	$600(0.12)\left(\frac{12}{365}\right) = 2.37$
July 12–August 20	39	400	$400(0.12)\left(\frac{39}{365}\right) = 5.13$
August 20–Sept. 30	41	300	$300(0.12)\left(\frac{41}{365}\right) = 4.04$
			Sept. 30 pay $11.54
Sept. 30-October 18	18	300	$300(0.12)\left(\frac{18}{365}\right) = 1.78$
			October 18 pay $1.78

Total interest paid = 28.28 + 19.63 + 11.54 + 1.78 = $61.23

13.

Interest Period	#of days	Balance	Rate	Interest
Feb.3-Feb.11	8	50 000	6%	$50\ 000(0.06)\left(\frac{8}{365}\right)=67.75$
				Feb. 11 Interest $65.75
Feb.11-Feb.20	9	50 000	6%	$50\ 000(0.06)(\frac{9}{365})=73.97$
Feb.20-March 11	19	40 000	6%	$40\ 000(0.06)(\frac{19}{365})=124.93$
				March 11 interest $198.90
March 11-March 20	9	40 000	6%	$40\ 000(0.06)(\frac{9}{365})=59.18$
March 20-April 1	12	30 000	6%	$30\ 000(0.06)(\frac{12}{365})=59.18$
April 1 –April 11	10	30 000	7%	$30\ 000(0.07)(\frac{10}{365})=57.53$
				April 11 interest $175.89
April 11-April 20	9	30 000	7%	$30\ 000(0.07)(\frac{9}{365})=51.78$
April 20-May 1	11	15 000	7%	$15\ 000(0.07)(\frac{11}{365})=31.64$
May 1-May11	10	15 000	6.5%	$15\ 000(0.065)(\frac{10}{365})=26.71$
				May 11 interest $110.13
May 11- May 20	9	15 000	6.5%	$15\ 000(0.065)(\frac{9}{365})=24.04$
				May 20 interest = $24.04

Total interest paid $= 65.75 + 198.90 + 175.89 + 110.13 + 24.04 =$ $574.71

14. Consider a $100 amount. Then $P = 98$, $I = 2$, $t = \frac{20}{365}$ and $r = 2/\left(98 \times \frac{20}{365}\right) = 37.24\%$

15. $r = 40/\left(1960 \times \frac{40}{365}\right) = 18.62\%$

16. $r = \frac{15}{485 \times \frac{20}{365}} = 56.44\%$

17. a) $r = \dfrac{50}{2450 \times \dfrac{45}{365}} = 16.55\,\%$

b) Interest on loan = $2450 \times 0.10 \times \dfrac{45}{365} = \$30.21$

Savings = $50 - 30.21 = \$19.79$

18. a) $S = 5000 + 5000(0.04)\dfrac{3}{12} + 5000(0.035)\dfrac{3}{12} + 5000(0.03)\dfrac{3}{12} + 5000(0.025)\dfrac{3}{12}$

$= 5000 + 162.50 = \$5162.50$

b) $S = 5000\left[1 + (0.04)\dfrac{3}{12}\right]\left[1 + (0.035)\dfrac{3}{12}\right]\left[1 + (0.03)\dfrac{3}{12}\right]\left[1 + (0.025)\dfrac{3}{12}\right] = \$5164.47$

EXERCISE 1.2

1. Maturity date is February 6
   Maturity value = \$3000.00
   Discount period = 67 days
   $$\text{Proceeds} = 3000\left[1+(0.1025)\left(\frac{67}{365}\right)\right]^{-1} = \$2944.60$$

2. Maturity date is November 3
   Maturity value = $1200[1 + (0.055)\ (\frac{63}{365})] = \$1211.39$
   Discount period = 27 days
   $$\text{Proceeds} = 1211.39\left[1+(0.0475)\left(\frac{27}{365}\right)\right]^{-1} = \$1207.15$$

3. Maturity date is June 27
   Maturity value = $500[1 + (0.07)\ (\frac{195}{365})] = \$518.70$
   Discount period = 185 days
   $$\text{Proceeds} = 518.70\left[1+(0.07)\left(\frac{185}{365}\right)\right]^{-1} = \$500.93$$

4. Maturity date is April 7
   Maturity value = \$4000.00
   Discount period = 37 days
   $$\text{Proceeds} = 4000\left[1+(0.085)\left(\frac{37}{365}\right)\right]^{-1} = \$3965.83$$

5. a) Maturity value = $2000[1 + (0.13)(\frac{93}{365})] = \$2066.25$
   $$\text{Proceeds} = 2066.25\left[1+(0.12)\left(\frac{42}{365}\right)\right]^{-1} = \$2038.11$$
   b) $r = 38.11/\left(2000\times\frac{51}{365}\right) = 13.64\%$
   c) Bank's profit = $2000\left[1+(0.13)\left(\frac{90}{365}\right)\right] - 2038.11 = \$26.00$
   $r = 26/\left(2038.11\times\frac{39}{365}\right) = 11.94\%$

6. $S = 5000\left[1+(0.085)\left(\frac{93}{365}\right)\right] = \$5108.29$
   $P = 5108.29\left[1+(0.08)\left(\frac{93}{365}\right)\right]^{-1} = \$5006.24$
   Investor's profit = 5006.24 – 5000 = \$6.42
   Bank's profit = 5108.29 – 5006.24 = \$102.05

7. $S = 800\left[1 + (0.6)\left(\frac{93}{365}\right)\right] = \$812.23$

   $P = 812.23\left[1 + (0.08)\left(\frac{60}{365}\right)\right]^{-1} = \$801.69$

8. $S = 1000\left[1 + (0.10)\left(\frac{93}{365}\right)\right] = \$1025.48$

9. Maturity date is 123 days after October 17,2014, i.e. February 17, 2015

   Maturity value = $5000[1 + (0.075)\ (\frac{123}{365})] = \$5126.37$

   Proceeds $= 5126.37\left[1 + (0.07)\left(\frac{33}{365}\right)\right]^{-1} = \$5094.13$

10. Proceeds $= 2000\left[1 + (0.13)\left(\frac{93}{365}\right)\right]^{-1} = \$1935.88$

    Profit $= \$435.88$

11. Needs 97% of 2000 = $1940.00

    $S = 1940\left[1 + (0.06)\left(\frac{63}{365}\right)\right] = \$1960.09$

12. Maturity date is 123 days after June 8, 2014, i.e. October 9, 2014

    Maturity value $S = 1500\left[1 + (0.065)(\frac{123}{365})\right] = \$1532.86$

    a) $P = 1532.86\left[1 + (0.085)\left(\frac{69}{365}\right)\right]^{-1} = \$1508.62$

    b) $r = \dfrac{8.62}{(1500)\left(\frac{54}{365}\right)} = 3.88\%$

    c) Banks profit = $1500\left[1 + (0.065)\left(\frac{120}{365}\right)\right] - 1508.62 = \$23.43$

    $r = 23.43 / (1508.62 \times \frac{66}{365}) = 8.59\%$

13. a) $P = 3500\left[1 + (0.06)\left(\frac{10}{12}\right)\right]^{-1} = \$3333.33$

    b) $P = 3500\left[1 + (0.06)\left(\frac{304}{365}\right)\right]^{-1} = \$3333.42$

    where 10 months after July 31 is May 31, which is 304 days

14. a) $P = 5000\left[1 + (0.0345)\left(\frac{91}{365}\right)\right]^{-1} = \$4957.36$

b) $P = 5000[1 + (0.0310)\left(\frac{51}{365}\right)]^{-1}$ = \$4978.44

c) $r = (4978.44 - 4957.36) / (4957.36 \times \frac{40}{365})$ = 3.88%

15. a) 23 892.06 $= 25000[1 + (r)\left(\frac{364}{365}\right)]^{-1}$

Solves for r = 4.65%

b) Sept 9, 2014 to Jan 25, 2015 is 138 days and
Jan 25, 2015 to Sept 8, 2015 is 226 days.

i) 24 102.25 $= 25000[1 + (r)\left(\frac{226}{365}\right)]^{-1}$

Solves for $r = 6.02\%$

ii) $r = (24\ 102.25 - 23\ 892.06) / (23\ 892.06 \times \frac{138}{365})$ = 2.33%

EXERCISE 1.3

1. $X = 1200[1 + (0.035)\left(\frac{80}{365}\right)]^{-1}$ = \$1190.86

2. $X = 100[1 + (0.06)\left(\frac{6}{12}\right)]^{-1} + 150[1 + (0.06)(1)]^{-1} = 97.09 + 141.51 = \$238.60$

3. $X = 200\left[1 + (0.08)\left(\frac{3}{12}\right)\right] + 800\left[1 + (0.08)\left(\frac{3}{12}\right)\right]^{-1} = 204 + 784.31 = \$988.31$

4. $500 + X[1 + (0.13)\left(\frac{6}{12}\right)]^{-1} = 600[1 + (0.13)\left(\frac{2}{12}\right)] + 400[1 + (0.13)\left(\frac{3}{12}\right)]^{-1}$

$500 + 0.938967136X = 613 + 387.41$

$0.938967136X = 500.41$

$X = \$532.94$

5. $X[1+(0.05)\left(\frac{3}{12}\right)]+X=800[1+(0.05)\left(\frac{7}{12}\right)]$

$1.0125X+X=823.33$

$2.0125X=823.33$

$X=\$409.11$

6. $X+X[1+(0.11)\left(\frac{60}{365}\right)]^{-1}+X[1+(0.11)\left(\frac{120}{365}\right)]^{-1}=2000[1+(0.11)\left(\frac{300}{365}\right)]$

$X+0.982238967X+0.965097832X=2180.82$

$2.947336798X=2180.82$

$X=\$739.93$

7. a) $X=500[1+(0.07)\left(\frac{4}{12}\right)]^{-1}+700[1+(0.07)\left(\frac{9}{12}\right)]^{-1}$

$=488.60+665.08=\$1153.68$

b) $X=500[1+(0.07)\left(\frac{2}{12}\right)]+700[1+(0.07)\left(\frac{3}{12}\right)]^{-1}$

$=505.83+687.96=\$1193.79$

c) $X=500[1+(0.07)\left(\frac{8}{12}\right)]+700[1+(0.07)\left(\frac{3}{12}\right)]$

$=523.33+712.25=\$1235.58$

8. First Debt $=500\left[1+(0.06)\left(\frac{3}{12}\right)\right]=\$507.50$

Second Debt $=200[1+(0.06)\left(\frac{6}{12}\right)]=\$206.00$

$X+300[1+(0.08)(1)]=507.50[1+(0.08)\left(\frac{9}{12}\right)]+206[1+(0.08)\left(\frac{6}{12}\right)]$

$X+324=537.95+214.221$

$X=\$428.19$

9. a) $X\left[1+(0.08)\left(\frac{6}{12}\right)\right]+X=1000\left[1+(0.08)(1)\right]$

$$1.04X+X=1080$$
$$2.04X=1080$$
$$X=\$529.41$$

b) $X\left[1+(0.08)(6/12)\right]^{-1}+X\left[1+(0.08)(1)\right]^{-1}=1000$

$$0.961538462X+0.925925926X=1000$$
$$1.887464387X=1000$$
$$X=\$529.81$$

10. $2000\left[1+(0.06)\left(\frac{247}{365}\right)\right]+2000\left[1+(0.06)\left(\frac{124}{365}\right)\right]+X=5000\left[1+(0.06)\left(\frac{359}{365}\right)\right]$

$$2081.21+2040.77+X=5295.07$$
$$X=\$1173.09$$

11. At the end of the 6 months:

$$X=\left[1+(0.12)\left(\frac{3}{12}\right)\right]+2X=200\left[1+(0.12)\left(\frac{2}{12}\right)\right]+300\left[1+(0.12)\left(\frac{4}{12}\right)\right]^{-1}$$
$$1.03X+2X=202+288.46$$
$$3.03X=490.46$$
$$X=\$161.87$$

At the end of 3 months:

$$X+2X\left[1+(0.12)\left(\frac{3}{12}\right)\right]^{-1}=200\left[1+(0.12)\left(\frac{2}{12}\right)\right]^{-1}+300\left[1+(0.12)\left(\frac{7}{12}\right)\right]^{-1}$$
$$X+1.941747573X=196.08+280.37$$
$$2.941747573X=476.45$$
$$X=\$161.96$$

12. At the present time:

$$3000+X\left[1+(0.08)(2)\right]^{-1}=2000\left[1+(0.08)(5)\right]^{-1}+8000\left[1+(0.08)(10)\right]^{-1}$$
$$3000+0.862068965X=1428.57+4444.44$$
$$0.862068965X=2873.02$$
$$X=\$3332.70$$

At the end of 2 years

$$3000+X\left[1+(0.08)(2)\right]+X=2000\left[1+(0.08)(3)\right]^{-1}+8000\left[1+(0.08)(8)\right]^{-1}$$
$$3480+X=1612.90+4878.05$$
$$X=\$3010.95$$

13. a) At the present time:

$$X[1+(0.04)\left(\frac{3}{12}\right)]^{-1}+X[1+(0.04)\left(\frac{6}{12}\right)]^{-1}+X[1+(0.04)\left(\frac{9}{12}\right)]^{-1}$$

$$+\ X[1+(0.04)(1)]^{-1}=2000$$

$$3.902903415\,X=2000$$

$$X=\$512.44$$

b) At the end of 1 year:

$$X[1+(0.04)\left(\frac{9}{12}\right)]+X[1+(0.04)\left(\frac{6}{12}\right)]$$

$$+X[1+(0.04)\left(\frac{3}{12}\right)]+X=2000[1+(0.04)(1)]$$

$$4.06\,X=2080$$

$$X=\$512.32$$

14. At the present time:

$$X[1+(0.10)\left(\frac{3}{12}\right)]^{-1}+2X[1+(0.10)\left(\frac{6}{12}\right)]^{-1}+4X[1+(0.10)\left(\frac{9}{12}\right)]^{-1}=800$$

$$6.601301893\,X=800$$

$$X=\$121.19$$

At the end of 3 months:

$$X+2X[1+(0.10)\left(\frac{3}{12}\right)]^{-1}+4X[1+(0.10)\left(\frac{6}{12}\right)]^{-1}=800[1+(0.10)\left(\frac{3}{12}\right)]$$

$$6.760743322\,X=820$$

$$X=\$121.29$$

At the end of 6 months:

$$X[1+(0.10)\left(\frac{3}{12}\right)]+2X+4X[1+(0.10)\left(\frac{3}{12}\right)]^{-1}=800[1+(0.10)\left(\frac{6}{12}\right)]$$

$$6.927439024\,X=840$$

$$X=\$121.26$$

At the end of 9 months:

$$X[1+(0.10)\left(\frac{6}{12}\right)]+2X[1+(0.10)\left(\frac{3}{12}\right)]+4X=800[1+(0.10)\left(\frac{9}{12}\right)]$$

$$7.10\,X=860$$

$$X=\$121.13$$

EXERCISE 1.4

1. a) Declining Balance Method:

$$\text{End of 3 months balance} = 1000\left[1+(0.09)\left(\frac{3}{12}\right)\right]-200 = \$822.50$$

$$\text{End of 7 months balance} = 822.50\left[1+(0.09)\left(\frac{4}{12}\right)\right]-400 = \$447.18$$

$$\text{End of year balance} = 447.18\left[1+(0.09)\left(\frac{5}{12}\right)\right] = \$463.95$$

b) Merchant's Rule:

$$\text{Balance due} = 1000(1.09)-200\left[1+(0.09)\left(\frac{9}{12}\right)\right]-400\left[1+(0.09)\left(\frac{5}{12}\right)\right]$$
$$= 1090-213.50-415 = \$461.50$$

2. a) Declining Balance Method:

$$\text{August 17/14 Balance} = 2000\left[1+(0.07)\left(\frac{77}{365}\right)\right]-800 = \$1229.53$$

$$\text{November 20/14 Balance} = 1229.53\left[1+(0.07)\left(\frac{95}{365}\right)\right]-400 = \$851.93$$

$$\text{February 2/15 Balance} = 851.93\left[1+(0.07)\left(\frac{74}{365}\right)\right]-500 = \$364.02$$

$$\text{April 18/15 Balance} = 364.02\left[1+(0.07)\left(\frac{75}{365}\right)\right] = \$369.26$$

b) Merchant's Rule: Balance due on April 18, 2015

$$= 2000\left[1+(0.07)\left(\frac{321}{365}\right)\right]-800\left[1+(0.07)\left(\frac{244}{365}\right)\right]-400\left[1+(0.07)\left(\frac{149}{365}\right)\right]$$
$$-500\left[1+(0.07)\left(\frac{75}{365}\right)\right]$$
$$= 2123.12-837.44-411.43-507.19 = \$367.07$$

3. a) Declining Balance Method:

$$\text{End of 2 months balance} = 5000\left[1+(0.05)\left(\frac{2}{12}\right)\right]-300 = \$2041.67$$

$$\text{End of 4 months balance} = 2041.67\left[1+(0.05)\left(\frac{2}{12}\right)\right]-1000 = \$1058.68$$

$$\text{End of 6 months balance} = 1058.68\left[1+(0.05)\left(\frac{2}{12}\right)\right] = \$1067.50$$

b) Merchant's Rule: Balance due

$$= 5000\left[1+(0.05)\left(\frac{6}{12}\right)\right]-3000\left[1+(0.05)\left(\frac{4}{12}\right)\right]-1000\left[1+(0.05)\left(\frac{2}{12}\right)\right]$$

$$= 5125 - 3050 - 1008.33 = \$1066.67$$

4. a) Declining Balance Method:

April 12 Balance $= 1000\left[1+(0.16)\left(\frac{94}{365}\right)\right]-350 = \$691.21$

Note that interest from April 12 to August 10 is larger than \$20

$\left(691.21 \times 0.16 \times \frac{120}{365} = \$36.36\right)$

October 3 Balance $= 691.21\left[1+(0.16)\left(\frac{174}{365}\right)\right]-420 = \$323.93$

December 15 Balance $= 323.93\left[1+(0.16)\left(\frac{73}{365}\right)\right] = \$334.30$

b) Merchant's Rule:

Balance due on December 15

$$= 1000\left[1+(0.16)\left(\frac{341}{365}\right)\right]-350\left[1+(0.16)\left(\frac{247}{365}\right)\right]-20\left[1+(0.16)\left(\frac{127}{365}\right)\right]$$
$$-400\left[1+(0.16)\left(\frac{73}{365}\right)\right]$$
$$= 1149.48-387.90-21.11-412.80 = \$327.67$$

5. End of 2 months balance $= 1400\left[1+(0.12)\left(\frac{2}{12}\right)\right]-400 = \$1028.00$

Note that interest for the next 4 months and is larger than the partial payment of \$30.00 ($\left(1028\right)(0.12)\left(\frac{4}{12}\right) = \$41.12$)

End of 8 months balance $= 1028\left[1+(0.12)\left(\frac{6}{12}\right)\right]-630 = \$459.68$

End of 12 months balance $= \$459.68\left[1+(0.12)\left(\frac{4}{12}\right)\right] = \$478.07$

6. a) Declining Balance Method:

August 16 balance $= 2500\left[1+(0.08)\left(\frac{106}{365}\right)\right]-1000 = \$1558.08$

Note that interest due on Nov 19 is smaller than partial payment of \$40

$\left(1558.05\left(0.08\right)\left(\frac{85}{365}\right) = \$29.03\right)$

Thus, Nov 19 Balance $= \left(1558.08+29.03\right)-40 = \$1547.11$

Jan 9 Balance $= 1547.11\left[1+(0.08)\left(\frac{51}{365}\right)\right]-350 = \$1214.40$

Mar 20 Balance $= 1214.40\left[1+(0.08)\left(\frac{70}{365}\right)\right] = \$1233.03$

b) Merchant's Rule:

Balance due on March 20

$$= 2500\left[1 + (0.08)\left(\frac{312}{265}\right)\right] - 1000\left[1 + (0.08)\left(\frac{206}{365}\right)\right] - 40\left[1 + (0.08)\left(\frac{121}{365}\right)\right]$$

$$- 350\left[1 + (0.08)\left(\frac{70}{365}\right)\right]$$

$$= 2670.96 - 1045.15 - 41.06 - 355.37 = \$1229.35$$

EXERCISE 1.5

1. a) $P = 2000[1 + (0.085)(\frac{130}{365})]^{-1} = \$1941.23$

   b) $P = 2000[1 - (0.085)(\frac{130}{365})] = \$1939.45$

2. a) $P = 5000[1 - (0.045)(\frac{272}{365})] = \$4832.33$

   b) $P = 5000[1 + (0.045)(\frac{272}{365})]^{-1} = \$5173.49$

   c) $r = \dfrac{0.045}{1 - (0.045)\left(\frac{272}{365}\right)} = 0.0465614 = 4.66\%$

3. $D = 700 \times 0.08 \times \frac{45}{365} = \$6.90$; Thus, $P = 700 - 6.90 = \$693.10$

4. Maturity value $= 2000[1 + (0.07)\left(\frac{178}{365}\right)] = \$2068.27$

   Discount $= 2068.27 - 2030.00 = \$38.27$

   $d = 38.27 / (2068.27 \times \frac{118}{365}) = 5.72\%$

5. $S = 800 / [1 + (0.10)\left(\frac{63}{365}\right)] = \$814.05$

6. Maturity date of the 1st note is August 4, 2014

   Interest due on the 1st note $= 50\ 000 \times 0.11 \times \frac{95}{365} = \$1431.51$

   Maturity date of the 2nd note is November 7, 2014

   $S = 50\ 000 / [1 - (0.12)\left(\frac{95}{365}\right)] = \$51\ 611.99$

7. $P = \$2869.11 \qquad S = \$3000 \qquad d = 0.065$

   $D = S \times d \times t \qquad Thus, \quad t = \dfrac{D}{S \times d} = \dfrac{130.89}{3000 \times 0.065} = 0.67123 = \dfrac{days}{365}$

   It is due to be paid in 245 days.

8. $P = \$1500; \quad S = \$1580; \quad D = \$80; \quad t = \frac{200}{365}$; Thus, $d = \dfrac{D}{S \times t} = \dfrac{80}{1580 \times \left(\frac{200}{365}\right)} = 9.24\%$

9. $P = \$1800 \qquad t = 1 \qquad d = 0.087$

   $S = P \times \frac{1}{1 - dt} = 1800 \times \frac{1}{1 - (0.087)(1)} = \$1971.52$

10. a) Mar 13 to Sep 2 $= 173$ *days*; $r = 0.10 / [\,1 - (0.12)\left(\frac{173}{365}\right)] = 10.50\,\%$

    b) $r = 0.065 / [\,1 - (0.065)(\frac{9}{12})\,] = 6.83\,\%$

    c) $r = 0.048 / [\,1 - (0.048)\left(\frac{15}{52}\right)] = 4.87\,\%$

11. a) Apr 25 to Aug 16 $= 113$ *days*; $d = 0.05 / [\,1 + (0.05)\left(\frac{113}{365}\right)] = 4.92\,\%$

    b) $d = 0.04 / [\,1 + (0.04)(\frac{11}{12})\,] = 3.86\,\%$

    c) $d = 0.075 / [\,1 + (0.075)\left(\frac{182}{365}\right)] = 7.23\,\%$

REVIEW EXERCISE 1.6

1. a) $t = \dfrac{I}{Pr} = \dfrac{100}{(1000)(0.06)} = 1\left(\dfrac{2}{3}\right) \textit{ years} = 20 \textit{ months}$

   b) $t = \dfrac{I}{Pr} = \dfrac{200}{(1000)(0.135)} = 1.481481481 \textit{ years} \doteq 541 \textit{ days}$

2. Ordinary interest: $S = 1000\left[1 + (0.07)\left(\dfrac{55}{360}\right)\right] = \$1010.69$

   $$P = 1000\left[1 + (0.07)\left(\frac{55}{360}\right)\right]^{-1} = \$989.42$$

   Exact interest: $S = 1000\left[1 + (0.07)\left(\dfrac{55}{365}\right)\right] = \$1010.55$

   $$P = 1000\left[1 + (0.07)\left(\frac{55}{365}\right)\right]^{-1} = \$989.56$$

3. $r = 57 / \left(323 \times \dfrac{52}{365}\right) = 123.87\%$

4. a) $r = \dfrac{I}{Pt} = \dfrac{240}{(7760)\left(\dfrac{30}{365}\right)} = 37.63\%$

   b) Interest on loan $= (7760)(0.12)\left(\dfrac{30}{365}\right) = \$76.54$

   Savings $= 240 - 76.54 = \$163.46$

5. a) $r = \dfrac{0.03x}{(0.97x)\left(\dfrac{40}{365}\right)} = 28.22\%$

   b) Interest on loan $= (4850)(0.08)\left(\dfrac{40}{365}\right) = \$42.52$

   Savings $= 150 - 42.52 = \$107.48$

6. $$X\left[1 + (0.09)\left(\frac{143}{365}\right)\right] + X\left[1 + (0.09)\left(\frac{87}{365}\right)\right] + X\left[1 + (0.09)\left(\frac{35}{365}\right)\right] + 500 = 2500\left[1 + (0.09)\left(\frac{196}{365}\right)\right]$$

   $$1.035260274X + 1.021452055X + 1.008630137X + 500 = 2620.82$$
   $$3.065342466X = 2120.82$$
   $$X = \$691.87$$

7. $$X\left[1 + (0.03)(1.5)\right] + x\left[1 + (0.03)(1)\right] + X\left[1 + (0.03)(0.5)\right] + X = 5000\left[1 + (0.03)(2)\right]$$
   $$1.045X + 1.03X + 1.015X + X = 5300$$
   $$4.09X = 5300$$

$$X = \$1295.84$$

8. $$X[1+(0.08)\left(\frac{4}{12}\right)]^{-1} + X[1+(0.08)\left(\frac{8}{12}\right)]^{-1} + X[1+(0.08)(1)]^{-1} = 2400$$

$$2.849318989\,X = 2400$$

$$X = \$842.31$$

9. Merchant's Rule:

$$1250\left[1+(0.09)\left(\frac{186}{365}\right)\right]+\ 2500\left[1+(0.09)\left(\frac{114}{365}\right)\right]+\ X=4500\left[1+(0.09)\left(\frac{302}{365}\right)\right]$$

$$1307.33+2570.33+X=4835.10$$

$$X=\$957.50$$

Declining Balance Method:

$$\text{October 27/14 Balance}=4500\left[1+(0.09)\left(\frac{116}{365}\right)\right]-1250=\$3378.71$$

$$\text{January 7/15 Balance}=3378.71\left[1+(0.09)\left(\frac{72}{365}\right)\right]-2500=\$938.69$$

$$\text{May 1/15 Balance}=938.69\left[1+(0.09)\left(\frac{114}{365}\right)\right]=\$965.08$$

10. a) Equation of value at the end of 6 months:

$$X+X\left[1+(0.075)\left(\frac{3}{12}\right)\right]^{-1}+\ 1000\left[1+(0.075)\left(\frac{3}{12}\right)\right]=4000\left[1+(0.075)\left(\frac{6}{12}\right)\right]$$

$$X+0.981595X+1018.75=4150.00$$

$$1.981595X=3131.25$$

$$X=\$1580.17$$

b) Equation of value at the present time:

$$X\left[1+(0.075)\left(\frac{6}{12}\right)\right]^{-1}+X\left[1+(0.075)\left(\frac{9}{12}\right)\right]^{-1}+1000\left[1+(0.075)\left(\frac{3}{12}\right)\right]^{-1}=4000$$

$$0.963855421X+0.946745562X+981.60=4000$$

$$1.910600984X=3081.40$$

$$X=\$1579.82$$

11. Merchant's Rule

On October 31, 2014:

$$500\left[1+(0.08)\left(\frac{106}{365}\right)\right]+400\left[1+(0.08)\left(\frac{32}{365}\right)\right]+X=1000\left[1+(0.08)\left(\frac{176}{365}\right)\right]$$

$$511.6164+402.8055+X=1038.5753$$

$$X=\$124.15$$

Declining Balance Method:

$$\text{July 17, 2014 balance}=1000\left[1+(0.08)\left(\frac{70}{365}\right)\right]-500=\$515.34$$

$$\text{September 29, 2014 balance}=515.34\left[1+(0.08)\left(\frac{74}{365}\right)\right]-400=\$123.70$$

$$\text{October 31, 2014 balance}=123.70\left[1+(0.08)\left(\frac{32}{265}\right)\right]=\$124.57$$

12. a) $P=1000\left[1-(0.045)\left(\frac{8}{12}\right)\right]=\$970.00$

b) $S = 1000 / [\,1 - (0.045)\left(\frac{8}{12}\right)] = \$1030.93$

c) $r = \dfrac{0.045}{\left[1 - (0.045)\left(\frac{8}{12}\right)\right]} = 4.64\,\%$

13. Maturity date: August 12, 2014

Maturity value $S = 1500\left[1 + (0.12)\left(\frac{93}{265}\right)\right] = \$1545.86$

a) $P = 1545.86\left[1 - (0.13)\left(\frac{41}{365}\right)\right] = 1523.29$

b) Prasad's profit $= \$23.29$, $r = \frac{23.29}{(1500)\left(\frac{52}{365}\right)} = 10.90\%$

c) Bank's profit $= 1545.86 - 1523.29 = \$22.57$

$r = \frac{22.57}{(1523.29)\left(\frac{41}{365}\right)} = 13.19\%$

d) Bank's profit $= 1500\left[1 + (0.12)\left(\frac{90}{365}\right)\right] - 1523.29 = \$21.09$

$r = \frac{21.09}{(1523.29)\left(\frac{41}{365}\right)} = 13.30\%$

The answer is different from c) since in c), the note was assumed to be paid back on the maturity date, while in d), the note was assumed to be paid back 3 days earlier on the due date. The rate of return is higher despite losing 3 days of interest because getting the note paid back three days earlier is more valuable to the bank than receiving the extra 3 days of interest.

14. Maturity value $S = 2000\left[1 + (0.07)\left(\frac{183}{365}\right)\right] = \$2070.19$

a) $P = 2070.19\left[1 - (0.07)\left(\frac{123}{365}\right)\right] = \$2021.36$

b) $r = \frac{21.36}{(2000)\left(\frac{60}{365}\right)} = 6.50\%$

c) Bank will receive $2000\left[1 + (0.07)\left(\frac{180}{365}\right)\right] = \$2069.04$

Discount $D = 2069.04 - 2021.36 = \$47.68$

$d = \frac{47.68}{(2069.04)\left(\frac{120}{365}\right)} = 7.01\%$

15. Interest due on the first note $= 800(0.06)\left(\frac{93}{365}\right) = \$12.23$

Face value of the second note $= 800 / \left[1 - (0.07)\left(\frac{63}{365}\right)\right] = \$809.78$

16. Assume last payment in X days

$$1200\left[1+(0.11)\left(\frac{X}{365}\right)\right]=500\left[1+(0.11)\left(\frac{X-45}{365}\right)\right]+300\left[1+(0.11)\left(\frac{X-100}{365}\right)\right]+436.92$$

$$1200+0.361643836X=500+0.150684932X-6.78+300+0.090410959X-9.04+436.92$$

$$0.120547945X=21.10$$

$$X=175 \text{ days}$$

17. Exact time = $365-105+35=295$ *days*

Banker's Rule: $I = 20\ 000 \times 0.09 \times \frac{295}{360} = \$1475.00$

Exact interest: $I = 20\ 000 \times 0.09 \times \frac{295}{365} = \$1454.79$

Difference = $1475.00-1454.79 = \$20.21$

18. At present time :

$$X\left[1+(0.0375)\left(\frac{3}{12}\right)\right]^{-1}+X\left[1+(0.0375)\left(\frac{7}{12}\right)\right]^{-1}+X\left[1+(0.0375)\left(\frac{12}{12}\right)\right]^{-1}=3000$$

$$0.990712074\,X+0.978593272\,X+0.963855422\,X=3000$$

$$2.93316077\,X=3000$$

$$X=\$1022.79$$

At the end of 12 months

$$X\left[1+(0.0375)\left(\frac{9}{12}\right)\right]+X\left[1+(0.0375)\left(\frac{5}{12}\right)\right]+X=3000\left[1+(0.0375)\left(\frac{12}{12}\right)\right]$$

$$1.28125\,X+\ 1.015625\,X+X=3112.50$$

$$3.04375\,X=3112.50$$

$$X=\$1022.59$$

Difference $=1022.79-1022.59=\$0.20$

19. Maturity Value $=1500\left[1+(0.095)\left(\frac{173}{365}\right)\right]=\$1567.54$

Proceeds $=1567.54\left[1-(0.14)\left(\frac{133}{365}\right)\right]=\$1487.57$

20. a) At the end of 8 months:

$$X\left[1+(0.08)\left(\frac{2}{12}\right)\right]+2X=1000\left[1+(0.08)\left(\frac{8}{12}\right)\right]+2000\left[1+(0.08)\left(\frac{4}{12}\right)\right]$$

$$1.0133333\,X+2\,X=1053.33+2053.33$$

$$3.01333333\,X=3106.67$$

$$X=\$1030.97$$

b) At the end of 8 months:

$$X\left[1-(0.08)\left(\frac{2}{12}\right)\right]^{-1}+2\,X=1000\left[1-(0.08)\left(\frac{8}{12}\right)\right]^{-1}+2000\left[1-(0.08)\left(\frac{4}{12}\right)\right]^{-1}$$

$$1.013513514\,X+2\,X=1056.34+2054.79$$

$$3.013513514\,X=3111.13$$

$$X=\$1032.39$$

Case Study I – Promissory Notes

a) Maturity Value of note $= 400\ \ 000\left[1+(0.075)\left(\frac{93}{365}\right)\right] = \$\,407\ \ 643.84$

<u>20 days</u>

Proceeds $= 407\ \ 643.84\left[1+(0.12)\left(\frac{93-20}{365}\right)\right]^{-1} = \$\,398\ \ 089.69$

<u>30 days</u>

Proceeds $= 407\ \ 643.84\left[1+(0.12)\left(\frac{93-30}{365}\right)\right]^{-1} = \$\,399\ \ 371.92$

<u>40 days</u>

Proceeds $= 407\ \ 643.84\left[1+(0.12)\left(\frac{93-40}{365}\right)\right]^{-1} = \$\,400\ \ 662.43$

<u>50 days</u>

Proceeds $= 407\ \ 643.84\left[1+(0.12)\left(\frac{93-50}{365}\right)\right]^{-1} = \$\,401\ \ 961.32$

Jerry should sell the note to Bank One sometime between 30 days and 40 days from now.

b) The exact time the note should be sold in order for the proceeds to be \$400,000:

$$\$\,400\ \ 000 = 407\ \ 643.84\left[1+(0.12)\left(\frac{93-t}{365}\right)\right]^{-1}$$

Which solves for t = 34.875 days

If Jerry sells the note to Bank One after 35 days, he will receive \$400,000 (actually, he will get a bit more than \$400,000).

c) If the bank uses d = 12%,

$$\$\,400\ \ 000 = 407\ \ 643.84\left[1-(0.12)\left(\frac{93-t}{365}\right)\right]$$

Which solves for t = 35.965 days

Jerry should sell the note to Bank One after 36 days.

Case Study II – Student Loans

The grace period is May 1 to October 31. From May 1 to September 15 is 258 – 121 = 137 days at 2.5% + 2.5% = 5%, while September 15 to October 31 (inclusive) is 304 – 258 + 1 = 47 days at 3.0% + 2.5% = 5.50%.

The accrued interest at the end of the grace period is

I = 12 500(0.05)(137/365) + 12 500(0.055)(47/365) = $323.12

The loan balance on October 31 is 12 500 + 323.12 = 12 823.12

Date	Days	Interest Rate	Interest ($)	Accrued Interest	Monthly Payment	Principal Portion	Outstanding Balance
Nov 1	--	--	--	--	--	--	12 823.12
Nov 30	30	5.50	57.97	57.97	200.00	142.03	12 681.09
Dec 31	31	5.50	59.24	59.24	200.00	140.76	12 540.33
Jan 19	19	5.50	35.90	35.90			
Jan 31	12	5.75	23.71	59.61	200.00	140.39	12 399.94
Feb 28	28	5.75	54.70	54.70	200.00	145.30	12 254.64

Case Study III – Solving for a simple interest rate

a) Set up an equation of value, using the end of 12 months as the focal date:

$220[1 + i] = 20[1 + i(11/12)] + 20[1 + i(10/12)] + \ldots + 20[1 + i(1/12)] + 20$

$220 + 220i = 20(12) + i(1 + 2 + \ldots + 11)/12$

$220 + 220i = 240 + i(66/12)$

$$i = \frac{240 - 220}{220 - 5.5} = \frac{20}{214.5} = 0.09324 = 9.324\,\%$$

b) Set up an equation of value, using the end of 6 months as the focal date:

$220[1 + i(6/12)] = 40[1 + i(5/12)] + 40[1 + i(4/12)] + \ldots + 40[1 + i(1/12)] + 40$

$220 + 110i = 40(6) + i(1 + 2 + \ldots + 5)/12$

$220 + 110i = 240 + i(15/12)$

$$i = \frac{240 - 220}{110 - 1.25} = \frac{20}{108.75} = 0.1839 = 18.39\,\%$$