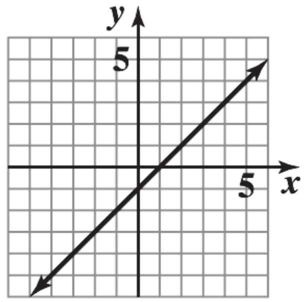
2 FUNCTIONS AND GRAPHS

EXERCISE 2-1

2.



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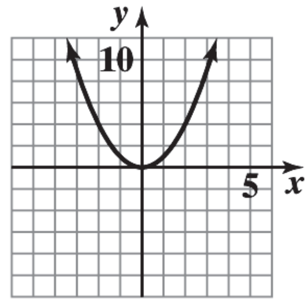

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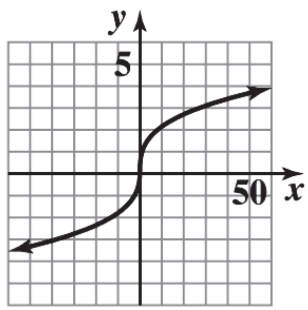

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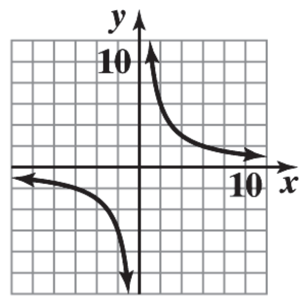


10. The table specifies a function, since for each domain value there corresponds one and only one range value.

12. The table does not specify a function, since more than one range value corresponds to a given domain value.
(Range values 1, 2 correspond to domain value 9.)

14. This is a function.

16. The graph specifies a function; each vertical line in the plane intersects the graph in at most one point.

18. The graph does not specify a function. There are vertical lines which intersect the graph in more than one point. For example, the y-axis intersects the graph in two points.

20. The graph does not specify a function.

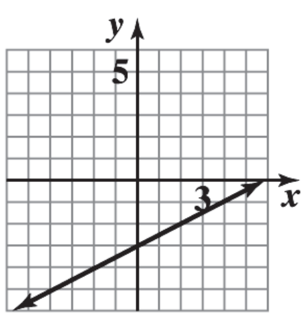
22. $y = 10 - 3x$ is linear.

24. $x^2 - y = 8$ is neither linear nor constant.

26. $y = \frac{2+x}{3} + \frac{2-x}{3} = \frac{2}{3} + \frac{x}{3} + \frac{2}{3} - \frac{x}{3}$
$= \frac{4}{3}$ which is constant.

28. $9x - 2y + 6 = 0$ is linear.

30.



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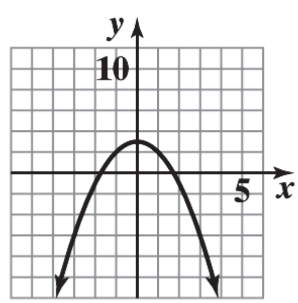

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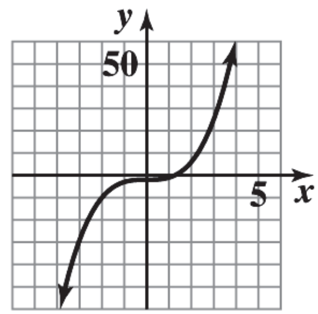

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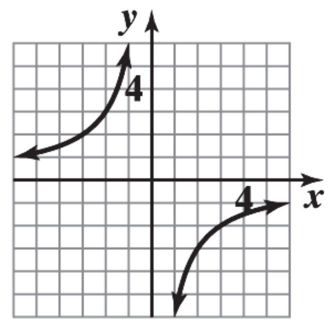
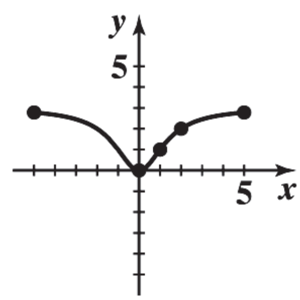


38. $f(x) = \dfrac{3x^2}{x^2+2}$. Since the denominator is bigger than 1, we note that the values of f are between 0 and 3.

Furthermore, the function f has the property that $f(-x) = f(x)$. So, adding points $x = 3$, $x = 4$, $x = 5$, we have:

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$F(x)$	2.78	2.67	2.45	2	1	0	1	2	2.45	2.67	2.78

The sketch is:



40. $y = f(4) = 0$

42. $y = f(-2) = 3$

44. $f(x) = 3$, $x < 0$ at $x = -4, -2$

46. $f(x) = 4$ at $x = 5$

48. All real numbers

50. All real numbers except $x = 2$

52. $x > -5$

54. Given $6x - 7y = 21$. Solving for y we have: $-7y = 21 - 6x$ and $y = \frac{6}{7}x - 3$.

This equation specifies a function. The domain is R, the set of real numbers.

56. Given $x(x + y) = 4$. Solving for y we have: $xy + x^2 = 4$ and $y = \dfrac{4 - x^2}{x}$.

This equation specifies a function. The domain is all real numbers except 0.

58. Given $x^2 + y^2 = 9$. Solving for y we have: $y^2 = 9 - x^2$ and $y = \pm\sqrt{9 - x^2}$.

This equation does not define y as a function of x. For example, when $x = 0$, $y = \pm 3$.

60. Given $\sqrt{x} - y^3 = 0$.. Solving for y we have: $y^3 = \sqrt{x}$ and $y = x^{1/6}$.

This equation specifies a function. The domain is all nonnegative real numbers, i.e., $x \ge 0$.

62. $f(-5) = (-5)^2 - 4 = 25 - 4 = 21$

64. $f(x-2) = (x-2)^2 - 4 = x^2 - 4x + 4 - 4 = x^2 - 4x$

66. $f(10x) = (10x)^2 - 4 = 100x^2 - 4$

68. $f(\sqrt{x}) = \left(\sqrt{x}\right)^2 - 4 = x - 4$

70. $f(-3) + f(h) = (-3)^2 - 4 + h^2 - 4 = 5 + h^2 - 4 = h^2 + 1$

72. $f(-3+h) = (-3+h)^2 - 4 = 9 - 6h + h^2 - 4 = 5 - 6h + h^2$

74. $f(-3+h) - f(-3) = \left[(-3+h)^2 - 4\right] - \left[(-3)^2 - 4\right] = (9 - 6h + h^2 - 4) - (9 - 4) = -6h + h^2$

76. (A) $f(x+h) = -3(x+h)+9 = -3x-3h+9$

(B) $f(x+h)-f(x) = (-3x-3h+9)-(-3x+9) = -3h$

(C) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-3h}{h} = -3$

78. (A) $f(x+h) = 3(x+h)^2+5(x+h)-8$

$= 3(x^2+2xh+h^2)+5x+5h-8$

$= 3x^2+6xh+3h^2+5x+5h-8$

(B) $f(x+h)-f(x) = (3x^2+6xh+3h^2+5x+5h-8)-(3x^2+5x-8)$

$= 6xh+3h^2+5h$

(C) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{6xh+3h^2+5h}{h} = 6x+3h+5$

80. (A) $f(x+h) = x^2 + 2xh + h^2 + 40x + 40h$

(B) $f(x+h) - f(x) = 2xh + h^2 + 40h$

(C) $\dfrac{f(x+h)-f(x)}{h} = 2x + h + 40$

82. Given $A = \ell w = 81$.

Thus, $w = \dfrac{81}{\ell}$. Now $P = 2\ell + 2w = 2\ell + 2\left(\dfrac{81}{\ell}\right) = 2\ell + \dfrac{162}{\ell}$.

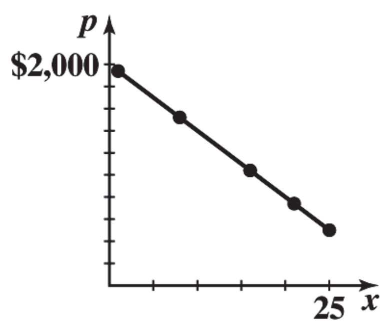
The domain is $\ell > 0$.

84. Given $P = 2\ell + 2w = 160$ or $\ell + w = 80$ and $\ell = 80 - w$.

Now $A = \ell w = (80 - w)w$ and $A = 80w - w^2$.

The domain is $0 < w < 80$. [Note: $w < 80$ since $w \geq 80$ implies $\ell \leq 0$.]

86. (A)



(B) $p(11) = 1{,}340$ dollars per computer

$p(18) = 920$ dollars per computer

88. (A) $R(x) = xp(x)$

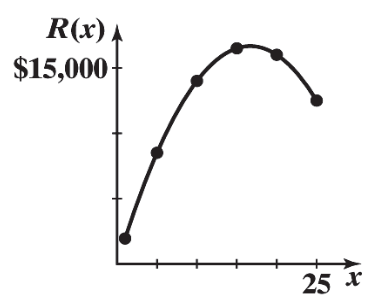
$= x(2{,}000 - 60x)$ thousands of dollars

Domain: $1 \leq x \leq 25$

(B) Table 11 Revenue

x(thousands)	$R(x)$(thousands)
1	$1,940
5	8,500
10	14,000
15	16,500
20	16,000
25	12,500

(C)



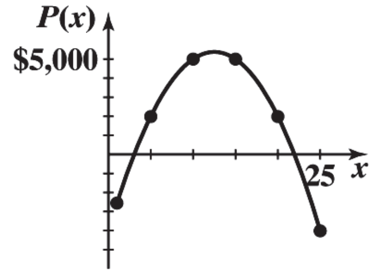
90. (A) $P(x) = R(x) - C(x)$
$= x(2{,}000 - 60x) - (4{,}000 + 500x)$ thousand dollars
$= 1{,}500x - 60x^2 - 4{,}000$

Domain: $1 \le x \le 25$

(B) Table 13 Profit

x (thousands)	$P(x)$ (thousands)
1	–$2,560
5	2,000
10	5,000
15	5,000
20	2,000
25	–4,000

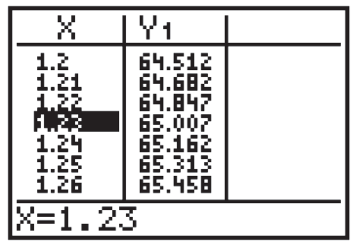
(C)



92. (A) 1.2 inches

(B) Evaluate the volume function for x = 1.21, 1.22, …, and choose the value of x whose volume is closest to 65.

(C) $x = 1.23$ to two decimal places



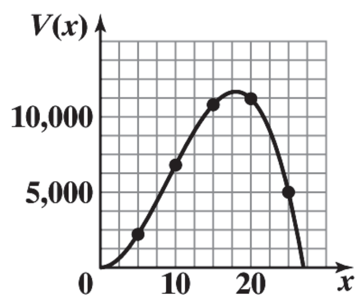
94. (A) $V(x) = x^2(108 - 4x)$

(B) $0 < x < 27$

(C) Table 16 Volume

x	$V(x)$
5	2,200
10	6,800
15	10,800
20	11,200
25	5,000

(D)



96. (A) Given $5v - 2s = 1.4$. Solving for v, we have:
$v = 0.4s + 0.28$.
If $s = 0.51$, then $v = 0.4(0.51) + 0.28 = 0.484$ or 48.4%.

(B) Solving the equation for s, we have:
$s = 2.5v - 0.7$.
If $v = 0.51$, then $s = 2.5(0.51) - 0.7 = 0.575$ or 57.5%.

EXERCISE 2-2

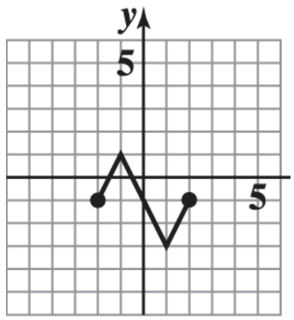
2. $f(x) = -4x + 12$ Domain: all real numbers; range: all real numbers.

4. $f(x) = 3 + \sqrt{x}$ Domain: $[0, \infty)$; range: $[3, \infty)$.

6. $f(x) = -5|x| + 2$ Domain: all real numbers; range: $(-\infty, 2]$.

8. $f(x) = 20 - 10\sqrt[3]{x}$ Domain: all real numbers; range: all real numbers.

10.



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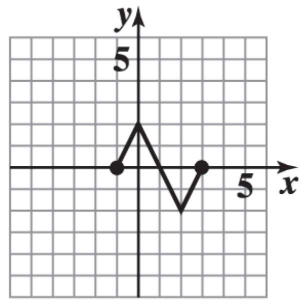

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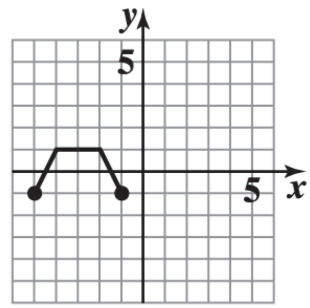

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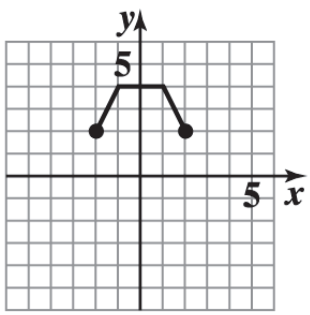

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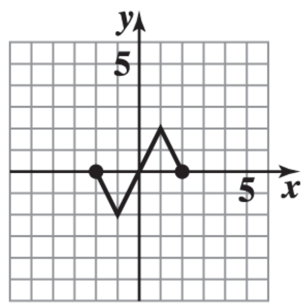

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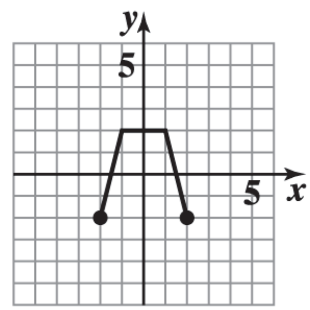
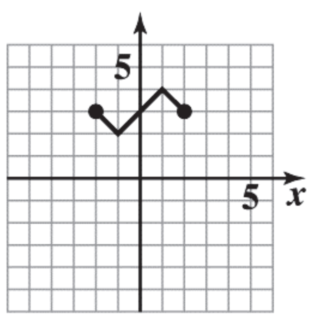
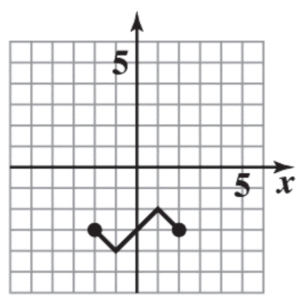


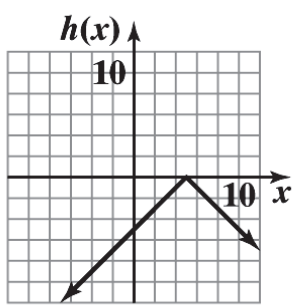
22.



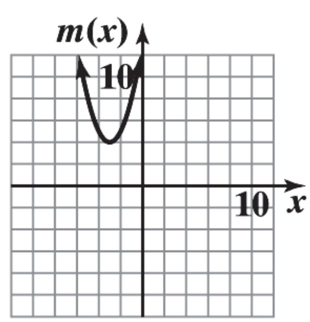
24.



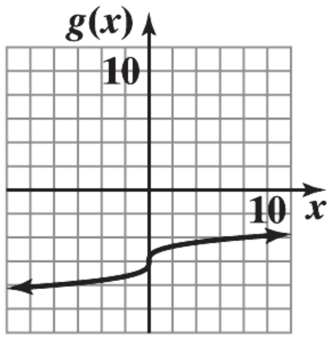
26. The graph of $h(x) = -|x - 5|$ is the graph of $y = |x|$ reflected in the x axis and shifted 5 units to the right.



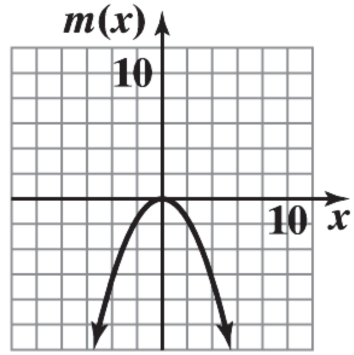
28. The graph of $m(x) = (x + 3)^2 + 4$ is the graph of $y = x^2$ shifted 3 units to the left and 4 units up.



30. The graph of $g(x) = -6 + \sqrt[3]{x}$ is the graph of $y = \sqrt[3]{x}$ shifted 6 units down.



32. The graph of $m(x) = -0.4x^2$ is the graph of $y = x^2$ reflected in the x axis and vertically contracted by a factor of 0.4.



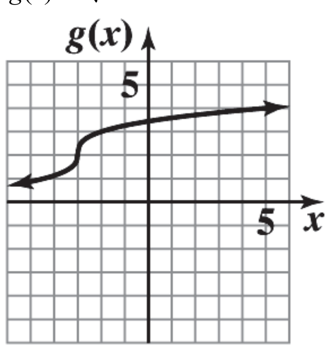
34. The graph of the basic function $y = |x|$ is shifted 3 units to the right and 2 units up. $y = |x - 3| + 2$

36. The graph of the basic function $y = |x|$ is reflected in the x axis, shifted 2 units to the left and 3 units up. Equation: $y = 3 - |x + 2|$

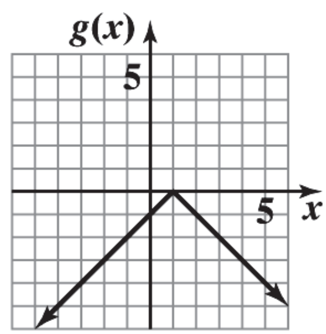
38. The graph of the basic function $\sqrt[3]{x}$ is reflected in the x axis and shifted up 2 units. Equation: $y = 2 - \sqrt[3]{x}$

40. The graph of the basic function $y = x^3$ is reflected in the x axis, shifted to the right 3 units and up 1 unit. Equation: $y = 1 - (x - 3)^3$

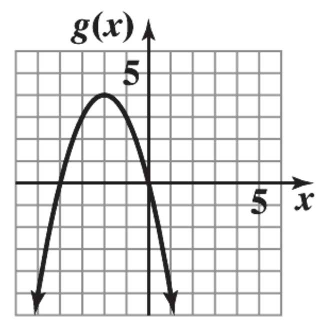
42. $g(x) = \sqrt[3]{x+3} + 2$



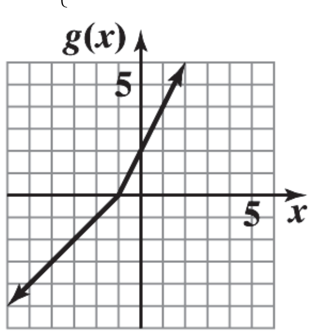
44. $g(x) = -|x - 1|$



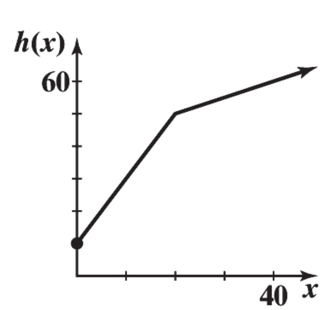
46. $g(x) = 4 - (x + 2)^2$



48. $g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ 2+2x & \text{if } x \geq -1 \end{cases}$

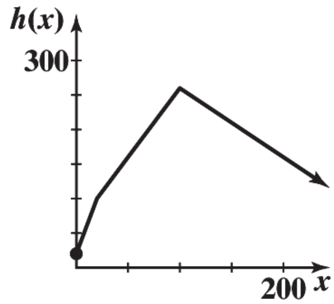


50. $h(x) = \begin{cases} 10+2x & \text{if } 0 \leq x \leq 20 \\ 40+0.5x & \text{if } x > 20 \end{cases}$



52.

$$h(x) = \begin{cases} 4x+20 & \text{if} \quad 0 \le x \le 20 \\ 2x+60 & \text{if} \quad 20 < x \le 100 \\ -x+360 & \text{if} \quad x > 100 \end{cases}$$



54. The graph of the basic function $y = x$ is reflected in the x axis and vertically expanded by a factor of 2. Equation: $y = -2x$

56. The graph of the basic function $y = |x|$ is vertically expanded by a factor of 4. Equation: $y = 4|x|$

58. The graph of the basic function $y = x^3$ is vertically contracted by a factor of 0.25. Equation: $y = 0.25x^3$.

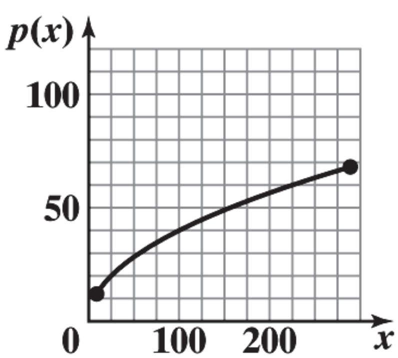
60. Vertical shift, reflection in y axis.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A vertical shift of k units followed by a reflection in y axis moves (a, b) to $(a, b + k)$ and then to $(-a, b + k)$. In the reverse order, a reflection in y axis followed by a vertical shift of k units moves (a, b) to $(-a, b)$ and then to $(-a, b + k)$. The results are the same.

62. Vertical shift, vertical expansion.
Reversing the order can change the result. For example, let (a, b) be a point in the plane. A vertical shift of k units followed by a vertical expansion of h $(h > 1)$ moves (a, b) to $(a, b + k)$ and then to $(a, bh + kh)$. In the reverse order, a vertical expansion of h followed by a vertical shift of k units moves (a, b) to (a, bh) and then to $(a, bh + k)$; $(a, bh + kh) \neq (a, bh + k)$.

64. Horizontal shift, vertical contraction.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A horizontal shift of k units followed by a vertical contraction of h $(0 < h < 1)$ moves (a, b) to $(a + k, b)$ and then to $(a + k, bh)$. In the reverse order, a vertical contraction of h followed by a horizontal shift of k units moves (a, b) to (a, bh) and then to $(a + k, bh)$. The results are the same.

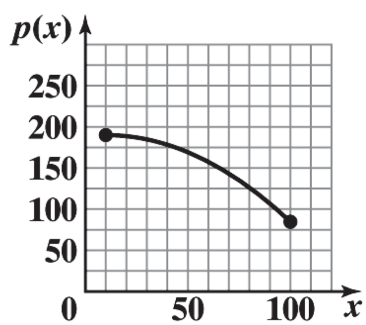
66. (A) The graph of the basic function $y = \sqrt{x}$ is vertically expanded by a factor of 4.

(B)



68. (A) The graph of the basic function $y = x^2$ is reflected in the x axis, vertically contracted by a factor of 0.013, and shifted 10 units to the right and 190 units up.

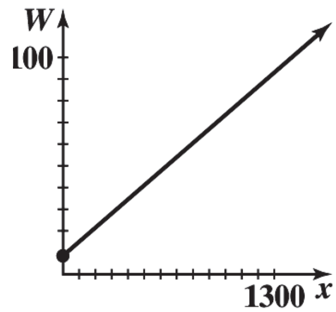
(B)



70. (A) Let x = number of kwh used in a winter month. For $0 \le x \le 700$, the charge is $8.5 + .065x$. At $x = 700$, the charge is \$54. For $x > 700$, the charge is $54 + .053(x - 700) = 16.9 + 0.053x$. Thus,

$$W(x) = \begin{cases} 8.5 + .065x & \text{if } 0 \le x \le 700 \\ 16.9 + 0.053x & \text{if } x > 700 \end{cases}$$

(B)

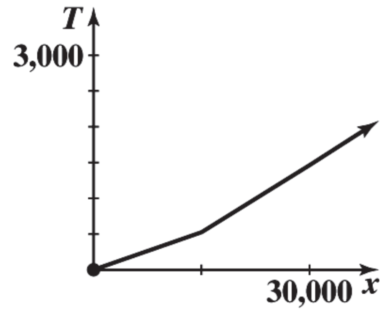


72. (A) Let x = taxable income. If $0 \le x \le 15{,}000$, the tax due is $\$.035x$. At $x = 15{,}000$, the tax due is \$525. For $15{,}000 < x \le 30{,}000$, the tax due is $525 + .0625(x - 15{,}000) = .0625x - 412.5$. For $x > 30{,}000$, the tax due is $1{,}462.5 + .0645(x - 30{,}000) = .0645x - 472.5$.

Thus,

$$T(x) = \begin{cases} 0.035x & \text{if } 0 \le x \le 15{,}000 \\ 0.0625x - 412.5 & \text{if } 15{,}000 < x \le 30{,}000 \\ 0.0645x - 472.5 & \text{if } x > 30{,}000 \end{cases}$$

(B)

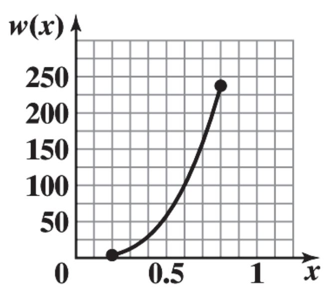


(C) $T(20{,}000) = \$837.50$

$T(35{,}000) = \$1{,}785$

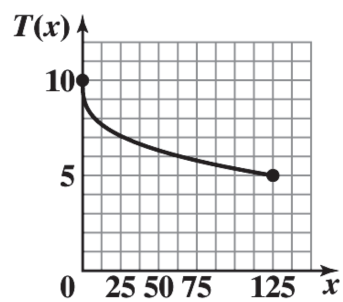
74. (A) The graph of the basic function $y = x^3$ is vertically expanded by a factor of 463.

(B)



76. (A) The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and shifted up 10 units.

(B)



EXERCISE 2-3

2. $x^2 + 16x$ (standard form)

$x^2 + 16x + 64 - 64$ (completing the square)

$(x + 8)^2 - 64$ (vertex form)

4. $x^2 - 12x - 8$ (standard form)

$(x^2 - 12x) - 8$

$(x^2 - 12x + 36) + 8 - 36$ (completing the square)

$(x - 6)^2 - 44$ (vertex form)

6. $3x^2 + 18x + 21$ (standard form)

$3(x^2 + 6x) + 21$

$3(x^2 + 6x + 9 - 9) + 21$ (completing the square)

$3(x + 3)^2 + 21 - 27$

$3(x + 3)^2 - 6$ (vertex form)

8. $-5x^2 + 15x - 11$ (standard form)

$-5(x^2 - 3x) - 11$

$-5(x^2 - 3x + \frac{9}{4} - \frac{9}{4}) - 11$ (completing the square)

$-5(x - \frac{3}{2})^2 - 11 + \frac{45}{4}$

$-5(x - \frac{3}{2})^2 + \frac{1}{4}$ (vertex form)

10. The graph of $g(x)$ is the graph of $y = x^2$ shifted right 1 unit and down 6 units; $g(x) = (x - 1)^2 - 6$.

12. The graph of $n(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 4 units and up 7 units; $n(x) = -(x - 4)^2 + 7$.

14. (A) g (B) m (C) n (D) f

16. (A) x intercepts: –5, –1; y intercept: –5 (B) Vertex: (–3, 4)
(C) Maximum: 4 (D) Range: $y \le 4$ or $(-\infty, 4]$

18. (A) x intercepts: 1, 5; y intercept: 5 (B) Vertex: (3, –4)
(C) Minimum: –4 (D) Range: $y \ge -4$ or $[-4, \infty)$

20. $g(x) = -(x + 2)^2 + 3$

(A) x intercepts:

$$-(x + 2)^2 + 3 = 0$$
$$(x + 2)^2 = 3$$
$$x + 2 = \pm\sqrt{3}$$
$$x = -2 - \sqrt{3}, -2 + \sqrt{3}$$

y intercept: –1

(B) Vertex: (–2, 3) (C) Maximum: 3 (D) Range: $y \le 3$ or $(-\infty, 3]$

22. $n(x) = (x - 4)^2 - 3$

(A) x intercepts:

$$(x - 4)^2 - 3 = 0$$
$$(x - 4)^2 = 3$$
$$x - 4 = \pm\sqrt{3}$$
$$x = 4 - \sqrt{3}, 4 + \sqrt{3}$$

y intercept: 13

(B) Vertex: (4, –3) (C) Minimum: –3 (D) Range: $y \ge -3$ or $[-3, \infty)$

24. $y = -(x - 4)^2 + 2$

26. $y = [x - (-3)]^2 + 1$ or $y = (x + 3)^2 + 1$

28. $g(x) = x^2 - 6x + 5 = x^2 - 6x + 9 - 4 = (x - 3)^2 - 4$

(A) x intercepts:
$$\begin{aligned}(x-3)^2 - 4 &= 0\\ (x-3)^2 &= 4\\ x - 3 &= \pm 2\\ x &= 1, 5\end{aligned}$$

y intercept: 5

(B) Vertex: $(3, -4)$ (C) Minimum: -4 (D) Range: $y \geq -4$ or $[-4, \infty)$

30. $s(x) = -4x^2 - 8x - 3 = -4\left[x^2 + 2x + \frac{3}{4}\right] = -4\left[x^2 + 2x + 1 - \frac{1}{4}\right]$

$$= -4\left[(x+1)^2 - \frac{1}{4}\right] = -4(x+1)^2 + 1$$

(A) x intercepts:
$$\begin{aligned}-4(x+1)^2 + 1 &= 0\\ 4(x+1)^2 &= 1\\ (x+1)^2 &= \frac{1}{4}\\ x + 1 &= \pm\frac{1}{2}\\ x &= -\frac{3}{2}, -\frac{1}{2}\end{aligned}$$

y intercept: -3

(B) Vertex: $(-1, 1)$ (C) Maximum: 1 (D) Range: $y \leq 1$ or $(-\infty, 1]$

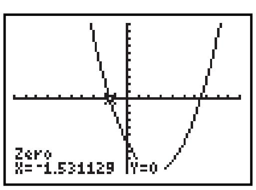
32. $v(x) = 0.5x^2 + 4x + 10 = 0.5[x^2 + 8x + 20] = 0.5[x^2 + 8x + 16 + 4]$
$$= 0.5[(x+4)^2 + 4]$$
$$= 0.5(x+4)^2 + 2$$

(A) x intercepts: none
y intercept: 10

(B) Vertex: $(-4, 2)$ (C) Minimum: 2 (D) Range: $y \geq 2$ or $[2, \infty)$

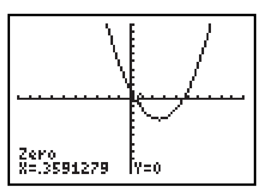
34. $g(x) = -0.6x^2 + 3x + 4$

(A) $g(x) = -2$: $-0.6x^2 + 3x + 4 = -2$
$$0.6x^2 - 3x - 6 = 0$$



$x = -1.53, 6.53$

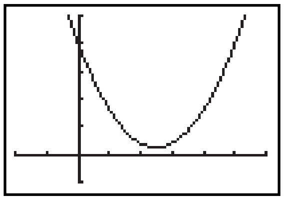
(B) $g(x) = 5$: $-0.6x^2 + 3x + 4 = 5$
$$-0.6x^2 + 3x - 1 = 0$$
$$0.6x^2 - 3x + 1 = 0$$



$x = 0.36, 4.64$

(C) $g(x) = 8$: $-0.6x^2 + 3x + 4 = 8$

$$-0.6x^2 + 3x - 4 = 0$$
$$0.6x^2 - 3x + 4 = 0$$



No solution

36. Using a graphing utility with $y = 100x - 7x^2 - 10$ and the calculus option with maximum command, we obtain 347.1429 as the maximum value.

38. $m(x) = 0.20x^2 - 1.6x - 1 = 0.20(x^2 - 8x - 5)$
$= 0.20[(x - 4)^2 - 21] = 0.20(x - 4)^2 - 4.2$

(A) x intercepts:

$0.20(x - 4)^2 - 4.2 = 0$
$(x - 4)^2 = 21$
$x - 4 = \pm\sqrt{21}$
$x = 4 - \sqrt{21} = -0.6, 4 + \sqrt{21} = 8.6;$
y intercept: -1

(B) Vertex: $(4, -4.2)$ (C) Minimum: -4.2 (D) Range: $y \ge -4.2$ or $[-4.2, \infty)$

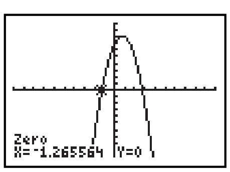
40. $n(x) = -0.15x^2 - 0.90x + 3.3 = -0.15(x^2 + 6x - 22) = -0.15[(x + 3)^2 - 31] = -0.15(x + 3)^2 + 4.65$

(A) x intercepts:

$-0.15(x + 3)^2 + 4.65 = 0$
$(x + 3)^2 = 31$
$x + 3 = \pm\sqrt{31}$
$x = -3 - \sqrt{31} = -8.6, -3 + \sqrt{31} = 2.6;$

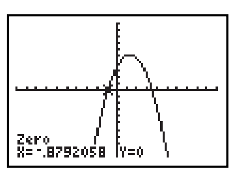
y intercept: 3.30
(B) Vertex: $(-3, 4.65)$ (C) Maximum: 4.65 (D) Range: $x \le 4.65$ or $(-\infty, 4.65]$

42.



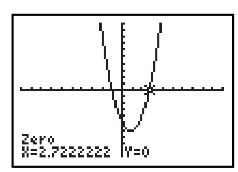
$x = -1.27, 2.77$

44.



$-0.88 \le x \le 3.52$

46.



$x < -1$ or $x > 2.72$

48. f is a quadratic function and max $f(x) = f(-3) = -5$

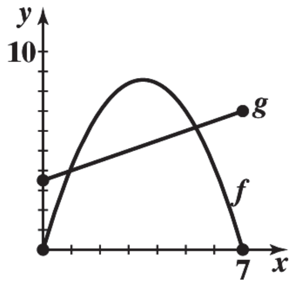
Axis: $x = -3$

Vertex: $(-3, -5)$

Range: $y \le -5$ or $(-\infty, -5]$

x intercepts: None

50. (A)



(B) $f(x) = g(x)$: $-0.7x(x-7) = 0.5x + 3.5$

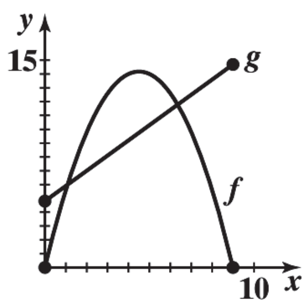
$$-0.7x^2 + 4.4x - 3.5 = 0$$

$$x = \frac{-4.4 \pm \sqrt{(4.4)^2 - 4(0.7)(3.5)}}{-1.4} = 0.93, 5.35$$

(C) $f(x) > g(x)$ for $0.93 < x < 5.35$

(D) $f(x) < g(x)$ for $0 \le x < 0.93$ or $5.35 < x \le 7$

52. (A)



(B) $f(x) = g(x)$: $-0.7x^2 + 6.3x = 1.1x + 4.8$

$$-0.7x^2 + 5.2x - 4.8 = 0$$

$$0.7x^2 - 5.2x + 4.8 = 0$$

$$x = \frac{-(-5.2) \pm \sqrt{(-5.2)^2 - 4(0.7)(4.8)}}{1.4} = 1.08, 6.35$$

(C) $f(x) > g(x)$ for $1.08 < x < 6.35$

(D) $f(x) < g(x)$ for $0 \le x < 1.08$ or $6.35 < x \le 9$

54. A quadratic with no real zeros will not intersect the x–axis.

56. Such an equation will have $b^2 - 4ac = 0$.

58. Such an equation will have $\frac{k}{a} < 0$.

60. $ax^2 + bx + c = a(x-h)^2 + k$

$= a(x^2 - 2hx + h^2) + k$

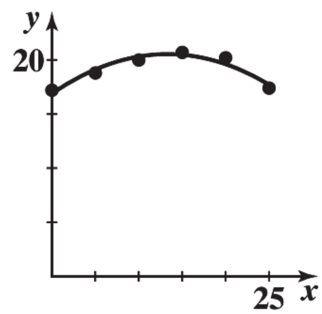
$= ax^2 - 2ahx + ah^2 + k$

Equating constant terms gives $k = c - ah^2$. Since h is the vertex, we have $h = -\dfrac{b}{2a}$. Substituting then gives

$$k = c - ah^2$$

$$= \frac{4ac - b}{4a}$$

(B)



$f(x) = -0.0169x^2 + 0.47x + 17.1$

(A)

x	Mkt Share	$f(x)$
0	17.2	17.1
5	18.8	19.0
10	20.0	20.1
15	20,7	20.3
20	20.2	19.7
25	17.4	18.3
30	16.4	16.0

(C) For 2020, $x = 40$ and $f(40) = -0.0169(30)^2 + 0.47(40) + 17.1 = 8.9\%$

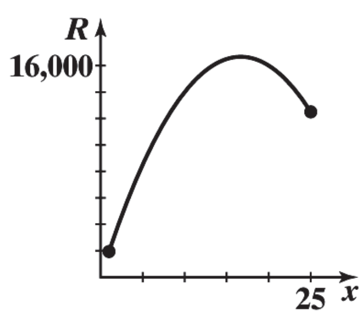
For 2025, $x = 45$ and $f(45) = -0.0169(45)^2 + 0.47(45) + 17.1 = 4.0\%$

(D) Market share rose from 17.2% in 1980 to a maximum of 20.7% in 1995 and then fell to 16.4% in 2010.

64. Verify

66.

(A)



(B) $R(x) = 2{,}000x - 60x^2$

$$= -60\left(x^2 - \frac{100}{3}x\right)$$

$$= -60\left[x^2 - \frac{100}{3}x + \frac{2500}{9} - \frac{2500}{9}\right]$$

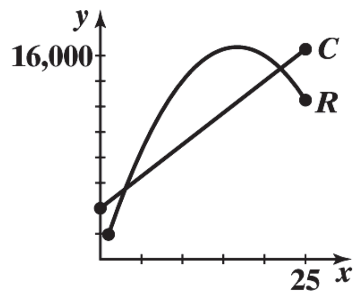
$$= -60\left[\left(x - \frac{50}{3}\right)^2 - \frac{2500}{9}\right]$$

$$= -60\left(x - \frac{50}{3}\right)^2 + \frac{50{,}000}{3}$$

16.667 thousand computers(16,667 computers);
16,666.667 thousand dollars ($16,666,667)

(C) $2000 - 60(50/3) =$ $1,000

68. (A) $p\left(\frac{50}{3}\right) = 2{,}000 - 60\left(\frac{50}{3}\right) = \$1{,}000$



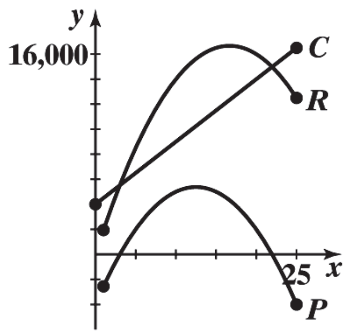
(C) Loss: $1 \le x < 3.035$ or $21.965 < x \le 25$;
Profit: $3.035 < x < 21.965$

(B) $R(x) = C(x)$

$$x(2{,}000 - 60x) = 4{,}000 + 500x$$
$$2{,}000x - 60x^2 = 4{,}000 + 500x$$
$$60x^2 - 1{,}500x + 4{,}000 = 0$$
$$6x^2 - 150x + 400 = 0$$
$$x = 3.035,\ 21.965$$

Break-even at 3.035 thousand (3,035) and 21.965 thousand (21,965)

70. (A) $P(x) = R(x) - C(x)$
$= 1{,}500x - 60x^2 - 4{,}000$



(B) and (C) Intercepts and break-even points: 3,035 computers and 21,965 computers

(D) and (E) Maximum profit is \$5,375,000 when 12,500
computers are produced. This is much smaller
than the maximum revenue of \$16,666,667.

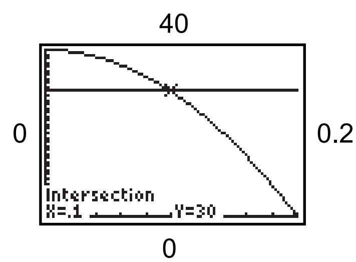
72. Solve: $f(x) = 1{,}000(0.04 - x^2) = 30$

$40 - 1000x^2 = 30$

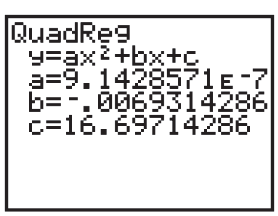
$1000x^2 = 10$

$x^2 = 0.01$

$x = 0.10$ cm



74.



For $x = 2{,}300$, the estimated fuel consumption is
$y = a(2{,}300)^2 + b(2{,}300) + c = 5.6$ mpg.

EXERCISE 2-4

2. $f(x) = 72 + 12x$

(A) Degree: 1

(B) $72+12x=0$

$12x=-72$

$x=-6$

x-intercept: $x=-6$

(C) $f(0)=72-12(0)=72$

y-intercept: 72

4. $f(x)=x^3(x+5)$

(A) Degree: 4

(B) $x^3(x+5)=0$

$x=0,-5$

x-intercepts: $0,-5$

(C) $f(0)=0(0+5)=0$

y-intercept: 0

6. $f(x)=x^2-4x-5$

(A) Degree: 2

(B) $(x-5)(x+1)=0$

$x=-1,5$

x-intercepts: $-1, 5$

(C) $f(0)=-5$

y-intercept: -5

8. $f(x)=(x^2-4)(x^3+27)$

(A) Degree: 5

(B) $(x^2-4)(x^3+27)=0$

$x=-2,2,-3$

x-intercepts: $x=-2,2,-3$

(C) $f(0)=-4(27)=-108$

y-intercept: -108

10. $f(x)=(x+3)^2(8x-4)^6$

(A) Degree: 8

(B) $(x+3)(8x-4)=0$

$x=-3,\frac{1}{2}$

x-intercepts: $-3, 1/2$

(C) $f(0)=3^2(-4)^6=36,864$

y-intercept: 36,864

12. (A) Minimum degree: 2

(B) Negative – it must have even degree, and positive values in the domain are mapped to negative values in the range.

14. (A) Minimum degree: 3

(B) Negative – it must have odd degree, and positive values in the domain are mapped to negative values in the range.

16. (A) Minimum degree: 4

(B) Positive – it must have even degree, and positive values in the domain are mapped to positive values in the range.

18. (A) Minimum degree: 5

(B) Positive – it must have odd degree, and positive values in the domain are mapped to positive values in the range.

20. A polynomial of degree 7 can have at most 7 x-intercepts.

22. A polynomial of degree 6 may have no x intercepts. For example, the polynomial $f(x) = x^6 + 1$ has no x- intercepts.

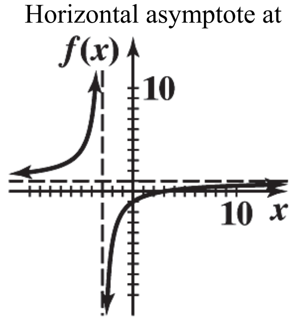
24. (A) Intercepts:

x-intercept(s):	y-intercept:
$x - 3 = 0$ $x = 3$ (3, 0)	$f(0) = \dfrac{0-3}{0+3} = -1$ (0, –1)

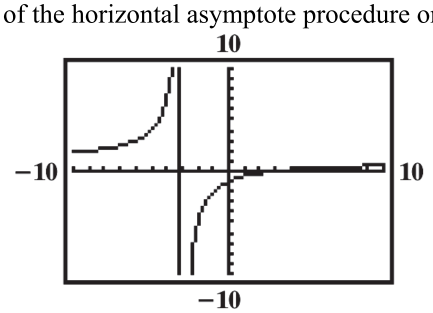
(B) Domain: all real numbers except $x = -3$

(C) Vertical asymptote at $x = -3$ by case 1 of the vertical asymptote procedure on page 90.
Horizontal asymptote at $y = 1$ by case 2 of the horizontal asymptote procedure on page 90.

(D)



(E)



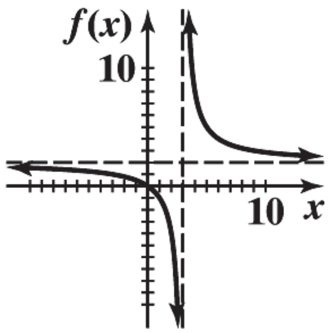
26. (A) Intercepts:

x-intercept(s):	y-intercept:
$2x = 0$ $x = 0$ (0, 0)	$f(0) = \dfrac{2(0)}{0-3} = 0$ (0, 0)

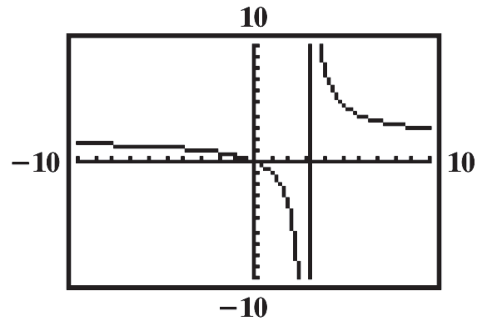
(B) Domain: all real numbers except $x = 3$.

(C) Vertical asymptote at $x = 3$ by case 1 of the vertical asymptote procedure on page 90.
Horizontal asymptote at $y = 2$ by case 2 of the horizontal asymptote procedure on page 90.

(D)



(E)



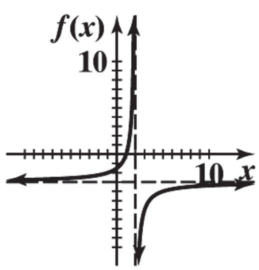
28. (A) Intercepts:

x-intercept:	y-intercept:
$3-3x=0$ $x=1$ (1, 0)	$f(0)=\frac{3-3(0)}{0-2}=-\frac{3}{2}$ $\left(0,-\frac{3}{2}\right)$

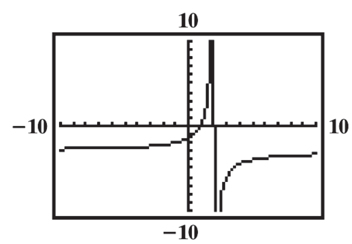
(B) Domain: all real numbers except $x=2$

(C) Vertical asymptote at $x=2$ by case 1 of the vertical asymptote procedure on page 90.
Horizontal asymptote at $y=-3$ by case 2 of the horizontal asymptote procedure on page 90.

(D)

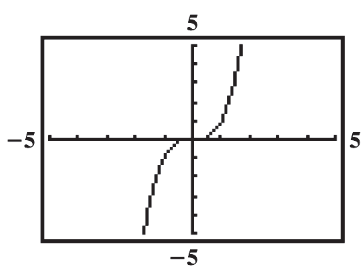


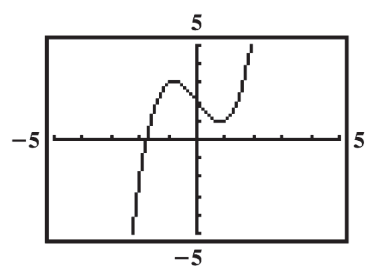
(E)



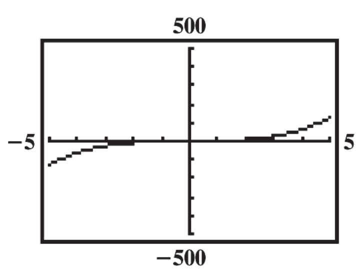
30.

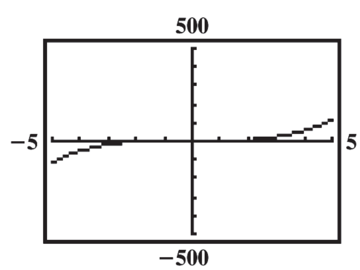
(A)





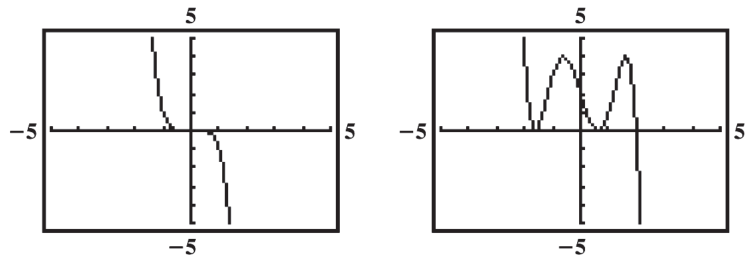
(B)



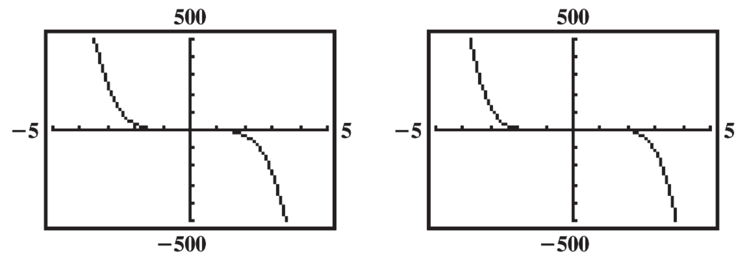


32.

(A)



(B)



34. $y = \frac{6}{4}$, by case 2 for horizontal asymptotes on page 90.

36. $y = -\frac{1}{2}$, by case 2 for horizontal asymptotes on page 90.

38. $y = 0$, by case 1 for horizontal asymptotes on page 90.

40. No horizontal asymptote, by case 3 for horizontal asymptotes on page 90.

42. Here we have denominator $(x^2-4)(x^2-16) = (x-2)(x+2)(x-4)(x+4)$. Since none of these linear terms are factors of the numerator, the function has vertical asymptotes at $x = 2$, $x = -2$, $x = 4$, and $x = -4$.

44. Here we have denominator $x^2+7x-8 = (x-1)(x+8)$. Also, we have numerator $x^2-8x+7 = (x-1)(x-7)$. By case 2 of the vertical asymptote procedure on page 90, we conclude that the function has a vertical asymptote at $x = -8$.

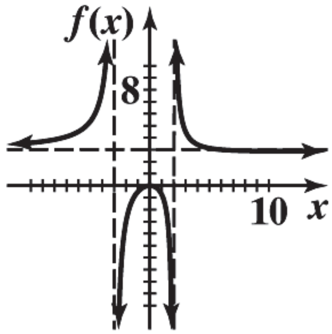
46. Here we have denominator $x^3-3x^2+2x = x(x^2-3x+2) = x(x-2)(x-1)$. We also have numerator $x^2+x-2 = (x+2)(x-1)$. By case 2 of the vertical asymptote procedure on page 90, we conclude that the function has a vertical asymptotes at $x = 0$ and $x = 2$.

48. (A) Intercepts:

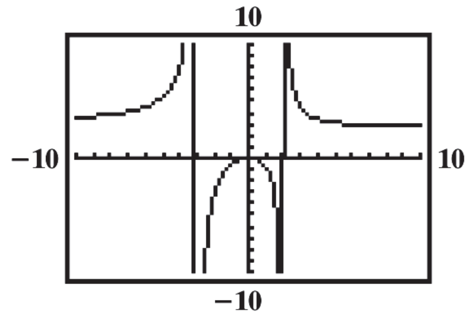
x-intercept(s):	y-intercept:
$3x^2 = 0$	$f(0) = 0$
$x = 0$	
(0, 0)	(0, 0)

(B) Vertical asymptote when $x^2+x-6 = (x-2)(x+3) = 0$; so, vertical asymptotes at $x = 2$, $x = -3$. Horizontal asymptote $y = 3$.

(C)



(D)

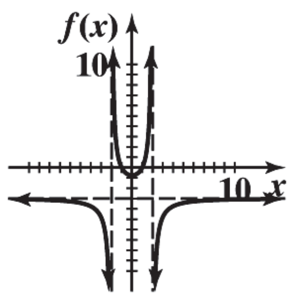


50. (A) Intercepts:

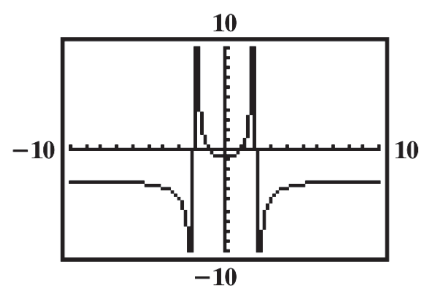
x-intercept(s):	y-intercept:
$3-3x^2=0$ $3x^2=3$ $x=\pm 1$ (1,0), (–1,0)	$f(0)=-\frac{3}{4}$ $\left(0,\frac{-3}{4}\right)$

(B) Vertical asymptotes when $x^2-4=0$; i.e. at $x=2$ and $x=-2$.
Horizontal asymptote at $y=-3$

(C)



(D)

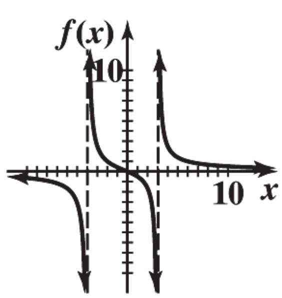


52. (A) Intercepts:

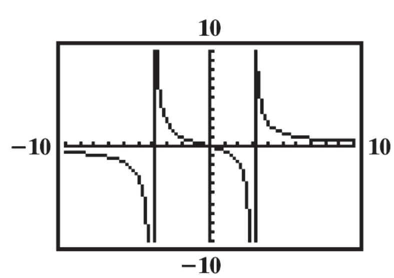
x-intercept(s):	y-intercept:
$5x-10=0$ $x=2$ (2,0)	$f(0)=\frac{-10}{-12}=\frac{5}{6}$ (0,5/6)

(B) Vertical asymptote when $x^2+x-12=(x+4)(x-3)=0$; i.e. when $x=-4$ and when $x=3$.
Horizontal asymptote at $y=0$.

(C)



(D)



54. $f(x) = -(x+2)(x-1) = -x^2 - x + 2$

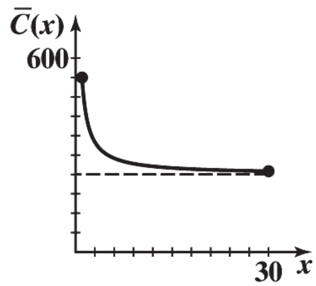
56. $f(x) = x(x+1)(x-1) = x\left(x^2-1\right) = x^3 - x$

58. (A) We want $C(x) = mx + b$. Fixed costs are $b = \$300$ per day. Given $C(20) = 5{,}100$ we have

$$m(20) + 300 = 5{,}100$$
$$20m = 4800$$
$$m = 240$$
$$C(x) = 240x + 300$$

(B) $\bar{C}(x) = \dfrac{C(x)}{x} = \dfrac{240x+300}{x} = 240 + \dfrac{300}{x}$

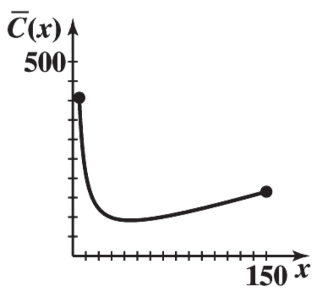
(C)



(D) Average cost tends towards \$240 as production increases.

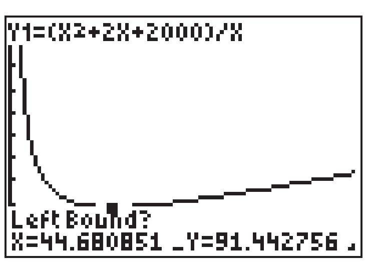
60. (A) $\bar{C}(x) = \dfrac{x^2 + 2x + 2{,}000}{x}$

(B)



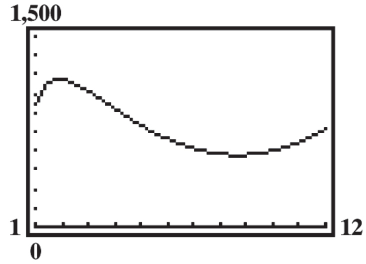
(C) A daily production level of $x = 45$ units per day, results in the lowest average cost of $\bar{C}(45) = \$91.44$ per unit.

(D)



62. (A) $\overline{C}(x) = \dfrac{20x^3 - 360x^2 + 2,300x - 1,000}{x}$

(B)



(C) A minimum average cost of $566.84 is achieved at a production level of $x = 8.67$ thousand cases per month.

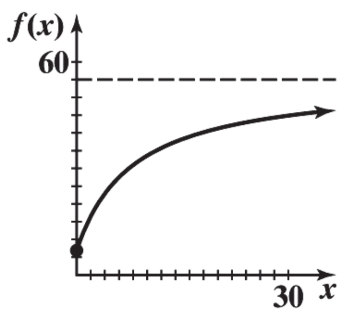
64. (A)

```
CubicReg
 y=ax³+bx²+cx+d
 a=⁻.0091111111
 b=.5004761905
 c=⁻7.655555556
 d=269.3571429
```

(B) $y(42) = 156$ eggs

66. (A) The horizontal asymptote is $y = 55$.

(B)



68. (A)

```
CubicReg
 y=ax³+bx²+cx+d
 a=4.4444444E-5
 b=⁻.0065833333
 c=.2471031746
 d=2.073809524
```

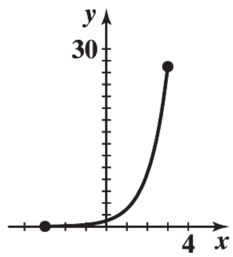
(B) This model gives an estimate of 2.5 divorces per 1,000 marriages.

EXERCISE 2-5

2. A. graph *g* B. graph *f* C. graph *h* D. graph *k*

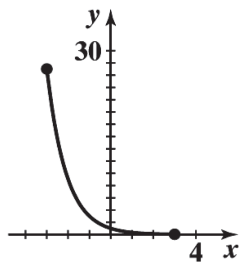
4. $y = 3^x$; $[-3, 3]$

x	y
–3	$\frac{1}{27}$
–1	$\frac{1}{3}$
0	1
1	3
3	27



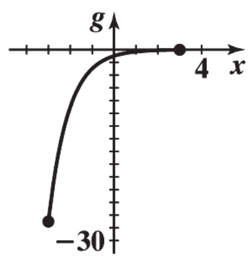
6. $y = 3^{-x}$; $[-3, 3]$

x	y
–3	27
–1	3
0	1
1	$\frac{1}{3}$
3	$\frac{1}{27}$



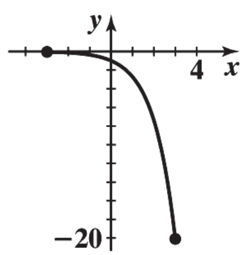
8. $g(x) = -3^{-x}$; $[-3, 3]$

x	$g(x)$
–3	–27
–1	–3
0	–1
1	$-\frac{1}{3}$
3	$-\frac{1}{27}$



10. $y = -e^x$; $[-3, 3]$

x	y
–3	≈ -0.05
–1	≈ -0.37
0	–1
1	≈ -2.72
3	≈ -20.09



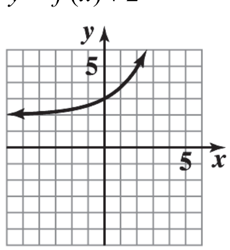
12. The graph of g is the graph of f shifted 2 units to the right.

14. The graph of g is the graph of f reflected in the x axis.

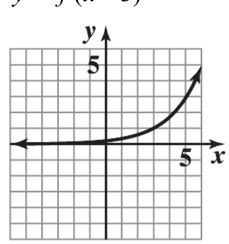
16. The graph of g is the graph of f shifted 2 units down.

18. The graph of g is the graph of f vertically contracted by a factor of 0.5 and shifted 1 unit to the right.

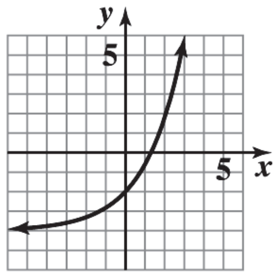
20. A. $y = f(x) + 2$



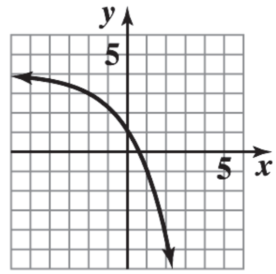
B. $y = f(x-3)$



C. $y = 2f(x) - 4$

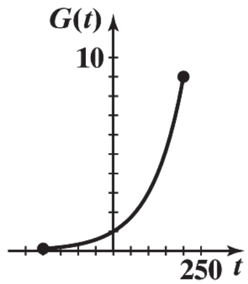


D. $y = 4 - f(x+2)$



22. $G(t) = 3^{t/100}$; $[-200, 200]$

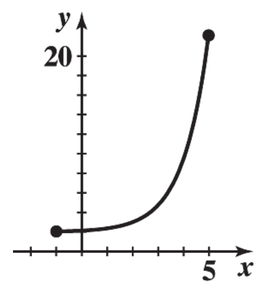
x	$G(t)$
-200	$\frac{1}{9}$
-100	$\frac{1}{3}$
0	1
100	3
200	9



24.

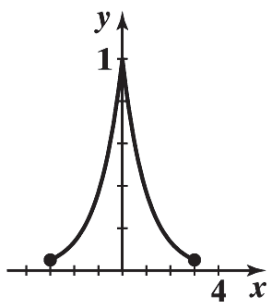
$y = 2 + e^{x-2}$; $[-1, 5]$

x	y
-1	≈ 2.05
0	≈ 2.14
1	≈ 2.37
3	≈ 4.72
5	≈ 22.09



26. $y = e^{-|x|}$; $[-3, 3]$

x	y
-3	≈ 0.05
-1	≈ 0.37
0	1
1	≈ 0.37
3	≈ 0.05



28. $a = 2$, $b = -2$ for example. The exponential function property: For $x \neq 0$, $a^x = b^x$ if and only if $a = b$ assumes $a > 0$ and $b > 0$.

30.
$$5^{3x} = 5^{4x-2}$$
$$3x = 4x - 2$$
$$-x = -2$$
$$x = 2$$

32.
$$7^{x^2} = 7^{2x+3}$$
$$x^2 = 2x + 3$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = -1, 3$$

34.
$$(1-x)^5 = (2x-1)^5$$
$$1 - x = 2x - 1$$
$$-3x = -2$$
$$x = \frac{2}{3}$$

36.
$$10xe^x - 5e^x = 0$$
$$e^x(10x - 5) = 0$$
$$10x - 5 = 0 \quad (\text{since } e^x \neq 0)$$
$$x = 1/2$$

38.
$$x^2e^{-x} - 9e^{-x} = 0$$
$$e^{-x}(x^2 - 9) = 0$$
$$(x^2 - 9) = 0 \quad (\text{since } e^{-x} \neq 0)$$
$$x = -3, 3$$

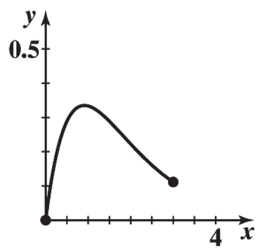
40. $e^{4x} + e > 0$ for all x;

$e^{4x} + e = 0$ has no solutions.

42.
$$e^{3x-1} - e = 0$$
$$e^{3x-1} = e^1$$
$$3x - 1 = 1$$
$$x = 2/3$$

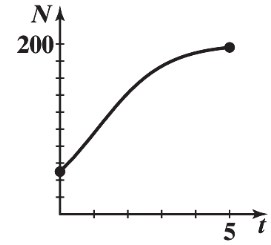
44. $m(x) = x(3^{-x})$; $[0, 3]$

x	$m(x)$
0	0
1	$\frac{1}{3}$
2	$\frac{2}{9}$
3	$\frac{1}{9}$



46. $N = \dfrac{200}{1+3e^{-t}}$; $[0, 5]$

x	N
0	50
1	≈ 95.07
2	≈ 142.25
3	≈ 174.01
4	≈ 189.58
5	≈ 196.04



48.
$$A = Pe^{rt}$$
$$A = (24{,}000)e^{(0.0435)(7)}$$
$$A = (24{,}000)e^{0.3045}$$
$$A = (24{,}000)(1.35594686)$$
$$A = \$32{,}542.72$$

50. (A)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 4000(1+\tfrac{0.06}{52})^{(52)(0.5)}$$
$$A = 4000(1.0011538462)^{26}$$
$$A = 4000(1.030436713)$$
$$A = \$4121.75$$

(B)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 4000(1+\tfrac{0.06}{52})^{(52)(10)}$$
$$A = 4000(1.0011538462)^{520}$$
$$A = 4000(1.821488661)$$
$$A = \$7285.95$$

52.
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$40{,}000 = P(1+\tfrac{0.055}{365})^{(365)(17)}$$
$$40{,}000 = P(1.0001506849)^{6205}$$
$$40{,}000 = P(2.547034043)$$
$$P = \$15{,}705$$

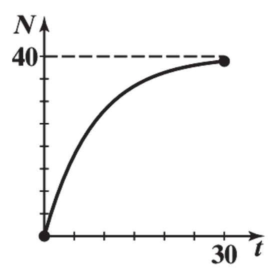
54. (A)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 10{,}000(1+\tfrac{0.0135}{4})^{(4)(5)}$$
$$A = 10{,}000(1.003375)^{20}$$
$$A = 10{,}000(1.069709)$$
$$A = \$10{,}697.09$$

(B)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 10{,}000(1+\tfrac{0.0130}{12})^{(12)(5)}$$
$$A = 10{,}000(1.00108333)^{60}$$
$$A = 10{,}000(1.067121479)$$
$$A = \$10{,}671.21$$

(C)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 10{,}000(1+\tfrac{0.0125}{365})^{(365)(5)}$$
$$A = 10{,}000(1.000034247)^{1825}$$
$$A = 10{,}000(1.06449332)$$
$$A = \$10{,}644.93$$

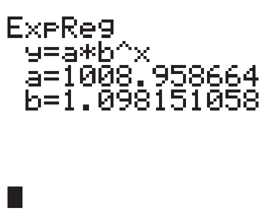
56. $N = 40(1-e^{-0.12t})$; $[0, 30]$

x	N
0	0
10	≈ 27.95
20	≈ 36.37
30	≈ 38.91



The maximum number of boards an average employee can be expected to produce in 1 day is 40.

58.



(A) The average salary in 2022: $y(32) \approx$ \$20,186,000.

(B) The model gives an average salary of $y(7) \approx$ \$1,943,000 in 1997.

60. (A) $I(50) = I_o e^{-0.00942(50)} \approx 62\ \%$ (B) $I(100) = I_o e^{-0.00942(100)} \approx 39\ \%$

62. (A) $P = 94e^{0.032t}$.

(B) Population in 2025: $P(13) = 94e^{0.032(13)} \approx 142{,}000{,}000$;

Population in 2035: $P(23) = 94e^{0.032(23)} \approx 196{,}000{,}000$.

64.

```
ExpReg
 y=a*b^x
 a=71.63144793
 b=1.002343596
```

Life expectancy for a person born in 2025: $y(55) \approx 81.5$ years.

EXERCISE 2-6

2. $\log_2 32 = 5 \Rightarrow 32 = 2^5$ **4.** $\log_e 1 = 0 \Rightarrow e^0 = 1$ **6.** $\log_9 27 = \frac{3}{2} \Rightarrow 27 = 9^{3/2}$ **8.** $36 = 6^2 \Rightarrow \log_6 36 = 2$

10. $9 = 27^{2/3} \Rightarrow \log_{27} 9 = \frac{2}{3}$ **12.** $M = b^x \Rightarrow \log_b M = x$ **14.** $\log_{10} 100{,}000 = \log_{10} 10^5 = 5$ **16.** $\log_3 \frac{1}{3} = \log_3 3^{-1} = -1$

18. $\log_4 1 = \log_4 4^0 = 0$ **20.** $\ln e^{-5} = -5$ **22.** $\log_b FG = \log_b F + \log_b G$ **24.** $\log_b w^{15} = 15 \log_b w$

26. $\frac{\log_3 P}{\log_3 R} = \log_R P$

28. $\log_2 x = 2$
$$2^2 = x$$
$$4 = x$$

30. $\log_3 27 = y$
$$3^y = 27$$
$$3^y = 3^3$$
$$y = 3$$

32. $\log_b e^{-2} = -2$
$$e^{-2} = b^{-2}$$
$$e = b$$

34. $\log_{25} x = \frac{1}{2}$
$$25^{1/2} = x$$
$$5 = x$$

36. False; an example of a polynomial function of odd degree that is not one–to–one is $f(x) = x^3 - x$.
$f(-1) = f(0) = f(1) = 0$.

38. True; the graph of every function (not necessarily one–to–one) intersects each vertical line exactly once.

40. False; $x = -1$ is in the domain of *f*, but cannot be in the range of *g*.

42. True; since *g* is the inverse of *f*, then (*a*, *b*) is on the graph of *f* if and only if (*b*, *a*) is on the graph of *g*. Therefore, *f* is also the inverse of *g*.

44. $\log_b x = \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$
$$\log_b x = \log_b 27^{2/3} + \log_b 2^2 - \log_b 3$$
$$\log_b x = \log_b 9 + \log_b 4 - \log_b 3$$
$$\log_b x = \log_b \frac{(9)(4)}{3}$$
$$\log_b x = \log_b 12$$
$$x = 12$$

46. $\log_b x = 3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20$
$$\log_b x = \log_b 2^3 + \log_b 25^{1/2} - \log_b 20$$
$$\log_b x = \log_b 8 + \log_b 5 - \log_b 20$$
$$\log_b x = \log_b \frac{(8)(5)}{20}$$
$$\log_b x = \log_b 2$$
$$x = 2$$

48. $\log_b(x+2)+\log_b x=\log_b 24$

$$\log_b(x+2)x=\log_b 24$$
$$\log_b(x^2+2x)=\log_b 24$$
$$x^2+2x=24$$
$$x^2+2x-24=0$$
$$(x+6)(x-4)=0$$
$$x=-6,4$$

Since the domain of a logarithmic function is $(0,\infty)$, omit the negative solution. Therefore, the solution is $x=4$.

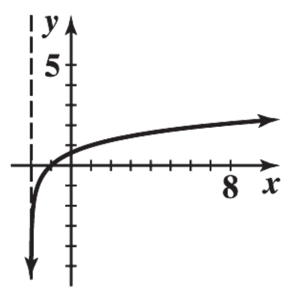
50. $\log_{10}(x+6)-\log_{10}(x-3)=1$

$$\log_{10}\frac{x+6}{x-3}=1$$
$$10^1=\frac{x+6}{x-3}$$
$$10(x-3)=x+6$$
$$10x-30=x+6$$
$$x=4$$

52. $y=\log_3(x+2)$

$$3^y=x+2$$
$$3^y-2=x$$

x	y
$-\frac{53}{27}$	-3
$-\frac{17}{9}$	-2
$-\frac{5}{3}$	-1
-1	0
1	1
7	2
25	3



54. The graph of $y=\log_3(x+2)$ is the graph of $y=\log_3 x$ shifted to the left 2 units.

56. The domain of logarithmic function is defined for positive values only. Therefore, the domain of the function is $x-1>0$ or $x>1$. The range of a logarithmic function is all real numbers. In interval notation the domain is $(1,\infty)$ and the range is $(-\infty,\infty)$.

58. A. $\log 72.604=1.86096$

B. $\log 0.033041=-1.48095$

C. $\ln 40,257=10.60304$

D. $\ln 0.0059263=-5.12836$

60. A. $\log x=2.0832$

$x=\log^{-1}(2.0832)=10^{2.0832}$

$x=121.1156$

B. $\log x=-1.1577$

$x=\log^{-1}(-1.1577)=10^{-1.1577}$

$x=0.0696$

C. $\ln x=3.1336$

$x=\ln^{-1}(3.1336)=e^{3.1336}$

$x=22.9565$

D. $\ln x = -4.3281$

$x = \ln^{-1}(-4.3281) = e^{-4.3281}$

$x = 0.0132$

62. $10^x = 153$

$\log 10^x = \log 153$

$x = 2.1847$

64. $e^x = 0.3059$

$\ln e^x = \ln 0.3059$

$x = -1.1845$

66. $1.02^{4t} = 2$

$\ln 1.02^{4t} = \ln 2$

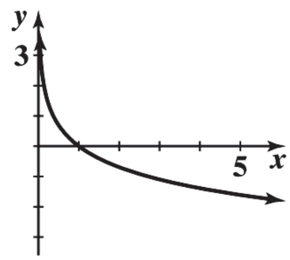
$4t \ln 1.02 = \ln 2$

$$t = \frac{\ln 2}{4 \ln 1.02}$$

$t = 8.7507$

68. $y = -\ln x;\ \ x > 0$

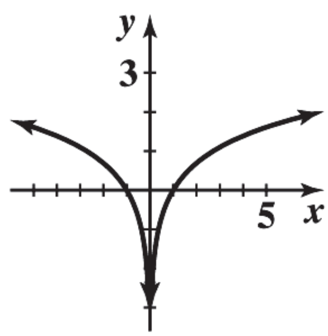
x	y
0.5	≈ 0.69
1	0
2	≈ -0.69
4	≈ -1.39
5	≈ -1.61



Based on the graph above, the function is decreasing on the interval $(0, \infty)$.

70. $y = \ln|x|$

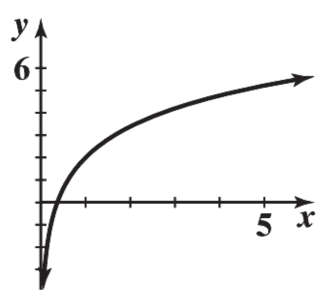
x	y
−5	≈ 1.61
−2	≈ 0.69
1	0
2	≈ 0.69
5	≈ 1.61



Based on the graph above, the function is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$.

72. $y = 2\ln x + 2$

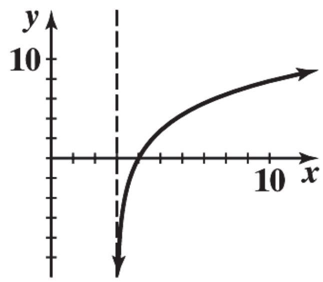
x	y
0.5	≈ 0.61
1	2
2	≈ 3.39
4	≈ 4.77
5	≈ 5.22



Based on the graph above, the function is increasing on the interval $(0, \infty)$.

74. $y = 4\ln(x-3)$

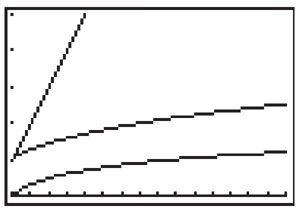
x	y
4	0
6	≈ 4.39
8	≈ 6.44
10	≈ 7.78
12	≈ 8.79



Based on the graph above, the function is increasing on the interval $(3, \infty)$.

76. It is not possible to find a power of 1 that is an arbitrarily selected real number, because 1 raised to any power is 1.

78.



A function *f* is "smaller than" a function *g* on an interval $[a, b]$ if $f(x) < g(x)$ for $a \le x \le b$. Based on the graph above, $\log x < \sqrt[3]{x} < x$ for $1 < x \le 16$.

80. Use the compound interest formula: $A = P(1+r)^t$. The problem is asking for the original amount to double, therefore $A = 2P$.

$$2P = P(1+0.0958)^t$$
$$2 = (1.0958)^t$$
$$\ln 2 = \ln(1.0958)^t$$
$$\ln 2 = t\ln(1.0958)$$
$$\frac{\ln 2}{\ln 1.0958} = t$$
$$7.58 \approx t$$

It will take approximately 8 years for the original amount to double.

82. Use the compound interest formula:
$A = P(1+\frac{r}{m})^{mt}$.

(A)
$$7500 = 5000(1+\tfrac{0.08}{2})^{2t}$$
$$1.5 = (1.04)^{2t}$$
$$\ln 1.5 = \ln(1.04)^{2t}$$
$$\ln 1.5 = 2t\ln(1.04)$$
$$\frac{\ln 1.5}{2\ln 1.04} = t$$
$$5.17 \approx t$$

It will take approximately 5.17 years for \$5000 to grow to \$7500 if compounded semiannually.

(B)
$$7500 = 5000(1+\tfrac{0.08}{12})^{12t}$$
$$1.5 = (1.0066667)^{12t}$$
$$\ln 1.5 = \ln(1.0066667)^{12t}$$
$$\ln 1.5 = 12t\ln(1.0066667)$$
$$\frac{\ln 1.5}{12\ln 1.0066667} = t$$
$$5.09 \approx t$$

It will take approximately 5.09 years for \$5000 to grow to \$7500 if compounded monthly.

It will take approximately 5.09 years for \$5000 to grow to \$7500 if compounded monthly.

84. Use the compound interest formula: $A = Pe^{rt}$.

$$41,000 = 17,000e^{0.0295t}$$
$$\tfrac{41}{17} = e^{0.0295t}$$
$$\ln\tfrac{41}{17} = \ln e^{0.0295t}$$
$$\ln\tfrac{41}{17} = 0.0295t$$
$$\frac{\ln\tfrac{41}{17}}{0.0295} = t$$
$$29.84 \approx t$$

It will take approximately 29.84 years for \$17,000 to grow to \$41,000 if compounded continuously.

86. Equilibrium occurs when supply and demand are equal. The models from Problem 85 have the demand and supply functions defined by $y = 256.4659159 - 24.03812068\ln x$ and $y = -127.8085281 + 20.01315349\ln x$, respectively. Set both equations equal to each other to yield:

$$256.4659159 - 24.03812068\ln x = -127.8085281 + 20.01315349\ln x$$
$$384.274444 = 44.05127417\ln x$$
$$\frac{384.274444}{44.05127417} = \ln x$$
$$e^{384.274444/44.05127417} = e^{\ln x}$$
$$6145 \approx x$$

Substitute the value above into either equation.

$$y = 256.4659159 - 24.03812068\ln x$$
$$y = 256.4659159 - 24.03812068\ln(6145)$$
$$y = 256.4659159 - 24.03812068(8.723394022)$$
$$y = 46.77$$

Therefore, equilibrium occurs when 6145 units are produced and sold at a price of \$46.77.

88. (A) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-13}}{10^{-16}} = 10\log 10^3 = 30$

(B) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{3.16\times 10^{-10}}{10^{-16}} = 10\log 3.16\times 10^6 \approx 65$

(C) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-8}}{10^{-16}} = 10\log 10^8 = 80$

(D) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-1}}{10^{-16}} = 10\log 10^{15} = 150$

90.

```
LnReg
 y=a+blnx
 a=-45845.97493
 b=12130.89096
```

2024: $t = 124$; $y(124) \approx 12,628$. Therefore, according to the model, the total production in the year 2024 will be approximately 12,628 million bushels.

92. $$\begin{aligned} A &= A_0 e^{-0.000124t} \\ 0.1A_0 &= A_0 e^{-0.000124t} \\ 0.1 &= e^{-0.000124t} \\ \ln 0.1 &= \ln e^{-0.000124t} \\ \ln 0.1 &= -0.000124t \\ 18{,}569 &\approx t \end{aligned}$$

If 10% of the original amount is still remaining, the skull would be approximately 18,569 years old.