

2 Review and Applications of Algebra

Exercise 2.1

1. $(-p) + (-3p) + 4p = -p - 3p + 4p = \underline{0}$
2. $(5s - 2t) - (2s - 4t) = 5s - 2t - 2s + 4t = \underline{3s + 2t}$
3. $4x^2y + (-3x^2y) - (-5x^2y) = 4x^2y - 3x^2y + 5x^2y = \underline{6x^2y}$
4. $1 - (7e^2 - 5 + 3e - e^3) = 1 - 7e^2 + 5 - 3e + e^3 = \underline{e^3 - 7e^2 - 3e + 6}$
5. $(6x^2 - 3xy + 4y^2) - (8y^2 - 10xy - x^2) = 6x^2 - 3xy + 4y^2 - 8y^2 + 10xy + x^2$
 $= \underline{7x^2 + 7xy - 4y^2}$
6. $(7m^3 - m - 6m^2 + 10) - (5m^3 - 9 + 3m - 2m^2)$
 $= 7m^3 - m - 6m^2 + 10 - 5m^3 + 9 - 3m + 2m^2$
 $= \underline{2m^3 - 4m^2 - 4m + 19}$
7. $2(7x - 3y) - 3(2x - 3y) = 14x - 6y - 6x + 9y = \underline{8x + 3y}$
8. $4(a^2 - 3a - 4) - 2(5a^2 - a - 6) = 4a^2 - 12a - 16 - 10a^2 + 2a + 12$
 $= \underline{-6a^2 - 10a - 4}$
9. $15x - [4 - 2(5x - 6)] = 15x - 4 + 10x - 12 = \underline{25x - 16}$
10. $6a - [3a - 2(2b - a)] = 6a - 3a + 4b - 2a = \underline{a + 4b}$
11. $\frac{2x+9}{4} - 1.2(x-1) = 0.5x + 2.25 - 1.2x + 1.2 = -\underline{0.7x + 3.45}$
12. $\frac{x}{2} - x^2 + \frac{4}{5} - 0.2x^2 - \frac{4}{5}x + \frac{1}{2} = 0.5x - x^2 + 0.8 - 0.2x^2 - 0.8x + 0.5$
 $= \underline{-1.2x^2 - 0.3x + 1.3}$
13. $\frac{8x}{0.5} + \frac{5.5x}{11} + 0.5(4.6x - 17) = 16x + 0.5x + 2.3x - 8.5 = \underline{18.8x - 8.5}$
14. $\frac{2x}{1.045} - \frac{2.016x}{3} + \frac{x}{2} = 1.9139x - 0.6720x + 0.5x = \underline{1.7419x}$
15. $\frac{P}{1+0.095 \times \frac{5}{12}} + 2P\left(1+0.095 \times \frac{171}{365}\right) = 0.96192P + 2.08901P = \underline{3.0509P}$
16. $y\left(1-0.125 \times \frac{213}{365}\right) + \frac{2y}{1+0.125 \times \frac{88}{365}} = 0.92706y + 1.94149y = \underline{2.8685y}$
17. $k(1+0.04)^2 + \frac{2k}{(1+0.04)^2} = 1.08160k + 1.84911k = \underline{2.9307k}$
18. $\frac{h}{(1+0.055)^2} - 3h(1+0.055)^3 = 0.89845h - 3.52272h = \underline{-2.6243h}$
19. $4a(3ab - 5a + 6b) = \underline{12a^2b - 20a^2 + 24ab}$

Exercise 2.1 (continued)

20. $9k(4 - 8k + 7k^2) = \underline{36k - 72k^2 + 63k^3}$
21. $-5xy(2x^2 - xy - 3y^2) = \underline{-10x^3y + 5x^2y^2 + 15xy^3}$
22. $-(p^2 - 4pq - 5p)\left(\frac{2q}{p}\right) = \underline{-2pq + 8q^2 + 10q}$
23. $(4r - 3t)(2t + 5r) = 8rt + 20r^2 - 6t^2 - 15rt = \underline{20r^2 - 7rt - 6t^2}$
24. $(3p^2 - 5p)(-4p + 2) = -12p^3 + 6p^2 + 20p^2 - 10p = \underline{-12p^3 + 26p^2 - 10p}$
25. $3(a - 2)(4a + 1) - 5(2a + 3)(a - 7) = 3(4a^2 + a - 8a - 2) - 5(2a^2 - 14a + 3a - 21)$
 $= 12a^2 - 21a - 6 - 10a^2 + 55a + 105$
 $= \underline{2a^2 + 34a + 99}$
26. $5(2x - y)(y + 3x) - 6x(x - 5y) = 5(2xy + 6x^2 - y^2 - 3xy) - 6x^2 + 30xy$
 $= -5xy + 30x^2 - 5y^2 - 6x^2 + 30xy$
 $= \underline{24x^2 + 25xy - 5y^2}$
27. $\frac{18x^2}{3x} = \underline{6x}$
28. $\frac{6a^2b}{-2ab^2} = \underline{-3\frac{a}{b}}$
29. $\frac{x^2y - xy^2}{xy} = \underline{x - y}$
30. $\frac{-4x + 10x^2 - 6x^3}{-0.5x} = \underline{8 - 20x + 12x^2}$
31. $\frac{12x^3 - 24x^2 + 36x}{48x} = \underline{\underline{\frac{x^2 - 2x + 3}{4}}}$
32. $\frac{32a^2b - 8ab + 14ab^2}{2ab} = \underline{16a - 4 + 7b}$
33. $\frac{4a^2b^3 - 6a^3b^2}{2ab^2} = \underline{2ab - 3a^2}$
34. $\frac{120(1+i)^2 + 180(1+i)^3}{360(1+i)} = \underline{\underline{\frac{2(1+i) + 3(1+i)^2}{6}}}$
35. $3d^2 - 4d + 15 = 3(2.5)^2 - 4(2.5) + 15$
 $= 18.75 - 10 + 15$
 $= \underline{23.75}$
36. $15g - 9h + 3 = 15(14) - 9(15) + 3 = \underline{78}$
37. $7x(4y - 8) = 7(3.2)(4 \times 1.5 - 8) = 22.4(6 - 8) = \underline{-44.8}$
38. $I \div Pr = \frac{\$13.75}{\$500 \times 0.11} = \underline{0.250}$
39. $\frac{I}{rt} = \frac{\$23.21}{0.095 \times \frac{283}{365}} = \frac{\$23.21}{0.073658} = \underline{\$315.11}$

Exercise 2.1 (continued)

$$40. \quad \frac{N}{1-d} = \frac{\$89.10}{1-0.10} = \underline{\underline{\$99.00}}$$

$$41. \quad L(1-d_1)(1-d_2)(1-d_3) = \$490(1-0.125)(1-0.15)(1-0.05) = \underline{\underline{\$346.22}}$$

$$42. \quad P(1+rt) = \$770 \left(1 + 0.013 \times \frac{223}{365} \right) = \$770(1.0079425) = \underline{\underline{\$776.12}}$$

$$43. \quad \frac{S}{1+rt} = \frac{\$2500}{1+0.085 \times \frac{123}{365}} = \frac{\$2500}{1.028644} = \underline{\underline{\$2430.38}}$$

$$44. \quad (1+i)^m - 1 = (1+0.0225)^4 - 1 = \underline{\underline{0.093083}}$$

$$45. \quad P(1+i)^n = \$1280(1+0.025)^3 = \underline{\underline{\$1378.42}}$$

$$46. \quad \frac{S}{(1+i)^n} = \frac{\$850}{(1+0.0075)^6} = \frac{\$850}{1.045852} = \underline{\underline{\$812.73}}$$

$$47. \quad R \left[\frac{(1+i)^n - 1}{i} \right] = \$550 \left(\frac{1.085^3 - 1}{0.085} \right) = \$550 \left(\frac{0.2772891}{0.085} \right) = \underline{\underline{\$1794.22}}$$

$$\begin{aligned} 48. \quad R \left[\frac{(1+i)^n - 1}{i} \right] (1+i) &= \$910 \left(\frac{1.1038129^4 - 1}{0.1038129} \right) (1.1038129) \\ &= \$910 \left(\frac{0.4845057}{0.1038129} \right) (1.1038129) \\ &= \underline{\underline{\$4687.97}} \end{aligned}$$

$$49. \quad \frac{R}{i} \left[1 - \frac{1}{(1+i)^n} \right] = \frac{\$630}{0.115} \left(1 - \frac{1}{1.115^2} \right) = \underline{\underline{\$1071.77}}$$

$$\begin{aligned} 50. \quad P(1+rt_1) + \frac{S}{1+rt_2} &= \$470 \left(1 + 0.075 \times \frac{104}{365} \right) + \frac{\$390}{1 + 0.075 \times \frac{73}{365}} \\ &= \$470(1.021370) + \frac{\$390}{1.01500} \\ &= \$480.044 + \$384.236 \\ &= \underline{\underline{\$864.28}} \end{aligned}$$

Exercise 2.2

$$\begin{aligned} 1. \quad I &= Prt \\ \$6.25 &= P(0.05)0.25 \\ \$6.25 &= 0.0125P \\ P &= \frac{\$6.25}{0.0125} = \underline{\underline{\$500.00}} \end{aligned}$$

Exercise 2.2 (continued)

$$\begin{aligned} 2. \quad PV &= \frac{PMT}{i} \\ \$150,000 &= \frac{\$900}{i} \\ \$150,000i &= \$900 \\ i &= \frac{\$900}{\$150,000} = \underline{\underline{0.00600}} \end{aligned}$$

$$\begin{aligned} 3. \quad S &= P(1 + rt) \\ \$3626 &= P(1 + 0.004 \times 9) \\ \$3626 &= 1.036P \\ P &= \frac{\$3626}{1.036} = \underline{\underline{\$3500.00}} \end{aligned}$$

$$\begin{aligned} 4. \quad N &= L(1 - d) \\ \$891 &= L(1 - 0.10) \\ \$891 &= 0.90L \\ L &= \frac{\$891}{0.90} = \underline{\underline{\$9900.00}} \end{aligned}$$

$$\begin{aligned} 5. \quad N &= L(1 - d) \\ \$410.85 &= \$498(1 - d) \\ \frac{\$410.85}{\$498} &= 1 - d \\ 0.825 &= 1 - d \\ d &= 1 - 0.825 = \underline{\underline{0.175}} \end{aligned}$$

$$\begin{aligned} 6. \quad S &= P(1 + rt) \\ \$5100 &= \$5000(1 + 0.0025t) \\ \$5100 &= \$5000 + \$12.5t \\ \$5100 - \$5000 &= \$12.5t \\ t &= \frac{\$100}{\$12.5} = \underline{\underline{8.00}} \end{aligned}$$

$$\begin{aligned} 7. \quad NI &= (CM)X - FC \\ \$15,000 &= CM(5000) - \$60,000 \\ \$15,000 + \$60,000 &= 5000CM \\ CM &= \frac{\$75,000}{5000} = \underline{\underline{\$15.00}} \end{aligned}$$

$$\begin{aligned} 8. \quad NI &= (CM)X - FC \\ -\$542.50 &= (\$13.50)X - \$18,970 \\ \$18,970 - \$542.50 &= (\$13.50)X \\ X &= \frac{\$18,427.50}{\$13.50} = \underline{\underline{1365}} \end{aligned}$$

$$\begin{aligned} 9. \quad N &= L(1 - d_1)(1 - d_2)(1 - d_3) \\ \$1468.80 &= L(1 - 0.20)(1 - 0.15)(1 - 0.10) \\ \$1468.80 &= L(0.80)(0.85)(0.90) \\ L &= \frac{\$1468.80}{0.6120} = \underline{\underline{\$2400.00}} \end{aligned}$$

Exercise 2.2 (continued)

$$10. \quad N = L(1-d_1)(1-d_2)(1-d_3)$$

$$\$70.29 = \$99.99(1-0.20)(1-d_2)(1-0.05)$$

$$\$70.29 = \$75.9924(1-d_2)$$

$$\frac{\$70.29}{\$75.9924} = (1-d_2)$$

$$d_2 = 1 - 0.92496 = \underline{0.0750}$$

$$11. \quad FV = PV(1+i_1)(1+i_2)(1+i_3)\cdots(1+i_n)$$

$$\$1094.83 = \$1000(1+i_1)(1+0.03)(1+0.035)$$

$$\$1094.83 = \$1066.05(1+i_1)$$

$$\frac{\$1094.83}{\$1066.05} = 1+i_1$$

$$i_1 = 1.02700 - 1 = \underline{0.0270}$$

$$12. \quad FV = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

$$\$1508.54 = PMT \left[\frac{(1+0.05)^4 - 1}{0.05} \right]$$

$$\$1508.54 = PMT \left(\frac{1.21550625 - 1}{0.05} \right)$$

$$PMT = \$1508.54 \times \frac{0.05}{0.21550625} = \underline{\$350.00}$$

$$13. \quad PV = PMT \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$\$6595.20 = PMT \left[\frac{1 - (1+0.06)^{-20}}{0.06} \right]$$

$$\$6595.20 = PMT \left[\frac{1 - 0.31180473}{0.06} \right]$$

$$PMT = \$6595.20 \times \frac{0.06}{0.68819527} = \underline{\$575.00}$$

$$14. \quad FV = PV(1+i)^n$$

$$\$9321.91 = \$2000(1+i)^{20}$$

$$\left(\frac{\$9321.91}{\$2000} \right)^{1/20} = 1+i$$

$$1.0800 = 1+i$$

$$i = 1.08000 - 1 = \underline{0.0800}$$

Exercise 2.2 (continued)

$$\begin{aligned}
 15. \quad PV &= FV(1+i)^{-n} \\
 \$5167.20 &= \$10,000 \\
 \frac{\$5167.20}{\$10,000} &= \frac{1}{(1+i)^{15}} \\
 (1+i)^{15} &= \frac{\$10,000}{\$5167.20} \\
 1+i &= (1.935284)^{1/15} = 1.0450 \\
 i &= \underline{\underline{0.0450}}
 \end{aligned}$$

$$16. \quad I = Prt$$

$$\begin{aligned}
 \frac{I}{Pr} &= \frac{Prt}{Pr} \\
 t &= \frac{I}{Pr}
 \end{aligned}$$

$$18. \quad N = L(1-d)$$

$$\frac{N}{L} = 1-d$$

$$d = 1 - \frac{N}{L}$$

$$20. \quad NI = (CM)X - FC$$

$$NI + FC = (CM)X$$

$$X = \frac{NI + FC}{CM}$$

$$22. \quad S = P(1+rt)$$

$$S = P + Prt$$

$$S - P = Prt$$

$$t = (S - P) / Pr$$

$$24. \quad N = L(1-d_1)(1-d_2)(1-d_3)$$

$$\frac{N}{L(1-d_1)(1-d_2)} = (1-d_3)$$

$$d_3 = 1 - \frac{N}{L(1-d_1)(1-d_2)}$$

$$25. \quad FV = PV(1+i)^n$$

$$\frac{FV}{(1+i)^n} = PV$$

$$PV = FV(1+i)^{-n}$$

$$17. \quad PV = \frac{PMT}{i}$$

$$i(PV) = PMT$$

$$i = \frac{PMT}{PV}$$

$$19. \quad NI = (CM)X - FC$$

$$NI + FC = (CM)X$$

$$CM = \frac{NI + FC}{X}$$

$$21. \quad S = P(1+rt)$$

$$S = P + Prt$$

$$S - P = Prt$$

$$r = (S - P) / Pt$$

$$23. \quad N = L(1-d_1)(1-d_2)(1-d_3)$$

$$\frac{N}{L(1-d_2)(1-d_3)} = (1-d_1)$$

$$d_1 = 1 - \frac{N}{L(1-d_2)(1-d_3)}$$

$$26. \quad FV = PV(1+i)^n$$

$$\left(\frac{FV}{PV} \right)^{1/n} = (1+i)$$

$$i = \left(\frac{FV}{PV} \right)^{1/n} - 1$$

Exercise 2.2 (continued)

27. $a^2 \times a^3 = \underline{\underline{a^5}}$

28. $(x^6)(x^{-4}) = \underline{\underline{x^2}}$

29. $b^{10} \div b^6 = b^{10-6} = \underline{\underline{b^4}}$

30. $h^7 \div h^{-4} = h^{7-(-4)} = \underline{\underline{h^{11}}}$

31. $(1+i)^4 \times (1+i)^9 = \underline{\underline{(1+i)^{13}}}$

32. $(1+i) \times (1+i)^n = \underline{\underline{(1+i)^{n+1}}}$

33. $(x^4)^7 = x^{4 \times 7} = \underline{\underline{x^{28}}}$

34. $(y^3)^3 = \underline{\underline{y^9}}$

35. $(t^6)^{\frac{1}{3}} = \underline{\underline{t^2}}$

36. $(n^{0.5})^8 = \underline{\underline{n^4}}$

37. $\frac{(x^5)(x^6)}{x^9} = x^{5+6-9} = \underline{\underline{x^2}}$

38. $\frac{(x^5)^6}{x^9} = x^{5 \times 6 - 9} = \underline{\underline{x^{21}}}$

39. $[2(1+i)]^2 = \underline{\underline{4(1+i)^2}}$

40. $\left(\frac{1+i}{3i}\right)^3 = \frac{(1+i)^3}{\underline{\underline{27i^3}}}$

41. $\frac{4r^5t^6}{(2r^2t)^3} = \frac{4r^5t^6}{8r^6t^3} = \frac{r^{5-6}t^{6-3}}{2} = \underline{\underline{\frac{t^3}{2r}}}$

42. $\frac{(-r^3)(2r)^4}{(2r^{-2})^2} = \frac{-r^3(16r^4)}{4r^{-4}} = -4r^{3+4-(-4)} = \underline{\underline{-4r^{11}}}$

43. $8^{\frac{4}{3}} = \left(8^{\frac{1}{3}}\right)^4 = 2^4 = \underline{\underline{16.0000}}$

44. $-27^{\frac{2}{3}} = -\left(27^{\frac{1}{3}}\right)^2 = \underline{\underline{-9.00000}}$

45. $7^{\frac{3}{2}} = 7^{1.5} = \underline{\underline{18.5203}}$

46. $5^{\frac{3}{4}} = 5^{-0.75} = \underline{\underline{0.299070}}$

47. $(0.001)^{-2} = \underline{\underline{1,000,000}}$

48. $0.893^{-\frac{1}{2}} = 0.893^{-0.5} = \underline{\underline{1.05822}}$

Exercise 2.2 (continued)

$$49. (1.0085)^5(1.0085)^3 = 1.0085^8 = \underline{\underline{1.07006}}$$

$$50. (1.005)^3(1.005)^{-6} = 1.005^{-3} = \underline{\underline{0.985149}}$$

$$51. \sqrt[3]{103} = 103^{0.\bar{3}} = \underline{\underline{100990}}$$

$$52. \sqrt[6]{105} = \underline{\underline{100816}}$$

$$53. (4^4)(3^{-3})\left(-\frac{3}{4}\right)^3 = \frac{4^4}{3^3}\left(-\frac{3^3}{4^3}\right) = \underline{\underline{4.00000}}$$

$$54. \left[\left(-\frac{3}{4}\right)^2\right]^{-2} = \left(-\frac{3}{4}\right)^{-4} = \left(-\frac{4}{3}\right)^4 = \frac{256}{81} = \underline{\underline{3.16049}}$$

$$55. \left(\frac{2}{3}\right)^3\left(-\frac{3}{2}\right)^2\left(-\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3\left(\frac{3}{2}\right)^2\left(-\frac{2}{3}\right)^3 = \frac{2}{3}\left(-\frac{2}{3}\right)^3 = -\frac{16}{81} = \underline{\underline{-0.197531}}$$

$$56. \left(-\frac{2}{3}\right)^3 + \left(\frac{3}{2}\right)^{-2} = \frac{\left(-\frac{2}{3}\right)^3}{\left(\frac{2}{3}\right)^2} = -\frac{2}{3} = \underline{\underline{-0.666667}}$$

$$57. \frac{103^{16} - 1}{0.03} = \underline{\underline{20.1569}}$$

$$58. \frac{(1008\bar{3})^{30} - 1}{0.008\bar{3}} = \frac{0.2826960}{0.008333333} = \underline{\underline{33.9235}}$$

$$59. \frac{1 - 10225^{-20}}{0.0225} = \frac{0.3591835}{0.0225} = \underline{\underline{15.9637}}$$

$$60. \frac{1 - (100\bar{6})^{-32}}{0.00\bar{6}} = \frac{0.1915410}{0.00\bar{6}} = \underline{\underline{28.7312}}$$

$$61. (1 + 0.0275)^{1/3} = \underline{\underline{1.00908}}$$

$$62. (1 + 0.055)^{1/6} - 1 = \underline{\underline{0.00896339}}$$

Exercise 2.3

$$\begin{aligned} 1. \quad & 10a + 10 = 12 + 9a \\ & 10a - 9a = 12 - 10 \\ & a = \underline{\underline{2}} \end{aligned}$$

$$\begin{aligned} 2. \quad & 29 - 4y = 2y - 7 \\ & 36 = 6y \\ & y = \underline{\underline{6}} \end{aligned}$$

$$\begin{aligned} 3. \quad & 0.5(x - 3) = 20 \\ & x - 3 = 40 \\ & x = \underline{\underline{43}} \end{aligned}$$

Exercise 2.3 (continued)

$$\begin{aligned}4. \quad \frac{1}{3}(x - 2) &= 4 \\ x - 2 &= 12 \\ x &= \underline{14}\end{aligned}$$

$$\begin{aligned}5. \quad y &= 192 + 0.04y \\ y - 0.04y &= 192 \\ y &= \frac{192}{0.96} = \underline{200}\end{aligned}$$

$$\begin{aligned}6. \quad x - 0.025x &= 341.25 \\ 0.975x &= 341.25 \\ x &= \frac{341.25}{0.975} = \underline{350}\end{aligned}$$

$$\begin{aligned}7. \quad 12x - 4(2x - 1) &= 6(x + 1) - 3 \\ 12x - 8x + 4 &= 6x + 6 - 3 \\ -2x &= -1 \\ x &= \underline{0.5}\end{aligned}$$

$$\begin{aligned}8. \quad 3y - 4 &= 3(y + 6) - 2(y + 3) \\ &= 3y + 18 - 2y - 6 \\ 2y &= 16 \\ y &= \underline{8}\end{aligned}$$

$$\begin{aligned}9. \quad 8 - 0.5(x + 3) &= 0.25(x - 1) \\ 8 - 0.5x - 1.5 &= 0.25x - 0.25 \\ -0.75x &= -6.75 \\ x &= \underline{9}\end{aligned}$$

$$\begin{aligned}10. \quad 5(2 - c) &= 10(2c - 4) - 6(3c + 1) \\ 10 - 5c &= 20c - 40 - 18c - 6 \\ -7c &= -56 \\ c &= \underline{8}\end{aligned}$$

$$\begin{aligned}11. \quad 3.1t + 145 &= 10 + 7.6t \\ -4.5t &= -135 \\ t &= \underline{30}\end{aligned}$$

$$\begin{aligned}12. \quad 1.25y - 20.5 &= 0.5y - 11.5 \\ 0.75y &= 9 \\ y &= \underline{12}\end{aligned}$$

$$\begin{aligned}13. \quad \frac{x}{1.1^2} + 2x(1.1)^3 &= \$1000 \\ 0.8264463x + 2.622x &= \$1000 \\ 3.488446x &= \$1000 \\ x &= \underline{\$286.66}\end{aligned}$$

$$\begin{aligned}14. \quad \frac{3x}{1.025^6} + x(1.025)^8 &= \$2641.35 \\ 2.586891x + 1.218403x &= \$2641.35 \\ x &= \underline{\$694.13}\end{aligned}$$

Exercise 2.3 (continued)

$$15. \quad \frac{2x}{1.03^7} + x + x(1.03^{10}) = \$1000 + \frac{\$2000}{1.03^4}$$

$$1.626183x + x + 1.343916x = \$1000 + \$1776.974$$

$$3.970099x = \$2776.974$$

$$x = \underline{\underline{\$699.47}}$$

$$16. \quad x(1.05)^3 + \$1000 + \frac{x}{1.05^7} = \frac{\$5000}{1.05^2}$$

$$1.157625x + 0.7106813x = \$4535.147 - \$1000$$

$$x = \underline{\underline{\$1892.17}}$$

$$17. \quad x\left(1 + 0.095 \times \frac{84}{365}\right) + \frac{2x}{1 + 0.095 \times \frac{108}{365}} = \$1160.20$$

$$1.021863x + 1.945318x = \$1160.20$$

$$2.967181x = \$1160.20$$

$$x = \underline{\underline{\$391.01}}$$

$$18. \quad \frac{x}{1 + 0.115 \times \frac{78}{365}} + 3x\left(1 + 0.115 \times \frac{121}{365}\right) = \$1000\left(1 + 0.115 \times \frac{43}{365}\right)$$

$$0.9760141x + 3.114370x = \$1013.548$$

$$x = \underline{\underline{\$247.79}}$$

$$19. \quad \begin{array}{rcl} x - y & = & 2 \quad \textcircled{1} \\ 3x + 4y & = & 20 \quad \textcircled{2} \\ \textcircled{1} \times 3: & \underline{3x - 3y = 6} & \\ \text{Subtract:} & 7y & = 14 \\ & y & = 2 \end{array}$$

Substitute into equation ①:

$$\begin{aligned} x - 2 &= 2 \\ x &= 4 \\ (x, y) &= \underline{\underline{(4, 2)}} \end{aligned}$$

Check: LHS of ② = $3(4) + 4(2) = 20$ = RHS of ②

$$20. \quad \begin{array}{rcl} y - 3x & = & 11 \quad \textcircled{1} \\ -4y + 5x & = & -30 \quad \textcircled{2} \\ \textcircled{1} \times 4: & \underline{4y - 12x = 44} & \\ \text{Add:} & -7x & = 14 \\ & x & = -2 \end{array}$$

Substitute into equation ①:

$$\begin{aligned} y - 3(-2) &= 11 \\ y &= 11 - 6 = 5 \\ (x, y) &= \underline{\underline{(-2, 5)}} \end{aligned}$$

Check: LHS of ② = $-4(5) + 5(-2) = -30$ = RHS of ②

Exercise 2.3 (continued)

$$21. \quad \begin{array}{rcl} 4a - 3b & = & -3 \quad \textcircled{1} \\ 5a - b & = & 10 \quad \textcircled{2} \end{array}$$

$$\textcircled{1} \times 1: \quad 4a - 3b = -3$$

$$\textcircled{2} \times 3: \quad \underline{15a - 3b = 30}$$

$$\text{Subtract:} \quad \underline{-11a} \quad = -33$$

$$a = 3$$

Substitute into equation $\textcircled{2}$:

$$5(3) - b = 10$$

$$b = 5$$

$$(a, b) = \underline{(3, 5)}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{1} = 4(3) - 3(5) = -3 = \text{RHS of } \textcircled{1}$$

$$22. \quad \begin{array}{rcl} 7p - 3q & = & 23 \quad \textcircled{1} \\ -2p - 3q & = & 5 \quad \textcircled{2} \end{array}$$

$$\text{Subtract:} \quad \underline{9p} \quad = 18$$

$$p = 2$$

Substitute into equation $\textcircled{1}$:

$$7(2) - 3q = 23$$

$$3q = -23 + 14$$

$$q = -3$$

$$(p, q) = \underline{(2, -3)}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{2} = -2(2) - 3(-3) = 5 = \text{RHS of } \textcircled{2}$$

$$23. \quad \begin{array}{rcl} y & = & 2x \quad \textcircled{1} \end{array}$$

$$\text{Add:} \quad \underline{\frac{7x - y}{7x}} = \underline{\frac{35}{2x + 35}} \quad \textcircled{2}$$

$$5x = 35$$

$$x = 7$$

Substitute into $\textcircled{1}$:

$$y = 2(7) = 14$$

$$(x, y) = \underline{(7, 14)}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{2} = 7(7) - 14 = 49 - 14 = 35 = \text{RHS of } \textcircled{2}$$

$$24. \quad \begin{array}{rcl} g - h & = & 17 \quad \textcircled{1} \end{array}$$

$$\frac{4}{3}g + \frac{3}{2}h = 0 \quad \textcircled{2}$$

$$1.3\overline{3}g + 1.5h = 0 \quad \textcircled{2}$$

$$\textcircled{1} \times 1.5: \quad \underline{1.5g - 1.5h = 25.5}$$

$$\text{Add:} \quad \underline{2.83\overline{3}g} \quad = 25.5$$

$$g = 9$$

Substitute into $\textcircled{2}$:

$$9 - h = 17$$

$$h = -8$$

$$(h, g) = \underline{(-8, 9)}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{2} = \frac{4}{3}(9) + \frac{3}{2}(-8) = 12 - 12 = 0 = \text{RHS of } \textcircled{2}$$

Exercise 2.3 (continued)

$$\begin{array}{rcl}
 25. & d = 3c - 500 & \textcircled{1} \\
 & 0.7c + 0.2d = 550 & \textcircled{2}
 \end{array}$$

To eliminate d,

$$\textcircled{1} \times 0.2: \quad -0.6c + 0.2d = -100$$

$$\textcircled{2}: \quad \underline{0.7c + 0.2d = 550}$$

$$\begin{array}{rcl}
 \text{Subtract:} & -1.3c + 0 & = -650 \\
 & c = & 500
 \end{array}$$

$$\text{Substitute into } \textcircled{1}: \quad d = 3(500) - 500 = 1000$$

$$(c, d) = \underline{(500, 1000)}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{2} = 0.7(500) + 0.2(1000) = 550 = \text{RHS of } \textcircled{2}$$

$$\begin{array}{rcl}
 26. & 0.03x + 0.05y = 51 & \textcircled{1} \\
 & 0.8x - 0.7y = 140 & \textcircled{2}
 \end{array}$$

To eliminate y,

$$\textcircled{1} \times 0.7: \quad 0.021x + 0.035y = 35.7$$

$$\textcircled{2} \times 0.05: \quad \underline{0.04x - 0.035y = 7}$$

$$\begin{array}{rcl}
 \text{Add:} & 0.061x + 0 & = 42.7 \\
 & x = & 700
 \end{array}$$

Substitute into $\textcircled{2}$:

$$0.8(700) - 0.7y = 140$$

$$-0.7y = -420$$

$$y = 600$$

$$(x, y) = \underline{(700, 600)}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{1} = 0.03(700) + 0.05(600) = 51 = \text{RHS of } \textcircled{1}$$

$$\begin{array}{rcl}
 27. & 2v + 6w = 1 & \textcircled{1} \\
 & 10v - 9w = 18 & \textcircled{2}
 \end{array}$$

To eliminate v,

$$\textcircled{1} \times 10: \quad 20v + 60w = 10$$

$$\textcircled{2} \times 2: \quad \underline{20v - 18w = 36}$$

$$\begin{array}{rcl}
 \text{Subtract:} & 0 + 78w & = -26 \\
 & w = & -\frac{1}{3}
 \end{array}$$

Substitute into $\textcircled{1}$:

$$2v + 6\left(-\frac{1}{3}\right) = 1$$

$$2v = 1 + 2$$

$$v = \frac{3}{2}$$

$$(v, w) = \underline{\underline{\left(\frac{3}{2}, -\frac{1}{3}\right)}}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{2} = 10\left(\frac{3}{2}\right) - 9\left(-\frac{1}{3}\right) = 18 = \text{RHS of } \textcircled{2}$$

Exercise 2.3 (continued)

$$\begin{array}{rcl} 28. & 2.5a + 2b = 11 & \textcircled{1} \\ & 8a + 3.5b = 13 & \textcircled{2} \end{array}$$

To eliminate b,

$$\begin{array}{rcl} \textcircled{1} \times 3.5: & 8.75a + 7b = 38.5 \\ \textcircled{2} \times 2: & \underline{16a + 7b = 26} \\ \text{Subtract:} & -7.25a + 0 = 12.5 \\ & a = -1.724 \end{array}$$

Substitute into $\textcircled{1}$:

$$\begin{array}{rcl} 2.5(-1.724) + 2b & = & 11 \\ 2b & = & 11 + 4.31 \\ b & = & 7.655 \\ (a, b) & = & \underline{(-1.72, 7.66)} \end{array}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{2} = 8(-1.724) + 3.5(7.655) = 13.00 = \text{RHS of } \textcircled{2}$$

$$\begin{array}{rcl} 29. & 37x - 63y = 235 & \textcircled{1} \\ & 18x + 26y = 468 & \textcircled{2} \end{array}$$

To eliminate x,

$$\begin{array}{rcl} \textcircled{1} \times 18: & 666x - 1134y = 4230 \\ \textcircled{2} \times 37: & \underline{666x + 962y = 17,316} \\ \text{Subtract:} & 0 - 2096y = -13,086 \\ & y = 6.243 \end{array}$$

Substitute into $\textcircled{1}$:

$$\begin{array}{rcl} 37x - 63(6.243) & = & 235 \\ 37x & = & 628.3 \\ x & = & 16.98 \\ (x, y) & = & \underline{(17.0, 6.24)} \end{array}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{2} = 18(16.98) + 26(6.243) = 468.0 = \text{RHS of } \textcircled{2}$$

$$\begin{array}{rcl} 30. & 68.9n - 38.5m = 57 & \textcircled{1} \\ & 45.1n - 79.4m = -658 & \textcircled{2} \end{array}$$

To eliminate n,

$$\begin{array}{rcl} \textcircled{1} \times 45.1: & 3107n - 1736.4m = 2571 \\ \textcircled{2} \times 68.9: & \underline{3107n - 5470.7m = -45,336} \\ \text{Subtract:} & 0 + 3734.3m = 47,907 \\ & m = 12.83 \end{array}$$

Substitute into $\textcircled{1}$:

$$\begin{array}{rcl} 68.9n - 38.5(12.83) & = & 57 \\ 68.9n & = & 551.0 \\ n & = & 7.996 \\ (m, n) & = & \underline{(12.8, 8.00)} \end{array}$$

$$\text{Check:} \quad \text{LHS of } \textcircled{2} = 45.1(7.996) - 79.4(12.83) = -658.1 = \text{RHS of } \textcircled{2}$$

Exercise 2.3 (continued)

$$\begin{array}{rcl} 31. & 0.33e + 1.67f = 292 & \textcircled{1} \\ & 1.2e + 0.61f = 377 & \textcircled{2} \end{array}$$

To eliminate e,

$$\begin{array}{rcl} \textcircled{1} \div 0.33: & e + 5.061f = 884.8 \\ \textcircled{2} \div 1.2: & \underline{e + 0.508f = 314.2} \\ \text{Subtract:} & 0 + 4.552f = 570.6 \\ & f = 125.4 \end{array}$$

Substitute into $\textcircled{1}$:

$$\begin{array}{rcl} 0.33e + 1.67(125.4) & = & 292 \\ 0.33e & = & 82.58 \\ e & = & 250.2 \\ (e, f) & = & \underline{(250.2, 125.4)} \end{array}$$

Check: LHS of $\textcircled{2} = 1.2(250.2) + 0.61(125.4) = 376.7 = \text{RHS of } \textcircled{2}$

$$\begin{array}{rcl} 32. & 318j - 451k = 7.22 & \textcircled{1} \\ & -249j + 193k = -18.79 & \textcircled{2} \end{array}$$

To eliminate k,

$$\begin{array}{rcl} \textcircled{1} \div 451: & 0.7051j - k = 0.01601 \\ \textcircled{2} \div 193: & \underline{-1.2902j + k = -0.09736} \\ \text{Add:} & -0.5851j + 0 = -0.08135 \\ & j = 0.1390 \end{array}$$

Substitute into $\textcircled{2}$:

$$\begin{array}{rcl} -249(0.1390) + 193k & = & -18.79 \\ 193k & = & 15.82 \\ k & = & 0.08197 \\ (j, k) & = & \underline{(0.139, 0.0820)} \end{array}$$

Check: LHS of $\textcircled{1} = 318(0.1390) - 451(0.08197) = 7.23 = \text{RHS of } \textcircled{1}$ (within rounding errors.)

Point of Interest (Section 2.4)

A "Trick" Question

The element of mathematical misdirection in the question is that it presumes (and attempts to get you thinking) that there really is a missing dollar, and that the \$3 difference between the \$90 originally paid and the net \$87 paid consists of the \$2 kept by the bellhop and the missing dollar.

But the \$3 refund sitting in the workers' pockets explains the difference between the \$90 and the \$87. The \$2 pilfered by the bellhop explains the \$2 difference between the net amount (\$87) paid by the workers and the amount (\$85) in the hotel's till. There is no missing \$1!

Exercise 2.4

1. Step 2: Hits last month = 2655 after the $\frac{2}{7}$ increase.

Let the number of hits 1 year ago be n .

Step 3: Hits last month = Hits 1 year ago + $\frac{2}{7}$ (Hits 1 year ago)

Step 4: $2655 = n + \frac{2}{7}n$

Step 5: $2655 = \frac{9}{7}n$

Multiply both sides by $\frac{7}{9}$.

$$n = 2655 \times \frac{7}{9} = 2065$$

The Web site had 2065 hits in the same month 1 year ago.

2. Step 2: Retail price = \$712; Markup = 60% of wholesale of cost.

Let the wholesale cost be C .

Step 3: Retail price = Cost + 0.60(Cost)

Step 4: $\$712 = C + 0.6C$

Step 5: $\$712 = 1.6C$

$$C = \frac{\$712}{1.6} = \underline{\underline{\$445.00.}} \quad \text{The wholesale cost is } \$445.00.$$

3. Step 2: Tag price = \$39.55 (including 13% HST). Let the plant's pretax price be P .

Step 3: Tag price = Pre-tax price + HST

Step 4: $\$39.55 = P + 0.13P$

Step 5: $\$39.55 = 1.13P$

$$P = \frac{\$39.55}{1.13} = \$35.00$$

The amount of HST is $\$39.55 - \$35.00 = \underline{\underline{\$4.55}}$

4. Step 2: Commission rate = 2.5% on the first \$5000 and 1.5% on the remainder

Commission amount = \$227. Let the transaction amount be x .

Step 3: Commission amount = $0.025(\$5000) + 0.015(\text{Remainder})$

Step 4: $\$227 = \$125.00 + 0.015(x - \$5000)$

Step 5: $\$102 = 0.015x - \75.00

$$\$102 + \$75 = 0.015x$$

$$x = \frac{\$177}{0.015} = \underline{\underline{\$11,800.00}}$$

The amount of the transaction was \$11,800.00.

5. Step 2: Let the basic price be P . First 20 meals at P .

Next 20 meals at $P - \$2$. Additional meals at $P - \$3$.

Step 3: Total price for 73 meals = \$1686

Step 4: $20P + 20(P - \$2) + (73 - 40)(P - \$3) = \$1686$

Step 5: $20P + 20P - \$40 + 33P - \$99 = \$1686$

$$73P = \$1686 + \$99 + \$40$$

$$P = \frac{\$1825}{73} = \underline{\underline{\$25.00}}$$

The basic price per meal is \$25.00.

Exercise 2.4 (continued)

6. Step 2: Rental Plan 1: \$295 per week + \$0.15 × (Distance in excess of 1000 km)
Rental Plan 2: \$389 per week
Let d represent the distance at which the costs of both plans are equal.

Step 3: Cost of Plan 1 = Cost of Plan 2

Step 4: $\$295 + \$0.15(d - 1000) = \$389$

Step 5: $\$295 + \$0.15d - \$150 = \389

$$\$0.15d = \$244$$

$$d = \underline{1627 \text{ km}}$$

The unlimited driving plan will be cheaper if you drive more than 1626.7 km in the one-week interval.

7. Step 2: Tax rate = 38%; Overtime hourly rate = $1.5(\$23.50) = \35.25
Cost of canoe = \$2750
Let h represent the hours of overtime Alicia must work.

Step 3: Gross overtime earnings – Income tax = Cost of the canoe

Step 4: $\$35.25h - 0.38(\$35.25h) = \$2750$

Step 5: $\$21.855h = \2750

$$h = 125.83 \text{ hours}$$

Alicia must work 125¾ hours of overtime to earn enough money to buy the canoe.

8. Step 2: Number of two-bedroom homes = $0.4(\text{Number of three-bedroom homes})$
Number of two-bedroom homes = $2(\text{Number of four-bedroom homes})$
Total number of homes = 96
Let h represent the number of two-bedroom homes

Step 3: # 2-bedroom homes + # 3-bedroom homes + # 4-bedroom homes = 96

Step 4: $h + \frac{h}{0.4} + \frac{h}{2} = 96$

Step 5: $h + 2.5h + 0.5h = 96$

$$4h = 96$$

$$h = 24$$

There should be 24 two-bedroom homes, $2.5(24) =$ 60 three-bedroom homes,
and $0.5(24) =$ 12 four-bedroom homes.

9. Step 2: Cost of radio advertising = $0.5(\text{Cost of newspaper advertising})$
Cost of TV advertising = $0.6(\text{Cost of radio advertising})$
Total advertising budget = \$160,000
Let r represent the amount allocated to radio advertising

Step 3: Radio advertising + TV advertising + Newspaper advertising = \$160,000

Step 4: $r + 0.6r + \frac{r}{0.5} = \$160,000$

Step 5: $3.6r = \$160,000$

$$r = \$44,444.44$$

The advertising budget allocations should be:

\$44,444 to radio advertising,

$0.6(\$44,444.44) = \$26,667$ to TV advertising, and

$2(\$44,444.44) = \$88,889$ to newspaper advertising.

Exercise 2.4 (continued)

10. Step 2: By-laws require: 5 parking spaces per 100 square meters,
4% of spaces for physically handicapped
In remaining 96%, # regular spaces = 1.4(# small car spaces)
Total area = 27,500 square meters

Let s represent the number of small car spaces.

Step 3: Total # spaces = # spaces for handicapped + # regular spaces + # small spaces

$$\text{Step 4: } \frac{27,500}{100} \times 5 = 0.04 \times \frac{27,500}{100} \times 5 + s + 1.4s$$

$$\text{Step 5: } 1375 = 55 + 2.4s$$
$$s = 550$$

The shopping centre must have 55 parking spaces for the physically handicapped,
550 small-car spaces, and 770 regular parking spaces.

11. Step 2: Overall portfolio's rate return = 1.1%, equity fund's rate of return = -3.3%,
bond fund's rate of return = 7.7%.
Let e represent the fraction of the portfolio initially invested in the equity fund.

Step 3: Overall rate of return = Weighted average rate of return
= (Equity fraction)(Equity return) + (Bond fraction)(Bond return)

$$\text{Step 4: } 1.1\% = e(-3.3\%) + (1 - e)(7.7\%)$$

$$\text{Step 5: } 1.1 = -3.3e + 7.7 - 7.7e$$
$$-6.6 = -11.0e$$
$$e = 0.600$$

Therefore, 60.0% of Erin's original portfolio was invested in the equity fund.

12. Step 2: Pile A steel is 5.25% nickel; pile B steel is 2.84% nickel.
We want a 32.5-tonne mixture from A and B averaging 4.15% nickel.
Let A represent the tonnes of steel required from pile A.

Step 3: Wt. of nickel in 32.5 tonnes of mixture
= Wt. of nickel in steel from pile A + Wt. of nickel in steel from pile B
= (% nickel in pile A)(Amount from A) + (% nickel in pile B)(Amount from B)

$$\text{Step 4: } 0.0415(32.5) = 0.0525A + 0.0284(32.5 - A)$$

$$\text{Step 5: } 1.34875 = 0.0525A + 0.9230 - 0.0284A$$
$$0.42575 = 0.0241A$$
$$A = 17.67 \text{ tonnes}$$

The recycling company should mix 17.67 tonnes from pile A with 14.83 tonnes from pile B.

13. Step 2: Total options = 100,000
of options to an executive = 2000 + # of options to a scientist or engineer
of options to a scientist or engineer = 1.5(# of options to a technician)
There are 3 executives, 8 scientists and engineers, and 14 technicians.
Let t represent the number of options to each technician.

Step 3: Total options = Total options to scientists and engineers
+ Total options to technicians + Total options to executives

$$\text{Step 4: } 100,000 = 8(1.5t) + 14t + 3(2000 + 1.5t)$$

$$\text{Step 5: } = 12t + 14t + 6000 + 4.5t$$
$$94,000 = 30.5t$$
$$t = 3082 \text{ options}$$

Each technician will receive 3082 options,
each scientist and engineer will receive $1.5(3082) = \underline{4623 \text{ options}}$,
and each executive will receive $2000 + 4623 = \underline{6623 \text{ options}}$.

Exercise 2.4 (continued)

14. Step 2: Plan X: 6.5 cents/minute (in business hours) and 4.5 cents/minute (at other times)
Plan Y: 5.3 cents/minute any time
Let b represent the fraction of business-hour usage at which costs are equal.

Step 3: Cost of Plan X = Cost of plan Y

Step 4: Pick any amount of usage in a month—say 1000 minutes.

$$b(1000)\$0.065 + (1 - b)(1000)\$0.045 = 1000(\$0.053)$$

$$\begin{aligned}\text{Step 5:} \quad \$65b + \$45 - \$45b &= \$53 \\ \$20b &= \$8 \\ b &= 0.40\end{aligned}$$

If business-hour usage exceeds 40% of overall usage, plan Y will be cheaper.

15. Step 2: Raisins cost \$3.75 per kg; peanuts cost \$2.89 per kg.
Cost per kg of ingredients in 50 kg of “trail mix” is to be \$3.20.
Let p represent the weight of peanuts in the mixture.

Step 3: Cost of 50 kg of trail mix = Cost of p kg peanuts + Cost of $(50 - p)$ kg of raisins

$$\text{Step 4: } 50(\$3.20) = p(\$2.89) + (50 - p)(\$3.75)$$

$$\begin{aligned}\text{Step 5: } \$160.00 &= \$2.89p + \$187.50 - \$3.75p \\ -\$27.50 &= -\$0.86p \\ p &= 31.98 \text{ kg}\end{aligned}$$

32.0 kg of peanuts should be mixed with 18.0 kg of raisins.

16. Step 2: Total bill = \$3310. Total hours = 41.
Hourly rate = \$120 for CGA
= \$50 for technician.

Let x represent the CGA's hours.

Step 3: Total bill = (CGA hours x CGA rate) + (Technician hours x Technician rate)

$$\text{Step 4: } \$3310 = x(\$120) + (41 - x)\$50$$

$$\begin{aligned}\text{Step 5: } \$3310 &= \$120x + \$2050 - \$50x \\ 1260 &= 70x \\ x &= 18\end{aligned}$$

The CGA worked 18 hours and the technician worked $41 - 18 =$ 23 hours.

17. Step 2: Total investment = \$32,760
Sue's investment = 1.2(Joan's investment)
Joan's investment = 1.2(Stella's investment)
Let L represent Stella's investment.

Step 3: Sue's investment + Joan's investment + Stella's investment = Total investment

$$\begin{aligned}\text{Step 4: } \text{Joan's investment} &= 1.2L \\ \text{Sue's investment} &= 1.2L(1.2L) = 1.44L \\ 1.44L + 1.2L + L &= \$32,760\end{aligned}$$

$$\begin{aligned}\text{Step 5:} \quad 3.64L &= \$32,760 \\ L &= \frac{\$32,760}{3.64} = \$9000\end{aligned}$$

Stella will invest \$9000, Joan will invest $1.2(\$9000) =$ \$10,800, and
Sue will invest $1.2(\$10,800) =$ \$12,960

Exercise 2.4 (continued)

18. Step 2: Sven receives 30% less than George (or 70% of George's share).
Robert receives 25% more than George (or 1.25 times George's share).
Net income = \$88,880
Let G represent George's share.
- Step 3: George's share + Robert's share + Sven's share = Net income
- Step 4: $G + 1.25G + 0.7G = \$88,880$
- Step 5: $2.95G = \$88,880$
 $G = \$30,128.81$
George's share is \$30,128.81, Robert's share is $1.25(\$30,128.81) = \underline{\$37,661.02}$,
and Sven's share is $0.7(\$30,128.81) = \underline{\$21,090.17}$.
19. Step 2: Time to make X is 20 minutes.
Time to make Y is 30 minutes.
Total time is 47 hours. Total units = 120. Let Y represent the number of units of Y.
- Step 3: Total time = (Number of X) $\times$ (Time for X) + (Number of Y) $\times$ (Time for Y)
- Step 4: $47 \times 60 = (120 - Y)20 + Y(30)$
- Step 5: $2820 = 2400 - 20Y + 30Y$
 $420 = 10Y$
 $Y = \underline{42}$
Forty-two units of product Y were manufactured.
20. Step 2: Price of blue ticket = \$19.00. Price of red ticket = \$25.50.
Total tickets = 4460. Total revenue = \$93,450.
Let the number of tickets in the red section be R.
- Step 3: Total revenue = (Number of red $\times$ Price of red) + (Number of blue $\times$ Price of blue)
- Step 4: $\$93,450 = R(\$25.50) + (4460 - R)\$19.00$
- Step 5: $93,450 = 25.5R + 84,740 - 19R$
 $6.5R = 8710$
 $R = 1340$
1340 seats were sold in the red section and $4460 - 1340 = \underline{3120 \text{ seats}}$ were sold in the blue section.
21. Step 2: $\frac{3}{5}$ of a $\frac{3}{7}$ interest was sold for \$27,000.
Let the V represent the implied value of the entire partnership.
- Step 3: $\frac{3}{5}$ of a $\frac{3}{7}$ interest is worth \$27,000.
- Step 4: $\frac{3}{5} \times \frac{3}{7} V = \$27,000$
- Step 5: $V = \frac{5 \times 7}{3 \times 3} \times \$27,000 = \$105,000$
- b. The implied value of the entire partnership is \$105,000.
- a. The implied value of Shirley's remaining interest is
 $\frac{2}{5} \times \frac{3}{7} V = \frac{6}{35} \times \$105,000 = \underline{\$18,000}$

Exercise 2.4 (continued)

22. Step 2: Regal owns a 58% interest in a mineral claim. Yukon owns the remainder (42%).
Regal sells one fifth of its interest for \$1.2 million.
Let the V represent the implied value of the entire mineral claim.

Step 3: $\frac{1}{5}$ (or 20%) of a 58% interest is worth \$1.2 million

Step 4: $0.20(0.58)V = \$1,200,000$

Step 5: $V = \frac{\$1,200,000}{0.20 \times 0.58} = \$10,344,828$

The implied value of Yukon's interest is

$$0.42V = 0.42 \times \$10,344,828 = \underline{\$4,344,828}$$

23. Step 2: $\frac{5}{7}$ of entrants complete Level 1. $\frac{2}{9}$ of Level 1 completers fail Level 2.
587 students completed Level 2 last year.
Let the N represent the original number who began Level 1.

Step 3: $\frac{7}{9}$ of $\frac{5}{7}$ of entrants will complete Level 2.

Step 4: $\frac{7}{9} \times \frac{5}{7} N = 587$

Step 5: $N = \frac{9 \times 7}{7 \times 5} \times 587 = 1056.6$

1057 students began Level 1.

24. Step 2: $\frac{4}{7}$ of inventory was sold at cost.

$\frac{3}{7}$ inventory was sold to liquidators at 45% of cost, yielding \$6700.

Let C represent the original cost of the entire inventory.

Step 3: $\frac{3}{7}$ of inventory was sold to liquidators at 45% of cost, yielding \$6700.

Step 4: $\frac{3}{7}(0.45C) = \$6700$

Step 5: $C = \frac{7 \times \$6700}{3 \times 0.45} = \$34,740.74$

- a. The cost of inventory sold to liquidators was

$$\frac{3}{7}(\$34,740.74) = \underline{\$14,888.89}$$

- b. The cost of the remaining inventory sold in the bankruptcy sale was

$$\$34,740.74 - \$14,888.89 = \underline{\$19,851.85}$$

25. Let r represent the number of regular members and s the number of student members.

Then $r + s = 583$ ①

Total revenue: $\$2140r + \$856s = \$942,028$ ②

① $\times \$856$: $\underline{\$856r + \$856s = \$499,048}$

Subtract: $\$1284r + 0 = \$442,980$

$$r = 345$$

Substitute into ①: $345 + s = 583$

$$s = 238$$

The club had 238 student members and 345 regular members.

Exercise 2.4 (continued)

26. Let c represent the number of children and a represent the number of adults.

$$\text{Then } c + a = 266 \quad \textcircled{1}$$

$$\$17.90c + \$25.90a = \$6609.40 \quad \textcircled{2}$$

$$\textcircled{1} \times \$25.90: \underline{\$25.90c + \$25.90a = \$6889.40}$$

$$\text{Subtract: } -\$8c + 0 = -\$280$$

$$c = 35$$

That is, 35 of the 266 customers were children.

27. Let s represent the distance travelled at the lower speed (50 km/h).
Let h represent the distance travelled at the higher speed (100 km/h).

Since the total distance = 1000 km,

$$\text{then } s + h = 1000 \quad \textcircled{1}$$

$$\text{Since travelling time} = \frac{\text{Distance}}{\text{Speed}},$$

$$\text{then Time at slower speed} = \frac{s}{50} \quad \text{and} \quad \text{Time at higher speed} = \frac{h}{100}$$

Since the total time = 12.3 hours,

$$\text{then } \frac{s}{50} + \frac{h}{100} = 12.3 \quad \textcircled{2}$$

$$\textcircled{2} \times 100: \quad 2s + h = 1230$$

$$\text{Repeat } \textcircled{1}: \quad \underline{s + h = 1000} \quad \textcircled{1}$$

$$\text{Subtract: } s + 0 = 230$$

Hence, Tina drive 230 km at 50 km/h and $1000 - 230 = \underline{770 \text{ km at } 100 \text{ km/h}}$.

28. Let a represent the adult airfare and c represent the child airfare.

$$\text{Mrs. Ramsey's cost: } a + 2c = \$610 \quad \textcircled{1}$$

$$\text{Chudnowskis' cost: } 2a + 3c = \$1050 \quad \textcircled{2}$$

$$\textcircled{1} \times 2: \quad \underline{2a + 4c = \$1220}$$

$$\text{Subtract: } 0 + -c = -\$170$$

$$\text{Substitute } c = \$170 \text{ into } \textcircled{1}: a + 2(\$170) = \$610$$

$$a = \$610 - \$340 = \$270$$

The airfare is \$270 per adult and \$170 per child.

29. Let h represent the rate per hour and k represent the rate per km.

$$\text{Vratislav's cost: } 2h + 47k = \$54.45 \quad \textcircled{1}$$

$$\text{Bryn's cost: } 5h + 93k = \$127.55 \quad \textcircled{2}$$

To eliminate x ,

$$\textcircled{1} \times 5: \quad 10h + 235k = \$272.25 \quad \textcircled{1}$$

$$\textcircled{2} \times 2: \quad \underline{10h + 186k = \$255.10} \quad \textcircled{2}$$

$$\text{Subtract: } 0 + 49k = \$17.15$$

$$k = \$0.35 \text{ per km}$$

Substitute into $\textcircled{1}$:

$$2h + 47(\$0.35) = \$54.45$$

$$2h = \$54.45 - \$16.45$$

$$= \$38.00 \text{ per hour}$$

$$h = \$19.00 \text{ per hour}$$

Budget Truck Rentals charged \$19.00 per hour plus \$0.35 per km.

Exercise 2.4 (continued)

30. Let s represent the weight of 6% nitrogen fertilizer.
Let t represent the weight of 22% nitrogen fertilizer.

$$\text{Total weight:} \quad s + t = 300 \quad ①$$

$$\text{Total nitrogen:} \quad 0.06s + 0.22t = 0.16(300)$$

$$\text{Multiply by 100:} \quad 6s + 22t = 4800 \quad ②$$

$$① \times 6: \quad 6s + 6t = 1800 \quad ①$$

$$\text{Subtract:} \quad 0 + 16t = 3000 \quad ②$$

$$t = 187.5 \text{ kg}$$

$$s = 300 - 187.5 = 112.5 \text{ kg}$$

Buckerfield's should mix 112.5 kg of 6% fertilizer with 187.5 kg of 22% fertilizer.

31. Let C represent the interest rate on Canada Savings Bonds.

Let O represent the interest rate on Ontario Savings Bonds.

$$\text{Year 1 interest:} \quad 4(\$1000)C + 6(\$1000)O = \$438 \quad ①$$

$$\text{Year 2 interest:} \quad 3(\$1000)C + 4(\$1000)O = \$306 \quad ②$$

$$① \times 3: \quad \$12,000C + \$18,000O = \$1314 \quad ①$$

$$② \times 4: \quad \$12,000C + \$16,000O = \$1224 \quad ②$$

$$\text{Subtract:} \quad 0 + \$2000O = \$90$$

$$O = \frac{\$90}{\$2000} = 0.045 = 4.5\%$$

$$\text{Substitute into } ②: \quad \$3000C + \$4000(0.045) = \$306$$

$$C = \frac{\$306 - \$180}{\$3000} = 0.042 = 4.2\%$$

The Canada Savings Bonds earn 4.2% per annum and
the Ontario Savings Bonds earn 4.5% per annum.

32. Let r represent the tax rate on residences and
let f represent the tax rate on land with farm buildings.

$$\text{LeClair tax:} \quad \$400,000r + \$300,000f = \$3870 \quad ①$$

$$\text{Bartoli tax:} \quad \$350,000r + \$380,000f = \$3774 \quad ②$$

$$① \times 7: \quad \$2,800,000r + \$2,100,000f = \$27,090 \quad ①$$

$$② \times 8: \quad \$2,800,000r + \$3,040,000f = \$30,192 \quad ②$$

$$\text{Subtract:} \quad 0 - \$940,000f = -\$3102$$

$$f = \frac{\$3102}{\$940,000} = 0.0033 = 0.33\%$$

$$\text{Substitute into } ①: \quad \$400,000r + \$300,000(0.0033) = \$3870$$

$$r = \frac{\$3870 - \$990}{\$400,000} = 0.0072 = 0.72\%$$

The tax rates are 0.72% on residences and 0.33% on land with farm buildings.

33. Let x represent the number of units of product X and
 y represent the number of units of product Y. Then

$$x + y = 93 \quad ①$$

$$0.5x + 0.75y = 60.5 \quad ②$$

$$① \times 0.5: \quad 0.5x + 0.5y = 46.5$$

$$\text{Subtract:} \quad 0 + 0.25y = 14$$

$$y = 56$$

$$\text{Substitute into } ①: \quad x + 56 = 93$$

$$x = 37$$

Therefore, 37 units of X and 56 units of Y were produced last week.

Exercise 2.4 (continued)

34. Let the price per litre of milk be m and the price per dozen eggs be e . Then

$$5m + 4e = \$19.51 \quad \textcircled{1}$$

$$9m + 3e = \$22.98 \quad \textcircled{2}$$

To eliminate e ,

$$\textcircled{1} \times 3: \quad 15m + 12e = \$58.53$$

$$\textcircled{2} \times 4: \quad \underline{36m + 12e = \$91.92}$$

$$\begin{array}{r} \text{Subtract:} \\ -21m + 0 = -\$33.39 \\ m = \$1.59 \end{array}$$

$$\begin{array}{l} \text{Substitute into } \textcircled{1}: 5(\$1.59) + 4e = \$19.51 \\ e = \$2.89 \end{array}$$

Milk costs \$1.59 per litre and eggs cost \$2.89 per dozen.

35. Let M be the number of litres of milk and J be the number of cans of orange juice per week.

$$\$1.50M + \$1.30J = \$57.00 \quad \textcircled{1}$$

$$\$1.60M + \$1.37J = \$60.55 \quad \textcircled{2}$$

To eliminate M ,

$$\textcircled{1} \times 1.6: \quad \$2.40M + \$2.080J = \$91.200$$

$$\textcircled{2} \times 1.5: \quad \underline{\$2.40M + \$2.055J = \$90.825}$$

$$\begin{array}{r} \text{Subtract:} \\ 0 + \$0.025J = \$0.375 \\ J = 15 \end{array}$$

Substitution of $J = 15$ into either equation will give $M = 25$. Hence, 25 litres of milk and 15 cans of orange juice are purchased each week.

36. Let S represent the selling price of a case of beer and R represent the refund per case of empties. Then

$$871S - 637R = \$12,632.10 \quad \textcircled{1}$$

$$932S - 805R = \$13,331.70 \quad \textcircled{2}$$

To eliminate S ,

$$\textcircled{1} \times 932: \quad 811,772S - 593,684R = \$11,773,117.20$$

$$\textcircled{2} \times 871: \quad \underline{811,772S - 701,155R = \$11,611,910.70}$$

$$\begin{array}{r} \text{Subtract:} \\ 0 + 107,471R = \$161,206.50 \\ R = \$1.50 \end{array}$$

The store paid a refund of \$1.50 per case.

37. Let S represent the number of people who bought single tickets and T represent the number of people who bought at three-for-\$5. Then

$$S + 3T = 3884 \quad \textcircled{1}$$

$$\$2S + \$5T = \$6925 \quad \textcircled{2}$$

To eliminate S ,

$$\textcircled{1} \times \$2: \quad \$2S + \$6T = \$7768$$

$$\textcircled{2}: \quad \underline{\$2S + \$5T = \$6925}$$

$$\begin{array}{r} \text{Subtract:} \\ 0 + \$1T = \$843 \\ T = 843 \end{array}$$

Hence, 843 people bought tickets at the three-for-\$5 discount.

Exercise 2.4 (continued)

38. Let P represent the number of six-packs and C represent the number of single cans sold.

$$\begin{array}{rcl} \text{Then} & \$4.35P + \$0.90C = \$178.35 & \textcircled{1} \\ & 6P + C = 225 & \textcircled{2} \end{array}$$

To eliminate C,

$$\begin{array}{rcl} \textcircled{1}: & \$4.35P + \$0.90C = \$178.35 & \\ \textcircled{2} \times \$0.90: & \underline{\$5.40P + \$0.90C = \$202.50} & \\ \text{Subtract:} & -\$1.05P + 0 = -\$24.15 & \\ & P = 23 & \end{array}$$

$$\begin{array}{rcl} \text{Substitute into } \textcircled{2}: & 6(23) + C = 225 & \\ & C = 87 & \end{array}$$

The store sold 23 six-packs and 87 single cans.

39. Let P represent the annual salary of a partner and T represent the annual salary of a technician. Then

$$\begin{array}{rcl} & 7P + 12T = \$1,629,000 & \textcircled{1} \\ & 1.05(7P) + 1.08(12T) = \$1,734,750 & \textcircled{2} \\ \textcircled{1} \times 1.05: & \underline{1.05(7P) + 1.05(12T) = \$1,710,450} & \\ \text{Subtract:} & 0 + 0.03(12T) = \$24,300 & \\ & T = \$67,500 & \end{array}$$

$$\begin{array}{rcl} \text{Substitute into } \textcircled{1}: & 7P + 12(\$67,500) = \$1,629,000 & \\ & P = \$117,000 & \end{array}$$

The current annual salary of a partner is \$117,000 and of a technician is \$67,500.

40. Let P represent the current number of production workers and A the current number of assembly workers. Then

$$\begin{array}{rcl} & \$5100P + \$4200A = \$380,700 & \textcircled{1} \\ & \$5100(0.8P) + \$4200(0.75A) = \$297,000 & \textcircled{2} \\ \text{To eliminate P,} & & \\ \textcircled{1} \times 0.8: & \$5100(0.8P) + \$4200(0.8A) = \$304,560 & \\ \textcircled{2}: & \underline{\$5100(0.8P) + \$4200(0.75A) = \$297,000} & \\ \text{Subtract:} & & \\ & \$4200(0.05A) = \$7560 & \\ & A = 36 & \end{array}$$

$$\begin{array}{rcl} \text{Substitute into } \textcircled{1}: & \$5100P + \$4200(36) = \$380,700 & \\ & P = 45 & \end{array}$$

Therefore, $0.2P = 9$ production workers and $0.25A = 9$ assembly workers will be laid off.

41. Step 2: Each of 4 children receive 0.5(Wife's share).

Each of 13 grandchildren receive $0.\bar{3}$ (Child's share).

Total distribution = \$759,000. Let w represent the wife's share.

Step 3: Total amount = Wife's share + 4(Child's share) + 13(Grandchild's share)

$$\text{Step 4: } \$759,000 = w + 4(0.5w) + 13(0.\bar{3})(0.5w)$$

$$\begin{array}{rcl} \text{Step 5: } \$759,000 & = w + 2w + 2.1\bar{6}w & \\ & = 5.1\bar{6}w & \end{array}$$

$$w = \$146,903.23$$

Each child will receive $0.5(\$146,903.23) = \underline{\$73,451.62}$

and each grandchild will receive $0.\bar{3}(\$73,451.62) = \underline{\$24,483.87}$.

Exercise 2.4 (continued)

42. Step 2: Stage B workers = $1.6(\text{Stage A workers})$
Stage C workers = $0.75(\text{Stage B workers})$
Total workers = 114. Let A represent the number of Stage A workers.
- Step 3: Total workers = A workers + B workers + C workers
- Step 4: $114 = A + 1.6A + 0.75(1.6A)$
- Step 5: $114 = 3.8A$
 $A = 30$
30 workers should be allocated to Stage A, $1.6(30) = \underline{48}$ workers to Stage B,
and $114 - 30 - 48 = \underline{36}$ workers to Stage C.
43. Step 2: Hillside charge = $2(\text{Barnett charge}) - \1000
Westside charge = Hillside charge + \$2000
Total charges = \$27,600. Let B represent the Barnett charge.
- Step 3: Total charges = Barnett charge + Hillside charge + Westside charge
- Step 4: $\$27,600 = B + 2B - \$1000 + 2B - \$1000 + \2000
- Step 5: $\$27,600 = 5B$
 $B = \$5520$
Hence, the Westside charge is $2(\$5520) - \$1000 + \$2000 = \underline{\$12,040}$
44. Step 2: There are 3 managers and 26 production workers. Total distribution = \$100,000.
Manager's share = 1.2 (Production worker's share).
Let p represent a production worker's share.
- Step 3: $3(\text{Manager's share}) + 26(\text{Production worker's share}) = \$100,000$
- Step 4: $3(1.2p) + 26p = \$100,000$
- Step 5: $29.6p = \$100,000$
 $p = \$3378.38$
Each production worker will receive \$3378.38 and each manager will receive
 $1.2(\$3378.38) = \underline{\$4054.05}$.
45. Step 2: Assembly time = $0.5(\text{Cutting time}) + 2$ minutes
Painting time = $0.5(\text{Assembly time}) + 0.5$ minutes
Total units = 72. Total time = 42 hours. Let C represent the cutting time.
- Step 3: Time to produce one toy = Cutting time + Assembly time + Painting time
- Step 4: $\frac{42 \times 60}{72} = C + 0.5C + 2 + 0.5(0.5C + 2) + 0.5$
- Step 5: $35 = 1.75C + 3.5$
 $C = 18$ minutes
Cutting requires 18 minutes (per unit), assembly requires $0.5(18)+2 = \underline{11}$ minutes,
and painting requires $0.5(11) + 0.5 = \underline{6}$ minutes.

Exercise 2.5

1. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$100 - \$95}{\$95} \times 100\% = \underline{\underline{5.26\%}}$
2. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$95 - \$100}{\$100} \times 100\% = \underline{\underline{-5.00\%}}$
3. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{135\text{kg} - 35\text{kg}}{35\text{kg}} \times 100\% = \underline{\underline{285.71\%}}$
4. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{35\text{kg} - 135\text{kg}}{135\text{kg}} \times 100\% = \underline{\underline{-74.07\%}}$
5. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{0.13 - 0.11}{0.11} \times 100\% = \underline{\underline{18.18\%}}$
6. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{0.085 - 0.095}{0.095} \times 100\% = \underline{\underline{-10.53\%}}$
7. $V_f = V_i(1 + c) = \$134.39[1 + (-0.12)] = \$134.39(0.88) = \underline{\underline{\$118.26}}$
8. $V_f = V_i(1 + c) = 112\text{g}(1 + 1.12) = \underline{\underline{237.44\text{g}}}$
9. $V_f = V_i(1 + c) = (26.3 \text{ cm})(1 + 3.00) = \underline{\underline{105.2 \text{ cm}}}$
10. $V_f = V_i(1 + c) = 0.043[1 + (-0.30)] = \underline{\underline{0.0301}}$
11. $V_i = \frac{V_f}{1 + c} = \frac{\$75}{1 + 2.00} = \underline{\underline{\$25.00}}$
12. $V_i = \frac{V_f}{1 + c} = \frac{\$75}{1 + (-0.50)} = \underline{\underline{\$150.00}}$
13. Given: $V_i = \$90$, $V_f = \$100$
 $c = \frac{\$100 - \$90}{\$90} \times 100\% = \underline{\underline{11.11\%}}$
\$100 is 11.11% more than \$90.
14. Given: $V_i = \$110$, $V_f = \$100$
 $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$100 - \$110}{\$110} \times 100\% = \underline{\underline{-9.09\%}}$
\$100 is 9.09% less than \$110.
15. Given: $c = 25\%$, $V_f = \$100$
 $V_i = \frac{V_f}{1 + c} = \frac{\$100}{1 + 0.25} = \underline{\underline{\$80.00}}$
\$80.00 increased by 25% equals \$100.00.
16. Given: $c = 7\%$, $V_f = \$52.43$
 $V_i = \frac{V_f}{1 + c} = \frac{\$52.43}{1 + 0.07} = \underline{\underline{\$49.00}}$
\$49.00 increased by 7% equals \$52.43.

Exercise 2.5 (continued)

17. Given: $V_f = \$75$, $c = 75\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$75}{1+0.75} = \underline{\underline{\$42.86}}$$

\$75 is 75% more than \$42.86.

18. Given: $V_i = \$56$, $c = 65\%$

$$V_f = V_i(1+c) = \$56(1.65) = \underline{\underline{\$92.40}}$$

\$56 after an increase of 65% is \$92.40.

19. Given: $V_i = \$759.00$, $V_f = \$754.30$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$754.30 - \$759.00}{\$759.00} \times 100\% = \underline{\underline{-0.62\%}}$$

\$754.30 is 0.62% less than \$759.00.

20. Given: $V_i = 77,400$, $V_f = 77,787$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{77,787 - 77,400}{77,400} \times 100\% = \underline{\underline{0.50\%}}$$

77,787 is 0.50% more than 77,400.

21. Given: $V_i = \$75$, $c = 75\%$

$$V_f = V_i(1+c) = \$75(1+0.75) = \underline{\underline{\$131.25}}$$

\$75.00 becomes \$131.25 after an increase of 75%.

22. Given: $V_f = \$100$, $c = -10\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.10)} = \underline{\underline{\$111.11}}$$

\$100.00 is 10% less than \$111.11.

23. Given: $V_f = \$100$, $c = -20\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.20)} = \underline{\underline{\$125.00}}$$

\$125 after a reduction of 20% equals \$100.

24. Given: $V_f = \$50$, $c = -25\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$50}{1+(-0.25)} = \underline{\underline{\$66.67}}$$

\$66.67 after a reduction of 25% equals \$50.

25. Given: $V_f = \$549$, $c = -16.\bar{6}\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$549}{1+(-0.1\bar{6})} = \underline{\underline{\$658.80}}$$

\$658.80 after a reduction of $16.\bar{6}\%$ equals \$549.

26. Given: $V_i = \$900$, $c = -90\%$

$$V_f = V_i(1+c) = \$900[1+(-0.9)] = \underline{\underline{\$90.00}}$$

\$900 after a decrease of 90% is \$90.00.

Exercise 2.5 (continued)

27. Given: $V_i = \$102$, $c = -2\%$
 $V_f = V_i(1 + c) = \$102(1 - 0.02) = \underline{\$99.96}$
\$102 after a decrease of 2% is \$99.96.
28. Given: $V_i = \$102$, $c = -100\%$
 $V_f = V_i(1 + c) = \$102[1 + (-1.00)] = \$102(0) = \underline{\$0.00}$
Any positive amount after a decrease of 100% is zero.
29. Given: $V_i = \$250$, $V_f = \$750$
$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$750 - \$250}{\$250} \times 100\% = \underline{\underline{200.00\%}}$$

\$750 is 200.00% more than \$250.
30. Given: $V_i = \$750$, $V_f = \$250$
$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$250 - \$750}{\$750} \times 100\% = \underline{\underline{-66.67\%}}$$

\$250 is 66.67% less than \$750.
31. Given: $c = 0.75\%$, $V_i = \$10,000$
 $V_f = V_i(1 + c) = \$10,000(1 + 0.0075) = \underline{\underline{\$10,075.00}}$
\$10,000 after an increase of $\frac{3}{4}\%$ is \$10,075.00.
32. Given: $V_i = \$1045$, $c = -0.5\%$
 $V_f = V_i(1 + c) = \$1045[1 + (-0.005)] = \underline{\underline{\$1039.78}}$
\$1045 after a decrease of 0.5% is \$1039.78.
33. Given: $c = 150\%$, $V_f = \$575$
$$V_i = \frac{V_f}{1 + c} = \frac{\$575}{1 + 1.5} = \underline{\underline{\$230.00}}$$

\$230.00 when increased by 150% equals \$575.
34. Given: $c = 210\%$, $V_f = \$465$
$$V_i = \frac{V_f}{1 + c} = \frac{\$465}{1 + 2.1} = \underline{\underline{\$150.00}}$$

\$150.00 after being increased by 210% equals \$465.
35. Given: $V_i = \$150$, $c = 150\%$
 $V_f = V_i(1 + c) = \$150(1 + 1.5) = \underline{\underline{\$375.00}}$
\$150 after an increase of 150% is \$375.00.
36. Let the retail price be p . Then
 $p + 0.13p = \$281.37$
$$p = \frac{\$281.37}{1.13} = \underline{\underline{\$249.00}}$$

The coat's sticker price was \$249.00.

Exercise 2.5 (continued)

37. Let the TV's pre-tax price be p . Then

$$p + 0.05p + 0.07p = \$2797.76$$

$$p = \frac{\$2797.76}{1.12} = \$2498.00$$

Then, $\text{GST} = 0.05p = 0.05(\$2498) = \underline{\underline{\$124.90}}$

and $\text{PST} = 0.07p = 0.07(\$2498) = \underline{\underline{\$174.86}}$

38. Let the population figure for 1999 be p . Then

$$p + 0.1056p = 33,710,000$$

$$p = \frac{\$33,710,000}{1.1056} = 30,490,232$$

Rounded to the nearest 10,000, the population in 1999 was 30,490,000.

39. a. . Given: $V_i = 32,400$, $V_f = 27,450$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{27,450 - 32,400}{32,400} \times 100\% = \underline{\underline{-15.28\%}}$$

The number of hammers sold declined by 15.28%.

- b. Given: $V_i = \$15.10$, $V_f = \$15.50$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$15.50 - \$15.10}{\$15.10} \times 100\% = \underline{\underline{2.65\%}}$$

The average selling price increased by 2.65%.

- c. Year 1 revenue = $32,400(\$15.10) = \$489,240$

Year 2 revenue = $27,450(\$15.50) = \$425,475$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$425,475 - \$489,240}{\$489,240} \times 100\% = \underline{\underline{-13.03\%}}$$

The revenue decreased by 13.03%.

40. a. Given: $V_i = \$0.55$, $V_f = \$1.55$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$1.55 - \$0.55}{\$0.55} \times 100\% = \underline{\underline{181.82\%}}$$

The share price rose by 181.82% in the first year.

- b. Given: $V_i = \$1.55$, $V_f = \$0.75$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$1.55}{\$1.55} \times 100\% = \underline{\underline{-51.61\%}}$$

The share price declined by 51.61% in the second year.

- c. Given: $V_i = \$0.55$, $V_f = \$0.75$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$0.55}{\$0.55} \times 100\% = \underline{\underline{36.36\%}}$$

The share price rose by 36.36% over 2 years.

Exercise 2.5 (continued)

41. Pick an arbitrary price, say \$1.00, for a bar of the soap.

The former unit price was $V_i = \frac{\$1.00}{100 \text{ g}} = \0.01 per gram.

The new unit price is $V_f = \frac{\$1.00}{90 \text{ g}} = \0.011111 per gram.

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.011111 - \$0.01}{\$0.01} \times 100\% = \underline{\underline{11.11\%}}$$

42. Initial unit price = $\frac{\$5.49}{1.65 \text{ l}} = \3.327 per litre

Final unit price = $\frac{\$7.98}{2.2 \text{ l}} = \3.627 per litre

The percent increase in the unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$3.627 - \$3.327}{\$3.327} \times 100\% = \underline{\underline{9.02\%}}$$

43. Initial unit price = $\frac{\$7.98}{3.6 \text{ kg}} = \2.2167 per kg

Final unit price = $\frac{\$6.98}{3 \text{ kg}} = \2.3267 per kg

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$2.3267 - \$2.2167}{\$2.2167} \times 100\% = \underline{\underline{4.96\%}}$$

44. Initial unit price = $\frac{1098 \text{ cents}}{700 \text{ g}} = 1.5686$ cents per g

Final unit price = $\frac{998 \text{ cents}}{600 \text{ g}} = 1.6633$ cents per g

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{1.6633 - 1.5686}{1.5686} \times 100\% = \underline{\underline{6.04\%}}$$

45. Current unit price = $\frac{449 \text{ cents}}{500 \text{ ml}} = 0.8980$ cents per ml

New unit price = $1.10(0.8980 \text{ cents per ml}) = 0.9878$ cents per ml

Price of a 425-ml container = $(425 \text{ ml}) \times (0.9878 \text{ cents per ml}) = 419.8 \text{ cents} = \underline{\underline{\$4.20}}$

46. Current unit price = $\frac{115 \text{ cents}}{100 \text{ g}} = 1.15$ cents per g

New unit price = $1.075(1.15 \text{ cents per g}) = 1.23625$ cents per g

Price of an 80-g bar = $(80 \text{ g}) \times (1.23625 \text{ cents per g}) = 98.9 \text{ cents} = \underline{\underline{\$0.99}}$

Exercise 2.5 (continued)

47. Given: $V_f = \$338,500$, $c = 8.7\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$338,500}{1.087} = \underline{\underline{\$311,400}}$$

The average price one year ago was \$311,400.

48. Given: $V_f = \$348.60$, $c = -0.30$

$$V_i = \frac{V_f}{1+c} = \frac{\$348.60}{1+(-0.30)} = \frac{\$348.60}{0.70} = \underline{\underline{\$498.00}}$$

The regular price of the boots is \$498.00.

49. For Year 1, $V_f = \$6$ and $V_f - V_i = -\$4$
Therefore, $V_i = V_f + \$4 = \$6 + \$4 = \10

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{-\$4}{\$10} \times 100\% = \underline{\underline{-40.00\%}}$$

For Year 2, $V_i = \$6$ and $V_f - V_i = \$4$

$$\text{Therefore, } c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$4}{\$6} \times 100\% = \underline{\underline{66.67\%}}$$

The percent change was -40.00% in Year 1 and 66.67% in Year 2.

50. Given: For Q2 of 2009, $V_f = 5.21$ million, $c = 626\%$

$$V_i = \frac{V_f}{1+c} = \frac{5.21 \text{ million}}{1+6.26} = 0.7176 \text{ million} = 717,600$$

Rounded to the nearest 10,000, Apple sold 720,000 iPhones in Q2 of 2008.

51. Given: In February of 2008, $V_i = 475,000$ visitors and $c = 1382\%$
In February of 2009, the number of visitors was

$$V_f = V_i(1+c) = 475,000(1+13.82) = 7,039,500$$

Rounded to the nearest 1000, Twitter.com had 7,040,000 visitors in February of 2009.

52. The fees to Fund A will be

$$\frac{(\text{Fees to Fund A}) - (\text{Fees to Fund B})}{(\text{Fees to Fund B})} \times 100\% = \frac{2.38\% - 1.65\%}{1.65\%} \times 100\% = \underline{\underline{44.24\%}}$$

more than the fees to Fund B.

53. Percent change in the GST rate

$$= \frac{(\text{Final GST rate}) - (\text{Initial GST rate})}{(\text{Initial GST rate})} \times 100\% = \frac{5\% - 6\%}{6\%} \times 100\% = \underline{\underline{-16.67\%}}$$

The GST paid by consumers was reduced by 16.67%.

54. Given: For February of 2009, $V_f = 65,704,000$ visitors, $c = 228.2\%$

$$\text{Then, } V_i = \frac{V_f}{1+c} = \frac{65,704,000}{1+2.282} = 20,019,500$$

That is, Facebook had 20,019,500 unique visitors in February of 2008

Therefore, the absolute increase from February of 2008 to February of 2009 was
 $65,704,000 - 20,019,500 = \underline{\underline{45,680,000}}$ (rounded to the nearest 10,000)

Exercise 2.5 (continued)

55. Given: $V_f = \$0.45$, $c = 76\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$0.45}{1+(-0.76)} = \$1.88$$

$$\text{Price decline} = V_i - V_f = \$1.88 - \$0.45 = \underline{\underline{\$1.43}}$$

The share price dropped by \$1.43.

56. Given: $V_f = \$24,300$, $c = -55\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$24,300}{1+(-0.55)} = \$54,000$$

$$\text{The amount of depreciation is } \$54,000 - \$24,300 = \underline{\underline{\$29,700}}.$$

57. Given: For the appreciation, $V_i = \text{Purchase price}$, $c = 140\%$, $V_f = \text{List price}$
For the price reduction, $V_i = \text{List price}$, $c = -10\%$, $V_f = \$172,800$

$$\text{List price} = \frac{V_f}{1+c} = \frac{\$172,800}{1+(-0.1)} = \$192,000$$

$$\text{Original purchase price} = \frac{V_f}{1+c} = \frac{\$192,000}{1+1.4} = \underline{\underline{\$80,000}}$$

The owner originally paid \$80,000 for the property.

58. Given: For the markup, $V_i = \text{Cost}$, $c = 22\%$, $V_f = \text{List price}$

For the markdown, $V_i = \text{List price}$, $c = -10\%$, $V_f = \$17,568$

$$\text{List price} = \frac{V_f}{1+c} = \frac{\$17,568}{1+(-0.10)} = \$19,520$$

$$\text{Cost (to dealer)} = \frac{V_f}{1+c} = \frac{\$19,520}{1+0.22} = \underline{\underline{\$16,000}}$$

The dealer paid \$16,000 for the car.

59. If General Paint's prices are marked down by 30%, then

General Paint's prices = 0.70(Cloverdale Paint's prices)

$$\text{Hence, Cloverdale's prices} = \frac{\text{General Paint's prices}}{0.70} = 1.4286(\text{General Paint's prices})$$

Therefore, you will pay 42.86% more at Cloverdale Paint.

60. If the Canadian dollar is worth 6.5% less than the US dollar,

Canadian dollar = $(1 - 0.065)(\text{US dollar}) = 0.935(\text{US dollar})$

$$\text{Hence, US dollar} = \frac{\text{Canadian dollar}}{0.935} = 1.0695(\text{Canadian dollar})$$

Therefore, the US dollar is worth 6.95% more than the Canadian dollar.

61. Canada's exports to US exceeded imports from the US by 23%.

That is, Exports = 1.23(Imports)

$$\text{Therefore, Imports} = \frac{\text{Exports}}{1.23} = 0.8130(\text{Exports})$$

That is, Canada's imports from US (= US exports to Canada) were

$$1 - 0.8130 = 0.1870 = \underline{\underline{18.70\%}}$$

less than Canada's exports to US (= US imports from Canada.)

Exercise 2.5 (continued)

62. Given: January sales were 17.4% less than December sales
Hence, January sales = $(1 - 0.174)(\text{December sales}) = 0.826(\text{December sales})$
Therefore, December sales = $\frac{\text{January sales}}{0.826} = 1.2107(\text{January sales})$
That is, December sales were 121.07% of January sales.
63. Suppose the initial ratio is $\frac{x}{y}$.
If the denominator is reduced by 20%, then
$$\text{Final ratio} = \frac{x}{y - 0.20y} = \frac{x}{0.8y} = 1.25 \frac{x}{y}$$

That is, the value of the ratio increases by 25%.
64. Next year there must be 15% fewer students per teacher.
With the same number of students,
$$\frac{\text{Students}}{\text{Teachers next year}} = 0.85 \left(\frac{\text{Students}}{\text{Teachers now}} \right)$$

Therefore, Teachers next year = $\frac{\text{Teachers now}}{0.85} = 1.1765(\text{Teachers now})$
That is, if the number of students does not change, the number of teachers must be increased by 17.65%.
65. Given: Operating expenses = 0.40(Revenue)
Then Revenue = $\frac{\text{Operating expenses}}{0.40} = 2.5(\text{Operating expenses})$
That is, Revenue is 250% of Operating expenses, or
Revenue exceeds Operating expenses by $250\% - 100\% = \underline{150\%}$.
66. Given: Equity = $(100\% - 50\%)$ of Debt = 50% of Debt = $0.50(\text{Debt})$
Therefore, $\frac{\text{Debt}}{\text{Equity}} = \frac{\text{Debt}}{0.5(\text{Debt})} = \frac{1}{0.5} = 2$
Since Debt is twice (or 200% of) Equity, then debt financing is 100% more than equity financing.
67. Use ppm as the abbreviation for "pages per minute".
Given: Lightning printer prints 30% more ppm than the Reliable printer.
That is, the Lightning's printing speed is 1.30 times the Reliable's printing speed.
Therefore, the Reliable's printing speed is
$$\frac{1}{1.3} = 0.7692 = 76.92\% \text{ of the Lightning's printing speed}$$

Therefore, the Reliable's printing speed is
 $100\% - 76.92\% = 23.08\%$ less than the Lightning's speed.
The Lightning printer will require 23.08% less time than the Reliable for a long printing job.
68. Given: Euro is worth 39% more than the Canadian dollar.
That is, Euro = $1.39(\text{Canadian dollar})$
Therefore, Canadian dollar = $\frac{\text{Euro}}{1.39} = 0.7914(\text{Euro}) = 79.14\%$ of a Euro.
That is, the Canadian dollar is worth $100\% - 79.14\% = \underline{28.06\% \text{ less}}$ than the Euro.

Exercise 2.5 (continued)

69. Let us use OT as an abbreviation for “overtime”.

The number of OT hours permitted by this year’s budget is

$$\text{OT hours (this year)} = \frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

The number of overtime hours permitted by next year’s budget is

$$\begin{aligned}\text{OT hours (next year)} &= \frac{\text{OT budget (next year)}}{\text{OT hourly rate (next year)}} = \frac{1.03 [\text{OT budget (this year)}]}{1.05 [\text{OT hourly rate (this year)}]} \\ &= 0.980952 \frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}} \\ &= 98.0952\% \text{ of this year's OT hours}\end{aligned}$$

The number of OT hours must be reduced by $100\% - 98.0952\% = \underline{1.90\%}$.

Review Problems

1. $4(3a + 2b)(2b - a) - 5a(2a - b) = 4(6ab - 3a^2 + 4b^2 - 2ab) - 10a^2 + 5ab$
 $= \underline{\underline{-22a^2 + 21ab + 16b^2}}$
2. a. Given: $c = 17.5\%$, $V_i = \$29.43$
 $V_f = V_i(1 + c) = \$29.43(1.175) = \underline{\underline{\$34.58}}$
\$34.58 is 17.5% more than \$29.43.
b. Given: $V_f = \$100$, $c = -80\%$
 $V_i = \frac{V_f}{1 + c} = \frac{\$100}{1 - 0.80} = \underline{\underline{\$500.00}}$
80% off \$500 leaves \$100.
c. Given: $V_f = \$100$, $c = -15\%$
 $V_i = \frac{V_f}{1 + c} = \frac{\$100}{1 - 0.15} = \underline{\underline{\$117.65}}$
\$117.65 reduced by 15% equals \$100.
d. Given: $V_i = \$47.50$, $c = 320\%$
 $V_f = V_i(1 + c) = \$47.50(1 + 3.2) = \underline{\underline{\$199.50}}$
\$47.50 after an increase of 320% is \$199.50.
e. Given: $c = -62\%$, $V_f = \$213.56$
 $V_i = \frac{V_f}{1 + c} = \frac{\$213.56}{1 - 0.62} = \underline{\underline{\$562.00}}$
\$562 decreased by 62% equals \$213.56.
f. Given: $c = 125\%$, $V_f = \$787.50$
 $V_i = \frac{V_f}{1 + c} = \frac{\$787.50}{1 + 1.25} = \underline{\underline{\$350.00}}$
\$350 increased by 125% equals \$787.50.
g. Given: $c = -30\%$, $V_i = \$300$
 $V_f = V_i(1 + c) = \$300(1 - 0.30) = \underline{\underline{\$210.00}}$
\$210 is 30% less than \$300.

Review Problems (continued)

3. a. $\frac{9y-7}{3} - 2.3(y-2) = 3y - 2.\bar{3} - 2.3y + 4.6 = \underline{\underline{0.7y + 2.2\bar{6}}}$
 b. $P\left(1 + 0.095 \times \frac{135}{365}\right) + \frac{2P}{1 + 0.095 \times \frac{75}{365}} = 1.035137P + 1.961706P = \underline{\underline{2.996843P}}$
4. a. $6(4y-3)(2-3y) - 3(5-y)(1+4y) = 6(8y-12y^2-6+9y) - 3(5+20y-y-4y^2)$
 $= \underline{\underline{-60y^2 + 45y - 51}}$
 b. $\frac{5b-4}{4} - \frac{25-b}{1.25} + \frac{7}{8}b = 1.25b - 1 - 20 + 0.8b + 0.875b = \underline{\underline{2.925b - 21}}$
 c. $\frac{x}{1 + 0.085 \times \frac{63}{365}} + 2x\left(1 + 0.085 \times \frac{151}{365}\right) = 0.985541x + 2.070329x = \underline{\underline{3.05587x}}$
 d. $\frac{96nm^2 - 72n^2m^2}{48n^2m} = \frac{4m-3nm}{2n} = \frac{4m}{2n} - \frac{3nm}{2n} = \underline{\underline{2\frac{m}{n} - 1.5m}}$
5. $P(1+i)^n + \frac{S}{1+rt} = \$2500(1.1025)^2 + \frac{\$1500}{1 + 0.09 \times \frac{93}{365}} = \$3038.766 + \$1466.374 = \underline{\underline{\$4505.14}}$
6. a. $L(1-d_1)(1-d_2)(1-d_3) = \$340(1-0.15)(1-0.08)(1-0.05) = \underline{\underline{\$252.59}}$
 b. $\frac{R}{i}\left[1 - \frac{1}{(1+i)^n}\right] = \frac{\$575}{0.085}\left[1 - \frac{1}{(1+0.085)^3}\right] = \$6764.706(1-0.7829081) = \underline{\underline{\$1468.56}}$
7. a. $\frac{(-3x^2)^3(2x^{-2})}{6x^5} = \frac{(-27x^6)(2x^{-2})}{6x^5} = \underline{\underline{-\frac{9}{x}}}$
 b. $\frac{(-2a^3)^{-2}(4b^4)^{3/2}}{(-2b^3)(0.5a)^3} = \frac{\left(\frac{1}{4a^6}\right)(8b^6)}{(-2b^3)(0.125a^3)} = \underline{\underline{-\frac{8b^3}{a^9}}}$
8. $\left(-\frac{2x^2}{3}\right)^{-2}\left(\frac{5^2}{6x^3}\right)\left(-\frac{15}{x^5}\right)^{-1} = \left(\frac{3}{2x^2}\right)^2\left(\frac{25}{6x^3}\right)\left(-\frac{x^5}{15}\right) = \underline{\underline{-\frac{5}{8x^2}}}$
9. a. $1.0075^{24} = \underline{\underline{1.19641}}$
 b. $(1.05)^{1/6} - 1 = \underline{\underline{0.00816485}}$
 c. $\frac{(1+0.0075)^{36} - 1}{0.0075} = \underline{\underline{41.1527}}$
 d. $\frac{1 - (1+0.045)^{-12}}{0.045} = \underline{\underline{9.11858}}$
10. a. $\frac{(1.006)^{240} - 1}{0.006} = \frac{4.926802 - 1}{0.006} = \underline{\underline{589.020}}$
 b. $(1+0.025)^{1/3} - 1 = \underline{\underline{0.00826484}}$

Review Problems (continued)

$$11. \quad a. \quad \frac{2x}{1 + 0.13 \times \frac{92}{365}} + x \left(1 + 0.13 \times \frac{59}{365} \right) = \$831$$

$$1.936545x + 1.021014x = \$831$$

$$2.957559x = \$831$$

$$x = \underline{\underline{\$280.97}}$$

$$b. \quad 3x(1.03^5) + \frac{x}{1.03^3} + x = \frac{\$2500}{1.03^2}$$

$$3.47782x + 0.91514x + x = \$2356.49$$

$$x = \underline{\underline{\$436.96}}$$

$$12. \quad a. \quad \frac{x}{1.08^3} + \frac{x}{2}(1.08)^4 = \$850$$

$$0.793832x + 0.680245x = \$850$$

$$x = \underline{\underline{\$576.63}}$$

Check: $\frac{\$576.63}{1.08^3} + \frac{\$576.63}{2}(1.08)^4 = \$457.749 + \$392.250 = \$850.00$

$$b. \quad 2x \left(1 + 0.085 \times \frac{77}{365} \right) + \frac{x}{1 + 0.085 \times \frac{132}{365}} = \$1565.70$$

$$2.03586x + 0.97018x = \$1565.70$$

$$x = \underline{\underline{\$520.85}}$$

Check:

$$2(\$520.85) \left(1 + 0.085 \times \frac{77}{365} \right) + \frac{\$520.85}{1 + 0.085 \times \frac{132}{365}} = \$1060.38 + \$505.32 = \$1565.70$$

$$13. \quad N = L(1 - d_1)(1 - d_2)(1 - d_3)$$

$$\$324.30 = \$498(1 - 0.20)(1 - d_2)(1 - 0.075)$$

$$\$324.30 = \$368.52(1 - d_2)$$

$$\frac{\$324.30}{\$368.52} = (1 - d_2)$$

$$d_2 = 1 - 0.8800 = \underline{\underline{0.120}} = \underline{\underline{12.0\%}}$$

$$14. \quad V_f = V_i(1 + c_1)(1 + c_2)(1 + c_3)$$

$$\$586.64 = \$500(1 + 0.17)(1 + c_2)(1 + 0.09)$$

$$\$586.64 = \$637.65(1 + c_2)$$

$$1 + c_2 = \frac{\$586.64}{\$637.65}$$

$$c_2 = 0.9200 - 1 = \underline{\underline{-0.0800}} = \underline{\underline{-8.00\%}}$$

$$15. \quad \begin{array}{ll} 3x + 5y = 11 & \textcircled{1} \\ 2x - y = 16 & \textcircled{2} \end{array}$$

To eliminate y,

$$\textcircled{1}: \quad 3x + 5y = 11$$

$$\textcircled{2} \times 5: \underline{10x - 5y = 80}$$

Add: $13x + 0 = 91$

$$x = 7$$

Substitute into equation $\textcircled{2}$: $2(7) - y = 16$

$$y = -2$$

Hence, $(x, y) = \underline{\underline{(7, -2)}}$

Review Problems (continued)

16. a. $4a - 5b = 30$ ①
 $2a - 6b = 22$ ②

To eliminate a,

$$\textcircled{1} \times 1: a - 5b = 30$$

$$\textcircled{2} \times 2: \underline{4a - 12b = 44}$$

$$\text{Subtract: } \quad \quad 7b = -14$$

$$\quad \quad \quad b = -2$$

$$\text{Substitute into } \textcircled{1}: 4a - 5(-2) = 30$$

$$4a = 30 - 10$$

$$a = 5$$

$$\text{Hence, } (a, b) = (5, -2)$$

b. $76x - 29y = 1050$ ①
 $-13x - 63y = 250$ ②

To eliminate ①,

$$\textcircled{1} \times 13: 988x - 377y = 13,650$$

$$\textcircled{2} \times 76: \underline{-988x - 4788y = 19,000}$$

$$\text{Add: } \quad \quad \quad -5165y = 32,650$$

$$\quad \quad \quad y = -6.321$$

$$\text{Substitute into } \textcircled{1}: 76x - 29(-6.321) = 1050$$

$$76x = 1050 - 183.31$$

$$x = 11.40$$

$$\text{Hence, } (x, y) = (11.40, -6.32)$$

17. $FV = PV(1 + i_1)(1 + i_2)$

$$\frac{FV}{PV(1 + i_2)} = (1 + i_1)$$

$$i_1 = \frac{FV}{PV(1 + i_2)} - 1$$

18. Given:

	<u>Year 1 value (V_i)</u>	<u>Year 2 value (V_f)</u>
Gold produced:	34,300 oz.	23,750 oz.
Average price:	\$1160	\$1280

a. Percent change in gold production = $\frac{23,750 - 34,300}{34,300} \times 100\% = \underline{\underline{-30.76\%}}$

b. Percent change in price = $\frac{\$1280 - \$1160}{\$1160} \times 100\% = \underline{\underline{10.34\%}}$

c. Year 1 revenue, $V_i = 34,300(\$1160) = \39.788 million
 Year 2 revenue, $V_f = 23,750(\$1280) = \30.400 million

$$\text{Percent change in revenue} = \frac{\$30.400 - \$39.788}{\$39.788} \times 100\% = \underline{\underline{-23.60\%}}$$

19. Given: For the first year, $V_i = \$3.40$, $V_f = \$11.50$.

For the second year, $V_i = \$11.50$, $c = -35\%$.

a. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$11.50 - \$3.40}{\$3.40} \times 100\% = \underline{\underline{238.24\%}}$

The share price increased by 238.24% in the first year.

b. Current share price, $V_f = V_i(1 + c) = \$11.50(1 - 0.35) = \underline{\underline{\$7.48}}$.

Review Problems (continued)

20. Given: For the first year, $c = 150\%$

For the second year, $c = -40\%$, $V_f = \$24$

The price at the beginning of the second year was

$$V_i = \frac{V_f}{1+c} = \frac{\$24}{1-0.40} = \$40.00 = V_f \text{ for the first year.}$$

The price at the beginning of the first year was

$$V_i = \frac{V_f}{1+c} = \frac{\$40.00}{1+1.50} = \underline{\underline{\$16.00}}$$

Barry bought the stock for \$16.00 per share.

21. Given: Last year's revenue = \$2,347,000

Last year's expenses = \$2,189,000

- a. Given: Percent change in revenue = 10%; Percent change in expenses = 5%

Anticipated revenues, $V_f = V_i(1+c) = \$2,347,000(1.1) = \$2,581,700$

Anticipated expenses = $\$2,189,000(1.05) = \underline{\underline{\$2,298,450}}$

Anticipated profit = $\$283,250$

Last year's profit = $\$2,347,000 - \$2,189,000 = \$158,000$

Percent increase in profit = $\frac{\$283,250 - \$158,000}{\$158,000} \times 100\% = \underline{\underline{79.27\%}}$

- b. Given: $c(\text{revenue}) = -10\%$; $c(\text{expenses}) = -5\%$

Anticipated revenues = $\$2,347,000(1 - 0.10) = \$2,112,300$

Anticipated expenses = $\$2,189,000(1 - 0.05) = \underline{\underline{\$2,079,550}}$

Anticipated profit = $\$32,750$

Percent change in profit = $\frac{\$32,750 - \$158,000}{\$158,000} \times 100\% = \underline{\underline{-79.27\%}}$

The operating profit will decline by 79.27%.

22. Given: Ken's share = $0.80(\text{Hugh's share}) + \$15,000$; Total distribution = \$98,430

Let H represent Hugh's share. Then

Hugh's share + Ken's share = Total distribution

$$H + 0.8H + \$15,000 = \$98,430$$

$$1.8H = \$83,430$$

$$H = \$46,350$$

Hugh should receive \$46,350 and Ken should receive \$98,430 - \$46,350 = \$52,080.

23. Given: Grace's share = $1.2(\text{Kajsa's share})$; Mary Anne's share = $\frac{5}{8}(\text{Grace's share})$

Total allocated = \$36,000

Let K represent Kajsa's share.

(Kajsa's share) + (Grace's share) + (Mary Anne's share) = \$36,000

$$K + 1.2K + \frac{5}{8}(1.2K) = \$36,000$$

$$2.95K = \$36,000$$

$$K = \underline{\underline{\$12,203.39}}$$

Kajsa's should receive \$12,203.39. Grace should receive $1.2K = \underline{\underline{\$14,644.07}}$.

Mary Anne should receive $\frac{5}{8}(\$14,644.07) = \underline{\underline{\$9152.54}}$.

Review Problems (continued)

24. Let R represent the price per kg for red snapper and let L represent the price per kg for ling cod. Then

$$370R + 264L = \$2454.20 \quad \textcircled{1}$$

$$255R + 304L = \$2124.70 \quad \textcircled{2}$$

To eliminate R,

$$\textcircled{1} \div 370: R + 0.71351L = \$6.6330$$

$$\textcircled{2} \div 255: \underline{R + 1.19216L = \$8.3322}$$

$$\begin{array}{r} \text{Subtract:} \\ -0.47865L = -\$1.6992 \\ L = \$3.55 \end{array}$$

$$\text{Substitute into } \textcircled{1}: 370R + 264(\$3.55) = \$2454.20$$

$$370R = \$1517.00$$

$$R = \$4.10$$

Nguyen was paid \$3.55 per kg for ling cod and \$4.10 per kg for red snapper.

25. Let b represent the base salary and r represent the commission rate. Then

$$r(\$27,000) + b = \$2815.00 \quad \textcircled{1}$$

$$\underline{r(\$35,500) + b = \$3197.50} \quad \textcircled{2}$$

$$\begin{array}{r} \text{Subtract:} \\ -\$8500r = \$382.50 \\ r = 0.045 \end{array}$$

$$\text{Substitute into } \textcircled{1}: 0.045(\$27,000) + b = \$2815$$

$$b = \$1600$$

Deanna's base salary is \$1600 per month and her commission rate is 4.5%.

26. Given: Total initial investment = \$7800; Value 1 year later = \$9310

Percent change in ABC portion = 15%

Percent change in XYZ portion = 25%

Let X represent the amount invested in XYZ Inc.

The solution "idea" is:

$$(\text{Amount invested in ABC})1.15 + (\text{Amount invested in XYZ})1.25 = \$9310$$

Hence,

$$(\$7800 - X)1.15 + (X)1.25 = \$9310$$

$$\$8970 - 1.15X + 1.25X = \$9310$$

$$0.10X = \$9310 - \$8970$$

$$X = \$3400$$

Rory invested \$3400 in XYZ Inc. and $\$7800 - \$3400 =$ \$4400 in ABC Ltd.

27. Let the regular season ticket prices be R for the red section and B for the blue section. Then

$$2500R + 4500B = \$50,250 \quad \textcircled{1}$$

$$2500(1.3R) + 4500(1.2B) = \$62,400 \quad \textcircled{2}$$

$$\textcircled{1} \times 1.2: \underline{2500(1.2R) + 4500(1.2B) = \$60,300}$$

$$\begin{array}{r} \text{Subtract:} \\ 2500(0.1R) + 0 = \$2100 \\ R = \$8.40 \end{array}$$

$$\text{Substitute into } \textcircled{1}: 2500(\$8.40) + 4500B = \$50,250$$

$$B = \$6.50$$

The ticket prices for the playoffs cost

$$1.3 \times \$8.40 = \underline{\$10.92} \text{ in the "reds"}$$

$$\text{and } 1.2 \times \$6.50 = \underline{\$7.80} \text{ in the "blues".}$$

Review Problems (*continued*)

28. 60% of a $\frac{3}{8}$ interest was purchased for \$25,000.

Let the V represent the implied value of the entire partnership.

Then $0.60 \times \frac{3}{8} V = \$25,000$

$$V = \frac{8 \times \$25,000}{0.60 \times 3} = \underline{\underline{\$111,111}}$$

The implied value of the chalet was \$111,111.

29. Let S represent the number of cucumbers sold individually and let F represent the number of four-cucumber packages sold in the promotion. Then

$$S + 4F = 541 \quad \text{①}$$

$$\$0.98S + \$2.94F = \$418.46 \quad \text{②}$$

To eliminate S,

$$\text{①} \times \$0.98: \$0.98S + \$3.92F = \$530.18$$

$$\text{②}: \underline{\underline{\$0.98S + \$2.94F = \$418.46}}$$

$$\begin{array}{r} \text{Subtract:} \quad \quad \quad 0 \quad + \$0.98F = \$111.72 \\ \quad \quad \quad \quad \quad \quad \quad \quad F = 114 \end{array}$$

Hence, a total of $4 \times 114 = \underline{\underline{456 \text{ cucumbers}}}$ were sold on the four-for-the-price-of-three promotion.