

- 31.** $F(x) = (x - a)^2(x - b)^2 + x$. Without loss of generality, we can assume that $a < b$. Being a polynomial, F is continuous on $[a, b]$. Also $F(a) = a$ and $F(b) = b$. Since $a < \frac{1}{2}(a + b) < b$, the Intermediate-Value Theorem guarantees that there is an x in (a, b) such that
 $F(x) = (a + b)/2$.