

- 32.** Let $g(x) = f(x) - x$. Since $0 \leq f(x) \leq 1$ if $0 \leq x \leq 1$, therefore, $g(0) \geq 0$ and $g(1) \leq 0$. If $g(0) = 0$ let $c = 0$, or if $g(1) = 0$ let $c = 1$. (In either case $f(c) = c$.) Otherwise, $g(0) > 0$ and $g(1) < 0$, and, by IVT, there exists c in $(0, 1)$ such that $g(c) = 0$, i.e., $f(c) = c$.