

18. To be proved: $\lim_{x \rightarrow -1} \frac{x+1}{x^2-1} = -\frac{1}{2}$.

Proof: Let $\epsilon > 0$ be given. If $x \neq -1$, we have

$$\left| \frac{x+1}{x^2-1} - \left(-\frac{1}{2}\right) \right| = \left| \frac{1}{x-1} - \left(-\frac{1}{2}\right) \right| = \frac{|x+1|}{2|x-1|}.$$

If $|x+1| < 1$, then $-2 < x < 0$, so $-3 < x-1 < -1$ and $|x-1| > 1$. Let $\delta = \min(1, 2\epsilon)$.

If $0 < |x - (-1)| < \delta$ then $|x-1| > 1$ and $|x+1| < 2\epsilon$. Thus

$$\left| \frac{x+1}{x^2-1} - \left(-\frac{1}{2}\right) \right| = \frac{|x+1|}{2|x-1|} < \frac{2\epsilon}{2} = \epsilon.$$

This completes the required proof.