31. To be proved: if $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} f(x) = M$, then $L = M$.
Proof: Suppose $L \neq M$. Let $\epsilon = |L - M|/3$. Then $\epsilon > 0$. Since $\lim_{x \to a} f(x) = L$, there exists $\delta_1 > 0$ such that $|f(x) - L| < \epsilon$ if $|x - a| < \delta_1$. Since $\lim_{x \to a} f(x) = M$, there exists $\delta_2 > 0$ such that $|f(x) - M| < \epsilon$ if $|x - a| < \delta_2$. Let $\delta = \min(\delta_1, \delta_2)$. If $|x - a| < \delta$, then

$$\begin{aligned} 3\epsilon = |L - M| &= |(f(x) - M) + (L - f(x)| \\ &\leq |f(x) - M| + |f(x) - L| < \epsilon + \epsilon = 2\epsilon. \end{aligned}$$

This implies that $3 < 2$, a contradiction. Thus the original assumption that $L \neq M$ must be incorrect. Therefore $L = M$.