

32. To be proved: if $\lim_{x \rightarrow a} g(x) = M$, then there exists $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|g(x)| < 1 + |M|$.

Proof: Taking $\epsilon = 1$ in the definition of limit, we obtain a number $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|g(x) - M| < 1$. It follows from this latter inequality that

$$|g(x)| = |(g(x) - M) + M| \leq |g(x) - M| + |M| < 1 + |M|.$$