

37. To be proved: if f is continuous at L and $\lim_{x \rightarrow c} g(x) = L$, then $\lim_{x \rightarrow c} f(g(x)) = f(L)$.

Proof: Let $\epsilon > 0$ be given. Since f is continuous at L , there exists a number $\gamma > 0$ such that if $|y - L| < \gamma$, then $|f(y) - f(L)| < \epsilon$. Since $\lim_{x \rightarrow c} g(x) = L$, there exists $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|g(x) - L| < \gamma$. Taking $y = g(x)$, it follows that if $0 < |x - c| < \delta$, then $|f(g(x)) - f(L)| < \epsilon$, so that $\lim_{x \rightarrow c} f(g(x)) = f(L)$.