1. Let $0 < a < b$. The average rate of change of x^3 over $[a, b]$ is

$$\frac{b^3 - a^3}{b - a} = b^2 + ab + a^2.$$

The instantaneous rate of change of x^3 at $x = c$ is

$$\lim_{h\to 0} \frac{(c+h)^3 - c^3}{h} = \lim_{h\to 0} \frac{3c^2h + 3ch^2 + h^3}{h} = 3c^2.$$

If $c = \sqrt{(a^2 + ab + b^2)/3}$, then $3c^2 = a^2 + ab + b^2$, so the average rate of change over $[a, b]$ is the instantaneous rate of change at $\sqrt{(a^2 + ab + b^2)/3}$.
Claim: $\sqrt{(a^2 + ab + b^2)/3} > (a + b)/2$.
Proof: Since $a^2 - 2ab + b^2 = (a - b)^2 > 0$, we have

$$\begin{aligned}
4a^2 + 4ab + 4b^2 &> 3a^2 + 6ab + 3b^2 \\
\frac{a^2 + ab + b^2}{3} &> \frac{a^2 + 2ab + b^2}{4} = \left(\frac{a+b}{2}\right)^2 \\
\sqrt{\frac{a^2 + ab + b^2}{3}} &> \frac{a+b}{2}.
\end{aligned}$$