

5. Use $a - b = \frac{a^3 - b^3}{a^2 + ab + b^2}$ to handle the denominator. We have

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{\sqrt{3+x} - 2}{\sqrt[3]{7+x} - 2} \\ &= \lim_{x \rightarrow 1} \frac{3+x-4}{\sqrt{3+x}+2} \times \frac{(7+x)^{2/3} + 2(7+x)^{1/3} + 4}{(7+x) - 8} \\ &= \lim_{x \rightarrow 1} \frac{(7+x)^{2/3} + 2(7+x)^{1/3} + 4}{\sqrt{3+x}+2} = \frac{4+4+4}{2+2} = 3. \end{aligned}$$