

20. To be proved: $\lim_{x \rightarrow 2} x^3 = 8$.

Proof: Let $\epsilon > 0$ be given. We have

$|x^3 - 8| = |x - 2||x^2 + 2x + 4|$. If $|x - 2| < 1$, then $1 < x < 3$ and $x^2 < 9$. Therefore $|x^2 + 2x + 4| \leq 9 + 2 \times 3 + 4 = 19$. If $|x - 2| < \delta = \min(1, \epsilon/19)$, then

$$|x^3 - 8| = |x - 2||x^2 + 2x + 4| < \frac{\epsilon}{19} \times 19 = \epsilon.$$

This completes the proof.