

35. To be proved: if $\lim_{x \rightarrow a} g(x) = M$ where $M \neq 0$, then $\lim_{x \rightarrow a} \frac{1}{g(x)} = \frac{1}{M}$.

Proof: Let $\epsilon > 0$ be given. Since $\lim_{x \rightarrow a} g(x) = M \neq 0$, there exists $\delta_1 > 0$ such that $|g(x) - M| < \epsilon|M|^2/2$ if $0 < |x - a| < \delta_1$. By Exercise 34, there exists $\delta_2 > 0$ such that $|g(x)| > |M|/2$ if $0 < |x - a| < \delta_2$. Let $\delta = \min(\delta_1, \delta_2)$. If $0 < |x - a| < \delta$, then

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \frac{|M - g(x)|}{|M||g(x)|} < \frac{\epsilon|M|^2}{2} \frac{2}{|M|^2} = \epsilon.$$

This completes the proof.