

8. a) To be proved: if  $f$  is a continuous function defined on a closed interval  $[a, b]$ , then the range of  $f$  is a closed interval.

Proof: By the Max-Min Theorem there exist numbers  $u$  and  $v$  in  $[a, b]$  such that  $f(u) \leq f(x) \leq f(v)$  for all  $x$  in  $[a, b]$ . By the Intermediate-Value Theorem,  $f(x)$  takes on all values between  $f(u)$  and  $f(v)$  at values of  $x$  between  $u$  and  $v$ , and hence at points of  $[a, b]$ . Thus the range of  $f$  is  $[f(u), f(v)]$ , a closed interval.

- b) If the domain of the continuous function  $f$  is an open interval, the range of  $f$  can be any interval (open, closed, half open, finite, or infinite).