CHAPTER P
Preparation for Calculus

Section P.1 Graphs and Models

1. $y = -\frac{3}{2}x + 3$

   x-intercept: $(2, 0)$

   y-intercept: $(0, 3)$

   Matches graph (b).

2. $y = \sqrt{9 - x^2}$

   x-intercepts: $(-3, 0), (3, 0)$

   y-intercept: $(0, 3)$

   Matches graph (d).

3. $y = 1 + x^2$

   x-intercept: none

   y-intercept: $(0, 1)$

   Matches graph (a).

4. $y = x^3 - x$

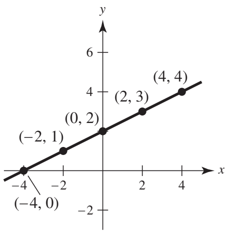
   x-intercepts: $(0, 0), (-1, 0), (1, 0)$

   y-intercept: $(0, 0)$

   Matches graph (c).

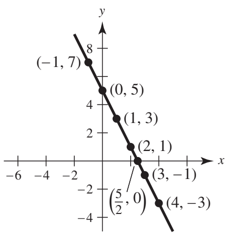
5. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



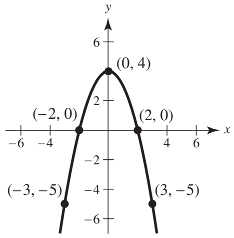
6. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



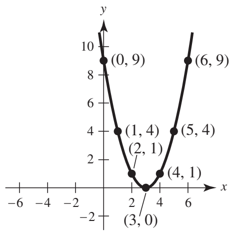
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



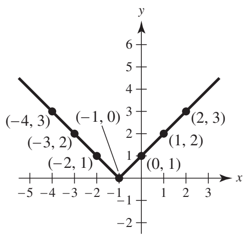
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



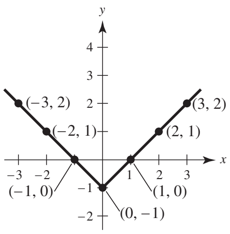
9. $y = |x + 1|$

x	-4	-3	-2	-1	0	1	2
y	3	2	1	0	1	2	3



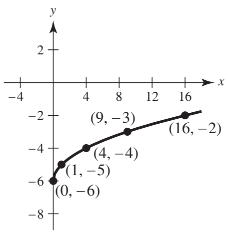
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



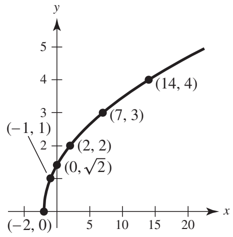
11. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



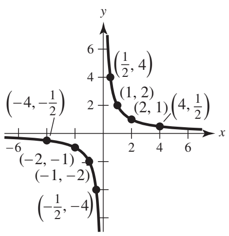
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



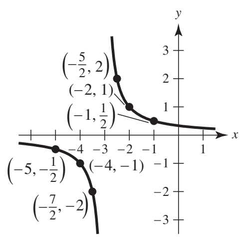
13. $y = \dfrac{2}{x}$

x	-4	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	4
y	$-\frac{1}{2}$	-1	-2	-4	Undef.	4	2	1	$\frac{1}{2}$

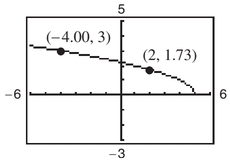


14. $y = \dfrac{1}{x + 3}$

x	-5	-4	$-\frac{7}{2}$	-3	$-\frac{5}{2}$	-2	-1
y	$-\frac{1}{2}$	-1	-2	Undef.	2	1	$\frac{1}{2}$



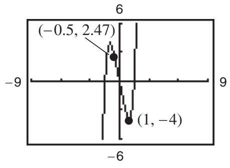
15. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73) \quad \left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3) \quad \left(3 = \sqrt{5 - (-4)}\right)$

16. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

17. $y = 4x - 3$

y-intercept: $y = 4(0) - 3 = -3;\ (0, -3)$

x-intercept: $0 = 4x - 3$

$3 = 4x$

$\frac{3}{4} = x;\ \left(\frac{3}{4}, 0\right)$

18. $y = 2x^2 + 5$

y-intercept: $y = 2(0)^2 + 5 = 5;\ (0, 5)$

x-intercept: $0 = 2x^2 + 5$

$-5 = 2x^2$

None (y cannot equal 0.)

19. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

20. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2;\ (0, 0), (\pm 2, 0)$

21. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0;\ (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4;\ (0, 0), (4, 0), (-4, 0)$

22. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1;\ (1, 0)$

23. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4;\ (4, 0)$

24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3;\ (0, 0), (-3, 0)$

25. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

26. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept:

$$\begin{aligned} 0 &= 2x - \sqrt{x^2 + 1} \\ 2x &= \sqrt{x^2 + 1} \\ 4x^2 &= x^2 + 1 \\ 3x^2 &= 1 \\ x^2 &= \frac{1}{3} \\ x &= \pm\frac{\sqrt{3}}{3} \\ x &= \frac{\sqrt{3}}{3};\ \left(\frac{\sqrt{3}}{3}, 0\right) \end{aligned}$$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

27. Symmetric with respect to the y-axis because
$y = (-x)^2 - 6 = x^2 - 6.$

28. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

29. Symmetric with respect to the x-axis because
$(-y)^2 = y^2 = x^3 - 8x.$

30. Symmetric with respect to the origin because

$$\begin{aligned} (-y) &= (-x)^3 + (-x) \\ -y &= -x^3 - x \\ y &= x^3 + x. \end{aligned}$$

31. Symmetric with respect to the origin because
$(-x)(-y) = xy = 4.$

32. Symmetric with respect to the x-axis because
$x(-y)^2 = xy^2 = -10.$

33. $y = 5 - \sqrt{x + 4}$

No symmetry with respect to either axis or the origin.

34. Symmetric with respect to the origin because

$$\begin{aligned} (-x)(-y) - \sqrt{16 - (-x)^2} &= 0 \\ xy - \sqrt{16 - x^2} &= 0. \end{aligned}$$

35. Symmetric with respect to the origin because

$$\begin{aligned} -y &= \frac{-x}{(-x)^2 + 1} \\ y &= \frac{x}{x^2 + 1}. \end{aligned}$$

36. Symmetric with respect to the y-axis because

$$y = \frac{(-x)^2}{(-x)^2 + 1} = \frac{x^2}{x^2 + 1}.$$

37. Symmetric with respect to the y-axis because
$y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|.$

38. Symmetric with respect to the x-axis because

$$\begin{aligned} |-y| - x &= 3 \\ |y| - x &= 3. \end{aligned}$$

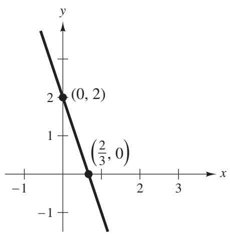
39. $y = 2 - 3x$

$y = 2 - 3(0) = 2$, y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$, x-intercept

Intercepts: $(0, 2), \left(\frac{2}{3}, 0\right)$

Symmetry: none



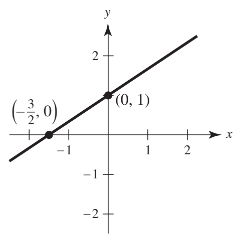
40. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1$, y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2}$, x-intercept

Intercepts: $(0, 1), \left(-\frac{3}{2}, 0\right)$

Symmetry: none



41. $y = 9 - x^2$

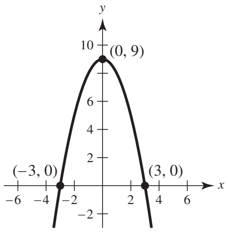
$y = 9 - (0)^2 = 9,$ y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3,$ x-intercepts

Intercepts: $(0, 9), (3, 0), (-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



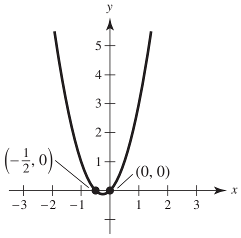
42. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0,$ y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2},$ x-intercepts

Intercepts: $(0, 0), \left(-\frac{1}{2}, 0\right)$

Symmetry: none



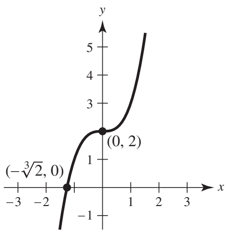
43. $y = x^3 + 2$

$y = 0^3 + 2 = 2,$ y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2},$ x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



44. $y = x^3 - 4x$

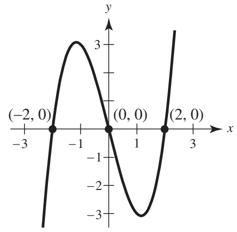
$y = 0^3 - 4(0) = 0,$ y-intercept

$$\begin{aligned} x^3 - 4x &= 0 \\ x(x^2 - 4) &= 0 \\ x(x + 2)(x - 2) &= 0 \\ x &= 0, \pm 2, \ x\text{-intercepts} \end{aligned}$$

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



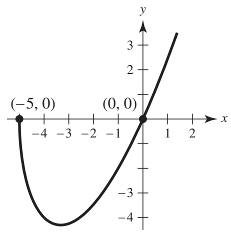
45. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0,$ y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5,$ x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



46. $y = \sqrt{25 - x^2}$

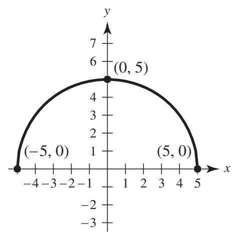
$y = \sqrt{25 - 0^2} = \sqrt{25} = 5,$ y-intercept

$$\begin{aligned} \sqrt{25 - x^2} &= 0 \\ 25 - x^2 &= 0 \\ (5 + x)(5 - x) &= 0 \\ x &= \pm 5, \ x\text{-intercept} \end{aligned}$$

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



47. $x = y^3$

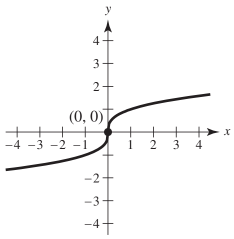
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: (0, 0)

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



48. $x = y^2 - 4$

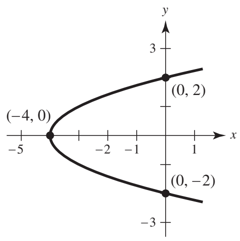
$$y^2 - 4 = 0$$
$$(y + 2)(y - 2) = 0$$
$$y = \pm 2, \text{ } y\text{-intercepts}$$

$x = 0^2 - 4 = -4$, x-intercept

Intercepts: $(0, 2)$, $(0, -2)$, $(-4, 0)$

$x = (-y)^2 - 4 = y^2 - 4$

Symmetry: x-axis



49. $y = \dfrac{6}{x}$

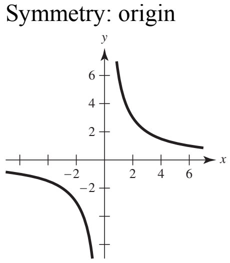
$y = \dfrac{6}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$0 = \dfrac{6}{x} \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{6}{-x} \Rightarrow y = \dfrac{6}{x}$

Symmetry: origin



50. $y = \dfrac{12}{x^2 + 1}$

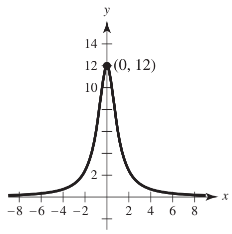
$y = \dfrac{12}{0 + 1} = 12$, y-intercept

$0 = \dfrac{12}{x^2 + 1} \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercept: (0, 12)

$y = \dfrac{12}{(-x)^2 + 1} = \dfrac{12}{x^2 + 1}$

Symmetry: y-axis



51. $y = 6 - |x|$

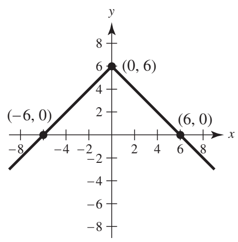
$y = 6 - |0| = 6$, y-intercept

$$6 - |x| = 0$$
$$6 = |x|$$
$$x = \pm 6, \text{ } x\text{-intercepts}$$

Intercepts: $(0, 6)$, $(-6, 0)$, $(6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



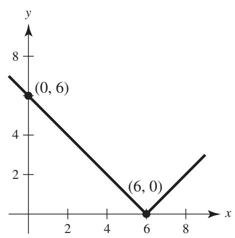
52. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

$$|6 - x| = 0$$
$$6 - x = 0$$
$$6 = x, \text{ } x\text{-intercept}$$

Intercepts: (0, 6), (6, 0)

Symmetry: none



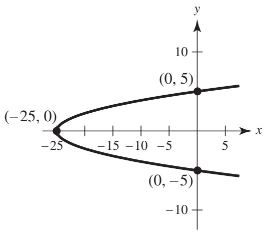
53. $y^2 - x = 25$

$$y^2 = x + 25$$
$$y = \pm\sqrt{x + 25}$$
$$y = \pm\sqrt{0 + 25} = \pm\sqrt{25} = \pm 5, \quad y\text{-intercepts}$$
$$\pm\sqrt{x + 25} = 0$$
$$x + 25 = 0$$
$$x = -25, \quad x\text{-intercept}$$

Intercepts: (0, 5), (0, –5), (–25, 0)

$(-y)^2 - x = 25 \Rightarrow y^2 - x = 25$

Symmetry: x-axis



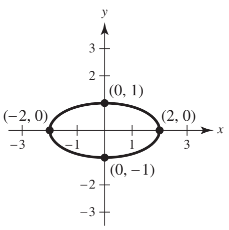
54. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

$$y = \pm\frac{\sqrt{4 - 0^2}}{2} = \pm\frac{\sqrt{4}}{2} = \pm 1, \ y\text{-intercepts}$$
$$x^2 + 4(0)^2 = 4$$
$$x^2 = 4$$
$$x = \pm 2, \ x\text{-intercepts}$$

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



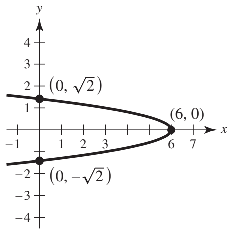
55. $x + 3y^2 = 6$

$$3y^2 = 6 - x$$
$$y = \pm\sqrt{\frac{6 - x}{3}}$$
$$y = \pm\sqrt{\frac{6 - 0}{3}} = \pm\sqrt{2}, \ y\text{-intercepts}$$
$$x + 3(0)^2 = 6$$
$$x = 6, \ x\text{-intercept}$$

Intercepts: $(6, 0), (0, \sqrt{2}), (0, -\sqrt{2})$

$x + 3(-y)^2 = 6 \Rightarrow x + 3y^2 = 6$

Symmetry: x-axis



56. $3x - 4y^2 = 8$

$$4y^2 = 3x - 8$$
$$y = \pm\sqrt{\tfrac{3}{4}x - 2}$$
$$y = \pm\sqrt{\frac{3}{4}(0) - 2} = \pm\sqrt{-2}$$

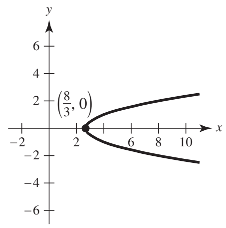
$\Rightarrow$ no solution $\Rightarrow$ no y-intercepts

$$3x - 4(0)^2 = 8$$
$$3x = 8$$
$$x = \frac{8}{3}, \ x\text{-intercept}$$

Intercept: $\left(\frac{8}{3}, 0\right)$

$3x - 4(-y)^2 = 8 \Rightarrow 3x - 4y^2 = 8$

Symmetry: x-axis



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$$\begin{aligned} 8 - x &= 4x - 7 \\ 15 &= 5x \\ 3 &= x \end{aligned}$$

The corresponding y-value is $y = 5$.

Point of intersection: (3, 5)

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$$\begin{aligned} \frac{3x + 4}{2} &= \frac{-4x - 10}{2} \\ 3x + 4 &= -4x - 10 \\ 7x &= -14 \\ x &= -2 \end{aligned}$$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 15 \Rightarrow y = -x^2 + 15$

$-3x + y = 11 \Rightarrow y = 3x + 11$

$$\begin{aligned} -x^2 + 15 &= 3x + 11 \\ 0 &= x^2 + 3x - 4 \\ 0 &= (x + 4)(x - 1) \\ x &= -4, 1 \end{aligned}$$

The corresponding y-values are $y = -1$ (for $x = -4$) and $y = 14$ (for $x = 1$).

Points of intersection: $(-4, -1)$, $(1, 14)$

60. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$-3x + y = 15 \Rightarrow y = 3x + 15$

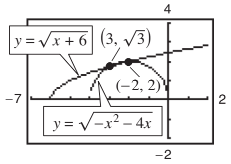
$$\begin{aligned} 25 - x^2 &= (3x + 15)^2 \\ 25 - x^2 &= 9x^2 + 90x + 225 \\ 0 &= 10x^2 + 90x + 200 \\ 0 &= x^2 + 9x + 20 \\ 0 &= (x + 5)(x + 4) \\ x &= -4 \text{ or } x = -5 \end{aligned}$$

The corresponding y-values are $y = 3$ (for $x = -4$) and $y = 0$ (for $x = -5$).

Points of intersection: $(-4, 3)$, $(-5, 0)$

61. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$

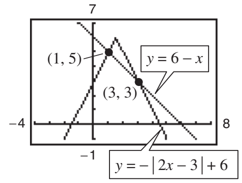


Points of intersection: $(-2, 2)$, $\left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically,
$$\begin{aligned} \sqrt{x + 6} &= \sqrt{-x^2 - 4x} \\ x + 6 &= -x^2 - 4x \\ x^2 + 5x + 6 &= 0 \\ (x + 3)(x + 2) &= 0 \\ x &= -3, -2. \end{aligned}$$

62. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: (3, 3), (1, 5)

Analytically, $-|2x - 3| + 6 = 6 - x$

$$|2x - 3| = x$$

$2x - 3 = x$ or $2x - 3 = -x$

$x = 3$ or $\quad x = 1$.

63. Replace x with $-x$ instead of y with $-y$.

$y^2 + 1 = (-x) \Rightarrow y^2 + 1 = -x$

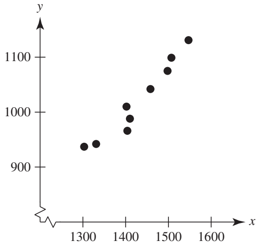
The graph of $y^2 + 1 = x$ is not symmetric about the y-axis.

64. The factored form of $x^2 - 2x - 3$ is $(x - 3)(x + 1)$.

$$\begin{aligned} -x + 1 &= -x^2 + x + 4 \\ x^2 - 2x - 3 &= 0 \\ (x - 3)(x + 1) &= 0 \\ x &= 3 \text{ or } -1 \end{aligned}$$

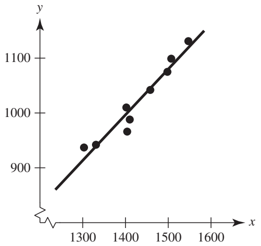
The points of intersection are $(-1, 2)$ and $(3, -2)$.

65. (a)



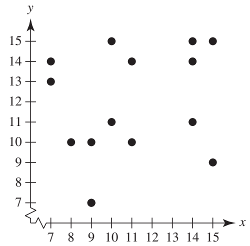
The data appear to be approximately linear.

(b) Models will vary. *Sample answer:* $y = \frac{5}{6}x - 170$ (found using a graphing utility)



When $x = 1475$, $y = \frac{5}{6}(1475) - 170 \approx \$1059$.

66. (a)

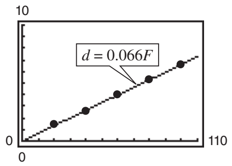


The data do not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, and so on. These variables may change from one quiz to the next.

67. (a) $d = 0.066F$

(b)

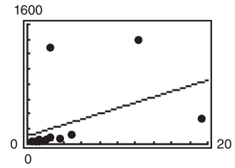


The model fits the data well. The correlation coefficient is $r \approx 0.9992$, so $|r|$ is close to 1, indicating that the linear model is a good fit for the data.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ centimeters.

68. (a) Using a graphing utility, $y = 37.27x + 108.4$. The correlation coefficient is $r = 0.4278$.

(b)



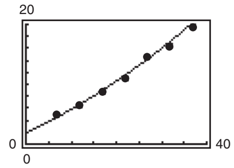
(c) Greater gross domestic product of a country tends to correspond to greater population of the country. The two points that differ most from the linear model are (12.238, 1390.1) and (2.597, 1283.6).

(d) Using a graphing utility, the new model is $y = 15.87x + 23.9$.

The correlation coefficient is $r = 0.9913$.

69. (a) Using a graphing utility, $y = 0.004t^2 + 0.35t + 1.9$.

(b)



The model is a good fit for the data.

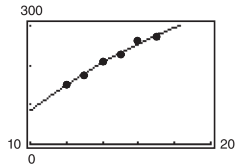
(c) When $t = 49$,

$y = 0.004(49)^2 + 0.35(49) + 1.9 \approx 28.7$.

So, in 2029, the GNP will be about \$28.7 trillion.

70. (a) Using a graphing utility, $y = -t^2 + 54.3t + 358$.

(b)



The model is a good fit for the data.

(c) When $t = 29$,

$y = -(29)^2 + 54.3(29) - 358 \approx 376$.

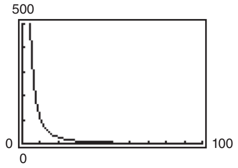
So, in 2029, there will be about 376 million smartphones in active use in the United States.

71.

$$\begin{aligned} R &= C \\ 2.98x &= 1.73x + 6500 \\ 2.98x - 1.73x &= 6500 \\ 1.25x &= 6500 \\ x &= \frac{6500}{1.25} \\ x &= 5200 \end{aligned}$$

To break even, 5200 units must be sold.

72. $y = \dfrac{10{,}370}{x^2}$

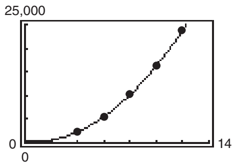


$$y = \frac{10{,}370}{(2x)^2} = \frac{10{,}370}{4x^2} = \frac{1}{4} \cdot \frac{10{,}370}{x^2}$$

If the diameter x is doubled, the resistance y is changed by a factor of $\frac{1}{4}$.

73. (a) Using a graphing utility,
$S = 180.89x^2 - 205.79x + 272.$

(b)



(c) When $x = 2$, $S \approx 583.98$ pounds.

(d) $\dfrac{2370}{584} \approx 4.06$

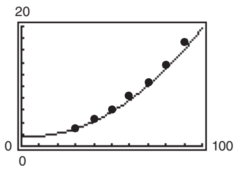
The breaking strength is approximately 4 times greater.

(e) $\dfrac{23{,}860}{5460} \approx 4.37$

When the width is doubled, the breaking strength increases approximately by a factor of 4.

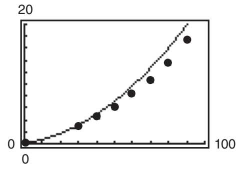
74. (a) Using a graphing utility,
$t = 0.002s^2 - 0.02s + 1.7.$

(b)



(c) According to the model, the times required to attain speeds of less than 30 miles per hour are all about the same. Further, it takes about 1.7 seconds to reach 0 miles per hour, which does not make sense.

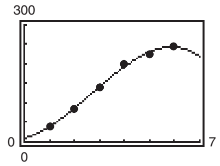
(d) Adding $(0, 0)$ to the data produces
$t = 0.002s^2 + 0.04s + 0.1.$



(e) Yes. Now the car starts at rest.

75. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

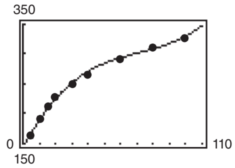
(b)



(c) If $x = 4.5$, $y \approx 214$ horsepower.

76. (a) $T = 0.00030p^3 - 0.0641p^2 + 5.285p + 143.06$

(b)



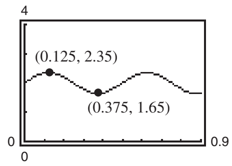
(c) For $T = 300°\text{F}$, $p \approx 67.49$ pounds per square inch.

(d) Sample answer: The model increases rapidly when $p > 100$.

77. (a) The amplitude is approximately
$(2.35 - 1.65)/2 = 0.35.$
The period is approximately
$2(0.375 - 0.125) = 0.5.$

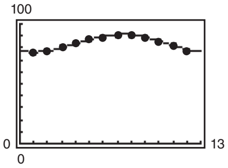
(b) One model is $y = 0.35 \sin(4\pi t) + 2.$

(c)

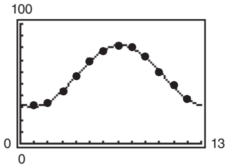


The model appears to fit the data well.

78. (a) $S(t) = 56.49 + 25.63\sin(0.5012t - 2.02)$

(b) 

The model is a good fit.

(c) 

The model is a good fit.

(d) Miami: 83.90°F

Syracuse: 56.49°F

The constant term gives the average annual maximum temperatures.

(e) Miami: $\dfrac{2\pi}{0.4957} \approx 12.7$

Syracuse: $\dfrac{2\pi}{0.5012} \approx 12.5$

In both cases, the period is approximately 12 months, or one year.

(f) Syracuse has greater variability because $25.63 > 7.42$.

79. $y = kx^3$

(a) $(1, 4)$: $4 = k(1)^3 \Rightarrow k = 4$

(b) $(-2, 1)$: $1 = k(-2)^3 = -8k \Rightarrow k = -\frac{1}{8}$

(c) $(0, 0)$: $0 = k(0)^3 \Rightarrow k$ can be any real number.

(d) $(-1, -1)$: $-1 = k(-1)^3 = -k \Rightarrow k = 1$

80. $y^2 = 4kx$

(a) $(1, 1)$:
$$1^2 = 4k(1)$$
$$1 = 4k$$
$$k = \tfrac{1}{4}$$

(b) $(2, 4)$:
$$(4)^2 = 4k(2)$$
$$16 = 8k$$
$$k = 2$$

(c) $(0, 0)$: $0^2 = 4k(0)$

k can be any real number.

(d) $(3, 3)$:
$$(3)^2 = 4k(3)$$
$$9 = 12k$$
$$k = \tfrac{9}{12} = \tfrac{3}{4}$$

81. Answers will vary. Sample answer:

$y = (x + 3)(x - 5)(x - 6)$ has intercepts at $x = -3$, $x = 5$, and $x = 6$.

82. Answers will vary. Sample answer:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

83. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

84. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0)\ (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because

$-y = (-x)^3 - (-x) = -x^3 + x.$

Symmetry with respect to the y-axis for $y = x^2 + 2$ because

$y = (-x)^2 + 2 = x^2 + 2.$

(c)
$$\begin{aligned} x^3 - x &= x^2 + 2 \\ x^3 - x^2 - x - 2 &= 0 \\ (x - 2)(x^2 + x + 1) &= 0 \\ x = 2 &\Rightarrow y = 6 \end{aligned}$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

85. False. Symmetry with respect to the x-axis means that if $(3, -4)$ is a point on the graph, then (3, 4) is also a point on the graph. For example, $(3, -4)$ is on the graph of $x = y^2 - 13$, but $(-3, -4)$ is not on the graph.

86. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

Section P.2 Linear Models and Rates of Change

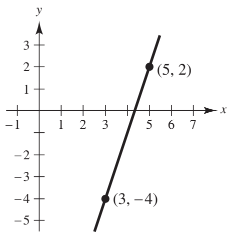
1. $m = 2$

2. $m = 0$

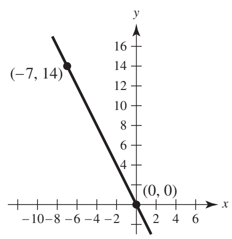
3. $m = -1$

4. $m = -12$

5. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

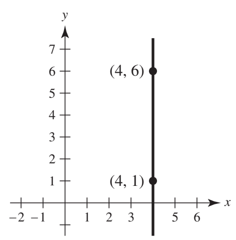


6. $m = \dfrac{14 - 0}{-7 - 0} = \dfrac{14}{-7} = -2$



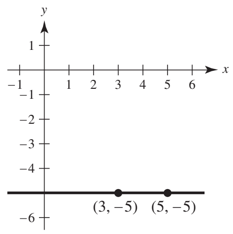
7. $m = \dfrac{1 - 6}{4 - 4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

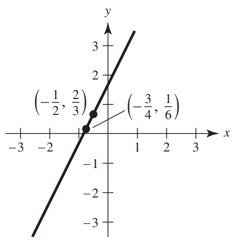


8. $m = \dfrac{-5 - (-5)}{5 - 3} = \dfrac{0}{2} = 0$

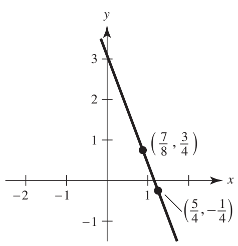
The line is horizontal.



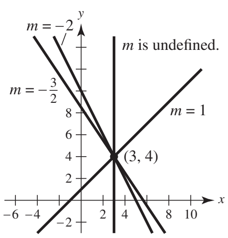
9. $m = \dfrac{\dfrac{2}{3} - \dfrac{1}{6}}{-\dfrac{1}{2} - \left(-\dfrac{3}{4}\right)} = \dfrac{\dfrac{1}{2}}{\dfrac{1}{4}} = 2$



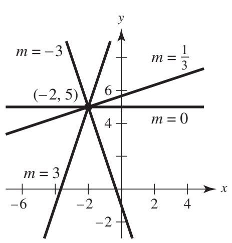
10. $m = \dfrac{\left(\dfrac{3}{4}\right) - \left(-\dfrac{1}{4}\right)}{\left(\dfrac{7}{8}\right) - \left(\dfrac{5}{4}\right)} = \dfrac{1}{-\dfrac{3}{8}} = -\dfrac{8}{3}$



11.



12.



13. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2)$, $(1, 2)$, and $(5, 2)$.

14. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0)$, $(-4, 1)$, and $(-4, 2)$.

15. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

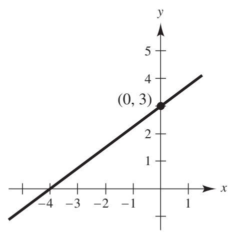
16. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

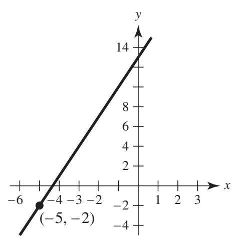
17.

$$y = \tfrac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$



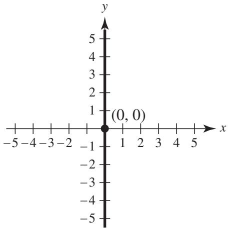
18.

$$y + 2 = 3(x + 5)$$
$$y + 2 = 3x + 15$$
$$y = 3x + 13$$
$$0 = 3x - y + 13$$

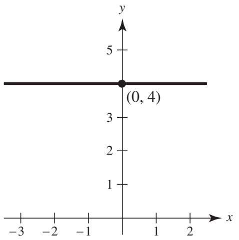


19. The slope is undefined, so the line is vertical.

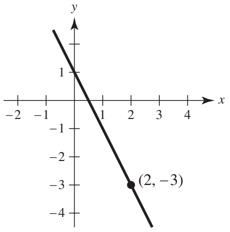
$x = 0$ (the y-axis)



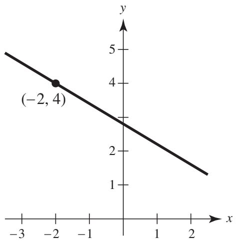
20.
$$y = 4$$
$$y - 4 = 0$$



21.
$$y + 3 = -2(x - 2)$$
$$y + 3 = -2x + 4$$
$$y = -2x + 1$$
$$2x + y - 1 = 0$$

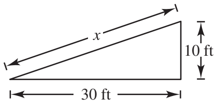


22.
$$y - 4 = -\tfrac{3}{5}(x + 2)$$
$$5y - 20 = -3x - 6$$
$$3x + 5y - 14 = 0$$



23. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

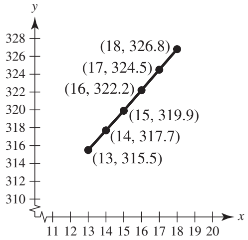
(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

24. (a)



(b) The slopes are:
$$\frac{317.7 - 315.5}{14 - 13} = 2.2$$
$$\frac{319.9 - 317.7}{15 - 14} = 2.2$$
$$\frac{322.2 - 319.9}{16 - 15} = 2.3$$
$$\frac{324.5 - 322.2}{17 - 16} = 2.3$$
$$\frac{326.8 - 324.5}{18 - 17} = 2.3$$

The population increased least rapidly from 2013 to 2014 and from 2014 to 2015.

(c) Average rate of change from 2013 through 2018:

$$\frac{326.8 - 315.5}{18 - 13} = \frac{11.3}{5}$$
$$= 2.26 \text{ million people per year}$$

(d) $t = 25$ is 7 years after $t = 18$.

$$P = 326.8 + 7(2.26) = 342.62$$

So, in 2025, the population will be about 342.62 million people.

25. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

26. $-x + y = 1$

$$y = x + 1$$

The slope is $m = 1$ and the y-intercept is (0, 1).

27. $5x + y = 20$

$$y = -5x + 20$$

The slope is $m = -5$ and the y-intercept is (0, 20).

28. $6x - 5y = 15$

$$y = \frac{6}{5}x - 3$$

The slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

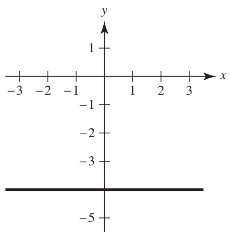
29. $x = -1$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

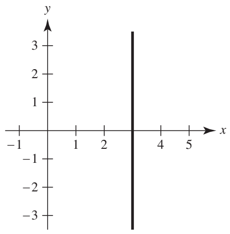
30. $y = 4$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, 4)$.

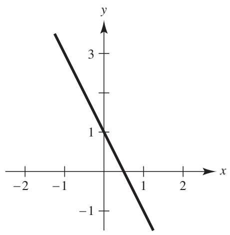
31. $y = -4$



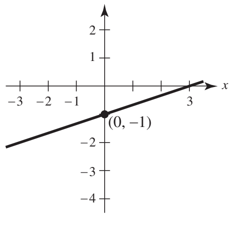
32. $x = 3$



33. $y = -2x + 1$

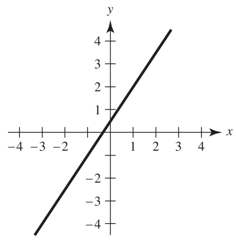


34. $y = \frac{1}{3}x - 1$



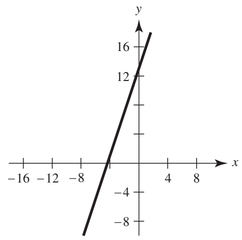
35. $y - 2 = \frac{3}{2}(x - 1)$

$$y = \frac{3}{2}x + \frac{1}{2}$$



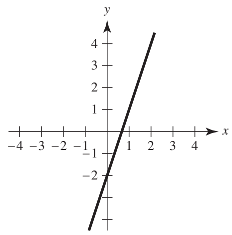
36. $y - 1 = 3(x + 4)$

$$y = 3x + 13$$



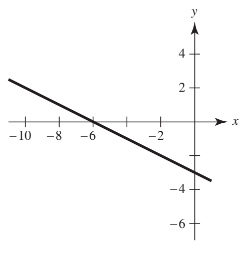
37. $3x - y - 2 = 0$

$$y = 3x - 2$$



38. $x + 2y + 6 = 0$

$$y = -\frac{1}{2}x - 3$$

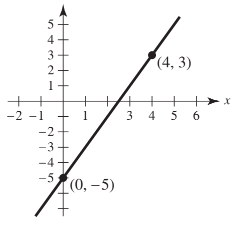


39. $m = \dfrac{-5 - 3}{0 - 4} = \dfrac{-8}{-4} = 2$

$$y - (-5) = 2(x - 0)$$

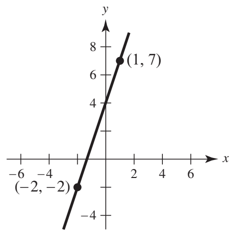
$$y + 5 = 2x$$

$$y = 2x - 5$$



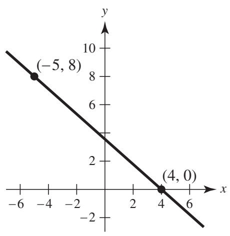
40. $m = \dfrac{7 - (-2)}{1 - (-2)} = \dfrac{9}{3} = 3$

$$y - (-2) = 3(x - (-2))$$
$$y + 2 = 3(x + 2)$$
$$y = 3x + 4$$
$$0 = 3x - y + 4$$



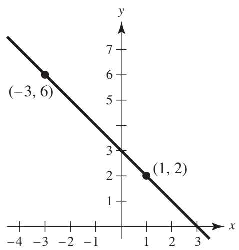
41. $m = \dfrac{8 - 0}{-5 - 4} = -\dfrac{8}{9}$

$$y - 0 = -\frac{8}{9}(x - 4)$$
$$y = -\frac{8}{9}x + \frac{32}{9}$$
$$8x + 9y - 32 = 0$$



42. $m = \dfrac{6 - 2}{-3 - 1} = \dfrac{4}{-4} = -1$

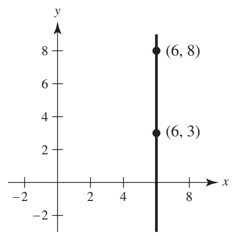
$$y - 2 = -1(x - 1)$$
$$y - 2 = -x + 1$$
$$x + y - 3 = 0$$



43. $m = \dfrac{8 - 3}{6 - 6} = \dfrac{5}{0}$, undefined

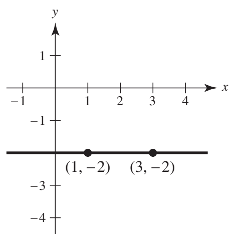
The line is horizontal.

$$x = 6$$
$$x - 6 = 0$$



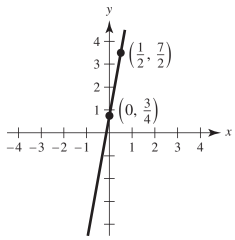
44. $m = \dfrac{-2 - (-2)}{3 - 1} = \dfrac{0}{2} = 0$

$$y = -2$$
$$y + 2 = 0$$



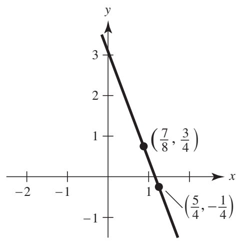
45. $m = \dfrac{\dfrac{7}{2} - \dfrac{3}{4}}{\dfrac{1}{2} - 0} = \dfrac{\dfrac{11}{4}}{\dfrac{1}{2}} = \dfrac{11}{2}$

$$y - \frac{3}{4} = \frac{11}{2}(x - 0)$$
$$y = \frac{11}{2}x + \frac{3}{4}$$
$$0 = 22x - 4y + 3$$



46. $m = \dfrac{\left(\dfrac{3}{4}\right) - \left(-\dfrac{1}{4}\right)}{\left(\dfrac{7}{8}\right) - \left(\dfrac{5}{4}\right)} = \dfrac{1}{-\dfrac{3}{8}} = -\dfrac{8}{3}$

$$y + \frac{1}{4} = \frac{-8}{3}\left(x - \frac{5}{4}\right)$$
$$12y + 3 = -32x + 40$$
$$32x + 12y - 37 = 0$$

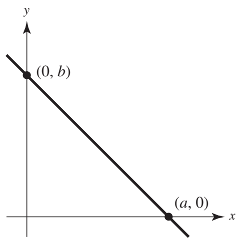


47. $m = \dfrac{c - b}{a - 0} = \dfrac{c - b}{a}$

$$y - b = \frac{c - b}{a}(x - 0)$$
$$ay - ab = (c - b)x$$
$$(c - b)x - ay + ab = 0$$

48. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$
$$\frac{b}{a}x + y = b$$
$$\frac{x}{a} + \frac{y}{b} = 1$$



49. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$$3x + 2y - 6 = 0$$

50. $$\begin{aligned} \frac{x}{-\frac{2}{3}} + \frac{y}{-2} &= 1 \\ \frac{-3x}{2} - \frac{y}{2} &= 1 \\ 3x + y &= -2 \\ 3x + y + 2 &= 0 \end{aligned}$$

51. $$\begin{aligned} \frac{x}{a} + \frac{y}{a} &= 1 \\ x + y &= a \\ -1 + 2 &= a \\ 1 &= a \\ x + y &= 1 \\ x + y - 1 &= 0 \end{aligned}$$

52. $$\begin{aligned} \frac{x}{a} + \frac{y}{a} &= 1 \\ x + y &= a \\ 3 + (-4) &= a \\ -1 &= a \\ x + y &= -1 \\ x + y + 1 &= 0 \end{aligned}$$

53. $$\begin{aligned} \frac{x}{2a} + \frac{y}{a} &= 1 \\ \frac{9}{2a} + \frac{-2}{a} &= 1 \\ \frac{9-4}{2a} &= 1 \\ 5 &= 2a \\ a &= \frac{5}{2} \\ \frac{x}{2\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{2}\right)} &= 1 \\ \frac{x}{5} + \frac{2y}{5} &= 1 \\ x + 2y &= 5 \\ x + 2y - 5 &= 0 \end{aligned}$$

54. $$\begin{aligned} \frac{x}{a} + \frac{y}{-a} &= 1 \\ \frac{\left(-\frac{2}{3}\right)}{a} + \frac{(-2)}{-a} &= 1 \\ -\frac{2}{3} + 2 &= a \\ a &= \frac{4}{3} \\ \frac{x}{\left(\frac{4}{3}\right)} + \frac{y}{\left(-\frac{4}{3}\right)} &= 1 \\ x - y &= \frac{4}{3} \\ 3x - 3y - 4 &= 0 \end{aligned}$$

55. The given line is vertical.

(a) $x = -7 \Rightarrow x + 7 = 0$

(b) $y = -2 \Rightarrow y + 2 = 0$

56. The given line is horizontal.

(a) $y = 0$

(b) $x = -1 \Rightarrow x + 1 = 0$

57. $$\begin{aligned} x + y &= 7 \\ y &= -x + 7 \\ m &= -1 \end{aligned}$$

(a) $$\begin{aligned} y - 2 &= -1(x + 3) \\ y - 2 &= -x - 3 \\ x + y + 1 &= 0 \end{aligned}$$

(b) $$\begin{aligned} y - 2 &= 1(x + 3) \\ y - 2 &= x + 3 \\ 0 &= x - y + 5 \end{aligned}$$

58. $$\begin{aligned} x - y &= -2 \\ y &= x + 2 \\ m &= 1 \end{aligned}$$

(a) $$\begin{aligned} y - 5 &= 1(x - 2) \\ y - 5 &= x - 2 \\ x - y + 3 &= 0 \end{aligned}$$

(b) $$\begin{aligned} y - 5 &= -1(x - 2) \\ y - 5 &= -x + 2 \\ x + y - 7 &= 0 \end{aligned}$$

59. $4x - 2y = 3$

$$y = 2x - \tfrac{3}{2}$$

$$m = 2$$

(a) $y - 1 = 2(x - 2)$

$$y - 1 = 2x - 4$$

$$0 = 2x - y - 3$$

(b) $y - 1 = -\tfrac{1}{2}(x - 2)$

$$2y - 2 = -x + 2$$

$$x + 2y - 4 = 0$$

60. $7x + 4y = 8$

$$4y = -7x + 8$$

$$y = \frac{-7}{4}x + 2$$

$$m = -\frac{7}{4}$$

(a) $y + \dfrac{1}{2} = \dfrac{-7}{4}\left(x - \dfrac{5}{6}\right)$

$$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$$

$$24y + 12 = -42x + 35$$

$$42x + 24y - 23 = 0$$

(b) $y + \dfrac{1}{2} = \dfrac{4}{7}\left(x - \dfrac{5}{6}\right)$

$$42y + 21 = 24x - 20$$

$$24x - 42y - 41 = 0$$

61. $5x - 3y = 0$

$$-3y = -5x$$

$$y = \frac{5}{3}x$$

$$m = \frac{5}{3}$$

(a) $y + 3 = \dfrac{5}{3}(x - 5)$

$$3y + 9 = 5x - 25$$

$$0 = 5x - 3y - 34$$

(b) $y + 3 = -\dfrac{3}{5}(x - 5)$

$$5y + 15 = -3x + 15$$

$$3x + 5y = 0$$

62. $3x + 4y = 7$

$$4y = -3x + 7$$

$$y = -\tfrac{3}{4}x + \tfrac{7}{4}$$

$$m = -\tfrac{3}{4}$$

(a) $y + 5 = -\tfrac{3}{4}(x + 4)$

$$4y + 20 = -3x - 12$$

$$3x + 4y + 32 = 0$$

(b) $y + 5 = \tfrac{4}{3}(x + 4)$

$$3y + 15 = 4x + 16$$

$$0 = 4x - 3y + 1$$

63. The variables $x = -1$ and $y = 4$ were substituted incorrectly into the point-slope form of the equation.

$$y - 4 = -\tfrac{5}{2}[x - (-1)]$$

$$y - 4 = -\tfrac{5}{2}x - \tfrac{5}{2}$$

$$y = -\tfrac{5}{2}x + \tfrac{3}{2}$$

64. The slope of the perpendicular line is the negative reciprocal of $-\tfrac{3}{4}$, which is $\tfrac{4}{3}$.

$$y - (-3) = \tfrac{4}{3}(x - 2)$$

$$3(y + 3) = 4(x - 2)$$

$$3y + 9 = 4x - 8$$

$$0 = 4x - 3y - 17$$

65. The slope is 250.

$V = 1850$ when $t = 9$.

$$V - 1850 = 250(t - 9)$$

$$V = 250t - 2250 + 1850$$

$$V = 250t - 400$$

66. The slope is 4.50.

$V = 156$ when $t = 9$.

$$V - 156 = 4.5(t - 9)$$

$$V - 156 = 4.5t - 40.5$$

$$V = 4.5t + 115.5$$

67. The slope is -1600.

$V = 17{,}200$ when $t = 9$.

$$V - 17{,}200 = -1600(t - 9)$$

$$V - 17{,}200 = -1600t + 14{,}400$$

$$V = -1600t + 31{,}600$$

68. The slope is $-16{,}500$.

$V = 524{,}000$ when $t = 9$.

$$V - 524{,}000 = -16{,}500(t - 9)$$
$$V - 524{,}000 = -16{,}500t + 148{,}500$$
$$V = -16{,}500t + 672{,}500$$

69. $m_1 = \dfrac{-6 - 4}{7 - 0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11 - 4}{-5 - 0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

70. $m_1 = \dfrac{1 - 0}{-2 - (-1)} = -1$

$m_2 = \dfrac{-2 - 0}{2 - (-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

71. Equations of perpendicular bisectors:

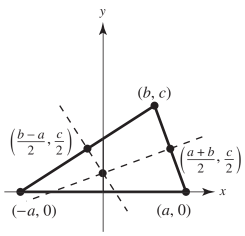
$$y - \frac{c}{2} = \frac{a - b}{c}\left(x - \frac{a + b}{2}\right)$$
$$y - \frac{c}{2} = \frac{a + b}{-c}\left(x - \frac{b - a}{2}\right)$$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

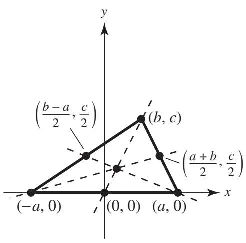
$$\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right).$$

This point lies on the third perpendicular bisector, $x = 0$.



72. Equations of medians:

$$y = \frac{c}{b}x$$
$$y = \frac{c}{3a + b}(x + a)$$
$$y = \frac{c}{-3a + b}(x - a)$$



Solving simultaneously, the point of intersection is $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$.

73. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(b) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$-5x + 8y = 4$

$$y = \tfrac{1}{8}(5x + 4) = \tfrac{5}{8}x + \tfrac{1}{2}$$

(c) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$5x + 2y = 4$

$$y = \tfrac{1}{2}(-5x + 4) = -\tfrac{5}{2}x + 2$$

(d) $a = \frac{5}{2}$ and $b = 3$.

$\frac{5}{2}x + 3y = 4$

$5x + 6y = 8$

74. (a) Lines c, d, e, and f have positive slopes.

(b) Lines a and b have negative slopes.

(c) Lines c and e appear parallel.

Lines d and f appear parallel.

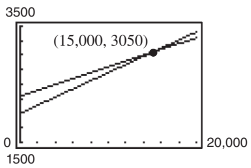
(d) Lines b and f appear perpendicular.

Lines b and d appear perpendicular.

75. (a) Current job: $W_1 = 0.07s + 2000$

New job offer: $W_2 = 0.05s + 2300$

(b)



Using a graphing utility, the point of intersection is (15,000, 3050).

Analytically, $W_1 = W_2$

$$0.07s + 2000 = 0.05s + 2300$$
$$0.02s = 300$$
$$s = 15{,}000$$

So, $W_1 = W_2 = 0.07(15{,}000) + 2000 = 3050$.

When sales exceed \$15,000, your current job pays more.

(c) No, if you can sell \$20,000 worth of goods, then $W_1 > W_2$.

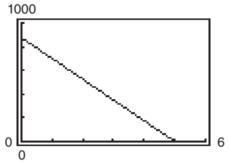
(**Note:** $W_1 = 3400$ and $W_2 = 3300$ when $s = 20{,}000$.)

76. (a) Depreciation per year:

$\frac{875}{5} = \$175$

$y = 875 - 175x,$

where $0 \le x \le 5$



(b) $y = 875 - 175(2) = \$525$

(c) $200 = 875 - 175x$

$175x = 675$

$x \approx 3.86$ years

77. (a) Two points are (50, 780) and (47, 825).

The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$

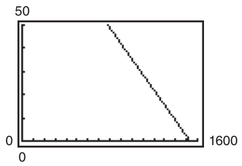
$$p - 780 = -15(x - 50)$$

$$p = -15x + 750 + 780 = -15x + 1530$$

or

$$x = \frac{1}{15}(1530 - p)$$

(b)



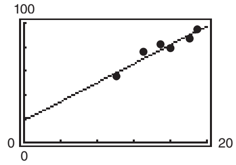
If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units.

78. (a) $y = 18.91 + 3.97x$

$(x = \text{quiz score}, \ y = \text{test score})$

(b)

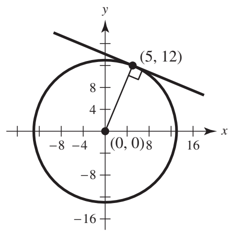


(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

79. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).



Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

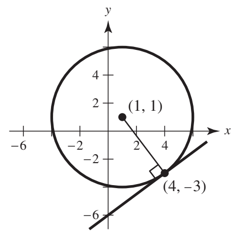
The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$

$$y = \frac{-5}{12}x + \frac{169}{12}$$

$$5x + 12y - 169 = 0.$$

80. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining $(1,1)$ and $(4,-3)$ is $\frac{1+3}{1-4} = \frac{-4}{3}$.

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4}x - 6$$

$$0 = 3x - 4y - 24$$

81. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$\begin{aligned} y - y_1 &= \frac{B}{A}(x - x_1) \\ Ay - Ay_1 &= Bx - Bx_1 \\ Bx_1 - Ay_1 &= Bx - Ay \end{aligned}$$

The point of intersection of these two lines is:

$$\begin{aligned} Ax + By = -C \quad &\Rightarrow \quad A^2x + ABy = -AC \qquad (1) \\ Bx - Ay = Bx_1 - Ay_1 \quad &\Rightarrow \quad \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \qquad (2) \\ &\qquad (A^2 + B^2)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))} \\ &\qquad x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} \end{aligned}$$

$$\begin{aligned} Ax + By = -C \quad &\Rightarrow \quad ABx + B^2y = -BC \qquad (3) \\ Bx - Ay = Bx_1 - Ay_1 \quad &\Rightarrow \quad \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} \qquad (4) \\ &\qquad (A^2 + B^2)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))} \\ &\qquad y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} \end{aligned}$$

$\left(\dfrac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \dfrac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right)$ point of intersection

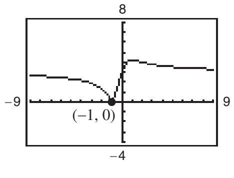
The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$\begin{aligned} d &= \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2} \\ &= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2} \\ &= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$

82. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).



83. $x - y - 2 = 0 \Rightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}}$

$= \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

84. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \dfrac{7}{5}$

85. A point on the line $x + y = 1$ is (0, 1). The distance from the point (0, 1) to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

86. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{|3(-1) - 4(-1) - 10|}{\sqrt{3^2 + (-4)^2}} = \frac{|-3 + 4 - 10|}{5} = \frac{9}{5}.$$

87. True.

$ax + by = c_1 \Rightarrow y = -\dfrac{a}{b}x + \dfrac{c_1}{b} \Rightarrow m_1 = -\dfrac{a}{b}$

$bx - ay = c_2 \Rightarrow y = \dfrac{b}{a}x - \dfrac{c_2}{a} \Rightarrow m_2 = \dfrac{b}{a}$

$m_2 = -\dfrac{1}{m_1}$

88. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

89. True. The slope must be positive.

90. True. The general form $Ax + By + C = 0$ includes both horizontal and vertical lines.

Section P.3 Functions and Their Graphs

1. $f(x) = 3x - 2$

(a) $f(0) = 3(0) - 2 = -2$

(b) $f(5) = 3(5) - 2 = 13$

(c) $f(b) = 3(b) - 2 = 3b - 2$

(d) $f(x - 1) = 3(x - 1) - 2 = 3x - 5$

2. (a) $f(4) = 4^2(4 - 4) = 0$

(b) $f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $f(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $f(t + 4) = (t + 4)^2(t + 4 - 4)$

$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

3. (a) $f(0) = \sin(2(0)) = \sin 0 = 0$

(b) $f\left(-\dfrac{\pi}{4}\right) = \sin\left(2\left(-\dfrac{\pi}{4}\right)\right) = \sin\left(-\dfrac{\pi}{2}\right) = -1$

(c) $f\left(\dfrac{\pi}{3}\right) = \sin\left(2\left(\dfrac{\pi}{3}\right)\right) = \sin\dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$

(d) $f(\pi) = \sin(2(\pi)) = \sin 2\pi = 0$

4. (a) $f(\pi) = \cos(\pi) = -1$

(b) $f\left(\dfrac{5\pi}{4}\right) = \cos\left(\dfrac{5\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

(c) $f\left(\dfrac{2\pi}{3}\right) = \cos\left(\dfrac{2\pi}{3}\right) = -\dfrac{1}{2}$

(d) $f\left(-\dfrac{\pi}{6}\right) = \cos\left(-\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

5. $\dfrac{f(x + \Delta x) - f(x)}{\Delta x} = \dfrac{(x + \Delta x)^3 - x^3}{\Delta x} = \dfrac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2,\ \Delta x \neq 0$

6. $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{3x - 1 - (3 - 1)}{x - 1} = \dfrac{3(x - 1)}{x - 1} = 3,\ x \neq 1$

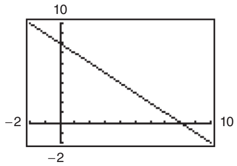
7. $\dfrac{f(x) - f(2)}{x - 2} = \dfrac{\left(1/\sqrt{x - 1} - 1\right)}{x - 2}$

$= \dfrac{1 - \sqrt{x - 1}}{(x - 2)\sqrt{x - 1}} \cdot \dfrac{1 + \sqrt{x - 1}}{1 + \sqrt{x - 1}} = \dfrac{2 - x}{(x - 2)\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)} = \dfrac{-1}{\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)},\ x \neq 2$

8. $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{x^3 - x - 0}{x - 1} = \dfrac{x(x + 1)(x - 1)}{x - 1} = x(x + 1),\ x \neq 1$

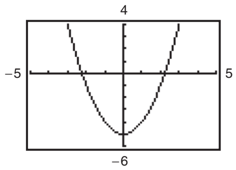
9. $f(x) = 8 - x$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. Because the graph of the function extends infinitely in both directions, the range is $(-\infty, \infty)$.



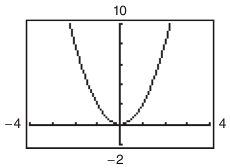
10. $g(x) = x^2 - 5$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. No matter what value of x is given, the value of $f(x)$ is never less than 5. So, the range is $[5, \infty)$.



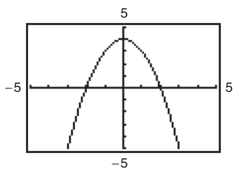
11. $f(x) = 2x^2$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. No matter what value of x is given, the value of $f(x)$ is never less than 0. So, the range is $[0, \infty)$.



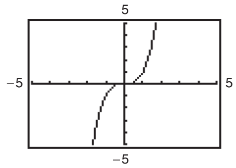
12. $h(x) = 4 - x^2$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. No matter what value of x is given, the value of $f(x)$ is never more than 4. So, the range is $(-\infty, 4]$.



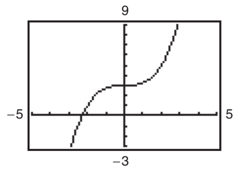
13. $f(x) = x^3$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. Because the graph of the function extends infinitely in both directions, the range is $(-\infty, \infty)$.



14. $f(x) = \frac{1}{4}x^3 + 3$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. Because the graph of the function extends infinitely in both directions, the range is $(-\infty, \infty)$.

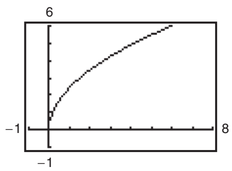


15. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$

$x \geq 0 \Rightarrow [0, \infty)$

Range: $\sqrt{6x} \geq 0 \Rightarrow [0, \infty)$



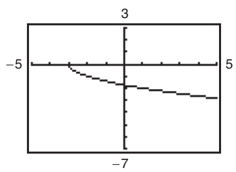
16. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \geq 0$

$x \geq -3 \Rightarrow [-3, \infty)$

Range: $-\sqrt{x + 3} \geq 0$

$\sqrt{x + 3} \leq 0 \Rightarrow (-\infty, 0]$



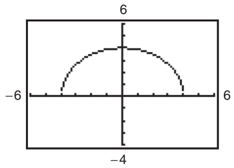
17. $f(x) = \sqrt{16 - x^2}$

Domain: $16 - x^2 \geq 0$

$$16 \geq x^2$$

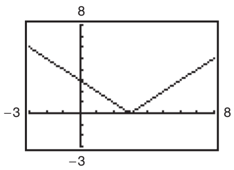
$$x \geq -4 \text{ and } x \leq 4 \Rightarrow [-4, 4]$$

Range: For $x \geq -4$ and $x \leq 4$, $0 \leq y \leq 4 \Rightarrow [0, 4]$



18. $f(x) = |x - 3|$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. No matter what value of x is given, the value of $f(x)$ is never negative. So, the range is $[0, \infty)$.

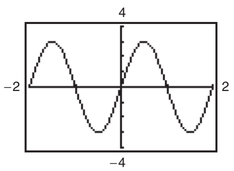


19. $g(t) = 3 \sin \pi t$

The function is defined for all real numbers t. So, the domain is $(-\infty, \infty)$.

Range: $-1 \leq \sin t \leq 1$

$$-3 \leq 3 \sin \pi t \leq 3 \Rightarrow [-3, 3]$$

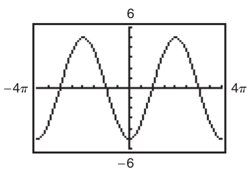


20. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

The function is defined for all real numbers θ. So, the domain is $(-\infty, \infty)$.

Range: $-1 \leq \cos \theta \leq 1$

$$5 \geq -5 \cos \frac{\theta}{2} \geq -5 \Rightarrow [-5, 5]$$

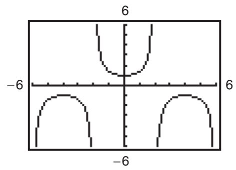


21. $f(t) = \sec \dfrac{\pi t}{4}$

$$\frac{\pi t}{4} \neq \frac{(2n + 1)\pi}{2} \Rightarrow t \neq 4n + 2$$

Domain: all $t \neq 4n + 2$, n is an integer

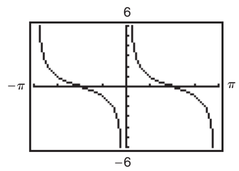
No matter what value of t is given within the domain, the value of $f(t)$ is either at least 1 or at most -1. So, the range is $(-\infty, -1] \cup [1, \infty)$.



22. $h(t) = \cot t$

Because h is undefined when $t = \pi$, this value and its multiples are not in the domain. So, the domain is the set of all t-values such that $t \neq n\pi$, where n is an integer.

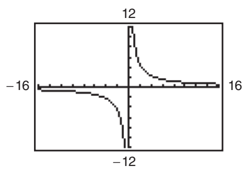
As t gets closer to the values $-\pi$ and π, $h(t)$ decreases without bound. So, the range is $(-\infty, \infty)$.



23. $f(x) = \dfrac{9}{x}$

Because division by 0 is undefined, the domain is all x-values such that $x \neq 0$, or $(-\infty, 0) \cup (0, \infty)$.

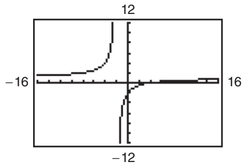
As x increases or decreases without bound, $f(x)$ approaches but does not reach 0. So, the range is $(-\infty, 0) \cup (0, \infty)$.



24. $f(x) = \dfrac{x - 4}{x + 2}$

Because division by 0 is undefined, the domain is all x-values such that $x \neq -2$, or $(-\infty, -2) \cup (-2, \infty)$.

As x increases or decreases without bound, $f(x)$ approaches but does not reach 1. So, the range is $(-\infty, 1) \cup (1, \infty)$.



25. $f(x) = \sqrt{x} + \sqrt{1 - x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

26. $f(x) = \sqrt{x^2 - 5x + 6}$

$x^2 - 5x + 6 \geq 0$

$(x - 3)(x - 2) \geq 0$

Domain: $x \geq 3$ or $x \leq 2$

Domain: $(-\infty, 2] \cup [3, \infty)$

27. $g(x) = \dfrac{2}{1 - \cos x}$

$1 - \cos x \neq 0$

$\cos x \neq 1$

Domain: all $x \neq 2n\pi$, n is an integer

28. $h(x) = \dfrac{1}{\sin x - (1/2)}$

$\sin x - \dfrac{1}{2} \neq 0$

$\sin x \neq \dfrac{1}{2}$

Domain: all $x \neq \dfrac{\pi}{6} + 2n\pi, \dfrac{5\pi}{6} + 2n\pi$, n is an integer

29. $f(x) = \dfrac{1}{|x + 3|}$

$|x + 3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

30. $g(x) = \dfrac{1}{|x^2 - 16|}$

$|x^2 - 16| \neq 0$

$(x - 4)(x + 4) \neq 0$

Domain: all $x \neq \pm 4$

Domain: $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

31. $f(x) = \begin{cases} 3x + 1, & x < 0 \\ 2x + 3, & x \geq 0 \end{cases}$

(a) $f(-1) = 3(-1) + 1 = -2$

(b) $f(0) = 2(0) + 3 = 3$

(c) $f(2) = 2(2) + 3 = 7$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 3 = 2t^2 + 5$

(**Note:** $t^2 + 1 > 0$ for all t.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [3, \infty)$

32. $f(x) = \begin{cases} x^2 + 1, & x \leq 1 \\ 2x^2 + 4, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 1 = 5$

(b) $f(0) = 0^2 + 1 = 1$

(c) $f(1) = 1^2 + 1 = 2$

(d) $f(x^2 + 2) = 2(x^2 + 2)^2 + 4 = 2x^4 + 8x^2 + 12$

(**Note:** $x^2 + 2 > 1$ for all x.)

Domain: $(-\infty, \infty)$

Range: $[1, \infty)$

33. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$