CHAPTER P
Preparation for Calculus

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Section P.1 Graphs and Models

1. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

2. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

3. $y = 3 - x^2$

x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y-intercept: $(0, 3)$

Matches graph (a).

4. $y = x^3 - x$

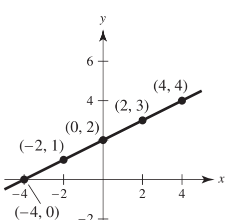
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

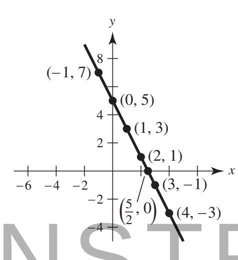
5. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



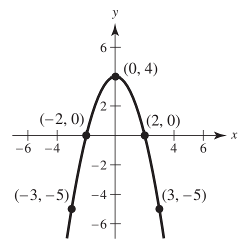
6. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



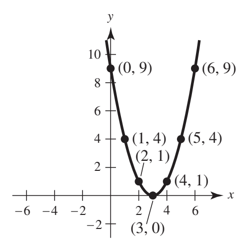
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



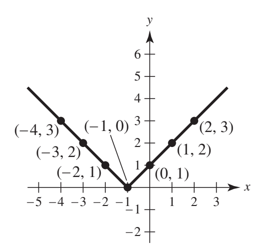
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



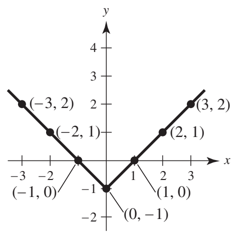
9. $y = |x + 1|$

x	-4	-3	-2	-1	0	1	2
y	3	2	1	0	1	2	3



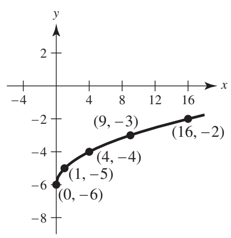
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



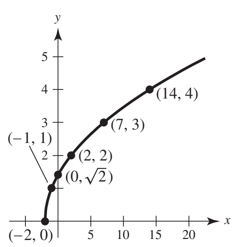
11. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



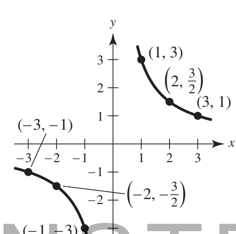
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



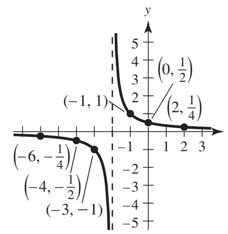
13. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

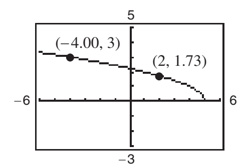


14. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



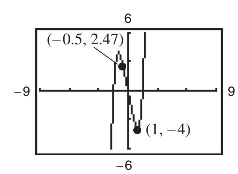
15. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

16. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

17. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5$; $(0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2}$; $\left(\frac{5}{2}, 0\right)$

18. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3$; $(0, 3)$

x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None (y cannot equal 0.)

19. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2; (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1; (-2, 0), (1, 0)$

20. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0; (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2; (0, 0), (\pm 2, 0)$

21. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0; (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4; (0, 0), (4, 0), (-4, 0)$

22. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1; (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1; (1, 0)$

23. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2; \ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4; \ (4, 0)$

24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0; (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3; (0, 0), (-3, 0)$

25. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0; (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0; (0, 0)$

26. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1; (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3}; \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

27. Symmetric with respect to the y-axis because $y = (-x)^2 - 6 = x^2 - 6$.

28. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

29. Symmetric with respect to the x-axis because $(-y)^2 = y^2 = x^3 - 8x$.

30. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

31. Symmetric with respect to the origin because $(-x)(-y) = xy = 4.$

32. Symmetric with respect to the x-axis because $x(-y)^2 = xy^2 = -10.$

33. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

34. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

35. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

36. Symmetric with respect to the y-axis because

$$y = \frac{(-x)^2}{(-x)^2 + 1} = \frac{x^2}{x^2 + 1}.$$

37. Symmetric with respect to the y-axis because

$$y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|.$$

38. Symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

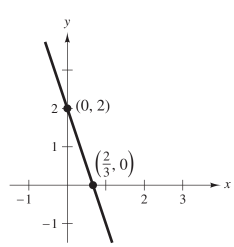
39. $y = 2 - 3x$

$y = 2 - 3(0) = 2,$ y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3},$ x-intercept

Intercepts: $(0, 2), \left(\frac{2}{3}, 0\right)$

Symmetry: none



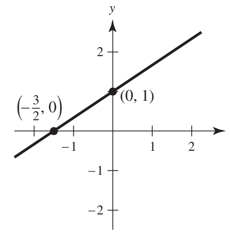
40. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1,$ y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2},$ x-intercept

Intercepts: $(0, 1), \left(-\frac{3}{2}, 0\right)$

Symmetry: none



41. $y = 9 - x^2$

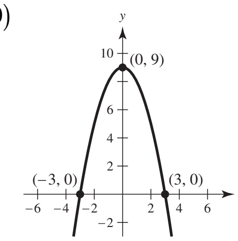
$y = 9 - (0)^2 = 9,$ y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3,$ x-intercepts

Intercepts: $(0, 9), (3, 0), (-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



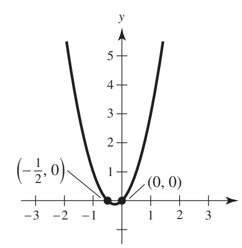
42. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0,$ y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2},$ x-intercepts

Intercepts: $(0, 0), \left(-\frac{1}{2}, 0\right)$

Symmetry: none



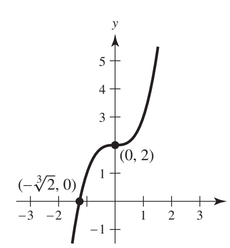
43. $y = x^3 + 2$

$y = 0^3 + 2 = 2,$ y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2},$ x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



44. $y = x^3 - 4x$

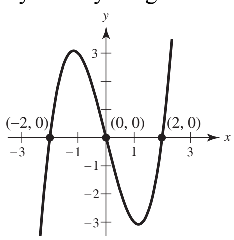
$y = 0^3 - 4(0) = 0$, y-intercept

$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x(x + 2)(x - 2) = 0$$
$$x = 0, \pm 2, \ x\text{-intercepts}$$

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



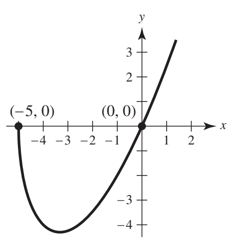
45. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



46. $y = \sqrt{25 - x^2}$

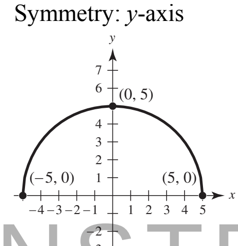
$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$$\sqrt{25 - x^2} = 0$$
$$25 - x^2 = 0$$
$$(5 + x)(5 - x) = 0$$
$$x = \pm 5, \ x\text{-intercept}$$

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



47. $x = y^3$

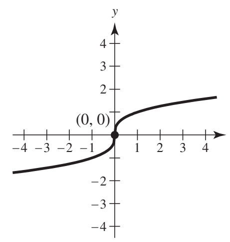
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: $(0, 0)$

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



48. $x = y^2 - 4$

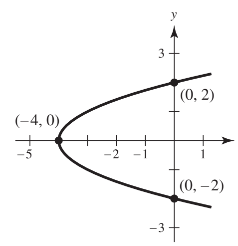
$$y^2 - 4 = 0$$
$$(y + 2)(y - 2) = 0$$
$$y = \pm 2, \ y\text{-intercepts}$$

$x = 0^2 - 4 = -4$, x-intercept

Intercepts: $(0, 2), (0, -2), (-4, 0)$

$x = (-y)^2 - 4 = y^2 - 4$

Symmetry: x-axis



49. $y = \dfrac{8}{x}$

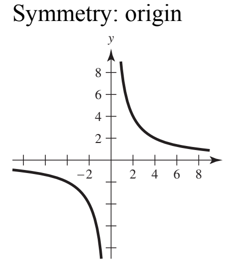
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



50. $y = \dfrac{10}{x^2 + 1}$

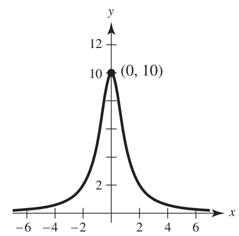
$y = \dfrac{10}{0^2 + 1} = 10$, y-intercept

$\dfrac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: $(0, 10)$

$y = \dfrac{10}{(-x)^2 + 1} = \dfrac{10}{x^2 + 1}$

Symmetry: y-axis



51. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

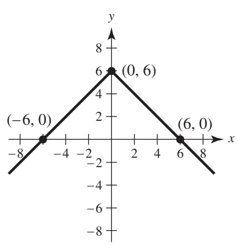
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6), (-6, 0), (6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



52. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

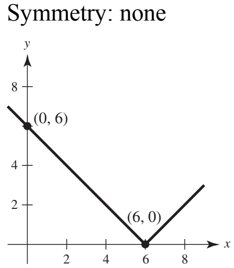
$|6 - x| = 0$

$6 - x = 0$

$6 = x$, x-intercept

Intercepts: $(0, 6), (6, 0)$

Symmetry: none



53. $y^2 - x = 9$

$y^2 = x + 9$

$y = \pm\sqrt{x + 9}$

$y = \pm\sqrt{0 + 9} = \pm\sqrt{9} = \pm 3$, y-intercepts

$\pm\sqrt{x + 9} = 0$

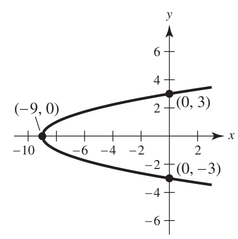
$x + 9 = 0$

$x = -9$, x-intercept

Intercepts: $(0, 3), (0, -3), (-9, 0)$

$(-y)^2 - x = 9 \Rightarrow y^2 - x = 9$

Symmetry: x-axis



54. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

$y = \pm\dfrac{\sqrt{4 - 0^2}}{2} = \pm\dfrac{\sqrt{4}}{2} = \pm 1$, y-intercepts

$x^2 + 4(0)^2 = 4$

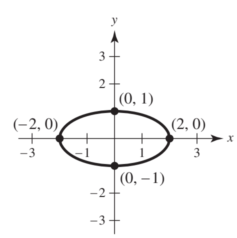
$x^2 = 4$

$x = \pm 2$, x-intercepts

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



55. $x + 3y^2 = 6$

$$3y^2 = 6 - x$$

$$y = \pm\sqrt{\frac{6-x}{3}}$$

$$y = \pm\sqrt{\frac{6-0}{3}} = \pm\sqrt{2}, \text{ } y\text{-intercepts}$$

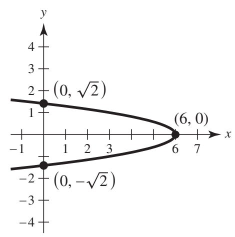
$$x + 3(0)^2 = 6$$

$$x = 6, \text{ } x\text{-intercept}$$

Intercepts: $(6, 0), (0, \sqrt{2}), (0, -\sqrt{2})$

$x + 3(-y)^2 = 6 \Rightarrow x + 3y^2 = 6$

Symmetry: x-axis



56. $3x - 4y^2 = 8$

$$4y^2 = 3x - 8$$

$$y = \pm\sqrt{\tfrac{3}{4}x - 2}$$

$$y = \pm\sqrt{\frac{3}{4}(0) - 2} = \pm\sqrt{-2}$$

$\Rightarrow$ no solution $\Rightarrow$ no y-intercepts

$$3x - 4(0)^2 = 8$$

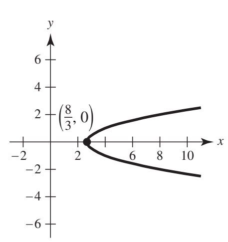
$$3x = 8$$

$$x = \frac{8}{3}, \text{ } x\text{-intercept}$$

Intercept: $\left(\frac{8}{3}, 0\right)$

$3x - 4(-y)^2 = 8 \Rightarrow 3x - 4y^2 = 8$

Symmetry: x-axis



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$$8 - x = 4x - 7$$

$$15 = 5x$$

$$3 = x$$

The corresponding y-value is $y = 5$.

Point of intersection: $(3, 5)$

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x+4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x-10}{2}$

$$\frac{3x+4}{2} = \frac{-4x-10}{2}$$

$$3x + 4 = -4x - 10$$

$$7x = -14$$

$$x = -2$$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 15 \Rightarrow y = -x^2 + 15$

$-3x + y = 11 \Rightarrow y = 3x + 11$

$$-x^2 + 15 = 3x + 11$$

$$0 = x^2 + 3x - 4$$

$$0 = (x + 4)(x - 1)$$

$$x = -4, 1$$

The corresponding y-values are $y = -1$ (for $x = -4$) and $y = 14$ (for $x = 1$).

Points of intersection: $(-4, -1), (1, 14)$

60. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$-3x + y = 15 \Rightarrow y = 3x + 15$

$$25 - x^2 = (3x + 15)^2$$

$$25 - x^2 = 9x^2 + 90x + 225$$

$$0 = 10x^2 + 90x + 200$$

$$0 = x^2 + 9x + 20$$

$$0 = (x + 5)(x + 4)$$

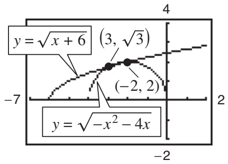
$$x = -4 \text{ or } x = -5$$

The corresponding y-values are $y = 3$ (for $x = -4$) and $y = 0$ (for $x = -5$).

Points of intersection: $(-4, 3), (-5, 0)$

61. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$

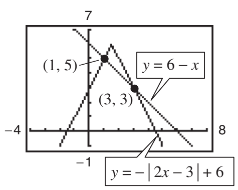


Points of intersection: $(-2, 2), \left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically,
$$\begin{aligned} \sqrt{x + 6} &= \sqrt{-x^2 - 4x} \\ x + 6 &= -x^2 - 4x \\ x^2 + 5x + 6 &= 0 \\ (x + 3)(x + 2) &= 0 \\ x &= -3, -2. \end{aligned}$$

62. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: (3, 3), (1, 5)

Analytically, $-|2x - 3| + 6 = 6 - x$

$$|2x - 3| = x$$

$2x - 3 = x$ or $2x - 3 = -x$

$x = 3$ or $x = 1.$

63. Replace x with $-x$ instead of y with $-y$.

$y^2 + 1 = (-x) \Rightarrow y^2 + 1 = -x$

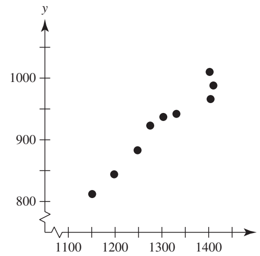
The graph of $y^2 + 1 = x$ is not symmetric about the y-axis.

64. The factored form of $x^2 - 2x - 3$ is $(x - 3)(x + 1)$.

$$\begin{aligned} -x + 1 &= -x^2 + x + 4 \\ x^2 - 2x - 3 &= 0 \\ (x - 3)(x + 1) &= 0 \\ x &= 3 \text{ or } -1 \end{aligned}$$

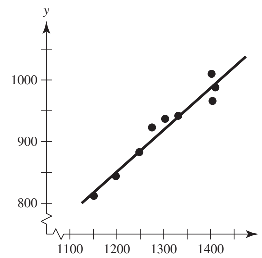
The points of intersection are $(-1, 2)$ and $(3, -2)$.

65. (a)



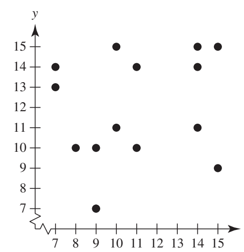
The data appear to be approximately linear.

(b) Models will vary. Sample answer: $y = 0.68x + 35$ (found using a graphing utility)



When $x = 1375$, $y = 0.68(1375) + 35 = \$970$.

66. (a)

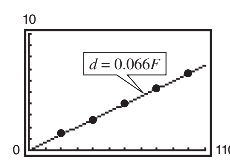


The data do not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, and so on. These variables may change from one quiz to the next.

67. (a) $d = 0.066F$

(b)

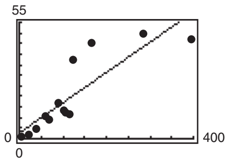


The model fits the data well. The correlation coefficient is $r \approx 0.9992$, so $|r|$ is close to 1, indicating that the linear model is a good fit for the data.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ centimeters.

68. (a) Using a graphing utility, $y = 0.143x + 2.696$.

The correlation coefficient is $r \approx 0.86$.

(b) 

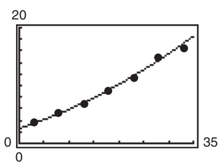
(c) Greater per capita energy consumption by a country tends to correspond to greater per capita gross national product of the country. The three countries that most differ from the linear model are Canada, Italy, and Japan.

(d) Using a graphing utility, the new model is $y = 0.174x - 2.14$.

The correlation coefficient is $r \approx 0.97$.

69. (a) Using a graphing utility,

$y = 0.005t^2 + 0.28t + 2.6$.

(b) 

The model is a good fit for the data.

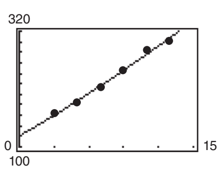
(c) When $t = 45$,

$y = 0.005(45)^2 + 0.28(45) + 2.6 \approx 25.3$.

So, in 2025, the GDP will be about $25.3 trillion.

70. (a) Using a graphing utility,

$y = 0.125t^2 + 12.8t + 120.475$.

(b) 

The model is a good fit for the data.

(c) When $t = 25$,

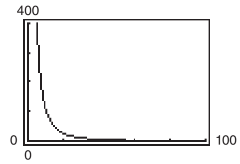
$y = 0.125(25)^2 + 12.8(25) + 120.475 = 518.6$.

So, in 2025, there will be about 518,600 cellular phone sites.

71.
$$\begin{aligned} C &= R \\ 2.04x + 5600 &= 3.29x \\ 5600 &= 3.29x - 2.04x \\ 5600 &= 1.25x \\ x &= \frac{5600}{1.25} = 4480 \end{aligned}$$

To break even, 4480 units must be sold.

72. $y = \dfrac{10{,}770}{x^2} - 0.37$

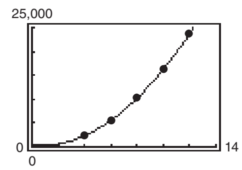


If the diameter is doubled, the resistance is changed by approximately a factor of $\frac{1}{4}$. For instance,

$y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

73. (a) Using a graphing utility,

$S = 180.89x^2 - 205.79x + 272$.

(b) 

(c) When $x = 2$, $S \approx 583.98$ pounds.

(d) $\dfrac{2370}{584} \approx 4.06$

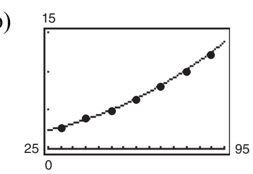
The breaking strength is approximately 4 times greater.

(e) $\dfrac{23{,}860}{5460} \approx 4.37$

When the height is doubled, the breaking strength increases approximately by a factor of 4.

74. (a) Using a graphing utility,

$t = 0.0013s^2 + 0.005s + 1.48$.

(b) 

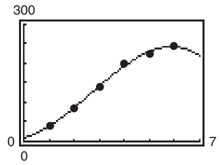
(c) According to the model, the times required to attain speeds of less than 20 miles per hour are all about the same. Furthermore, it takes 1.48 seconds to reach 0 miles per hour, which does not make sense.

(d) Adding $(0, 0)$ to the data produces

$t = 0.0009s^2 + 0.053s + 0.10$.

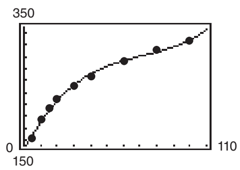
(e) Yes. Now the car starts at rest.

75. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

(b) 

(c) If $x = 4.5$, $y \approx 214$ horsepower.

76. (a) $T = 0.0003p^3 - 0.064p^2 + 5.28p + 143.1$

(b) 

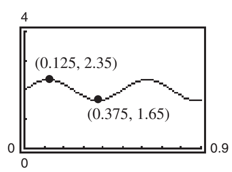
(c) For $T = 300°\text{F}$, $p \approx 68.29$ pounds per square inch.

(d) The model is based on data up to 100 pounds per square inch.

77. (a) The amplitude is approximately $(2.35 - 1.65)/2 = 0.35$.

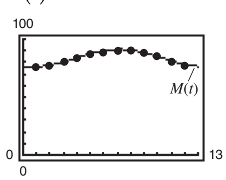
The period is approximately $2(0.375 - 0.125) = 0.5$.

(b) One model is $y = 0.35 \sin(4\pi t) + 2$.

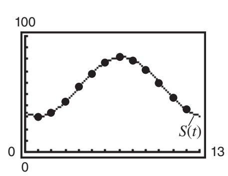
(c) 

The model appears to fit the data well.

78. (a) $S(t) = 56.37 + 25.47 \sin(0.5080t - 2.07)$

(b) 

The model is a good fit.

(c) 

The model is a good fit.

(d) The average is the constant term in each model. $83.70°\text{F}$ for Miami and $56.37°\text{F}$ for Syracuse.

(e) The period for Miami is $2\pi/0.4912 \approx 12.8$. The period for Syracuse is $2\pi/0.5080 \approx 12.4$. In both cases the period is approximately 12, or one year.

(f) Syracuse has greater variability because $25.47 > 7.46$.

79. $y = kx^3$

(a) $(1, 4)$: $\quad 4 = k(1)^3 \Rightarrow k = 4$

(b) $(-2, 1)$: $\quad 1 = k(-2)^3 = -8k \Rightarrow k = -\frac{1}{8}$

(c) $(0, 0)$: $\quad 0 = k(0)^3 \Rightarrow k$ can be any real number.

(d) $(-1, -1)$: $\quad -1 = k(-1)^3 = -k \Rightarrow k = 1$

80. $y^2 = 4kx$

(a) $(1, 1)$:
$$1^2 = 4k(1)$$
$$1 = 4k$$
$$k = \tfrac{1}{4}$$

(b) $(2, 4)$:
$$(4)^2 = 4k(2)$$
$$16 = 8k$$
$$k = 2$$

(c) $(0, 0)$: $\quad 0^2 = 4k(0)$

k can be any real number.

(d) $(3, 3)$:
$$(3)^2 = 4k(3)$$
$$9 = 12k$$
$$k = \tfrac{9}{12} = \tfrac{3}{4}$$

81. Answers will vary. Sample answer:

$y = (x + 4)(x - 3)(x - 8)$ has intercepts at $x = -4$, $x = 3$, and $x = 8$.

82. Answers will vary. Sample answer:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

83. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

84. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0)\ (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because

$-y = (-x)^3 - (-x) = -x^3 + x.$

Symmetry with respect to the y-axis for $y = x^2 + 2$ because

$y = (-x)^2 + 2 = x^2 + 2.$

(c)
$$\begin{aligned} x^3 - x &= x^2 + 2 \\ x^3 - x^2 - x - 2 &= 0 \\ (x - 2)(x^2 + x + 1) &= 0 \\ x = 2 &\Rightarrow y = 6 \end{aligned}$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

85. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

86. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

Section P.2 Linear Models and Rates of Change

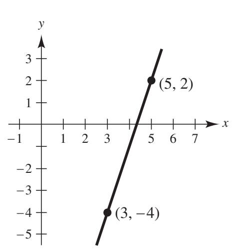
1. $m = 2$

2. $m = 0$

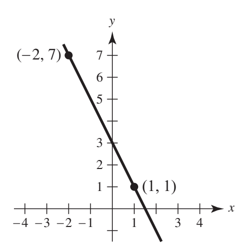
3. $m = -1$

4. $m = -12$

5. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

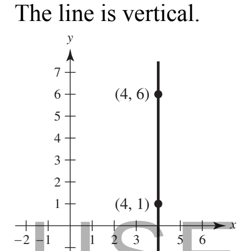


6. $m = \dfrac{7 - 1}{-2 - 1} = \dfrac{6}{-3} = -2$



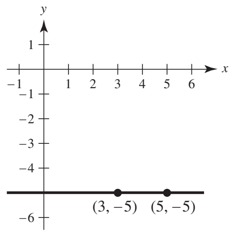
7. $m = \dfrac{1 - 6}{4 - 4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

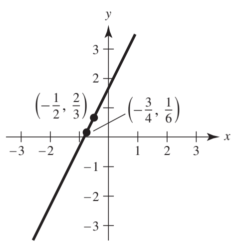


8. $m = \dfrac{-5 - (-5)}{5 - 3} = \dfrac{0}{2} = 0$

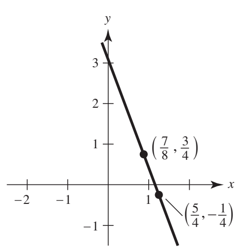
The line is horizontal.



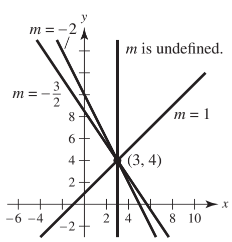
9. $m = \dfrac{\frac{2}{3} - \frac{1}{6}}{-\frac{1}{2} - \left(-\frac{3}{4}\right)} = \dfrac{\frac{1}{2}}{\frac{1}{4}} = 2$



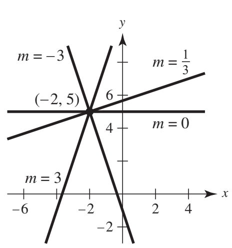
10. $m = \dfrac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \dfrac{1}{-\frac{3}{8}} = -\dfrac{8}{3}$



11.



12.



13. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2)$, $(1, 2)$, $(5, 2)$.

14. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0)$, $(-4, 1)$, $(-4, 2)$.

15. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

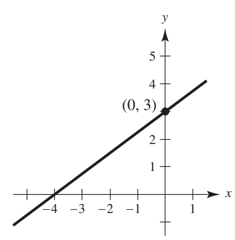
Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

16. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

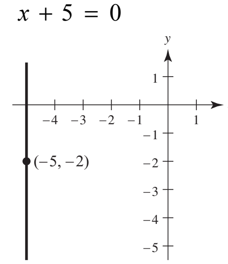
Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

17.
$$y = \tfrac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$

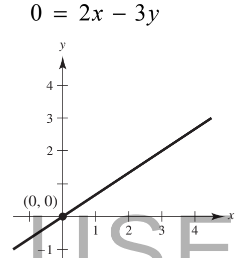


18. The slope is undefined, so the line is vertical.

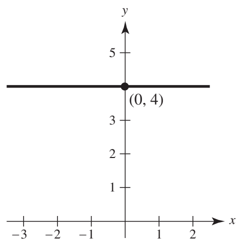
$$x = -5$$
$$x + 5 = 0$$



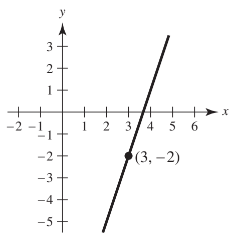
19.
$$y = \tfrac{2}{3}x$$
$$3y = 2x$$
$$0 = 2x - 3y$$



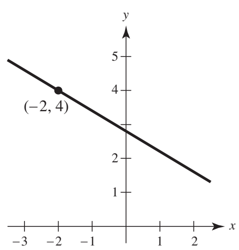
20. $$\begin{aligned} y &= 4 \\ y - 4 &= 0 \end{aligned}$$



21. $$\begin{aligned} y + 2 &= 3(x - 3) \\ y + 2 &= 3x - 9 \\ y &= 3x - 11 \\ 0 &= 3x - y - 11 \end{aligned}$$

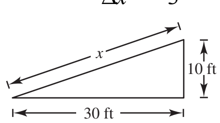


22. $$\begin{aligned} y - 4 &= -\tfrac{3}{5}(x + 2) \\ 5y - 20 &= -3x - 6 \\ 3x + 5y - 14 &= 0 \end{aligned}$$



23. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

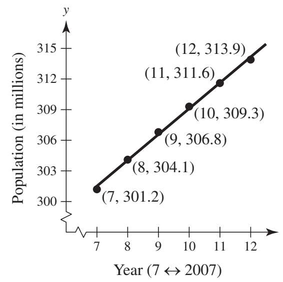
(b)



By the Pythagorean Theorem,

$$\begin{aligned} x^2 &= 30^2 + 10^2 = 1000 \\ x &= 10\sqrt{10} \approx 31.623 \text{ feet.} \end{aligned}$$

24. (a)



(b) The slopes are: $\dfrac{304.1 - 301.2}{8 - 7} = 2.9$

$\dfrac{306.8 - 304.1}{9 - 8} = 2.7$

$\dfrac{309.3 - 306.8}{10 - 9} = 2.5$

$\dfrac{311.6 - 309.3}{11 - 10} = 2.3$

$\dfrac{313.9 - 311.6}{12 - 11} = 2.3$

The population increased least rapidly from 2010 to 2011 and from 2011 to 2012.

(c) Average rate of change from 2007 through 2012:

$$\begin{aligned} \frac{313.9 - 301.2}{12 - 7} &= \frac{12.7}{5} \\ &= 2.54 \text{ million per year} \end{aligned}$$

(d) When $t = 20$, $y = 2.54(20) + 283.42 = 334.22$.

So, in 2020, the population will be 334.22 million people.

25. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

26. $$\begin{aligned} -x + y &= 1 \\ y &= x + 1 \end{aligned}$$

The slope is $m = 1$ and the y-intercept is (0, 1).

27. $$\begin{aligned} 5x + y &= 20 \\ y &= -5x + 20 \end{aligned}$$

The slope is $m = -5$ and the y-intercept is (0, 20).

28. $$\begin{aligned} 6x - 5y &= 15 \\ y &= \tfrac{6}{5}x - 3 \end{aligned}$$

The slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

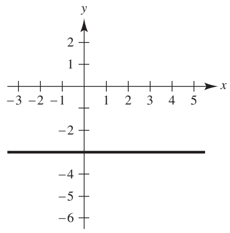
29. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

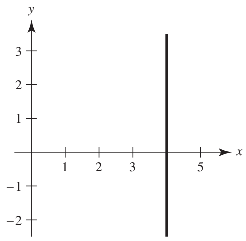
30. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

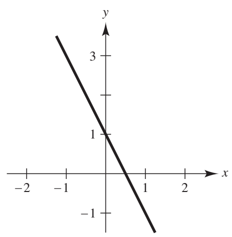
31. $y = -3$



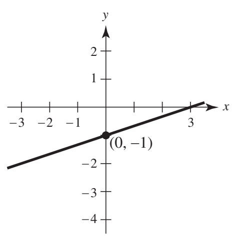
32. $x = 4$



33. $y = -2x + 1$

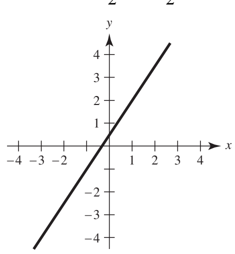


34. $y = \frac{1}{3}x - 1$



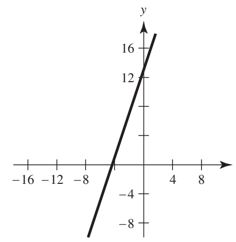
35. $y - 2 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{1}{2}$



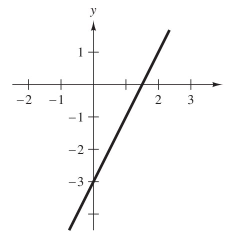
36. $y - 1 = 3(x + 4)$

$y = 3x + 13$



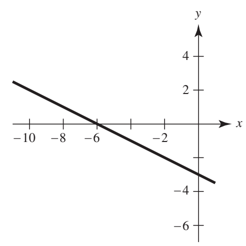
37. $2x - y - 3 = 0$

$y = 2x - 3$



38. $x + 2y + 6 = 0$

$y = -\frac{1}{2}x - 3$

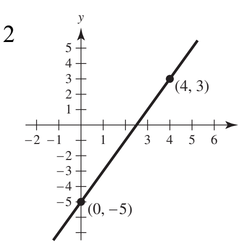


39. $m = \dfrac{-5 - 3}{0 - 4} = \dfrac{-8}{-4} = 2$

$y - (-5) = 2(x - 0)$

$y + 5 = 2x$

$y = 2x - 5$



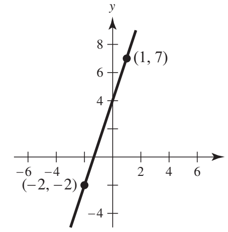
40. $m = \dfrac{7 - (-2)}{1 - (-2)} = \dfrac{9}{3} = 3$

$y - (-2) = 3(x - (-2))$

$y + 2 = 3(x + 2)$

$y = 3x + 4$

$0 = 3x - y + 4$

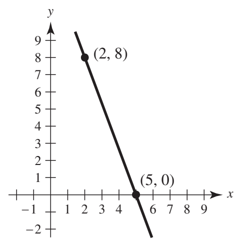


41. $m = \dfrac{8-0}{2-5} = -\dfrac{8}{3}$

$$y - 0 = -\frac{8}{3}(x - 5)$$

$$y = -\frac{8}{3}x + \frac{40}{3}$$

$$8x + 3y - 40 = 0$$

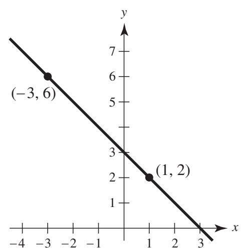


42. $m = \dfrac{6-2}{-3-1} = \dfrac{4}{-4} = -1$

$$y - 2 = -1(x - 1)$$

$$y - 2 = -x + 1$$

$$x + y - 3 = 0$$

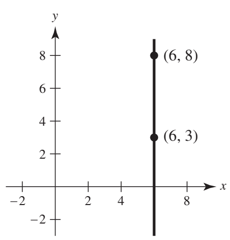


43. $m = \dfrac{8-3}{6-6} = \dfrac{5}{0}$, undefined

The line is horizontal.

$$x = 6$$

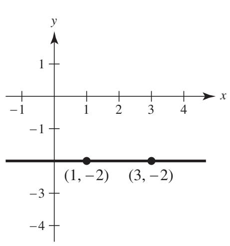
$$x - 6 = 0$$



44. $m = \dfrac{-2-(-2)}{3-1} = \dfrac{0}{2} = 0$

$$y = -2$$

$$y + 2 = 0$$

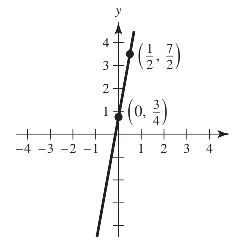


45. $m = \dfrac{\frac{7}{2} - \frac{3}{4}}{\frac{1}{2} - 0} = \dfrac{\frac{11}{4}}{\frac{1}{2}} = \dfrac{11}{2}$

$$y - \frac{3}{4} = \frac{11}{2}(x - 0)$$

$$y = \frac{11}{2}x + \frac{3}{4}$$

$$0 = 22x - 4y + 3$$

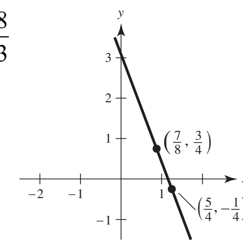


46. $m = \dfrac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \dfrac{1}{-\frac{3}{8}} = -\dfrac{8}{3}$

$$y + \frac{1}{4} = \frac{-8}{3}\left(x - \frac{5}{4}\right)$$

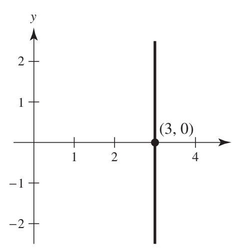
$$12y + 3 = -32x + 40$$

$$32x + 12y - 37 = 0$$



47. $x = 3$

$$x - 3 = 0$$

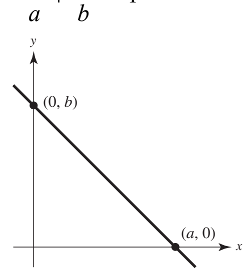


48. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$

$$\frac{b}{a}x + y = b$$

$$\frac{x}{a} + \frac{y}{b} = 1$$



49. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

50. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$\dfrac{-3x}{2} - \dfrac{y}{2} = 1$

$3x + y = -2$

$3x + y + 2 = 0$

51. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{1}{a} + \dfrac{2}{a} = 1$

$\dfrac{3}{a} = 1$

$a = 3 \Rightarrow x + y = 3$

$x + y - 3 = 0$

52. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{-3}{a} + \dfrac{4}{a} = 1$

$\dfrac{1}{a} = 1$

$a = 1 \Rightarrow x + y = 1$

$x + y - 1 = 0$

53. $\dfrac{x}{2a} + \dfrac{y}{a} = 1$

$\dfrac{9}{2a} + \dfrac{-2}{a} = 1$

$\dfrac{9 - 4}{2a} = 1$

$5 = 2a$

$a = \dfrac{5}{2}$

$\dfrac{x}{2\left(\frac{5}{2}\right)} + \dfrac{y}{\left(\frac{5}{2}\right)} = 1$

$\dfrac{x}{5} + \dfrac{2y}{5} = 1$

$x + 2y = 5$

$x + 2y - 5 = 0$

54. $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$\dfrac{\left(-\frac{2}{3}\right)}{a} + \dfrac{(-2)}{-a} = 1$

$-\dfrac{2}{3} + 2 = a$

$a = \dfrac{4}{3}$

$\dfrac{x}{\left(\frac{4}{3}\right)} + \dfrac{y}{\left(-\frac{4}{3}\right)} = 1$

$x - y = \dfrac{4}{3}$

$3x - 3y - 4 = 0$

55. The given line is vertical.

(a) $x = -7 \Rightarrow x + 7 = 0$

(b) $y = -2 \Rightarrow y + 2 = 0$

56. The given line is horizontal.

(a) $y = 0$

(b) $x = -1 \Rightarrow x + 1 = 0$

57. $x + y = 7$

$y = -x + 7$

$m = -1$

(a) $y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$x + y + 1 = 0$

(b) $y - 2 = 1(x + 3)$

$y - 2 = x + 3$

$0 = x - y + 5$

58. $x - y = -2$

$y = x + 2$

$m = 1$

(a) $y - 5 = 1(x - 2)$

$y - 5 = x - 2$

$x - y + 3 = 0$

(b) $y - 5 = -1(x - 2)$

$y - 5 = -x + 2$

$x + y - 7 = 0$

59. $4x - 2y = 3$

$$y = 2x - \tfrac{3}{2}$$
$$m = 2$$

(a) $$y - 1 = 2(x - 2)$$
$$y - 1 = 2x - 4$$
$$0 = 2x - y - 3$$

(b) $$y - 1 = -\tfrac{1}{2}(x - 2)$$
$$2y - 2 = -x + 2$$
$$x + 2y - 4 = 0$$

60. $7x + 4y = 8$

$$4y = -7x + 8$$
$$y = \frac{-7}{4}x + 2$$
$$m = -\frac{7}{4}$$

(a) $$y + \frac{1}{2} = \frac{-7}{4}\left(x - \frac{5}{6}\right)$$
$$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$$
$$24y + 12 = -42x + 35$$
$$42x + 24y - 23 = 0$$

(b) $$y + \frac{1}{2} = \frac{4}{7}\left(x - \frac{5}{6}\right)$$
$$42y + 21 = 24x - 20$$
$$24x - 42y - 41 = 0$$

61. $5x - 3y = 0$

$$y = \tfrac{5}{3}x$$
$$m = \tfrac{5}{3}$$

(a) $$y - \tfrac{7}{8} = \tfrac{5}{3}\left(x - \tfrac{3}{4}\right)$$
$$24y - 21 = 40x - 30$$
$$0 = 40x - 24y - 9$$

(b) $$y - \tfrac{7}{8} = -\tfrac{3}{5}\left(x - \tfrac{3}{4}\right)$$
$$40y - 35 = -24x + 18$$
$$24x + 40y - 53 = 0$$

62. $3x + 4y = 7$

$$4y = -3x + 7$$
$$y = -\tfrac{3}{4}x + \tfrac{7}{4}$$
$$m = -\tfrac{3}{4}$$

(a) $$y - (-5) = -\tfrac{3}{4}(x - 4)$$
$$y + 5 = -\tfrac{3}{4}x + 3$$
$$4y + 20 = -3x + 12$$
$$3x + 4y + 8 = 0$$

(b) $$y - (-5) = \tfrac{4}{3}(x - 4)$$
$$y + 5 = \tfrac{4}{3}x - \tfrac{16}{3}$$
$$3y + 15 = 4x - 16$$
$$0 = 4x - 3y - 31$$

63. The variables $x = -1$ and $y = 4$ were substituted incorrectly into the point-slope form of the equation.

$$y - 4 = -\tfrac{5}{2}[x - (-1)]$$
$$y - 4 = -\tfrac{5}{2}x - \tfrac{5}{2}$$
$$y = -\tfrac{5}{2}x + \tfrac{3}{2}$$

64. The slope of the perpendicular line is the negative reciprocal of $-\tfrac{3}{4}$, which is $\tfrac{4}{3}$.

$$y - (-3) = \tfrac{4}{3}(x - 2)$$
$$3(y + 3) = 4(x - 2)$$
$$3y + 9 = 4x - 8$$
$$0 = 4x - 3y - 17$$

65. The slope is 250.

$V = 1850$ when $t = 5$.

$$V = 250(t - 5) + 1850$$
$$= 250t + 600$$

66. The slope is 4.50.

$V = 156$ when $t = 5$.

$$V = 4.5(t - 5) + 156$$
$$= 4.5t + 133.5$$

67. The slope is -1600.

$V = 17{,}200$ when $t = 5$.

$$V = -1600(t - 5) + 17{,}200$$
$$= -1600t + 25{,}200$$

68. The slope is -5600.

$V = 245{,}000$ when $t = 5$.

$$V = -5600(t - 5) + 245{,}000$$
$$= -5600t + 273{,}000$$

69. $m_1 = \dfrac{-6-4}{7-0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11-4}{-5-0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

70. $m_1 = \dfrac{1-0}{-2-(-1)} = -1$

$m_2 = \dfrac{-2-0}{2-(-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

71. (a) Equations of perpendicular bisectors:

$$y - \frac{c}{2} = \frac{a-b}{c}\left(x - \frac{a+b}{2}\right)$$

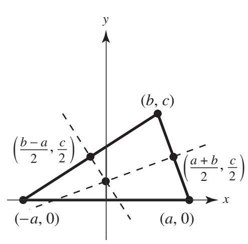
$$y - \frac{c}{2} = \frac{a+b}{-c}\left(x - \frac{b-a}{2}\right)$$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

$$\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right).$$

This point lies on the third perpendicular bisector, $x = 0$.

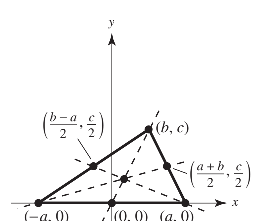


(b) Equations of medians:

$$y = \frac{c}{b}x$$

$$y = \frac{c}{3a+b}(x+a)$$

$$y = \frac{c}{-3a+b}(x-a)$$



Solving simultaneously, the point of intersection is $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$.

72. The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(b, \dfrac{a^2-b^2}{c}\right)$ is:

$$m_1 = \frac{\left[\left(a^2-b^2\right)/c\right] - (c/3)}{b - (b/3)} = \frac{\left(3a^2 - 3b^2 - c^2\right)/(3c)}{(2b)/3} = \frac{3a^2 - 3b^2 - c^2}{2bc}$$

The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(0, \dfrac{-a^2+b^2+c^2}{2c}\right)$ is:

$$m_2 = \frac{\left[\left(-a^2+b^2+c^2\right)/(2c)\right] - (c/3)}{0 - (b/3)} = \frac{\left(-3a^2+3b^2+3c^2-2c^2\right)/(6c)}{-b/3} = \frac{3a^2-3b^2-c^2}{2bc}$$

$m_1 = m_2$

Therefore, the points are collinear.

73. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(b) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$-5x + 8y = 4$

$y = \frac{1}{8}(5x+4) = \frac{5}{8}x + \frac{1}{2}$

(c) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$5x + 2y = 4$

$y = \frac{1}{2}(-5x+4) = -\frac{5}{2}x + 2$

(d) $a = \frac{5}{2}$ and $b = 3$.

$\frac{5}{2}x + 3y = 4$

$5x + 6y = 8$

74. (a) Lines c, d, e, and f have positive slopes.

(b) Lines a and b have negative slopes.

(c) Lines c and e appear parallel.

Lines d and f appear parallel.

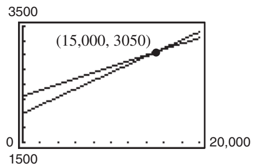
(d) Lines b and f appear perpendicular.

Lines b and d appear perpendicular.

75. (a) Current job: $W_1 = 0.07s + 2000$

New job offer: $W_2 = 0.05s + 2300$

(b)



Using a graphing utility, the point of intersection is (15,000, 3050).

Analytically, $W_1 = W_2$

$$0.07s + 2000 = 0.05s + 2300$$
$$0.02s = 300$$
$$s = 15{,}000$$

So, $W_1 = W_2 = 0.07(15{,}000) + 2000 = 3050.$

When sales exceed \$15,000, your current job pays more.

(c) No, if you can sell \$20,000 worth of goods, then $W_1 > W_2$.

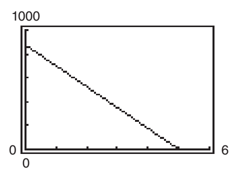
(**Note:** $W_1 = 3400$ and $W_2 = 3300$ when $s = 20{,}000$.)

76. (a) Depreciation per year:

$\frac{875}{5} = \$175$

$y = 875 - 175x,$

where $0 \le x \le 5$



(b) $y = 875 - 175(2) = \$525$

(c) $200 = 875 - 175x$

$175x = 675$

$x \approx 3.86$ years

77. (a) Two points are (50, 780) and (47, 825).

The slope is

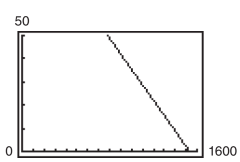
$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$

$$p - 780 = -15(x - 50)$$
$$p = -15x + 750 + 780 = -15x + 1530$$

or

$$x = \frac{1}{15}(1530 - p)$$

(b)



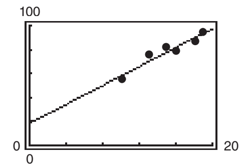
If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units.

78. (a) $y = 18.91 + 3.97x$

$(x = \text{quiz score}, y = \text{test score})$

(b)

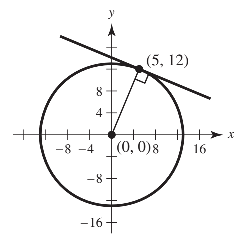


(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

79. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).

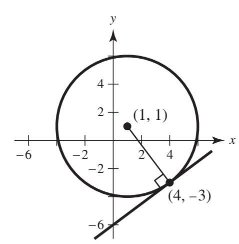


Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$
$$y = \frac{-5}{12}x + \frac{169}{12}$$
$$5x + 12y - 169 = 0.$$

80. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is

$$\frac{1 + 3}{1 - 4} = \frac{-4}{3}.$$

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$
$$y = \frac{3}{4}x - 6$$
$$0 = 3x - 4y - 24$$

81. $x - y - 2 = 0 \Rightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}}$

$$= \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

82. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \dfrac{7}{5}$

83. A point on the line $x + y = 1$ is (0, 1). The distance from the point (0, 1) to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

84. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{|3(-1) - 4(-1) - 10|}{\sqrt{3^2 + (-4)^2}} = \frac{|-3 + 4 - 10|}{5} = \frac{9}{5}.$$

85. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$\begin{aligned} y - y_1 &= \frac{B}{A}(x - x_1) \\ Ay - Ay_1 &= Bx - Bx_1 \\ Bx_1 - Ay_1 &= Bx - Ay \end{aligned}$$

The point of intersection of these two lines is:

$$\begin{aligned} &Ax + By = -C &&\Rightarrow A^2x + ABy = -AC && (1) \\ &Bx - Ay = Bx_1 - Ay_1 &&\Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} && (2) \\ & &&(A^2 + B^2)x = -AC + B^2x_1 - ABy_1 && \text{(By adding equations (1) and (2))} \\ & &&x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} \end{aligned}$$

$$\begin{aligned} &Ax + By = -C &&\Rightarrow ABx + B^2y = -BC && (3) \\ &Bx - Ay = Bx_1 - Ay_1 &&\Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} && (4) \\ & &&(A^2 + B^2)y = -BC - ABx_1 + A^2y_1 && \text{(By adding equations (3) and (4))} \\ & &&y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} \end{aligned}$$

$\left(\dfrac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \dfrac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right)$ point of intersection

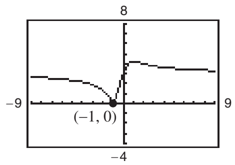
The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$\begin{aligned} d &= \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2} \\ &= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2} \\ &= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$

86. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).



87. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

88. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

Section P.3 Functions and Their Graphs

1. $f(x) = 3x - 2$

(a) $f(0) = 3(0) - 2 = -2$

(b) $f(5) = 3(5) - 2 = 13$

(c) $f(b) = 3(b) - 2 = 3b - 2$

(d) $f(x - 1) = 3(x - 1) - 2 = 3x - 5$

2. (a) $g(4) = 4^2(4 - 4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $g(t + 4) = (t + 4)^2(t + 4 - 4)$
$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

3. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

(d) $f(\pi) = \cos(2(\pi)) = 1$

4. (a) $f(\pi) = \sin \pi = 0$

(b) $f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

(c) $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

(d) $f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

5. $$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2,\ \Delta x \neq 0$$

6. $$\frac{f(x) - f(1)}{x - 1} = \frac{3x - 1 - (3 - 1)}{x - 1} = \frac{3(x - 1)}{x - 1} = 3,\ x \neq 1$$

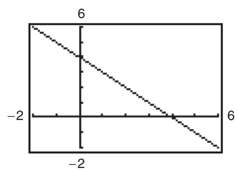
7. $$\frac{f(x) - f(2)}{x - 2} = \frac{\left(1/\sqrt{x - 1} - 1\right)}{x - 2}$$

$$= \frac{1 - \sqrt{x - 1}}{(x - 2)\sqrt{x - 1}} \cdot \frac{1 + \sqrt{x - 1}}{1 + \sqrt{x - 1}} = \frac{2 - x}{(x - 2)\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)} = \frac{-1}{\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)},\ x \neq 2$$

8. $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{x^3 - x - 0}{x - 1} = \dfrac{x(x + 1)(x - 1)}{x - 1} = x(x + 1),\ x \neq 1$

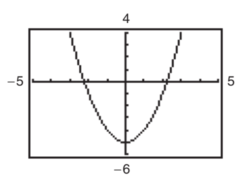
9. $f(x) = 4 - x$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. Because the graph of the function extends infinitely in both directions, the range is $(-\infty, \infty)$.



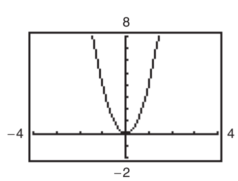
10. $g(x) = x^2 - 5$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. No matter what value of x is given, the value of $f(x)$ is never less than 5. So, the range is $[5, \infty)$.



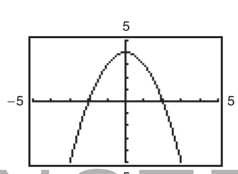
11. $f(x) = 4x^2$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. No matter what value of x is given, the value of $f(x)$ is never less than 0. So, the range is $[0, \infty)$.



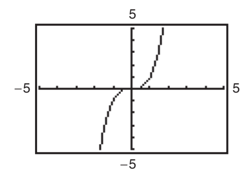
12. $h(x) = 4 - x^2$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. No matter what value of x is given, the value of $f(x)$ is never more than 4. So, the range is $(-\infty, 4]$.



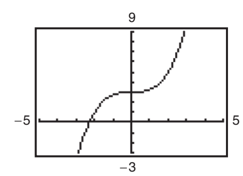
13. $f(x) = x^3$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. Because the graph of the function extends infinitely in both directions, the range is $(-\infty, \infty)$.



14. $f(x) = \frac{1}{4}x^3 + 3$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. Because the graph of the function extends infinitely in both directions, the range is $(-\infty, \infty)$.

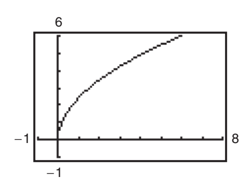


15. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$

$x \geq 0 \Rightarrow [0, \infty)$

Range: $\sqrt{6x} \geq 0 \Rightarrow [0, \infty)$



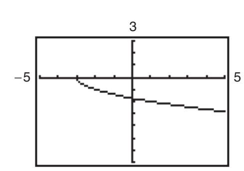
16. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \geq 0$

$x \geq -3 \Rightarrow [-3, \infty)$

Range: $-\sqrt{x + 3} \geq 0$

$\sqrt{x + 3} \leq 0 \Rightarrow (-\infty, 0]$



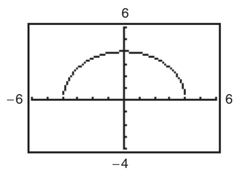
17. $f(x) = \sqrt{16 - x^2}$

Domain: $16 - x^2 \geq 0$

$16 \geq x^2$

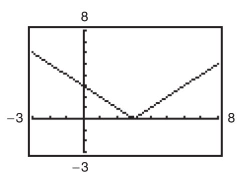
$x \geq -4$ and $x \leq 4 \Rightarrow [-4, 4]$

Range: For $x \geq -4$ and $x \leq 4$, $0 \leq y \leq 4 \Rightarrow [0, 4]$



18. $f(x) = |x - 3|$

The function is defined for all real numbers x. So, the domain is $(-\infty, \infty)$. No matter what value of x is given, the value of $f(x)$ is never negative. So, the range is $[0, \infty)$.

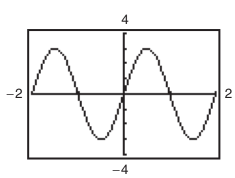


19. $g(t) = 3 \sin \pi t$

The function is defined for all real numbers t. So, the domain is $(-\infty, \infty)$.

Range: $-1 \leq \sin t \leq 1$

$-3 \leq 3 \sin \pi t \leq 3 \Rightarrow [-3, 3]$

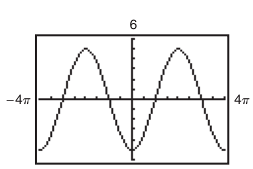


20. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

The function is defined for all real numbers θ. So, the domain is $(-\infty, \infty)$.

Range: $-1 \leq \cos \theta \leq 1$

$5 \geq -5 \cos \dfrac{\theta}{2} \geq -5 \Rightarrow [-5, 5]$

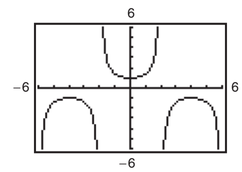


21. $f(t) = \sec \dfrac{\pi t}{4}$

$\dfrac{\pi t}{4} \neq \dfrac{(2n + 1)\pi}{2} \Rightarrow t \neq 4n + 2$

Domain: all $t \neq 4n + 2$, n is an integer

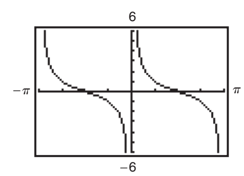
No matter what value of t is given within the domain, the value of $f(t)$ is either at least 1 or at most -1. So, the range is $(-\infty, -1] \cup [1, \infty)$.



22. $h(t) = \cot t$

Because h is undefined when $t = \pi$, this value and its multiples are not in the domain. So, the domain is the set of all t-values such that $t \neq n\pi$, where n is an integer.

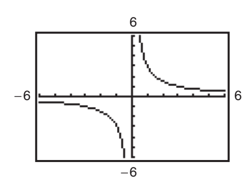
As t gets closer to the values $-\pi$ and π, $h(t)$ decreases without bound. So, the range is $(-\infty, \infty)$.



23. $f(x) = \dfrac{3}{x}$

Because division by 0 is undefined, the domain is all x-values such that $x \neq 0$, or $(-\infty, 0) \cup (0, \infty)$.

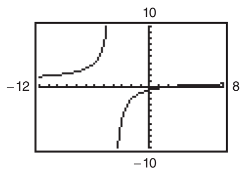
As x increases or decreases without bound, $f(x)$ approaches but does not reach 0. So, the range is $(-\infty, 0) \cup (0, \infty)$.



24. $f(x) = \dfrac{x-2}{x+4}$

Because division by 0 is undefined, the domain is all x-values such that $x \neq -4$, or $(-\infty, -4) \cup (-4, \infty)$.

As x increases or decreases without bound, $f(x)$ approaches but does not reach 2. So, the range is $(-\infty, 2) \cup (2, \infty)$.



25. $f(x) = \sqrt{x} + \sqrt{1-x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

26. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \geq 0$

$(x-2)(x-1) \geq 0$

Domain: $x \geq 2$ or $x \leq 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

27. $g(x) = \dfrac{2}{1 - \cos x}$

$1 - \cos x \neq 0$

$\cos x \neq 1$

Domain: all $x \neq 2n\pi$, n is an integer

28. $h(x) = \dfrac{1}{\sin x - (1/2)}$

$\sin x - \dfrac{1}{2} \neq 0$

$\sin x \neq \dfrac{1}{2}$

Domain: all $x \neq \dfrac{\pi}{6} + 2n\pi, \dfrac{5\pi}{6} + 2n\pi$, n is an integer

29. $f(x) = \dfrac{1}{|x+3|}$

$|x+3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

30. $g(x) = \dfrac{1}{|x^2 - 4|}$

$|x^2 - 4| \neq 0$

$(x-2)(x+2) \neq 0$

Domain: all $x \neq \pm 2$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

31. $f(x) = \begin{cases} 2x+1, & x < 0 \\ 2x+2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all t.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

32. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(x^2 + 2) = 2(x^2 + 2)^2 + 2 = 2x^4 + 8x^2 + 10$

(**Note:** $x^2 + 2 > 1$ for all x.)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

33. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

34. $f(x) = \begin{cases} \sqrt{x+4}, & x \le 5 \\ (x-5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3+4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0+4} = 2$

(c) $f(5) = \sqrt{5+4} = 3$

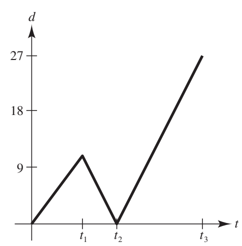
(d) $f(10) = (10-5)^2 = 25$

Domain: $[-4, \infty)$

Range: $[0, \infty)$

35. The student travels $\dfrac{2-0}{4-0} = \dfrac{1}{2}$ mile per minute during the first 4 minutes, is stationary for the next 2 minutes, and travels $\dfrac{6-2}{10-6} = 1$ mile per minute during the final 4 minutes.

36.



37. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

38. $\sqrt{x^2-4} - y = 0 \Rightarrow y = \sqrt{x^2-4}$

y is a function of x. Vertical lines intersect the graph at most once.

39. $y = \begin{cases} x+1, & x \le 0 \\ -x+2 & x > 0 \end{cases}$

y is a function of x. Vertical lines intersect the graph at most once.

40. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4-x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

41. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16-x^2}$

y is not a function of x because there are two values of y for some x.

42. $x^2 + y = 16 \Rightarrow y = 16 - x^2$

y is a function of x because there is one value of y for each x.

43. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2-1}$

y is not a function of x because there are two values of y for some x.

44. $x^2y - x^2 + 4y = 0 \Rightarrow y = \dfrac{x^2}{x^2+4}$

y is a function of x because there is one value of y for each x.

45. The transformation is a horizontal shift two units to the right.

Equation: $y = \sqrt{x-2}$

46. The transformation is a vertical shift 4 units upward.

Equation: $y = \sin x + 4$

47. The transformation is a horizontal shift 2 units to the right and a vertical shift 1 unit downward.

Equation: $y = (x-2)^2 - 1$

48. The transformation is a horizontal shift 1 unit to the left and a vertical shift 2 units upward.

Equation: $y = (x+1)^3 + 2$

49. $y = f(x+5)$ is a horizontal shift 5 units to the left. Matches *d*.

50. $y = f(x) - 5$ is a vertical shift 5 units downward. Matches *b*.

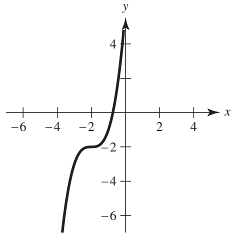
51. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches *c*.

52. $y = -f(x-4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches *a*.

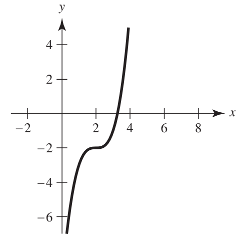
53. $y = f(x+6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches *e*.

54. $y = f(x-1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches *g*.

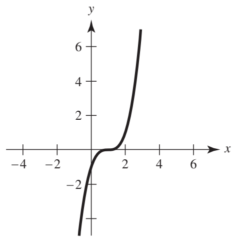
55. (a) The graph is shifted 3 units to the left.



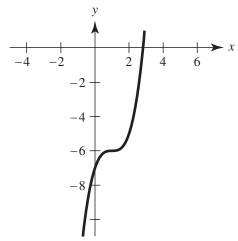
(b) The graph is shifted 1 unit to the right.



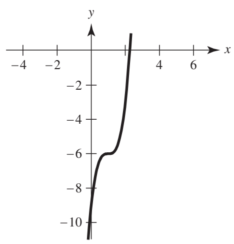
(c) The graph is shifted 2 units upward.



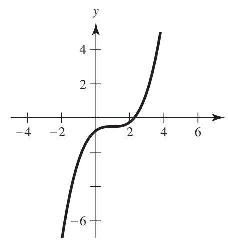
(d) The graph is shifted 4 units downward.



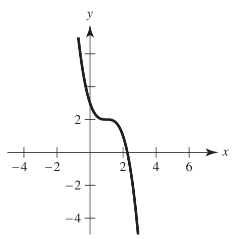
(e) The graph is stretched vertically by a factor of 3.



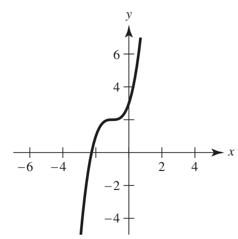
(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.



(g) The graph is a reflection in the *x*-axis.



(h) The graph is a reflection about the origin.

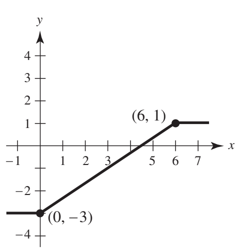


56. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

$g(0) = f(-4) = -3$

The graph is shifted 4 units to the right.

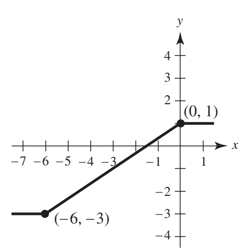


(b) $g(x) = f(x + 2)$

$g(0) = f(2) = 1$

$g(-6) = f(-4) = -3$

The graph is shifted 2 units to the left.

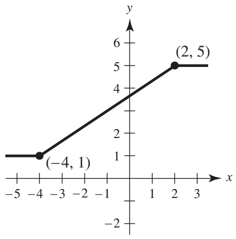


(c) $g(x) = f(x) + 4$

$g(2) = f(2) + 4 = 5$

$g(-4) = f(-4) + 4 = 1$

The graph is shifted 4 units upward.

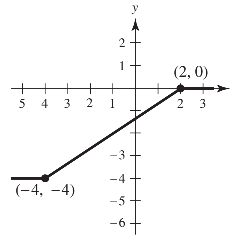


(d) $g(x) = f(x) - 1$

$g(2) = f(2) - 1 = 0$

$g(-4) = f(-4) - 1 = -4$

The graph is shifted 1 unit downward.

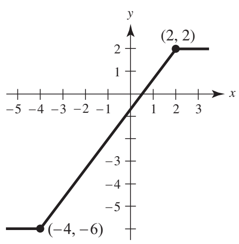


(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

$g(-4) = 2f(-4) = -6$

The graph is stretched vertically by a factor of 2.

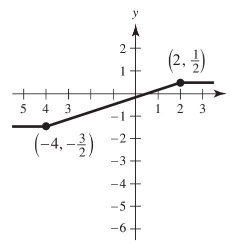


(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$

The graph is stretched vertically by a factor of $\frac{1}{2}$.

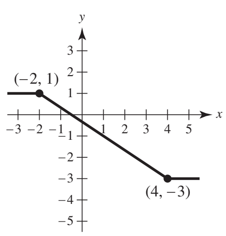


(g) $g(x) = f(-x)$

$g(-2) = f(2) = 1$

$g(4) = f(-4) = -3$

The graph is a reflection in the y-axis.

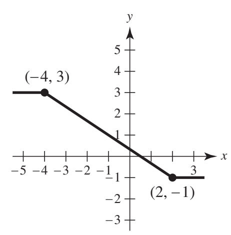


(h) $g(x) = -f(x)$

$g(2) = f(2) = -1$

$g(-4) = f(-4) = 3$

The graph is a reflection in the x-axis.



57. $f(x) = 2x - 5, \quad g(x) = 4 - 3x$

(a) $f(x) + g(x) = (2x - 5) + (4 - 3x) = -x - 1$

(b) $f(x) - g(x) = (2x - 5) - (4 - 3x) = 5x - 9$

(c) $f(x) \cdot g(x) = (2x - 5)(4 - 3x)$

$$= -6x^2 + 8x + 15x - 20$$

$$= -6x^2 + 23x - 20$$

(d) $f(x)/g(x) = \dfrac{2x - 5}{4 - 3x}$

58. $f(x) = x^2 + 5x + 4, \quad g(x) = x + 1$

(a) $f(x) + g(x) = (x^2 + 5x + 4) + (x + 1) = x^2 + 6x + 5$

(b) $f(x) - g(x) = (x^2 + 5x + 4) - (x + 1) = x^2 + 4x + 3$

(c) $f(x) \cdot g(x) = (x^2 + 5x + 4)(x + 1)$

$$= x^3 + 5x^2 + 4x + x^2 + 5x + 4$$
$$= x^3 + 6x^2 + 9x + 4$$

(d) $f(x)/g(x) = \dfrac{x^2 + 5x + 4}{x + 1} = \dfrac{(x + 4)(x + 1)}{x + 1} = x + 4, x \neq -1$

59. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \ x \geq 0$

60. $f(x) = \sin x, g(x) = \pi x$

(a) $f(g(2)) = f(2\pi) = \sin(2\pi) = 0$

(b) $f\left(g\left(\dfrac{1}{2}\right)\right) = f\left(\dfrac{\pi}{2}\right) = \sin\left(\dfrac{\pi}{2}\right) = 1$

(c) $g(f(0)) = g(0) = 0$

(d) $g\left(f\left(\dfrac{\pi}{4}\right)\right) = g\left(\sin\left(\dfrac{\pi}{4}\right)\right)$

$$= g\left(\frac{\sqrt{2}}{2}\right) = \pi\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi\sqrt{2}}{2}$$

(e) $f(g(x)) = f(\pi x) = \sin(\pi x)$

(f) $g(f(x)) = g(\sin x) = \pi \sin x$

61. The expression $(p - 4)^2$ is equal to $p^2 - 8p + 16$.

$$f(p - 4) = (p - 4)^2 + 3(p - 4) - 5$$
$$= p^2 - 8p + 16 + 3p - 12 - 5$$
$$= p^2 - 5p - 1$$

62. The expression $g(-6.25)$ should be $\sqrt{11 - 4(-6.25)} = \sqrt{11 + 25} = \sqrt{36}$.

$$(g \circ f)(-2.5) = g(f(-2.5))$$
$$= g(-6.25)$$
$$= \sqrt{36}$$

63. $f(x) = x^2, g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x))$

$$= f(\sqrt{x}) = (\sqrt{x})^2 = x, \ x \geq 0$$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No, their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

64. $f(x) = x^2 - 1, g(x) = \cos x$

$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = \cos(x^2 - 1)$

Domain: $(-\infty, \infty)$

No, $f \circ g \neq g \circ f$.

65. $f(x) = \dfrac{3}{x}, g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \dfrac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{3}{x}\right) = \left(\frac{3}{x}\right)^2 - 1 = \frac{9}{x^2} - 1 = \frac{9 - x^2}{x^2}$$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

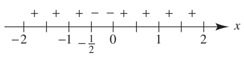
No, $f \circ g \neq g \circ f$.

66. $(f \circ g)(x) = f\left(\sqrt{x+2}\right) = \dfrac{1}{\sqrt{x+2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 2} = \sqrt{\dfrac{1+2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right] \cup (0, \infty)$

No, $f \circ g \neq g \circ f$.

67. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined because the graph of g does not exist at $x = -5$.

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined because the graph of f does not exist at $x = -4$.

68. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$,

which represents the area of the circle at time t.

69. Answers will vary. Sample answer: $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x$, $g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then, $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

70. Answers will vary. Sample answer: $F(x) = -4 \sin(1 - x)$

Let $f(x) = -4x$, $g(x) = \sin x$ and $h(x) = 1 - x$. Then,

$(f \circ g \circ h)(x) = f(g(1 - x)) = f(\sin(1 - x)) = -4 \sin(1 - x) = F(x)$.

71. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

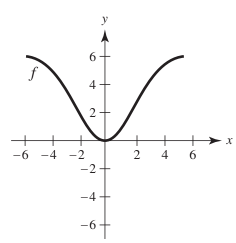
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

72. (a) If f is even, then $(-4, 9)$ is on the graph.

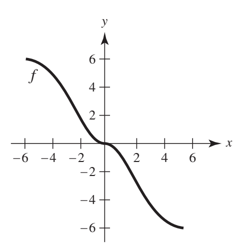
(b) If f is odd, then $(-4, -9)$ is on the graph.

73. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

74. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.



75. $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2\left(4 - (-x)^2\right) = x^2(4 - x^2) = f(x)$

f is even.

$f(x) = x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

Zeros: $x = -2, 0, 2$

76. $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{(-x)} = -\sqrt[3]{x} = -f(x)$

f is odd.

$f(x) = \sqrt[3]{x} = 0 \Rightarrow$

Zero: $x = 0$

77. $f(x) = x\cos x$

$f(-x) = (-x)\cos(-x) = -x\cos x = -f(x)$

f is odd.

$f(x) = x\cos x = 0$

Zeros: $x = 0, \dfrac{\pi}{2} + n\pi$, where n is an integer

78. $f(x) = \sin^2 x$

$f(-x) = \sin^2(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x$

f is even.

$\sin^2 x = 0 \Rightarrow \sin x = 0$

Zeros: $x = n\pi$, where n is an integer

79. Slope $= \dfrac{4 - (-6)}{-2 - 0} = \dfrac{10}{-2} = -5$

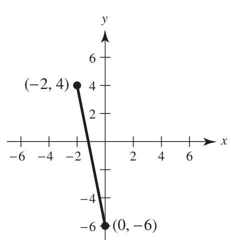
$y - 4 = -5\left(x - (-2)\right)$

$y - 4 = -5x - 10$

$y = -5x - 6$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, -2 \le x \le 0$



80. Slope $= \dfrac{8 - 1}{5 - 3} = \dfrac{7}{2}$

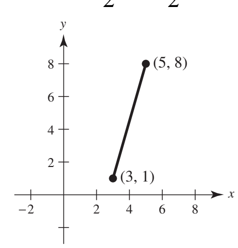
$y - 1 = \dfrac{7}{2}(x - 3)$

$y - 1 = \dfrac{7}{2}x - \dfrac{21}{2}$

$y = \dfrac{7}{2}x - \dfrac{19}{2}$

For the line segment, you must restrict the domain.

$f(x) = \dfrac{7}{2}x - \dfrac{19}{2}, 3 \le x \le 5$

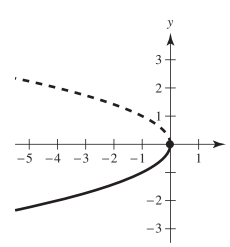


81. $x + y^2 = 0$

$y^2 = -x$

$y = -\sqrt{-x}$

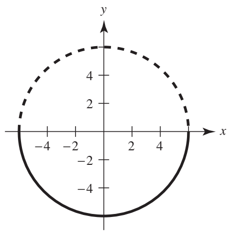
$f(x) = -\sqrt{-x}, x \le 0$



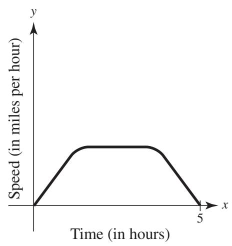
82. $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

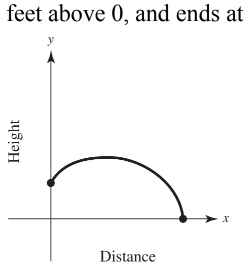
$y = -\sqrt{36 - x^2},\ -6 \le x \le 6$



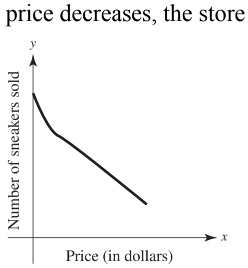
83. Answers will vary. Sample answer: Speed begins and ends at 0. The speed might be constant in the middle:



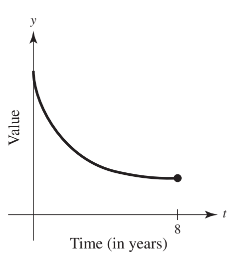
84. Answers will vary. Sample answer: Height begins a few feet above 0, and ends at 0.



85. Answers will vary. Sample answer: In general, as the price decreases, the store will sell more.



86. Answers will vary. Sample answer: As time goes on, the value of the car will decrease



87. $y = \sqrt{c - x^2}$

$y^2 = c - x^2$

$x^2 + y^2 = c$, a circle.

For the domain to be $[-5, 5]$, $c = 25$.

88. For the domain to be the set of all real numbers, you must require that $x^2 + 3cx + 6 \ne 0$. So, the discriminant must be less than zero:

$$(3c)^2 - 4(6) < 0$$
$$9c^2 < 24$$
$$c^2 < \tfrac{8}{3}$$
$$-\sqrt{\tfrac{8}{3}} < c < \sqrt{\tfrac{8}{3}}$$
$$-\tfrac{2}{3}\sqrt{6} < c < \tfrac{2}{3}\sqrt{6}$$

89. (a) $T(4) = 16°\text{C}$, $T(15) \approx 23°\text{C}$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

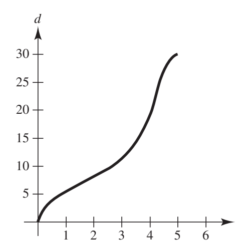
(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

90. (a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

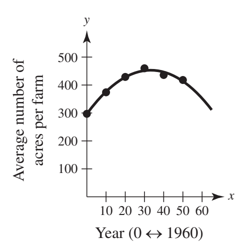
Range: $0 \le d \le 30$

(c)



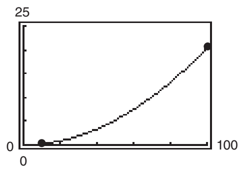
(d) $d(4) \approx 18$. After 4 seconds, the depth is approximately 18 centimeters.

91. (a)



(b) Answers will vary. Sample answer: $A(25) \approx 445$

92. (a)



(b) $H\left(\frac{x}{1.6}\right) = 0.002\left(\frac{x}{1.6}\right)^2 + 0.005\left(\frac{x}{1.6}\right) - 0.029$

$= 0.00078125x^2 + 0.003125x - 0.029$

93. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2$.

If $0 \le x \le 2$, then $f(x) = x - (x - 2) = 2$.

If $x > 2$, then $f(x) = x + (x - 2) = 2x - 2$.

So,

$$f(x) = \begin{cases} -2x + 2, & x < 0 \\ 2, & 0 \le x \le 2. \\ 2x - 2, & x > 2 \end{cases}$$

94. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, you have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Because the graph has no breaks, the graph must cross the x-axis at least one time.

95. $f(-x) = a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x)$

$= -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right]$

$= -f(x)$

So, $f(x)$ is odd.

96. $f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$

$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$= f(x)$

So, $f(x)$ is even.

97. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.

So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$F(-x) = f(-x)g(-x) = \left[-f(x)\right]\left[-g(x)\right] = f(x)g(x) = F(x)$.

So, $F(x)$ is even.

98. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then

$F(-x) = f(-x)g(-x) = f(x)\left[-g(x)\right] = -f(x)g(x) = -F(x)$.

So, $F(x)$ is odd.

99. By equating slopes, $\frac{y-2}{0-3} = \frac{0-2}{x-3}$

$y - 2 = \frac{6}{x-3}$

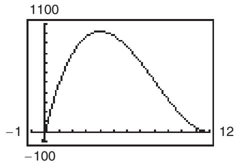
$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3}$,

$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}$.

100. (a) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(b)



Maximum volume occurs at $x = 4$. So, the dimensions of the box would be $4 \times 16 \times 16$ cm.

x	*length and width*	*volume*
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The dimensions of the box that yield a maximum volume appear to be $4 \times 16 \times 16$ cm.

101. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

102. False. If $f(x) = x^2$, then $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So, $3f(x) \neq f(3x)$.

103. False. The constant function $f(x) = 0$ has symmetry with respect to the x-axis.

104. True. If the domain is $\{a\}$, then the range is $\{f(a)\}$.

Section P.4 Inverse Functions

1. (a)

$$f(x) = 5x + 1$$

$$g(x) = \frac{x-1}{5}$$

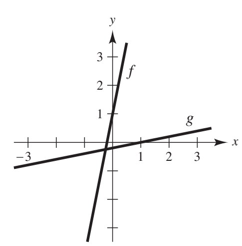
$$f(g(x)) = f\left(\frac{x-1}{5}\right) = 5\left(\frac{x-1}{5}\right) + 1 = x$$

$$g(f(x)) = g(5x+1) = \frac{(5x+1)-1}{5} = x$$

(b)

x	-3	-2	-1	0	1	2	3
$f(x)$	-14	-9	-4	1	6	11	16
$g(x)$	$-\frac{4}{5}$	$-\frac{3}{5}$	$-\frac{2}{5}$	$-\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{2}{5}$
$f(g(x))$	-3	-2	-1	0	1	2	3
$g(f(x))$	-3	-2	-1	0	1	2	3

(c)



2. (a) $f(x) = 3 - 4x$

$$g(x) = \frac{3 - x}{4}$$

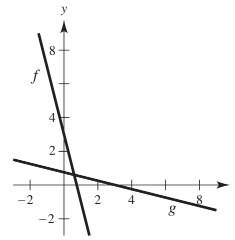
$$f(g(x)) = f\left(\frac{3 - x}{4}\right) = 3 - 4\left(\frac{3 - x}{4}\right) = x$$

$$g(f(x)) = g(3 - 4x) = \frac{3 - (3 - 4x)}{4} = x$$

(b)

x	-2	-1	0	1	2	3	4
$f(x)$	11	7	3	-1	-5	-9	-13
$g(x)$	$\frac{5}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	$-\frac{1}{4}$
$f(g(x))$	-2	-1	0	1	2	3	4
$g(f(x))$	-2	-1	0	1	2	3	4

(c)



3. (a) $f(x) = x^3$

$$g(x) = \sqrt[3]{x}$$

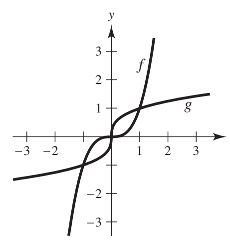
$$f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$$

$$g(f(x)) = g\left(x^3\right) = \sqrt[3]{x^3} = x$$

(b)

x	-3	-2	-1	0	1	2	3
$f(x)$	-27	-8	-1	0	1	8	27
$g(x)$	-1.44	-1.26	-1	0	1	1.26	1.44
$f(g(x))$	-3	-2	-1	0	1	2	3
$g(f(x))$	-3	-2	-1	0	1	2	3

(c)



4. (a) $f(x) = 1 - x^3$

$$g(x) = \sqrt[3]{1 - x}$$

$$f(g(x)) = f\left(\sqrt[3]{1 - x}\right) = 1 - \left(\sqrt[3]{1 - x}\right)^3$$

$$= 1 - (1 - x) = x$$

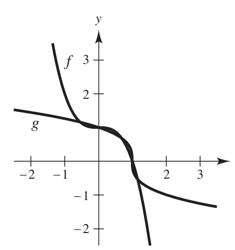
$$g(f(x)) = g\left(1 - x^3\right)$$

$$= \sqrt[3]{1 - \left(1 - x^3\right)} = \sqrt[3]{x^3} = x$$

(b)

x	-2	-1	0	1	2	3
$f(x)$	9	2	1	0	-7	-26
$g(x)$	1.44	1.26	1	0	-1	-1.26
$f(g(x))$	-2	-1	0	1	2	3
$g(f(x))$	-2	-1	0	1	2	3

(c)

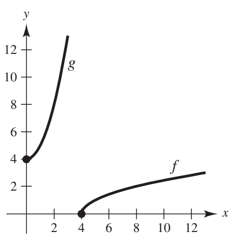


5. (a)
$$f(x) = \sqrt{x-4}$$
$$g(x) = x^2 + 4, \quad x \ge 0$$
$$f(g(x)) = f(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$$
$$g(f(x)) = g(\sqrt{x-4}) = (\sqrt{x-4})^2 + 4 = x - 4 + 4 = x$$

(b)

x	0	1	2	3	4	5	6
$f(x)$	undef.	undef.	undef.	undef.	0	1	1.41
$g(x)$	4	5	8	13	20	29	40
$f(g(x))$	0	1	2	3	4	5	6
$g(f(x))$	undef.	undef.	undef.	undef.	4	5	6

(c)

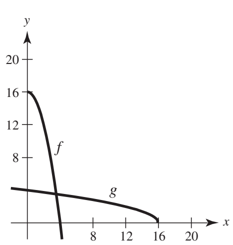


6. (a)
$$f(x) = 16 - x^2, \quad x \ge 0$$
$$g(x) = \sqrt{16-x}$$
$$f(g(x)) = f(\sqrt{16-x}) = 16 - (\sqrt{16-x})^2 = 16 - (16-x) = x$$
$$g(f(x)) = g(16-x^2) = \sqrt{16-(16-x^2)} = \sqrt{x^2} = x$$

(b)

x	0	1	2	3	4	5	6
$f(x)$	16	15	12	7	0	−9	−20
$g(x)$	4	3.87	3.74	3.61	3.46	3.32	3.16
$f(g(x))$	0	1	2	3	4	5	6
$g(f(x))$	0	1	2	3	4	5	6

(c)

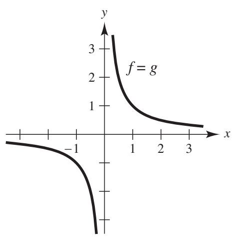


7. (a)
$$f(x) = \frac{1}{x}$$
$$g(x) = \frac{1}{x}$$
$$f(g(x)) = \frac{1}{1/x} = x$$
$$g(f(x)) = \frac{1}{1/x} = x$$

(b)

x	-3	-2	-1	0	1	2	3
$f(x)$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	undef.	1	$\frac{1}{2}$	$\frac{1}{3}$
$g(x)$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	undef.	1	$\frac{1}{2}$	$\frac{1}{3}$
$f(g(x))$	-3	-2	-1	undef.	1	2	3
$g(f(x))$	-3	-2	-1	undef.	1	2	3

(c)

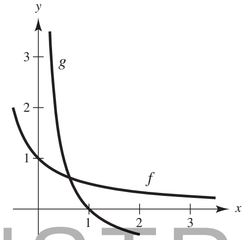


8. (a)
$$f(x) = \frac{1}{1+x}, \qquad x \geq 0$$
$$g(x) = \frac{1-x}{x}, \qquad 0 < x \leq 1$$
$$f(g(x)) = f\left(\frac{1-x}{x}\right) = \frac{1}{1 + \dfrac{1-x}{x}} = \frac{1}{\dfrac{1}{x}} = x$$
$$g(f(x)) = g\left(\frac{1}{1+x}\right) = \frac{1 - \dfrac{1}{1+x}}{\dfrac{1}{1+x}} = \frac{x}{1+x} \cdot \frac{1+x}{1} = x$$

(b)

x	-1	0	1	2	3	4
$f(x)$	undef.	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$g(x)$	-2	undef.	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{3}{4}$
$f(g(x))$	-1	undef.	1	2	3	4
$g(f(x))$	undef.	0	1	2	3	4

(c)



9. Matches (c)

10. Matches (b)

11. Matches (a)

12. Matches (d)

13. $f(x) = \frac{3}{4}x + 6$

One-to-one; has an inverse

14. $f(x) = 5x - 3$

One-to-one; has an inverse

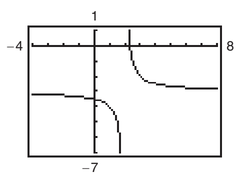
15. $f(\theta) = \sin\theta$

Not one-to-one; does not have an inverse

16. $f(x) = \dfrac{x^2}{x^2 + 4}$

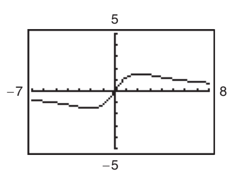
Not one-to-one; does not have an inverse

17. $h(s) = \dfrac{1}{s - 2} - 3$



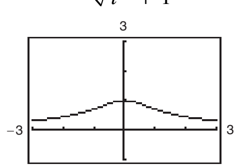
One-to-one; has an inverse

18. $f(x) = \dfrac{6x}{x^2 + 4}$



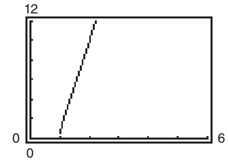
Not one-to-one; does not have an inverse

19. $g(t) = \dfrac{1}{\sqrt{t^2 + 1}}$



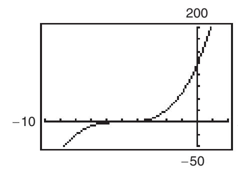
Not one-to-one; does not have an inverse

20. $f(x) = 5x\sqrt{x - 1}$



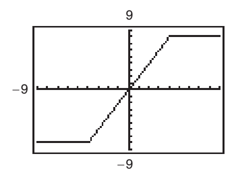
One-to-one; has an inverse

21. $g(x) = (x + 5)^3$



One-to-one; has an inverse

22. $h(x) = |x + 4| - |x - 4|$



Not one-to-one; does not have an inverse

23. $f(x) = \dfrac{x^4}{4} - 2x^2$

The function is not one-to-one, so f does not have an inverse.

24. $f(x) = \sin\dfrac{3x}{2}$

The function is not one-to-one, so f does not have an inverse.

25. $f(x) = 2 - x - x^3$

The function is one-to-one, so f has an inverse.

26. $f(x) = \sqrt[3]{x + 1}$

The function is one-to-one, so f has an inverse.

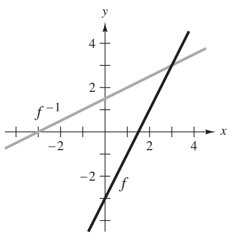
27. (a) $f(x) = 2x - 3 = y$

$$x = \frac{y + 3}{2}$$

$$y = \frac{x + 3}{2}$$

$$f^{-1}(x) = \frac{x + 3}{2}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $(-\infty, \infty)$

Range of f: $(-\infty, \infty)$

Domain of f^{-1}: $(-\infty, \infty)$

Range of f^{-1}: $(-\infty, \infty)$

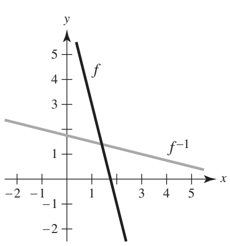
28. (a) $f(x) = 7 - 4x = y$

$$x = \frac{7 - y}{4}$$

$$y = \frac{7 - x}{4}$$

$$f^{-1}(x) = \frac{7 - x}{4}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: $(-\infty, \infty)$

Range of f: $(-\infty, \infty)$

Domain of f^{-1}: $(-\infty, \infty)$

Range of f^{-1}: $(-\infty, \infty)$

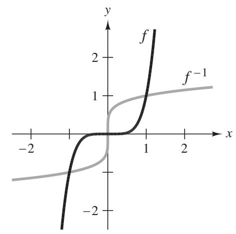
29. (a) $f(x) = x^5 = y$

$$x = \sqrt[5]{y}$$

$$y = \sqrt[5]{x}$$

$$f^{-1}(x) = \sqrt[5]{x} = x^{1/5}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $(-\infty, \infty)$

Range of f: $(-\infty, \infty)$

Domain of f^{-1}: $(-\infty, \infty)$

Range of f^{-1}: $(-\infty, \infty)$

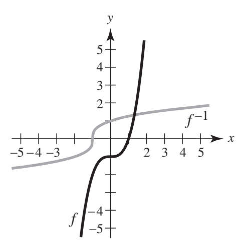
30. (a) $f(x) = x^3 - 1 = y$

$$x = \sqrt[3]{y + 1}$$

$$y = \sqrt[3]{x + 1}$$

$$f^{-1}(x) = \sqrt[3]{x + 1} = (x + 1)^{1/3}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $(-\infty, \infty)$

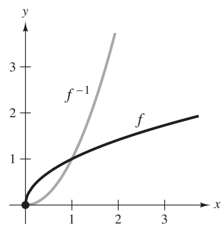
Range of f: $(-\infty, \infty)$

Domain of f^{-1}: $(-\infty, \infty)$

Range of f^{-1}: $(-\infty, \infty)$

31. (a) $f(x) = \sqrt{x} = y$

$$x = y^2$$
$$y = x^2$$
$$f^{-1}(x) = x^2, \quad x \geq 0$$

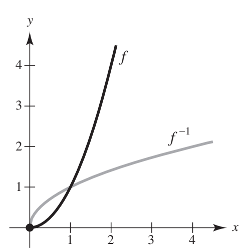
(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $[0, \infty)$
Range of f: $[0, \infty)$
Domain of f^{-1}: $[0, \infty)$
Range of f^{-1}: $[0, \infty)$

32. (a) $f(x) = x^2 = y, \quad x \geq 0$

$$x = \sqrt{y}$$
$$y = \sqrt{x}$$
$$f^{-1}(x) = \sqrt{x}$$

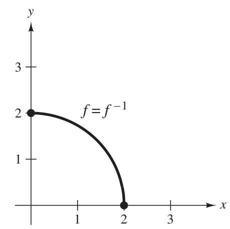
(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $[0, \infty)$
Range of f: $[0, \infty)$
Domain of f^{-1}: $[0, \infty)$
Range of f^{-1}: $[0, \infty)$

33. (a) $f(x) = \sqrt{4 - x^2} = y, \quad 0 \leq x \leq 2$

$$4 - x^2 = y^2$$
$$x^2 = 4 - y^2$$
$$x = \sqrt{4 - y^2}$$
$$y = \sqrt{4 - x^2}$$
$$f^{-1}(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$$

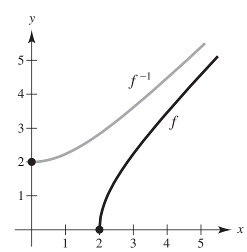
(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$. In fact, the graphs are identical.

(d) Domain of f: $[0, 2]$
Range of f: $[0, 2]$
Domain of f^{-1}: $[0, 2]$
Range of f^{-1}: $[0, 2]$

34. (a) $f(x) = \sqrt{x^2 - 4} = y, \quad x \geq 2$

$$x^2 = y^2 + 4$$
$$x = \sqrt{y^2 + 4}$$
$$y = \sqrt{x^2 + 4}$$
$$f^{-1}(x) = \sqrt{x^2 + 4}, \quad x \geq 0$$

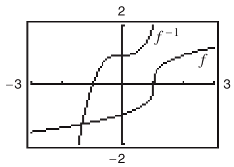
(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $[2, \infty)$
Range of f: $[0, \infty)$
Domain of f^{-1}: $[0, \infty)$
Range of f^{-1}: $[2, \infty)$

35. (a) $f(x) = \sqrt[3]{x-1} = y$

$$x - 1 = y^3$$
$$x = y^3 + 1$$
$$y = x^3 + 1$$
$$f^{-1}(x) = x^3 + 1$$

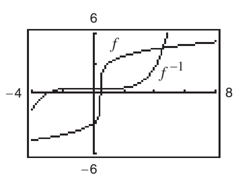
(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $(-\infty, \infty)$
Range of f: $(-\infty, \infty)$
Domain of f^{-1}: $(-\infty, \infty)$
Range of f^{-1}: $(-\infty, \infty)$

36. (a) $f(x) = 3\sqrt[5]{2x-1} = y$

$$2x - 1 = \left(\frac{y}{3}\right)^5 = \frac{y^5}{243}$$
$$2x = \frac{y^5 + 243}{243}$$
$$x = \frac{y^5 + 243}{486}$$
$$y = \frac{x^5 + 243}{486}$$
$$f^{-1}(x) = \frac{x^5 + 243}{486}$$

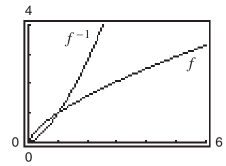
(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $(-\infty, \infty)$
Range of f: $(-\infty, \infty)$
Domain of f^{-1}: $(-\infty, \infty)$
Range of f^{-1}: $(-\infty, \infty)$

37. (a) $f(x) = x^{2/3} = y, \quad x \ge 0$

$$x = y^{3/2}$$
$$y = x^{3/2}$$
$$f^{-1}(x) = x^{3/2}, \; x \ge 0$$

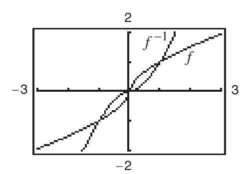
(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $[0, \infty)$
Range of f: $[0, \infty)$
Domain of f^{-1}: $[0, \infty)$
Range of f^{-1}: $[0, \infty)$

38. (a) $f(x) = x^{3/5} = y$

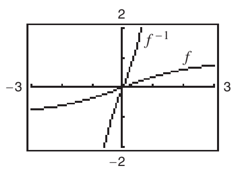
$$x = y^{5/3}$$
$$y = x^{5/3}$$
$$f^{-1}(x) = x^{5/3}$$

(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $(-\infty, \infty)$
Range of f: $(-\infty, \infty)$
Domain of f^{-1}: $(-\infty, \infty)$
Range of f^{-1}: $(-\infty, \infty)$

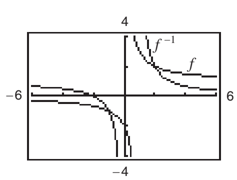
39. (a)
$$f(x) = \frac{x}{\sqrt{x^2 + 7}} = y$$
$$x = y\sqrt{x^2 + 7}$$
$$x^2 = y^2(x^2 + 7) = y^2x^2 + 7y^2$$
$$x^2(1 - y^2) = 7y^2$$
$$x = \frac{\sqrt{7}y}{\sqrt{1 - y^2}}$$
$$y = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}$$
$$f^{-1}(x) = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $(-\infty, \infty)$
Range of f: $(-1, 1)$
Domain of f^{-1}: $(-1, 1)$
Range of f^{-1}: $(-\infty, \infty)$

40. (a)
$$f(x) = \frac{x + 2}{x} = y, \quad x \neq 0$$
$$x + 2 = yx$$
$$x(1 - y) = -2$$
$$x = \frac{2}{y - 1}$$
$$y = \frac{2}{x - 1}$$
$$f^{-1}(x) = \frac{2}{x - 1}, \quad x \neq 1$$

(b) 

(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $(-\infty, 0) \cup (0, \infty)$
Range of f: $(-\infty, 1) \cup (1, \infty)$
Domain of f^{-1}: $(-\infty, 1) \cup (1, \infty)$
Range of f^{-1}: $(-\infty, 0) \cup (0, \infty)$

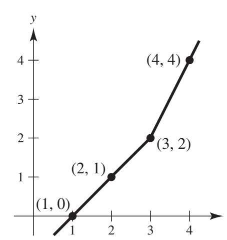
41. In Step 4, dividing by negative one also negates y^3.
$$f(x) = \sqrt[3]{5 - x}$$
$$\sqrt[3]{5 - x} = y$$
$$5 - x = y^3$$
$$-x = y^3 - 5$$
$$x = -y^3 + 5$$
$$y = -x^3 + 5$$
$$f^{-1}(x) = -x^3 + 5$$

42. In Step 3, add x to each side of the equation.
$$g(x) = \frac{4 - x}{x}$$
$$\frac{4 - x}{x} = y, \ x \neq 0$$
$$4 - x = yx$$
$$4 = yx + x$$
$$4 = x(y + 1)$$
$$\frac{4}{y + 1} = x$$
$$\frac{4}{x + 1} = y$$
$$\frac{4}{x + 1} = g^{-1}(x), \ x \neq -1$$

43.

x	0	1	2	3
$f(x)$	1	2	3	4

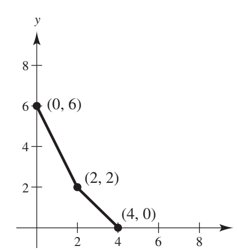
x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



44.

x	0	2	6
$f(x)$	4	2	0

x	0	2	4
$f^{-1}(x)$	6	2	0



45. (a) Let x be the number of pounds of the commodity that costs \$1.25 per pound. Because there are 50 pounds total, the amount of the second commodity is $50 - x$. The total cost is

$$y = 1.25x + 1.60(50 - x)$$
$$= -0.35x + 80, \quad 0 \le x \le 50.$$

(b) Find the inverse of the original function.

$$y = -0.35x + 80$$
$$0.35x = 80 - y$$
$$x = \tfrac{100}{35}(80 - y)$$

Inverse: $y = \tfrac{100}{35}(80 - x) = \tfrac{20}{7}(80 - x)$

x represents cost and y represents pounds.

(c) The domain of the inverse is $62.5 \le x \le 80$.

The total cost will be between \$62.50 and \$80.00.

(d) If $x = 73$ in the inverse function, then

$y = \tfrac{100}{35}(80 - 73) = \tfrac{100}{5} = 20$ pounds.

46. $C = \tfrac{5}{9}(F - 32), \quad F \ge -459.6$

(a) $\tfrac{9}{5}C = F - 32$

$F = 32 + \tfrac{9}{5}C$

(b) The inverse function gives the Fahrenheit temperature F corresponding to the Celsius temperature C.

(c) For $F \ge -459.6, C = \tfrac{5}{9}(F - 32) \ge -273.\overline{1}$.

So, the domain is $C \ge -273.\overline{1} = -273\tfrac{1}{9}$.

(d) If $C = 22°$, then $F = 32 + \tfrac{9}{5}(22) = 71.6°\text{F}$.

47. $f(x) = \sqrt{x - 2},\ x \ge 2$

f is one-to-one and has an inverse.

$$y = \sqrt{x - 2},\ x \ge 2,\ y \ge 0$$
$$y^2 = x - 2$$
$$x = y^2 + 2$$
$$f^{-1}(x) = x^2 + 2,\ x \ge 0$$

48. $f(x) = \sqrt{9 - x^2}$ is not one-to-one, so it does not have an inverse.

49. $f(x) = -3$

Not one-to-one so it does not have an inverse.

50. $f(x) = |x - 2|,\ x \le 2$

$$= -(x - 2)$$
$$= 2 - x$$

f is one-to-one and has an inverse.

$$2 - x = y$$
$$2 - y = x$$
$$f^{-1}(x) = 2 - x, \quad x \ge 0$$

51. $f(x) = ax + b$

f is one-to-one and has an inverse.

$$ax + b = y$$
$$x = \frac{y - b}{a}$$
$$y = \frac{x - b}{a}$$
$$f^{-1}(x) = \frac{x - b}{a}, \quad a \ne 0$$

52. $f(x) = (x + a)^3 + b$

f is one-to-one and has an inverse.

$$y = (x + a)^3 + b$$
$$y - b = (x + a)^3$$
$$x + a = \sqrt[3]{y - b}$$
$$x = \sqrt[3]{y - b} - a$$
$$f^{-1}(x) = \sqrt[3]{x - b} - a$$

53. $f(x) = (x - 4)^2$ on $[4, \infty)$

f passes the Horizontal Line Test on $[4, \infty)$, so it is one-to-one on $[4, \infty)$.

54. $f(x) = |x + 2|$ on $[-2, \infty)$

f passes the Horizontal Line Test on $[-2, \infty)$, so it is one-to-one on $[-2, \infty)$.

55. $f(x) = \dfrac{4}{x^2}$ on $(0, \infty)$

f passes the Horizontal Line Test on $(0, \infty)$, so it is one-to-one on $(0, \infty)$.

56. $f(x) = \cot x$ on $(0, \pi)$

f passes the Horizontal Line Test on $(0, \pi)$, so it is one-to-one on $(0, \pi)$.

57. $f(x) = \cos x$ on $[0, \pi]$

f passes the Horizontal Line Test on $[0, \pi]$, so it is one-to-one on $[0, \pi]$.

58. $f(x) = \sec x$ on $\left[0, \dfrac{\pi}{2}\right)$

f passes the Horizontal Line Test on $[0, \pi/2)$, so it is one-to-one on $[0, \pi/2]$.

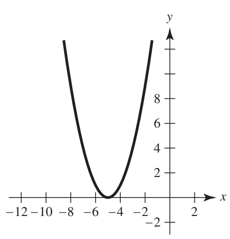
59. Answers will vary. Sample answer: $f(x) = (x - 3)^2$ is one-to-one for $x \ge 3$.

$$\begin{aligned}(x - 3)^2 &= y\\ x - 3 &= \sqrt{y}\\ x &= \sqrt{y} + 3\\ y &= \sqrt{x} + 3\\ f^{-1}(x) &= \sqrt{x} + 3, \quad x \ge 0\end{aligned}$$

60. Answers will vary. Sample answer: $f(x) = |x - 3|$ is one-to-one for $x \ge 3$.

$$\begin{aligned}x - 3 &= y\\ x &= y + 3\\ y &= x + 3\\ f^{-1}(x) &= x + 3, \quad x \ge 0\end{aligned}$$

61. (a) $f(x) = (x + 5)^2$



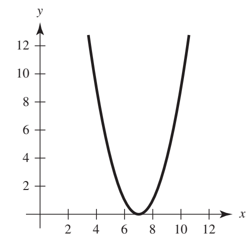
(b) f is one-to-one on $[-5, \infty)$.

Note: f is also one-to-one on $(-\infty, -5]$.

(c) $$\begin{aligned}f(x) = (x + 5)^2 &= y, \qquad x \ge -5\\ x + 5 &= \sqrt{y}\\ x &= \sqrt{y} - 5\\ y &= \sqrt{x} - 5\\ f^{-1}(x) &= \sqrt{x} - 5\end{aligned}$$

(d) Domain of f^{-1}: $[0, \infty)$

62. (a) $f(x) = (7 - x)^2 = (x - 7)^2$



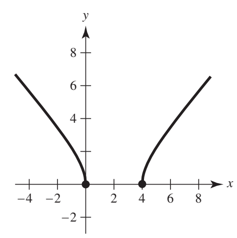
(b) f is one-to-one on $[7, \infty)$.

Note: f is also one-to-one on $(-\infty, 7]$.

(c) $$\begin{aligned}f(x) = (x - 7)^2 &= y, \qquad x \ge 7\\ x - 7 &= \sqrt{y}\\ x &= 7 + \sqrt{y}\\ y &= 7 + \sqrt{x}\\ f^{-1}(x) &= 7 + \sqrt{x}\end{aligned}$$

(d) Domain of f^{-1}: $[0, \infty)$

63. (a) $f(x) = \sqrt{x^2 - 4x}$



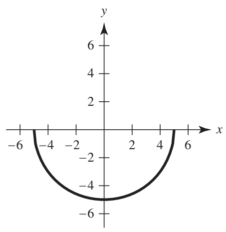
(b) Answers will vary. Sample answer: f is one-to-one on $[4, \infty)$.

Note: f is also one-to-one on $(-\infty, 0]$.

(c) $$\begin{aligned}f(x) = \sqrt{x^2 - 4x} &= y, \ x \ge 4\\ x^2 - 4x &= y^2\\ x^2 - 4x + 4 &= y^2 + 4\\ (x - 2)^2 &= y^2 + 4\\ x - 2 &= \sqrt{y^2 + 4}\\ x &= 2 + \sqrt{y^2 + 4}\\ y &= 2 + \sqrt{x^2 + 4}\\ f^{-1}(x) &= 2 + \sqrt{x^2 + 4}\end{aligned}$$

(d) Domain of f^{-1}: $[0, \infty)$

64. (a) $f(x) = -\sqrt{25 - x^2}$



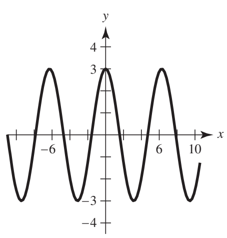
(b) Answers will vary. Sample answer: f is one-to-one on $[0, 5]$.

Note: f is also one-to-one on $[-5, 0]$.

(c)
$$\begin{aligned} f(x) = -\sqrt{25 - x^2} &= y, \qquad 0 \le x \le 5, -5 \le y \le 0 \\ 25 - x^2 &= y^2 \\ x^2 &= 25 - y^2 \\ x &= \sqrt{25 - y^2} \\ y &= \sqrt{25 - x^2} \\ f^{-1}(x) &= \sqrt{25 - x^2} \end{aligned}$$

(d) Domain of f^{-1}: $[-5, 0]$

65. (a) $f(x) = 3\cos x$

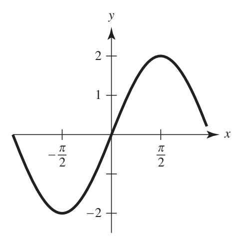


(b) Answers will vary. Sample answer: f is one-to-one on $[0, \pi]$.

(c)
$$\begin{aligned} f(x) = 3\cos x &= y \\ \cos x &= \frac{y}{3} \\ x &= \arccos\left(\frac{y}{3}\right) \\ y &= \arccos\left(\frac{x}{3}\right) \\ f^{-1}(x) &= \arccos\left(\frac{x}{3}\right) \end{aligned}$$

(d) Domain of f^{-1}: $[-3, 3]$

66. (a) $f(x) = 2\sin x$



(b) Answers will vary. Sample answer: f is one-to-one on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(c)
$$\begin{aligned} f(x) = 2\sin x &= y \\ \sin x &= \frac{y}{2} \\ x &= \arcsin\left(\frac{y}{2}\right) \\ y &= \arcsin\left(\frac{x}{2}\right) \\ f^{-1}(x) &= \arcsin\left(\frac{x}{2}\right) \end{aligned}$$

(d) Domain of f^{-1}: $[-2, 2]$

67. $f(x) = x^3 + 2x - 1$

$f(1) = 2 = a \Rightarrow f^{-1}(2) = 1$

68. $f(x) = 2x^5 + x^3 + 1$

$f(-1) = -2 = a \Rightarrow f^{-1}(-2) = -1$

69. $f(x) = \sin x$

$f\left(\frac{\pi}{6}\right) = \frac{1}{2} = a \Rightarrow f^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

70. $f(x) = \cos 2x$

$f(0) = 1 = a \Rightarrow f^{-1}(1) = 0$

71. $f(x) = x^3 - \frac{4}{x}$

$f(2) = 6 = a \Rightarrow f^{-1}(6) = 2$

72. $f(x) = \sqrt{x - 4}$

$f(8) = 2 = a \Rightarrow f^{-1}(2) = 8$

In Exercises 73–76, use the following.

$\boldsymbol{f(x) = \frac{1}{8}x - 3}$ **and** $\boldsymbol{g(x) = x^3}$

$\boldsymbol{f^{-1}(x) = 8(x + 3)}$ **and** $\boldsymbol{g^{-1}(x) = \sqrt[3]{x}}$

73. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1)) = f^{-1}(1) = 32$

74. $(g^{-1} \circ f^{-1})(-3) = g^{-1}(f^{-1}(-3)) = g^{-1}(0) = 0$

75. $(f^{-1} \circ f^{-1})(6) = f^{-1}(f^{-1}(6)) = f^{-1}(72) = 600$

76. $(g^{-1} \circ g^{-1})(-4) = g^{-1}(g^{-1}(-4)) = g^{-1}(\sqrt[3]{-4})$

$= \sqrt[3]{\sqrt[3]{-4}} = -\sqrt[9]{4}$

In Exercises 77–80, use the following.

$\boldsymbol{f(x) = x + 4}$ **and** $\boldsymbol{g(x) = 2x - 5}$

$\boldsymbol{f^{-1}(x) = x - 4}$ **and** $\boldsymbol{g^{-1}(x) = \frac{x + 5}{2}}$

77. $(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x))$

$= g^{-1}(x - 4)$

$= \frac{(x - 4) + 5}{2}$

$= \frac{x + 1}{2}$

78. $(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x))$

$= f^{-1}\left(\frac{x + 5}{2}\right)$

$= \frac{x + 5}{2} - 4$

$= \frac{x - 3}{2}$

79. $(f \circ g)(x) = f(g(x))$

$= f(2x - 5)$

$= (2x - 5) + 4$

$= 2x - 1$

So, $(f \circ g)^{-1}(x) = \frac{x + 1}{2}$.

Note: $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

80. $(g \circ f)(x) = g(f(x))$

$= g(x + 4)$

$= 2(x + 4) - 5$

$= 2x + 3$

So, $(g \circ f)^{-1}(x) = \frac{x - 3}{2}$.

Note: $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

81. (a) f is one-to-one because it passes the Horizontal Line Test.

(b) The domain of f^{-1} is the range of f: $[-2, 2]$.

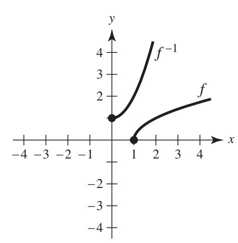
(c) $f^{-1}(2) = -4$ because $f(-4) = 2$.

82. (a) f is one-to-one because it passes the Horizontal Line Test.

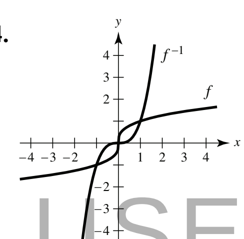
(b) The domain of f^{-1} is the range of f: $[-3, 3]$.

(c) $f^{-1}(2) \approx 1.73$ because $f(1.73) \approx 2$.

83.



84.

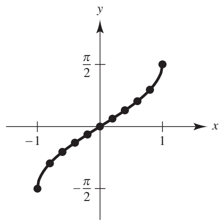


85. $y = \arcsin x$

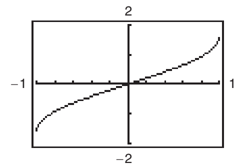
(a)

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
y	-1.571	-0.927	-0.644	-0.412	-0.201	0	0.201	0.412	0.644	0.927	1.571

(b)



(c)



(d) Symmetric about origin: $\arcsin(-x) = -\arcsin x$

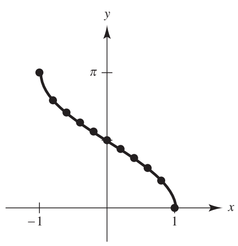
Intercept: $(0, 0)$

86. $y = \arccos x$

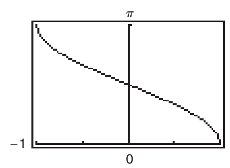
(a)

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
y	3.142	2.498	2.214	1.982	1.772	1.571	1.369	1.159	0.927	0.644	0

(b)



(c)



(d) Intercepts: $\left(0, \frac{\pi}{2}\right)$ and $(1, 0)$

87. $y = \arccos x$

$\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$ because $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

$\left(\frac{1}{2}, \frac{\pi}{3}\right)$ because $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$ because $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

88. No, g is not the inverse of f. $f(x) = \sin x$ is not one-to-one. The graph of g is not the graph of a function.

89. $\arcsin \frac{1}{2} = \frac{\pi}{6}$

90. $\arcsin 0 = 0$

91. $\arccos \frac{1}{2} = \frac{\pi}{3}$

92. $\arccos 1 = 0$

93. $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$

94. $\operatorname{arccot}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$

95. $\operatorname{arccsc}\left(-\sqrt{2}\right) = -\frac{\pi}{4}$

96. $\operatorname{arcsec}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$

97. $\arccos(0.8) \approx 2.50$

98. $\arcsin(-0.39) \approx -0.40$

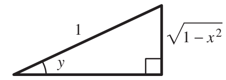
99. $\operatorname{arcsec}(1.269) = \arccos\left(\frac{1}{1.269}\right) \approx 0.66$

100. $\arctan(-5) \approx -1.37$

101. $\cos\left[\arccos(-0.1)\right] = -0.1$

102. $\arcsin(\sin 3\pi) = \arcsin(0) = 0$

In Exercises 103–108, use the triangle.



103. $y = \arccos x$

$\cos y = x$

104. $\sin y = \sqrt{1 - x^2}$

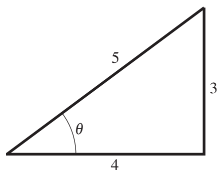
105. $\tan y = \dfrac{\sqrt{1 - x^2}}{x}$

106. $\cot y = \dfrac{x}{\sqrt{1 - x^2}}$

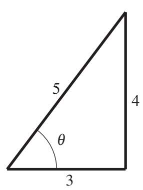
107. $\sec y = \dfrac{1}{x}$

108. $\csc y = \dfrac{1}{\sqrt{1 - x^2}}$

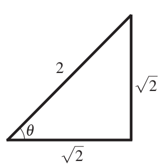
109. (a) $\sin\left(\arctan\dfrac{3}{4}\right) = \dfrac{3}{5}$



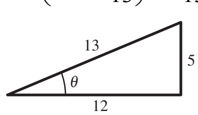
(b) $\sec\left(\arcsin\dfrac{4}{5}\right) = \dfrac{5}{3}$



110. (a) $\tan\left(\arccos\dfrac{\sqrt{2}}{2}\right) = \tan\left(\dfrac{\pi}{4}\right) = 1$



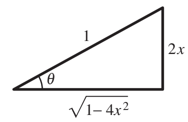
(b) $\cos\left(\arcsin\dfrac{5}{13}\right) = \dfrac{12}{13}$



111. $y = \cos(\arcsin 2x)$

$\theta = \arcsin 2x$

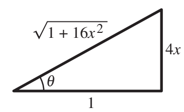
$y = \cos\theta = \sqrt{1 - 4x^2}$



112. $\sec(\arctan 4x)$

$\theta = \arctan 4x$

$y = \sec\theta = \sqrt{16x^2 + 1}$

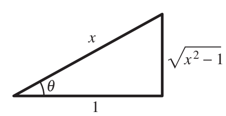


113. $y = \sin(\operatorname{arcsec} x)$

$\theta = \operatorname{arcsec} x,\ 0 \le \theta \le \pi,\ \theta \ne \dfrac{\pi}{2}$

$y = \sin\theta = \dfrac{\sqrt{x^2 - 1}}{|x|}$

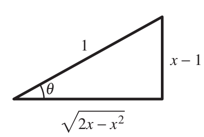
The absolute value bars on x are necessary because of the restriction $0 \le \theta \le \pi$, $\theta \ne \pi/2$, and $\sin\theta$ for this domain must always be nonnegative.



114. $y = \sec[\arcsin(x - 1)]$

$\theta = \arcsin(x - 1)$

$y = \sec\theta = \dfrac{1}{\sqrt{2x - x^2}}$



115. $\arcsin(3x - \pi) = \frac{1}{2}$

$3x - \pi = \sin\frac{1}{2}$

$x = \frac{1}{3}\left[\sin\frac{1}{2} + \pi\right] \approx 1.207$

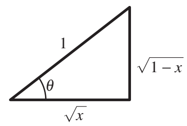
116. $\arctan(2x - 5) = -1$

$2x - 5 = \tan(-1)$

$x = \frac{1}{2}(5 + \tan(-1)) \approx 1.721$

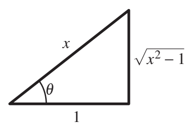
117. $\arcsin\sqrt{2x} = \arccos\sqrt{x}$

$$\sqrt{2x} = \sin\left(\arccos\sqrt{x}\right)$$
$$\sqrt{2x} = \sqrt{1-x},\ 0 \le x \le 1$$
$$2x = 1 - x$$
$$3x = 1$$
$$x = \tfrac{1}{3}$$



118. $\arccos x = \operatorname{arcsec} x$

$$x = \cos(\operatorname{arcsec} x)$$
$$x = \frac{1}{x}$$
$$x^2 = 1$$
$$x = \pm 1$$



119. The trigonometric functions are not one-to-one. So, their domains must be restricted to define the inverse trigonometric functions.

120. You could graph $f(x) = \operatorname{arccot} x$ as follows.

$$f(x) = \begin{cases} \arctan(1/x) + \pi, & -\infty < x < 0 \\ \pi/2, & x = 0 \\ \arctan(1/x), & 0 < x < \infty \end{cases}$$

121. (a) $\operatorname{arccsc} x = \arcsin\dfrac{1}{x},\ |x| \ge 1$

Let $y = \operatorname{arccsc} x$.

Then for $-\dfrac{\pi}{2} \le y < 0$ and $0 < y \le \dfrac{\pi}{2}$,

$\csc y = x \Rightarrow \sin y = \dfrac{1}{x}.$

So, $y = \arcsin\left(\dfrac{1}{x}\right)$. Therefore,

$\operatorname{arccsc} x = \arcsin\left(\dfrac{1}{x}\right).$

(b) $\arctan x + \arctan\dfrac{1}{x} = \dfrac{\pi}{2},\ x > 0$

Let $y = \arctan x + \arctan(1/x)$.

$$\text{Then } \tan y = \frac{\tan(\arctan x) + \tan[\arctan(1/x)]}{1 - \tan(\arctan x)\tan[\arctan(1/x)]}$$
$$= \frac{x + (1/x)}{1 - x(1/x)}$$
$$= \frac{x + (1/x)}{0} \text{ (which is undefined).}$$

So, $y = \pi/2$. Therefore,

$\arctan x + \arctan(1/x) = \pi/2.$

122. (a) $\arcsin(-x) = -\arcsin x,\ |x| \le 1$

Let $y = \arcsin(-x)$.

Then $-x = \sin y \Rightarrow x = -\sin y \Rightarrow x = \sin(-y)$.

So, $-y = \arcsin x \Rightarrow y = -\arcsin x$.

Therefore, $\arcsin(-x) = -\arcsin x$.

(b) $\arccos(-x) = \pi - \arccos x,\ |x| \le 1$

Let $y = \arccos(-x)$. Then

$-x = \cos y \Rightarrow x = -\cos y \Rightarrow x = \cos(\pi - y)$.

So, $\pi - y = \arccos x \Rightarrow y = \pi - \arccos x$.

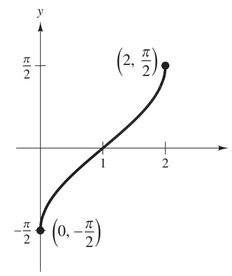
Therefore, $\arccos(-x) = \pi - \arccos x$.

123. $f(x) = \arcsin(x - 1)$

$$x - 1 = \sin y$$
$$x = 1 + \sin y$$

Domain: $[0, 2]$

Range: $[-\pi/2, \pi/2]$



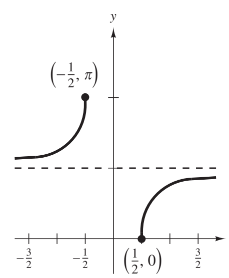
$f(x)$ is the graph of arcsin x shifted right one unit.

124. $f(x) = \operatorname{arcsec} 2x$

$$2x = \sec y$$
$$x = \frac{1}{2}\sec y$$

Domain: $(-\infty, -1/2], [1/2, \infty)$

Range: $[0, \pi/2), (\pi/2, \pi]$

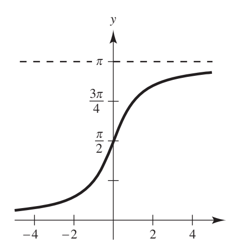


125. $f(x) = \arctan x + \dfrac{\pi}{2}$

$$x = \tan\left(y - \frac{\pi}{2}\right)$$

Domain: $(-\infty, \infty)$

Range: $[0, \pi]$



$f(x)$ is the graph of arctan x shifted $\pi/4$ unit upward.