Chapter 1

Prerequisites for Calculus

Section 1.1 Linear Functions (pp. 3–12)

Quick Review 1.1

1. $y = -2 + 4(3 - 3) = -2 + 4(0) = -2 + 0 = -2$

2. $$\begin{aligned} 3 &= 3 - 2(x+1) \\ 3 &= 3 - 2x - 2 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

3. $m = \dfrac{2-3}{5-4} = \dfrac{-1}{1} = -1$

4. $m = \dfrac{2-(-3)}{3-(-1)} = \dfrac{5}{4}$

5. **(a)** $3(2) - 4\left(\dfrac{1}{4}\right) \overset{?}{=} 5$
$6 - 1 = 5$ Yes

(b) $3(3) - 4(-1) \overset{?}{=} 5$
$13 \neq 5$ No

6. **(a)** $7 \overset{?}{=} -2(-1) + 5$
$7 = 2 + 5$ Yes

(b) $1 = -2(-2) + 5$
$1 \neq 9$ No

7. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0-1)^2 + (1-0)^2} \\ &= \sqrt{2} \end{aligned}$$

8. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-2)^2 + \left(-\frac{1}{3} - 1\right)^2} \\ &= \sqrt{(-1)^2 + \left(-\frac{4}{3}\right)^2} \\ &= \sqrt{1 + \frac{16}{9}} \\ &= \sqrt{\frac{25}{9}} \\ &= \frac{5}{3} \end{aligned}$$

9. $$\begin{aligned} 4x - 3y &= 7 \\ -3y &= -4x + 7 \\ y &= \frac{4}{3}x - \frac{7}{3} \end{aligned}$$

10. $$\begin{aligned} -2x + 5y &= -3 \\ 5y &= 2x - 3 \\ y &= \frac{2}{5}x - \frac{3}{5} \end{aligned}$$

Section 1.1 Exercises

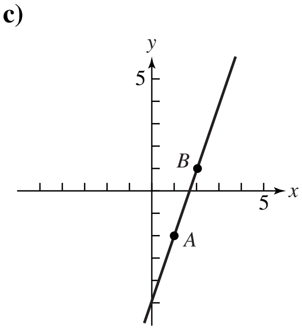
1. $\Delta x = -1 - 1 = -2$
$\Delta y = -1 - 2 = -3$

2. $\Delta x = -1 - (-3) = 2$
$\Delta y = -2 - 2 = -4$

3. $\Delta x = -8 - (-3) = -5$
$\Delta y = 1 - 1 = 0$

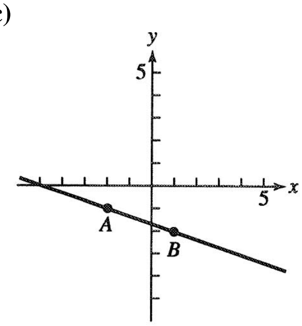
4. $\Delta x = 0 - 0 = 0$
$\Delta y = -2 - 4 = -6$

5. **(a, c)**



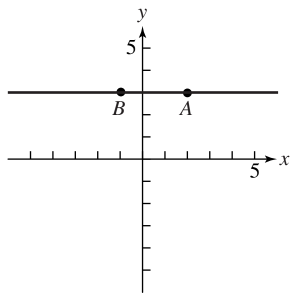
(b) $m = \dfrac{1-(-2)}{2-1} = \dfrac{3}{1} = 3$

6. **(a, c)**



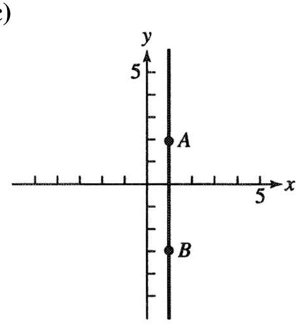
(b) $m = \dfrac{-2-(-1)}{1-(-2)} = \dfrac{-1}{3} = -\dfrac{1}{3}$

7. **(a, c)**



(b) $m = \dfrac{3-3}{-1-2} = \dfrac{0}{-3} = 0$

8. **(a, c)**



(b) $m = \dfrac{-3-2}{1-1} = \dfrac{-5}{0}$ (undefined)
This line has no slope.

9. $P(3, 5)$, $m = 2$, $x = 4.5$
$\Delta x = 4.5 - 3 = 1.5$
Since the ratio of the increments is 2,
$\Delta y = 2 \cdot \Delta x = 3$.
The y-coordinate is $5 + 3 = 8$.

10. $P(-2, 1)$, $m = 3$, $x = 2$
$\Delta x = 2 - (-2) = 4$
Since the ratio of the increments is 3,
$\Delta y = 3 \cdot \Delta x = 12$.
The y-coordinate is $1 + 12 = 13$.

11. $P(3, 2)$, $m = -3$, $x = 5$
$\Delta x = 5 - 3 = 2$
Since the ratio of the increments is -3,
$\Delta y = -3 \cdot \Delta x = -6$.
The y-coordinate is $2 + (-6) = -4$.

12. $P(-1, -2)$, $m = 0.8$, $x = 1$
$\Delta x = 1 - (-1) = 2$
Since the ratio of the increments is 0.8,
$\Delta y = 0.8 \cdot \Delta x = 1.6$.
The y-coordinate is $-2 + 1.6 = -0.4$.

13. Because this is a linear function, the ratio of the increments is constant:
$\dfrac{\Delta d}{\Delta t} = \dfrac{2}{5} = 0.4$ km per min.
From $t = 6$ to 8, $\Delta t = 2$, so $\Delta d = 2(0.4) = 0.8$.
The distance at 8 min is $5 + 0.8 = 5.8$ km.

14. Because this is a linear function, the ratio of the increments is constant:
$\dfrac{\Delta d}{\Delta t} = \dfrac{2}{5} = 0.4$ km per min.
From $t = 6$ to 3, $\Delta t = -3$, so
$\Delta d = -3(0.4) = -1.2$.
The distance at 3 min is $5 + (-1.2) = 3.8$ km.

15. Because this is a linear function, the ratio of the increments is constant:
$\dfrac{\Delta d}{\Delta t} = \dfrac{2}{5} = 0.4$ km per min.
From $t = 6$ to 12, $\Delta t = 6$, so $\Delta d = 6(0.4) = 2.4$.
The distance at 12 min is $5 + 2.4 = 7.4$ km.

16. Because this is a linear function, the ratio of the increments is constant:
$\dfrac{\Delta d}{\Delta t} = \dfrac{2}{5} = 0.4$ km per min.
From $t = 6$ to 20, $\Delta t = 14$, so
$\Delta d = 14(0.4) = 5.6$.
The distance at 20 min is $5 + 5.6 = 10.6$ km.

17. If d is the position at time t, we get the same constant ratio of increments as the position changes from 5 to d and the time changes from 6 to t. We can replace Δd by $d - 5$ and Δt by $t - 6$. Solve for d.

$$\frac{d-5}{t-6} = 0.4$$
$$d - 5 = 0.4(t-6)$$
$$d = 0.4(t-6) + 5$$

18. The monthly fee F is the sum of the flat fee of \$65 and \$20 per half hour (or \$40 per hour t, since t is in hours).
$F = 65 + 40t$

19. $y = 1(x - 1) + 1$

20. $y = -1[x - (-1)] + 1$
$y = -1(x + 1) + 1$

21. $y = 2(x - 0) + 3$

22. $y = -2[x - (-4)] + 0$
$y = -2(x + 4) + 0$

23. The line contains (0, 0) and (10, 25).

$$m = \frac{25-0}{10-0} = \frac{25}{10} = \frac{5}{2}$$

$$y = \frac{5}{2}x$$

24. The line contains (0, 0) and (5, 2).

$$m = \frac{2-0}{5-0} = \frac{2}{5}$$

$$y = \frac{2}{5}x$$

25. $3x + 4y = 12$

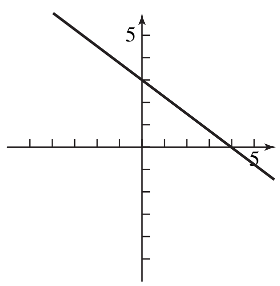
$$4y = -3x + 12$$

$$y = -\frac{3}{4}x + 3$$

(a) Slope: $-\frac{3}{4}$

(b) y-intercept: 3

(c)



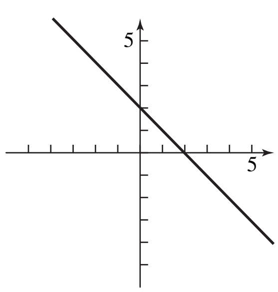
26. $x + y = 2$

$$y = -x + 2$$

(a) Slope: -1

(b) y-intercept: 2

(c)



27. $\frac{x}{3} + \frac{y}{4} = 1$

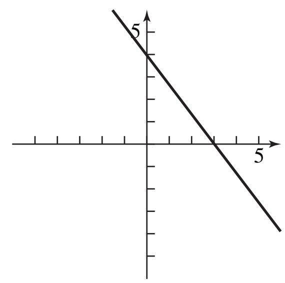
$$\frac{y}{4} = -\frac{x}{3} + 1$$

$$y = -\frac{4}{3}x + 4$$

(a) Slope: $-\frac{4}{3}$

(b) y-intercept: 4

(c)

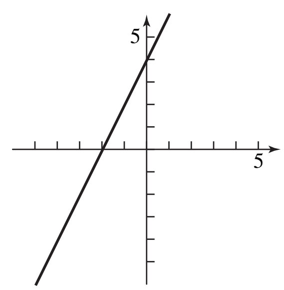


28. $y = 2x + 4$

(a) Slope: 2

(b) y-intercept: 4

(c)



29. The Line L: $y = -x + 2$ has slope -1.

(a) The desired line has slope -1 and passes through (0, 0): $y = -1(x - 0) + 0$ or $y = -x$.

(b) The desired line has slope $\frac{-1}{-1} = 1$ and passes through (0, 0): $y = 1(x - 0) + 0$ or $y = x$.

30. The given equation is equivalent to $y = -2x + 4$, so L has slope -2.

(a) The desired line has slope -2 and passes through $(-2, 2)$:
$y = -2(x + 2) + 2$ or $y = -2x - 2$.

(b) The desired line has slope $\frac{-1}{-2} = \frac{1}{2}$ and passes through $(-2, 2)$:
$y = \frac{1}{2}(x+2)+2$ or $y = \frac{1}{2}x+3$.

31. The line L: $x = 5$ is vertical and has no slope.

(a) We seek a vertical line through $(-2, 4)$: $x = -2$.

(b) We seek a horizontal line through $(-2, 4)$: $y = 4$.

32. The line L: $y = 3$ is horizontal and has slope 0.

(a) We seek a horizontal line through $\left(-1, \frac{1}{2}\right)$: $y = \frac{1}{2}$.

(b) We seek a vertical line through $\left(-1, \frac{1}{2}\right)$:
$x = -1$.

33. Solving using substitution is shown here.

$$x - 2y = 13$$
$$3x + y = 4$$

Solve the first equation for x.
$x = 2y + 13$
Substitute the expression into the second equation and solve for y.

$$\begin{aligned} 3(2y+13)+y &= 4 \\ 6y+39+y &= 4 \\ 7y &= -35 \\ y &= -5 \end{aligned}$$

Plug $y = -5$ into either original equation to get $x = 3$. The answer is $(3, -5)$.

34. Solving using elimination is shown here.

$$\begin{array}{rl} 2x + y &= 11 \\ +\ 6x - y &= 5 \\ \hline 8x \quad &= 16 \\ x &= 2 \end{array}$$

Plug $x = 2$ into either original equation to get $y = 7$. The answer is $(2, 7)$.

35. Solving using substitution is shown here.

$$20x + 7y = 22$$
$$y - 5x = 11$$

Solve the second equation for y.
$y = 5x + 11$
Substitute the expression into the first equation and solve for x.

$$\begin{aligned} 20x+7(5x+11) &= 22 \\ 20x+35x+77 &= 22 \\ 55x &= -55 \\ x &= -1 \end{aligned}$$

Plug $x = -1$ into either original equation to get $y = 6$. The answer is $(-1, 6)$.

36. Solving using elimination is shown here.

$$\begin{array}{rcrl} 2y-5x=0 & \Rightarrow & 5x-2y &= 0 \\ 4x+y=26 & \Rightarrow & +\ 8x+2y &= 52 \\ \hline & & 13x \qquad &= 52 \\ & & x &= 4 \end{array}$$

Plug $x = 4$ into either original equation to get $y = 10$. The answer is $(4, 10)$.

37. Solving using elimination is shown here.

$$\begin{array}{rcrl} 4x-y=4 & \Rightarrow & 12x-3y &= 12 \\ 14x+3y=1 & \Rightarrow & +\ 14x+3y &= 1 \\ \hline & & 26x \qquad &= 13 \\ & & x &= \frac{1}{2} \end{array}$$

Plug $x = \frac{1}{2}$ into either original equation to get $y = -2$. The answer is $\left(\frac{1}{2}, -2\right)$.

38. Solving using elimination is shown here.

$$\begin{array}{rcrl} 3x+2y=4 & \Rightarrow & -12x-8y &= -16 \\ 12x-5y=3 & \Rightarrow & +\ 12x-5y &= 3 \\ \hline & & -13y &= -13 \\ & & y &= 1 \end{array}$$

Plug $y = 1$ into either original equation to get $x = \frac{2}{3}$. The answer is $\left(\frac{2}{3}, 1\right)$.

39. Let b = cost of a burger and
f = cost of an order of fries.
Find a unique pair (b, f) that satisfies both equations simultaneously.

$$5b + 4f = 30.76$$
$$8b + 6f = 48.28$$

Solving using elimination is shown here.
Multiply equation (1) by -8 and multiply equation (2) by 5. Add the results.

$$\begin{array}{r} -40b-32f=-246.08 \\ +\quad 40b+30f=241.40 \\ \hline -2f=-4.68 \\ f=2.34 \end{array}$$

Plug $f = 2.34$ into either original equation to get $b = 4.28$.
A burger costs \$4.28 and an order of fries costs \$2.34.

40. **(a)** When $y = 0$, we have $\frac{x}{c}=1$, so $x = c$.

When $x = 0$, we have $\frac{y}{d}=1$, so $y = d$.

(b) When $y = 0$, we have $\frac{x}{c}=2$, so $x = 2c$.

When $x = 0$, we have $\frac{y}{d}=2$, so $y = 2d$.

The x-intercept is $2c$ and the y-intercept is $2d$.

41. The given equations are equivalent to $y=-\frac{2}{k}x+\frac{3}{k}$ and $y = -x + 1$, respectively, so the slopes are $-\frac{2}{k}$ and -1.

(a) The lines are parallel when $-\frac{2}{k}=-1$, so $k = 2$.

(b) The lines are perpendicular when $-\frac{2}{k}=\frac{-1}{-1}$, so $k = -2$.

42. **(a)** $m=\frac{68-69.5}{0.4-0}$
$=\frac{-1.5}{0.4}$
$=-3.75$ degrees/inch

(b) $m=\frac{9-68}{4-0.4}=\frac{-59}{3.6}=-16.4$ degrees/inch

(c) $m=\frac{4-9}{4.7-4}=\frac{-5}{0.7}=-7.1$ degrees/inch

(d) Best insulator: Fiberglass insulation
Poorest insulator: Gypsum wallboard
The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

43. Let l be the level of seed (in inches).
Let t be time, where $t = 0$ is 10:00 A.M.
(note: $t = 4$ at 2:00 P.M.)

$\frac{\Delta l}{\Delta t}=\frac{7-12}{4-0}=-\frac{5}{4}$

$l=-\frac{5}{4}t+12$

Let $l = 0$ (the seed is gone) and solve for t.

$0=-\frac{5}{4}t+12$

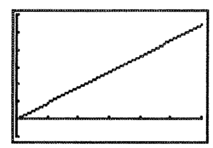
$-12=-\frac{5}{4}t$

$9.6=t$

The seed will be gone 9.6 hours after 10:00 A.M., or at 7:36 P.M.

44. **(a)** $d(t) = 45t$

(b)



[0, 6] by [−50, 300]

(c) The slope is 45, which is the speed in miles per hour.

(d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t = 0$.

(e) The car starts at time $t = 0$ at a point 30 miles past P.

45. False: $m=\frac{\Delta y}{\Delta x}$ and $\Delta x = 0$, so it is undefined, or has no slope.

46. False: perpendicular lines satisfy the equation $m_1m_2=-1$, or $m_1=-\frac{1}{m_2}$.

47. A: $y=m(x-x_1)+y_1$
$y=\frac{1}{2}(x+3)+4$ or $y-4=\frac{1}{2}(x+3)$

48. E

49. D: $y=2x-5$
$0=2x-5$
$5=2x$
$x=\frac{5}{2}$

50. B: $m = -3$
$y = -3(x - (-2)) + (-1)$
$y = -3(x + 2) - 1$
$y = -3x - 7$

51. Find the slope of the radius. The radius passes through (0, 0) and (3, 4).

$$\text{slope} = \frac{4-0}{3-0} = \frac{4}{3}$$

Since a line tangent to the circle at (3, 4) is perpendicular to the radius at this point, the slope of the tangent line will be the negative reciprocal of the slope of the radius, or $-\frac{3}{4}$.

The equation of the line is $y - 4 = -\frac{3}{4}(x - 3)$.

52. $A(-3, 10)$, $B(1, 3)$, $C(15, 11)$
Find the slope of the three sides of the triangle. If two slopes are negative reciprocals, then the lines are perpendicular to each other.

$$\text{segment } AB \text{ has slope} = \frac{3-10}{1-(-3)} = \frac{-7}{4}$$

$$\text{segment } AC \text{ has slope} = \frac{11-10}{15-(-3)} = \frac{1}{18}$$

$$\text{segment } BC \text{ has slope} = \frac{11-3}{15-1} = \frac{8}{14} = \frac{4}{7}$$

Thus, $AB \perp BC$ and the segment AC is the hypotenuse.

53. **(a)** Solving by elimination is shown.
Multiply the first equation by 3.

$$\begin{array}{rcr} 3x - 5y = 3 & \Rightarrow & 9x - 15y = 9 \\ -9x + 15y = 8 & \Rightarrow & + \ -9x + 15y = 8 \\ \hline & & 0 = 17 \end{array}$$

Since 0 = 17 is impossible, the conclusion is that there is no pair (x, y) that satisfies both equations simultaneously.

(b) A graph shows the two lines to be parallel.

(c) Two linear equations that are dependent and inconsistent have parallel graphs that do not intersect. Therefore, there is no pair (x, y) that can satisfy both equations simultaneously.

54. **(a)** Solving by elimination is shown.
Multiply the first equation by −3.

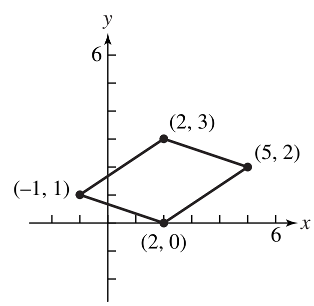
$$\begin{array}{rcr} 2x - 5y = 3 & \Rightarrow & -6x + 15y = -9 \\ 6x - 15y = 9 & \Rightarrow & + \ 6x - 15y = 9 \\ \hline & & 0 = 0 \end{array}$$

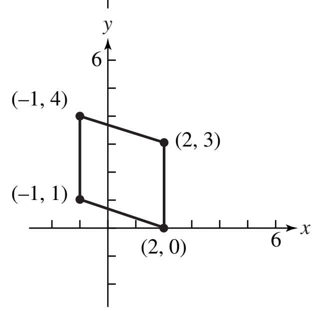
Since the equation 0 = 0 cannot be false, the conclusion is that any pair (x, y) that satisfies one equation must also satisfy the other.

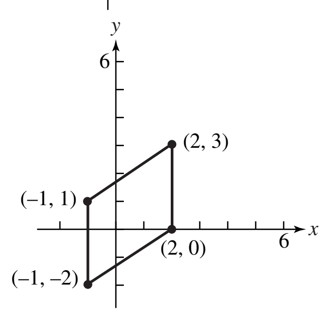
(b) A graph shows that the two linear equations have the same graph.

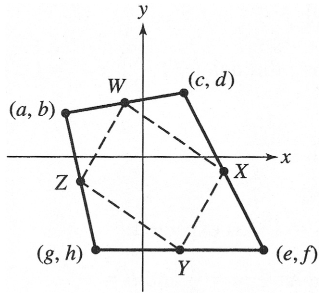
(c) Two linear equations that are dependent and consistent have graphs that are the same line. Therefore, every pair (x, y) on the line satisfies both equations simultaneously.

55. The coordinates of the three missing vertices are (5, 2), (−1, 4) and (−1, −2), as shown below.







56. 

Suppose that the vertices of the given quadrilateral are (a, b), (c, d), $e, f)$, and (g, h). Then the midpoints of the consecutive sides are $W\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, $X\left(\frac{c+e}{2}, \frac{d+f}{2}\right)$, $Y\left(\frac{e+g}{2}, \frac{f+h}{2}\right)$, and $Z\left(\frac{g+a}{2}, \frac{h+b}{2}\right)$.

When these four points are connected, the slopes of the sides of the resulting figure are:

WX: $\dfrac{\frac{d+f}{2}-\frac{b+d}{2}}{\frac{c+e}{2}-\frac{a+c}{2}} = \dfrac{f-b}{e-a}$

XY: $\dfrac{\frac{f+h}{2}-\frac{d+f}{2}}{\frac{e+g}{2}-\frac{c+e}{2}} = \dfrac{h-d}{g-c}$

ZY: $\dfrac{\frac{f+h}{2}-\frac{h+b}{2}}{\frac{e+g}{2}-\frac{g+a}{2}} = \dfrac{f-b}{e-a}$

WZ: $\dfrac{\frac{h+b}{2}-\frac{b+d}{2}}{\frac{g+a}{2}-\frac{a+c}{2}} = \dfrac{h-d}{g-c}$

Opposite sides have the same slope and are parallel.

57. The radius through $(-2, 6)$ has slope $\dfrac{6-2}{-2-1} = -\dfrac{4}{3}$.

The tangent line is perpendicular to the radius, so its slope is $\dfrac{-1}{-\frac{4}{3}} = \dfrac{3}{4}$. We seek the line of slope $\dfrac{3}{4}$ that passes through $(-2, 6)$.

$$y-6 = \frac{3}{4}(x+2)$$

58. **(a)** The equation for line L can be written as $y = -\dfrac{A}{B}x + \dfrac{C}{B}$, so its slope is $-\dfrac{A}{B}$. The perpendicular line has slope $\dfrac{-1}{-\frac{A}{B}} = \dfrac{B}{A}$ and passes through (a, b), so its equation is $y = \dfrac{B}{A}(x-a)+b$.

(b) Substituting $\dfrac{B}{A}(x-a)+b$ for y in the equation for line L gives:

$$Ax + B\left[\frac{B}{A}(x-a)+b\right] = C$$

$$A^2x + B^2(x-a) + ABb = AC$$

$$(A^2+B^2)x = B^2a + AC - ABb$$

$$x = \frac{B^2a + AC - ABb}{A^2+B^2}$$

Substituting the expression for x in the equation for line L gives:

$$A\left(\frac{B^2a+AC-ABb}{A^2+B^2}\right)+By=C$$

$$By=\frac{-A(B^2a+AC-ABb)}{A^2+B^2}+\frac{C(A^2+B^2)}{A^2+B^2}$$

$$By=\frac{-AB^2a-A^2C+A^2Bb+A^2C+B^2C}{A^2+B^2}$$

$$By=\frac{A^2Bb+B^2C-AB^2a}{A^2+B^2}$$

$$y=\frac{A^2b+BC-ABa}{A^2+B^2}$$

The coordinates of Q are $\left(\frac{B^2a+AC-ABb}{A^2+B^2}, \frac{A^2b+BC-ABa}{A^2+B^2}\right)$.

(c) Distance $=\sqrt{(x-a)^2+(y-b)^2}$

$$=\sqrt{\left(\frac{B^2a+AC-ABb}{A^2+B^2}-a\right)^2+\left(\frac{A^2b+BC-ABa}{A^2+B^2}-b\right)^2}$$

$$=\sqrt{\left(\frac{B^2a+AC-ABb-a(A^2+B^2)}{A^2+B^2}\right)^2+\left(\frac{A^2b+BC-ABa-b(A^2+B^2)}{A^2+B^2}\right)^2}$$

$$=\sqrt{\left(\frac{AC-ABb-A^2a}{A^2+B^2}\right)^2+\left(\frac{BC-ABa-B^2b}{A^2+B^2}\right)^2}$$

$$=\sqrt{\left(\frac{A(C-Bb-Aa)}{A^2+B^2}\right)^2+\left(\frac{B(C-Aa-Bb)}{A^2+B^2}\right)^2}$$

$$=\sqrt{\frac{A^2(C-Aa-Bb)^2}{(A^2+B^2)^2}+\frac{B^2(C-Aa-Bb)^2}{(A^2+B^2)^2}}$$

$$=\sqrt{\frac{(A^2+B^2)(C-Aa-Bb)^2}{(A^2+B^2)^2}}$$

$$=\sqrt{\frac{(C-Aa-Bb)^2}{A^2+B^2}}$$

$$=\frac{|C-Aa-Bb|}{\sqrt{A^2+B^2}}$$

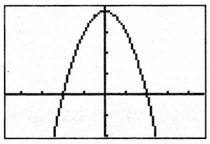
$$=\frac{|Aa+Bb-C|}{\sqrt{A^2+B^2}}$$

Section 1.2 Functions and Graphs
(pp. 13–22)

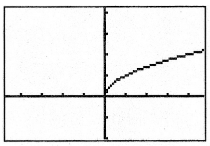
Exploration 1 Composing Functions

1. $y_3 = g \circ f,\ y_4 = f \circ g$

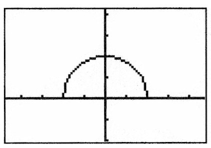
2. Domain of y_3: $[-2, 2]$
   Range of y_3: $[0, 2]$



y_1: $[-4.7, 4.7]$ by $[-2, 4.2]$

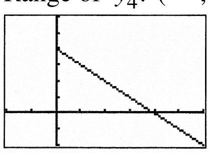


y_2: $[-4.7, 4.7]$ by $[-2, 4.2]$



y_3: $[-4.7, 4.7]$ by $[-2, 4.2]$

3. Domain of y_4: $[0, \infty)$
   Range of y_4: $(-\infty, 4]$



$[-2, 6]$ by $[-2, 6]$

4. $y_3 = y_2(y_1(x)) = \sqrt{y_1(x)} = \sqrt{4 - x^2}$
   $y_4 = y_1(y_2(x))$
   $= 4 - (y_2(x))^2$
   $= 4 - \left(\sqrt{x}\right)^2$
   $= 4 - x,\ x \ge 0$

Quick Review 1.2

1. $3x - 1 \le 5x + 3$
   $-2x \le 4$
   $x \ge -2$
   Solution: $[-2, \infty)$

2. $x(x - 2) > 0$
   Solutions to $x(x - 2) = 0$: $x = 0$, $x = 2$
   Test $x = -1$: $-1(-1 - 2) = 3 > 0$
   $x(x - 2) > 0$ is true when $x < 0$.
   Test $x = 1$: $1(1 - 2) = -1 < 0$
   $x(x - 2) > 0$ is false when $0 < x < 2$.
   Test $x = 3$: $3(3 - 2) = 3 > 0$
   $x(x - 2) > 0$ is true when $x > 2$.
   Solution set: $(-\infty, 0) \cup (2, \infty)$

3. $|x - 3| < 4$
   $-4 \le x - 3 \le 4$
   $-1 \le x \le 7$
   Solution set: $[-1, 7]$

4. $|x - 2| \ge 5$
   $x - 2 \le -5$ or $x - 2 \ge 5$
   $x \le -3$ or $x \ge 7$
   Solution set: $(-\infty, -3] \cup [7, \infty)$

5. $x^2 < 16$
   Solutions to $x^2 = 16$: $x = -4$, $x = 4$
   Test $x = -6$: $(-6)^2 = 36 > 16$
   $x^2 < 16$ is false when $x < -4$.
   Test $x = 0$: $0^2 = 0 < 16$
   $x^2 < 16$ is true when $-4 < x < 4$.
   Test $x = 6$: $6^2 = 36 > 16$
   $x^2 < 16$ is false when $x > 4$.
   Solution set: $(-4, 4)$

6. $9 - x^2 \ge 0$
   Solutions to $9 - x^2 = 0$: $x = -3$, $x = 3$
   Test $x = -4$: $9 - (-4)^2 = 9 - 16 = -7 < 0$
   $9 - x^2 \ge 0$ is false when $x < -3$.
   Test $x = 0$: $9 - 0^2 = 9 > 0$
   $9 - x^2 \ge 0$ is true when $-3 < x < 3$.
   Test $x = 4$: $9 - 4^2 = 9 - 16 = 7 < 0$
   $9 - x^2 \ge 0$ is false when $x > 3$.
   Solution set: $[-3, 3]$

7. Translate the graph of f 2 units left and 3 units downward.

8. Translate the graph of f 5 units right and 2 units upward.

9. **(a)**
$$\begin{aligned} f(x) &= 4 \\ x^2 - 5 &= 4 \\ x^2 - 9 &= 0 \\ (x+3)(x-3) &= 0 \end{aligned}$$
$x = -3$ or $x = 3$

(b)
$$\begin{aligned} f(x) &= -6 \\ x^2 - 5 &= -6 \\ x^2 &= -1 \end{aligned}$$
No real solution

10. **(a)**
$$\begin{aligned} f(x) &= -5 \\ \frac{1}{x} &= -5 \\ x &= -\frac{1}{5} \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= 0 \\ \frac{1}{x} &= 0 \end{aligned}$$
No solution

11. **(a)**
$$\begin{aligned} f(x) &= 4 \\ \sqrt{x+7} &= 4 \\ x+7 &= 16 \\ x &= 9 \end{aligned}$$
Check: $\sqrt{9+7} = \sqrt{16} = 4$; it checks.

(b)
$$\begin{aligned} f(x) &= 1 \\ \sqrt{x+7} &= 1 \\ x+7 &= 1 \\ x &= -6 \end{aligned}$$
Check: $\sqrt{-6+7} = 1$; it checks.

12. **(a)**
$$\begin{aligned} f(x) &= -2 \\ \sqrt[3]{x-1} &= -2 \\ x-1 &= -8 \\ x &= -7 \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= 3 \\ \sqrt[3]{x-1} &= 3 \\ x-1 &= 27 \\ x &= 28 \end{aligned}$$

Section 1.2 Exercises

1. $A(d) = \pi\left(\frac{d}{2}\right)^2$

$A(4) = \pi\left(\frac{4 \text{ in.}}{2}\right)^2 = \pi(2 \text{ in.})^2 = 4\pi \text{ in}^2$

2. $h(s) = \frac{\sqrt{3}}{2}s$

$h(3) = \frac{\sqrt{3}}{2} \cdot 3 \text{ m} = 1.5\sqrt{3} \text{ m}$

3. $S(e) = 6e^2$

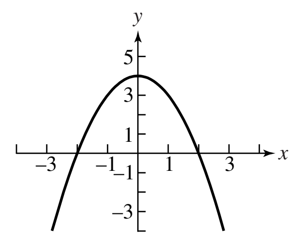
$S(5) = 6(5 \text{ ft})^2 = 6(25 \text{ ft}^2) = 150 \text{ ft}^2$

4. $V(r) = \frac{4}{3}\pi r^3$

$V(3) = \frac{4}{3}\pi(3 \text{ cm})^3 = \frac{4}{3}\pi(27 \text{ cm}^3) = 36\pi \text{ cm}^3$

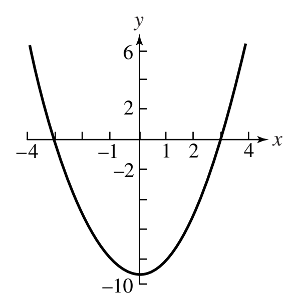
5. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
Range: $(-\infty, 4]$

(b)



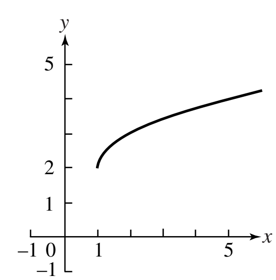
6. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
Range: $[-9, \infty)$

(b)



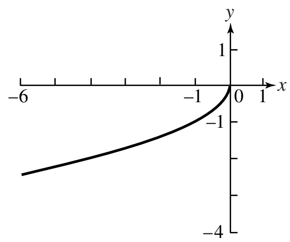
7. **(a)** Since we require $x - 1 \geq 0$, the domain is $[1, \infty)$. Range: $[2, \infty)$

(b)



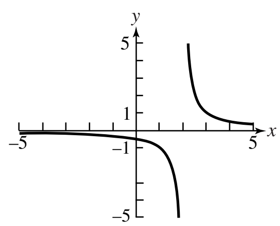
8. **(a)** Since we require $-x \geq 0$, the domain is $(-\infty, 0]$. Range: $(-\infty, 0]$

(b)



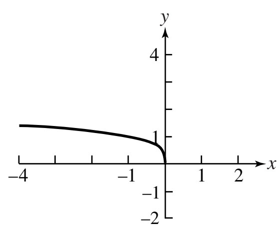
9. **(a)** Since we require $x - 2 \neq 0$, the domain is $(-\infty, 2) \cup (2, \infty)$. Since $\dfrac{1}{x-2}$ can assume any value except 0, the range is $(-\infty, 0) \cup (0, \infty)$.

(b)



10. **(a)** Since we require $-x \geq 0$, the domain is $(-\infty, 0]$. Range: $[0, \infty)$

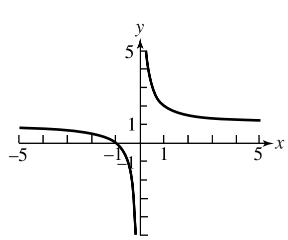
(b)



11. **(a)** Since we require $x \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

Note that $\dfrac{1}{x}$ can assume any value except 0, so $1+\dfrac{1}{x}$ can assume any value except 1. The range is $(-\infty, 1) \cup (1, \infty)$.

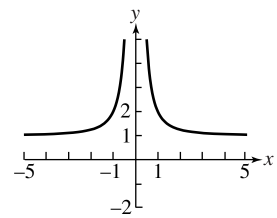
(b)



12. **(a)** Since we require $x^2 \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

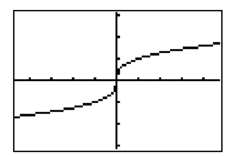
Since $\dfrac{1}{x^2} > 0$ for all x, the range is $(1, \infty)$.

(b)



13. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
Range: $(-\infty, \infty)$ or all real numbers

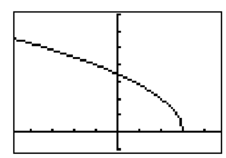
(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

14. **(a)** Since we require $3 - x \geq 0$, the domain is $(-\infty, 3]$. Range: $[0, \infty)$

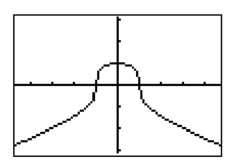
(b)



$[-4.7, 4.7]$ by $[-1, 7]$

15. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
The maximum function value is attained at the point $(0, 1)$, so the range is $(-\infty, 1]$.

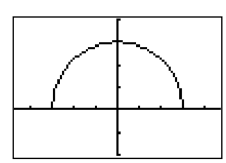
(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

16. **(a)** Since we require $9 - x^2 \geq 0$ the domain is $[-3, 3]$. Range: $[0, 3]$

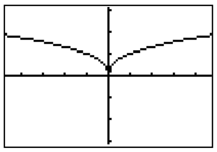
(b)



$[-4.7, 4.7]$ by $[-2, 4]$

17. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
Since $x^{2/5}$ is equivalent to $\left(\sqrt[5]{x}\right)^2$ the range is $[0, \infty)$.

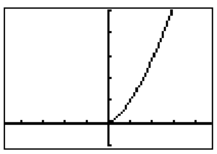
(b)



[–4.7, 4.7] by [–3.1, 3.1]

18. **(a)** This function is equivalent to $y = \left(\sqrt{x}\right)^3$, so its domain is $[0, \infty)$.
Range: $[0, \infty)$

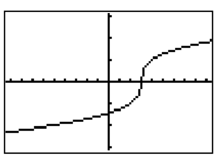
(b)



[–4.7, 4.7] by [–1, 5]

19. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
Range: $(-\infty, \infty)$ or all real numbers

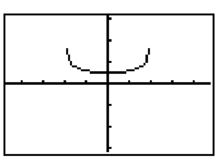
(b)



[–9.4, 9.4] by [–3.1, 3.1]

20. **(a)** Since we require $4 - x^2 > 0$, the domain is $(-2, 2)$. Domain: $[0.5, \infty)$

(b)



[–4.7, 4.7] by [–3.1, 3.1]

(Note: this is an example of grapher failure because the graph really has vertical asymptotes at $x = \pm 2$ that do not show up in the graph above.)

21. Even, since the function is an even power of x.

22. Neither, since the function is a sum of even and odd powers of x.

23. Neither, since the function is a sum of even and odd powers of x, $(x^1 + 2x^0)$.

24. Even, since the function is a sum of even powers of x, $(x^2 - 3x^0)$.

25. Even, since the function involves only even powers of x.

26. Odd, since the function is a sum of odd powers of x.

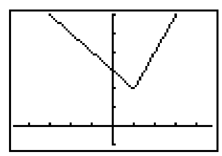
27. Odd, since the function is a quotient of an odd function (x^3) and an even function $(x^2 - 1)$.

28. Neither, since, (for example), $y(-2) = 4^{1/3}$ and $y(2) = 0$.

29. Neither, since, (for example), $y(-1)$ is defined and $y(1)$ is undefined.

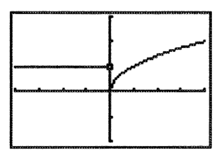
30. Even, since the function involves only even powers of x.

31.



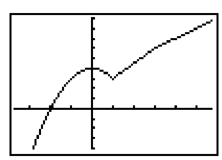
[–4.7, 4.7] by [–1, 6]

32.



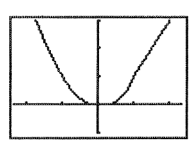
[−4, 4] by [−2, 3]

33.



[–3.7, 5.7] by [–4, 9]

34.



[−2.35, 2.35] by [−1, 3]

35. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate would correspond to the value assigned to the x-coordinate. Since there is only one y-coordinate, the assignment would be unique.

36. If the curve is not $y = 0$, there must be a point (x, y) on the curve where $y \neq 0$. That would mean that (x, y) and $(x, -y)$ are two different points on the curve and it is not the graph of a function, since it fails the vertical line test.

37. No

38. Yes

39. Yes

40. No

41. Line through (0, 0) and (1, 1): $y = x$
Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$$

42. $f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$

43. Line through (0, 2) and (2, 0): $y = -x + 2$
Line through (2, 1) and (5, 0):

$m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$ so

$y = -\frac{1}{3}(x-2)+1 = -\frac{1}{3}x+\frac{5}{3}.$

$$f(x) = \begin{cases} -x+2, & 0 < x \le 2 \\ -\frac{1}{3}x+\frac{5}{3}, & 2 < x \le 5 \end{cases}$$

44. Line through (−1, 0) and (0, −3):

$m = \frac{-3-0}{0-(-1)} = \frac{-3}{1} = -3$, so $y = -3x - 3$.

Line through (0, 3) and (2, −1):

$m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$.

$$f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$$

45. Line through (−1, 1) and (0, 0): $y = -x$
Line through (0, 1) and (1, 1): $y = 1$
Line through (1, 1) and (3, 0):

$m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so

$y = -\frac{1}{2}(x-1)+1 = -\frac{1}{2}x+\frac{3}{2}.$

$$f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ -\frac{1}{2}x+\frac{3}{2}, & 1 < x < 3 \end{cases}$$

46. Line through (−2, −1) and (0, 0): $y = \frac{1}{2}x$
Line through (0, 2) and (1, 0): $y = -2x + 2$
Line through (1, −1) and (3, −1): $y = -1$

$$f = \begin{cases} \frac{1}{2}x, & -2 \le x \le 0 \\ -2x+2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$$

47. Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$:

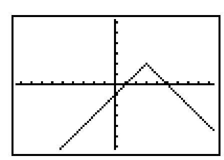
$m = \frac{1-0}{T-\left(\frac{T}{2}\right)} = \frac{2}{T}$, so

$y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1.$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x-1, & \frac{T}{2} < x \le T \end{cases}$$

48. $f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A & \frac{3T}{2} \le x \le 2T \end{cases}$

49. **(a)**



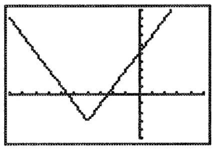
[−9.4, 9.4] by [−6.2, 6.2]

Note that $f(x) = -|x-3| + 2$, so its graph is the graph of the absolute value function reflected across the x-axis and then shifted 3 units right and 2 units upward.

(b) $(-\infty, \infty)$

(c) $(-\infty, 2]$

50. **(a)** The graph of $f(x)$ is the graph of the absolute value function stretched vertically by a factor of 2 and then shifted 4 units to the left and 3 units downward.



[−10, 5] by [−5, 10]

(b) $(-\infty, \infty)$ or all real numbers

(c) $[-3, \infty)$

51. **(a)** $f(g(x)) = (x^2 - 3) + 5 = x^2 + 2$

(b)
$$\begin{aligned} g(f(x)) &= (x+5)^2 - 3 \\ &= (x^2 + 10x + 25) - 3 \\ &= x^2 + 10x + 22 \end{aligned}$$

(c) $f(g(0)) = 0^2 + 2 = 2$

(d) $g(f(0)) = 0^2 + 10 \cdot 0 + 22 = 22$

(e) $g(g(-2)) = [(-2)^2 - 3]^2 - 3 = 1^2 - 3 = -2$

(f) $f(f(x)) = (x + 5) + 5 = x + 10$

52. **(a)** $f(g(x)) = (x - 1) + 1 = x$

(b) $g(f(x)) = (x + 1) - 1 = x$

(c) $f(g(0)) = 0$

(d) $g(f(0)) = 0$

(e) $g(g(-2)) = (-2 - 1) - 1 = -3 - 1 = -4$

(f) $f(f(x)) = (x + 1) + 1 = x + 2$

53. **(a)** Since $(f \circ g)(x) = \sqrt{g(x) - 5} = \sqrt{x^2 - 5}$, $g(x) = x^2$.

(b) Since $(f \circ g)(x) = 1 + \dfrac{1}{g(x)} = x$, we know that $\dfrac{1}{g(x)} = x - 1$, so $g(x) = \dfrac{1}{x-1}$.

(c) $(f \circ g)(x) = f\left(\dfrac{1}{x}\right) = x$, $f(x) = \dfrac{1}{x}$.

(d) Since $(f \circ g)(x) = f\left(\sqrt{x}\right) = |x|$, $f(x) = x^2$.

Completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
x^2	$\sqrt{x-5}$	$\sqrt{x^2 - 5}$
$\frac{1}{x-1}$	$1 + \frac{1}{x}$	$x, x \neq 1$
$\frac{1}{x}$	$\frac{1}{x}$	$x, x \neq 0$
$\sqrt{x}$	x^2	$\lvert x \rvert, x \geq 0$

54. The radius is $\dfrac{1}{2}$ of the diameter, so $r = \dfrac{h}{2}$.

(a) $V = \pi r^2 h = \pi\left(\dfrac{h}{2}\right)^2 \cdot h = \pi\left(\dfrac{h^2}{4}\right) \cdot h = \dfrac{\pi h^3}{4}$

(b)
$$\begin{aligned} A &= 2\pi rh + 2\pi r^2 \\ &= 2\pi\left(\frac{h}{2}\right) \cdot h + 2\pi\left(\frac{h}{2}\right)^2 \\ &= \frac{2\pi h^2}{2} + \frac{\pi h^2}{2} \\ &= \frac{3\pi h^2}{2} \end{aligned}$$

(c)
$$\begin{aligned} A &= \frac{3\pi h^2}{2} \\ 54\pi &= \frac{3\pi h^2}{2} \\ 36 &= h^2 \\ 6 &= h \end{aligned}$$

$$\begin{aligned} V &= \frac{\pi h^3}{4} \\ V &= \frac{\pi \cdot 6^3}{4} \\ V &= 54\pi \end{aligned}$$

55. **(a)** Because the circumference of the original circle was 8π and a piece of length x was removed.

(b) $r = \dfrac{8\pi - x}{2\pi} = 4 - \dfrac{x}{2\pi}$

(c) $h = \sqrt{16 - r^2}$

$= \sqrt{16 - \left(4 - \dfrac{x}{2\pi}\right)^2}$

$= \sqrt{16 - \left(16 - \dfrac{4x}{\pi} + \dfrac{x^2}{4\pi^2}\right)}$

$= \sqrt{\dfrac{4x}{\pi} - \dfrac{x^2}{4\pi^2}}$

$= \dfrac{\sqrt{16\pi x - x^2}}{2\pi}$

(d) $V = \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi\left(\dfrac{8\pi - x}{2\pi}\right)^2 \cdot \dfrac{\sqrt{16\pi x - x^2}}{2\pi}$

$= \dfrac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$

56. **(a)** Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10{,}560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$.

(b) $C(0) = \$1{,}200{,}000$
$C(500) \approx \$1{,}175{,}812$
$C(1000) \approx \$1{,}186{,}512$
$C(1500) \approx \$1{,}212{,}000$
$C(2000) \approx \$1{,}243{,}732$
$C(2500) \approx \$1{,}278{,}479$
$C(3000) \approx \$1{,}314{,}870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 ft from point *P*.

57. False: $x^4 + x^2 + x \neq (-x)^4 + (-x)^2 + (-x)$.

58. True: $(-x)^{-3} = -x^{-3}$

59. B: Since $9 - x^2 > 0$, the domain is $(-3, 3)$.

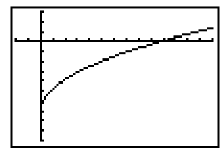
60. A: $y \neq 1$

61. D: $(f \circ g)(x) = 2(g(x)) - 1$
$= 2(x + 3) - 1$
$= 2x + 5$
$(f \circ g) = (2) = 2(2) + 5 = 9$

62. C: $A(W) = LW$
$L = 2W$
$A(W) = 2W^2$

63. **(a)** Enter $y_1 = f(x) = x - 7$, $y_2 = g(x) = \sqrt{x}$, $y_3 = (f \circ g)(x) = y_1(y_2(x))$, and $y_4 = (g \circ f)(x) = y_2(y_1(x))$

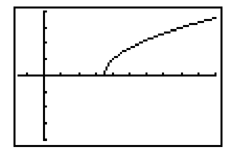
$f \circ g$:



[–10, 70] by [–10, 3]

Domain: $[0, \infty)$
Range: $[-7, \infty)$

$g \circ f$:



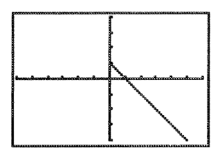
[–3, 20] by [–4, 4]

Domain: $[7, \infty)$
Range: $[0, \infty)$

(b) $(f \circ g)(x) = \sqrt{x} - 7$
$(g \circ f)(x) = \sqrt{x - 7}$

64. **(a)** Enter $y_1 = f(x) = 1 - x^2$, $y_2 = g(x) = \sqrt{x}$, $y_3 = (f \circ g)(x) = y_1(y_2(x))$, and $y_4 = (g \circ f)(x) = y_2(y_1(x))$.

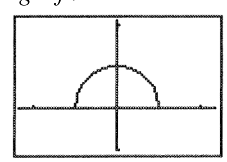
$f \circ g$:



[–6, 6] by [–4, 4]

Domain: $[0, \infty)$
Range: $(-\infty, 1]$

$g \circ f$:



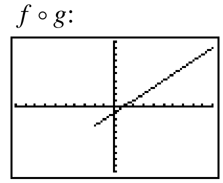
[–2.35, 2.35] by [–1, 2.1]

Domain: $[-1, 1]$
Range: $[0, 1]$

(b) $(f \circ g)(x) = 1 - \left(\sqrt{x}\right)^2 = 1 - x,\ x \geq 0$
$(g \circ f)(x) = \sqrt{1 - x^2}$

65. **(a)** Enter $y_1 = f(x) = x^2 - 3$, $y_2 = g(x) = \sqrt{x + 2}$, $y_3 = (f \circ g)(x) = y_1(y_2(x))$, and $y_4 = (g \circ f)(x) = y_2(y_1(x))$.

$f \circ g$:

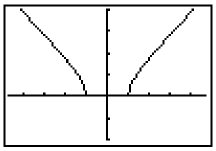


[–10, 10] by [–10, 10]

Domain: $[-2, \infty)$

Range: $[-3, \infty)$

$g \circ f$:



$[-4.7, 4.7]$ by $[-2, 4]$

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[0, \infty)$

(b) $(f \circ g)(x) = \left(\sqrt{x+2}\right)^2 - 3$
$= (x+2) - 3,\ x \ge -2$
$= x - 1,\ x \ge -2$

$(g \circ f)(x) = \sqrt{(x^2 - 3) + 2} = \sqrt{x^2 - 1}$

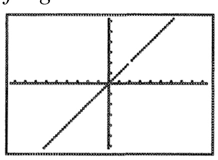
66. **(a)** Enter $y_1(x) = f(x) = \dfrac{2x-1}{x+3}$, $y_2 = \dfrac{3x+1}{2-x}$.

$y_3 = (f \circ g)(x) = y_1(y_2(x))$, and

$y_4 = (g \circ f)(x) = y_2(y_1(x))$.

Use a "decimal window" such as the one shown.

$f \circ g$:

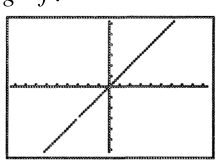


$[-9.4, 9.4]$ by $[-6.2, 6.2]$

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, 2) \cup (2, \infty)$

$g \circ f$:



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

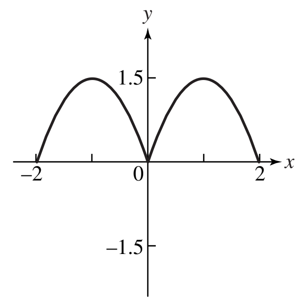
Domain: $(-\infty, -3) \cup (-3, \infty)$

Range: $(-\infty, -3) \cup (-3, \infty)$

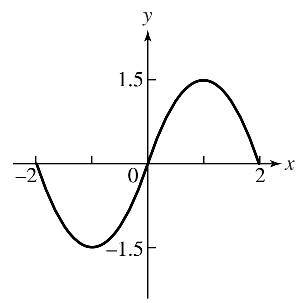
(b)
$$(f \circ g)(x) = \frac{2\left(\frac{3x+1}{2-x}\right) - 1}{\frac{3x+1}{2-x} + 3}$$
$$= \frac{2(3x+1) - (2-x)}{(3x+1) + 3(2-x)},\ x \ne 2$$
$$= \frac{7x}{7},\ x \ne 2$$
$$= x,\ x \ne 2$$

$$(g \circ f)(x) = \frac{3\left(\frac{2x-1}{x+3}\right) + 1}{2 - \frac{2x-1}{x+3}}$$
$$= \frac{3(2x-1) + (x+3)}{2(x+3) - (2x-1)},\ x \ne -3$$
$$= \frac{7x}{7},\ x \ne -3$$
$$= x,\ x \ne -3$$

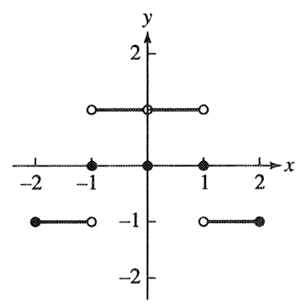
67. **(a)**



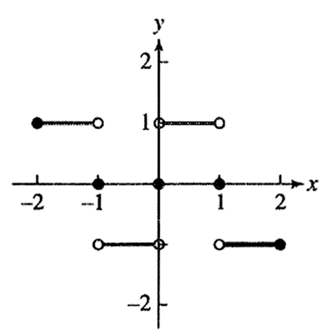
(b)



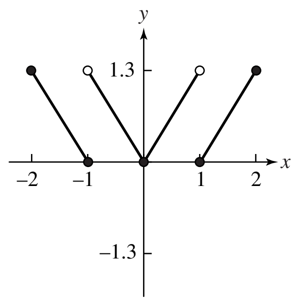
68. **(a)**



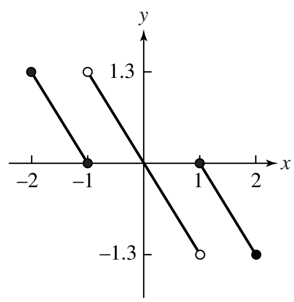
(b)



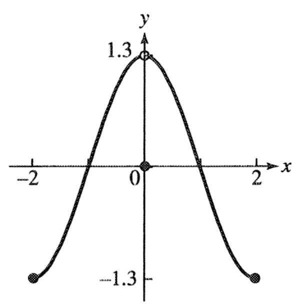
69. **(a)**



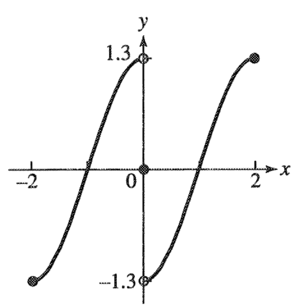
(b)



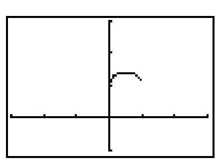
70. **(a)**



(b)



71. **(a)**



[−3, 3] by [−1, 3]

(b) Domain of y_1: $[0, \infty)$
Domain of y_2: $(-\infty, 1]$
Domain of y_3: $[0, 1]$

(c) The functions $y_1 - y_2$, $y_2 - y_1$, and $y_1 \cdot y_2$ all have domain [0, 1], the same as the domain of $y_1 + y_2$ found in part (b).

Domain of $\dfrac{y_1}{y_2}$: $[0, 1)$

Domain of $\dfrac{y_2}{y_1}$: $(0, 1]$

(d) The domain of a sum, difference, or product of two functions is the intersection of their domains. The domain of a quotient of two functions is the intersection of their domains with any zeros of the denominator removed.

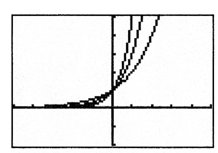
72. **(a)** Yes, since
$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x)\\ &= f(x) \cdot g(x)\\ &= (f \cdot g)(x),\end{aligned}$$
function $(f \cdot g)(x)$ will also be even.

(b) The product will be even, since
$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x)\\ &= (-f(x)) \cdot (-g(x))\\ &= f(x) \cdot g(x)\\ &= (f \cdot g)(x).\end{aligned}$$

Section 1.3 Exponential Functions (pp. 23–28)

Exploration 1 Exponential Functions

1.



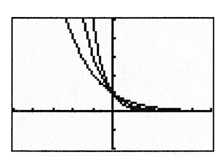
[−5, 5] by [−2, 5]

2. $x > 0$

3. $x < 0$

4. $x = 0$

5.



[−5, 5] by [−2, 5]

6. $2^{-x} < 3^{-x} < 5^{-x}$ for $x < 0$; $2^{-x} > 3^{-x} > 5^{-x}$ for $x > 0$; $2^{-x} = 3^{-x} = 5^{-x}$ for $x = 0$.

Quick Review 1.3

1. Using a calculator, $5^{2/3} \approx 2.924$.

2. Using a calculator, $3^{\sqrt{2}} \approx 4.729$.

3. Using a calculator, $3^{-1.5} \approx 0.192$.

4. $x^3 = 17$
$x = \sqrt[3]{17}$
$x \approx 2.5713$

5. $x^5 = 24$
$x = \sqrt[5]{24}$
$x \approx 1.8882$

6. $x^{10} = 1.4567$
$x = \pm\sqrt[10]{1.4567}$
$x \approx \pm 1.0383$

7. $\$500(1.0475)^5 \approx \$630.58$

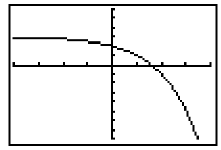
8. $\$1000(1.063)^3 \approx \$1201.16$

9. $$\frac{(x^{-3}y^2)^2}{(x^4y^3)^3} = \frac{x^{-6}y^4}{x^{12}y^9} = x^{-6-12}y^{4-9} = x^{-18}y^{-5} = \frac{1}{x^{18}y^5}$$

10. $$\left(\frac{a^3b^{-2}}{c^4}\right)^2\left(\frac{a^4c^{-2}}{b^3}\right)^{-1} = \frac{a^6b^{-4}}{c^8}\cdot\frac{b^3}{a^4c^{-2}} = \frac{a^6}{b^4c^8}\cdot\frac{b^3c^2}{a^4} = a^{6-4}b^{-4+3}c^{-8+2} = a^2b^{-1}c^{-6} = \frac{a^2}{bc^6}$$

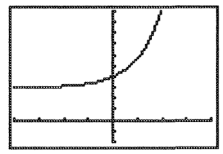
Section 1.3 Exercises

1.



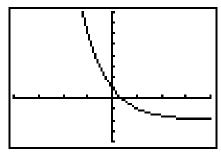
[–4, 4] by [–8, 6]
Domain: $(-\infty, \infty)$
Range: $(-\infty, 3)$

2.



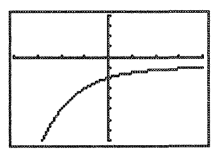
[−4, 4] by [−2, 10]
Domain: $(-\infty, \infty)$
Range: $(3, \infty)$

3.



[–4, 4] by [–4, 8]
Domain: $(-\infty, \infty)$
Range: $(-2, \infty)$

4.



[−4, 4] by [−8, 4]
Domain: $(-\infty, \infty)$
Range: $(-\infty, -1)$

5. $9^{2x} = (3^2)^{2x} = 3^{4x}$

6. $16^{3x} = (2^4)^{3x} = 2^{12x}$

7. $\left(\frac{1}{8}\right)^{2x} = (2^{-3})^{2x} = 2^{-6x}$

8. $\left(\frac{1}{27}\right)^{x} = (3^{-3})^{x} = 3^{-3x}$

9. zero: ≈ 2.322

10. zero: ≈ 1.386

11. zero: ≈ -0.631

12. zero: ≈ 1.585

13. The graph of $y = 2^x$ is increasing from left to right and has the negative x-axis as an asymptote. (a)

14. The graph of $y = 3^{-x}$ or, equivalently, $y = \left(\frac{1}{3}\right)^x$, is decreasing from left to right and has the positive x-axis as an asymptote. (d)

15. The graph of $y = -3^{-x}$ is the reflection about the x-axis of the graph in Exercise 14. (e)

16. The graph of $y = -0.5^{-x}$ or, equivalently, $y = -2^x$, is the reflection about the x-axis of the graph in Exercise 13. (c)

17. The graph of $y = 2^{-x} - 2$ is decreasing from left to right and has the line $y = -2$ as an asymptote. (b)

18. The graph of $y = 1.5^x - 2$ is increasing from left to right and has the line $y = -2$ as an asymptote. (f)

19. Let t be the number of years. Solving $500,000(1.0375)^t = 1,000,000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

20. **(a)** The population is given by $P(t) = 6250(1.0275)^t$, where t is the number of years after 1890.
Population in 1915: $P(25) \approx 12,315$
Population in 1940: $P(50) \approx 24,265$

(b) Solving $P(t) = 50,000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

21. The half-life is the amount of time it takes the substance to go from its original state to half (50%) of its original state.

(a) Therefore, it will take 63 years for a sample to lose 50% of its titanium-44.

(b) It will lose 50% and then 50% of 50% (or $0.5 \cdot 0.5 = 0.25$, which is 25%). Therefore, it will take 63 years + 63 years or 126 years for a sample to lose 75% of its titanium-44.

22. The half-life is the amount of time it takes the substance to go from its original state to half (50%) of its original state.
Let x = the number of years of the half-life. Therefore, it will take x years to go from 28 grams to 14 grams and then another x years to go from 14 grams to 7 grams. So, the half-life of silicon-32 is $2x = 340$ years or $x = 170$ years.

23. **(a)** $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$

(b) Solving $A(t) = 1$ graphically, we find that $t \approx 38.1145$. There will be 1 gram remaining after about 38.1145 days.

24. Let t be the number of years. Solving $2300(1.06)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

25. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

26. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1+\frac{0.0625}{12}\right)^{12t} = 2A$, which is equivalent to $\left(1+\frac{0.0625}{12}\right)^{12t} = 2$. Solving graphically, we find that $t \approx 11.119$. It will take about 11.119 years. (If the interest is credited at the end of each month, it will take 11 years, 2 months.)

27. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0625t} = 2A$, which is equivalent to $e^{0.0625t} = 2$. Solving graphically, we find that $t \approx 11.090$. It will take about 11.090 years.

28. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0575)^t = 3A$, which is equivalent to $1.0575^t = 3$. Solving graphically, we find that $t \approx 19.650$. It will take about 19.650 years. (If the interest is credited at the end of each year, it will take 20 years.)

29. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1+\frac{0.0575}{365}\right)^{365t} = 3A$, which is equivalent to $\left(1+\frac{0.0575}{365}\right)^{365t} = 3$. Solving graphically, we find that $t \approx 19.108$. It will take about 19.108 years.

30. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0575t} = 3A$, which is equivalent to $e^{0.0575t} = 3$. Solving graphically, we find that $t \approx 19.106$. It will take about 19.106 years.

31. After t hours, the population is $P(t) = 2^{t/0.5}$ or, equivalently, $P(t) = 2^{2t}$. After 24 hours, the population is $P(24) = 2^{48} \approx 2.815 \times 10^{14}$ bacteria.

32. **(a)** Each year, the number of cases is 100% − 20% = 80% of the previous year's number of cases. After t years, the number of cases will be $C(t) = 10,000(0.8)^t$. Solving $C(t) = 1000$ graphically, we find that $t \approx 10.319$. It will take about 10.319 years.

(b) Solving $C(t) = 1$ graphically, we find that $t \approx 41.275$. It will take about 41.275 years.

33.

x	y	Δy
1	−1	
		2
2	1	
		2
3	3	
		2
4	5	

34.

x	y	Δy
1	1	
		−3
2	−2	
		−3
3	−5	
		−3
4	−8	

35.

x	y	Δy
1	1	
		3
2	4	
		5
3	9	
		7
4	16	

36.

x	y	ratio
1	8.155	
		2.718
2	22.167	
		2.718
3	60.257	
		2.718
4	163.794	

37. Since $\Delta x = 1$, the corresponding value of Δy is equal to the slope of the line. If the changes in x are constant for a linear function, then the corresponding changes in y are constant as well.

38. **(a)** When $t = 0$, $B = 100e^0 = 100$. There were 100 bacteria present initially.

(b) When $t = 6$, $B = 100e^{0.639(6)} \approx 6394.351$. After 6 hours, there are about 6394 bacteria.

(c) Solving $100e^{0.639t} = 200$ graphically, we find that $t \approx 1.000$. The population will be 200 after about 1 hour. Since the population doubles (from 100 to 200) in about 1 hour, the doubling time is about 1 hour.

39. (1, 6)

$$y = a \cdot b^x$$
$$6 = a \cdot b^1$$
$$\frac{6}{b} = a$$

(2, 9)

$$y = a \cdot b^x$$
$$9 = a \cdot b^2$$
$$\frac{9}{b^2} = a$$

$$\frac{6}{b} = \frac{9}{b^2}$$
$$6b^2 = 9b$$
$$6b = 9$$
$$b = \frac{3}{2}$$

$$a = \frac{6}{b}$$
$$a = \frac{6}{\frac{3}{2}}$$
$$a = 4$$

40. (−1, 8)

$$y = a \cdot b^x$$
$$8 = a \cdot b^{-1}$$
$$8b = a$$

(2, 0.125)

$$y = a \cdot b^x$$
$$0.125 = a \cdot b^2$$
$$\frac{0.125}{b^2} = a$$

$$8b = \frac{0.125}{b^2}$$
$$b^3 = 0.015625$$
$$b = \frac{1}{4}$$

$$a = 8b$$
$$a = 8 \cdot \frac{1}{4}$$
$$a = 2$$

41. False; $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$; it is positive.

42. True; $4^3 = (2^2)^3 = 2^6$

43. D: Let t be the number of years. We wish to solve $100(1.045)^t = 200$, which is equivalent to $1.045^t = 2$. Solving graphically, we find $t \approx 15.747$ years.

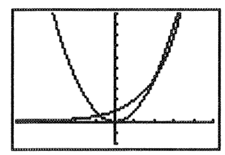
44. A

45. B

46. C

$$0 = 4 - e^x$$
$$e^x = 4$$
$$x = \ln 4$$
$$x = 1.386$$

47. **(a)**



[−5, 5] by [−2, 10]

In this window, it appears they cross twice, although a third crossing off-screen appears likely.

(b)

x	change in $y1$	change in $y2$
1		
	3	2
2		
	5	4
3		
	7	8
4		

It happens by the time $x = 4$.

(c) Solving graphically, $x \approx -0.7667$, $x = 2$, $x = 4$.

(d) The solution set is approximately $(-0.7667, 2) \cup (4, \infty)$.

48. Since $f(1) = 4.5$ we have $ka = 4.5$, and since $f(-1) = 0.5$ we have $ka^{-1} = 0.5$.

$$\frac{ka}{ka^{-1}} = \frac{4.5}{0.5}$$

Dividing, we have $a^2 = 9$

$$a = \pm 3$$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 3$. Then $ka = 4.5$ gives $3k = 4.5$, so $k = 1.5$. The values are $a = 3$ and $k = 1.5$.

49. Since $f(1) = 1.5$ we have $ka = 1.5$, and since $f(-1) = 6$ we have $ka^{-1} = 6$. Dividing, we have

$$\frac{ka}{ka^{-1}} = \frac{1.5}{6}$$
$$a^2 = 0.25$$
$$a = \pm 0.5$$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 0.5$. Then $ka = 1.5$ gives $0.5k = 1.5$, so $k = 3$. The values are $a = 0.5$ and $k = 3$.

Quick Quiz (Sections 1.1–1.3)

1. C, $m = -2$
$$y + 1 = -2(x - 3)$$
$$y = -2x + 6 - 1$$

2. D, $g(2) = 2(2) - 1 = 4 - 1 = 3$
$$f(3) = (3)^2 + 1 = 9 + 1 = 10$$

3. E, $A(t) = 5\left(\frac{1}{2}\right)^{t/8}$

We need to solve $A(t) = 1$. Solving graphically, we find $t \approx 18.6$ years.

4. **(a)** $(-\infty, \infty)$

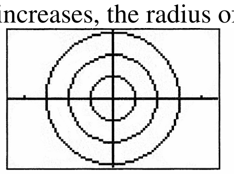
(b) $(-2, \infty)$

(c)
$$0 = e^{-x} - 2$$
$$e^{-x} = 2$$
$$-x = \ln 2$$
$$x = -0.693$$

Section 1.4 Parametric Equations (pp. 29–35)

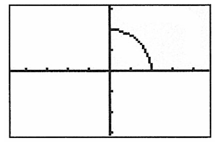
Exploration 1 Parametrizing Circles

1. Each is a circle with radius $|a|$. As $|a|$ increases, the radius of the circle increases.



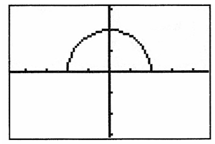
[−4.7, 4.7] by [−3.1, 3.1]

2. $0 \le t \le \frac{\pi}{2}$:



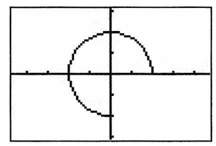
[−4.7, 4.7] by [−3.1, 3.1]

$0 \le t \le \pi$:



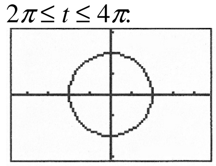
[−4.7, 4.7] by [−3.1, 3.1]

$0 \le t \le \frac{3\pi}{2}$:



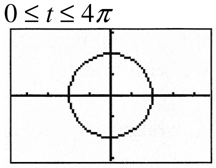
[−4.7, 4.7] by [−3.1, 3.1]

$2\pi \le t \le 4\pi$:



[−4.7, 4.7] by [−3.1, 3.1]

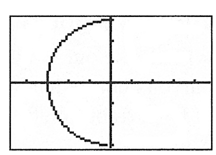
$0 \le t \le 4\pi$



[−4.7, 4.7] by [−3.1, 3.1]

Let d be the length of the parametric interval. If $d < 2\pi$, you get $\frac{d}{2\pi}$ of a complete circle. If $d = 2\pi$, you get the complete circle. If $d > 2\pi$, you get the complete circle but portions of the circle will be traced out more than once. For example, if $d = 4\pi$ the entire circle is traced twice.

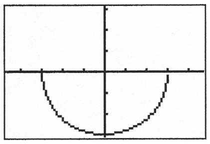
3.



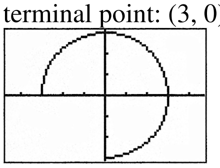
$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$

initial point: (0, 3)

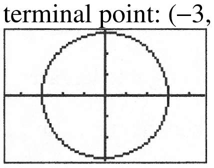
terminal point: (0, –3)



$\pi \le t \le 2\pi$
initial point: (–3, 0)
terminal point: (3, 0)



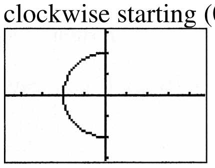
$\frac{3\pi}{2} \le t \le 3\pi$
initial point: (0, –3)
terminal point: (–3, 0)



$\pi \le t \le 5\pi$
initial point: (–3, 0)
terminal point: (–3, 0)

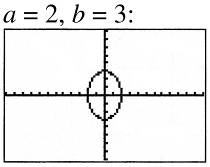
4. For $0 \le t \le 2\pi$, the complete circle is traced once clockwise beginning and ending at (2, 0). For $\pi \le t \le 3\pi$, the complete circle is traced once clockwise beginning and ending at (–2, 0).

For $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$ the half circle below is traced clockwise starting (0, –2) and ending at (0, 2).



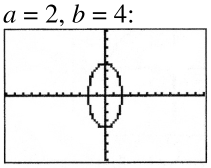
Exploration 2 Parametrizing Ellipses

1. $a = 2, b = 3$:



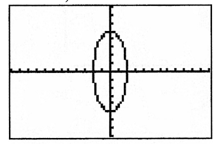
[−12, 12] by [−8, 8]

$a = 2, b = 4$:



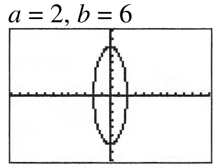
[−12, 12] by [−8, 8]

$a = 2, b = 5$:



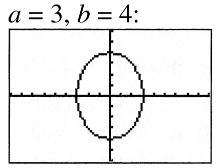
[−12, 12] by [−8, 8]

$a = 2, b = 6$



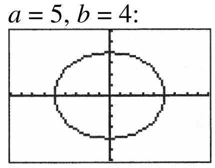
[−12, 12] by [−8, 8]

2. $a = 3, b = 4$:



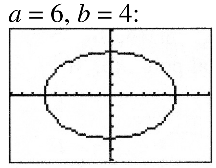
[−9, 9] by [−6, 6]

$a = 5, b = 4$:



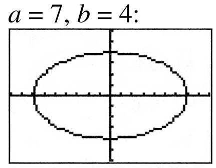
[−9, 9] by [−6, 6]

$a = 6, b = 4$:



[−9, 9] by [−6, 6]

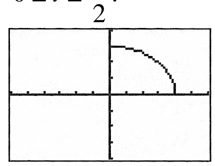
$a = 7, b = 4$:



[−9, 9] by [−6, 6]

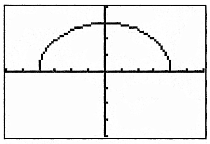
3. If $|a| > |b|$, then the major axis is on the x-axis and the minor on the y-axis. If $|a| < |b|$, then the major axis is on the y-axis and the minor on the x-axis.

4. $0 \le t \le \frac{\pi}{2}$:



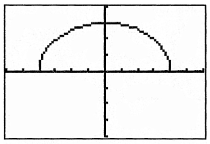
[−6, 6] by [−4, 4]

$0 \le t \le \pi$:



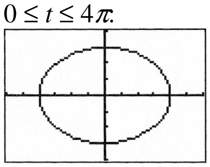
[−6, 6] by [−4, 4]

$0 \le t \le \dfrac{3\pi}{2}$:



[−6, 6] by [−4, 4]

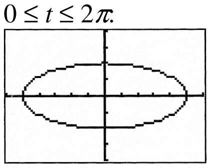
$0 \le t \le 4\pi$:



[−6, 6] by [−4, 4]

Let d be the length of the parametric interval. If $d < 2\pi$, you get $\dfrac{d}{2\pi}$ of a complete ellipse. If $d = 2\pi$, you get the complete ellipse. If $d > 2\pi$, you get the complete ellipse but portions of the ellipse will be traced out more than once. For example, if $d = 4\pi$ the entire ellipse is traced twice.

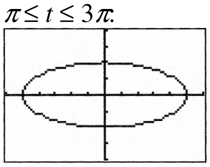
5. $0 \le t \le 2\pi$:



[−6, 6] by [−4, 4]

initial point: (5, 0)
terminal point: (5, 0)

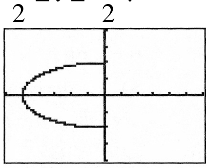
$\pi \le t \le 3\pi$:



[−6, 6] by [−4, 4]

initial point: (−5, 0)
terminal point: (−5, 0)

$\dfrac{\pi}{2} \le t \le \dfrac{3\pi}{2}$:



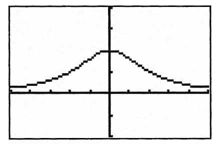
[−6, 6] by [−4, 4]

initial point: (0, −2)
terminal point: (0, 2)
Each curve is traced clockwise from the initial point to the terminal point.

Exploration 3 Graphing the Witch of Agnesi

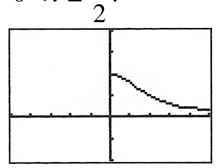
1. We used the parameter interval $[0, \pi]$ because our graphing calculator ignored the fact that the curve is not defined when $t = 0$ or π. The curve is traced from right to left across the screen. x ranges from $-\infty$ to ∞.

2. $-\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$:



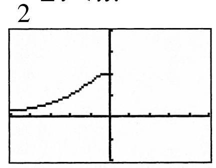
[−5, 5] by [−2, 4]

$0 < t \le \dfrac{\pi}{2}$:



[−5, 5] by [−2, 4]

$\dfrac{\pi}{2} \le t < \pi$:



[−5, 5] by [−2, 4]

For $-\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$, the entire graph described in part 1 is drawn. The left branch is drawn from right to left across the screen starting at the point (0, 2). Then the right branch is drawn from right to left across the screen stopping at the point (0, 2). If you leave out $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$, then the point (0, 2) is not drawn. For $0 < t \le \dfrac{\pi}{2}$, the right branch is drawn from right to left across the screen stopping at the point (0, 2). If you leave out $\dfrac{\pi}{2}$, then the point (0, 2) is not drawn.

For $\frac{\pi}{2} \le t < \pi$, the left branch is drawn from right to left across the screen starting at the point (0, 2). If you leave out $\frac{\pi}{2}$, then the point (0, 2) is not drawn.

3. If you replace $x = 2 \cot t$ by $x = -2 \cot t$, the same graph is drawn except it is traced from left to right across the screen. If you replace $x = 2 \cot t$ by $x = 2 \cot (\pi - t)$, the same graph is drawn except it is traced from left to right across the screen.

Quick Review 1.4

1. $m = \frac{3-8}{4-1} = \frac{-5}{3} = -\frac{5}{3}$

$y = -\frac{5}{3}(x-1)+8$

$y = -\frac{5}{3}x + \frac{29}{3}$

2. $y = -4$

3. $x = 2$

4. When $y = 0$, we have $\frac{x^2}{9} = 1$, so the x-intercepts are -3 and 3. When $x = 0$, we have $\frac{y^2}{16} = 1$, so the y-intercepts are -4 and 4.

5. When $y = 0$, we have $\frac{x^2}{16} = 1$, so the x-intercepts are -4 and 4. When $x = 0$, we have $-\frac{y^2}{9} = 1$, which has no real solution, so there are no y-intercepts.

6. When $y = 0$, we have $0 = x + 1$, so the x-intercept is -1. When $x = 0$, we have $2y^2 = 1$, so the y-intercepts are $-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

7. **(a)** $2(1)^2(1)+1^2 \stackrel{?}{=} 3$

$3 = 3$ Yes

(b) $2(-1)^2(-1)+(-1)^2 \stackrel{?}{=} 3$

$-2+1 \stackrel{?}{=} 3$

$-1 \ne 3$ No

(c) $2\left(\frac{1}{2}\right)^2(-2)+(-2)^2 \stackrel{?}{=} 3$

$-1+4 \stackrel{?}{=} 3$

$3 = 3$ Yes

8. **(a)** $9(1)^2 - 18(1) + 4(3)^2 = 27$

$9 - 18 + 36 \stackrel{?}{=} 27$

$27 = 27$ Yes

(b) $9(1)^2 - 18(1) + 4(-3)^2 \stackrel{?}{=} 27$

$9 - 18 + 36 \stackrel{?}{=} 27$

$27 = 27$ Yes

(c) $9(-1)^2 - 18(-1) + 4(3)^2 \stackrel{?}{=} 27$

$9 + 18 + 36 \stackrel{?}{=} 27$

$63 \ne 27$ No

9. **(a)** $2x + 3t = -5$

$3t = -2x - 5$

$t = \frac{-2x-5}{3}$

(b) $3y - 2t = -1$

$-2t = -3y - 1$

$2t = 3y + 1$

$t = \frac{3y+1}{2}$

10. **(a)** The equation is true for $a \ge 0$.

(b) The equation is equivalent to "$\sqrt{a^2} = a$ or $\sqrt{a^2} = -a$." Since $\sqrt{a^2} = a$ is true for $a \ge 0$ and $\sqrt{a^2} = -a$ is true for $a \le 0$, at least one of the two equations is true for all real values of a. Therefore, the given equation $\sqrt{a^2} = \pm a$ is true for all real values of a.

(c) The equation is true for all real values of a.

Section 1.4 Exercises

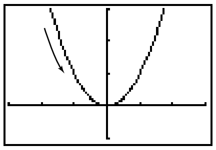
1. Graph (c); window: $[-4, 4]$ by $[-3, 3]$, $0 \le t \le 2\pi$

2. Graph (a); window: [–2, 2] by [–2, 2], $0 \le t \le 2\pi$

3. Graph (d); window: [–10, 10] by [–10, 10], $0 \le t \le 2\pi$

4. Graph (b); window: [–15, 15] by [–15, 15], $0 \le t \le 2\pi$

5. **(a)**



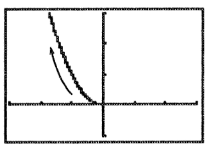
[–3, 3] by [–1, 3]

No initial or terminal point

(b) $y = 9t^2 = (3t)^2 = x^2$

The parametrized curve traces all of the parabola defined by $y = x^2$.

6. **(a)**



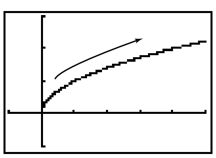
[–3, 3] by [–1, 3]

Initial point: (0, 0)
Terminal point: None

(b) $y = t = \left(-\sqrt{t}\right)^2 = x^2$

The parametrized curve traces the left half of the parabola defined by $y = x^2$ (or all of the curve defined by $x = -\sqrt{y}$).

7. **(a)**



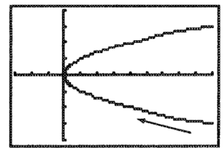
[–1, 5] by [–1, 3]

Initial point: (0, 0)
Terminal point: None

(b) $y = \sqrt{t} = \sqrt{x}$

The parametrized curve traces all of the curve defined by $y = \sqrt{x}$ (or the upper half of the parabola defined by $x = y^2$).

8. **(a)**



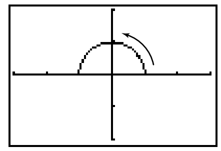
[–3, 9] by [–4, 4]

No initial or terminal point.

(b) $x = \sec^2 t - 1 = \tan^2 t = y^2$

The parametrized curve traces all of the parabola defined by $x = y^2$.

9. **(a)**



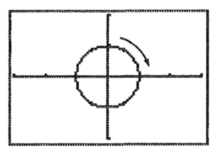
[–3, 3] by [–2, 2]

Initial point: (1, 0)
Terminal point: (–1, 0)

(b) $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

The parametrized curve traces the upper half of the circle defined by $x^2 + y^2 = 1$ (or all of the semicircle defined by $y = \sqrt{1 - x^2}$).

10. **(a)**



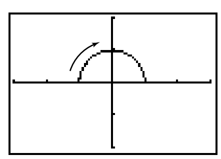
[–3, 3] by [–2, 2]

Initial and terminal point: (0, 1)

(b) $x^2 + y^2 = \sin^2(2\pi t) + \cos^2(2\pi t) = 1$

The parametrized curve traces all of the circle defined by $x^2 + y^2 = 1$.

11. **(a)**



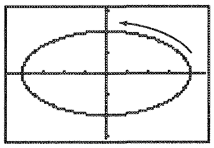
[–3, 3] by [–2, 2]

Initial point: (–1, 0)
Terminal point: (1, 0)

(b) $x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1$

The parametrized curve traces the upper half of the circle defined by $x^2 + y^2 = 1$ (or all of the semicircle defined by $y = \sqrt{1 - x^2}$).

12. (a)



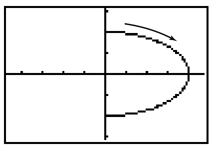
[−4.7, 4.7] by [−3.1, 3.1]

Initial and terminal point: (4, 0)

(b) $\left(\frac{x}{4}\right)^2+\left(\frac{y}{2}\right)^2=\cos^2 t+\sin^2 t=1$

The parametrized curve traces all of the ellipse defined by $\left(\frac{x}{4}\right)^2+\left(\frac{y}{2}\right)^2=1.$

13. (a)



[–4.7, 4.7] by [–3.1, 3.1]

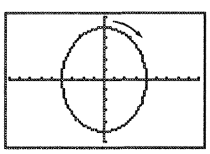
Initial point: (0, 2)
Terminal point: (0, –2)

(b) $\left(\frac{x}{4}\right)^2+\left(\frac{y}{2}\right)^2=\sin^2 t+\cos^2 t=1$

The parametrized curve traces the right half of the ellipse defined by

$\left(\frac{x}{4}\right)^2+\left(\frac{y}{2}\right)^2=1$ (or all of the curve defined by $x=2\sqrt{4-y^2}$).

14. (a)



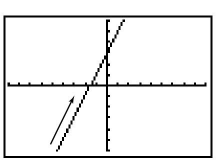
[−9, 9] by [−6, 6]

Initial and terminal point: (0, 5)

(b) $\left(\frac{x}{4}\right)^2+\left(\frac{y}{5}\right)^2=\sin^2 t+\cos^2 t=1$

The parametrized curve traces all of the ellipse defined by $\left(\frac{x}{4}\right)^2+\left(\frac{y}{5}\right)^2=1.$

15. (a)



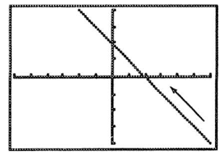
[–9, 9] by [–6, 6]

No initial or terminal point.

(b) $y = 4t - 7 = 2(2t - 5) + 3 = 2x + 3$

The parametrized curve traces all of the line defined by $y = 2x + 3$.

16. (a)



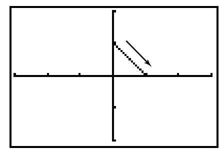
[−6, 6] by [−4, 4]

No initial or terminal point.

(b) $y = 1 + t = 2 - (1 - t) = 2 - x = -x + 2$

The parametrized curve traces all of the line defined by $y = -x + 2$.

17. (a)



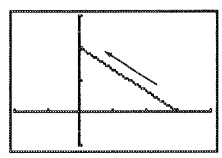
[–3, 3] by [–2, 2]

Initial point: (0, 1)
Terminal point: (1, 0)

(b) $y = 1 - t = 1 - x = -x + 1$

The Cartesian equation is $y = -x + 1$. The portion traced by the parametrized curve is the segment from (0, 1) to (1, 0).

18. (a)



[−2, 4] by [−1, 3]

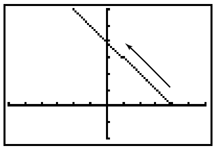
Initial point: (3, 0)
Terminal point: (0, 2)

(b)
$$\begin{aligned} y &= 2t \\ &= (2t-2)+2 \\ &= -\frac{2}{3}(3-3t)+2 \\ &= -\frac{2}{3}x+2 \end{aligned}$$

The Cartesian equation is $y=-\frac{2}{3}x+2.$

The portion traced by the curve is the segment from (3, 0) to (0, 2).

19. **(a)**



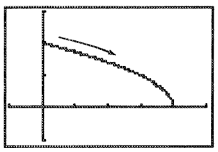
[–6, 6] by [–2, 6]

Initial point: (4, 0)
Terminal point: None

(b) $y = \sqrt{t} = 4 - \left(4 - \sqrt{t}\right) = 4 - x = -x + 4$

The parametrized curve traces the portion of the line defined by $y = -x + 4$ to the left of (4, 0), that is, for $x \le 4$.

20. **(a)**



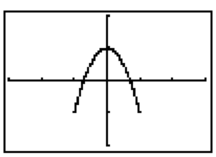
[–1, 5] by [–1, 3]

Initial point: (0, 2)
Terminal point: (4, 0)

(b) $y = \sqrt{4 - t^2} = \sqrt{4 - x}$

The parametrized curve traces the right portion of the curve defined by $y = \sqrt{4 - x}$, that is, for $x \ge 0$.

21. **(a)**



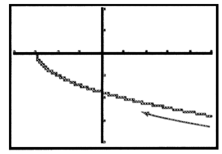
[–3, 3] by [–2, 2]

No initial or terminal point, since the t-interval has no beginning or end. The curve is traced and retraced in both directions.

(b)
$$
\begin{aligned}
y &= \cos 2t \\
&= \cos^2 t - \sin^2 t \\
&= 1 - 2\sin^2 t \\
&= 1 - 2x^2 \\
&= -2x^2 + 1
\end{aligned}
$$

The parametrized curve traces the portion of the parabola defined by $y = -2x^2 + 1$ corresponding to $-1 \le x \le 1$.

22. **(a)**



[–4, 5] by [–4, 2]

Initial point: None
Terminal point: (–3, 0)

(b) $x = t^2 - 3 = y^2 - 3$

The parametrized curve traces the lower half of the parabola defined by $x = y^2 - 3$ (or all of the curve defined by $y = -\sqrt{x + 3}$).

23. Using (–1, –3) we create the parametric equations $x = -1 + at$ and $y = -3 + bt$, representing a line which goes through (–1, –3) at $t = 0$. We determine a and b so that the line goes through (4, 1) when $t = 1$.
Since $4 = -1 + a$, $a = 5$.
Since $1 = -3 + b$, $b = 4$.
Therefore, one possible parametrization is $x = -1 + 5t$, $y = -3 + 4t$, $0 \le t \le 1$.

24. Using (–1, 3) we create the parametric equations $x = -1 + at$ and $y = 3 + bt$, representing a line which goes through (–1, 3) at $t = 0$. We determine a and b so that the line goes through (3, –2) at $t = 1$.
Since $3 = -1 + a$, $a = 4$.
Since $-2 = 3 + b$, $b = -5$.
Therefore, one possible parametrization is $x = -1 + 4t$, $y = 3 - 5t$, $0 \le t \le 1$.

25. The lower half of the parabola is given by $x = y^2 + 1$ for $y \le 0$. Substituting t for y, we obtain one possible parametrization: $x = t^2 + 1$, $y = t$, $t \le 0$.

26. The vertex of the parabola is at (–1, –1), so the left half of the parabola is given by $y = x^2 + 2x$ for $x \le -1$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = t^2 + 2t$, $t \le -1$.

27. For simplicity, we assume that x and y are linear functions of t and that point (x, y) starts at (2, 3) for $t = 0$ and passes through (–1, –1) at $t = 1$. Then $x = 2 + at$ and $y = 3 + bt$.
Since $-1 = 2 + a$, $a = -3$
Since $-1 = 3 + b$, $b = -4$
Therefore, one possible parameterization is $x = 2 - 3t$, $y = 3 - 4t$, $t \ge 0$.

28. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at $(-1, 2)$ for $t = 0$ and passes through $(0, 0)$ at $t = 1$. Then $x = -1 + at$ and $y = 2 + bt$.
Since $0 = -1 + a$, $a = 1$
Since $0 = 2 + b$, $b = -2$
Therefore, one possible parametrization is:
$x = -1 + t$, $y = 2 - 2t$, $t \geq 0$.

29. The graph is in Quadrant I when $0 < y < 2$, which corresponds to $1 < t < 3$. To confirm, note that $x(1) = 2$ and $x(3) = 0$.

30. The graph is in Quadrant II when $2 < y \leq 4$, which corresponds to $3 < t \leq 5$. To confirm, note that $x(3) = 0$ and $x(5) = -2$.

31. The graph is in Quadrant III when $-6 \leq y < -4$, which corresponds to $-5 \leq t < -3$. To confirm, note that $x(-5) = -2$ and $x(-3) = 0$.

32. The graph is in Quadrant IV when $-4 < y < 0$, which corresponds to $-3 < t < 1$. To confirm, note that $x(-3) = 0$ and $x(1) = 2$.

33. The graph of $y = x^2 + 2x + 2$ lies in Quadrant I for all $x > 0$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = t^2 + 2t + 2$, $t > 0$.

34. The graph of $y = \sqrt{x+3}$ lies in Quadrant I for all $x > 0$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = \sqrt{t+3}$, $t > 0$.

35. Possible answers:

(a) $x = a \cos t$, $y = -a \sin t$, $0 \leq t \leq 2\pi$

(b) $x = a \cos t$, $y = a \sin t$, $0 \leq t \leq 2\pi$

(c) $x = a \cos t$, $y = -a \sin t$, $0 \leq t \leq 4\pi$

(d) $x = a \cos t$, $y = a \sin t$, $0 \leq t \leq 4\pi$

36. Possible answers:

(a) $x = -a \cos t$, $y = b \sin t$, $0 \leq t \leq 2\pi$

(b) $x = -a \cos t$, $y = -b \sin t$, $0 \leq t \leq 2\pi$

(c) $x = -a \cos t$, $y = b \sin t$, $0 \leq t \leq 4\pi$

(d) $x = -a \cos t$, $y = -b \sin t$, $0 \leq t \leq 4\pi$

37. False. It is an ellipse.

38. True; circle starting at $(2, 0)$ and ending at $(2, 0)$.

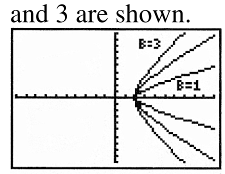
39. D

40. C

41. A

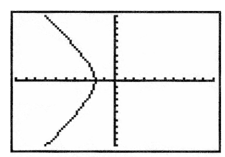
42. E

43. **(a)** The resulting graph appears to be the right half of a hyperbola in the first and fourth quadrants. The parameter a determines the x-intercept. The parameter b determines the shape of the hyperbola. If b is smaller, the graph has less steep slopes and appears "sharper." If b is larger, the slopes are steeper and the graph appears more "blunt." The graphs for $a = 2$ and $b = 1, 2,$ and 3 are shown.



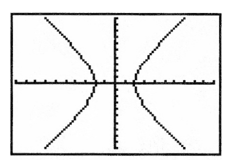
$[-10, 10]$ by $[-10, 10]$

(b)



$[-10, 10]$ by $[-10, 10]$
This appears to be the left half of the same hyberbola.

(c)



$[-10, 10]$ by $[-10, 10]$
One must be careful because both sec t and tan t are discontinuous at these points. This might cause the grapher to include extraneous lines (the asymptotes of the hyperbola) in its graph. The extraneous lines can be avoided by using the grapher's dot mode instead of connected mode.

(d) Note that $\sec^2 t - \tan^2 t = 1$ by a standard trigonometric identity. Substituting $\frac{x}{a}$ for $\sec t$ and $\frac{y}{b}$ for $\tan t$ gives

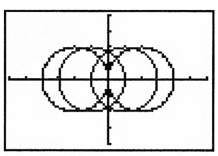
$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1.$$

(e) This changes the orientation of the hyperbola. In this case, b determines the y-intercept of the hyperbola, and a determines the shape. The parameter interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ gives the upper half of the hyperbola. The parameter interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ gives the lower half. The same values of t cause discontinuities and may add extraneous lines to the graph.

Substituting $\frac{y}{b}$ for sec t and $\frac{x}{a}$ for tan t in the identity $\sec^2 t - \tan^2 t = 1$ gives

$$\left(\frac{y}{b}\right)^2 - \left(\frac{x}{a}\right)^2 = 1.$$

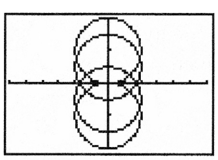
44. **(a)**



[−6, 6] by [−4, 4]

The graph is a circle of radius 2 centered at $(h, 0)$. As h changes, the graph shifts horizontally.

(b)



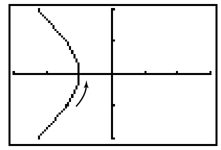
[−6, 6] by [−4, 4]

The graph is a circle of radius 2 centered at $(0, k)$. As k changes, the graph shifts vertically.

(c) Since the circle is to be centered at $(2, -3)$, we use $h = 2$ and $k = -3$. Since a radius of 5 is desired, we need to change the coefficients of cos t and sin t to 5.
$x = 5\cos t + 2$, $y = 5\sin t - 3$, $0 \le t \le 2\pi$

(d) $x = 5\cos t - 3$, $y = 2\sin t + 4$, $0 \le t \le 2\pi$

45. **(a)**



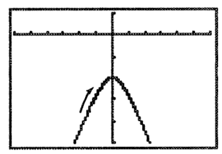
[−3, 3] by [−2, 2]

No initial or terminal point. Note that it may be necessary to use a t-interval such as [−1.57, 1.57] or use dot mode in order to avoid "asymptotes" showing on the calculator screen.

(b) $x^2 - y^2 = \sec^2 t - \tan^2 t = 1$

The parametrized curve traces the left branch of the hyperbola defined by $x^2 - y^2 = 1$ (or all of the curve defined by $x = -\sqrt{y^2 + 1}$).

46. **(a)**



[−6, 6] by [−5, 1]

No initial or terminal point. Note that it may be necessary to use a t-interval such as [−1.57, 1.57] or use dot mode in order to avoid "asymptotes" showing on the calculator screen.

(b) $\left(\frac{y}{2}\right)^2 - x^2 = \sec^2 t - \tan^2 t = 1$

The parametrized curve traces the lower branch of the hyperbola defined by $\left(\frac{y}{2}\right)^2 - x^2 = 1$ (or all of the curve defined by $y = -2\sqrt{x^2 + 1}$)

47. Note that $m\angle OAQ = t$, since alternate interior angles formed by a transversal of parallel lines are congruent. Therefore, $\tan r = \frac{OQ}{AQ} = \frac{2}{x}$, so $x \frac{2}{\tan t} = 2\cot t$. Now, by equation (iii), we know that

$$AB = \frac{(AQ)^2}{AO} = \left(\frac{AQ}{AO}\right)(AQ) = (\cos t)(x) = (\cos t)(2\cot t) = \frac{2\cos^2 t}{\sin t}.$$

Then equation (ii) gives $y = 2 - AB\sin t = 2 - \frac{2\cos^2 t}{\sin t} \cdot \sin t = 2 - 2\cos^2 t = 2\sin^2 t.$

The parametric equations are:

$x = 2\cot t,\ y = 2\sin^2 t,\ 0 < t < \pi$

Note: Equation (iii) may not be immediately obvious, but it may be justified as follows. Sketch segment QB. Then $\angle OBQ$ is a right angle, so $\Delta ABQ \sim \Delta AQO$, which gives $\frac{AB}{AQ} = \frac{AQ}{AO}$.

48. **(a)** If $x_2 = x_1$ then the line is a vertical line and the first parametric equation gives $x = x_1$, while the second will give all real values for y since it cannot be the case that $y_2 = y_1$ as well. Otherwise, solving the first equation for t gives $t = \frac{(x - x_1)}{(x_2 - x_1)}$.

Substituting that into the second equation gives $y = y_1 + \left[\frac{(y_2 - y_1)}{(x_2 - x_1)}\right](x - x_1)$ which is the point-slope form of the equation for the line through $(x_1,\ y_1)$ and $(x_2,\ y_2)$.

Note that the first equation will cause x to take on all real values, because $(x_2 - x_1)$ is not zero. Therefore, all of the points on the line will be traced out.

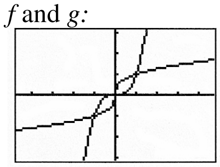
(b) Use the equations for x and y given in part (a), with $0 \le t \le 1$.

Section 1.5 Inverse Functions and Logarithms (pp. 36–44)

Exploration 1 Testing for Inverses Graphically

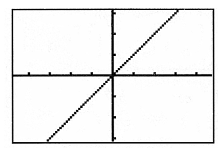
1. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverses of each other.

(a) f and g:



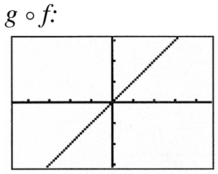
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(b) $f \circ g$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

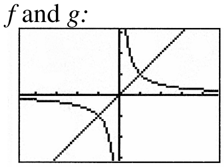
(c) $g \circ f$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

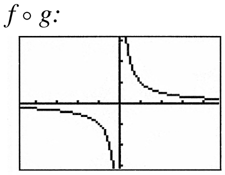
2. It appears that $f \circ g = g \circ f = g$, suggesting that f may be the identity function.

(a) f and g:



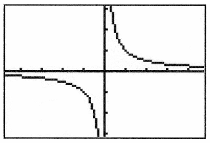
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(b) $f \circ g$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

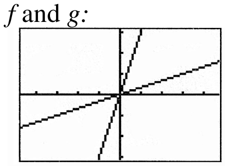
(c) $g \circ f$:



[−4.7, 4.7] by [−3.1, 3.1]

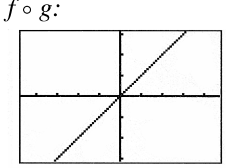
3. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverses of each other.

(a) f and g:



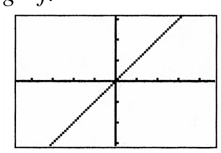
[−4.7, 4.7] by [−3.1, 3.1]

(b) $f \circ g$:



[−4.7, 4.7] by [−3.1, 3.1]

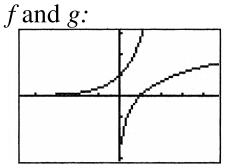
(c) $g \circ f$:



[−4.7, 4.7] by [−3.1, 3.1]

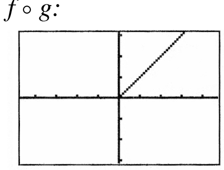
4. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverses of each other. (Notice that the domain of $f \circ g$ is $(0, \infty)$ and the domain of $g \circ f$ is $(-\infty, \infty)$.)

(a) f and g:



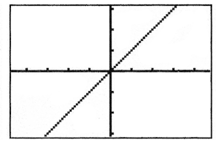
[−4.7, 4.7] by [−3.1, 3.1]

(b) $f \circ g$:



[−4.7, 4.7] by [−3.1, 3.1]

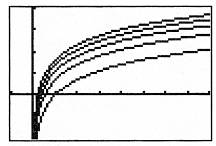
(c) $g \circ f$:



[−4.7, 4.7] by [−3.1, 3.1]

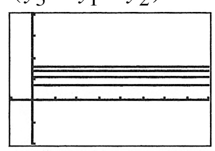
Exploration 2 Supporting the Product Rule

1. They appear to be vertical translations of each other.



[−1, 8] by [−2, 4]

2. This graph suggests that each difference $(y_3 = y_1 - y_2)$ is a constant.



[−1, 8] by [−2, 4]

3.
$$\begin{aligned} y_3 &= y_1 - y_2 \\ &= \ln(ax) - \ln x \\ &= \ln a + \ln x - \ln x \\ &= \ln a \end{aligned}$$

Thus, the difference $y_3 = y_1 - y_2$ is the constant $\ln a$.

Quick Review 1.5

1. $(f \circ g)(1) = f(g(1)) = f(2) = 1$

2. $(g \circ f)(-7) = g(f(-7)) = g(-2) = 5$

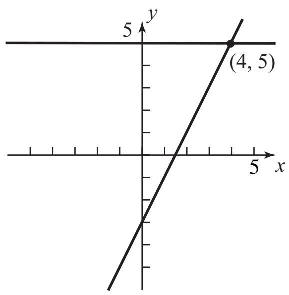
3.
$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(x^2+1) \\ &= \sqrt[3]{(x^2+1)-1} \\ &= \sqrt[3]{x^2} \\ &= x^{2/3} \end{aligned}$$

4.
$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g\left(\sqrt[3]{3x-1}\right) \\ &= \left(\sqrt[3]{3x-1}\right)^2 + 1 \\ &= (x-1)^{2/3} + 1 \end{aligned}$$

5. Substituting t for x, one possible answer is:
$x = t,\ y = \dfrac{1}{t-1},\ t \ge 2.$

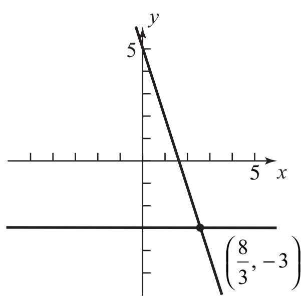
6. Substituting t for x, one possible answer is:
$x = t, y = t, t < -3$

7.



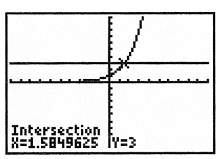
(4, 5)

8.



$\left(\dfrac{8}{3}, -3\right)$

9. **(a)**

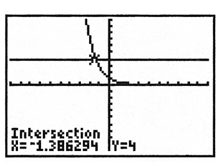


[−10, 10] by [−10, 10]

(1.58, 3)

(b) No points of intersection, since $2^x > 0$ for all values of x.

10. **(a)**



[−10, 10] by [−10, 10]

(–1.39, 4)

(b) No points of intersection, since $e^{-x} > 0$ for all values of x.

Section 1.5 Exercises

1. No, since (for example) the horizontal line $y = 2$ intersects the graph twice.

2. Yes, since each horizontal line intersects the graph only once.

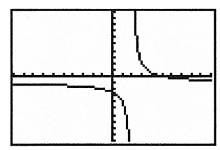
3. Yes, since each horizontal line intersects the graph at most once.

4. No, since (for example) the horizontal line $y = 0.5$ intersects the graph twice.

5. Yes, since each horizontal line intersects the graph only once.

6. No, since (for example) the horizontal line $y = 2$ intersects the graph at more than one point.

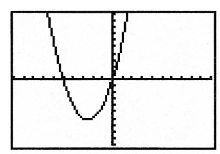
7.



[−10, 10] by [−10, 10]

Yes, the function is one-to-one since each horizontal line intersects the graph at most once, so it has an inverse function.

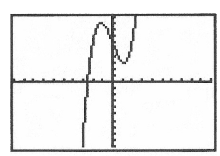
8.



[−10, 10] by [−10, 10]

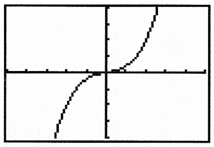
No, the function is not one-to-one since (for example) the horizontal line $y = 0$ intersects the graph twice, so it does not have an inverse function.

9.



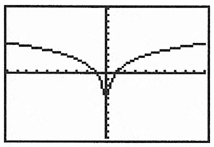
[−10, 10] by [−10, 10]

No, the function is not one-to-one since (for example) the horizontal line $y = 5$ intersects the graph more than once, so it does not have an inverse function.

10. 

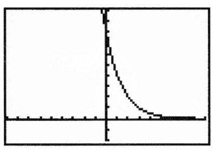
[−5, 5] by [−20, 20]

Yes, the function is one-to-one since each horizontal line intersects the graph only once, so it has an inverse function.

11. 

[−10, 10] by [−10, 10]

No, the function is not one-to-one since each horizontal line intersects the graph twice, so it does not have an inverse function.

12. 

[−9, 9] by [−2, 10]

Yes, the function is one-to-one since each horizontal line intersects the graph at most once, so it has an inverse function.

13.
$$y = 2x+3$$
$$y-3 = 2x$$
$$\frac{y-3}{2} = x$$
Interchange x and y.
$$\frac{x-3}{2} = y$$
$$f^{-1}(x) = \frac{x-3}{2}$$
Verify.
$$\begin{aligned}(f \circ f^{-1})(x) &= f\left(\frac{x-3}{2}\right)\\ &= 2\left(\frac{x-3}{2}\right)+3\\ &= (x-3)+3\\ &= x\end{aligned}$$
$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(2x+3)\\ &= \frac{(2x+3)-3}{2}\\ &= \frac{2x}{2}\\ &= x\end{aligned}$$

14.
$$y = 5-4x$$
$$4x = 5-y$$
$$x = \frac{5-y}{4}$$
Interchange x and y.
$$y = \frac{5-x}{4}$$
$$f^{-1}(x) = \frac{5-x}{4}$$
Verify.
$$\begin{aligned}(f \circ f^{-1})(x) &= f\left(\frac{5-x}{4}\right)\\ &= 5-4\left(\frac{5-x}{4}\right)\\ &= 5-(5-x)\\ &= x\end{aligned}$$
$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(5-4x)\\ &= \frac{5-(5-4x)}{4}\\ &= \frac{4x}{4}\\ &= x\end{aligned}$$

15.
$$y = x^3-1$$
$$y+1 = x^3$$
$$(y+1)^{1/3} = x$$
Interchange x and y.
$$(x+1)^{1/3} = y$$
$$f^{-1}(x) = (x+1)^{1/3} \text{ or } \sqrt[3]{x+1}$$
Verify.
$$\begin{aligned}(f \circ f^{-1})(x) &= f\left(\sqrt[3]{x+1}\right)\\ &= \left(\sqrt[3]{x+1}\right)^3-1\\ &= (x+1)-1\\ &= x\end{aligned}$$
$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(x^3-1)\\ &= \sqrt[3]{(x^3-1)+1}\\ &= \sqrt[3]{x^3}\\ &= x\end{aligned}$$

16.
$$y = x^2+1,\ x \ge 0$$
$$y-1 = x^2,\ x \ge 0$$
$$\sqrt{y-1} = x$$
Interchange x and y.
$$\sqrt{x-1} = y$$
$$f^{-1}(x) = \sqrt{x-1} \text{ or } (x-1)^{1/2}$$
Verify. For $x \ge 1$ (the domain of f^{-1}),

$$(f \circ f^{-1})(x) = f\left(\sqrt{x-1}\right)$$
$$= \left(\sqrt{x-1}\right)^2 + 1$$
$$= (x-1) + 1$$
$$= x$$

For $x > 0$, (the domain of f),

$$(f^{-1} \circ f)(x) = f^{-1}(x^2 + 1)$$
$$= \sqrt{(x^2+1) - 1}$$
$$= \sqrt{x^2}$$
$$= |x|$$
$$= x$$

17. $y = x^2,\ x \le 0$

$y = -\sqrt{y}$

Interchange x and y.

$y = -\sqrt{x}$

$f^{-1}(x) = -\sqrt{x}$ or $-x^{1/2}$

Verify.

For $x \ge 0$ (the domain of f^{-1}),

$$(f \circ f^{-1})(x) = f\left(-\sqrt{x}\right) = \left(-\sqrt{x}\right)^2 = x$$

For $x \le 0$, (the domain of f),

$$(f^{-1} \circ f)(x) = f^{-1}(x^2)$$
$$= -\sqrt{x^2}$$
$$= -|x|$$
$$= x$$

18. $y = x^{2/3},\ x \ge 0$

$y^{3/2} = (x^{2/3})^{3/2},\ x \ge 0$

$y^{3/2} = x$

Interchange x and y.

$x^{3/2} = y$

$f^{-1}(x) = x^{3/2}$

Verify.

For $x \ge 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f(x^{3/2}) = (x^{3/2})^{2/3} = x$ for $x \ge 0$, (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(x^{2/3}) = (x^{2/3})^{3/2} = |x| = x$

19. $y = -(x-2)^2,\ x \le 2$

$(x-2)^2 = -y,\ x \le 2$

$x - 2 = -\sqrt{-y}$

$x = 2 - \sqrt{-y}$

Interchange x and y.

$y = 2 - \sqrt{-x}$

$f^{-1}(x) = 2 - \sqrt{-x}$ or $2 - (-x)^{1/2}$

Verify. For $x \le 0$ (the domain of f^{-1}),

$$(f \circ f^{-1})(x) = f\left(2 - \sqrt{-x}\right)$$
$$= -\left[\left(2 - \sqrt{-x}\right) - 2\right]^2$$
$$= -\left(-\sqrt{-x}\right)^2$$
$$= -|x|$$
$$= x$$

For $x \le 2$ (the domain of f),

$$(f^{-1} \circ f)(x) = f^{-1}(-(x-2)^2)$$
$$= 2 - \sqrt{(x-2)^2}$$
$$= 2 - |x-2|$$
$$= 2 + (x-2)$$
$$= x$$

20. $y = (x^2 + 2x + 1),\ x \ge -1$

$y = (x+1)^2,\ x \ge -1$

$\sqrt{y} = x + 1$

$\sqrt{y} - 1 = x$

Interchange x and y.

$\sqrt{x} - 1 = y$

$f^{-1}(x) = \sqrt{x} - 1$ or $x^{1/2} - 1$

Verify. For $x \ge 0$ (the domain of f^{-1}),

$$(f \circ f^{-1})(x) = f\left(\sqrt{x} - 1\right)$$
$$= \left[\left(\sqrt{x}-1\right)^2 + 2\left(\sqrt{x}-1\right) + 1\right]$$
$$= \left(\sqrt{x}\right)^2 - 2\sqrt{x} + 1 + 2\sqrt{x} - 2 + 1$$
$$= \left(\sqrt{x}\right)^2$$
$$= x$$

For $x \ge -1$ (the domain of f),

$$(f^{-1} \circ f)(x) = f^{-1}(x^2 + 2x + 1)$$
$$= \sqrt{x^2 + 2x + 1} - 1$$
$$= \sqrt{(x+1)^2} - 1$$
$$= |x+1| - 1$$
$$= (x+1) - 1$$
$$= x$$

21. $y = \dfrac{1}{x^2},\ x > 0$

$x^2 = \dfrac{1}{y},\ x > 0$

$x = \sqrt{\dfrac{1}{y}} = \dfrac{1}{\sqrt{y}}$

Interchange x and y.

$$y = \frac{1}{\sqrt{x}}$$

$$f^{-1}(x) = \frac{1}{\sqrt{x}} \text{ or } \frac{1}{x^{1/2}}$$

Verify. For $x > 0$ (the domain of f^{-1}),

$$(f \circ f^{-1})(x) = f\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\left(\frac{1}{\sqrt{x}}\right)^2} = x$$

For $x > 0$ (the domain of f),

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}\left(\frac{1}{x^2}\right)\\ &= \frac{1}{\sqrt{\frac{1}{x^2}}}\\ &= \sqrt{x^2}\\ &= |x|\\ &= x\end{aligned}$$

22. $y = \dfrac{1}{x^3}$

$$x^3 = \frac{1}{y}$$

$$x = \sqrt[3]{\frac{1}{y}} = \frac{1}{\sqrt[3]{y}}$$

Interchange x and y.

$$y = \frac{1}{\sqrt[3]{x}}$$

$$f^{-1}(x) = \frac{1}{\sqrt[3]{x}} \text{ or } \frac{1}{x^{1/3}}$$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{1}{\sqrt[3]{x}}\right) = \frac{1}{\left(\frac{1}{\sqrt[3]{x}}\right)^3} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{1}{x^3}\right) = \frac{1}{\sqrt[3]{\frac{1}{x^3}}} = x$$

23. $y = \dfrac{2x+1}{x+3}$

$$\begin{aligned}xy + 3y &= 2x+1\\ xy - 2x &= 1 - 3y\\ (y-2)x &= 1-3y\\ x &= \frac{1-3y}{y-2}\end{aligned}$$

Interchange x and y.

$$y = \frac{1-3x}{x-2}$$

$$f^{-1}(x) = \frac{1-3x}{x-2}$$

Verify.

$$\begin{aligned}(f \circ f^{-1})(x) &= f\left(\frac{1-3x}{x-2}\right)\\ &= \frac{2\left(\frac{1-3x}{x-2}\right)+1}{\frac{1-3x}{x-2}+3}\\ &= \frac{2(1-3x)+(x-2)}{(1-3x)+3(x-2)}\\ &= \frac{-5x}{-5}\\ &= x\end{aligned}$$

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}\left(\frac{2x+1}{x+3}\right)\\ &= \frac{1-3\left(\frac{2x+1}{x+3}\right)}{\frac{2x+1}{x+3}-2}\\ &= \frac{(x+3)-3(2x+1)}{(2x+1)-2(x+3)}\\ &= \frac{-5x}{-5}\\ &= x\end{aligned}$$

24. $y = \dfrac{x+3}{x-2}$

$$\begin{aligned}xy - 2y &= x+3\\ xy - x &= 2y+3\\ x(y-1) &= 2y+3\\ x &= \frac{2y+3}{y-1}\end{aligned}$$

Interchange x and y.

$$y = \frac{2x+3}{x-1}$$

$$f^{-1}(x) = \frac{2x+3}{x-1}$$

Verify.

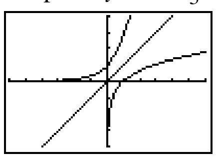
$$\begin{aligned}(f \circ f^{-1})(x) &= f\left(\frac{2x+3}{x-1}\right)\\ &= \frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}\\ &= \frac{(2x+3)+3(x-1)}{(2x+3)-2(x-1)}\\ &= \frac{5x}{5}\\ &= x\end{aligned}$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{x+3}{x-2}\right)$$
$$= \frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}$$
$$= \frac{2(x+3)+3(x-2)}{(x+3)-(x-2)}$$
$$= \frac{5x}{5}$$
$$= x$$

25. Graph of f: $x_1 = t,\ y_1 = e^t$

Graph of f^{-1}: $x_2 = e^t,\ \ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

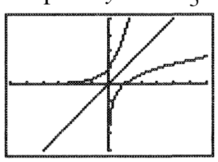


[–6, 6] by [–4, 4]

26. Graph of f: $x_1 = t,\ y_1 = 3^t$

Graph of f^{-1}: $x_2 = 3^t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

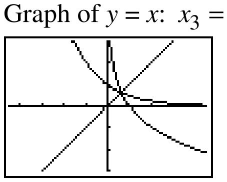


[−6, 6] by [−4, 4]

27. Graph of f: $x_1 = t,\ y_1 = 2^{-t}$

Graph of f^{-1}: $x_2 = 2^{-t},\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

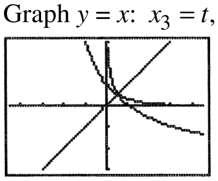


[–4.5, 4.5] by [–3, 3]

28. Graph of f: $x_1 = t,\ y_1 = 3^{-t}$

Graph of f^{-1}: $x_2 = 3^{-t},\ y_2 = t$

Graph $y = x$: $x_3 = t,\ y_3 = t$

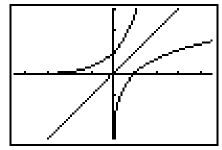


[−4.5, 4.5] by [−3, 3]

29. Graph of f: $x_1 = t,\ y_1 = \ln t$

Graph of f^{-1}: $x_2 = \ln t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

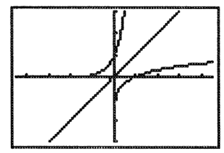


[–4.5, 4.5] by [–3, 3]

30. Graph of f: $x_1 = t,\ y_1 = \log t$

Graph of f^{-1}: $x_2 = \log t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

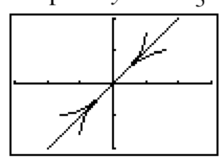


[−4.5, 4.5] by [−3, 3]

31. Graph of f: $x_1 = t,\ y_1 = \sin^{-1} t$

Graph of f^{-1}: $x_2 = \sin^{-1} t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

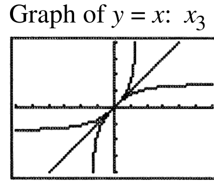


[–3, 3] by [–2, 2]

32. Graph of f: $x_1 = t,\ y_1 = \tan^{-1} t$

Graph of f^{-1}: $x_2 = \tan^{-1} t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$



[−6, 6] by [−4, 4]

33.
$$(1.045)^t = 2$$
$$\ln(1.045)^t = \ln 2$$
$$t \ln 1.045 = \ln 2$$
$$t = \frac{\ln 2}{\ln 1.045} \approx 15.75$$

34.
$$\begin{aligned} e^{0.05t} &= 3 \\ \ln e^{0.05t} &= \ln 3 \\ 0.05t &= \ln 3 \\ t &= \frac{\ln 3}{0.05} = 20\ln 3 \approx 21.97 \end{aligned}$$

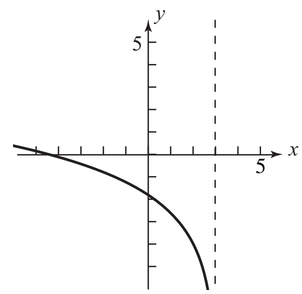
35.
$$\begin{aligned} e^x + e^{-x} &= 3 \\ e^x - 3 + e^{-x} &= 0 \\ e^x(e^x - 3 + e^{-x}) &= e^x(0) \\ (e^x)^2 - 3e^x + 1 &= 0 \\ e^x &= \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} \\ e^x &= \frac{3 \pm \sqrt{5}}{2} \\ x &= \ln\left(\frac{3 \pm \sqrt{5}}{2}\right) \approx -0.96 \text{ or } 0.96 \end{aligned}$$

36.
$$\begin{aligned} 2^x + 2^{-x} &= 5 \\ 2^x - 5 + 2^{-x} &= 0 \\ 2^x(2^x - 5 + 2^{-x}) &= 2^x(0) \\ (2^x)^2 - 5(2^x) + 1 &= 0 \\ 2^x &= \frac{5 \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)} \\ 2^x &= \frac{5 \pm \sqrt{21}}{2} \\ x &= \log_2\left(\frac{5 \pm \sqrt{21}}{2}\right) \approx -2.26 \text{ or } 2.26 \end{aligned}$$

37.
$$\begin{aligned} \ln y &= 2t + 4 \\ e^{\ln y} &= e^{2t+4} \\ y &= e^{2t+4} \end{aligned}$$

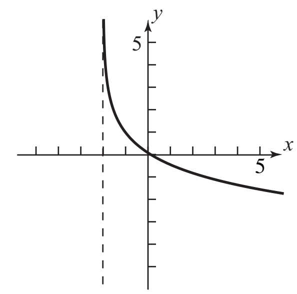
38.
$$\begin{aligned} \ln(y-1) - \ln 2 &= x + \ln x \\ \ln(y-1) &= x + \ln x + \ln 2 \\ e^{\ln(y-1)} &= e^{x + \ln x + \ln 2} \\ y - 1 &= e^x(x)(2) \\ y &= 2xe^x + 1 \end{aligned}$$

39.



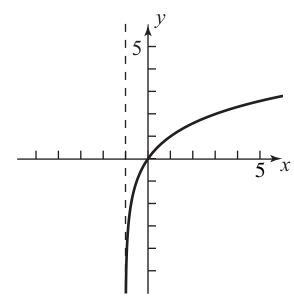
Domain: $(-\infty, 3)$
Range: $(-\infty, \infty)$

40.



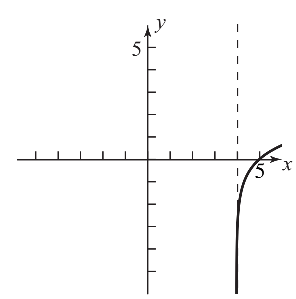
Domain: $(-2, \infty)$
Range: $(-\infty, \infty)$

41.



Domain: $(-1, \infty)$
Range: $(-\infty, \infty)$

42.



Domain: $(4, \infty)$
Range: $(-\infty, \infty)$

43.

$$y = \frac{100}{1+2^{-x}}$$

$$1+2^{-x} = \frac{100}{y}$$

$$2^{-x} = \frac{100}{y} - 1$$

$$\log_2(2^{-x}) = \log_2\left(\frac{100}{y} - 1\right)$$

$$-x = \log_2\left(\frac{100}{y} - 1\right)$$

$$x = -\log_2\left(\frac{100}{y} - 1\right)$$

$$= -\log_2\left(\frac{100-y}{y}\right)$$

$$= \log_2\left(\frac{y}{100-y}\right)$$

Interchange x and y.

$$y = \log_2\left(\frac{x}{100-x}\right)$$

$$f^{-1}(x) = \log_2\left(\frac{x}{100-x}\right)$$

Verify.

$$(f \circ f^{-1})(x) = f\left(\log_2 \frac{x}{100-x}\right)$$

$$= \frac{100}{1+2^{-\log_2\left(\frac{x}{100-x}\right)}}$$

$$= \frac{100}{1+2^{\log_2\left(\frac{100-x}{x}\right)}}$$

$$= \frac{100}{1+\frac{100-x}{x}}$$

$$= \frac{100x}{x+(100-x)}$$

$$= \frac{100x}{100}$$

$$= x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{100}{1+2^{-x}}\right)$$

$$= \log_2\left(\frac{\frac{100}{1+2^{-x}}}{100 - \frac{100}{1+2^{-x}}}\right)$$

$$= \log_2\left(\frac{100}{100(1+2^{-x})-100}\right)$$

$$= \log_2\left(\frac{1}{2^{-x}}\right)$$

$$= \log_2(2^x)$$

$$= x$$

44.

$$y = \frac{50}{1+1.1^{-x}}$$

$$1+1.1^{-x} = \frac{50}{y}$$

$$1.1^{-x} = \frac{50}{y} - 1$$

$$\log_{1.1}(1.1^{-x}) = \log_{1.1}\left(\frac{50}{y} - 1\right)$$

$$-x = \log_{1.1}\left(\frac{50}{y} - 1\right)$$

$$x = -\log_{1.1}\left(\frac{50}{y} - 1\right)$$

$$= -\log_{1.1}\left(\frac{50-y}{y}\right)$$

$$= \log_{1.1}\left(\frac{y}{50-y}\right)$$

Interchange x and y:

$$y = \log_{1.1}\left(\frac{x}{50-x}\right)$$

$$f^{-1}(x) = \log_{1.1}\left(\frac{x}{50-x}\right)$$

Verify.

$$(f \circ f^{-1})(x) = f\left(\log_{1.1}\left(\frac{x}{50-x}\right)\right)$$
$$= \frac{50}{1+1.1^{-\log_{1.1}\left(\frac{x}{50-x}\right)}}$$
$$= \frac{50}{1+1.1^{\log_{1.1}\left(\frac{50-x}{x}\right)}}$$
$$= \frac{50}{1+\frac{50-x}{x}}$$
$$= \frac{50x}{x+(50-x)}$$
$$= \frac{50x}{50}$$
$$= x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{50}{1+1.1^{-x}}\right)$$
$$= \log_{1.1}\left(\frac{\frac{50}{1+1.1^{-x}}}{50-\frac{50}{1+1.1^{-x}}}\right)$$
$$= \log_{1.1}\left(\frac{50}{50(1+1.1^{-x})-50}\right)$$
$$= \log_{1.1}\left(\frac{1}{1.1^{-x}}\right)$$
$$= \log_{1.1}(1.1^x)$$
$$= x$$

45. **(a)** $f(f(x)) = \sqrt{1-(f(x))^2}$
$$= \sqrt{1-(1-x^2)}$$
$$= \sqrt{x^2}$$
$$= |x|$$
$$= x, \text{ since } x \ge 0$$

(b) $f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$ for all $x \ne 0$

46. **(a)** Amount $= 8\left(\frac{1}{2}\right)^{t/12}$

(b) $8\left(\frac{1}{2}\right)^{t/12} = 1$
$$\left(\frac{1}{2}\right)^{t/12} = \frac{1}{8}$$
$$\left(\frac{1}{2}\right)^{t/12} = \left(\frac{1}{2}\right)^3$$
$$\frac{t}{12} = 3$$
$$t = 36$$
There will be 1 gram remaining after 36 hours.

47. $500(1.0475)^t = 1000$
$$1.0475^t = 2$$
$$\ln(1.0475^t) = \ln 2$$
$$t \ln 1.0475 = \ln 2$$
$$t = \frac{\ln 2}{\ln 1.0475} \approx 14.936$$
It will take about 14.936 years. (If the interest is paid at the end of each year, it will take 15 years.)

48. $375{,}000(1.0225)^t = 1{,}000{,}000$
$$1.0225^t = \frac{8}{3}$$
$$\ln(1.0225^t) = \ln\left(\frac{8}{3}\right)$$
$$t \ln 1.0225 = \ln\left(\frac{8}{3}\right)$$
$$t = \frac{\ln\left(\frac{8}{3}\right)}{\ln 1.0225} \approx 44.081$$
It will take about 44.081 years.

49. **(a)** The values of t must be $-2 \le t \le 2$ because all other values of t will make one of the expressions under the radicals negative. Note: If $t = 3$, then $\sqrt{2-t} = \sqrt{2-3} = \sqrt{-1}$. If $t = -3$, then $\sqrt{2+t} = \sqrt{2+(-3)} = \sqrt{-1}$.

(b) $(x, y) = \left(\sqrt{2-t}, \sqrt{2+t}\right)$
$$x^2 + y^2 = \left(\sqrt{2-t}\right)^2 + \left(\sqrt{2+t}\right)^2$$
$$= 2-t+2+t$$
$$= 4$$
$$\sqrt{x^2+y^2} = \sqrt{4} = 2$$
Every point of the form $\left(\sqrt{2-t}, \sqrt{2+t}\right)$ is at distance 2 from the origin.

(c) At $t=-2$,

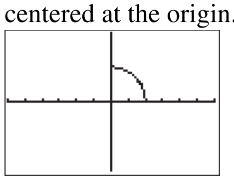
$$\left(\sqrt{2-t},\ \sqrt{2+t}\right)=\left(\sqrt{2-(-2)},\ \sqrt{2+(-2)}\right)=\left(\sqrt{4},\ \sqrt{0}\right)=(2,\ 0)$$

At $t=2$,

$$\left(\sqrt{2-t},\ \sqrt{2+t}\right)=\left(\sqrt{2-2},\ \sqrt{2+2}\right)=\left(\sqrt{0},\ \sqrt{4}\right)=(0,\ 2)$$

(d) Since both radicals are positive, all other points on this curve must lie in the first quadrant.

(e) The curve is a quarter-circle of radius 2 centered at the origin.



[–6, 6] by [–4, 4]

50. **(a)**

$$4\ln\sqrt{e^x}=26$$
$$\ln\sqrt{e^x}=\frac{26}{4}$$
$$\ln e^{\frac{1}{2}x}=\frac{26}{4}$$
$$\frac{1}{2}x\ln e=\frac{26}{4}$$
$$\frac{1}{2}x=\frac{26}{4}$$
$$x=13$$

(b)

$$x-\log(100)=\ln(e^3)$$
$$x-2=3\ln e$$
$$x-2=3$$
$$x=5$$

(c)

$$\log_x(7x-10)=2$$
$$x^2=7x-10$$
$$x^2-7x+10=0$$
$$(x-5)(x-2)=0$$
$$x-5=0 \quad\text{or}\quad x-2=0$$
$$x=5 \quad\text{or}\quad x=2$$
$$\{2,\ 5\}$$

(d)

$$2\log_3 x-\log_3(x-2)=2$$
$$\log_3 x^2-\log_3(x-2)=2$$
$$\log_3\left(\frac{x^2}{x-2}\right)=2$$
$$3^2=\frac{x^2}{x-2}$$
$$9(x-2)=x^2$$
$$0=x^2-9x+18$$
$$0=(x-6)(x-3)$$
$$x-6=0 \quad\text{or}\quad x-3=0$$
$$x=6 \quad\text{or}\quad x=3$$
$$\{3,\ 6\}$$

51. **(a)** Suppose that $f(x_1)=f(x_2)$. Then $mx_1+b=mx_2+b$ so $mx_1=mx_2$. Since $m\neq 0$, this gives $x_1=x_2$.

(b)

$$y=mx+b$$
$$y-b=mx$$
$$\frac{y-b}{m}=x$$

Interchange x and y.

$$\frac{x-b}{m}=y$$
$$f^{-1}(x)=\frac{x-b}{m}=\frac{1}{m}x=\frac{b}{m}$$

The slopes are reciprocals.

(c) If the original functions both have slope m, each of the inverse functions will have slope $\frac{1}{m}$. The graphs of the inverses will be parallel lines with nonzero slope.

(d) If the original functions have slopes m and $-\frac{1}{m}$, respectively, then the inverse functions will have slopes $\frac{1}{m}$ and $-m$, respectively. Since each of $\frac{1}{m}$ and $-m$ is the negative reciprocal of the other, the graphs of the inverses will be perpendicular lines with nonzero slopes.

52. False; for example, the horizontal line test finds three answers for $x=0$.

53. False; must satisfy $(f\circ g)(x)=(g\circ f)(x)=x$, not just $(f\circ g)(x)=x$.

54. C; ln(x + 2) is defined only if $x + 2 > 0$, or $x > -2$.

55. A; the range of the logarithm function is $(-\infty, \infty)$.

56. E;
$$\begin{aligned} f(x) &= 3x-2 \\ y &= 3x-2 \\ 3x &= y+2 \\ x &= \frac{y+2}{3} \end{aligned}$$
Interchange x and y: $y = \dfrac{x+2}{3}$

57. B;
$$\begin{aligned} 2-3^{-x} &= -1 \\ -3^{-x} &= -3 \\ -3^{-x} &= 3 \\ -x &= 1 \\ x &= -1 \end{aligned}$$

58. **(a)** y_2 is a vertical shift (upward) of y_1, although it's difficult to see that near the vertical asymptote at $x = 0$. One might use "trace" or "table" to verify this.

(b) Each graph of y_3 is a horizontal line.

(c) The graphs of y_4 and $y = a$ are the same.

(d) $e^{y_2-y_1} = a$, $\ln(e^{y_2-y_1}) = \ln a$, $y_2 - y_1 = \ln a$, $y_1 = y_2 - \ln a = \ln x - \ln a$

59. If the graph of $f(x)$ passes the horizontal line test, so will the graph of $g(x) = -f(x)$ since it's the same graph reflected about the x-axis.
Alternate answer: If $g(x_1) = g(x_2)$ then $-f(x_1) = -f(x_2)$, $f(x_1) = f(x_2)$, and $x_1 = x_2$ since f is one-to-one.

60. Suppose that $g(x_1) = g(x_2)$. Then $\dfrac{1}{f(x_1)} = \dfrac{1}{f(x_2)}$, $f(x_1) = f(x_2)$, and x_1 and x_2 since f is one-to-one.

61. **(a)** The expression $a(b^{c-x}) + d$ is defined for all values of x, so the domain is $(-\infty, \infty)$. Since b^{c-x} attains all positive values, the range is (d, ∞) if $a > 0$ and the range is $(-\infty, d)$ if $a < 0$.

(b) The expression $a\log_b(x-c)+d$ is defined when $x - c > 0$, so the domain is (c, ∞). Since $a\log_b(x-c)+d$ attains every real value for some value of x, the range is $(-\infty, \infty)$.

62. **(a)** Suppose $f(x_1) = f(x_2)$. Then:
$$\begin{aligned} \frac{ax_1+b}{cx_1+d} &= \frac{ax_2+b}{cx_2+d} \\ (ax_1+b)(cx_2+d) &= (ax_2+b)(cx_1+d) \\ acx_1x_2 + adx_1 + bcx_2 + bd &= acx_1x_2 + adx_2 + bcx_1 + bd \\ adx_1 + bcx_2 &= adx_2 + bcx_1 \\ (ad-bc)x_1 &= (ad-bc)x_2 \end{aligned}$$
Since $ad - bc \neq 0$, this means that $x_1 = x_2$.

(b) $$y = \frac{ax+b}{cx+d}$$
$$cxy + dy = ax + b$$
$$(cy - a)x = -dy + b$$
$$x = \frac{-dy+b}{cy-a}$$

Interchanging x and y:

$$y = \frac{-dx+b}{cx-a}$$
$$f^{-1}(x) = \frac{-dx+b}{cx-a}$$

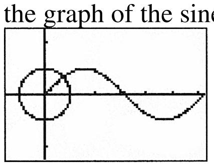
(c) As $x \to \pm\infty$, $f(x) = \frac{ax+b}{cx+d} \to \frac{a}{c}$, so the horizontal asymptote is $y = \frac{a}{c}$ $(c \neq 0)$. Since $f(x)$ is undefined at $x = -\frac{d}{c}$, the vertical asymptote is $x = -\frac{d}{c}$.

(d) As $x \to \pm\infty$, $f^{-1}(x) = \frac{-dx+b}{cx-a} \to -\frac{d}{c}$, so the horizontal asymptote is $y = -\frac{d}{c}$ $(c \neq 0)$. Since $f^{-1}(x)$ is undefined at $x = \frac{a}{c}$, the vertical asymptote is $x = \frac{a}{c}$. The horizontal asymptote of f becomes the vertical asymptote of f^{-1} and vice versa due to the reflection of the graph about the line $y = x$.

Section 1.6 Trigonometric Functions (pp. 45–53)

Exploration 1 Unwrapping Trigonometric Functions

1. (x_1, y_1) is the circle of radius 1 centered at the origin (unit circle). (x_2, y_2) is one period of the graph of the sine function.

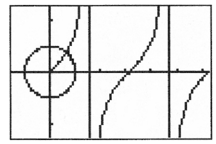


2. The y values are the same in the interval $0 \le t \le 2\pi$.

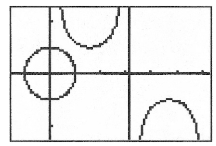
3. The y values are the same in the interval $0 \le t \le 4\pi$.

4. The x_1 values and the y_2 values are the same in each interval.

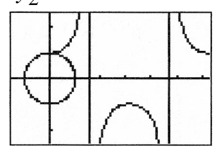
5. $y_2 = \tan t$:



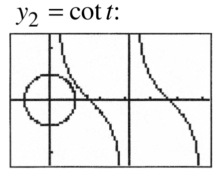
$y_2 = \csc t$:



$y_2 = \sec t$:



$y_2 = \cot t$:



For each value of t, the value of $y_2 = \tan t$ is equal to the ratio $\frac{y_1}{x_1}$.

For each value of t, the value of $y_2 = \csc t$ is equal to the ratio $\frac{1}{y_1}$.

For each value of t, the value of $y_2 = \sec t$ is equal to the ratio $\frac{1}{x_1}$.

For each value of t, the value of $y_2 = \cot t$ is equal to the ratio $\frac{x_1}{y_1}$.

Quick Review 1.6

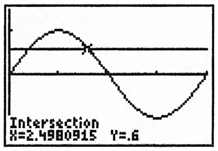
1. $\frac{\pi}{3} \cdot \frac{180°}{\pi} = 60°$

2. $-2.5 \cdot \frac{180°}{\pi} = \left(-\frac{450}{\pi}\right)^{\circ} \approx -143.24°$

3. $-40° \cdot \dfrac{\pi}{180°} = -\dfrac{2\pi}{9}$

4. $45° \cdot \dfrac{\pi}{180°} = \dfrac{\pi}{4}$

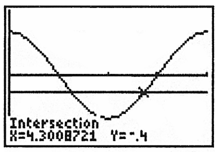
5.



$[0, 2\pi]$ by $[-1.5, 1.5]$

$x \approx 0.6435$, $x \approx 2.4981$

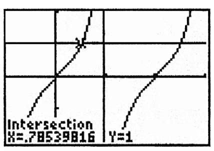
6.



$[0, 2\pi]$ by $[-1.5, 1.5]$

$x \approx 1.9823$, $x \approx 4.3009$

7.



$\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ by $[-2, 2]$

$x \approx 0.7854\left(\text{or } \dfrac{\pi}{4}\right)$, $x \approx 3.9270 \left(\text{or } \dfrac{5\pi}{4}\right)$

8. $f(-x) = 2(-x)^2 - 3 = 2x^2 - 3 = f(x)$

The graph is symmetric about the y-axis because if a point (a, b) is on the graph, then so is the point $(-a, b)$.

9.
$$\begin{aligned} f(-x) &= (-x)^3 - 3(-x) \\ &= -x^3 + 3x \\ &= -(x^3 - 3x) \\ &= -f(x) \end{aligned}$$

The graph is symmetric about the origin because if a point (a, b) is on the graph, then so is the point $(-a, -b)$.

10. $x \ge 0$. Alternatively, $x \le 0$.

Section 1.6 Exercises

1. Arc length $= \left(\dfrac{5\pi}{8}\right)(2) = \dfrac{5\pi}{4}$

2. Radius $= \dfrac{10}{175°\left(\frac{\pi}{180°}\right)} = \dfrac{72}{7\pi} \approx 3.274$

3. Angle $= \dfrac{7}{14} = \dfrac{1}{2}$ radian or about 28.65°.

4. Angle $= \dfrac{\frac{3\pi}{2}}{6} = \dfrac{\pi}{4}$ radian or 45°.

5. Even; $\sec(-\theta) = \dfrac{1}{\cos(-\theta)} = \dfrac{1}{\cos(\theta)} = \sec(\theta)$

6. Odd; $\tan(-\theta) = \dfrac{\sin(-\theta)}{\cos(-\theta)} = \dfrac{-\sin(\theta)}{\cos(\theta)} = -\tan\theta$

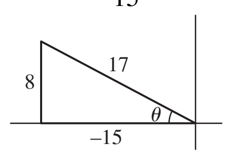
7. Odd; $\csc(-\theta) = \dfrac{1}{\sin(-\theta)} = \dfrac{1}{-\sin(\theta)} = -\csc\theta$

8. Odd; $\cot(-\theta) = \dfrac{\cos(-\theta)}{\sin(-\theta)} = \dfrac{\cos(\theta)}{-\sin(\theta)} = -\cot(\theta)$

9. Using a triangle with sides: –15, 17 and 8;

$\sin\theta = \dfrac{8}{17}$, $\tan\theta = -\dfrac{8}{15}$, $\csc\theta = \dfrac{17}{8}$,

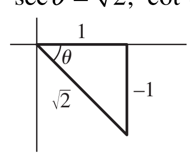
$\sec\theta = -\dfrac{17}{15}$, $\cot\theta = -\dfrac{15}{8}$



10. Using a triangle with sides: –1, 1, and $\sqrt{2}$;

$\sin\theta = -\dfrac{\sqrt{2}}{2}$, $\cos\theta = \dfrac{\sqrt{2}}{2}$, $\csc\theta = -\sqrt{2}$,

$\sec\theta = \sqrt{2}$, $\cot\theta = -1$



11. **(a)** Period $= \dfrac{2\pi}{3}$

(b) Domain: Since $\csc(3x + \pi) = \dfrac{1}{\sin(3x+\pi)}$,

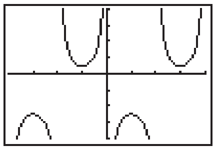
we require $3x + \pi \ne k\pi$, or $x \ne \dfrac{(k-1)\pi}{3}$.

This requirement is equivalent to $x \ne \dfrac{k\pi}{3}$

for integers k.

(c) Since $|\csc(3x + \pi)| \geq 1$, the range excludes numbers between $-3 - 2 = -5$ and $3 - 2 = 1$. The range is $(-\infty, -5] \cup [1, \infty)$.

(d)



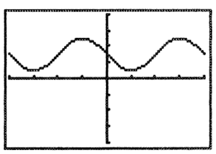
$\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ by $[-8, 8]$

12. **(a)** Period $= \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

(b) Domain: $(-\infty, \infty)$

(c) Since $|\sin(4x + \pi)| \leq 1$, the range extends from $-2 + 3 = 1$ to $2 + 3 = 5$. The range is $[1, 5]$.

(d)



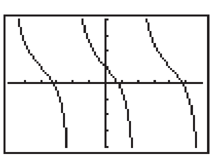
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $[-8, 8]$

13. **(a)** Period $= \dfrac{\pi}{3}$

(b) Domain: We require $3x + \pi \neq \dfrac{k\pi}{2}$ for odd integers k. Therefore, $x \neq \dfrac{(k-2)\pi}{6}$ for odd integers k. This requirement is equivalent to $x \neq \dfrac{k\pi}{6}$ for odd integers k.

(c) Since the tangent function attains all real values, the range is $(-\infty, \infty)$.

(d)



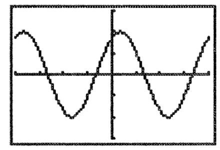
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $[-8, 8]$

14. **(a)** Period $= \dfrac{2\pi}{2} = \pi$

(b) Domain: $(-\infty, \infty)$

(c) Range: Since $\left|\sin\left(2x + \dfrac{\pi}{3}\right)\right| \leq 1$, the range is $[-2, 2]$.

(d)



$[-\pi, \pi]$ by $[-3, 3]$

15. **(a)** The period of $y = \sec x$ is 2π, so the window should have length 4π.
One possible answer: $[0, 4\pi]$ by $[-3, 3]$

(b) The period of $y = \csc x$ is 2π, so the window should have length 4π.
One possible answer: $[0, 4\pi]$ by $[-3, 3]$

(c) The period of $y = \cot x$ is π, so the window should have length 2π.
One possible answer: $[0, 2\pi]$ by $[-3, 3]$

16. **(a)** The period $y = \sin x$ is 2π, so the window should have length 4π.
One possible answer: $[0, 4\pi]$ by $[-2, 2]$

(b) The period of $y = \cos x$ is 2π, so the window should have length 4π.
One possible answer: $[0, 4\pi]$ by $[-2, 2]$

(c) The period of $y = \tan x$ is π, so the window should have length 2π.
One possible answer: $[0, 2\pi]$ by $[-3, 3]$

17. **(a)** Period $= \dfrac{2\pi}{2} = \pi$

(b) Amplitude = 1.5

(c) $[-2\pi, 2\pi]$ by $[-2, 2]$

18. **(a)** Period $= \dfrac{2\pi}{3}$

(b) Amplitude = 2

(c) $\left[-\dfrac{2\pi}{3}, \dfrac{2\pi}{3}\right]$ by $[-4, 4]$

19. **(a)** Period $= \frac{2\pi}{2} = \pi$

(b) Amplitude = 3

(c) $[-2\pi, 2\pi]$ by $[-4, 4]$

20. **(a)** Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

(b) Amplitude = 5

(c) $[-4\pi, 4\pi]$ by $[-10, 10]$

21. **(a)** Period $= \frac{2\pi}{\frac{\pi}{3}} = 6$

(b) Amplitude = 4

(c) $[-3, 3]$ by $[-5, 5]$

22. **(a)** Period $= \frac{2\pi}{\pi} = 2$

(b) Amplitude = 1

(c) $[-4, 4]$ by $[-2, 2]$

23. $y = 1.23\sin(2073.55x - 0.49) + 0.44$

(a) The period is $\frac{2\pi}{2073.55}$, so the frequency is $\frac{2073.55}{2\pi}$, which is about 330 Hz.

(b) From the table, the note produced is an E.

24. **(a)** $b = \frac{2\pi}{12} = \frac{\pi}{6}$

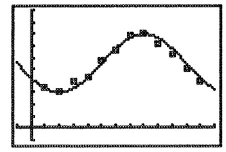
(b) It's half of the difference, so $a = \frac{80-30}{2} = 25.$

(c) $k = \frac{80+30}{2} = 55$

(d) The function should have its minimum at $t = 2$ (when the temperature is 30°F) and its maximum at $t = 8$ (when the temperature is 80°F). The value of h is $\frac{2+8}{2} = 5.$ Equation:

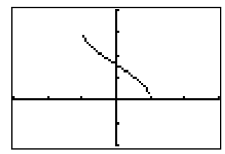
$$y = 25\sin\left[\frac{\pi}{6}(x-5)\right] + 55$$

(e)



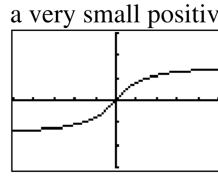
$[-1, 13]$ by $[-10, 100]$

25. The portion of the curve $y = \cos x$ between $0 \le x \le \pi$ passes the horizontal line test, so it is one-to-one.



$[-3, 3]$ by $[-2, 4]$

26. The portion of the curve $y = \tan x$ between $-\frac{\pi}{2} < x < \frac{\pi}{2}$ passes the horizontal line test, so it is one-to-one. [In parametric mode, use $T_{\min} = -\frac{\pi}{2} + \varepsilon$ and $T_{\max} = \frac{\pi}{2} - \varepsilon$, where ε is a very small positive number, say 0.00001.]



$[-5, 5]$ by $[-3, 3]$

27. Since $\frac{\pi}{6}$ is in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ or $y = \sin^{-1} x$ and $\sin\frac{\pi}{6} = 0.5,$

$\sin^{-1}(0.5) = \frac{\pi}{6}$ radian or $\frac{\pi}{6} \cdot \frac{180°}{\pi} = 30°.$

28. Since $-\frac{\pi}{4}$ is in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ of $y = \sin^{-1} x$ and $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2},$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ radian or

$-\frac{\pi}{4} \cdot \frac{180°}{\pi} = -45°.$

29. Using a calculator, $\tan^{-1}(-5) \approx -1.3734$ radians or $-78.6901°.$

30. Using a calculator, $\cos^{-1}(0.7) = 0.7954$ radian or $45.5730°.$

31. The angle $\tan^{-1}(2.5) \approx 1.190$ is the solution to this equation in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Another solution in $0 \le x \le 2\pi$ is $\tan^{-1}(2.5) + \pi \approx 4.332$. The solutions are $x \approx 1.190$ and $x \approx 4.332$.

32. The angle $\cos^{-1}(-0.7) \approx 2.346$ is the solution to this equation in the interval $0 \le x \le \pi$. Since the cosine function is even, the value $-\cos^{-1}(-0.7) \approx -2.346$ is also a solution, so any value of the form $\pm\cos^{-1}(-0.7) + 2k\pi$ is a solution, where k is an integer. In $2\pi \le x < 4\pi$ the solutions are $x = \cos^{-1}(-0.7) + 2\pi \approx 8.629$ and $x = -\cos^{-1}(-0.7) + 4\pi \approx 10.200$.

33. The equation is equivalent to $\sin x = \frac{1}{2}$, so the solutions in the interval $0 \le x < 2\pi$ are $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$.

34. This equation is equivalent to $\cos x = -\frac{1}{3}$, so the solution in the interval $0 \le x \le \pi$ is $y = \cos^{-1}\left(-\frac{1}{3}\right) \approx 1.911$. Since the cosine function is even, the solutions in the interval $-\pi \le x < \pi$ are $x \approx -1.911$ and $x \approx 1.911$.

35. The solutions in interval $0 \le x < 2\pi$ are $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$. Since $y = \sin x$ has period 2π, the solutions are all of the form $x = \frac{7\pi}{6} + 2k\pi$ or $x = \frac{11\pi}{6} + 2k\pi$, where k is any integer.

36. The equation is equivalent to $\tan x = \frac{1}{-1} = -1$, to the solution in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$ is $x = \tan^{-1}(-1) = -\frac{\pi}{4}$. Since the period of $y = \tan x$ is π, all solutions are of the form $x = -\frac{\pi}{4} = k\pi$, where k is any integer. This is equivalent to $x = \frac{3\pi}{4} + k\pi$, where k is any integer.

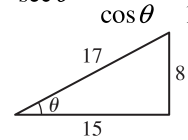
37. Note that $\sqrt{8^2 + 15^2} = 17$. Since $\sin\theta = \frac{8}{17}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$,

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{8}{17}\right)^2} = \frac{15}{17}.$$

Therefore: $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{8}{15},\quad \cot\theta = \frac{1}{\tan\theta} = \frac{15}{8},$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{17}{15},\quad \csc\theta = \frac{1}{\sin\theta} = \frac{17}{8}$$



38. Note that $\sqrt{5^2 + 12^2} = 13$.

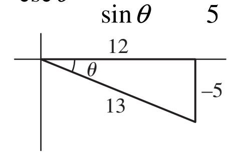
Since $\tan\theta = -\frac{5}{12} = \frac{-\frac{5}{13}}{\frac{12}{13}} = \frac{\sin\theta}{\cos\theta}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, we have $\sin\theta = -\frac{5}{13}$ and $\cos\theta = \frac{12}{13}$.

In summary:

$$\sin\theta = -\frac{5}{13},\ \cos\theta = \frac{12}{13},\ \tan\theta = -\frac{5}{12}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{12}{5},\quad \sec\theta = \frac{1}{\cos\theta} = \frac{13}{12},$$

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{13}{5}$$

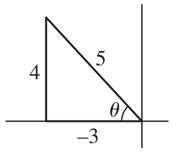


39. Note that $r = \sqrt{(-3)^2 + 4^2} = 5$. Then:

$$\sin\theta = \frac{y}{r} = \frac{4}{5},\ \cos\theta = \frac{x}{r} = -\frac{3}{5},$$

$$\tan\theta = \frac{y}{x} = -\frac{4}{3},\quad \cot\theta = \frac{x}{y} = -\frac{3}{4},$$

$\sec\theta = \frac{r}{x} = -\frac{5}{3}, \quad \csc\theta = \frac{r}{y} = \frac{5}{4}$



40. Note that $r = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$. Then:

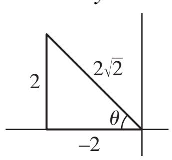
$\sin\theta = \frac{y}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}},$

$\cos\theta = \frac{x}{r} = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}},$

$\tan\theta = \frac{y}{x} = \frac{2}{-2} = -1, \quad \cot\theta = \frac{x}{y} = \frac{-2}{2} = -1,$

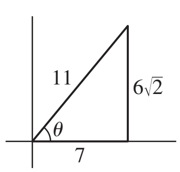
$\sec\theta = \frac{r}{x} = \frac{2\sqrt{2}}{-2} = -\sqrt{2},$

$\csc\theta = \frac{r}{y} = \frac{2\sqrt{2}}{2} = \sqrt{2}$



41. Let $\theta = \cos^{-1}\left(\frac{7}{11}\right)$. Then $0 \le \theta \le \pi$ and

$$\cos\theta = \frac{7}{11}, \text{ so } \sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right) = \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{7}{11}\right)^2} = \frac{\sqrt{72}}{11} = \frac{6\sqrt{2}}{11} \approx 0.771$$

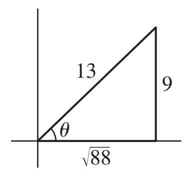


42. Let $\theta = \sin^{-1}\left(\frac{9}{13}\right)$. Then $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and

$\sin\theta = \frac{9}{13}$, so

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{9}{13}\right)^2} = \frac{\sqrt{88}}{13}.$

Therefore, $\tan\left(\sin^{-1}\left(\frac{9}{13}\right)\right) = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{9}{13}}{\frac{\sqrt{88}}{13}} = \frac{9}{\sqrt{88}} \approx 0.959.$



43. Amplitude $= \frac{\text{high point} - \text{low point}}{2} = \frac{80 - 40.5}{2} = 19.75$

Reflected about the x-axis, so $A = -19.75$.

Period = 12 months, so $B = \frac{2\pi}{12} = \frac{\pi}{6}$.

$C = 40.5 + 19.75 = 60.25$

$y = A\cos(Bx) + C$

$y = -19.75\cos\left(\frac{\pi}{6}x\right) + 60.25$

44. Amplitude $= \frac{\text{high point} - \text{low point}}{2} = \frac{82.6 - 28.9}{2} = 26.85$

Reflected about the x-axis, so $A = -26.85$.

Period = 12 months, so $B = \frac{2\pi}{12} = \frac{\pi}{6}$.

$C = 28.9 + 26.85 = 55.75$

$y = A\cos(Bx) + C$

$y = -26.85\cos\left(\frac{\pi}{6}x\right) + 55.75$

45. **(a)** $\cot(-x) = \dfrac{\cos(-x)}{\sin(-x)} = \dfrac{\cos(x)}{-\sin(x)} = -\cot(x)$

(b) Assume that f is even and g is odd.
Then $\dfrac{f(-x)}{g(-x)} = \dfrac{f(x)}{-g(x)} = -\dfrac{f(x)}{g(x)}$ so $\dfrac{f}{g}$ is odd. The situation is similar for $\dfrac{g}{f}$.

46. **(a)** $\csc(-x) = \dfrac{1}{\sin(-x)} = \dfrac{1}{-\sin(x)} = -\csc(x)$

(b) Assume that f is odd. Then
$\dfrac{1}{f(-x)} = \dfrac{1}{-f(x)} = -\dfrac{1}{f(x)}$ so $\dfrac{1}{f}$ is odd.

47. Assume that f is even and g is odd. Then $f(-x)g(-x) = (f(x))(-g(x)) = -f(x)g(x)$ so fg is odd.

48. If (a, b) is the point on the unit circle corresponding to the angle θ, then $(-a, -b)$ is the point on the unit circle corresponding to the angle $(\theta + \pi)$ since it is exactly half way around the circle. This means that both $\tan(\theta)$ and $\tan(\theta + \pi)$ have the same value, $\dfrac{b}{a}$.

49. **(a)** $y = (\sin x)(\sin 2x)$ is not sinusoidal. The period is 2π.

(b) $y = (\sin x)(\cos x)$ is sinusoidal. The period is π.

(c) $y = (\sin x)(\cos x)$ where $A = \dfrac{1}{2}$ and $b = \partial$, so $y = \dfrac{1}{2}\sin 2x$.

50. False; it is 4π because $\dfrac{2\pi}{B} = \dfrac{1}{2}$ implies the period B is 4π.

51. False; the amplitude is $\dfrac{1}{2}$.

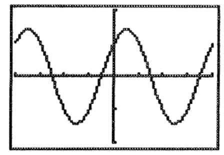
52. D

53. B; the curve oscillates between –3 and 1.

54. E

55. A

56. **(a)**



$[-2\pi, 2\pi]$ by $[-2, 2]$

The graph is a sine/cosine type graph, but it is shifted and has an amplitude greater than 1.

(b) Amplitude ≈ 1.414 $\left(\text{that is, } \sqrt{2}\right)$

Period = 2π

Horizontal shift = –0.785 $\left(\text{that is, } -\dfrac{\pi}{4}\right)$ or 5.498 $\left(\text{that is, } \dfrac{7\pi}{4}\right)$

Vertical shift = 0

(c) $\sin\left(x + \dfrac{\pi}{4}\right)$

$= (\sin x)\left(\cos\dfrac{\pi}{4}\right) + (\cos x)\left(\sin\dfrac{\pi}{4}\right)$

$= (\sin x)\left(\dfrac{1}{\sqrt{2}}\right) + (\cos x)\left(\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{1}{\sqrt{2}}(\sin x + \cos x)$

Therefore, $\sin x + \cos x = \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)$.

57. **(a)** $\sqrt{2}\sin\left(ax + \dfrac{\pi}{4}\right)$

(b) See part (a).

(c) It works.

(d) $\sin\left(ax + \dfrac{\pi}{4}\right)$

$= (\sin ax)\left(\cos\dfrac{\pi}{4}\right) + (\cos ax)\left(\sin\dfrac{\pi}{4}\right)$

$= (\sin ax)\left(\dfrac{1}{\sqrt{2}}\right) + (\cos ax)\left(\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{1}{\sqrt{2}}(\sin ax + \cos ax)$

So, $\sin(ax) + \cos(ax) = \sqrt{2}\sin\left(ax + \dfrac{\pi}{4}\right)$.

58. **(a)** One possible answer: $y = \sqrt{a^2 + b^2}\,\sin\left(x + \tan^{-1}\left(\frac{b}{a}\right)\right)$

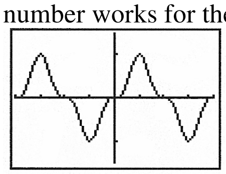
(b) See part (a).

(c) It works.

(d)
$$\sin\left(x+\tan^{-1}\left(\frac{b}{a}\right)\right) = \sin(x)\cos\left(\tan^{-1}\left(\frac{b}{a}\right)\right) + \cos(x)\sin\left(\tan^{-1}\left(\frac{b}{a}\right)\right)$$
$$= \sin(x)\left(\frac{a}{\sqrt{a^2+b^2}}\right) + \cos(x)\left(\frac{b}{\sqrt{a^2+b^2}}\right)$$
$$= \frac{1}{\sqrt{a^2+b^2}} \cdot (a\sin x + b\cos x)$$

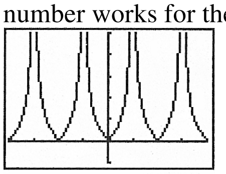
and multiplying through by the square root gives the desired result. Note that the substitutions $\cos\left(\tan^{-1}\frac{b}{a}\right) = \frac{a}{\sqrt{a^2+b^2}}$ and $\sin\left(\tan^{-1}\frac{b}{a}\right) = \frac{b}{\sqrt{a^2+b^2}}$ depend on the requirement that a is positive. If a is negative, the formula does not work.

59. Since $\sin x$ has period 2π, $\sin^3(x+2\pi) = \sin^3(x)$. This function has period 2π. A graph shows that no smaller number works for the period.



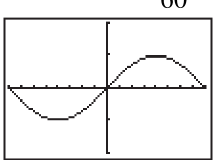
$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

60. Since $\tan x$ has period π, $|\tan(x+\pi)| = |\tan x|$. This function has period π. A graph shows that no smaller number works for the period.



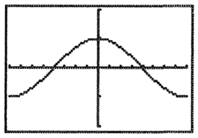
$[-2\pi, 2\pi]$ by $[-1, 5]$

61. The period is $\frac{2\pi}{60} = \frac{\pi}{30}$. One possible graph:



$\left[-\frac{\pi}{60}, \frac{\pi}{60}\right]$ by $[-2, 2]$

62. The period is $\frac{2\pi}{60\pi}=\frac{1}{30}$. One possible graph:



$\left[-\frac{1}{60}, \frac{1}{60}\right]$ by $[-2, 2]$

Quick Quiz (Sections 1.4–1.6)

1. C

2. D

3. E

4. **(a)**
$$\begin{aligned} f(x) &= 5x-3 \\ y &= 5x-3 \\ x &= 5y-3 \\ x+3 &= 5y \\ \frac{x+3}{5} &= y \\ g(x) &= \frac{x+3}{5} \end{aligned}$$

(b)
$$\begin{aligned} (f \circ g)(x) &= f\left(\frac{x+3}{5}\right) \\ &= 5\left(\frac{x+3}{5}\right)-3 \\ &= x+3-3 \\ &= x \end{aligned}$$

(c)
$$\begin{aligned} (g \circ f)(x) &= g(5x-3) \\ &= \frac{(5x-3)+3}{5} \\ &= \frac{5x}{5} \\ &= x \end{aligned}$$

Chapter 1 Review Exercises (pp. 54–56)

1. $y = 3(x-1) + (-6)$
$y = 3x - 9$

2. $y=-\frac{1}{2}(x+1)+2$
$y=-\frac{1}{2}x+\frac{3}{2}$

3. $x = 0$

4. $m=\frac{-2-6}{1-(-3)}=\frac{-8}{4}=-2$
$y = -2(x + 3) + 6$
$y = -2x$

5. $y = 2$

6. $m=\frac{5-3}{-2-3}=\frac{2}{-5}=-\frac{2}{5}$
$y=-\frac{2}{5}(x-3)+3$
$y=-\frac{2}{5}x+\frac{21}{5}$

7. $y = -3x + 3$

8. Since $2x - y = -2$ is equivalent to $y = 2x + 2$, the slope of the given line (and hence the slope of the desired line) is 2.
$y = 2(x - 3) + 1$
$y = 2x - 5$

9. Since $4x + 3y = 12$ is equivalent to $y=-\frac{4}{3}x+4$, the slope of the given line (and hence the slope of the desired line) is $-\frac{4}{3}$.
$y=-\frac{4}{3}(x-4)-12$
$y=-\frac{4}{3}x-\frac{20}{3}$

10. Since $3x - 5y = 1$ is equivalent to $y=\frac{3}{5}x-\frac{1}{5}$, the slope of the given line is $\frac{3}{5}$ and the slope of the perpendicular line is $-\frac{5}{3}$.
$y=-\frac{5}{3}(x+2)-3$
$y=-\frac{5}{3}x-\frac{19}{3}$

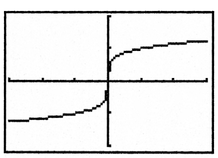
11. Since $\frac{1}{2}x+\frac{1}{3}y=1$ is equivalent to $y=-\frac{3}{2}x+3$, the slope of the given line is $-\frac{3}{2}$ and the slope of the perpendicular line is $\frac{2}{3}$.

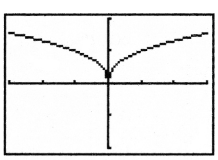
$$y = \frac{2}{3}(x+1)+2$$
$$y = \frac{2}{3}x + \frac{8}{3}$$

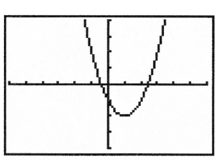
12. The line passes through (0, –5) and (3, 0).
$$m = \frac{0-(-5)}{3-0} = \frac{5}{3}$$
$$y = \frac{5}{3}x - 5$$

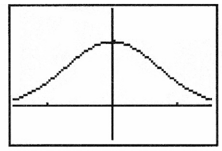
13. $m = \dfrac{2-4}{2-(-2)} = \dfrac{-2}{4} = -\dfrac{1}{2}$
$$f(x) = -\frac{1}{2}(x+2)+4$$
$$f(x) = -\frac{1}{2}x + 3$$
Check: $f(4) = -\frac{1}{2}(4)+3 = 1,$ as expected.

14. The line passes through (4, –2) and (–3, 0).
$$m = \frac{0-(-2)}{-3-4} = \frac{2}{-7} = -\frac{2}{7}$$
$$y = -\frac{2}{7}(x-4)-2$$
$$y = -\frac{2}{7}x - \frac{6}{7}$$

15.

[−3, 3] by [−2, 2]
Symmetric about the origin.

16.

[−3, 3] by [−2, 2]
Symmetric about the y-axis.

17.

[−6, 6] by [−4, 4]
Neither

18.

[−1.5, 1.5] by [−0.5, 1.5]
Symmetric about the y-axis.

19. $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$
Even

20.
$$\begin{aligned} y(-x) &= (-x)^5 - (-x)^3 - (-x) \\ &= -x^5 + x^3 + x \\ &= -y(x) \end{aligned}$$
Odd

21. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$
Even

22.
$$\begin{aligned} y(-x) &= \sec(-x)\tan(-x) \\ &= \frac{\sin(-x)}{\cos^2(-x)} \\ &= \frac{-\sin x}{\cos^2 x} \\ &= -\sec x \tan x \\ &= -y(x) \end{aligned}$$
Odd

23.
$$\begin{aligned} y(-x) &= \frac{(-x)^4+1}{(-x)^3-2(-x)} \\ &= \frac{x^4+1}{-x^3+2x} \\ &= -\frac{x^4+1}{x^3-2x} \\ &= -y(x) \end{aligned}$$
Odd

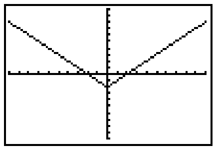
24. $y(-x) = 1 - \sin(-x) = 1 + \sin x$
Neither even nor odd

25. $y(-x) = -x + \cos(-x) = -x + \cos x$
Neither even nor odd

26. $y(-x) = \sqrt{(-x)^4 - 1} = \sqrt{x^4 - 1}$
Even

27. **(a)** The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

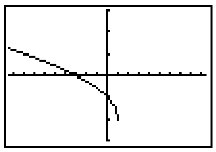
(b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.

(c) 

[–10, 10] by [–10, 10]

28. (a) Since the square root requires $1 - x \geq 0$, the domain is $(-\infty, 1]$.

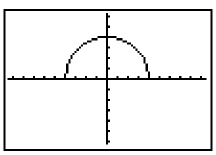
(b) Since $\sqrt{1-x}$ attains all nonnegative values, the range is $[-2, \infty)$.

(c) 

[–9.4, 9.4] by [–3, 3]

29. (a) Since the square root requires $16 - x^2 \geq 0$, the domain is $[-4, 4]$.

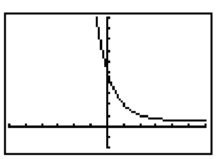
(b) For values of x in the domain, $0 \leq 16 - x^2 \leq 16$, so $0 \leq \sqrt{16 - x^2} \leq 4$. The range is $[0, 4]$.

(c) 

[–9.4, 9.4] by [–6.2, 6.2]

30. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

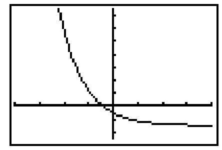
(b) Since 3^{2-x} attains all positive values, the range is $(1, \infty)$.

(c) 

[–6, 6] by [–4, 20]

31. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

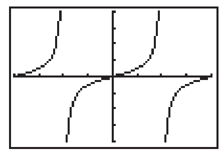
(b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.

(c) 

[–4, 4] by [–5, 15]

32. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k. The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k.

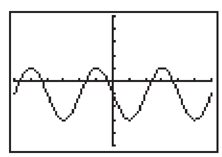
(b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.

(c) 

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by [–8, 8]

33. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

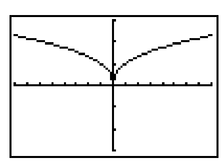
(b) The sine function attains values from –1 to 1, so $-2 \leq 2\sin(3x + \pi) \leq 2$, and hence $-3 \leq 2\sin(3x + \pi) - 1 \leq 1$. The range is $[-3, 1]$.

(c) 

[$-\pi$, π] by [–5, 5]

34. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.

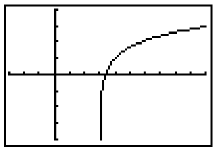
(c) 

[–8, 8] by [–3, 3]

35. **(a)** The logarithm requires $x - 3 > 0$, so the domain is $(3, \infty)$.

(b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.

(c)

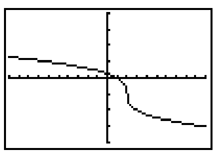


[–3, 10] by [–4, 4]

36. **(a)** The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The cube root attains all real values, so the range is $(-\infty, \infty)$.

(c)

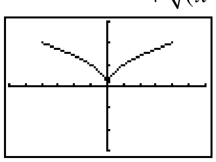


[–10, 10] by [–4, 4]

37. **(a)** The function is defined for $-4 \le x \le 4$, so the domain is $[-4, 4]$.

(b) The function is equivalent to $y = \sqrt{|x|}$, $-4 \le x \le 4$, which attains values from 0 to 2 for x in the domain. The range is $[0, 2]$.

(c) $\text{Y1} = \sqrt{(-x \cdot (-4 \le x \text{ and } x \le 0))}$
$\qquad + \sqrt{(x \cdot (0 \le x \text{ and } x \le 4))}$

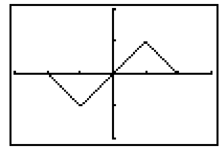


[–6, 6] by [–3, 3]

38. **(a)** The function is defined for $-2 \le x \le 2$, so the domain is $[-2, 2]$.

(b) See the graph in part (c). The range is $[-1, 1]$.

(c) $\text{Y1} = (-x-2) \cdot (-2 \le x \text{ and } x \le -1)$
$\qquad + x \cdot (-1 \le x \text{ and } x \le 1)$
$\qquad + (-x+2) \cdot (-1 < x \text{ and } x \le 2)$



[–3, 3] by [–2, 2]

39. First piece: Line through (0, 1) and (1, 0)

$$m = \frac{0-1}{1-0} = \frac{-1}{1} = -1$$

$y = -x + 1$ or $1 - x$

Second piece: Line through (1, 1) and (2, 0)

$$m = \frac{0-1}{2-1} = \frac{-1}{1} = -1$$

$y = -(x - 1) + 1$

$y = -x + 2$ or $2 - x$

$$f(x) = \begin{cases} 1-x, & 0 \le x < 1 \\ 2-x, & 1 \le x \le 2 \end{cases}$$

40. First piece: Line through (0, 0) and (2, 5)

$$m = \frac{5-0}{2-0} = \frac{5}{2}$$

$$y = \frac{5}{2}x$$

Second piece: Line through (2, 5) and (4, 0)

$$m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2}$$

$$y = -\frac{5}{2}(x-2) + 5$$

$$y = -\frac{5}{2}x + 10 \text{ or } 10 - \frac{5x}{2}$$

$$f(x) = \begin{cases} \dfrac{5x}{2}, & 0 \le x < 2 \\ 10 - \dfrac{5x}{2}, & 2 \le x \le 4 \end{cases}$$

(Note: $x = 2$ can be included on either piece.)

41. **(a)**
$$\begin{aligned}(f \circ g)(-1) &= f(g(-1)) \\ &= f\left(\frac{1}{\sqrt{-1+2}}\right) \\ &= f(1) \\ &= \frac{1}{1} \\ &= 1\end{aligned}$$

(b) $(g \circ f)(2) = g(f(2))$
$$= g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{1}{2}+2}} = \frac{1}{\sqrt{2.5}} \text{ or } \sqrt{\frac{2}{5}}$$

(c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x,$

$x \neq 0$

(d) $(g \circ g)(x) = g(g(x))$
$$= g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x+2}}+2}} = \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x+2}}}$$

42. (a) $(f \circ g)(-1) = f(g(-1))$
$$= f\left(\sqrt[3]{-1+1}\right) = f(0) = 2-0 = 2$$

(b) $(g \circ f)(2) = g(f(2))$
$$= g(2-2) = g(0) = \sqrt[3]{0+1} = 1$$

(c) $(f \circ f)(x) = f(f(x))$
$$= f(2-x) = 2-(2-x) = x$$

(d) $(g \circ g)(x) = g(g(x))$
$$= g\left(\sqrt[3]{x+1}\right) = \sqrt[3]{\sqrt[3]{x+1}+1}$$

43. (a) $(f \circ g)(x) = f(g(x))$
$$= f\left(\sqrt{x+2}\right) = 2-\left(\sqrt{x+2}\right)^2 = -x,\ x \geq -2$$

$(g \circ f)(x) = g(f(x))$
$$= g(2-x^2) = \sqrt{(2-x^2)+2} = \sqrt{4-x^2}$$

(b) Domain of $(f \circ g)(x)$: $[-2, \infty)$
Domain of $(g \circ f)(x)$: $[-2, 2]$

(c) Range of $(f \circ g)(x)$: $(-\infty, 2]$
Range of $(g \circ f)(x)$: $[0, 2]$

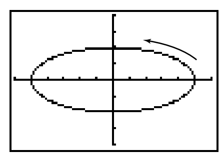
44. (a) $(f \circ g)(x) = f(g(x))$
$$= f\left(\sqrt{1-x}\right) = \sqrt{\sqrt{1-x}} = \sqrt[4]{1-x}$$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \sqrt{1-\sqrt{x}}$

(b) Domain of $(f \circ g)(x)$: $(-\infty, 1]$
Domain of $(g \circ f)(x)$: $[0, 1]$

(c) Range of $(f \circ g)(x)$: $[0, \infty)$
Range of $(g \circ f)(x)$: $[0, 1]$

45. (a)

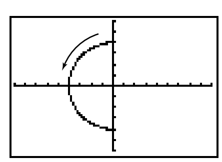


[–6, 6] by [–4, 4]

Initial point: (5, 0)
Terminal point: (5, 0)
The ellipse is traced exactly once in a counterclockwise direction starting and ending at the point (5, 0).

(b) Substituting $\cos t = \frac{x}{5}$ and $\sin t = \frac{y}{2}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1.$
The entire ellipse is traced by the curve.

46. (a)



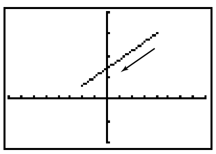
[–9, 9] by [–6, 6]

Initial point: (0, 4)
Terminal point: None (since the endpoint

$\frac{3\pi}{2}$ is not included in the t-interval)
The semicircle is traced in a counterclockwise direction starting at (0, 4) and extending to, but not including, (0, −4).

(b) Substituting $\cos t = \frac{x}{4}$ and $\sin t = \frac{y}{4}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$, or $x^2 + y^2 = 16$. The left half of the circle is traced by the parametrized curve.

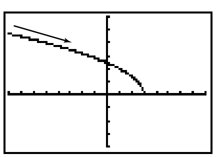
47. **(a)**



[−8, 8] by [−10, 20]
Initial point: (4, 15)
Terminal point: (−2, 3)
The line segment is traced from right to left starting at (4, 15) and ending at (−2, 3).

(b) Substituting $t = 2 - x$ into $y = 11 - 2t$ gives the Cartesian equation $y = 11 - 2(2 - x)$, or $y = 2x + 7$.
The part of the line from (4, 15) to (−2, 3) is traced by the parametrized curve.

48. **(a)**



[−8, 8] by [−4, 6]
Initial point: None
Terminal point: (3, 0)
The curve is traced from left to right ending at the point (3, 0).

(b) Substituting $t = x - 1$ into $y = \sqrt{4-2t}$ gives the Cartesian equation $y = \sqrt{4-2(x-1)}$, or $y = \sqrt{6-2x}$. The entire curve is traced by the parametrized curve.

49. For simplicity, we assume that x and y are linear functions of t, and that the point (x, y) starts at (−2, 5) for $t = 0$ and ends at (4, 3) for $t = 1$. Then $x = f(t)$, where $f(0) = -2$ and $f(1) = 4$.
Since slope $= \frac{\Delta x}{\Delta t} = \frac{4-(-2)}{1-0} = 6$,
$x = f(t) = 6t - 2 = -2 + 6t$.
Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 3$.
Since slope $= \frac{\Delta y}{\Delta t} = \frac{3-5}{1-0} = -2$,
$y = g(t) = -2t + 5 = 5 - 2t$.
One possible parametrization is:
$x = -2 + 6t,\ y = 5 - 2t,\ 0 \le t \le 1$

50. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) passes through (−3, −2) for $t = 0$ and (4, −1) for $t = 1$. Then $x = f(t)$, where $f(0) = -3$ and $f(1) = 4$. Since slope $= \frac{\Delta x}{\Delta t} = \frac{4-(-3)}{1-0} = 7$,
$x = f(t) = 7t - 3 = -3 + 7t$.
Also, $y = g(t)$, where $g(0) = -2$ and $g(1) = -1$.
Since slope $= \frac{\Delta y}{\Delta t} = \frac{-1-(-2)}{1-0} = 1$,
$y = g(t) = t - 2 = -2 + t$.
One possible parametrization is:
$x = -3 + 7t,\ y = -2 + t,\ -\infty < t < \infty$.

51. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at (2, 5) for $t = 0$ and passes through (−1, 0) for $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$. Since slope $= \frac{\Delta x}{\Delta t} = \frac{-1-2}{1-0} = -3$,
$x = f(t) = -3t + 2 = 2 - 3t$. Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 0$. Since
slope $= \frac{\Delta y}{\Delta t} = \frac{0-5}{1-0} = -5$,
$y = g(t) = -5t + 5 = 5 - 5t$.
One possible parametrization is:
$x = 2 - 3t,\ y = 5 - 5t,\ t \ge 0$.

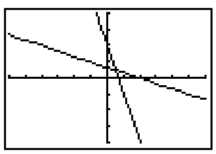
52. One possible parametrization is:
$x = t,\ y = t(t - 4),\ t \le 2$.

53. **(a)**
$$y = 2 - 3x$$
$$3x = 2 - y$$
$$x = \frac{2-y}{3}$$
Interchange x and y.
$$y = \frac{2-x}{3}$$
$$f^{-1}(x) = \frac{2-x}{3}$$
Verify:

$$\begin{aligned}(f\circ f^{-1})(x) &= f(f^{-1}(x))\\ &= f\left(\frac{2-x}{3}\right)\\ &= 2-3\left(\frac{2-x}{3}\right)\\ &= 2-(2-x)\\ &= x\end{aligned}$$

$$\begin{aligned}(f^{-1}\circ f)(x) &= f^{-1}(f(x))\\ &= f^{-1}(2-3x)\\ &= \frac{2-(2-3x)}{3}\\ &= \frac{3x}{3}\\ &= x\end{aligned}$$

(b)



[–6, 6] by [–4, 4]

54. (a) $y=(x+2)^2,\ x\ge -2$

$\sqrt{y}=x+2$

$x=\sqrt{y}-2$

Interchange x and y.

$y=\sqrt{x}-2$

$f^{-1}(x)=\sqrt{x}-2$

Verify.

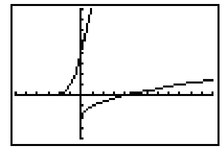
For $x\ge 0$ (the domain of f^{-1})

$$\begin{aligned}(f\circ f^{-1})(x) &= f(f^{-1}(x))\\ &= f\left(\sqrt{x}-2\right)\\ &= \left[\left(\sqrt{x}-2\right)+2\right]^2\\ &= \left(\sqrt{x}\right)^2\\ &= x\end{aligned}$$

For $x\ge -2$ (the domain of f),

$$\begin{aligned}(f^{-1}\circ f)(x) &= f^{-1}(f(x))\\ &= f^{-1}((x+2)^2)\\ &= \sqrt{(x+2)^2}-2\\ &= |x+2|-2\\ &= (x+2)-2\\ &= x\end{aligned}$$

(b)



[–6, 12] by [–4, 8]

55. Using a calculator,
$\sin^{-1}(0.6)\approx 0.6435$ radians or $36.8699°$.

56. Using a calculator,
$\tan^{-1}(-2.3)\approx -1.1607$ radians or $-66.5014°$.

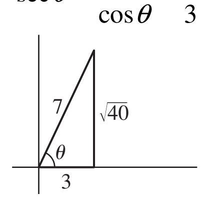
57. Since $\cos\theta=\frac{3}{7}$ and $0\le\theta\le\pi$,

$$\begin{aligned}\sin\theta &= \sqrt{1-\cos^2\theta}\\ &= \sqrt{1-\left(\frac{3}{7}\right)^2}\\ &= \sqrt{\frac{40}{49}}\\ &= \frac{\sqrt{40}}{7}.\end{aligned}$$

Therefore, $\sin\theta=\frac{\sqrt{40}}{7}$, $\cos\theta=\frac{3}{7}$,

$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\sqrt{40}}{3}$, $\cot\theta=\frac{1}{\tan\theta}=\frac{3}{\sqrt{40}}$,

$\sec\theta=\frac{1}{\cos\theta}=\frac{7}{3}$, $\csc\theta=\frac{1}{\sin\theta}=\frac{7}{\sqrt{40}}$



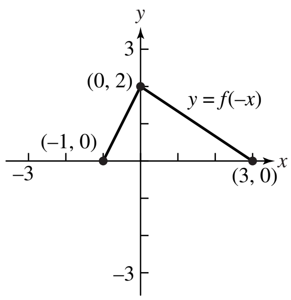
58. (a) Note that $\sin^{-1}(-0.2)\approx -0.2014$. In $[0, 2\pi)$, the solutions are
$x=\pi-\sin^{-1}(-0.2)\approx 3.3430$ and
$x=\sin^{-1}(-0.2)+2\pi\approx 6.0818$.

(b) Since the period of $\sin x$ is 2π, the solutions are $x\approx 3.3430+2k\pi$ and $x\approx 6.0818+2k\pi$, k any integer.

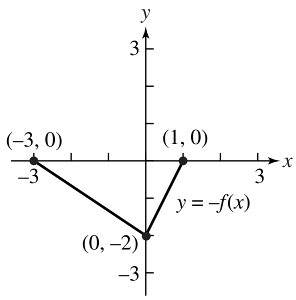
59.

$$\begin{aligned}e^{-0.2x} &= 4\\ \ln e^{-0.2x} &= \ln 4\\ -0.2x &= \ln 4\\ x &= \frac{\ln 4}{-0.2}=-5\ln 4\end{aligned}$$

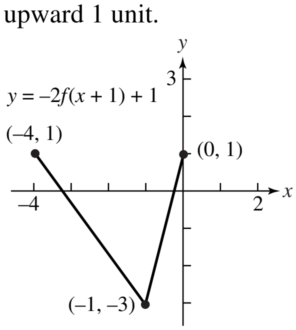
60. (a) The given graph is reflected about the y-axis.



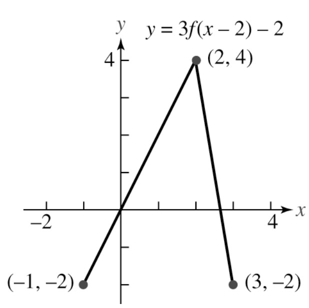
(b) The given graph is reflected about the x-axis.



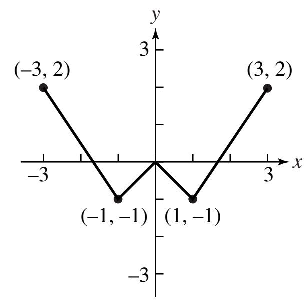
(c) The given graph is shifted left 1 unit, stretched vertically by a factor of 2, reflected about the x-axis, and then shifted upward 1 unit.



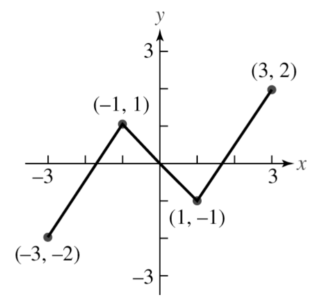
(d) The given graph is shifted right 2 units, stretched vertically by a factor of 3, and then shifted downward 2 units.



61. (a)



(b)



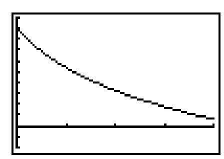
62. (a) $V = 100{,}000 - 10{,}000x,\ 0 \le x \le 10$

(b)
$$\begin{aligned} V &= 55{,}000 \\ 100{,}000 - 10{,}000x &= 55{,}000 \\ -10{,}000x &= -45{,}000 \\ x &= 4.5 \end{aligned}$$
The value is \$55,000 after 4.5 years.

63. (a) $f(0) = 90$ units

(b) $f(2) = 90 - 52 \ln 3 \approx 32.8722$ units

(c)



[0, 4] by [–20, 100]

64.
$$\begin{aligned} 150(1.08)^t &= 5000 \\ 1.08^t &= \frac{5000}{1500} = \frac{10}{3} \\ \ln(1.08)^t &= \ln\frac{10}{3} \\ t\ln 1.08 &= \ln\frac{10}{3} \\ t &= \frac{\ln\left(\frac{10}{3}\right)}{\ln 1.08} \\ t &\approx 15.6439 \end{aligned}$$
It will take about 15.6439 years. (If the bank only pays interest at the end of the year, it will take 16 years.)

65. **(a)** $N(t) = 4 \cdot 2^t$

(b) 4 days: $4 \cdot 2^4 = 64$ guppies

1 week: $4 \cdot 2^7 = 512$ guppies

(c)
$$\begin{aligned} N(t) &= 2000 \\ 4 \cdot 2^t &= 2000 \\ 2^t &= 500 \\ \ln 2^t &= \ln 500 \\ t \ln 2 &= \ln 500 \\ t &= \frac{\ln 500}{\ln 2} \approx 8.9658 \end{aligned}$$
There will be 2000 guppies after 8.9658 days, or after nearly 9 days.

(d) Because it suggests the number of guppies will continue to double indefinitely and become arbitrarily large, which is impossible due to the finite size of the tank and the oxygen supply in the water.

66. **(a)**
$$\begin{aligned} Pe^{rt} &= 2P \\ e^{rt} &= 2 \\ \ln e^{rt} &= \ln 2 \\ rt \ln e &= \ln 2 \\ rt &= \ln 2 \\ t &= \frac{\ln 2}{r} \approx \frac{0.69}{r} \end{aligned}$$

(b) Note that $r = \frac{R}{100}$, so
$$t = \frac{\ln 2}{\frac{R}{100}} = \frac{100 \ln 2}{R} \approx \frac{69}{R}.$$

(c) Doubling time $t \approx \frac{69+1}{R} = \frac{70}{R}$.

67. Since 72 is evenly divisible by so many integer factors, people find it easier to approximate the doubling time by using $\frac{72}{R}$ than by using $\frac{70}{R}$.

68. **(a)** $m = -1$

(b) $y = -x - 1$

(c) $y = x + 3$

(d) 2

69. **(a)** $(2, \infty)$, since $x - 2 > 0$

(b) $(-\infty, \infty)$, or all real numbers

(c)
$$\begin{aligned} f(x) &= 1 - \ln(x-2) \\ 0 &= 1 - \ln(x-2) \\ 1 &= \ln(x-2) \\ e^1 &= x - 2 \\ x &= e + 2 \approx 4.718 \end{aligned}$$

(d)
$$\begin{aligned} y &= 1 - \ln(x-2) \\ y - 1 &= -\ln(x-2) \\ 1 - y &= \ln(x-2) \\ e^{1-y} &= x - 2 \\ x &= 2 + e^{1-y} \\ y &= 2 + e^{1-x} \\ f^{-1}(x) &= 2 + e^{1-x} \end{aligned}$$

(e)
$$\begin{aligned} (f \circ f^{-1})(x) &= f(f^{-1}(x)) \\ &= f(2 + e^{1-x}) \\ &= 1 - \ln(2 + e^{1-x} - 2) \\ &= 1 - \ln(e^{1-x}) \\ &= 1 - (1 - x) \\ &= x \end{aligned}$$
$$\begin{aligned} (f^{-1} \circ f)(x) &= f^{-1}(f(x)) \\ &= f^{-1}(1 - \ln(x-2)) \\ &= 2 + e^{1-(1-\ln(x-2))} \\ &= 2 + e^{\ln(x-2)} \\ &= 2 + (x - 2) \\ &= x \end{aligned}$$

70. **(a)** $(-\infty, \infty)$, or all real numbers

(b) $[-2, 4]$, $1 - 3\cos(2x)$ oscillates between -2 and 4.

(c) π

(d) Even; $\cos(-\theta) = \cos(\theta)$

(e) $x \approx 2.526$