

Chapter 1 Equations and Inequalities

Section 1.1 Linear Equations and Rational Equations

1. linear

2. first

3. solution

4. conditional

5. identity

6. contradiction

7. rational

8. empty (or null); $\{ \} \text{ or } \phi$

$$-2x = 8$$

$$\frac{-2x}{-2} = \frac{8}{-2}$$

9. a. Linear; $x = -4$

$$\{-4\}$$

b. Nonlinear

$$-\frac{1}{2}x = 8$$

$$-2\left(-\frac{1}{2}x\right) = -2(8)$$

c. Linear; $x = -16$

$$\{-16\}$$

d. Nonlinear

$$x - 2 = 8$$

$$x - 2 + 2 = 8 + 2$$

e. Linear; $x = 10$

$$\{10\}$$

$$12 = 4x$$

$$\frac{12}{4} = \frac{4x}{4}$$

10. a. Linear; $3 = x$

$$\{3\}$$

b. Nonlinear

$$12 = \frac{1}{4}x$$

$$4(12) = 4\left(\frac{1}{4}x\right)$$

c. Linear; $48 = x$

$$\{48\}$$

d. Nonlinear

$$12 = 4 + x$$

$$12 - 4 = 4 + x - 4$$

e. Linear; $8 = x$

$$\{8\}$$

$$-6x - 4 = 20$$

$$-6x = 24$$

11. $x = -4$

$$\{-4\}$$

$$-8y + 6 = 22$$

$$-8y = 16$$

12. $y = -2$

$$\{-2\}$$

$$4 = 7 - 3(4t + 1)$$

$$4 = 7 - 12t - 3$$

$$4 = 4 - 12t$$

$$0 = -12t$$

13. $0 = t$

$$\{0\}$$

$$11 = 7 - 2(5p - 2)$$

$$11 = 7 - 10p + 4$$

$$11 = 11 - 10p$$

$$0 = -10p$$

14. $0 = p$

$$\{0\}$$

$$-6(v-2)+3=9-(v+4)$$

$$-6v+12+3=9-v-4$$

$$-6v+15=5-v$$

$$-5v=-10$$

$$15. \quad v=2$$

$$\{2\}$$

$$-5(u-4)+2=11-(u-3)$$

$$-5u+20+2=11-u+3$$

$$-5u+22=14-u$$

$$-4u=-8$$

$$16. \quad u=2$$

$$\{2\}$$

$$2.3=4.5x+30.2$$

$$-27.9=4.5x$$

$$17. \quad -6.2=x$$

$$\{-6.2\}$$

$$9.4=3.5p-0.4$$

$$9.8=3.5p$$

$$18. \quad 2.8=p$$

$$\{2.8\}$$

$$0.05y+0.02(6000-y)=270$$

$$0.05y+120-0.02y=270$$

$$0.03y+120=270$$

$$0.03y=150$$

$$19. \quad y=5000$$

$$\{5000\}$$

$$0.06x+0.04(10,000-x)=520$$

$$0.06x+400-0.04x=520$$

$$0.02x=120$$

$$20. \quad x=6000$$

$$\{6000\}$$

$$2(5x-6)=4[x-3(x-10)]$$

$$10x-12=4(x-3x+30)$$

$$10x-12=4(-2x+30)$$

$$10x-12=-8x+120$$

$$18x=132$$

$$21. \quad x=\frac{132}{18}=\frac{22}{3}$$

$$\left\{\frac{22}{3}\right\}$$

$$4(y-3)=3[y+2(y-2)]$$

$$4y-12=3(y+2y-4)$$

$$4y-12=3(3y-4)$$

$$4y-12=9y-12$$

$$-5y=0$$

$$22. \quad y=0$$

$$\{0\}$$

$$\frac{1}{4}x-\frac{3}{2}=2$$

$$4\left(\frac{1}{4}x-\frac{3}{2}\right)=4(2)$$

$$x-6=8$$

$$23. \quad x=14$$

$$\{14\}$$

$$\frac{1}{6}x-\frac{5}{3}=1$$

$$6\left(\frac{1}{6}x-\frac{5}{3}\right)=6(1)$$

$$x-10=6$$

$$24. \quad x=16$$

$$\{16\}$$

$$\frac{1}{2}w-\frac{3}{4}=\frac{2}{3}w+2$$

$$12\left(\frac{1}{2}w-\frac{3}{4}\right)=12\left(\frac{2}{3}w+2\right)$$

$$6w-9=8w+24$$

$$-2w=33$$

$$25. \quad w=-\frac{33}{2}$$

$$\left\{-\frac{33}{2}\right\}$$

$$\begin{aligned}\frac{2}{5}p - \frac{3}{10} &= \frac{7}{15}p - 1 \\ 30\left(\frac{2}{5}p - \frac{3}{10}\right) &= 30\left(\frac{7}{15}p - 1\right) \\ 12p - 9 &= 14p - 30 \\ -2p &= -21 \\ p &= \frac{21}{2}\end{aligned}$$

26.

$$\left\{\frac{21}{2}\right\}$$

$$\begin{aligned}\frac{y-1}{5} + \frac{y}{4} &= \frac{y+3}{2} + 1 \\ 20\left(\frac{y-1}{5} + \frac{y}{4}\right) &= 20\left(\frac{y+3}{2} + 1\right) \\ 4(y-1) + 5y &= 10(y+3) + 20 \\ 4y - 4 + 5y &= 10y + 30 + 20 \\ 9y - 4 &= 10y + 50 \\ -y &= 54 \\ y &= -54\end{aligned}$$

27.

$$\{-54\}$$

$$\begin{aligned}\frac{x-6}{3} + \frac{x}{7} &= \frac{x+1}{3} + 2 \\ 21\left(\frac{x-6}{3} + \frac{x}{7}\right) &= 21\left(\frac{x+1}{3} + 2\right) \\ 7(x-6) + 3x &= 7(x+1) + 42 \\ 7x - 42 + 3x &= 7x + 7 + 42 \\ 10x - 42 &= 7x + 49 \\ 3x &= 91 \\ x &= \frac{91}{3}\end{aligned}$$

28.

$$\left\{\frac{91}{3}\right\}$$

$$\frac{n+3}{4} - \frac{n-2}{5} = \frac{n+1}{10} - 1$$

29.

$$\begin{aligned}20\left(\frac{n+3}{4} - \frac{n-2}{5}\right) &= 20\left(\frac{n+1}{10} - 1\right) \\ 5(n+3) - 4(n-2) &= 2(n+1) - 20 \\ 5n + 15 - 4n + 8 &= 2n + 2 - 20 \\ n + 23 &= 2n - 18 \\ -n &= -41 \\ n &= 41\end{aligned}$$

$$\{41\}$$

$$\begin{aligned}\frac{t-2}{3} - \frac{t+7}{5} &= \frac{t-4}{10} + 2 \\ 30\left(\frac{t-2}{3} - \frac{t+7}{5}\right) &= 30\left(\frac{t-4}{10} + 2\right) \\ 10(t-2) - 6(t+7) &= 3(t-4) + 60 \\ 10t - 20 - 6t - 42 &= 3t - 12 + 60 \\ 4t - 62 &= 3t + 48 \\ t &= 110\end{aligned}$$

30.

$$\{110\}$$

$$\begin{aligned}T &= -1.83a + 212 \\ T &= -1.83(4) + 212 \\ &= 204.68^\circ\text{F} \\ &\approx 205^\circ\text{F}\end{aligned}$$

31. a.

$$\begin{aligned}T &= -1.83a + 212 \\ 193 &= -1.83a + 212 \\ -19 &= -1.83a\end{aligned}$$

$$\text{b. } 10.4 \approx a$$

$$\begin{aligned}10.4 \times 10^3 &= 10,400 \\ \text{Approximately } 10,400 \text{ ft} \\ C &= 167.95x + 94 \\ C &= 167.95(9) + 94 \\ &= \$1605.55\end{aligned}$$

32. a.

$$\begin{aligned}C &= 167.95x + 94 \\ 2445.30 &= 167.95x + 94 \\ 2351.30 &= 167.95x\end{aligned}$$

$$\text{b. } 14 = x$$

14 credit-hours

- $S = 14.2t + 149$
 $362 = 14.2t + 149$
 $213 = 14.2t$
33. $15 = t$
 $2004 + 15 = 2019$
 In 2019
 $S = 18t + 232$
 $628 = 18t + 232$
 $396 = 18t$
34. $22 = t$
 $2000 + 22 = 2022$
 In 2022
35. a. $C = 7x$
 $C = 105$
 $7x = 105$
b. $x = 15$
 The motorist will save money
 beginning on the 16th working day.
36. a. $C = 2.25x$
 $C = 89$
 $2.25x = 89$
b. $x \approx 39.6$
 The commuter will save money on the
 40th ride.
37. a. $S_1 = 45,000 + 2250x$
b. $S_2 = 48,000 + 2000x$
 $S_1 = S_2$
 $45,000 + 2250x = 48,000 + 2000x$
 $250x = 3000$
 $x = 12 \text{ yr}$
c.
38. a. $S_1 = 25,000 + 0.16x$
b. $S_2 = 30,000 + 0.15x$
 $S_1 = S_2$
 $25,000 + 0.16x = 30,000 + 0.15x$
 $0.01x = 5000$
 $x = \$500,000$
c.

- $2x - 3 = 4(x - 1) - 1 - 2x$
 $2x - 3 = 4x - 4 - 1 - 2x$
 $2x - 3 = 2x - 5$
39. $-3 = -5$
 Contradiction
 $4(3 - 5n) + 1 = -4n - 8 - 16n$
 $12 - 20n + 1 = -4n - 8 - 16n$
 $-20n + 13 = -20n - 8$
40. $13 = -8$
 Contradiction
 $-(6 - 2w) = 4(w + 1) - 2w - 10$
 $-6 + 2w = 4w + 4 - 2w - 10$
 $-6 + 2w = 2w - 6$
41. $0 = 0$
 Identity; $\square$
 $-5 + 3x = 3(x - 1) - 2$
 $-5 + 3x = 3x - 3 - 2$
 $-5 + 3x = 3x - 5$
42. $0 = 0$
 Identity; $\square$
 $\frac{1}{2}x + 3 = \frac{1}{4}x + 1$
 $4\left(\frac{1}{2}x + 3\right) = 4\left(\frac{1}{4}x + 1\right)$
 $2x + 12 = x + 4$
43. $x = -8$
 Conditional equation; $\{-8\}$
 $\frac{2}{3}y - 5 = \frac{1}{6}y - 4$
 $6\left(\frac{2}{3}y - 5\right) = 6\left(\frac{1}{6}y - 4\right)$
 $4y - 30 = y - 24$
 $3y = 6$
 $y = 2$
44.
 Conditional equation; $\{2\}$
 $\frac{3}{x - 5} + \frac{2}{x + 4} = \frac{5}{7}$
45. $x \neq 5, x \neq -4$

$$46. \frac{2}{x+1} - \frac{5}{x-7} = \frac{2}{3}$$

$$x \neq -1, x \neq 7$$

$$\frac{5}{2x-3} - \frac{3}{5x} = \frac{1}{3-x}$$

$$\frac{5}{2\left(x-\frac{3}{2}\right)} - \frac{3}{5x} = \frac{1}{3-x}$$

$$47. x \neq \frac{3}{2}, x \neq 0, x \neq 3$$

$$\frac{1}{2x} - \frac{3}{6-x} = \frac{2}{4x-5}$$

$$\frac{1}{2x} - \frac{3}{6-x} = \frac{2}{4\left(x-\frac{5}{4}\right)}$$

$$48. x \neq 0, x \neq 6, x \neq \frac{5}{4}$$

$$\frac{1}{2} - \frac{7}{2y} = \frac{5}{y}$$

$$2y\left(\frac{1}{2} - \frac{7}{2y}\right) = 2y\left(\frac{5}{y}\right)$$

$$y-7=10$$

$$y=17$$

$$49. \{17\}$$

$$\frac{1}{3} - \frac{4}{3t} = \frac{7}{t}$$

$$3\left(\frac{1}{3} - \frac{4}{3t}\right) = 3\left(\frac{7}{t}\right)$$

$$t-4=21$$

$$50. t=25$$

$$\{25\}$$

$$\frac{w+3}{4w} + 1 = \frac{w-5}{w}$$

$$4w\left(\frac{w+3}{4w} + 1\right) = 4w\left(\frac{w-5}{w}\right)$$

$$w+3+4w=4(w-5)$$

$$5w+3=4w-20$$

$$w=-23$$

$$51. \{-23\}$$

$$\frac{x+2}{6x} + 1 = \frac{x-7}{x}$$

$$6x\left(\frac{x+2}{6x} + 1\right) = 6x\left(\frac{x-7}{x}\right)$$

$$x+2+6x=6(x-7)$$

$$7x+2=6x-42$$

$$52. x=-44$$

$$\{-44\}$$

$$53. \frac{c}{c-3} = \frac{3}{c-3} - \frac{3}{4}$$

$$4(c-3)\left(\frac{c}{c-3}\right) = 4(c-3)\left(\frac{3}{c-3} - \frac{3}{4}\right)$$

$$4c=12-3(c-3)$$

$$4c=12-3c+9$$

$$7c=21$$

$$c=3$$

$\{ \}$; The value 3 does not check.

$$\frac{7}{d-7} - \frac{7}{8} = \frac{d}{d-7}$$

$$54. 8(d-7)\left(\frac{7}{d-7} - \frac{7}{8}\right) = 8(d-7)\left(\frac{d}{d-7}\right)$$

$$56-7(d-7)=8d$$

$$56-7d+49=8d$$

$$-15d=-105$$

$$d=7$$

$\{ \}$; The value 7 does not check.

$$\begin{aligned}\frac{1}{t-1} &= \frac{3}{t^2-1} \\ \frac{1}{t-1} &= \frac{3}{(t+1)(t-1)} \\ (t+1)(t-1)\left(\frac{1}{t-1}\right) &= (t+1)(t-1)\left[\frac{3}{(t+1)(t-1)}\right] \\ t+1 &= 3 \\ t &= 2\end{aligned}$$

55. $\{2\}$

$$\begin{aligned}\frac{1}{w+2} &= \frac{5}{w^2-4} \\ \frac{1}{w+2} &= \frac{5}{(w+2)(w-2)} \\ (w+2)(w-2)\left(\frac{1}{w+2}\right) &= (w+2)(w-2)\left[\frac{5}{(w+2)(w-2)}\right] \\ w-2 &= 5 \\ w &= 7\end{aligned}$$

56. $\{7\}$ **57.**

$$\begin{aligned}\frac{2}{x-5} - \frac{1}{x+5} &= \frac{11}{x^2-25} \\ \frac{2}{x-5} - \frac{1}{x+5} &= \frac{11}{(x+5)(x-5)} \\ (x+5)(x-5)\left(\frac{2}{x-5} - \frac{1}{x+5}\right) &= (x+5)(x-5)\left[\frac{11}{(x+5)(x-5)}\right] \\ 2(x+5) - 1(x-5) &= 11 \\ 2x+10-x+5 &= 11 \\ x+15 &= 11 \\ x &= -4\end{aligned}$$

 $\{-4\}$

58.
$$\frac{2}{c+3} - \frac{1}{c-3} = \frac{10}{c^2-9}$$

$$\frac{2}{c+3} - \frac{1}{c-3} = \frac{10}{(c+3)(c-3)}$$

$$(c+3)(c-3)\left(\frac{2}{c+3} - \frac{1}{c-3}\right) = (c+3)(c-3)\left[\frac{10}{(c+3)(c-3)}\right]$$

$$2(c-3) - 1(c+3) = 10$$

$$2c - 6 - c - 3 = 10$$

$$c - 9 = 10$$

$$c = 19$$

{19}

59.
$$\frac{-14}{x^2-x-12} - \frac{1}{x-4} = \frac{4}{x+3}$$

$$\frac{-14}{(x-4)(x+3)} - \frac{1}{(x-4)} = \frac{2}{(x+3)}$$

$$(x-4)(x+3)\left[\frac{-14}{(x-4)(x+3)} - \frac{1}{(x-4)}\right] = (x-4)(x+3)\left[\frac{2}{(x+3)}\right]$$

$$-14 - (x+3) = 2(x-4)$$

$$-14 - x - 3 = 2x - 8$$

$$-17 - x = 2x - 8$$

$$-3 = x$$

{ } ; The value -3 does not check.

60.
$$\frac{2}{x^2+5x+6} - \frac{2}{x+2} = \frac{1}{x+3}$$

$$\frac{2}{(x+2)(x+3)} - \frac{2}{(x+2)} = \frac{1}{(x+3)}$$

$$(x+2)(x+3)\left[\frac{2}{(x+2)(x+3)} - \frac{2}{(x+2)}\right] = (x+2)(x+3)\left[\frac{1}{(x+3)}\right]$$

$$2 - 2(x+3) = (x+2)$$

$$2 - 2x - 6 = x + 2$$

$$-4 - 2x = x + 2$$

$$-2 = x$$

{ } ; The value -2 does not check.

61.

$$\frac{5}{x^2-x-2} - \frac{2}{x^2-4} = \frac{4}{x^2+3x+2}$$

$$\frac{5}{(x-2)(x+1)} - \frac{2}{(x-2)(x+2)} = \frac{4}{(x+2)(x+1)}$$

$$(x+2)(x-2)(x+1) \left[\frac{5}{(x-2)(x+1)} - \frac{2}{(x-2)(x+2)} \right] = (x+2)(x-2)(x+1) \left[\frac{4}{(x+2)(x+1)} \right]$$

$$5(x+2) - 2(x+1) = 4(x-2)$$

$$5x+10-2x-2=4x-8$$

$$3x+8=4x-8$$

$$16=x$$

{16}

62.

$$\frac{4}{x^2-2x-8} - \frac{1}{x^2-16} = \frac{2}{x^2+6x+8}$$

$$\frac{4}{(x-4)(x+2)} - \frac{1}{(x-4)(x+4)} = \frac{2}{(x+4)(x+2)}$$

$$(x+4)(x-4)(x+2) \left[\frac{4}{(x-4)(x+2)} - \frac{1}{(x-4)(x+4)} \right] = (x+4)(x-4)(x+2) \left[\frac{2}{(x+4)(x+2)} \right]$$

$$4(x+4) - 1(x+2) = 2(x-4)$$

$$4x+16-x-2=2x-8$$

$$3x+14=2x-8$$

$$x=-22$$

{-22}

63.

$$\frac{5}{m-2} = \frac{3m}{m^2+2m-8} - \frac{2}{m+4}$$

$$\frac{5}{m-2} = \frac{3m}{(m+4)(m-2)} - \frac{2}{m+4}$$

$$(m+4)(m-2) \left(\frac{5}{m-2} \right) = (m+4)(m-2) \left[\frac{3m}{(m+4)(m-2)} - \frac{2}{m+4} \right]$$

$$5(m+4) = 3m - 2(m-2)$$

$$5m+20 = 3m - 2m + 4$$

$$5m+20 = m+4$$

$$4m = -16$$

$$m = -4$$

{ } ; The value -4 does not check.

$$\begin{aligned}
 \mathbf{64.} \quad & \frac{10}{n-6} = \frac{15n}{n^2-2n-24} - \frac{6}{n+4} \\
 & \frac{10}{n-6} = \frac{15n}{(n-6)(n+4)} - \frac{6}{n+4} \\
 & (n-6)(n+4) \left(\frac{10}{n-6} \right) = (n-6)(n+4) \left[\frac{15n}{(n-6)(n+4)} - \frac{6}{n+4} \right] \\
 & 10(n+4) = 15n - 6(n-6) \\
 & 10n + 40 = 15n - 6n + 36 \\
 & 10n + 40 = 9n + 36 \\
 & n = -4
 \end{aligned}$$

$\{ \}$; The value -4 does not check.

$$\begin{aligned}
 \mathbf{65.} \quad & \frac{5x}{3x^2-5x-2} - \frac{1}{3x+1} = \frac{3}{2-x} \\
 & \frac{5x}{(3x+1)(x-2)} - \frac{1}{3x+1} = \frac{-3}{x-2} \\
 & (3x+1)(x-2) \left[\frac{5x}{(3x+1)(x-2)} - \frac{1}{3x+1} \right] = (3x+1)(x-2) \left(\frac{-3}{x-2} \right) \\
 & 5x - 1(x-2) = -3(3x+1) \\
 & 5x - x + 2 = -9x - 3 \\
 & 4x + 2 = -9x - 3 \\
 & 13x = -5 \\
 & x = -\frac{5}{13}
 \end{aligned}$$

$$\left\{ -\frac{5}{13} \right\}$$

$$\begin{aligned}
 66. \quad & \frac{3x}{2x^2+x-3} - \frac{2}{2x+3} = \frac{4}{1-x} \\
 & \frac{3x}{(2x+3)(x-1)} - \frac{2}{2x+3} = \frac{-4}{x-1} \\
 & (2x+3)(x-1) \left[\frac{3x}{(2x+3)(x-1)} - \frac{2}{2x+3} \right] = (2x+3)(x-1) \left(\frac{-4}{x-1} \right) \\
 & 3x - 2(x-1) = -4(2x+3) \\
 & 3x - 2x + 2 = -8x - 12 \\
 & x + 2 = -8x - 12 \\
 & 9x = -14 \\
 & x = -\frac{14}{9}
 \end{aligned}$$

$$\left\{ -\frac{14}{9} \right\}$$

$$A = lw \text{ for } l$$

$$\frac{A}{w} = \frac{lw}{w}$$

$$67. \quad \frac{A}{w} = l \text{ or } l = \frac{A}{w}$$

$$E = IR \text{ for } R$$

$$\frac{E}{I} = \frac{IR}{I}$$

$$68. \quad \frac{E}{I} = R \text{ or } R = \frac{E}{I}$$

$$P = a + b + c \text{ for } c$$

$$69. \quad P - a - b = c \text{ or } c = P - a - b$$

$$W = K - T \text{ for } K$$

$$70. \quad W + T = K \text{ or } K = W + T$$

$$\Delta s = s_2 - s_1 \text{ for } s_1$$

$$\Delta s - s_2 = -s_1$$

$$71. \quad s_1 = s_2 - \Delta s$$

$$\Delta t = t_f - t_i \text{ for } t_i$$

$$\Delta t - t_f = -t_i$$

$$72. \quad t_i = t_f - \Delta t$$

$$7x + 2y = 8 \text{ for } y$$

$$2y = -7x + 8$$

$$\frac{2y}{2} = \frac{-7x + 8}{2}$$

$$73. \quad y = \frac{-7x + 8}{2} \text{ or } y = -\frac{7}{2}x + 4$$

$$3x + 5y = 15 \text{ for } y$$

$$5y = -3x + 15$$

$$\frac{5y}{5} = \frac{-3x + 15}{5}$$

$$74. \quad y = \frac{-3x + 15}{5} \text{ or } y = -\frac{3}{5}x + 3$$

$$5x - 4y = 2 \text{ for } y$$

$$-4y = -5x + 2$$

$$\frac{-4y}{-4} = \frac{-5x + 2}{-4}$$

$$75. \quad y = \frac{5x - 2}{4} \text{ or } y = \frac{5}{4}x - \frac{1}{2}$$

$$7x - 2y = 5 \text{ for } y$$

$$-2y = -7x + 5$$

$$\frac{-2y}{-2} = \frac{-7x + 5}{-2}$$

$$76. \quad y = \frac{7x - 5}{2} \text{ or } y = \frac{7}{2}x - \frac{5}{2}$$

$$77. \quad \frac{1}{2}x + \frac{1}{3}y = 1 \text{ for } y$$

$$6\left(\frac{1}{2}x + \frac{1}{3}y\right) = 6(1)$$

$$3x + 2y = 6$$

$$2y = -3x + 6$$

$$\frac{2y}{2} = \frac{-3x + 6}{2}$$

$$y = \frac{-3x + 6}{2} \text{ or } y = -\frac{3}{2}x + 3$$

$$78. \quad \frac{1}{4}x - \frac{2}{3}y = 2 \text{ for } y$$

$$12\left(\frac{1}{4}x - \frac{2}{3}y\right) = 12(2)$$

$$3x - 8y = 24$$

$$-8y = -3x + 24$$

$$\frac{-8y}{-8} = \frac{-3x + 24}{-8}$$

$$y = \frac{3x - 24}{8} \text{ or } y = \frac{3}{8}x - 3$$

$$79. \quad S = \frac{n}{2}(a + d) \text{ for } d$$

$$2(S) = 2\left[\frac{n}{2}(a + d)\right]$$

$$2S = n(a + d)$$

$$2S = na + nd$$

$$2S - na = nd$$

$$\frac{2S - na}{n} = \frac{nd}{n}$$

$$\frac{2S - na}{n} = d$$

$$d = \frac{2S - na}{n} \text{ or } d = \frac{2S}{n} - a$$

$$80. \quad S = \frac{n}{2}[2a + (n - 1)d] \text{ for } a$$

$$2(S) = 2\left\{\frac{n}{2}[2a + (n - 1)d]\right\}$$

$$2S = n[2a + (n - 1)d]$$

$$2S = n(2a + nd - d)$$

$$2S = 2an + n^2d - nd$$

$$2S - n^2d + nd = 2an$$

$$\frac{2S - n^2d + nd}{2n} = \frac{2an}{2n}$$

$$\frac{2S - n^2d + nd}{2n} = a \text{ or } a = \frac{2S - n^2d + nd}{2n}$$

$$V = \frac{1}{3}\pi r^2 h \text{ for } h$$

$$3(V) = 3\left(\frac{1}{3}\pi r^2 h\right)$$

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$81. \quad \frac{3V}{\pi r^2} = h \text{ or } h = \frac{3V}{\pi r^2}$$

$$V = \frac{1}{3}Bh \text{ for } B$$

$$3(V) = 3\left(\frac{1}{3}Bh\right)$$

$$3V = Bh$$

$$\frac{3V}{h} = \frac{Bh}{h}$$

$$82. \quad \frac{3V}{h} = B \text{ or } B = \frac{3V}{h}$$

$$6 = 4x + tx \text{ for } x$$

$$6 = x(4 + t)$$

$$\frac{6}{4 + t} = \frac{x(4 + t)}{4 + t}$$

$$83. \quad \frac{6}{4 + t} = x \text{ or } x = \frac{6}{4 + t}$$

$$8 = 3x + kx \text{ for } x$$

$$8 = x(3 + k)$$

$$\frac{8}{3 + k} = \frac{x(3 + k)}{3 + k}$$

$$84. \quad \frac{8}{3 + k} = x \text{ or } x = \frac{8}{3 + k}$$

$$\begin{aligned}
 6x + ay &= bx + 5 \text{ for } x \\
 6x - bx &= 5 - ay \\
 x(6 - b) &= 5 - ay \\
 \frac{x(6 - b)}{6 - b} &= \frac{5 - ay}{6 - b} \\
 x &= \frac{5 - ay}{6 - b} \text{ or } x = \frac{ay - 5}{b - 6}
 \end{aligned}$$

85.

$$\begin{aligned}
 3x + 2y &= cx + d \text{ for } x \\
 3x - cx &= d - 2y \\
 x(3 - c) &= d - 2y \\
 \frac{x(3 - c)}{3 - c} &= \frac{d - 2y}{3 - c} \\
 x &= \frac{d - 2y}{3 - c} \text{ or } x = \frac{2y - d}{c - 3}
 \end{aligned}$$

86.

$$\begin{aligned}
 \frac{5}{2n+1} &= \frac{-2}{3n-4} \\
 (2n+1)(3n-4) \left(\frac{5}{2n+1} \right) &= (2n+1)(3n-4) \left(\frac{-2}{3n-4} \right) \\
 5(3n-4) &= -2(2n+1) \\
 15n - 20 &= -4n - 2 \\
 19n &= 18 \\
 n &= \frac{18}{19} \\
 \left\{ \frac{18}{19} \right\}
 \end{aligned}$$

$$\begin{aligned}
 \frac{4}{5z-3} &= \frac{-2}{4z+7} \\
 (5z-3)(4z+7) \left(\frac{4}{5z-3} \right) &= (5z-3)(4z+7) \left(\frac{-2}{4z+7} \right) \\
 4(4z+7) &= -2(5z-3) \\
 16z + 28 &= -10z + 6 \\
 26z &= -22 \\
 z &= -\frac{22}{26} = -\frac{11}{13} \\
 \left\{ -\frac{11}{13} \right\}
 \end{aligned}$$

90.

$$\begin{aligned}
 A &= P + Prt \text{ for } P \\
 A &= P(1 + rt) \\
 \frac{A}{1 + rt} &= \frac{P(1 + rt)}{1 + rt} \\
 \frac{A}{1 + rt} &= P \text{ or } P = \frac{A}{1 + rt} \\
 C &= A + Ar \text{ for } A \\
 C &= A(1 + r) \\
 \frac{C}{1 + r} &= \frac{A(1 + r)}{1 + r} \\
 \frac{C}{1 + r} &= A \text{ or } A = \frac{C}{1 + r}
 \end{aligned}$$

87.**88.**

$$91. \quad 5 - 2\{3 - [5v + 3(v - 7)]\} = 8v + 6(3 - 4v) - 61$$

$$5 - 2[3 - (5v + 3v - 21)] = 8v + 18 - 24v - 61$$

$$5 - 2[3 - (8v - 21)] = -16v - 43$$

$$5 - 2(3 - 8v + 21) = -16v - 43$$

$$5 - 2(24 - 8v) = -16v - 43$$

$$5 - 48 + 16v = -16v - 43$$

$$16v - 43 = -16v - 43$$

$$32v = 0$$

$$v = 0$$

$$\{0\}$$

$$92. \quad 6 - \{4 - 2[8u - 2(u - 3)]\} = -4u + 3(2 - u) + 8$$

$$6 - [4 - 2(8u - 2u + 6)] = -4u + 6 - 3u + 8$$

$$6 - [4 - 2(6u + 6)] = -7u + 14$$

$$6 - (4 - 12u - 12) = -7u + 14$$

$$6 - (12u - 8) = -7u + 14$$

$$6 - 12u + 8 = -7u + 14$$

$$-5u = 0$$

$$u = 0$$

$$\{0\}$$

$$(x - 7)(x + 2) = x^2 + 4x + 13$$

$$x^2 + 2x - 7x - 14 = x^2 + 4x + 13$$

$$x^2 - 5x - 14 = x^2 + 4x + 13$$

$$-9x = 27$$

$$93. \quad x = -3$$

$$\{-3\}$$

$$(m + 3)(2m - 5) = 2m^2 + 4m - 3$$

$$2m^2 + 6m - 5m - 15 = 2m^2 + 4m - 3$$

$$2m^2 + m - 15 = 2m^2 + 4m - 3$$

$$-3m = 12$$

$$94. \quad m = -4$$

$$\{-4\}$$

$$95. \quad \frac{3}{c^2 - 4c} - \frac{9}{2c^2 + 3c} = \frac{2}{2c^2 - 5c - 12}$$

$$\frac{3}{c(c-4)} - \frac{9}{c(2c+3)} = \frac{2}{(2c+3)(c-4)}$$

$$c(2c+3)(c-4) \left[\frac{3}{c(c-4)} - \frac{9}{c(2c+3)} \right] = c(2c+3)(c-4) \left[\frac{2}{(2c+3)(c-4)} \right]$$

$$3(2c+3) - 9(c-4) = 2c$$

$$6c + 9 - 9c + 36 = 2c$$

$$-3c + 45 = 2c$$

$$-5c = -45$$

$$c = 9$$

{9}

96.

$$\frac{4}{d^2-d} - \frac{5}{2d^2+5d} = \frac{2}{2d^2+3d-5}$$

$$\frac{4}{d(d-1)} - \frac{5}{d(2d+5)} = \frac{2}{(2d+5)(d-1)}$$

$$d(2d+5)(d-1) \left[\frac{4}{d(d-1)} - \frac{5}{d(2d+5)} \right] = d(2d+5)(d-1) \left[\frac{2}{(2d+5)(d-1)} \right]$$

$$4(2d+5) - 5(d-1) = 2d$$

$$8d + 20 - 5d + 5 = 2d$$

$$3d + 25 = 2d$$

$$d = -25$$

{-25}

$$\frac{1}{3}x + \frac{1}{2} = \frac{1}{2}(x+1) - \frac{1}{6}x$$

$$6\left(\frac{1}{3}x + \frac{1}{2}\right) = 6\left[\frac{1}{2}(x+1) - \frac{1}{6}x\right]$$

$$2x + 3 = 3(x+1) - x$$

$$2x + 3 = 3x + 3 - x$$

$$2x + 3 = 2x + 3$$

97.

$$0 = 0$$

□

$$\frac{1}{2}x + \frac{2}{5} = \frac{2}{5}(x+1) + \frac{1}{10}x$$

$$10\left(\frac{1}{2}x + \frac{2}{5}\right) = 10\left[\frac{2}{5}(x+1) + \frac{1}{10}x\right]$$

$$5x + 4 = 4(x+1) + x$$

$$5x + 4 = 4x + 4 + x$$

$$5x + 4 = 5x + 4$$

98.

$$0 = 0$$

□

$$(t+2)^2 = (t-4)^2$$

$$t^2 + 4t + 4 = t^2 - 8t + 16$$

$$12t = 12$$

$$t = 1$$

99.

{1}

$$(y-3)^2 = (y+1)^2$$

$$y^2 - 6y + 9 = y^2 + 2y + 1$$

$$-8y = -8$$

$$y = 1$$

100.

{1}

$$\begin{aligned}
 101. \quad \frac{3}{3a+4} &= \frac{5}{5a-1} \\
 \left[\frac{(3a+4)(5a-1)}{\left(\frac{3}{3a+4}\right)} \right] &= \left[\frac{(3a+4)(5a-1)}{\left(\frac{5}{5a-1}\right)} \right] \\
 3(5a-1) &= 5(3a+4) \\
 15a-3 &= 15a+20 \\
 -3 &= 20 \\
 \{ \}
 \end{aligned}$$

$$\begin{aligned}
 102. \quad \frac{8}{8x-3} &= \frac{2}{2x+5} \\
 \left[\frac{(8x-3)(2x+5)}{\left(\frac{8}{8x-3}\right)} \right] &= \left[\frac{(8x-3)(2x+5)}{\left(\frac{2}{2x+5}\right)} \right] \\
 8(2x+5) &= 2(8x-3) \\
 16x+40 &= 16x-6 \\
 40 &= -6 \\
 \{ \}
 \end{aligned}$$

$$P = \frac{40+20x}{1+0.05x}$$

103.

$$\begin{aligned}
 200 &= \frac{40+20x}{1+0.05x} \\
 (1+0.05x)(200) &= (1+0.05x)\left(\frac{40+20x}{1+0.05x}\right) \\
 200+10x &= 40+20x \\
 -10x &= -160 \\
 x &= 16 \text{ yr} \\
 v &= \frac{180t}{2t+10} \\
 60 &= \frac{180t}{2t+10} \\
 (2t+10)(60) &= (2t+10)\left(\frac{180t}{2t+10}\right) \\
 120t+600 &= 180t \\
 -60t &= -600 \\
 t &= 10 \text{ sec}
 \end{aligned}$$

104.

$$\begin{aligned}
 105. \quad A &= \frac{1}{22}c + \frac{1}{30}h \\
 7 &= \frac{1}{22}c + \frac{1}{30}(165) \\
 7 &= \frac{1}{22}c + \frac{11}{2} \\
 22(7) &= 22\left(\frac{1}{22}c + \frac{11}{2}\right) \\
 154 &= c + 121 \\
 c &= 33 \text{ mi}
 \end{aligned}$$

$$\begin{aligned}
 106. \quad A &= \frac{1}{24}c + \frac{1}{32}h \\
 9 &= \frac{1}{24}(60) + \frac{1}{32}h \\
 9 &= \frac{5}{2} + \frac{1}{32}h \\
 32(9) &= 32\left(\frac{5}{2} + \frac{1}{32}h\right) \\
 288 &= 80 + h \\
 h &= 208 \text{ mi}
 \end{aligned}$$

107. The value 5 is not defined within the expressions in the equation. Substituting 5 into the equation would result in division by 0.

108. The equation is an identity. The solution set is all real numbers.

109. The equation cannot be written in the form $ax + b = 0$. The term $\frac{3}{x} = 3x^{-1}$. Therefore, the term $\frac{3}{x}$ is not first degree and the equation is not a first-degree equation.

110. The equation cannot be written in the form $ax + b = 0$. The term $2\sqrt{x} = 2x^{1/2}$. Therefore, the term $2\sqrt{x}$ is not first degree and the equation is not a first-degree equation.

111. The equation is a contradiction. There is no real number x to which we add 1 that will equal the same real number x to which we add 2.

112. In each case, we can clear fractions by multiplying both sides of the equation by the LCD. For the

equation $\frac{x}{3} + \frac{1}{2} = 1$, the LCD is 6,

whereas for $\frac{3}{x} + \frac{1}{2} = 1$, the LCD is $2x$.

$$\begin{aligned} ax + 6 &= 4x + 14 \\ a(4) + 6 &= 4(4) + 14 \\ 4a + 6 &= 16 + 14 \\ 4a + 6 &= 30 \\ 4a &= 24 \end{aligned}$$

113.

$$\begin{aligned} a &= 6 \\ ax - 3 &= 2x + 9 \\ a(3) - 3 &= 2(3) + 9 \\ 3a - 3 &= 6 + 9 \\ 3a - 3 &= 15 \\ 3a &= 18 \end{aligned}$$

114.

$$\begin{aligned} a &= 6 \\ a(2x - 5) + 6 &= 5x + 7 \\ a[2(16) - 5] + 6 &= 5(16) + 7 \\ a(32 - 5) + 6 &= 80 + 7 \\ 27a + 6 &= 87 \\ 27a &= 81 \end{aligned}$$

115. $a = 3$

116.

$$\begin{aligned} a(2x + 4) + 12x &= 3(2 - x) \\ a[2(34) + 4] + 12(34) &= 3[2 - (34)] \\ a(68 + 4) + 408 &= 3(-32) \\ 72a + 408 &= -96 \\ 72a &= -504 \\ a &= -7 \end{aligned}$$

Section 1.2 Applications with Linear and Rational Equations

1. \$900

2. $0.08(2) = 0.16$ L

3. $\frac{d}{r}$

4. $\frac{d}{t}$

5. $P = 2l + 2w$

6. 180°

7. Let x represent the amount borrowed at 3%. Then, $(5000 - x)$ is the amount borrowed at 2.5%.

	3% Interest	2.5% Interest	Total
Principal	x	$5000 - x$	
Interest ($I =$	$x(0.03)(1)$	$(5000 - x)(0.025)(1)$	132.50

$$\begin{aligned}
x(0.03) + (5000 - x)(0.025) &= 132.50 \\
0.03x + 125 - 0.025x &= 132.50 \\
0.005x + 125 &= 132.50 \\
0.005x &= 7.50 \\
x &= 1500 \\
5000 - x &= 5000 - 1500 \\
&= 3500
\end{aligned}$$

Rocco borrowed \$1500 at 3% and \$3500 at 2.5%.

8. Let x represent the amount borrowed at 4%. Then, $(22,000 - x)$ is the amount borrowed at 5.5%.

	4% Interest	5.5% Interest Loan	Tot
Principal	x	$22,000 - x$	
Interest ($I =$	$x(0.04)(1)$	$(22,000 - x)(0.055)(1)$	910

$$\begin{aligned}
x(0.04) + (22,000 - x)(0.055) &= 910 \\
0.04x + 1210 - 0.055x &= 910 \\
-0.015x + 1210 &= 910 \\
-0.015x &= -300 \\
x &= 20,000 \\
22,000 - x &= 22,000 - 20,000 \\
&= 2000
\end{aligned}$$

Laura borrowed \$20,000 from the bank charging 4% interest, and \$2000 from the bank charging 5.5% interest.

9. Let x represent the amount invested in the 3-yr CD. Then, $x - 2000$ is the amount invested in the 18-month CD.

	3-yr CD	18-month (1.5-yr)	Total
Principal	x	$x - 2000$	
Interest ($I =$	$x(0.044)(3)$	$(x - 2000)(0.03)(1.5)$	706.50

$$\begin{aligned}
x(0.044)(3) + (x - 2000)(0.03)(1.5) &= 706.50 \\
0.132x + 0.045x - 90 &= 706.50 \\
0.177x - 90 &= 706.50 \\
0.177x &= 796.50 \\
x &= 4500 \\
x - 2000 &= 4500 - 2000 \\
&= 2500
\end{aligned}$$

Fernando invested \$4500 in the 3-yr CD and \$2500 in the 18-month CD.

10. Let x represent the amount invested in the 5-yr Treasury note. Then, $(x + 5000)$ is the amount invested in the 10-yr bond.

	5-yr Note	10-yr Bond	Tot
Principal	x	$x + 5000$	

Interest (I =	$x(0.028)(5)$	$(x + 5000)(0.036)(10)$	5300
-----------------------------------	---------------	-------------------------	------

$$\begin{aligned}
 x(0.028)(5) + (x + 5000)(0.036)(10) &= 5300 \\
 0.14x + 0.36x + 1800 &= 5300 \\
 0.5x + 1800 &= 5300 \\
 0.5x &= 3500 \\
 x &= 7000 \\
 x + 5000 &= 7000 + 5000 \\
 &= 12,000
 \end{aligned}$$

Ebony invested \$7000 in the Treasury note and \$12,000 in the bond.

- 11.** Let x represent the amount of the 5% solution (in gallons). 5000 gal is the amount of the 10% solution. Therefore, $x + 5000$ is the amount of the resulting 9% solution.

	5%	10%	9% Solution
Amount of	x	5000	$x + 5000$
Pure Ethanol	$0.05x$	$0.1(5000)$	$0.09(x + 5000)$

$$\begin{aligned}
 0.05x + 0.1(5000) &= 0.09(x + 5000) \\
 0.05x + 500 &= 0.09x + 450 \\
 50 &= 0.04x \\
 1250 &= x
 \end{aligned}$$

1250 gal of E5 should be mixed with the E10.

- 12.** Let x represent the amount of the 10% solution (in cubic centimeters). 60 cc is the amount of the 50% solution. Therefore, $x + 60$ is the amount of the resulting 25% solution.

	10%	50%	25%
Amount of	x	60	$x + 60$
Pure Saline	$0.1x$	$0.5(60)$	$0.25(x + 60)$

$$\begin{aligned}
 0.1x + 0.5(60) &= 0.25(x + 60) \\
 0.1x + 30 &= 0.25x + 15 \\
 15 &= 0.15x \\
 100 &= x
 \end{aligned}$$

100 cc of 10% saline solution should be mixed with the 50% saline solution.

- 13.** Let x represent the amount of the pure sand (in cubic feet). 480 ft² is the amount of the concrete mix that is 70% sand. Therefore, $(x + 480)$ is the amount of the resulting 75% sand mixture.

	100%	70%	75% Sand
Amount of	x	480	$x + 480$
Pure Sand	x	$0.7(480)$	$0.75(x + 480)$

$$x + 0.7(480) = 0.75(x + 480)$$

$$x + 336 = 0.75x + 360$$

$$0.25x = 24$$

$$x = 96$$

96 ft² of sand should be mixed with the 70% sand mixture.

- 14.** Let x represent the amount of 50% antifreeze solution (in gallons) to be drained (and therefore the amount of 100% antifreeze solution to be added). 4 gal is the amount of the resulting 65% antifreeze solution. Therefore, $(4 - x)$ is the amount of 50% antifreeze solution that is not drained.

	100%	50%	65%
Amount of	x	$4 - x$	4
Pure Antifreeze	x	$0.5(4 - x)$	$0.65(4)$

$$x + 0.5(4 - x) = 0.65(4)$$

$$x + 2 - 0.5x = 2.6$$

$$0.5x = 0.6$$

$$x = 1.2$$

1.2 gal of 50% antifreeze solution should be drained and replaced with 100% antifreeze.

- 15.** Let x represent the speed of the plane flying to Los Angeles. Then, $(x + 60)$ is the speed of the plane flying to New York City.

	Distance	Rate	Time
Los Angeles Flight	$3.4x$	x	3.4
New York City Flight	$2.4(x + 60)$	$x + 60$	2.4

$$3.4x + 2.4(x + 60) = 2464$$

$$3.4x + 2.4x + 144 = 2464$$

$$5.8x = 2320$$

$$x = 400$$

$$x + 60 = 400 + 60$$

$$= 460$$

The plane to Los Angeles travels 400 mph and the plane to New York City travels 460 mph.

- 16.** Let x represent the speed of the plane flying to Seattle. Then, $(x - 44)$ is the speed of the plane flying to Boston.

	Distance	Rate	Time
Seattle Flight	$5.2x$	x	5.2
Boston Flight	$2.5(x - 44)$	$x - 44$	2.5

$$5.2x + 2.5(x - 44) = 3124$$

$$5.2x + 2.5x - 110 = 3124$$

$$7.7x = 3234$$

$$x = 420$$

$$x - 44 = 420 - 44$$

$$= 376$$

The plane to Seattle travels 420 mph, and the plane to Boston travels 376 mph.

17. Let x represent the distance from Darren's home to his school.

	Distan	Rat	Tim
To School	x	32	$\frac{x}{32}$
To Home	x	48	$\frac{x}{48}$

$$\frac{x}{32} + \frac{x}{48} = 1.25$$

$$96\left(\frac{x}{32} + \frac{x}{48}\right) = 96(1.25)$$

$$3x + 2x = 120$$

$$5x = 120$$

$$x = 24$$

The distance is 24 mi.

18. Let x represent the distance of the loop.

	Distan	Rat	Tim
Runni ng	x	8	$\frac{x}{8}$
Riding	$5x$	16	$\frac{5x}{16}$

$$\frac{x}{8} + \frac{5x}{16} = 1.75$$

$$16\left(\frac{x}{8} + \frac{5x}{16}\right) = 16(1.75)$$

$$2x + 5x = 28$$

$$7x = 28$$

$$x = 4$$

The loop is 4 mi.

- 19.**
- Let
- t
- represent the time it takes for the

runners to cover $\frac{1}{4}$ mile.

$$\frac{1 \text{ lap}}{66 \text{ sec}} + \frac{1 \text{ lap}}{60 \text{ sec}} = \frac{1 \text{ lap}}{t \text{ sec}}$$

$$660t \left(\frac{1}{66} + \frac{1}{60} \right) = 660t \left(\frac{1}{t} \right)$$

$$10t + 11t = 660$$

$$21t = 660$$

$$t = \frac{220}{7} \approx 31.4 \text{ sec}$$

- 20.**
- Let
- t
- represent the time it takes Marta and her daughter to vacuum the house together.

$$\frac{1 \text{ job}}{40 \text{ min}} + \frac{1 \text{ job}}{60 \text{ min}} = \frac{1 \text{ job}}{t \text{ min}}$$

$$120t \left(\frac{1}{40} + \frac{1}{60} \right) = 120t \left(\frac{1}{t} \right)$$

$$3t + 2t = 120$$

$$5t = 120$$

$$t = 24 \text{ min}$$

- 21.**
- Let
- t
- represent the time it takes the second pump to fill the pool by itself.

$$\frac{1 \text{ job}}{10 \text{ hr}} + \frac{1 \text{ job}}{t \text{ hr}} = \frac{1 \text{ job}}{6 \text{ hr}}$$

$$30t \left(\frac{1}{10} + \frac{1}{t} \right) = 30t \left(\frac{1}{6} \right)$$

$$3t + 30 = 5t$$

$$30 = 2t$$

$$t = 15 \text{ hr}$$

- 22.**
- Let
- t
- represent the time it takes Angelina to mow the lawn by herself.

$$\frac{1 \text{ job}}{50 \text{ min}} + \frac{1 \text{ job}}{t \text{ min}} = \frac{1 \text{ job}}{30 \text{ min}}$$

$$150t \left(\frac{1}{50} + \frac{1}{t} \right) = 150t \left(\frac{1}{30} \right)$$

$$3t + 150 = 5t$$

$$150 = 2t$$

$$t = 75 \text{ min or } 1 \text{ hr } 15 \text{ min}$$

- 23.**
- Let
- x
- represent the amount of cement and
- y
- represent the amount of gravel.

$$\frac{1}{2.4} = \frac{x}{150}$$

$$2.4x = 150$$

$$x = 62.5$$

$$\frac{2.4}{3.6} = \frac{150}{y}$$

$$2.4y = 540$$

$$y = 225$$

62.5 lb of cement and 225 lb of gravel

- 24.**
- Let
- x
- represent the property tax on a house that is \$240,000.

$$\frac{180,000}{1296} = \frac{240,000}{x}$$

$$180,000x = 1296(240,000)$$

$$x = \frac{1296(240,000)}{180,000}$$

$$x = \$1728$$

- 25.**
- Let
- x
- represent the patient's LDL cholesterol level. The HDL cholesterol level is 60 g/dL, and the total cholesterol is
- $(x + 60)$
- .

$$\frac{x + 60}{60} = 3.4$$

$$60 \left(\frac{x + 60}{60} \right) = 60(3.4)$$

$$x + 60 = 204$$

$$x = 144$$

$$x + 60 = 144 + 60$$

$$= 204$$

LDL is 144 mg/dL and the total cholesterol is 204 mg/dL.

- 26.**
- Let
- x
- represent the number of Democrats. Then,
- $(x - 10)$
- represents the number of Republicans.

$$\frac{x}{x - 10} = \frac{11}{9}$$

$$9(x - 10) \left(\frac{x}{x - 10} \right) = 9(x - 10) \left(\frac{11}{9} \right)$$

$$9x = 11(x - 10)$$

$$9x = 11x - 110$$

$$110 = 2x$$

$$55 = x$$

$$x - 10 = 55 - 10 = 45$$

$$\frac{x}{x-10} = \frac{11}{9}$$

$$9(x-10)\left(\frac{x}{x-10}\right) = 9(x-10)\left(\frac{11}{9}\right)$$

$$9x = 11(x-10)$$

$$9x = 11x - 110$$

$$110 = 2x$$

$$55 = x$$

$$x - 10 = 55 - 10 = 45$$

There were 55 Democrat and 45 Republican senators.

- 27.** Let x represent the number of deer in the population.

$$\frac{30}{x} = \frac{5}{80}$$

$$5x = 2400$$

$$x = 480 \text{ deer}$$

- 28.** Let x represent the number of bass in the lake.

$$\frac{24}{x} = \frac{4}{40}$$

$$4x = 960$$

$$x = 240 \text{ bass}$$

- 29.** Let x represent the distance from the epicenter to the station.

	Distanc	Rat	Tim
P Waves	x	5	$\frac{x}{5}$
S Waves	x	3	$\frac{x}{3}$

$$\frac{x}{3} - \frac{x}{5} = 40$$

$$15\left(\frac{x}{3} - \frac{x}{5}\right) = 15(40)$$

$$5x - 3x = 600$$

$$2x = 600$$

$$x = 300 \text{ km}$$

- 30.** Let x represent the distance from the epicentre to the station.

	Distanc	Rat	Tim
P Wave s	x	8	$\frac{x}{8}$

S Wave s	x	4.8	$\frac{x}{4.8}$
-----------------	-----	-----	-----------------

$$\frac{x}{4.8} - \frac{x}{8} = 20$$

$$24\left(\frac{x}{4.8} - \frac{x}{8}\right) = 24(20)$$

$$5x - 3x = 480$$

$$2x = 480$$

$$x = 240 \text{ km}$$

- 31.** Let x represent the price set by the merchant.

$$x - 0.25x = 180 + 0.40(180)$$

$$0.75x = 180 + 72$$

$$0.75x = 252$$

$$x = \$336$$

- 32.** Let x represent the price set by the bookstore.

$$x - 0.10x = 80 + 0.35(80)$$

$$0.90x = 80 + 28$$

$$0.90x = 108$$

$$x = \$120$$

- 33. a.** $C = 110 + 60x$

$$C = 350$$

$$110 + 60x = 350$$

$$60x = 240$$

b. $x = 4 \text{ hr}$

- 34. a.** $C = 2400 + 80x$

$$C = 5520$$

$$2400 + 80x = 5520$$

$$80x = 3120$$

b. $x = 39 \text{ hr}$

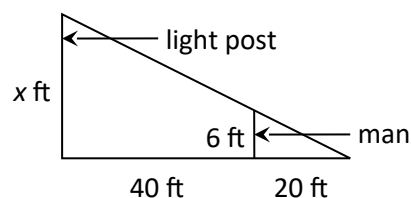
- 35.** Let x represent the height of the Washington Monument.

$$\frac{5}{4} = \frac{x}{444}$$

$$4x = 2220$$

$$x = 555 \text{ ft}$$

- 36.** Let x represent the height of the light post.



$$\begin{aligned}\frac{6}{20} &= \frac{x}{40+20} \\ \frac{6}{20} &= \frac{x}{60} \\ 20x &= 360 \\ x &= 18 \text{ ft}\end{aligned}$$

37. Let x represent the height of the pole.

Then, $\frac{1}{8}x$ is the length of the pole that is in the ground, and $\frac{2}{3}x$ is the length of the pole that is in the snow.

$$\begin{aligned}x &= 1.5 + \frac{2}{3}x + \frac{1}{8}x \\ x - \frac{2}{3}x - \frac{1}{8}x &= 1.5 \\ 24\left(x - \frac{2}{3}x - \frac{1}{8}x\right) &= 24(1.5) \\ 24x - 16x - 3x &= 36 \\ 5x &= 36 \\ x &= 7.2\end{aligned}$$

The pole is 7.2 ft long, and the snow is 4.8 ft deep.

$$\begin{aligned}C &= \frac{5}{9}(F - 32) \\ F &= \frac{5}{9}(F - 32) \\ 9(F) &= 9\left[\frac{5}{9}(F - 32)\right] \\ \mathbf{38.} \quad 9F &= 5F - 160 \\ 4F &= -160 \\ F &= -40 \\ -40^\circ\text{C} &= -40^\circ\text{F}\end{aligned}$$

- 39.** Let x represent the amount of 20% fertilizer solution (in litres) to be drained (and therefore the amount of water to be added). 40 L is the amount of the resulting 15% fertilizer solution. Therefore, $(40 - x)$ is the amount of 20% fertilizer solution that is not drained.

	0%	20%	15%
Amount of	x	$40 - x$	40
Pure fertilizer	$0(x)$	$0.20(40 - x)$	$0.15(40)$

$$0(x) + 0.20(40 - x) = 0.15(40)$$

$$8 - 0.20x = 6$$

$$-0.20x = -2$$

$$x = 10$$

10 L should be drained and replaced by water.

- 40.** Let x represent the amount of water (in litres) to be evaporated. Therefore, $(200 - x)$ is the amount of the final 25% salt solution.

	0%	5%	25%
Amount of	x	200	$200 - x$
Pure salt	$0(x)$	$0.05(200)$	$0.25(200 - x)$

$$0(x) + 0.05(200) = 0.25(200 - x)$$

$$10 = 50 - 0.25x$$

$$-40 = -0.25x$$

$$160 = x$$

160 mL should be evaporated.

- 41.** The length of the lot is $l = 128 + 2x$.

The width of the lot is $w = 60 + 2x$.

$$P = 2l + 2w$$

$$440 = 2(128 + 2x) + 2(60 + 2x)$$

$$440 = 256 + 4x + 120 + 4x$$

$$440 = 8x + 376$$

$$64 = 8x$$

$$8 = x$$

The width of the easement is 8 ft.

- 42.** The length of the play area is $l = 78 + 2x$. The width of the play area is $w = 36 + 2x$.

$$P = 2l + 2w$$

$$396 = 2(78 + 2x) + 2(36 + 2x)$$

$$396 = 156 + 4x + 72 + 4x$$

$$396 = 8x + 228$$

$$168 = 8x$$

$$21 = x$$

The width of the border is 21 ft.

- 43. a.** The width of the kitchen is w . The length of the kitchen is $l = w + 4$.

$$P = 2l + 2w$$

$$48 = 2(w + 4) + 2w$$

$$48 = 2w + 8 + 2w$$

$$48 = 4w + 8$$

$$40 = 4w$$

$$10 = w$$

The kitchen is 14 ft by 10 ft.

$$A = lw + 0.1lw$$

$$= 1.1lw$$

$$A = 1.1(14)(10)$$

$$\mathbf{b.} \quad = 154 \text{ ft}^2$$

$$C = 1.06(12)(154)$$

$$\mathbf{c.} \quad = \$1958.88$$

- 44. a.** The width of the porch is w . The length of the porch is $l = 2w + 2$.

$$P = 2l + 2w$$

$$64 = 2(2w + 2) + 2w$$

$$64 = 4w + 4 + 2w$$

$$64 = 6w + 4$$

$$60 = 6w$$

$$10 = w$$

$$l = 2w + 2$$

$$= 2(10) + 2$$

$$= 22$$

The porch is 22 ft by 10 ft.

$$A = lw + 0.1lw$$

$$= 1.1lw$$

$$A = 1.1(22)(10)$$

$$\mathbf{b.} \quad = 242 \text{ ft}^2$$

$$C = 1.075(5.85)(242)$$

$$\mathbf{c.} \quad \approx \$1521.88$$

- 45.** Aliyah had $8000 - 0.28(8000) = 8000 - 2240 = 5760$ to invest. Let x represent the amount she invested at 11%. Then, $(5760 - x)$ is the amount she invested at 5%.

	11%	5% Investment	Total
Principal	x	$5760 - x$	
Interest ($I =$	$x(0.11)(1)$	$(5760 - x)(0.05)(1)$	453.60

$$x(0.11) + (5760 - x)(0.05) = 453.60$$

$$0.11x + 288 - 0.05x = 453.60$$

$$0.06x + 288 = 453.60$$

$$0.06x = 165.60$$

$$x = 2760$$

$$5760 - x = 5760 - 2760$$

$$= 3000$$

Aliyah invested \$2760 in the stock returning 11% and \$3000 in the stock returning 5%.

- 46.** Let x represent the amount Caitlin invested in the balanced fund. Then, $(2x)$ is the amount she invested in the stock fund.

	Balanced Fund	Stock Fund	Tot
Principal	x	$2x$	
Interest ($I =$	$x(0.035)(1)$	$(2x)(0.17)(1)$	1125

$$x(0.035) + (2x)(0.17) = 1125$$

$$0.035x + 0.34x = 1125$$

$$0.375x = 1125$$

$$x = 3000$$

$$\begin{aligned} 2x &= 2(3000) \\ &= 6000 \end{aligned}$$

Caitlin invested \$3000 in the balanced fund and \$6000 in the stock fund.

$$\frac{7}{8} = \frac{x}{12.8}$$

$$8x = 89.6$$

$$x = 11.2$$

$$\frac{8}{y} = \frac{12.8}{12}$$

$$12.8y = 96$$

$$47. \quad y = 7.5$$

$$x = 11.2 \text{ ft and } y = 7.5 \text{ cm}$$

$$\frac{1.2}{0.96} = \frac{x}{1.04}$$

$$0.96x = 1.248$$

$$x = 1.3$$

$$\frac{0.5}{y} = \frac{1.2}{0.96}$$

$$1.2y = 0.48$$

$$48. \quad y = 0.4$$

$$x = 1.3 \text{ m and } y = 0.4 \text{ in.}$$

49. No. If x represents the measure of the smallest angle, then the equation

$$x + (x + 2) + (x + 4) = 180$$

does not result in an odd integer value for x .

Instead the measures of the angles would be even integers.

50. No. If x represents the number of each type of bill, then the solution to the equation $20x + 10x + 5x = 100$ is not a whole number.

51. Let x represent the smaller number.

Then, $(x + 16)$ is the larger number.

$$\frac{x + 16}{x} = 3 + \frac{2}{x}$$

$$x \left(\frac{x + 16}{x} \right) = x \left(3 + \frac{2}{x} \right)$$

$$x + 16 = 3x + 2$$

$$14 = 2x$$

$$7 = x$$

$$x + 16 = 7 + 16$$

$$= 23$$

The numbers are 7 and 23.

52. Let x represent the smaller number.

Then, $(x + 25)$ is the larger number.

$$\frac{x + 25}{x} = 4 + \frac{1}{x}$$

$$x \left(\frac{x + 25}{x} \right) = x \left(4 + \frac{1}{x} \right)$$

$$x + 25 = 4x + 1$$

$$24 = 3x$$

$$8 = x$$

$$x + 25 = 8 + 25$$

$$= 33$$

The numbers are 8 and 33.

53. Let x represent the tens digit of the number. Then, $(14 - x)$ is the ones digit.

$$10(14 - x) + 1(x) = 10(x) + 1(14 - x) + 18$$

$$140 - 10x + x = 10x + 14 - x + 18$$

$$140 - 9x = 9x + 32$$

$$108 = 18x$$

$$6 = x$$

$$14 - x = 14 - 6$$

$$= 8$$

The original number is 68.

54. Let x represent the tens digit of the number. Then, $(9 - x)$ is the ones digit.

$$10(9 - x) + 1(x) = 10(x) + 1(9 - x) - 45$$

$$90 - 10x + x = 10x + 9 - x - 45$$

$$90 - 9x = 9x - 36$$

$$126 = 18x$$

$$7 = x$$

$$9 - x = 9 - 7$$

$$= 2$$

The original number is 72.

$$m_1x_1 + m_2x_2 = 0$$

$$(30)(-1.2) + (20)x_2 = 0$$

$$20x_2 = 36$$

$$x_2 = 1.8 \text{ m}$$

55.

$$m_1x_1 + m_2x_2 = 0$$

$$(64)x_1 + (80)(2) = 0$$

$$64x_1 = -160$$

$$x_1 = -2.5 \text{ m}$$

56.

$$\begin{aligned}
 m_1x_1 + m_2x_2 &= 0 \\
 (10)(-3.2) + m_2(8) &= 0 \\
 8m_2 &= -32 \\
 m_2 &= 4 \text{ kg}
 \end{aligned}$$

57.

$$\begin{aligned}
 m_1x_1 + m_2x_2 &= 0 \\
 m_1(-10) + (6)(7) &= 0 \\
 -10m_1 &= -42
 \end{aligned}$$

58.

$$m_1 = 4.2 \text{ kg}$$

Section 1.3 Complex Numbers

1. -1

2. $i\sqrt{b}$

3. real; imaginary**4.** conjugate

5. $\sqrt{-121} = i\sqrt{121} = 11i$

6. $\sqrt{-100} = i\sqrt{100} = 10i$

7. $\sqrt{-98} = i\sqrt{98} = 7i\sqrt{2}$

8. $\sqrt{-63} - i\sqrt{63} = 3i\sqrt{7}$

9. $\sqrt{-19} = i\sqrt{19}$

10. $\sqrt{-23} = i\sqrt{23}$

11. $-\sqrt{-16} = -i\sqrt{16} = -4i$

12. $-\sqrt{-25} = -i\sqrt{25} = -5i$

$$\begin{aligned}
 \sqrt{-4}\sqrt{-9} &= i\sqrt{4} \cdot i\sqrt{9} \\
 &= 2i \cdot 3i = 6i^2 \\
 &= 6(-1) = -6
 \end{aligned}$$

13.

$$\begin{aligned}
 \sqrt{-1}\sqrt{-36} &= i\sqrt{1} \cdot i\sqrt{36} \\
 &= 1i \cdot 6i = 6i^2 \\
 &= 6(-1) = -6
 \end{aligned}$$

14.

$$\begin{aligned}
 \sqrt{-10}\sqrt{-5} &= i\sqrt{10} \cdot i\sqrt{5} \\
 &= i^2\sqrt{50} \\
 &= (-1)\sqrt{5^2 \cdot 2} \\
 &= -5\sqrt{2}
 \end{aligned}$$

15.

$$\begin{aligned}
 \sqrt{-6}\sqrt{-15} &= i\sqrt{6} \cdot i\sqrt{15} \\
 &= i^2\sqrt{90} \\
 &= (-1)\sqrt{3^2 \cdot 10} \\
 &= -3\sqrt{10}
 \end{aligned}$$

16.

$$\begin{aligned}
 \sqrt{-6}\sqrt{-14} &= i\sqrt{6} \cdot i\sqrt{14} \\
 &= i^2\sqrt{84} \\
 &= (-1)\sqrt{2^2 \cdot 21} \\
 &= -2\sqrt{21}
 \end{aligned}$$

17.

$$\begin{aligned}
 \sqrt{-10}\sqrt{-15} &= i\sqrt{10} \cdot i\sqrt{15} \\
 &= i^2\sqrt{150} \\
 &= (-1)\sqrt{5^2 \cdot 6} \\
 &= -5\sqrt{6}
 \end{aligned}$$

18.

$$\begin{aligned}
 \frac{\sqrt{-98}}{\sqrt{-2}} &= \frac{i\sqrt{98}}{i\sqrt{2}} \\
 &= \sqrt{\frac{98}{2}} \\
 &= \sqrt{49} = 7
 \end{aligned}$$

19.

$$\begin{aligned}
 \frac{\sqrt{-45}}{\sqrt{-5}} &= \frac{i\sqrt{45}}{i\sqrt{5}} \\
 &= \sqrt{\frac{45}{5}} \\
 &= \sqrt{9} = 3
 \end{aligned}$$

20.

$$\begin{aligned}
 \frac{\sqrt{-63}}{\sqrt{7}} &= \frac{i\sqrt{63}}{\sqrt{7}} \\
 &= i\sqrt{\frac{63}{7}} \\
 &= i\sqrt{9} = 3i
 \end{aligned}$$

21.

$$\begin{aligned}
 \frac{\sqrt{-80}}{\sqrt{5}} &= \frac{i\sqrt{80}}{\sqrt{5}} \\
 &= i\sqrt{\frac{80}{5}} \\
 &= i\sqrt{16} \\
 &= 4i
 \end{aligned}$$

22.**23.** Real part: 3; Imaginary part: -7 **24.** Real part: 2; Imaginary part: -4 **25.** Real part: 0; Imaginary part: 19**26.** Real part: 0; Imaginary part: 40

27. Real part: $-\frac{1}{4}$; Imaginary part: 0

28. Real part: $-\frac{4}{7}$; Imaginary part: 0
 $4\sqrt{-4} = 4 \cdot 2i$

29. $= 8i = 0 + 8i$

$2\sqrt{-144} = 2 \cdot 12i$

30. $= 24i = 0 + 24i$

31. $2 + \sqrt{-12} = 2 + 2\sqrt{3}i$ or $2 + 2i\sqrt{3}$

32. $6 - \sqrt{-24} = 6 + (-2\sqrt{6})i$ or $6 - 2i\sqrt{6}$

$\frac{8+3i}{14} = \frac{8}{14} + \frac{3}{14}i$
 $= \frac{4}{7} + \frac{3}{14}i$

33.

$\frac{4+5i}{6} = \frac{4}{6} + \frac{5}{6}i$
 $= \frac{2}{3} + \frac{5}{6}i$

34.

$\frac{-4-6i}{-2} = \frac{-4}{-2} + \frac{-6}{-2}i$
 $= 2 + 3i$

35.

$\frac{9-15i}{-3} = \frac{9}{-3} - \frac{15}{-3}i$
 $= -3 + 5i$

36.

37.

$\frac{-18+\sqrt{-48}}{4} = \frac{-18+4\sqrt{3}i}{4}$
 $= -\frac{18}{4} + \frac{4\sqrt{3}i}{4}$
 $= -\frac{9}{2} + \sqrt{3}i$ or $-\frac{9}{2} + i\sqrt{3}$

$\frac{-20+\sqrt{-50}}{-10} = \frac{-20+5\sqrt{2}i}{-10}$
 $= \frac{-20}{-10} + \frac{5\sqrt{2}i}{-10}$
 $= 2 - \frac{\sqrt{2}}{2}i$ or $2 - i\frac{\sqrt{2}}{2}$

38.

$\frac{14-\sqrt{-98}}{-7} = \frac{14-7\sqrt{2}i}{-7}$
 $= -\frac{14}{7} + \frac{7\sqrt{2}i}{7}$
 $= -2 + \sqrt{2}i$ or $-2 + i\sqrt{2}$

39.

40.

$\frac{-10+\sqrt{-125}}{5} = \frac{-10+5\sqrt{5}i}{5}$
 $= -\frac{10}{5} + \frac{5\sqrt{5}i}{5}$
 $= -2 + \sqrt{5}i$ or $-2 + i\sqrt{5}$

41. a. $i^{20} = 1$

$i^{29} = i^{28} \cdot i^1$

$= (1) \cdot i^1 = i$

b.

$i^{50} = i^{48} \cdot i^2$

$= (1) \cdot i^2 = -1$

c.

$i^{-41} = i^{-44} \cdot i^3$

$= (1) \cdot i^3 = -i$

d.

42. a. $i^{32} = 1$

$i^{47} = i^{44} \cdot i^3$

$= (1) \cdot i^3 = -i$

b.

$i^{66} = i^{64} \cdot i^2$

$= (1) \cdot i^2 = -1$

c.

$i^{-27} = i^{-28} \cdot i^1$

$= (1) \cdot i^1 = i$

d.

$i^{37} = i^{36} \cdot i^1$

$= i$

43. a.

$i^{-37} = i^{-40} \cdot i^3$

$= (1) \cdot i^3 = -i$

b.

$i^{82} = i^{80} \cdot i^2$

$= (1) \cdot i^2 = -1$

c.

- $j^{-82} = j^{-84} \cdot j^2$
 $= (1) \cdot j^2 = -1$
d. $j^{103} = j^{100} \cdot j^3$
 $= (1) \cdot j^3 = -j$
44. a. $j^{-103} = j^{-104} \cdot j^1$
 $= (1) \cdot j^1 = j$
b.
c. $j^{52} = 1$
d. $j^{-52} = 1$
 $(2 - 7i) + (8 - 3i)$
45.
 $= (2 + 8) + (-7 - 3)i$
 $= 10 - 10i$
46. $(6 - 10i) + (8 + 4i)$
 $= (6 + 8) + (-10 + 4)i$
 $= 14 - 6i$
47. $(15 + 21i) - (18 - 40i)$
 $= (15 - 18) + [21 - (-40)]i$
 $= -3 + 61i$
48. $(250 + 100i) - (80 + 25i)$
 $= (250 - 80) + (100 - 25)i$
 $= 170 + 75i$
 $\left(\frac{1}{2} + \frac{2}{3}i\right) - \left(\frac{5}{6} + \frac{1}{12}i\right)$
49.
 $= \left(\frac{1}{2} - \frac{5}{6}\right) + \left(\frac{2}{3} - \frac{1}{12}\right)i$
 $= \left(\frac{3}{6} - \frac{5}{6}\right) + \left(\frac{8}{12} - \frac{1}{12}\right)i$
 $= -\frac{2}{6} + \frac{7}{12}i$
 $= -\frac{1}{3} + \frac{7}{12}i$
 $\left(\frac{3}{5} - \frac{1}{8}i\right) - \left(\frac{7}{10} + \frac{1}{6}i\right)$
50.
- $= \left(\frac{3}{5} - \frac{7}{10}\right) + \left(-\frac{1}{8} - \frac{1}{6}\right)i$
 $= \left(\frac{6}{10} - \frac{7}{10}\right) + \left(-\frac{3}{24} - \frac{4}{24}\right)i$
 $= -\frac{1}{10} - \frac{7}{24}i$
 $(2.3 + 4i) - (8.1 - 2.7i) + (4.6 - 6.7i)$
 $= (2.3 - 8.1 + 4.6) + (4 + 2.7 - 6.7)i$
 $= -1.2 + 0i$
51. $\begin{pmatrix} 0.05 \\ -0.03i \end{pmatrix} + \begin{pmatrix} -0.12 \\ +0.08i \end{pmatrix} - \begin{pmatrix} 0.07 \\ +0.05i \end{pmatrix}$
 $= (0.05 - 0.12 - 0.07)$
 $+ (-0.03 + 0.08 - 0.05)i$
52. $= -0.14 + 0i$
 $-\frac{1}{8}(16 + 24i) = -2 - 3i$
53.
 $-\frac{1}{6}(60 - 30i) = -10 + 5i$
54. $2i(5 + i) = 10i + 2i^2$
 $= 10i + 2(-1)$
 $= -2 + 10i$
55. $4i(6 + 5i) = 24i + 20i^2$
 $= 24i + 20(-1)$
 $= -20 + 24i$
56. $\sqrt{-3}(\sqrt{11} - \sqrt{-7}) = i\sqrt{3}(\sqrt{11} - i\sqrt{7})$
 $= i\sqrt{33} - i^2\sqrt{21}$
 $= i\sqrt{33} - (-1)\sqrt{21}$
 $= \sqrt{21} + i\sqrt{33}$
57. $\sqrt{-2}(\sqrt{13} + \sqrt{-5}) = i\sqrt{2}(\sqrt{13} + i\sqrt{5})$
 $= i\sqrt{26} + i^2\sqrt{10}$
 $= i\sqrt{26} + (-1)\sqrt{10}$
 $= -\sqrt{10} + i\sqrt{26}$
58.
 $(3 - 6i)(10 + i)$
59.

$$\begin{aligned}
&= 3(10) + 3(i) + (-6i)(10) + (-6i)(i) \\
&= 30 + 3i - 60i - 6i^2 \\
&= 30 - 57i - 6(-1) \\
&= 36 - 57i \\
&\quad (2 - 5i)(8 + 2i)
\end{aligned}$$

$$\begin{aligned}
\mathbf{60.} \quad &= 2(8) + 2(2i) + (-5i)(8) + (-5i)(2i) \\
&= 16 + 4i - 40i - 10i^2 \\
&= 16 - 36i - 10(-1) \\
&= 26 - 36i \\
&\quad (3 - 7i)^2 = (3)^2 - 2(3)(7i) + (7i)^2 \\
&\quad \quad = 9 - 42i + 49i^2 \\
&\quad \quad = 9 - 42i + 49(-1) \\
&\quad \quad = 9 - 42i - 49
\end{aligned}$$

$$\begin{aligned}
\mathbf{61.} \quad &= -40 - 42i \\
&\quad (10 - 3i)^2 = (10)^2 - 2(10)(3i) + (3i)^2 \\
&\quad \quad = 100 - 60i + 9i^2 \\
&\quad \quad = 100 - 60i + 9(-1) \\
&\quad \quad = 100 - 60i - 9 \\
&\quad \quad = 91 - 60i
\end{aligned}$$

$$\mathbf{62.} \quad (3 - \sqrt{-5})(4 + \sqrt{-5})$$

$$\begin{aligned}
\mathbf{63.} \quad &= (3 - i\sqrt{5})(4 + i\sqrt{5}) \\
&= 3(4) + 3(i\sqrt{5}) + (-i\sqrt{5})(4) \\
&\quad \quad + (-i\sqrt{5})(i\sqrt{5}) \\
&= 12 + 3i\sqrt{5} - 4i\sqrt{5} - 5i^2 \\
&= 12 - i\sqrt{5} - 5(-1) \\
&= 17 - i\sqrt{5} \\
&\quad (2 + \sqrt{-7})(10 + \sqrt{-7})
\end{aligned}$$

64.

$$\begin{aligned}
&= (2 + i\sqrt{7})(10 + i\sqrt{7}) \\
&= 2(10) + 2(i\sqrt{7}) + i\sqrt{7}(10) \\
&\quad \quad \quad + i\sqrt{7}(i\sqrt{7}) \\
&= 20 + 2i\sqrt{7} + 10i\sqrt{7} + 7i^2 \\
&= 20 + 12i\sqrt{7} + 7(-1) \\
&= 13 + 12i\sqrt{7} \\
&\quad 4(6 + 2i) - 5i(3 - 7i)
\end{aligned}$$

$$\begin{aligned}
\mathbf{65.} \quad &= 24 + 8i - 15i + 35i^2 \\
&= 24 - 7i + 35(-1) \\
&= -11 - 7i \\
&\quad -3(8 - 3i) - 6i(2 + i)
\end{aligned}$$

$$\begin{aligned}
\mathbf{66.} \quad &= -24 + 9i - 12i - 6i^2 \\
&= -24 - 3i - 6(-1) \\
&= -18 - 3i \\
&\quad (2 - i)^2 + (2 + i)^2
\end{aligned}$$

$$\begin{aligned}
\mathbf{67.} \quad &= (2)^2 - 2(2)(i) + i^2 + (2)^2 \\
&\quad \quad \quad + 2(2)(i) + i^2 \\
&= 4 - 4i + i^2 + 4 + 4i + i^2 \\
&= 8 + 2i^2 \\
&= 8 + 2(-1) = 6 \\
&\quad (3 - 2i)^2 + (3 + 2i)^2
\end{aligned}$$

$$\begin{aligned}
\mathbf{68.} \quad &= (3)^2 - 2(3)(2i) + (2i)^2 + (3)^2 \\
&\quad \quad \quad + 2(3)(2i) + (2i)^2 \\
&= 9 - 12i + 4i^2 + 9 + 12i + 4i^2 \\
&= 18 + 8i^2 \\
&= 18 + 8(-1) = 10
\end{aligned}$$

$$\mathbf{69. a.} \quad 3 + 6i$$

$$\begin{aligned}
&(3 - 6i)(3 + 6i) = (3)^2 + (6)^2 \\
&\quad \quad \quad = 9 + 36 \\
&\quad \quad \quad = 45
\end{aligned}$$

b.

$$\mathbf{70. a.} \quad 4 + 5i$$

$$\begin{aligned}(4-5i)(4+5i) &= (4)^2 + (5)^2 \\ &= 16 + 25 \\ &= 41\end{aligned}$$

b.
71. a. $0-8i$

$$\begin{aligned}(0-8i)(0+8i) &= (0)^2 + (8)^2 \\ &= 0 + 64 \\ &= 64\end{aligned}$$

b.
72. a. $0-9i$

$$\begin{aligned}(0-9i)(0+i) &= (0)^2 + (9)^2 \\ &= 0 + 81 \\ &= 81\end{aligned}$$

b.
 $(10-4i)(10+4i) = (10)^2 + (4)^2$
 $= 100 + 16$

73. $= 116$

$$\begin{aligned}(3-9i)(3+9i) &= (3)^2 + (9)^2 \\ &= 9 + 81 \\ &= 90\end{aligned}$$

74. $(7i)(-7i) = 7^2 = 49$

75. $(-5i)(5i) = (5)^2 = 25$

76. $(\sqrt{2} + \sqrt{3}i)(\sqrt{2} + \sqrt{3}i)$

77.
 $= (\sqrt{2})^2 + (\sqrt{3})^2$
 $= 2 + 3 = 5$

78. $(\sqrt{5} + \sqrt{7}i)(\sqrt{5} - \sqrt{7}i)$
 $= (\sqrt{5})^2 + (\sqrt{7})^2$
 $= 5 + 7 = 12$

79. $\frac{6+2i}{3-i} = \frac{(6+2i)(3+i)}{(3-i)(3+i)}$
 $= \frac{18+6i+6i+2i^2}{(3)^2+(1)^2}$
 $= \frac{18+12i+2(-1)}{9+1}$
 $= \frac{16+12i}{10}$
 $= \frac{16}{10} + \frac{12}{10}i$
 $= \frac{8}{5} + \frac{6}{5}i$

$$\frac{5+i}{4-i} = \frac{(5+i)(4+i)}{(4-i)(4+i)}$$

80.
 $= \frac{20+5i+4i+i^2}{(4)^2+(1)^2}$
 $= \frac{20+9i+1(-1)}{16+1}$
 $= \frac{19+9i}{17}$
 $= \frac{19}{17} + \frac{9}{17}i$

81. $\frac{8-5i}{13+2i} = \frac{(8-5i)(13-2i)}{(13+2i)(13-2i)}$
 $= \frac{104-16i-65i+10i^2}{(13)^2+(2)^2}$
 $= \frac{104-81i+10(-1)}{169+4}$
 $= \frac{94-81i}{173}$
 $= \frac{94}{173} - \frac{81}{173}i$

$$\begin{aligned}
 \mathbf{82.} \quad \frac{10-3i}{11+4i} &= \frac{(10-3i)(11-4i)}{(11+4i)(11-4i)} \\
 &= \frac{110-40i-33i+12i^2}{(11)^2+(4)^2} \\
 &= \frac{110-73i+12(-1)}{121+16} \\
 &= \frac{98-73i}{137} \\
 &= \frac{98}{137} - \frac{73}{137}i \\
 (6+\sqrt{5}i)^{-1} &= \frac{1}{6+\sqrt{5}i} \\
 &= \frac{1(6-\sqrt{5}i)}{(6+\sqrt{5}i)(6-\sqrt{5}i)} \\
 \mathbf{83.} \quad &= \frac{6-\sqrt{5}i}{(6)^2+(\sqrt{5})^2} \\
 &= \frac{6-\sqrt{5}i}{36+5} \\
 &= \frac{6-\sqrt{5}i}{41} \\
 &= \frac{6}{41} - \frac{\sqrt{5}}{41}i \\
 (4-\sqrt{3}i)^{-1} &= \frac{1}{4-\sqrt{3}i} \\
 &= \frac{1(4+\sqrt{3}i)}{(4-\sqrt{3}i)(4+\sqrt{3}i)} \\
 \mathbf{84.} \quad &= \frac{4+\sqrt{3}i}{(4)^2+(\sqrt{3})^2} \\
 &= \frac{4+\sqrt{3}i}{16+3} \\
 &= \frac{4+\sqrt{3}i}{19} \\
 &= \frac{4}{19} + \frac{\sqrt{3}}{19}i
 \end{aligned}$$

$$\begin{aligned}
 \frac{5}{13i} &= \frac{5 \cdot i}{13i \cdot i} \\
 &= \frac{5i}{13i^2} = \frac{5i}{13(-1)} \\
 &= \frac{5i}{-13} = -\frac{5}{13}i \\
 &= 0 - \frac{5}{13}i \\
 \mathbf{85.} \quad \frac{6}{7i} &= \frac{6(-i)}{7i(-i)} \\
 &= \frac{-6i}{-7i^2} = \frac{-6i}{-7(-1)} \\
 &= -\frac{6}{7}i = 0 - \frac{6}{7}i \\
 \mathbf{86.} \quad \frac{-1}{\sqrt{-3}} &= \frac{-1}{\sqrt{3}i} \\
 &= \frac{-1 \cdot \sqrt{3}i}{\sqrt{3}i \cdot \sqrt{3}i} = \frac{-\sqrt{3}i}{3i^2} \\
 &= \frac{-\sqrt{3}i}{3(-1)} = \frac{-\sqrt{3}i}{-3} \\
 &= \frac{\sqrt{3}i}{3} = 0 + \frac{\sqrt{3}}{3}i \\
 \mathbf{87.} \quad \frac{-2}{\sqrt{-11}} &= \frac{-2}{\sqrt{11}i} \\
 &= \frac{-2 \cdot \sqrt{11}i}{\sqrt{11}i \cdot \sqrt{11}i} \\
 &= \frac{-2\sqrt{11}i}{11i^2} = \frac{-2\sqrt{11}i}{11(-1)} \\
 &= \frac{2\sqrt{11}}{11}i = 0 + \frac{2\sqrt{11}}{11}i \\
 \mathbf{88.} \quad \sqrt{b^2-4ac} &= \sqrt{(4)^2-4(2)(6)} \\
 &= \sqrt{16-48} \\
 &= \sqrt{-32} = i\sqrt{32} \\
 &= 4i\sqrt{2} \\
 \mathbf{89.} \quad \sqrt{b^2-4ac} &= \sqrt{(-5)^2-4(5)(10)} \\
 &= \sqrt{25-200} \\
 &= \sqrt{-175} = i\sqrt{175} \\
 &= 5i\sqrt{7} \\
 \mathbf{90.} \quad &
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{b^2 - 4ac} &= \sqrt{(-6)^2 - 4(2)(5)} \\
 &= \sqrt{36 - 40} \\
 &= \sqrt{-4} = i\sqrt{4} \\
 &= 2i
 \end{aligned}$$

91.

$$\begin{aligned}
 \sqrt{b^2 - 4ac} &= \sqrt{(4)^2 - 4(2)(4)} \\
 &= \sqrt{16 - 32} \\
 &= \sqrt{-16} = i\sqrt{16} \\
 &= 4i
 \end{aligned}$$

92.

$$\begin{aligned}
 x^2 + 25 &= 0 \\
 (5i)^2 + 25 &= 0 \\
 25(-1) + 25 &= 0 \\
 -25 + 25 &= 0 \\
 0 &= 0 \checkmark
 \end{aligned}$$

93. a.

$$\begin{aligned}
 x^2 + 25 &= 0 \\
 (-5i)^2 + 25 &= 0 \\
 25(-1) + 25 &= 0 \\
 -25 + 25 &= 0 \\
 0 &= 0 \checkmark
 \end{aligned}$$

b.

$$\begin{aligned}
 x^2 + 49 &= 0 \\
 (7i)^2 + 49 &= 0 \\
 49(-1) + 49 &= 0 \\
 -49 + 49 &= 0 \\
 0 &= 0 \checkmark
 \end{aligned}$$

94. a.

$$\begin{aligned}
 x^2 + 49 &= 0 \\
 (-7i)^2 + 49 &= 0 \\
 49(-1) + 49 &= 0 \\
 -49 + 49 &= 0 \\
 0 &= 0 \checkmark
 \end{aligned}$$

b.**95. a.**

$$\begin{aligned}
 x^2 - 4x + 7 &= 0 \\
 (2 + i\sqrt{3})^2 - 4(2 + i\sqrt{3}) + 7 &= 0 \\
 4 + 4i\sqrt{3} + 3i^2 - 8 - 4i\sqrt{3} + 7 &= 0 \\
 4 + 3(-1) - 8 + 7 &= 0 \\
 4 - 3 - 1 &= 0 \\
 0 &= 0 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 x^2 - 4x + 7 &= 0 \\
 (2 - i\sqrt{3})^2 - 4(2 - i\sqrt{3}) + 7 &= 0 \\
 4 - 4i\sqrt{3} + 3i^2 - 8 + 4i\sqrt{3} + 7 &= 0 \\
 4 + 3(-1) - 8 + 7 &= 0 \\
 4 - 3 - 1 &= 0 \\
 0 &= 0 \checkmark
 \end{aligned}$$

b.**96. a.**

$$\begin{aligned}
 x^2 - 6x + 11 &= 0 \\
 (3 + i\sqrt{2})^2 - 6(3 + i\sqrt{2}) + 11 &= 0 \\
 9 + 6i\sqrt{2} + 2i^2 - 18 - 6i\sqrt{2} + 11 &= 0 \\
 9 + 2(-1) - 18 + 11 &= 0 \\
 9 - 2 - 7 &= 0 \\
 0 &= 0 \checkmark
 \end{aligned}$$

b.

$$\begin{aligned}
 x^2 - 6x + 11 &= 0 \\
 (3 - i\sqrt{2})^2 - 6(3 - i\sqrt{2}) + 11 &= 0 \\
 9 - 6i\sqrt{2} + 2i^2 - 18 + 6i\sqrt{2} + 11 &= 0 \\
 9 + 2(-1) - 18 + 11 &= 0 \\
 9 - 2 - 7 &= 0 \\
 0 &= 0 \checkmark
 \end{aligned}$$

$$\mathbf{97.} \quad (a + bi)(c + di)$$

$$\begin{aligned}
 &= ac + adi + bci + bdi^2 \\
 &= ac + (ad + bc)i + bd(-1) \\
 &= (ac - bd) + (ad + bc)i
 \end{aligned}$$

$$\begin{aligned}
 (a+bi)^2 &= (a)^2 + 2(a)(bi) + (bi)^2 \\
 &= a^2 + (2ab)i + b^2i^2 \\
 &= a^2 + (2ab)i + b^2(-1) \\
 &= (a^2 - b^2) + (2ab)i
 \end{aligned}$$

98.

99. The second step does not follow because the multiplication property of radicals can be applied only if the individual radicals are real numbers.

Because $\sqrt{-9}$ and $\sqrt{-4}$ are imaginary numbers, the correct logic for simplification would be

$$\sqrt{-9} \cdot \sqrt{-4} = i\sqrt{9} \cdot i\sqrt{4}$$

$$= i^2 \sqrt{36}$$

$$= -1 \cdot 6 = -6$$

100. The product $(a+b)(a-b)$

simplifies to $a^2 - b^2$. The product

$(a+bi)(a-bi)$ simplifies to

$a^2 - (bi)^2$, which simplifies to $a^2 + b^2$.

101. Any real number. For example: 5.

102. Any complex number and its conjugate. For example: $2+5i$ and $2-5i$. In general, for real numbers, a and b , $(a+bi)(a-bi) = a^2 + b^2$, which is a real number.

103. $z \cdot \bar{z} = (a+bi)(a-bi) = a^2 + b^2$

104. $z^2 - \bar{z}^2$

$$\begin{aligned}
 &= (a+bi)^2 - (a-bi)^2 \\
 &= a^2 + 2abi + (bi)^2 - [a^2 - 2abi + (bi)^2] \\
 &= a^2 + 2abi + b^2i^2 - a^2 + 2abi - b^2i^2 \\
 &= (4ab)i
 \end{aligned}$$

105. a. $x^2 - 9 = (x+3)(x-3)$

b. $x^2 + 9 = (x+3i)(x-3i)$

106. a. $x^2 - 100 = (x+10)(x-10)$

b. $x^2 + 100 = (x+10i)(x-10i)$

107. a. $x^2 - 64 = (x+8)(x-8)$

b. $x^2 + 64 = (x+8i)(x-8i)$

108. a. $x^2 - 25 = (x+5)(x-5)$

b. $x^2 + 25 = (x+5i)(x-5i)$

109. a. $x^2 - 3 = (x+\sqrt{3})(x-\sqrt{3})$

b. $x^2 + 3 = (x+i\sqrt{3})(x-i\sqrt{3})$

110. a. $x^2 - 11 = (x+\sqrt{11})(x-\sqrt{11})$

b. $x^2 + 11 = (x+i\sqrt{11})(x-i\sqrt{11})$

111.

$$\begin{array}{l} \sqrt{(-16)} \\ (4-5i)-(2+3i) \quad 4i \\ (12-15i)(-2+9i) \quad 2-8i \\ 11i+138i \end{array}$$

112.

$$\begin{array}{l} \sqrt{(-169)} \\ (-11-2i)+(-4+9i) \quad 13i \\ (8+12i)(-3-7i) \quad -15+7i \\ 60-92i \end{array}$$

113.

$$\begin{array}{l} (4-9i)^2 \\ 7/(2i) \cdot \text{Frac} \quad -65-72i \\ (14+8i)/(3-i) \cdot \text{Fr} \quad -7/2i \\ ac \quad 17/5+19/5i \end{array}$$

114.

$$\begin{array}{l} (11+4i)^2 \\ 11/(10i) \cdot \text{Frac} \quad 105+88i \\ (5+7i)/(6+8i) \cdot \text{Fr} \quad -11/10i \\ ac \quad 43/50+1/50i \end{array}$$

Section 1.4 Quadratic Equation

1. quadratic

2. linear

3. $\pm\sqrt{k}$

4. 100

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5.

6. $b^2 - 4ac$

$$(x-3)(x+7)=0$$

$$x-3=0 \quad \text{or} \quad x+7=0$$

$$x=3 \quad x=-7$$

7.

$$\{3, -7\}$$

$$(t+4)(t-1)=0$$

$$t+4=0 \quad \text{or} \quad t-1=0$$

$$t=-4 \quad t=1$$

8.

$$\{-4, 1\}$$

$$n^2 + 5n = 24$$

$$n^2 + 5n - 24 = 0$$

$$(n+8)(n-3)=0$$

$$n+8=0 \quad \text{or} \quad n-3=0$$

$$n=-8 \quad n=3$$

9.

$$\{-8, 3\}$$

$$y^2 = 18 - 7y$$

$$y^2 + 7y - 18 = 0$$

$$(y+9)(y-2)=0$$

$$y+9=0 \quad \text{or} \quad y-2=0$$

$$y=-9 \quad y=2$$

10.

$$\{-9, 2\}$$

11.

$$8(t+3)=2t-5$$

$$8t^2 + 24t = 2t - 5$$

$$8t^2 + 22t + 5 = 0$$

$$(2t+5)(4t+1)=0$$

$$2t+5=0 \quad \text{or} \quad 4t+1=0$$

$$2t=-5 \quad 4t=-1$$

$$t=-\frac{5}{2} \quad t=-\frac{1}{4}$$

$$\left\{-\frac{5}{2}, -\frac{1}{4}\right\}$$

$$6m(m+4)=m-15$$

$$6m^2 + 24m = m - 15$$

$$6m^2 + 23m + 15 = 0$$

$$12. \quad (6m+5)(m+3)=0$$

$$6m+5=0$$

$$6m=-5$$

$$m=-\frac{5}{6} \quad \text{or}$$

$$m+3=0$$

$$m=-3$$

$$\left\{-\frac{5}{6}, -3\right\}$$

$$13. \quad 40p^2 - 90 = 0$$

$$10(4p^2 - 9) = 0$$

$$10(2p-3)(2p+3) = 0$$

$$2p-3=0 \quad \text{or} \quad 2p+3=0$$

$$2p=3 \quad 2p=-3$$

$$p=\frac{3}{2} \quad p=-\frac{3}{2}$$

$$\left\{\frac{3}{2}, -\frac{3}{2}\right\}$$

$$14. \quad 32n^2 - 162 = 0$$

$$2(16n^2 - 81) = 0$$

$$2(4n-9)(4n+9) = 0$$

$$4n-9=0 \quad \text{or} \quad 4n+9=0$$

$$4n=9 \quad 4n=-9$$

$$n=\frac{9}{4} \quad n=-\frac{9}{4}$$

$$\left\{\frac{9}{4}, -\frac{9}{4}\right\}$$

$$3x^2 = 12x$$

$$3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$3x=0 \quad \text{or} \quad x-4=0$$

$$15. \quad x=0 \quad x=4$$

$$\{0, 4\}$$

$$z^2 = 25z$$

$$z^2 - 25z = 0$$

$$16. \quad z(z-25) = 0$$

$$z=0 \quad \text{or} \quad z-25=0$$

$$z=25$$

$$\{0, 25\}$$

17.

$$(m+4)(m-5) = -8$$

$$m^2 + 4m - 5m - 20 = -8$$

$$m^2 - m - 12 = 0$$

$$(m+3)(m-4) = 0$$

$$m+3=0 \quad \text{or} \quad m-4=0$$

$$m=-3 \quad m=4$$

$$\{-3, 4\}$$

$$(n+2)(n-4) = 27$$

$$n^2 - 4n + 2n - 8 = 27$$

$$n^2 - 2n - 35 = 0$$

$$(n+5)(n-7) = 0$$

$$n+5=0 \quad \text{or} \quad n-7=0$$

$$n=-5 \quad n=7$$

18.

$$\{-5, 7\}$$

$$x^2 = 81$$

$$x = \pm\sqrt{81}$$

$$= \pm 9$$

19.

$$\{9, -9\}$$

$$w^2 = 121$$

$$w = \pm\sqrt{121}$$

$$= \pm 11$$

20.

$$\{11, -11\}$$

$$5y^2 - 35 = 0$$

$$5y^2 = 35$$

$$y^2 = 7$$

$$y = \pm\sqrt{7}$$

21.

$$\{\sqrt{7}, -\sqrt{7}\}$$

$$6v^2 - 30 = 0$$

$$6v^2 = 30$$

$$v^2 = 5$$

$$v = \pm\sqrt{5}$$

22.

$$\{\sqrt{5}, -\sqrt{5}\}$$

- $4u^2 + 64 = 0$
 $4u^2 = -64$
 $u^2 = -16$
23. $u = \pm\sqrt{-16} = \pm 4i$
 $\{4i, -4i\}$
- $8p^2 + 72 = 0$
 $8p^2 = -72$
 $p^2 = -9$
24. $p = \pm\sqrt{-9} = \pm 3i$
 $\{3i, -3i\}$
- $(k+2)^2 = 28$
 $k+2 = \pm\sqrt{28}$
 $k = -2 \pm \sqrt{28}$
25. $= -2 \pm 2\sqrt{7}$
 $\{-2 \pm 2\sqrt{7}\}$
- $3(z+11)^2 - 10 = 110$
 $3(z+11)^2 = 120$
 $(z+11)^2 = 40$
 $z+11 = \pm\sqrt{40}$
 $z = -11 \pm \sqrt{40}$
26. $= -11 \pm 2\sqrt{10}$
 $\{-11 \pm 2\sqrt{10}\}$
- $2(w-5)^2 + 5 = 23$
 $2(w-5)^2 = 18$
 $w-5 = \pm\sqrt{9}$
 $w = 5 \pm \sqrt{9}$
 $w = 5 \pm 3$
 $w = 5+3$ or $w = 5-3$
27. $w = 8$ $w = 2$
 $\{8, 2\}$
- $(c-3)^2 = 49$
 $c-3 = \pm\sqrt{49}$
 $c = 3 \pm \sqrt{49}$
 $c = 3 \pm 7$
 $c = 3+7$ or $c = 3-7$
28. $c = 10$ $c = -4$
 $\{10, -4\}$
- $\left(t - \frac{1}{2}\right)^2 = -\frac{17}{4}$
29. $t - \frac{1}{2} = \pm\sqrt{-\frac{17}{4}}$
 $t = \frac{1}{2} \pm \sqrt{-\frac{17}{4}}$
 $= \frac{1}{2} \pm \frac{i\sqrt{17}}{2}$
 $= \frac{1}{2} \pm \frac{\sqrt{17}}{2}i$
 $\left\{\frac{1}{2} \pm \frac{\sqrt{17}}{2}i\right\}$
- $\left(a - \frac{1}{3}\right)^2 = -\frac{47}{9}$
 $a - \frac{1}{3} = \pm\sqrt{-\frac{47}{9}}$
30. $a = \frac{1}{3} \pm \sqrt{-\frac{47}{9}}$
 $= \frac{1}{3} \pm \frac{i\sqrt{47}}{3}$
 $= \frac{1}{3} \pm \frac{\sqrt{47}}{3}i$
 $\left\{\frac{1}{3} \pm \frac{\sqrt{47}}{3}i\right\}$

$$\begin{aligned}
 x^2 + 14x + n &= x^2 + 14x + \left[\frac{1}{2}(14)\right]^2 \\
 &= x^2 + 14x + (7)^2 \\
 &= x^2 + 14x + 49 \\
 &= (x + 7)^2
 \end{aligned}$$

31.

$$n = 49; (x + 7)^2$$

$$\begin{aligned}
 y^2 + 22y + n &= y^2 + 22y + \left[\frac{1}{2}(22)\right]^2 \\
 &= y^2 + 22y + (11)^2 \\
 &= y^2 + 22y + 121 \\
 &= (y + 11)^2
 \end{aligned}$$

32.

$$n = 121; (y + 11)^2$$

$$\begin{aligned}
 p^2 - 26p + n &= p^2 - 26p + \left[\frac{1}{2}(-26)\right]^2 \\
 &= p^2 - 26p + (-13)^2 \\
 &= p^2 - 26p + 169 \\
 &= (p - 13)^2
 \end{aligned}$$

33.

$$n = 169; (p - 13)^2$$

$$\begin{aligned}
 u^2 - 4u + n &= u^2 - 4u + \left[\frac{1}{2}(-4)\right]^2 \\
 &= u^2 - 4u + (-2)^2 \\
 &= u^2 - 4u + 4 \\
 &= (u - 2)^2
 \end{aligned}$$

34.

$$n = 4; (u - 2)^2$$

$$\begin{aligned}
 w^2 - 3w + n &= w^2 - 3w + \left[\frac{1}{2}(-3)\right]^2 \\
 &= w^2 - 3w + \left(-\frac{3}{2}\right)^2 \\
 &= w^2 - 3w + \frac{9}{4} \\
 &= \left(w - \frac{3}{2}\right)^2
 \end{aligned}$$

35.

$$n = \frac{9}{4}; \left(w - \frac{3}{2}\right)^2$$

$$\begin{aligned}
 v^2 - 11v + n &= v^2 - 11v + \left[\frac{1}{2}(-11)\right]^2 \\
 &= v^2 - 11v + \left(-\frac{11}{2}\right)^2 \\
 &= v^2 - 11v + \frac{121}{4} \\
 &= \left(v - \frac{11}{2}\right)^2
 \end{aligned}$$

36.

$$n = \frac{121}{4}; \left(v - \frac{11}{2}\right)^2$$

$$\begin{aligned}
 m^2 + \frac{2}{9}m + n &= m^2 + \frac{2}{9}m + \left[\frac{1}{2}\left(\frac{2}{9}\right)\right]^2 \\
 &= m^2 + \frac{2}{9}m + \left(\frac{1}{9}\right)^2 \\
 &= m^2 + \frac{2}{9}m + \frac{1}{81} \\
 &= \left(m + \frac{1}{9}\right)^2
 \end{aligned}$$

37.

$$n = \frac{1}{81}; \left(m + \frac{1}{9}\right)^2$$

$$\begin{aligned}
 k^2 + \frac{2}{5}k + n &= k^2 + \frac{2}{5}k + \left[\frac{1}{2}\left(\frac{2}{5}\right)\right]^2 \\
 &= k^2 + \frac{2}{5}k + \left(\frac{1}{5}\right)^2 \\
 &= k^2 + \frac{2}{5}k + \frac{1}{25} \\
 &= \left(k + \frac{1}{5}\right)^2
 \end{aligned}$$

38.

$$n = \frac{1}{25}; \left(k + \frac{1}{5}\right)^2$$

$$39. \quad y^2 + 22y - 4 = 0$$

$$y^2 + 22y = 4$$

$$y^2 + 22y + \left[\frac{1}{2}(22)\right]^2 = 4 + \left[\frac{1}{2}(22)\right]^2$$

$$y^2 + 22y + 121 = 4 + 121$$

$$(y + 11)^2 = 125$$

$$y + 11 = \pm\sqrt{125}$$

$$y = -11 \pm 5\sqrt{5}$$

$$\{-11 \pm 5\sqrt{5}\}$$

$$40. \quad x^2 + 14x - 3 = 0$$

$$x^2 + 14x = 3$$

$$x^2 + 14x + \left[\frac{1}{2}(14)\right]^2 = 3 + \left[\frac{1}{2}(14)\right]^2$$

$$x^2 + 14x + 49 = 3 + 49$$

$$(x + 7)^2 = 52$$

$$x + 7 = \pm\sqrt{52}$$

$$x = -7 \pm 2\sqrt{13}$$

$$\{-7 \pm 2\sqrt{13}\}$$

$$t^2 - 8t = -24$$

$$t^2 - 8t + \left[\frac{1}{2}(-8)\right]^2 = -24 + \left[\frac{1}{2}(-8)\right]^2$$

$$t^2 - 8t + 16 = -24 + 16$$

$$(t - 4)^2 = -8$$

$$t - 4 = \pm\sqrt{-8}$$

$$41. \quad t = 4 \pm 2i\sqrt{2}$$

$$\{4 \pm 2i\sqrt{2}\}$$

$$p^2 - 24p = -156$$

$$42.$$

$$p^2 - 24p + \left[\frac{1}{2}(-24)\right]^2 = -156 + \left[\frac{1}{2}(-24)\right]^2$$

$$p^2 - 24p + 144 = -156 + 144$$

$$(p - 12)^2 = -12$$

$$p - 12 = \pm\sqrt{-12}$$

$$p = 12 \pm 2i\sqrt{3}$$

$$\{12 \pm 2i\sqrt{3}\}$$

$$43. \quad 4z^2 + 24z = -160$$

$$\frac{4z^2}{4} + \frac{24z}{4} = \frac{-160}{4}$$

$$z^2 + 6z = -40$$

$$z^2 + 6z + \left[\frac{1}{2}(6)\right]^2 = -40 + \left[\frac{1}{2}(6)\right]^2$$

$$z^2 + 6z + 9 = -40 + 9$$

$$(z + 3)^2 = -31$$

$$z + 3 = \pm\sqrt{-31}$$

$$z = -3 \pm i\sqrt{31}$$

$$\{-3 \pm i\sqrt{31}\}$$

$$44. \quad 2m^2 + 20m = -70$$

$$\frac{2m^2}{2} + \frac{20m}{2} = \frac{-70}{2}$$

$$m^2 + 10m = -35$$

$$m^2 + 10m + \left[\frac{1}{2}(10)\right]^2 = -35 + \left[\frac{1}{2}(10)\right]^2$$

$$m^2 + 10m + 25 = -35 + 25$$

$$(m + 5)^2 = -10$$

$$m + 5 = \pm\sqrt{-10}$$

$$m = -5 \pm i\sqrt{10}$$

$$\{-5 \pm i\sqrt{10}\}$$

$$45.$$

$$2x(x - 3) = 4 + x$$

$$2x^2 - 6x = 4 + x$$

$$2x^2 - 7x = 4$$

$$\frac{2x^2}{2} - \frac{7x}{2} = \frac{4}{2}$$

$$x^2 - \frac{7}{2}x = 2$$

$$x^2 - \frac{7}{2}x + \left[\frac{1}{2}\left(-\frac{7}{2}\right)\right]^2 = 2 + \left[\frac{1}{2}\left(-\frac{7}{2}\right)\right]^2$$

$$x^2 - \frac{7}{2}x + \frac{49}{16} = \frac{32}{16} + \frac{49}{16}$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{81}{16}$$

$$x - \frac{7}{4} = \pm\sqrt{\frac{81}{16}}$$

$$x = \frac{7}{4} \pm \frac{9}{4}$$

$$x = \frac{7}{4} + \frac{9}{4} \quad \text{or} \quad x = \frac{7}{4} - \frac{9}{4}$$

$$x = \frac{16}{4} = 4 \quad x = \frac{-2}{4} = -\frac{1}{2}$$

$$\left\{4, -\frac{1}{2}\right\}$$

$$5c(c-2) = 6 + 3c$$

$$5c^2 - 10c = 6 + 3c$$

$$5c^2 - 13c = 6$$

$$\frac{5c^2}{5} - \frac{13c}{5} = \frac{6}{5}$$

$$c^2 - \frac{13}{5}c = \frac{6}{5}$$

46.

$$c^2 - \frac{13}{5}c + \left[\frac{1}{2}\left(-\frac{13}{5}\right)\right]^2 = \frac{6}{5} + \left[\frac{1}{2}\left(-\frac{13}{5}\right)\right]^2$$

$$c^2 - \frac{13}{5}c + \frac{169}{100} = \frac{120}{100} + \frac{169}{100}$$

$$\left(c - \frac{13}{10}\right)^2 = \frac{289}{100}$$

$$c - \frac{13}{10} = \pm \sqrt{\frac{289}{100}}$$

$$c = \frac{30}{10} = 3 \quad \text{or} \quad c = \frac{-4}{10} = -\frac{2}{5}$$

$$\left\{3, -\frac{2}{5}\right\}$$

$$\mathbf{47.} \quad -4y^2 - 12y + 5 = 0$$

$$\frac{-4y^2}{-4} - \frac{12y}{-4} + \frac{5}{-4} = \frac{0}{-4}$$

$$y^2 + 3y = \frac{5}{4}$$

$$y^2 + 3y + \left[\frac{1}{2}(3)\right]^2 = \frac{5}{4} + \left[\frac{1}{2}(3)\right]^2$$

$$y^2 + 3y + \frac{9}{4} = \frac{5}{4} + \frac{9}{4}$$

$$\left(y + \frac{3}{2}\right)^2 = \frac{7}{2}$$

$$y + \frac{3}{2} = \pm \sqrt{\frac{7}{2}}$$

$$y + \frac{3}{2} = \pm \frac{\sqrt{7} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$$y + \frac{3}{2} = \pm \frac{\sqrt{14}}{2}$$

$$y = -\frac{3}{2} \pm \frac{\sqrt{14}}{2}$$

$$\left\{-\frac{3}{2} \pm \frac{\sqrt{14}}{2}\right\}$$

$$\mathbf{48.} \quad -2x^2 - 14x + 5 = 0$$

$$\frac{-2x^2}{-2} - \frac{14x}{-2} + \frac{5}{-2} = \frac{0}{-2}$$

$$x^2 + 7x = \frac{5}{2}$$

$$x^2 + 7x + \left[\frac{1}{2}(7)\right]^2 = \frac{5}{2} + \left[\frac{1}{2}(7)\right]^2$$

$$x^2 + 7x + \frac{49}{4} = \frac{10}{4} + \frac{49}{4}$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{59}{4}$$

$$x + \frac{7}{2} = \pm \sqrt{\frac{59}{4}}$$

$$x + \frac{7}{2} = \pm \frac{\sqrt{59}}{2}$$

$$x = -\frac{7}{2} \pm \frac{\sqrt{59}}{2}$$

$$\left\{-\frac{7}{2} \pm \frac{\sqrt{59}}{2}\right\}$$

$$49. \quad 3x^2 + 5x - 6 = 0$$

$$\frac{3x^2}{3} + \frac{5x}{3} - \frac{6}{3} = \frac{0}{3}$$

$$x^2 + \frac{5}{3}x = 2$$

$$x^2 + \frac{5}{3}x + \left[\frac{1}{2}\left(\frac{5}{3}\right)\right]^2 = 2 + \left[\frac{1}{2}\left(\frac{5}{3}\right)\right]^2$$

$$x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{72}{36} + \frac{25}{36}$$

$$\left(x + \frac{5}{6}\right)^2 = \frac{97}{36}$$

$$x + \frac{5}{6} = \pm \sqrt{\frac{97}{36}}$$

$$x + \frac{5}{6} = \pm \frac{\sqrt{97}}{6}$$

$$x = -\frac{5}{6} \pm \frac{\sqrt{97}}{6}$$

$$\left\{-\frac{5}{6} \pm \frac{\sqrt{97}}{6}\right\}$$

$$50. \quad 4x^2 + 3x - 8 = 0$$

$$\frac{4x^2}{4} + \frac{3x}{4} - \frac{8}{4} = \frac{0}{4}$$

$$x^2 + \frac{3}{4}x = 2$$

$$x^2 + \frac{3}{4}x + \left[\frac{1}{2}\left(\frac{3}{4}\right)\right]^2 = 2 + \left[\frac{1}{2}\left(\frac{3}{4}\right)\right]^2$$

$$x^2 + \frac{3}{4}x + \frac{9}{64} = \frac{128}{64} + \frac{9}{64}$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{137}{64}$$

$$x + \frac{3}{8} = \pm \sqrt{\frac{137}{64}}$$

$$x + \frac{3}{8} = \pm \frac{\sqrt{137}}{8}$$

$$x = -\frac{3}{8} \pm \frac{\sqrt{137}}{8}$$

$$\left\{-\frac{3}{8} \pm \frac{\sqrt{137}}{8}\right\}$$

$$x^2 = 7x - 4$$

$$x^2 - 7x + 4 = 0$$

$$ax^2 + bx + c = 0$$

$$51. \quad a = 1, b = -7, c = 4$$

$$x^2 = 3(x - 2)$$

$$x^2 = 3x - 6$$

$$x^2 - 3x + 6 = 0$$

$$ax^2 + bx + c = 0$$

$$52. \quad a = 1, b = -3, c = 6$$

$$5x^2 + 3x = 0$$

$$5x^2 + 3x = 0$$

$$ax^2 + bx + c = 0$$

$$53. \quad a = 5, b = 3, c = 0$$

$$2x^2 - 18 = 0$$

$$ax^2 + bx + c = 0$$

$$54. \quad a = 2, b = 0, c = -18$$

$$55. \quad x^2 - 3x - 7 = 0$$

$$a = 1, b = -3, c = -7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9 + 28}}{2}$$

$$= \frac{3 \pm \sqrt{37}}{2}$$

$$\left\{\frac{3 \pm \sqrt{37}}{2}\right\}$$

$$56. \quad x^2 - 5x - 9 = 0$$

$$a=1, b=-5, c=-9$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-9)}}{2(1)} \\ &= \frac{5 \pm \sqrt{25 + 36}}{2} \\ &= \frac{5 \pm \sqrt{61}}{2} \\ &\left\{ \frac{5 \pm \sqrt{61}}{2} \right\} \end{aligned}$$

$$y^2 = -4y - 6$$

$$57. y^2 + 4y + 6 = 0$$

$$a=1, b=4, c=6$$

$$\begin{aligned} y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(6)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{16 - 24}}{2} \\ &= \frac{-4 \pm \sqrt{-8}}{2} \\ &= \frac{-4 \pm 2i\sqrt{2}}{2} \\ &= -2 \pm i\sqrt{2} \\ &\{-2 \pm i\sqrt{2}\} \end{aligned}$$

$$z^2 = -8z - 19$$

$$58. z^2 + 8z + 19 = 0$$

$$a=1, b=8, c=19$$

$$\begin{aligned} z &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(19)}}{2(1)} \\ &= \frac{-8 \pm \sqrt{64 - 76}}{2} \\ &= \frac{-8 \pm \sqrt{-12}}{2} \\ &= \frac{-8 \pm 2i\sqrt{3}}{2} \\ &= -4 \pm i\sqrt{3} \\ &\{-4 \pm i\sqrt{3}\} \end{aligned}$$

$$t(t-6) = -10$$

$$59. t^2 - 6t + 10 = 0$$

$$a=1, b=-6, c=10$$

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)} \\ &= \frac{6 \pm \sqrt{36 - 40}}{2} \\ &= \frac{6 \pm \sqrt{-4}}{2} \\ &= \frac{6 \pm 2i}{2} \\ &= 3 \pm i \\ &\{3 \pm i\} \end{aligned}$$

$$m(m+10) = -34$$

$$60. m^2 + 10m + 34 = 0$$

$$a=1, b=10, c=34$$

$$\begin{aligned} m &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(10) \pm \sqrt{(10)^2 - 4(1)(34)}}{2(1)} \\ &= \frac{-10 \pm \sqrt{100 - 136}}{2} \\ &= \frac{-10 \pm \sqrt{-36}}{2} \\ &= \frac{-10 \pm 6i}{2} \\ &= -5 \pm 3i \end{aligned}$$

$$\begin{aligned}
 m &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(10) \pm \sqrt{(10)^2 - 4(1)(34)}}{2(1)} \\
 &= \frac{-10 \pm \sqrt{100 - 136}}{2} \\
 &= \frac{-10 \pm \sqrt{-36}}{2} \\
 &= \frac{-10 \pm 6i}{2} \\
 &= -5 \pm 3i \\
 &\quad \{-5 \pm 3i\} \\
 &\quad -7c + 3 = -5c^2
 \end{aligned}$$

61. $5c^2 - 7c + 3 = 0$

$a = 5, b = -7, c = 3$

$$\begin{aligned}
 c &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(3)}}{2(5)} \\
 &= \frac{7 \pm \sqrt{49 - 60}}{10} \\
 &= \frac{7 \pm \sqrt{-11}}{10} \\
 &= \frac{7 \pm i\sqrt{11}}{10} \\
 &\quad \left\{ \frac{7 \pm i\sqrt{11}}{10} \right\} \\
 &\quad -5d + 2 = -6d^2
 \end{aligned}$$

62. $6d^2 - 5d + 2 = 0$

$a = 6, b = -5, c = 2$

$$\begin{aligned}
 d &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(6)(2)}}{2(6)} \\
 &= \frac{5 \pm \sqrt{25 - 48}}{12} \\
 &= \frac{5 \pm \sqrt{-23}}{12} \\
 &= \frac{5 \pm i\sqrt{23}}{12} \\
 &\quad \left\{ \frac{5 \pm i\sqrt{23}}{12} \right\}
 \end{aligned}$$

63.

$(6x + 5)(x - 3) = -2x(7x + 5) + x - 12$

$6x^2 - 18x + 5x - 15 = -14x^2 - 10x + x - 12$

$6x^2 - 13x - 15 = -14x^2 - 9x - 12$

$20x^2 - 4x - 3 = 0$

$a = 20, b = -4, c = -3$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(20)(-3)}}{2(20)} \\
 &= \frac{4 \pm \sqrt{16 + 240}}{40} \\
 &= \frac{4 \pm \sqrt{256}}{40} \\
 &= \frac{4 \pm 16}{40} \\
 x &= \frac{4 + 16}{40} \quad \text{or} \quad x = \frac{4 - 16}{40} \\
 x &= \frac{20}{40} = \frac{1}{2} \quad x = \frac{-12}{40} = -\frac{3}{10} \\
 &\quad \left\{ \frac{1}{2}, -\frac{3}{10} \right\}
 \end{aligned}$$

64.

$(5c + 7)(2c - 3) = -2c(c + 15) - 35$

$$10c^2 - 15c + 14c - 21 = -2c^2 - 30c - 35$$

$$10c^2 - c - 21 = -2c^2 - 30c - 35$$

$$12c^2 + 29c + 14 = 0$$

$$a = 12, b = 29, c = 14$$

$$\begin{aligned} c &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(29) \pm \sqrt{(29)^2 - 4(12)(14)}}{2(12)} \\ &= \frac{-29 \pm \sqrt{841 - 672}}{24} \\ &= \frac{-29 \pm \sqrt{169}}{24} \\ &= \frac{-29 \pm 13}{24} \end{aligned}$$

$$c = \frac{-29+13}{24} \quad \text{or} \quad c = \frac{-29-13}{24}$$

$$c = \frac{-16}{24} = -\frac{2}{3} \quad c = \frac{-42}{24} = -\frac{7}{4}$$

$$\left\{ -\frac{2}{3}, -\frac{7}{4} \right\}$$

$$\mathbf{65.} \quad 9x^2 + 49 = 0$$

$$a = 9, b = 0, c = 49$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(0) \pm \sqrt{(0)^2 - 4(9)(49)}}{2(9)} \\ &= \frac{\pm \sqrt{-1764}}{18} \\ &= \frac{\pm 42i}{18} \\ &= \pm \frac{7}{3}i \\ &\quad \left\{ \pm \frac{7}{3}i \right\} \end{aligned}$$

$$\mathbf{66.} \quad 121x^2 + 4 = 0$$

$$a = 121, b = 0, c = 4$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(0) \pm \sqrt{(0)^2 - 4(121)(4)}}{2(121)} \\ &= \frac{\pm \sqrt{-1936}}{242} \\ &= \frac{\pm 44i}{242} \\ &= \pm \frac{2}{11}i \\ &\quad \left\{ \pm \frac{2}{11}i \right\} \end{aligned}$$

$$\frac{1}{2}x^2 - \frac{2}{7} = \frac{5}{14}x$$

$$14\left(\frac{1}{2}x^2 - \frac{2}{7}\right) = 14\left(\frac{5}{14}x\right)$$

$$7x^2 - 4 = 5x$$

$$\mathbf{67.} \quad 7x^2 - 5x - 4 = 0$$

$$a = 7, b = -5, c = -4$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(7)(-4)}}{2(7)} \\ &= \frac{5 \pm \sqrt{25 + 112}}{14} \\ &= \frac{5 \pm \sqrt{137}}{14} \\ &\quad \left\{ \frac{5 \pm \sqrt{137}}{14} \right\} \end{aligned}$$

$$\frac{1}{3}x^2 - \frac{7}{6} = \frac{3}{2}x$$

$$6\left(\frac{1}{3}x^2 - \frac{7}{6}\right) = 6\left(\frac{3}{2}x\right)$$

$$2x^2 - 7 = 9x$$

$$\mathbf{68.} \quad 2x^2 - 9x - 7 = 0$$

$$a = 2, b = -9, c = -7$$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-7)}}{2(2)} \\
 &= \frac{9 \pm \sqrt{81 + 56}}{4} \\
 &= \frac{9 \pm \sqrt{137}}{4} \\
 &\left\{ \frac{9 \pm \sqrt{137}}{4} \right\}
 \end{aligned}$$

$$0.4y^2 = 2y - 2.5$$

$$10(0.4y^2) = 10(2y - 2.5)$$

$$4y^2 = 20y - 25$$

$$4y^2 - 20y + 25 = 0$$

69.

$$a = 4, b = -20, c = 25$$

$$\begin{aligned}
 y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)} \\
 &= \frac{20 \pm \sqrt{400 - 400}}{8} \\
 &= \frac{20 \pm \sqrt{0}}{8} \\
 &= \frac{20}{8} = \frac{5}{2} \\
 &\left\{ \frac{5}{2} \right\}
 \end{aligned}$$

$$0.09n^2 = 0.42n - 0.49$$

$$100(0.09n^2) = 100(0.42n - 0.49)$$

$$9n^2 = 42n - 49$$

$$\mathbf{70.} \quad 9n^2 - 42n + 49 = 0$$

$$a = 9, b = -42, c = 49$$

$$\begin{aligned}
 n &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-42) \pm \sqrt{(-42)^2 - 4(9)(49)}}{2(9)} \\
 &= \frac{42 \pm \sqrt{1764 - 1764}}{18} \\
 &= \frac{42 \pm \sqrt{0}}{18} \\
 &= \frac{42}{18} = \frac{7}{3} \\
 &\left\{ \frac{7}{3} \right\}
 \end{aligned}$$

71. Linear

$$2y + 4 = 0$$

$$2y = -4$$

$$y = \frac{-4}{2}$$

$$= -2$$

$$\{-2\}$$

72. Linear

$$3z - 9 = 0$$

$$3z = 9$$

$$z = 3$$

$$\{3\}$$

73. Quadratic

$$2y^2 + 4y = 0$$

$$\frac{2y^2}{2} + \frac{4y}{2} = \frac{0}{2}$$

$$y^2 + 2y = 0$$

$$y(y + 2) = 0$$

$$y = 0 \quad \text{or} \quad y + 2 = 0$$

$$y = -2$$

$$\{0, -2\}$$

74. Quadratic

$$3z^2 - 9z = 0$$

$$\frac{3z^2}{3} - \frac{9z}{3} = \frac{0}{3}$$

$$z^2 - 3z = 0$$

$$z(z - 3) = 0$$

$$z = 0 \quad \text{or} \quad z - 3 = 0$$

$$z = 3$$

$$\{0, 3\}$$

75. Linear

$$5x(x + 6) = 5x^2 + 27x + 3$$

$$5x^2 + 30x = 5x^2 + 27x + 3$$

$$3x = 3$$

$$x = 1$$

$$\{1\}$$

76. Linear

$$3x(x - 4) = 3x^2 - 11x + 4$$

$$3x^2 - 12x = 3x^2 - 11x + 4$$

$$-x = 4$$

$$x = -4$$

$$\{-4\}$$

77. Neither**78. Neither**

$$(3x - 4)^2 = 0$$

$$3x - 4 = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

79.

$$\left\{\frac{4}{3}\right\}$$

$$(2x + 1)^2 = 0$$

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

80.

$$\left\{-\frac{1}{2}\right\}$$

$$m^2 + 4m = -2$$

$$m^2 + 4m + \left[\frac{1}{2}(4)\right]^2 = -2 + \left[\frac{1}{2}(4)\right]^2$$

$$m^2 + 4m + 4 = -2 + 4$$

$$(m + 2)^2 = 2$$

$$m + 2 = \pm\sqrt{2}$$

$$m = -2 \pm \sqrt{2}$$

81.

$$\{-2 \pm \sqrt{2}\}$$

$$n^2 + 8n = -3$$

$$n^2 + 8n + \left[\frac{1}{2}(8)\right]^2 = -3 + \left[\frac{1}{2}(8)\right]^2$$

$$n^2 + 8n + 16 = -3 + 16$$

$$(n + 4)^2 = 13$$

$$n + 4 = \pm\sqrt{13}$$

$$n = -4 \pm \sqrt{13}$$

82.

$$\{-4 \pm \sqrt{13}\}$$

$$\frac{x^2 - 4x}{6} - \frac{5x}{3} = 1$$

$$6\left(\frac{x^2 - 4x}{6} - \frac{5x}{3}\right) = 6(1)$$

83.

$$x^2 - 4x - 10x = 6$$

$$x^2 - 14x = 6$$

$$x^2 - 14x + \left[\frac{1}{2}(-14)\right]^2 = 6 + \left[\frac{1}{2}(-14)\right]^2$$

$$x^2 - 14x + 49 = 6 + 49$$

$$(x - 7)^2 = 55$$

$$x - 7 = \pm\sqrt{55}$$

$$x = 7 \pm \sqrt{55}$$

$$\{7 \pm \sqrt{55}\}$$

$$\frac{m^2 + 2m}{7} - \frac{9m}{14} = \frac{3}{2}$$

$$14\left(\frac{m^2 + 2m}{7} - \frac{9m}{14}\right) = 14\left(\frac{3}{2}\right)$$

$$2m^2 + 4m - 9m = 21$$

$$2m^2 - 5m - 21 = 0$$

$$a = 2, b = -5, c = -21$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-21)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 + 168}}{4} = \frac{5 \pm \sqrt{193}}{4}$$

$$\left\{ \frac{5 \pm \sqrt{193}}{4} \right\}$$

$$2(x + 4) + x^2 = x(x + 2) + 8$$

$$2x + 8 + x^2 = x^2 + 2x + 8$$

$$0 = 0$$

□

$$3(y - 5) + y^2 = y(y + 3) - 15$$

$$3y - 15 + y^2 = y^2 + 3y - 15$$

$$0 = 0$$

□

$$\frac{3}{5}x^2 - \frac{1}{10}x = \frac{1}{2}$$

87.

$$10\left(\frac{3}{5}x^2 - \frac{1}{10}x\right) = 10\left(\frac{1}{2}\right)$$

$$6x^2 - x = 5$$

$$6x^2 - x - 5 = 0$$

$$(x - 1)(6x + 5) = 0$$

$$x - 1 = 0 \quad \text{or} \quad 6x + 5 = 0$$

$$x = 1 \quad 6x = -5$$

$$x = -\frac{5}{6}$$

$$\left\{ 1, -\frac{5}{6} \right\}$$

$$\frac{1}{12}x^2 - \frac{11}{24}x = -\frac{1}{2}$$

$$24\left(\frac{1}{12}x^2 - \frac{11}{24}x\right) = 24\left(-\frac{1}{2}\right)$$

$$2x^2 - 11x = -12$$

$$2x^2 - 11x + 12 = 0$$

$$(2x - 3)(x - 4) = 0$$

$$2x - 3 = 0 \quad \text{or} \quad x - 4 = 0$$

$$2x = 3 \quad x = 4$$

$$x = \frac{3}{2}$$

$$\left\{ \frac{3}{2}, 4 \right\}$$

$$x^2 - 5x = 5x(x - 1) - 4x^2 + 1$$

$$x^2 - 5x = 5x^2 - 5x - 4x^2 + 1$$

$$x^2 - 5x = x^2 - 5x + 1$$

$$0 = 1$$

89.

$$\{ \}$$

$$p^2 - 4p = 4p(p - 1) - 3p^2 + 2$$

$$p^2 - 4p = 4p^2 - 4p - 3p^2 + 2$$

$$p^2 - 4p = p^2 - 4p + 2$$

$$0 = 2$$

90.

$$\{ \}$$

$$(2y + 7)(y + 1) = 2y^2 - 11$$

$$2y^2 + 2y + 7y + 7 = 2y^2 - 11$$

$$2y^2 + 9y + 7 = 2y^2 - 11$$

$$9y = -18$$

$$y = -2$$

91.

$$\{-2\}$$

$$(3z - 8)(z + 2) = 3z^2 + 10$$

$$3z^2 + 6z - 8z - 16 = 3z^2 + 10$$

$$3z^2 - 2z - 16 = 3z^2 + 10$$

$$-2z = 26$$

$$z = -13$$

92.

$$\{-13\}$$

$7d^2 + 5 = 0$
 $7d^2 = -5$
 $d^2 = -\frac{5}{7}$
93. $d = \pm \sqrt{-\frac{5}{7}} = \pm i \sqrt{\frac{5}{7}}$
 $= \pm \frac{\sqrt{5}}{\sqrt{7}} i = \pm \frac{\sqrt{5} \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} i$
 $= \pm \frac{\sqrt{35}}{7} i$
 $\left\{ \pm \frac{\sqrt{35}}{7} i \right\}$

94. $11t^2 + 3 = 0$
 $11t^2 = -3$
 $t^2 = -\frac{3}{11}$
 $t = \pm \sqrt{-\frac{3}{11}} = \pm i \sqrt{\frac{3}{11}}$
 $= \pm \frac{\sqrt{3}}{\sqrt{11}} i = \pm \frac{\sqrt{3} \cdot \sqrt{11}}{\sqrt{11} \cdot \sqrt{11}} i$
 $= \pm \frac{\sqrt{33}}{11} i$
 $\left\{ \pm \frac{\sqrt{33}}{11} i \right\}$

$x^2 - \sqrt{5} = 0$
 $x^2 = \sqrt{5}$
95. $x = \pm \sqrt[4]{5}$
 $\left\{ \pm \sqrt[4]{5} \right\}$

$y^2 - \sqrt{11} = 0$
 $y^2 = \sqrt{11}$
96. $y = \pm \sqrt[4]{11}$
 $\left\{ \pm \sqrt[4]{11} \right\}$

97. a. $3x^2 - 4x + 6 = 0$
 $b^2 - 4ac = (-4)^2 - 4(3)(6)$
 $= 16 - 72$
 $= -56$

b. $-56 < 0$; there are two nonreal solutions.

98. a. $5x^2 - 2x + 4 = 0$
 $b^2 - 4ac = (-2)^2 - 4(5)(4)$
 $= 4 - 80$
 $= -76$
b. $-76 < 0$; there are two nonreal solutions.
 $-2w^2 + 8w = 3$

99. a. $-2w^2 + 8w - 3 = 0$
 $b^2 - 4ac = (8)^2 - 4(-2)(-3)$
 $= 64 - 24$
 $= 40$
b. $40 > 0$; there are two real solutions.
 $-6d^2 + 9d = 2$
 $-6d^2 + 9d - 2 = 0$

100. a. $6d^2 - 9d + 2 = 0$
 $b^2 - 4ac = (-9)^2 - 4(6)(2)$
 $= 81 - 48$
 $= 33$
b. $33 > 0$; there are two real solutions.
 $3x(x - 4) = x - 4$
 $3x^2 - 12x = x - 4$

101. a. $3x^2 - 13x + 4 = 0$
 $b^2 - 4ac = (-13)^2 - 4(3)(4)$
 $= 169 - 48$
 $= 121$
b. $121 > 0$; there are two real solutions.
 $2x(x - 2) = x + 3$
 $2x^2 - 4x = x + 3$

102. a. $2x^2 - 5x - 3 = 0$
 $b^2 - 4ac = (-5)^2 - 4(2)(-3)$
 $= 25 + 24$
 $= 49$

b. $49 > 0$; there are two real solutions.

103. a.

$$-1.4m + 0.1 = -4.9m^2$$

$$10(-1.4m + 0.1) = 10(-4.9m^2)$$

$$-14m + 1 = -49m^2$$

$$49m^2 - 14m + 1 = 0$$

$$b^2 - 4ac = (-14)^2 - 4(49)(1)$$

$$= 196 - 196$$

$$= 0$$

b. The discriminant is 0; there is one real solution.

104. a.

$$3.6n + 0.4 = -8.1n^2$$

$$10(3.6n + 0.4) = 10(-8.1n^2)$$

$$36n + 4 = -81n^2$$

$$81n^2 + 36n + 4 = 0$$

$$b^2 - 4ac = (36)^2 - 4(81)(4)$$

$$= 1296 - 1296$$

$$= 0$$

b. The discriminant is 0; there is one real solution.

$$A = \pi r^2$$

$$\frac{A}{\pi} = \frac{\pi r^2}{\pi}$$

$$\frac{A}{\pi} = r^2$$

$$r = \sqrt{\frac{A}{\pi}} \text{ or } r = \frac{\sqrt{A\pi}}{\pi}$$

105.

$$A = \pi r^2 h$$

$$\frac{A}{\pi h} = \frac{\pi r^2 h}{\pi h}$$

$$\frac{A}{\pi h} = r^2$$

$$r = \sqrt{\frac{A}{\pi h}} \text{ or } r = \frac{\sqrt{A\pi h}}{\pi h}$$

106.

$$s = \frac{1}{2}gt^2$$

107.

$$2(s) = 2\left(\frac{1}{2}gt^2\right)$$

$$2s = gt^2$$

$$\frac{2s}{g} = \frac{gt^2}{g}$$

$$\frac{2s}{g} = t^2$$

$$t = \sqrt{\frac{2s}{g}} \text{ or } t = \frac{\sqrt{2sg}}{g}$$

$$c = \frac{d^2t}{2}$$

108.

$$2c = 2\left(\frac{d^2t}{2}\right)$$

$$2c = d^2t$$

$$\frac{2c}{t} = \frac{d^2t}{t}$$

$$\frac{2c}{t} = d^2$$

$$d = \sqrt{\frac{2c}{t}} \text{ or } d = \frac{\sqrt{2ct}}{t}$$

$$a^2 + b^2 = c^2$$

$$a^2 = c^2 - b^2$$

$$a = \sqrt{c^2 - b^2}$$

109.

$$a^2 + b^2 + c^2 = d^2$$

$$c^2 = d^2 - a^2 - b^2$$

110.

$$c = \sqrt{d^2 - a^2 - b^2}$$

111.

$$L = c^2 l^2 Rt$$

$$\frac{L}{c^2 Rt} = \frac{c^2 l^2 Rt}{c^2 Rt}$$

$$\frac{L}{c^2 Rt} = l^2$$

$$l = \sqrt{\frac{L}{c^2 Rt}} = \frac{1}{c} \sqrt{\frac{L}{Rt}} \text{ or } \frac{\sqrt{LRt}}{cRt}$$

112.

$$I = cN^2 r^2 s$$

$$\frac{I}{cr^2s} = \frac{cN^2r^2s}{cr^2s}$$

$$\frac{I}{cr^2s} = N^2$$

$$N = \sqrt{\frac{I}{cr^2s}}$$

$$N = \frac{1}{r} \sqrt{\frac{I}{cs}} \text{ or } N = \frac{\sqrt{Ics}}{crs}$$

$$kw^2 - cw = r$$

$$\mathbf{113.} \quad kw^2 - cw - r = 0$$

$$a = k, b = -c, c = -r$$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-(-c) \pm \sqrt{(-c)^2 - 4(k)(-r)}}{2(k)}$$

$$w = \frac{c \pm \sqrt{c^2 + 4kr}}{2k}$$

$$dy^2 + my = p$$

$$\mathbf{114.} \quad dy^2 + my - p = 0$$

$$a = d, b = m, c = -p$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(m) \pm \sqrt{(m)^2 - 4(d)(-p)}}{2(d)}$$

$$y = \frac{-m \pm \sqrt{m^2 + 4dp}}{2k}$$

$$s = v_0 t + \frac{1}{2} at^2$$

$$2(s) = 2\left(v_0 t + \frac{1}{2} at^2\right)$$

$$2s = 2v_0 t + at^2$$

$$\mathbf{115.} \quad at^2 + 2v_0 t - 2s = 0$$

$$a = a, b = 2v_0, c = -2s$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(2v_0) \pm \sqrt{(2v_0)^2 - 4(a)(-2s)}}{2(a)}$$

$$t = \frac{-2v_0 \pm \sqrt{4v_0^2 + 8as}}{2a}$$

$$t = \frac{-2v_0 \pm 2\sqrt{v_0^2 + 2as}}{2a}$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2as}}{a}$$

$$S = 2\pi rh + \pi r^2 h$$

$$\mathbf{116.} \quad \pi hr^2 + 2\pi hr - S = 0$$

$$a = \pi h, b = 2\pi h, c = -S$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-(2\pi h) \pm \sqrt{(2\pi h)^2 - 4(\pi h)(-S)}}{2(\pi h)}$$

$$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 4\pi h S}}{2\pi h}$$

$$r = \frac{-2\pi h \pm 2\sqrt{\pi^2 h^2 + \pi h S}}{2\pi h}$$

$$r = \frac{-\pi h \pm \sqrt{\pi^2 h^2 + \pi h S}}{\pi h}$$

$$LI^2 + RI + \frac{1}{C} = 0$$

$$C\left(LI^2 + RI + \frac{1}{C}\right) = C(0)$$

$$\mathbf{117.} \quad CLI^2 + CRI + 1 = 0$$

$$a = CL, b = CR, c = 1$$

$$I = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$I = \frac{-(CR) \pm \sqrt{(CR)^2 - 4(CL)(1)}}{2(CL)}$$

$$I = \frac{-CR \pm \sqrt{C^2 R^2 - 4CL}}{2CL}$$

$$A = \pi r^2 + \pi rS$$

$$\mathbf{118.} \quad \pi r^2 + \pi rS - A = 0$$

$$a = \pi, b = \pi S, c = -A$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-(\pi S) \pm \sqrt{(\pi S)^2 - 4(\pi)(-A)}}{2(\pi)}$$

$$r = \frac{-\pi S \pm \sqrt{\pi^2 S^2 + 4\pi A}}{2\pi}$$

119. The right side of the equation is not equal to zero.

120. Given $ax^2 + bx + c = 0$, where $a \neq 0$, the discriminant is $b^2 - 4ac$. The discriminant indicates the number and type of solutions to the equation.

121.

$$x^2 - xy - 2y^2 = 0$$

$$(x - 2y)(x + y) = 0$$

$$x - 2y = 0 \quad \text{or} \quad x + y = 0$$

$$x = 2y \quad \quad \quad x = -y$$

$$3a^2 + 2ab - b^2 = 0$$

$$(3a - b)(a + b) = 0$$

$$3a - b = 0 \quad \text{or} \quad a + b = 0$$

$$3a = b \quad \quad \quad a = -b$$

$$a = \frac{b}{3}$$

122.

$$(x - 4)(x + 2) = 0$$

$$x^2 + 2x - 4x - 8 = 0$$

$$\mathbf{123.} \quad x^2 - 2x - 8 = 0$$

$$\begin{aligned}
 &(x-7)(x+1)=0 \\
 &x^2+x-7x-7=0 \\
 \mathbf{124.} \quad &x^2-6x-7=0 \\
 &\left(x-\frac{2}{3}\right)\left(x-\frac{1}{4}\right)=0 \\
 &3\left(x-\frac{2}{3}\right) \cdot 4\left(x-\frac{1}{4}\right)=12 \cdot 0 \\
 &(3x-2)(4x-1)=0 \\
 &12x^2-3x-8x+2=0 \\
 &12x^2-11x+2=0
 \end{aligned}$$

125.

$$\begin{aligned}
 &\left(x-\frac{3}{5}\right)\left(x-\frac{1}{7}\right)=0 \\
 &5\left(x-\frac{3}{5}\right) \cdot 7\left(x-\frac{1}{7}\right)=35 \cdot 0 \\
 &(5x-3)(7x-1)=0 \\
 &35x^2-5x-21x+3=0
 \end{aligned}$$

$$\mathbf{126.} \quad 35x^2-26x+3=0$$

$$\begin{aligned}
 &(x-\sqrt{5})(x+\sqrt{5})=0 \\
 &(x)^2-(\sqrt{5})^2=0
 \end{aligned}$$

$$\mathbf{127.} \quad x^2-5=0$$

$$\begin{aligned}
 &(x-\sqrt{2})(x+\sqrt{2})=0 \\
 &(x)^2-(\sqrt{2})^2=0
 \end{aligned}$$

$$\mathbf{128.} \quad x^2-2=0$$

$$\begin{aligned}
 &(x-2i)(x+2i)=0 \\
 &(x)^2+(2)^2=0
 \end{aligned}$$

$$\mathbf{129.} \quad x^2+4=0$$

$$\begin{aligned}
 &(x-9i)(x+9i)=0 \\
 &(x)^2+(9)^2=0
 \end{aligned}$$

$$\mathbf{130.} \quad x^2+81=0$$

$$\begin{aligned}
 &[x-(1+2i)][x-(1-2i)]=0 \\
 &(x-1-2i)(x-1+2i)=0 \\
 &[(x-1)-2i][(x-1)+2i]=0 \\
 &(x-1)^2+(2)^2=0 \\
 &x^2-2x+1+4=0
 \end{aligned}$$

$$\mathbf{131.} \quad x^2-2x+5=0$$

$$\begin{aligned}
 &[x-(2+9i)][x-(2-9i)]=0 \\
 &(x-2-9i)(x-2+9i)=0 \\
 &[(x-2)-9i][(x-2)+9i]=0 \\
 &(x-2)^2+(9)^2=0 \\
 &x^2-4x+4+81=0
 \end{aligned}$$

$$\mathbf{132.} \quad x^2-4x+85=0$$

$$\mathbf{133.} \quad x_1+x_2$$

$$\begin{aligned}
 &= \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} \\
 &= \frac{-b+\sqrt{b^2-4ac}+(-b)-\sqrt{b^2-4ac}}{2a} \\
 &= \frac{-2b}{2a} = -\frac{b}{a}
 \end{aligned}$$

$$x_1x_2$$

134.

$$\begin{aligned}
 &= \left(\frac{-b+\sqrt{b^2-4ac}}{2a} \right) \cdot \left(\frac{-b-\sqrt{b^2-4ac}}{2a} \right) \\
 &= \frac{(-b)^2 - (\sqrt{b^2-4ac})^2}{(2a)^2} \\
 &= \frac{b^2 - (b^2-4ac)}{4a^2} \\
 &= \frac{4ac}{4a^2} = \frac{c}{a}
 \end{aligned}$$

Problem Recognition Exercises:
Simplifying Expressions versus
Solving Equations

1. a. Expression

$$\begin{aligned}
 (2x-5)(3x+1) &= 6x^2+2x-15x-5 \\
 &= 6x^2-13x-5
 \end{aligned}$$

b. Equation

$$\begin{aligned}(2x-5)(3x+1) &= 0 \\ 2x-5 &= 0 \quad \text{or} \quad 3x+1=0 \\ 2x &= 5 & 3x &= -1 \\ x &= \frac{5}{2} & x &= -\frac{1}{3}\end{aligned}$$

$$\left\{\frac{5}{2}, -\frac{1}{3}\right\}$$

2. a. Expression

$$\begin{aligned}& \frac{5}{x-3} - \frac{1}{x+7} - \frac{2}{x^2+4x-21} \\ &= \frac{5}{x-3} - \frac{1}{x+7} - \frac{2}{(x-3)(x+7)} \\ &= \frac{5}{x-3} \cdot \frac{x+7}{x+7} - \frac{1}{x+7} \cdot \frac{x-3}{x-3} - \frac{2}{(x-3)(x+7)} \\ &= \frac{5(x+7) - 1(x-3) - 2}{(x-3)(x+7)} \\ &= \frac{5x+35-x+3-2}{(x-3)(x+7)} \\ &= \frac{4x+36}{(x-3)(x+7)}\end{aligned}$$

b. Equation

$$\begin{aligned}& \frac{5}{x-3} - \frac{1}{x+7} = \frac{2}{x^2+4x-21} \\ & \left[\frac{(x-3)(x+7)}{\left(\frac{5}{x-3} - \frac{1}{x+7}\right)} \right] = \left[\frac{(x-3)(x+7)}{\left(\frac{2}{(x-3)(x+7)}\right)} \right] \\ & 5(x+7) - 1(x-3) = 2 \\ & 5x+35-x+3 = 2 \\ & 4x+38 = 2 \\ & 4x = -36 \\ & x = -9 \\ & \{-9\}\end{aligned}$$

3. a. Equation

$$\begin{aligned}(2x-3)^2 &= 8 \\ 2x-3 &= \pm\sqrt{8} \\ 2x-3 &= \pm 2\sqrt{2} \\ 2x &= 3 \pm \sqrt{2} \\ x &= \frac{3 \pm \sqrt{2}}{2}\end{aligned}$$

$$\left\{\frac{3 \pm \sqrt{2}}{2}\right\}$$

b. Expression

$$\begin{aligned}(2x-3)^2 - 8 &= (2x)^2 - 2(2x)(3) + (3)^2 - 8 \\ &= 4x^2 - 12x + 9 - 8 \\ &= 4x^2 - 12x + 1\end{aligned}$$

4. a. Equation

$$\begin{aligned}5 - \{6 + 3[2 - 5(y-2)] + 1\} &= 7 \\ 5 - \{6 + 3[2 - 5y + 10] + 1\} &= 7 \\ 5 - \{6 + 3[-5y + 12] + 1\} &= 7 \\ 5 - \{6 - 15y + 36 + 1\} &= 7 \\ 5 - \{-15y + 43\} &= 7 \\ 5 + 15y - 43 &= 7 \\ 15y - 38 &= 7 \\ 15y &= 45 \\ y &= 3\end{aligned}$$

$$\{3\}$$

b. Expression

$$\begin{aligned}5 - \{6 + 3[2 - 5(y-2)] + 1\} &= 5 - \{6 + 3[2 - 5y + 10] + 1\} \\ &= 5 - \{6 + 3[-5y + 12] + 1\} \\ &= 5 - \{6 - 15y + 36 + 1\} \\ &= 5 - \{-15y + 43\} \\ &= 5 + 15y - 43 \\ &= 15y - 38\end{aligned}$$

5. a. Equation

$$\begin{aligned}
 x^2 - 11x + 28 &= 0 \\
 (x - 7)(x - 4) &= 0 \\
 x - 7 &= 0 \quad \text{or} \quad x - 4 = 0 \\
 x &= 7 \qquad \qquad x = 4 \\
 \{7, 4\}
 \end{aligned}$$

b. Equation

$$\begin{aligned}
 x^2 - 11x - 28 &= 0 \\
 a &= 1, b = -11, c = -28 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(-28)}}{2(1)} \\
 &= \frac{11 \pm \sqrt{121 + 112}}{2} = \frac{11 \pm \sqrt{233}}{2} \\
 &\left\{ \frac{11 \pm \sqrt{233}}{2} \right\}
 \end{aligned}$$

6. a. Equation

$$\begin{aligned}
 3x(x + 9) &= 20 - x \\
 3x^2 + 27x &= 20 - x \\
 3x^2 + 28x - 20 &= 0 \\
 (3x - 2)(x + 10) &= 0 \\
 3x - 2 &= 0 \quad \text{or} \quad x + 10 = 0 \\
 3x &= 2 \qquad \qquad x = -10 \\
 x &= \frac{2}{3} \\
 \left\{ \frac{2}{3}, -10 \right\}
 \end{aligned}$$

b. Equation

$$\begin{aligned}
 3(x + 9) &= 20 - x \\
 3x + 27 &= 20 - x \\
 4x &= -7 \\
 x &= -\frac{7}{4} \\
 \left\{ -\frac{7}{4} \right\}
 \end{aligned}$$

7. a. Equation

$$\begin{aligned}
 \frac{35}{x} + 12 + x &= 0 \\
 x \left(\frac{35}{x} + 12 + x \right) &= x(0) \\
 35 + 12x + x^2 &= 0 \\
 x^2 + 12x + 35 &= 0 \\
 (x + 7)(x + 5) &= 0 \\
 x + 7 &= 0 \quad \text{or} \quad x + 5 = 0 \\
 x &= -7 \qquad \qquad x = -5 \\
 \{-7, -5\}
 \end{aligned}$$

b. Expression

$$\begin{aligned}
 \frac{35}{x} + 12 + x &= \frac{35}{x} + 12 \cdot \frac{x}{x} + x \cdot \frac{x}{x} \\
 &= \frac{35 + 12x + x^2}{x}
 \end{aligned}$$

8. a. Equation

$$\begin{aligned}
 \frac{x}{x-2} + \frac{2}{3} &= \frac{2}{x-2} \\
 3(x-2) \left(\frac{x}{x-2} + \frac{2}{3} \right) &= 3(x-2) \left(\frac{2}{x-2} \right) \\
 3x + 2(x-2) &= 6 \\
 3x + 2x - 4 &= 6 \\
 5x &= 10 \\
 x &= 2 \\
 \{ \}; &\text{The value 2 does not check.}
 \end{aligned}$$

b. Expression

$$\begin{aligned}
 \frac{x}{x-2} + \frac{2}{3} - \frac{2}{x-2} &= \frac{x-2}{x-2} + \frac{2}{3} \\
 &= 1 + \frac{2}{3} = \frac{5}{3}; \text{ for } x \neq 2
 \end{aligned}$$

Section 1.5 Applications of Quadratic Equations

1. $A = \frac{1}{2}bh$

2. $A = \pi r^2$

3. $V = lwh$

4. $a^2 + b^2 = c^2$

5. a. Let x represents the width of the rectangle (in yd). Then $(2x + 3)$ is the length of the rectangle.

$$lw = A$$

$$x(2x + 3) = 629$$

$$x(2x + 3) = 629$$

$$2x^2 + 3x = 629$$

b. $2x^2 + 3x - 629 = 0$

$$x^2 + \frac{3}{2}x - \frac{629}{2} = 0$$

$$x^2 + \frac{3}{2}x + \left(\frac{1}{2}\left(\frac{3}{2}\right)\right)^2 = \frac{629}{2} + \left(\frac{1}{2}\left(\frac{3}{2}\right)\right)^2$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{5041}{16}$$

$$\left(x + \frac{3}{4}\right) = \pm \frac{71}{4}$$

$$x + \frac{3}{4} = \frac{71}{4} \text{ or } x + \frac{3}{4} = -\frac{71}{4}$$

$$x = 17 \quad x \neq -\frac{37}{2}$$

$$2x + 3 = 37 \quad 2x + 3 \neq -34$$

The width is 17 yd and the length is 37 yd.

6. a. Let x represents the length of the

rectangle (in meter). Then $\left(\frac{1}{4}x - 2\right)$ is the width of the rectangle.

$$lw = A$$

$$x\left(\frac{1}{4}x - 2\right) = 252$$

b. $x\left(\frac{1}{4}x - 2\right) = 252$

$$\frac{1}{4}x^2 - 2x = 252$$

$$x^2 - 8x - 1008 = 0$$

$$(x - 36)(x + 28) = 0$$

$$x - 36 = 0 \text{ or } x + 28 = 0$$

$$x = 36 \quad x \neq -28$$

$$\frac{1}{4}x - 2 = 7 \quad \frac{1}{4}x - 2 \neq -9$$

The width is 7 m and the length is 36 m.

7. a. Let x represents the base of the

triangle (in feet). Then $(x - 2)$ is the height of the triangle.

$$\frac{1}{2}bh = A$$

$$\frac{1}{2}x(x - 2) = 40$$

b. $\frac{1}{2}x(x - 2) = 40$

$$\frac{1}{2}x^2 - x = 40$$

$$x^2 - 2x - 80 = 0$$

$$(x - 10)(x + 8) = 0$$

$$x - 10 = 0 \text{ or } x + 8 = 0$$

$$x = 10 \quad x \neq -8$$

$$x - 2 = 8 \quad x - 2 \neq -10$$

The base is 10 ft and the height is 8 ft.

- 8. a.** Let x represents the base of the triangle (in feet). Then $(x + 4)$ is the height of the triangle.

$$\frac{1}{2}bh = A$$

$$\frac{1}{2}x(x + 4) = 70$$

$$\frac{1}{2}x(x + 4) = 70$$

$$\frac{1}{2}x^2 + 2x = 70$$

$$x^2 + 4x - 140 = 0$$

$$(x - 10)(x + 14) = 0$$

$$x - 10 = 0 \quad \text{or} \quad x + 14 = 0$$

$$x = 10 \quad x \neq -14$$

$$x + 4 = 14 \quad x + 4 \neq -10$$

The base is 10 yd and the height is 14 yd.

- 9. a.** Let x represents length of a

rectangular box (in inches). Then $\frac{x}{5}$ is the height of the box.

$$lwh = V$$

$$x \cdot 8 \cdot \frac{x}{5} = 640$$

$$\frac{8}{5}x^2 = 640$$

$$\frac{8}{5}x^2 = 640$$

$$x^2 = 400$$

$$x = \pm 20$$

$$x = 20 \quad \text{or} \quad x \neq -20$$

$$\frac{x}{5} = 4 \quad x \neq -4$$

b.

The length is 20 in., the width is 8 in., and the height is 4 in.

- 10. a.** Let x represents width of a rectangular box (in feet). Then $(2x + 1)$ is the length of the box.

$$lwh = V$$

$$4x(2x + 1) = 312$$

$$4x(2x + 1) = 312$$

b.

$$8x^2 + 4x = 312$$

$$x^2 + \frac{1}{2}x = 39$$

$$x^2 + \frac{1}{2}x + \left(\frac{1}{2}\left(\frac{1}{2}\right)\right)^2 = 39 + \left(\frac{1}{2}\left(\frac{1}{2}\right)\right)^2$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{625}{16}$$

$$\left(x + \frac{1}{4}\right) = \pm \frac{25}{4}$$

$$x + \frac{1}{4} = \frac{25}{4} \quad \text{or} \quad x + \frac{1}{4} = -\frac{25}{4}$$

$$x = 6 \quad x \neq -\frac{13}{2}$$

$$2x + 1 = 13 \quad 2x + 1 \neq -12$$

The width is 6 ft, the length is 13 ft, and the height is 4 ft.

- 11. a.** Let x represents length of the shorter leg of a right triangle (in feet). Then $(x + 2)$ is the length of the longer leg and $(2x - 2)$ is the hypotenuse of the triangle.

$$a^2 + b^2 = c^2$$

$$x^2 + (x + 2)^2 = (2x - 2)^2$$

$$x^2 + (x + 2)^2 = (2x - 2)^2$$

b.

$$2x^2 + 4x + 4 = 4x^2 - 8x + 4$$

$$-2x^2 + 12x = 0$$

$$x^2 - 6x = 0$$

$$x(x - 6) = 0$$

$$x = 6 \quad \text{or} \quad x \neq 0$$

$$x + 2 = 8 \quad x + 2 \neq 2$$

$$2x - 2 = 10 \quad 2x - 2 \neq -2$$

The legs are 6 ft and 8 ft, and the hypotenuse is 10 ft.

- 12. a.** Let x represents length of the shorter leg of a right triangle (in cm). Then $(x + 7)$ is the length of the longer leg.

$$a^2 + b^2 = c^2$$

$$x^2 + (x + 7)^2 = (17)^2$$

$$\mathbf{b.} \quad x^2 + (x + 7)^2 = (17)^2$$

$$2x^2 + 14x + 49 = 289$$

$$2x^2 + 14x - 240 = 0$$

$$x^2 + 7x - 120 = 0$$

$$(x - 8)(x + 15) = 0$$

$$x - 8 = 0 \quad \text{or} \quad x + 15 = 0$$

$$x = 8 \quad \quad \quad x \times -15$$

$$x + 7 = 15 \quad \quad \quad x + 7 \times -8$$

The legs are 8 cm and 15 cm, and the hypotenuse is 17 cm.

- 13.** Let x represents width of a rectangular garden (in yd). Then, $(x + 2)$ is the length and 40 yd^2 is the area of the garden.

$$x(x + 2) = 40$$

$$x^2 + 2x = 40$$

$$x^2 + 2x + 1 = 40 + 1$$

$$(x + 1)^2 = 41$$

$$(x + 1) = \pm\sqrt{41}$$

$$x = -1 + \sqrt{41} \quad \text{or} \quad x = -1 - \sqrt{41}$$

$$x = 5.403 \approx 5.4 \quad \quad \quad x \times -7.403$$

$$x + 2 = 7.403 \approx 7.4 \quad \quad \quad x + 2 \times -5.403$$

The width is approximately 5.4 yd and the length is approximately 7.4 yd.

- 14.** Let x represents length of a rectangular piece of carpet (in yd). Then, $(x - 9)$ is the width and 200 yd^2 is the area of the piece of carpet.

$$x(x - 9) = 200$$

$$x^2 - 9x = 200$$

$$x^2 - 9x + \left(\frac{1}{2}(9)\right)^2 = 200 + \left(\frac{1}{2}(9)\right)^2$$

$$\left(x - \frac{9}{2}\right)^2 = \frac{881}{4}$$

$$\left(x - \frac{9}{2}\right) = \pm \frac{\sqrt{881}}{2}$$

$$x = \frac{9}{2} + \frac{\sqrt{881}}{2} \quad \text{or} \quad x = \frac{9}{2} - \frac{\sqrt{881}}{2}$$

$$x = 19.341 \approx 19.3 \quad \quad \quad x \times -10.341 \approx 10.3$$

$$x - 9 = 10.341 \approx 10.3 \quad \quad \quad x - 9 \times -19.341$$

The length is approximately 19.3 yd and the width is approximately 10.3 yd.

- 15.** Let x represents the base of a triangular truss (in feet). Then, $(x - 8)$ is the height of the truss and 86 ft^2 is the area of the truss.

$$\frac{1}{2}x(x - 8) = 86$$

$$x^2 - 8x = 172$$

$$x^2 - 8x + 16 = 172 + 16$$

$$(x - 4)^2 = 188$$

$$(x - 4) = \pm\sqrt{188}$$

$$x = 4 + \sqrt{188} \quad \text{or} \quad x = 4 - \sqrt{188}$$

$$x = 17.711 \approx 17.7 \quad \quad \quad x \times -9.711$$

$$x - 8 = 9.711 \approx 9.7 \quad \quad \quad x - 8 \times -17.711$$

The length is approximately 17.7 ft, and the height is approximately 9.7 ft.

- 16.** Let x represents the height of a triangular piece of fabric (in inches). Then, $(x + 6)$ is the base and 600 in.^2 is the area of the piece of fabric.

$$\begin{aligned}\frac{1}{2}x(x+6) &= 600 \\ x^2 + 6x &= 1200 \\ x^2 + 6x + 9 &= 1200 + 9 \\ (x+3)^2 &= 1209 \\ (x+3) &= \pm\sqrt{1209} \\ x &= -3 + \sqrt{1209} \quad \text{or} \quad x = -3 - \sqrt{1209} \\ x &= 31.771 \approx 31.8 \quad x \neq -37.771 \\ x + 6 &= 37.771 \approx 37.8 \quad x + 6 \neq -31.771 \\ \text{The height is approximately 31.8 in.,} \\ \text{and the base is approximately 37.8 in.}\end{aligned}$$

17. a. $x(x+2) = 120$

$$\begin{aligned}x(x+2) &= 120 \\ x^2 + 2x &= 120 \\ x^2 + 2x - 120 &= 0\end{aligned}$$

b. $(x-10)(x+12) = 0$

$$\begin{aligned}x-10 &= 0 & \text{or} & & x+12 &= 0 \\ x &= 10 & & & x &= -12 \\ x+2 &= 12 & & & x+2 &= -10\end{aligned}$$

The integers are 10 and 12 or -10 and -12 .

18. a. $x(x+2) = 35$

$$\begin{aligned}x(x+2) &= 35 \\ x^2 + 2x &= 35 \\ x^2 + 2x - 35 &= 0\end{aligned}$$

b. $(x-5)(x+7) = 0$

$$\begin{aligned}x-5 &= 0 & \text{or} & & x+7 &= 0 \\ x &= 5 & & & x &= -7 \\ x+2 &= 7 & & & x+2 &= -5\end{aligned}$$

The integers are 5 and 7 or -5 and -7 .

19. a. $x^2 + (x+1)^2 = 113$

b. $x^2 + (x+1)^2 = 113$

$$\begin{aligned}x^2 + x^2 + 2x + 1 &= 113 \\ 2x^2 + 2x - 112 &= 0 \\ \frac{2x^2}{2} + \frac{2x}{2} - \frac{112}{2} &= \frac{0}{2} \\ x^2 + x - 56 &= 0 \\ (x-7)(x+8) &= 0 \\ x-7 &= 0 & \text{or} & & x+8 &= 0 \\ x &= 7 & & & x &= -8 \\ x+1 &= 8 & & & x+1 &= -7 \\ \text{The integers are 7 and 8 or } -7 & \text{ and } -8.\end{aligned}$$

20. a. $x^2 + (x+1)^2 = 181$

b. $x^2 + (x+1)^2 = 181$

$$\begin{aligned}x^2 + x^2 + 2x + 1 &= 181 \\ 2x^2 + 2x - 180 &= 0 \\ \frac{2x^2}{2} + \frac{2x}{2} - \frac{180}{2} &= \frac{0}{2} \\ x^2 + x - 90 &= 0 \\ (x-9)(x+10) &= 0 \\ x-9 &= 0 & \text{or} & & x+10 &= 0 \\ x &= 9 & & & x &= -10 \\ x+1 &= 10 & & & x+1 &= -9 \\ \text{The integers are 9 and 10 or } -9 & \text{ and } -10.\end{aligned}$$

21. Let x represents the width of the cargo space. The length is 12 ft and the height is $x-1$ ft.

$$\begin{aligned}V &= lwh \\ 504 &= (12)(x)(x-1) \\ 504 &= 12(x^2 - x) \\ 504 &= 12x^2 - 12x \\ 12x^2 - 12x - 504 &= 0 \\ 12(x^2 - x - 42) &= 0 \\ (x-7)(x+6) &= 0 \\ x-7 &= 0 & \text{or} & & x+6 &= 0 \\ x &= 7 & & & x &= -6\end{aligned}$$

$$\begin{aligned}x - 1 &= 7 - 1 \\ &= 6\end{aligned}$$

The dimensions of the cargo space are 6 ft by 7 ft by 12 ft.

- 22.** Let x represents the width of the cardboard sheet. Then $(x + 12)$ in. is the length of the cardboard sheet. The width of the box is $(x - 12)$ in. The length of the box is $(x + 12 - 12) = x$ in. The height of the box is 6 in.

$$V = lwh$$

$$1728 = (x)(x - 12)(6)$$

$$1728 = 6(x^2 - 12x)$$

$$1728 = 6x^2 - 72x$$

$$6x^2 - 72x - 1728 = 0$$

$$6(x^2 - 12x - 288) = 0$$

$$(x - 24)(x + 12) = 0$$

$$x - 24 = 0 \quad \text{or} \quad x + 12 = 0$$

$$x = 24 \quad \quad \quad \cancel{x = -12}$$

$$\begin{aligned}x + 12 &= 24 + 12 \\ &= 36\end{aligned}$$

The dimensions of the cardboard sheet are 24 in. by 36 in.

- 23.** Let r represent the radius (in yards) of the region watered.

$$A = \pi r^2$$

$$2000 = \pi r^2$$

$$\frac{2000}{\pi} = r^2$$

$$r = \pm \sqrt{\frac{2000}{\pi}} \approx \pm 25$$

The radius is approximately 25 yd.

- 24.** Let r represent the radius (in miles) of the area in which the earthquake could be felt.

$$A = \pi r^2$$

$$46,000 = \pi r^2$$

$$\frac{46,000}{\pi} = r^2$$

$$\begin{aligned}r &= \pm \sqrt{\frac{46,000}{\pi}} \\ &\approx \pm 121\end{aligned}$$

The earthquake could be felt up to 121mi from the epicentre.

- 25.** Let x represent the height of the triangle (in feet) and $(x - 3)$ represent the base of the triangle.

$$lw + 2\left(\frac{1}{2}bh\right) = A$$

$$(20)(x) + 2\left[\frac{1}{2}(x - 3)(x)\right] = 348$$

$$20x + x^2 - 3x = 348$$

$$x^2 + 17x - 348 = 0$$

$$(x + 29)(x - 12) = 0$$

$$x + 29 = 0 \quad \text{or} \quad x - 12 = 0$$

$$\cancel{x = -29} \quad \quad \quad x = 12$$

$$x - 3 = 12 - 3$$

$$= 9$$

The base is 9 ft and the height is 12 ft.

- 26.** Let x represent the height of the triangle and $3x$ represent the base of the triangle.

$$lw + \frac{1}{2}bh = A$$

$$(3x)(x + 2) + \frac{1}{2}(3x)(x) = 336$$

$$3x^2 + 6x + \frac{3}{2}x^2 = 336$$

$$6x^2 + 12x + 3x^2 = 672$$

$$9x^2 + 12x - 672 = 0$$

$$(9x + 84)(x - 8) = 0$$

$$9x + 84 = 0 \quad \text{or} \quad x - 8 = 0$$

$$9x = -84 \quad \quad \quad x = 8$$

$$\cancel{x = -\frac{84}{9}}$$

$$3x = 3(8) = 24$$

$$x + 2 = 8 + 2 = 10$$

The length is 24 ft and the height is 10 ft.

- 27.** Let x represent the distance (in feet) from home plate to second base.

$$a^2 + b^2 = c^2$$

$$(90)^2 + (90)^2 = (x)^2$$

$$8100 + 8100 = x^2$$

$$16,200 = x^2$$

$$x = \pm\sqrt{16,200}$$

$$= \pm 90\sqrt{2}$$

$$\approx \pm 127.3$$

The distance is $90\sqrt{2}$ ft or approximately 127.3 ft.

$$a^2 + b^2 = c^2$$

$$(6)^2 + (6)^2 = c^2$$

$$36 + 36 = c^2$$

$$72 = c^2$$

$$c = \pm\sqrt{72}$$

$$= \pm 6\sqrt{2} \text{ in.}$$

28. a.

$$a^2 + b^2 = c^2$$

$$(6\sqrt{2})^2 + (6)^2 = d^2$$

$$72 + 36 = d^2$$

$$108 = d^2$$

$$d = \pm\sqrt{108}$$

$$= \pm 6\sqrt{3} \text{ in.}$$

b.

- 29. a.** Let x represent the length (in feet) of the middle leg.

$$a^2 + b^2 = c^2$$

$$(x)^2 + (x-2)^2 = (x+2)^2$$

$$x^2 + x^2 - 4x + 4 = x^2 + 4x + 4$$

$$2x^2 - 4x + 4 = x^2 + 4x + 4$$

$$x^2 - 8x = 0$$

$$x(x-8) = 0$$

$$x - 8 = 0$$

$$x = 0 \quad \text{or} \quad x = 8$$

$$x - 2 = 8 - 2 = 6$$

$$x + 2 = 8 + 2 = 10$$

The lengths of the sides of the lower triangle are 6 ft, 8 ft, and 10 ft.

$$A = A_T + A_B$$

$$= \frac{1}{2}b_T h_T + \frac{1}{2}b_B h_B$$

b.

$$= \frac{1}{2}(b_T h_T + b_B h_B)$$

$$= \frac{1}{2}[(10)(4) + (8)(6)]$$

$$= \frac{1}{2}(40 + 48)$$

$$= \frac{1}{2}(88) = 44$$

The total area is 44 ft².

- 30. a.** Let x represent the length (in feet) of the shorter leg.

$$a^2 + b^2 = c^2$$

$$(x)^2 + (x+7)^2 = (2x+1)^2$$

$$x^2 + x^2 + 14x + 49 = 4x^2 + 4x + 1$$

$$2x^2 + 14x + 49 = 4x^2 + 4x + 1$$

$$-2x^2 + 10x + 48 = 0$$

$$\frac{-2x^2}{-2} + \frac{10x}{-2} + \frac{48}{-2} = \frac{0}{-2}$$

$$x^2 - 5x - 24 = 0$$

$$(x+3)(x-8) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 8 = 0$$

$$\cancel{x = -3}$$

$$x = 8$$

The lengths of the sides of the triangle on the left are 8 ft, 15 ft, and 17 ft.

- b.** Let x represent the length (in feet) of the unlabeled side.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 (x)^2 + (9)^2 &= (15)^2 \\
 x^2 + 81 &= 225 \\
 x^2 &= 144 \\
 x &= \pm\sqrt{144} \\
 &= \pm 12
 \end{aligned}$$

The lengths of the sides of the triangle on the right are 9 ft, 12 ft, and 15 ft.

- 31. a.** Let x represent the width (in inches) of the cell phone. Then $1.5x$ is the length of the phone.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 (x)^2 + (1.5x)^2 &= (3.5)^2 \\
 x^2 + 2.25x^2 &= 12.25 \\
 3.25x^2 &= 12.25 \\
 x^2 &= \frac{12.25}{3.25} \\
 x &= \pm\sqrt{\frac{12.25}{3.25}} \\
 &\approx \pm 1.94 \\
 1.5x &\approx 1.5(1.94) \\
 &\approx 2.91
 \end{aligned}$$

The length is approximately 2.91 in. and the width is approximately 1.94 in.

$$2.91(326) \approx 949$$

$$1.94(326) \approx 632$$

b.

Using the rounded values from part (a), the screen is approximately 949 pixels by 632 pixels.

- 32.** Let x represent the width (in inches) of the display. Then $1.6x$ is the length of the display.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 (x)^2 + (1.6x)^2 &= (15)^2 \\
 x^2 + 2.56x^2 &= 225 \\
 3.56x^2 &= 225 \\
 x^2 &= \frac{225}{3.56} \\
 x &= \pm\sqrt{\frac{225}{3.56}} \\
 &\approx \pm 7.95 \\
 1.6x &\approx 1.6(7.95) \\
 &\approx 12.72
 \end{aligned}$$

The length is approximately 12.72 in., and the width is approximately 7.95 in.

- 33.** Let n represent the number of players.

$$\begin{aligned}
 N &= \frac{1}{2}n(n-1) \\
 28 &= \frac{1}{2}n(n-1) \\
 56 &= n^2 - n \\
 n^2 - n - 56 &= 0 \\
 (n-8)(n+7) &= 0 \\
 n-8 &= 0 \quad \text{or} \quad n+7=0 \\
 n &= 8 \quad \quad \quad \cancel{n=-7}
 \end{aligned}$$

There were 8 players.

34.
$$S = \frac{1}{2}n(n+1)$$

$$\begin{aligned}
 171 &= \frac{1}{2}n(n+1) \\
 342 &= n^2 + n
 \end{aligned}$$

$$\begin{aligned}
 n^2 + n - 342 &= 0 \\
 (n-18)(n+19) &= 0 \\
 n-18 &= 0 \quad \text{or} \quad n+19=0 \\
 n &= 18 \quad \quad \quad \cancel{n=-19}
 \end{aligned}$$

The value of n is 18.

- 35.** Let t represent the time(s) at which the

population was 600,000.

$$\begin{aligned}
 P &= -1718t^2 + 82,000t + 10,000 \\
 600,000 &= -1718t^2 + 82,000t + 10,000 \\
 0 &= -1718t^2 + 82,000t - 590,000 \\
 0 &= 1718t^2 - 82,000t + 590,000
 \end{aligned}$$

$$\begin{aligned}
 t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-82,000) \pm \sqrt{(-82,000)^2 - 4(1718)(590,000)}}{2(1718)} \\
 &= \frac{82,000 \pm \sqrt{2,669,520,000}}{3436} \\
 &\approx 9 \text{ or } 39
 \end{aligned}$$

There were 600,000 organisms approximately 9 hr and 39 hr after the culture was started.

- 36.** Let x represent the speed of the vehicle in mph.

$$\begin{aligned}
 m &= -0.04x^2 + 3.6x - 49 \\
 30 &= -0.04x^2 + 3.6x - 49 \\
 0 &= 0.04x^2 - 3.6x + 79 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-3.6) \pm \sqrt{(-3.6)^2 - 4(0.04)(79)}}{2(0.04)} \\
 &= \frac{3.6 \pm \sqrt{0.32}}{0.08}
 \end{aligned}$$

$$x \approx 38 \text{ or } x \approx 52$$

The gas mileage will be 30 mpg for speeds of 38 mph and 52 mph.

$$\begin{aligned}
 d &= 0.05v^2 + 2.2v \\
 d &= 0.05(50)^2 + 2.2(50) \\
 &= 0.05(2500) + 110 \\
 &= 125 + 110 \\
 &= 235 \text{ ft}
 \end{aligned}$$

37. a.

$$\begin{aligned}
 d &= 0.05v^2 + 2.2v \\
 330 &= 0.05v^2 + 2.2v
 \end{aligned}$$

b. $d = 0.05v^2 + 2.2v - 330$

$$\begin{aligned}
 v &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(2.2) \pm \sqrt{(2.2)^2 - 4(0.05)(-330)}}{2(0.05)} \\
 &= \frac{-2.2 \pm \sqrt{70.84}}{0.1} \\
 &\approx 62 \text{ or } \cancel{106}
 \end{aligned}$$

The car can travel at 62 mph and stop in time.

38. a.

$$\begin{aligned}
 c &= 219x^2 - 26.7x + 1.64 \\
 &= 219(0.22)^2 - 26.7(0.22) + 1.64 \\
 &= 219(0.0484) - 5.874 + 1.64 \\
 &= 10.5996 - 5.874 + 1.64 \\
 &\approx 6.4 \text{ ng/mL}
 \end{aligned}$$

$$\begin{aligned}
 c &= 219x^2 - 26.7x + 1.64 \\
 3 &= 219x^2 - 26.7x + 1.64
 \end{aligned}$$

b. $0 = 219x^2 - 26.7x - 1.36$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-26.7) \pm \sqrt{(-26.7)^2 - 4(219)(-1.36)}}{2(219)} \\
 &= \frac{26.7 \pm \sqrt{1904.25}}{438} \\
 &\approx 0.16 \text{ or } \cancel{0.04} \\
 &= 16\%
 \end{aligned}$$

$$\begin{aligned}
 s &= -\frac{1}{2}gt^2 + v_0t + s_0 \\
 s &= -\frac{1}{2}(32)t^2 + (16)t + 0
 \end{aligned}$$

39. a. $s = -16t^2 + 16t$

b. $4 = -16t^2 + 16t$

$$\begin{aligned}
 16t^2 - 16t + 4 &= 0 \\
 4(4t^2 - 4t + 1) &= 0 \\
 4(2t - 1)^2 &= 0 \\
 2t - 1 &= 0 \\
 2t &= 1 \\
 t &= \frac{1}{2}
 \end{aligned}$$

It would take Michael Jordan 0.5

sec to reach his maximum height of 4 ft.

$$s = -\frac{1}{2}gt^2 + v_0t + s_0$$

$$s = -\frac{1}{2}(32)t^2 + (8\sqrt{5})t + 0$$

40. a. $= -16t^2 + 8\sqrt{5}t$

b. $5 = -16t^2 + 8\sqrt{5}t$

$$16t^2 - 8\sqrt{5}t + 5 = 0$$

$$(4t - \sqrt{5})^2 = 0$$

$$4t - \sqrt{5} = 0$$

$$4t = \pm\sqrt{5}$$

$$t = \pm\frac{\sqrt{5}}{4}$$

$$\approx \pm 0.56$$

It would take 0.56 sec to reach a height of 5 ft.

$$s = -\frac{1}{2}gt^2 + v_0t + s_0$$

$$s = -\frac{1}{2}(32)t^2 + (75)t + 4$$

41. a. $= -16t^2 + 75t + 4$

$$80 = -16t^2 + 75t + 4$$

b. $0 = 16t^2 - 75t + 76$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-75) \pm \sqrt{(-75)^2 - 4(16)(76)}}{2(16)}$$

$$t = \frac{75 \pm \sqrt{761}}{32}$$

$$\approx 1.5 \text{ or } 3.2$$

The ball will be at an 80-ft height 1.5 sec and 3.2 sec after being kicked.

$$s = -\frac{1}{2}gt^2 + v_0t + s_0$$

$$s = -\frac{1}{2}(9.8)t^2 + (18)t + 1$$

42. a. $= -4.9t^2 + 18t + 1$

$$16 = -4.9t^2 + 18t + 10$$

b. $= -4.9t^2 + 18t - 15$

$$0 = 4.9t^2 - 18t + 15$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(4.9)(15)}}{2(4.9)}$$

$$t = \frac{18 \pm \sqrt{30}}{9.8}$$

$$\approx 1.3 \text{ or } 2.4$$

George will catch the bread 1.3 sec after release.

$$\frac{L}{W} = \frac{L+W}{L}$$

$$\frac{L}{1} = \frac{L+1}{L}$$

$$L(L) = L\left(\frac{L+1}{L}\right)$$

$$L^2 = L + 1$$

43. a. $L^2 - L - 1 = 0$

$$L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$L = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$L = \frac{1 \pm \sqrt{5}}{2}$$

$$\approx \cancel{0.62} \text{ or } 1.62$$

$$\frac{1.62}{1} = \frac{l}{9}$$

$$l = 9 \cdot 1.62$$

b. $\approx 14.6 \text{ ft}$

44. Since the length of the sides of the square is 18 in., the length of the sides of the right triangles shown in the

figure is $\left(\frac{18-x}{2}\right)$. Use the Pythagorean theorem,

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 \left(\frac{18-x}{2}\right)^2 + \left(\frac{18-x}{2}\right)^2 &= x^2 \\
 2\left(\frac{18-x}{2}\right)^2 &= x^2 \\
 \frac{(18-x)^2}{2} &= x^2 \\
 (18-x)^2 &= 2x^2 \\
 324 - 36x + x^2 &= 2x^2 \\
 x^2 + 36x - 324 &= 0
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-36) \pm \sqrt{(36)^2 - 4(1)(-324)}}{2(1)} \\
 &= \frac{-36 \pm \sqrt{2592}}{2} \\
 &\approx 7.46 \text{ or } -43.46
 \end{aligned}$$

The sides are approximately 7.46 in.

$$\begin{aligned}
 y &= \frac{80-2x}{3} \\
 y &= \frac{80-2(25)}{3} = \frac{30}{3} = 10 \\
 \text{or} \\
 y &= \frac{80-2(15)}{3} = \frac{50}{3}
 \end{aligned}$$

Each pen can be 25 yd by 10 yd, or

it can be 15 yd by $\frac{50}{3}$ yd.

- 46.** Let t represent the time when the ships are 100 nautical miles apart. Then the distance travelled by the first ship is $(10t)$ and the distance travelled by the second ship is $15(t-2)$.

45. a.

$$\begin{aligned}
 4x + 6y &= 160 \\
 6y &= 160 - 4x \\
 y &= \frac{160-4x}{6} \text{ or } y = \frac{80-2x}{3}
 \end{aligned}$$

$$A = lw$$

$$A = x \left(\frac{80-2x}{3} \right)$$

b.

$$A = x \left(\frac{80-2x}{3} \right)$$

c.

$$250 = x \left(\frac{80-2x}{3} \right)$$

$$750 = 80x - 2x^2$$

$$375 = 40x - x^2$$

$$x^2 - 40x + 375 = 0$$

$$(x-25)(x-15) = 0$$

$$x-25=0 \text{ or } x-15=0$$

$$x=25 \quad x=15$$

$$a^2 + b^2 = c^2$$

$$(10t)^2 + [15(t-2)]^2 = (100)^2$$

$$100t^2 + (15t-30)^2 = 10,000$$

$$100t^2 + 225t^2 - 900t + 900 = 10,000$$

$$325t^2 - 900t - 9100 = 0$$

$$13t^2 - 36t - 364 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-36) \pm \sqrt{(-36)^2 - 4(13)(-364)}}{2(13)}$$

$$= \frac{36 \pm \sqrt{20,224}}{26}$$

$$\approx 6.85 \text{ or } -4.09$$

$$6.85 \text{ hours} = 6 \text{ hours} + \left(\frac{85}{100} \text{ min} \right) \left(\frac{60 \text{ min}}{1 \text{ hr}} \right)$$

$$\approx 6 \text{ hours} + 51 \text{ min}$$

The ships will be 100 nautical miles apart at approximately 6:51 PM.

Section 1.6 More Equations and Applications

1. absolute; $\{k, -k\}$

2. $u = w$ or $u = -w$

3. quadratic; $m^{\frac{1}{3}}$

4. $4x^2 + 1$

$$\begin{aligned} -3x(2x-1)(x+6)^2 &= 0 \\ -6x\left(x-\frac{1}{2}\right)(x+6)(x+6) &= 0 \\ x\left(x-\frac{1}{2}\right)(x+6)(x+6) &= 0 \end{aligned}$$

5.

$$x = 0 \text{ or } x - \frac{1}{2} = 0 \text{ or } x + 6 = 0 \text{ or } x + 6 = 0$$

$$x = 0 \quad x = \frac{1}{2} \quad x = -6$$

$$\left\{0, \frac{1}{2}, -6\right\}$$

$$\begin{aligned} 5y(3-y)(4y+1)^2 &= 0 \\ 60y\left(1-\frac{y}{3}\right)\left(y+\frac{1}{4}\right)\left(y+\frac{1}{4}\right) &= 0 \\ y\left(1-\frac{y}{3}\right)\left(y+\frac{1}{4}\right)\left(y+\frac{1}{4}\right) &= 0 \end{aligned}$$

6.

$$y = 0 \text{ or } 1 - \frac{y}{3} = 0 \text{ or } y + \frac{1}{4} = 0 \text{ or } y + \frac{1}{4} = 0$$

$$y = 0 \quad y = 3 \quad y = -\frac{1}{4}$$

$$\left\{0, 3, -\frac{1}{4}\right\}$$

$$7. \quad 4(w^2 - 7)(w^2 + 4) = 0$$

$$w^2 - 7 = 0 \text{ or } w^2 + 4 = 0$$

$$w^2 = 7 \quad w^2 = -4$$

$$w = \pm\sqrt{7} \quad w = \pm\sqrt{-4}$$

$$w = \pm 2i$$

$$\{\pm\sqrt{7}, \pm 2i\}$$

$$8. \quad -2(t^2 + 1)(t^2 - 5) = 0$$

$$t^2 + 1 = 0 \text{ or } t^2 - 5 = 0$$

$$t^2 = -1 \quad \text{or} \quad t^2 = 5$$

$$t = \pm\sqrt{-1} \quad t = \pm\sqrt{5}$$

$$t = \pm i$$

$$\{\pm\sqrt{5}, \pm i\}$$

$$75y^3 + 100y^2 - 3y - 4 = 0$$

$$75y^3 - 3y + 100y^2 - 4 = 0$$

$$3y(25y^2 - 1) + 4(25y^2 - 1) = 0$$

$$(25y^2 - 1)(3y + 4) = 0$$

$$9. \quad (5y + 1)(5y - 1)(3y + 4) = 0$$

$$5y + 1 = 0 \text{ or } 5y - 1 = 0 \text{ or } 3y + 4 = 0$$

$$y = -\frac{1}{5} \quad y = \frac{1}{5} \quad y = -\frac{4}{3}$$

$$\left\{\pm\frac{1}{5}, -\frac{4}{3}\right\}$$

$$98t^3 - 49t^2 - 8t + 4 = 0$$

$$49t^2(2t - 1) - 4(2t - 1) = 0$$

$$(49t^2 - 4)(2t - 1) = 0$$

$$10. \quad (7t + 2)(7t - 2)(2t - 1) = 0$$

$$7t + 2 = 0 \text{ or } 7t - 2 = 0 \text{ or } 2t - 1 = 0$$

$$t = -\frac{2}{7} \quad t = \frac{2}{7} \quad t = \frac{1}{2}$$

$$\left\{\pm\frac{2}{7}, \frac{1}{2}\right\}$$

$$x^3 + 7x^2 = 4(x + 7)$$

$$x^3 + 7x^2 - 4x - 28 = 0$$

$$x^3 - 4x + 7x^2 - 28 = 0$$

$$x(x^2 - 4) + 7(x^2 - 4) = 0$$

$$(x + 7)(x^2 - 4) = 0$$

$$11. \quad (x + 7)(x + 2)(x - 2) = 0$$

$$x + 7 = 0 \text{ or } x + 2 = 0 \text{ or } x - 2 = 0$$

$$x = -7 \quad x = -2 \quad x = 2$$

$$\{-7, -2, 2\}$$

$$2m^3 + 3m^2 = 9(2m + 3)$$

$$2m^3 + 3m^2 - 18m - 27 = 0$$

$$2m^3 - 18m + 3m^2 - 27 = 0$$

$$2m(m^2 - 9) + 3(m^2 - 9) = 0$$

$$(2m + 3)(m^2 - 9) = 0$$

$$12. \quad (2m + 3)(m + 3)(m - 3) = 0$$

$$2m + 3 = 0 \text{ or } m + 3 = 0 \text{ or } m - 3 = 0$$

$$m = -\frac{3}{2} \quad m = -3 \quad m = 3$$

$$\left\{-\frac{3}{2}, -3, 3\right\}$$

$$2x^4 - 32 = 0$$

$$2(x^4 - 16) = 0$$

$$2(x^2 - 4)(x^2 + 4) = 0$$

$$13. \quad 2(x - 2)(x + 2)(x^2 + 4) = 0$$

$$x - 2 = 0 \text{ or } x + 2 = 0 \text{ or } x^2 + 4 = 0$$

$$x = 2 \quad x = -2 \quad x^2 = -4$$

$$x = \pm\sqrt{-4}$$

$$x = \pm 2i$$

$$\{\pm 2i, \pm 2\}$$

$$5m^4 - 5 = 0$$

$$5(m^4 - 1) = 0$$

$$5(m^2 - 1)(m^2 + 1) = 0$$

$$14. \quad 5(m - 1)(m + 1)(m^2 + 1) = 0$$

$$m-1=0 \text{ or } m+1=0 \text{ or } m^2+1=0$$

$$m=1 \quad m=-1 \quad m^2=-1$$

$$m=\pm\sqrt{-1}$$

$$m=\pm i$$

$$\{\pm i, \pm 1\}$$

$$2x^4 = -128x$$

$$2x^4 + 128x = 0$$

$$2x(x^3 + 64) = 0$$

$$\mathbf{15.} \quad 2x(x+4)(x^2-4x+16) = 0$$

$$2x = 0 \text{ or } x+4 = 0 \text{ or } x^2-4x+16 = 0$$

$$x = 0 \text{ or } x = -4 \text{ or}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-48}}{2}$$

$$= \frac{4 \pm 4i\sqrt{3}}{2}$$

$$= 2 \pm 2i\sqrt{3}$$

$$\{0, -4, 2 \pm 2i\sqrt{3}\}$$

$$10x^5 = -1250x^2$$

$$10x^5 + 1250x^2 = 0$$

$$10x^2(x^3 + 125) = 0$$

$$\mathbf{16.} \quad 10x^2(x+5)(x^2-5x+25) = 0$$

$$10x^2 = 0 \text{ or } x+5 = 0 \text{ or } x^2-5x+25 = 0$$

$$x = 0 \text{ or } x = -5 \text{ or}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{-75}}{2} = \frac{5 \pm 5i\sqrt{3}}{2}$$

$$\left\{0, -5, \frac{5 \pm 5i\sqrt{3}}{2}\right\}$$

$$3n^2(n^2+3) = 20-2n^2$$

$$3n^4 + 9n^2 = 20-2n^2$$

$$3n^4 + 11n^2 - 20 = 0$$

$$(3n^2-4)(n^2+5) = 0$$

17.

$$3n^2-4=0$$

or

$$n^2+5=0$$

$$3n^2=4$$

$$n^2=-5$$

$$n^2 = \frac{4}{3}$$

$$n = \pm\sqrt{-5}$$

$$n = \pm\sqrt{\frac{4}{3}} = \pm\frac{2\sqrt{3}}{3}$$

$$n = \pm i\sqrt{5}$$

$$\left\{\pm\frac{2\sqrt{3}}{3}, \pm i\sqrt{5}\right\}$$

$$2y^2(y^2-2) = 18+y^2$$

$$2y^4 - 4y^2 = 18+y^2$$

$$2y^4 - 5y^2 - 18 = 0$$

$$\mathbf{18.} \quad (2y^2-9)(y^2+2) = 0$$

$$2y^2-9=0$$

$$2y^2=9$$

$$y^2 = \frac{9}{2}$$

$$y = \pm\sqrt{\frac{9}{2}}$$

$$= \pm\frac{3\sqrt{2}}{2}$$

or

$$y^2+2=0$$

$$y^2=-2$$

$$y = \pm\sqrt{-2}$$

$$y = \pm i\sqrt{2}$$

$$\left\{\pm\frac{3\sqrt{2}}{2}, \pm i\sqrt{2}\right\}$$

19.

$$x^3-8=x-2$$

$$(x-2)(x^2+2x+4) = (x-2)$$

$$(x-2)(x^2+2x+4) - (x-2) = 0$$

$$(x-2)(x^2+2x+4-1) = 0$$

$$(x-2)(x^2+2x+3) = 0$$

$$\begin{aligned}
 x^2 + 2x + 3 &= 0 \\
 x^2 + 2x + 1 &= -3 + 1 \\
 (x+1)^2 &= -2 \\
 x+1 &= \pm i\sqrt{2} \\
 x-2 &= 0 & x &= -1 \pm i\sqrt{2} \\
 x &= 2 \quad \text{or} \\
 \{2, -1 \pm i\sqrt{2}\}
 \end{aligned}$$

20.

$$\begin{aligned}
 x^3 - 64 &= x - 4 \\
 (x-4)(x^2 + 4x + 16) &= (x-4) \\
 (x-4)(x^2 + 4x + 16) - (x-4) &= 0 \\
 (x-4)(x^2 + 4x + 16 - 1) &= 0 \\
 (x-4)(x^2 + 4x + 15) &= 0 \\
 x-4 &= 0 \\
 x &= 4 \quad \text{or} \\
 x^2 + 4x + 15 &= 0 \\
 x^2 + 4x + 4 &= -15 + 4 \\
 (x+2)^2 &= -11 \\
 x+2 &= \pm i\sqrt{11} \\
 x &= -2 \pm i\sqrt{11} \\
 \{4, -2 \pm i\sqrt{11}\}
 \end{aligned}$$

21.

$$\begin{aligned}
 \frac{3x}{x+2} - \frac{5}{x-4} &= \frac{2x^2 - 14x}{x^2 - 2x - 8} \\
 \frac{3x}{x+2} - \frac{5}{x-4} &= \frac{2x^2 - 14x}{(x+2)(x-4)} \\
 (x+2)(x-4) \left(\frac{3x}{x+2} - \frac{5}{x-4} \right) &= (x+2)(x-4) \left[\frac{2x^2 - 14x}{(x+2)(x-4)} \right] \\
 3x(x-4) - 5(x+2) &= 2x^2 - 14x \\
 3x^2 - 12x - 5x - 10 &= 2x^2 - 14x \\
 x^2 - 3x - 10 &= 0 \\
 (x+2)(x-5) &= 0 \\
 \cancel{x = -2} \quad \text{or} \quad x &= 5 \\
 \{5\}; &\text{ The value } -2 \text{ does not check.}
 \end{aligned}$$

22.

$$\frac{4c}{c-5} - \frac{1}{c+1} = \frac{3c^2+3}{c^2-4c-5}$$

$$\frac{4c}{c-5} - \frac{1}{c+1} = \frac{3c^2+3}{(c-5)(c+1)}$$

$$(c-5)(c+1)\left(\frac{4c}{c-5} - \frac{1}{c+1}\right) = (c-5)(c+1)\left[\frac{3c^2+3}{(c-5)(c+1)}\right]$$

$$4c(c+1) - 1(c-5) = 3c^2+3$$

$$4c^2 + 4c - c + 5 = 3c^2 + 3$$

$$c^2 + 3c + 2 = 0$$

$$(c+1)(c+2) = 0$$

$$\cancel{c = -1} \text{ or } c = -2$$

$\{-2\}$; The value -1 does not check.

$$\frac{m}{2m+1} + 1 = \frac{2}{m-3}$$

$$(2m+1)(m-3)\left(\frac{m}{2m+1} + 1\right) = (2m+1)(m-3)\left(\frac{2}{m-3}\right)$$

$$m(m-3) + 1(2m+1)(m-3) = 2(2m+1)$$

$$m^2 - 3m + 2m^2 - 6m + m - 3 = 4m + 2$$

23.

$$3m^2 - 12m - 5 = 0$$

$$m = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-5)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{204}}{6}$$

$$= \frac{12 \pm 2\sqrt{51}}{6}$$

$$= \frac{6 \pm \sqrt{51}}{3}$$

$$\left\{ \frac{6 \pm \sqrt{51}}{3} \right\}$$

$$24. \quad \frac{n}{n-3} + 2 = \frac{3}{2n-1}$$

$$(n-3)(2n-1)\left(\frac{n}{n-3} + 2\right) = (n-3)(2n-1)\left(\frac{3}{2n-1}\right)$$

$$n(2n-1) + 2(n-3)(2n-1) = 3(n-3)$$

$$2n^2 - n + 2(2n^2 - 7n + 3) = 3n - 9$$

$$2n^2 - n + 4n^2 - 14n + 6 = 3n - 9$$

$$6n^2 - 18n + 15 = 0$$

$$2n^2 - 6n + 5 = 0$$

$$n = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{-4}}{4}$$

$$= \frac{6 \pm 2i}{4}$$

$$= \frac{3 \pm i}{2}$$

$$\left\{ \frac{3 \pm i}{2} \right\}$$

$$25. \quad 2 - \frac{3}{y} = \frac{5}{y^2}$$

$$y^2 \left(2 - \frac{3}{y} \right) = y^2 \left(\frac{5}{y^2} \right)$$

$$2y^2 - 3y = 5$$

$$2y^2 - 3y - 5 = 0$$

$$(2y-5)(y+1) = 0$$

$$2y-5=0 \quad \text{or} \quad y+1=0$$

$$2y=5 \quad y=-1$$

$$y = \frac{5}{2}$$

$$\left\{ \frac{5}{2}; -1 \right\}$$

$$26. \quad 7 + \frac{20}{z} = \frac{3}{z^2}$$

$$z^2 \left(7 + \frac{20}{z} \right) = z^2 \left(\frac{3}{z^2} \right)$$

$$7z^2 + 20z = 3$$

$$7z^2 + 20z - 3 = 0$$

$$(7z-1)(z+3) = 0$$

$$7z-1=0 \quad \text{or} \quad z+3=0$$

$$7z=1 \quad z=-3$$

$$z = \frac{1}{7}$$

$$\left\{ \frac{1}{7}; -3 \right\}$$

$$27. \quad \frac{18}{m^2 - 3m} + 2 = \frac{6}{m - 3}$$

$$\frac{18}{m(m-3)} + 2 = \frac{6}{m-3}$$

$$m(m-3) \left[\frac{18}{m(m-3)} + 2 \right] = m(m-3) \left(\frac{6}{m-3} \right)$$

$$18 + 2m(m-3) = 6m$$

$$18 + 2m^2 - 6m = 6m$$

$$2m^2 - 12m + 18 = 0$$

$$m^2 - 6m + 9 = 0$$

$$(m-3)^2 = 0$$

$$m-3 = 0$$

$$\cancel{m=3}$$

$\{ \}$; The value 3 does not check.

$$28. \quad \frac{48}{m^2 - 4m} + 3 = \frac{12}{m - 4}$$

$$\frac{48}{m(m-4)} + 3 = \frac{12}{m-4}$$

$$m(m-4) \left[\frac{48}{m(m-4)} + 3 \right] = m(m-4) \left(\frac{12}{m-4} \right)$$

$$48 + 3m(m-4) = 12m$$

$$48 + 3m^2 - 12m = 12m$$

$$3m^2 - 24m + 48 = 0$$

$$m^2 - 8m + 16 = 0$$

$$(m-4)^2 = 0$$

$$m-4 = 0$$

$$\cancel{m=4}$$

$\{ \}$; The value 4 does not check.

29. Let x represent the speed of the boat in still water.

	Distance	Rate	Time
With current	72	$x + 2$	$\frac{72}{x + 2}$
Against current	72	$x - 2$	$\frac{72}{x - 2}$

$$\frac{72}{x-2} - \frac{72}{x+2} = 9$$

$$(x-2)(x+2) \left(\frac{72}{x-2} - \frac{72}{x+2} \right) = (x-2)(x+2)(9)$$

$$72(x+2) - 72(x-2) = 9(x^2 - 4)$$

$$72x + 144 - 72x + 144 = 9x^2 - 36$$

$$288 = 9x^2 - 36$$

$$324 = 9x^2$$

$$36 = x^2$$

$$x = \pm\sqrt{36} = 6 \text{ or } \cancel{6}$$

Jesse travels 6 km/hr in still water.

30. Let x represent the speed of the plane in still air.

	Distance	Rate	Time
With wind	800	$x + 40$	$\frac{800}{x + 40}$

Against wind	800	$x - 40$	$\frac{800}{x - 40}$
-------------------------	-----	----------	----------------------

$$\begin{aligned}\frac{800}{x - 40} - \frac{800}{x + 40} &= 0.5 \\ (x - 40)(x + 40) \left(\frac{800}{x - 40} - \frac{800}{x + 40} \right) &= (x - 40)(x + 40)(0.5) \\ 800(x + 40) - 800(x - 40) &= 0.5(x^2 - 1600) \\ 800x + 32,000 - 800x + 32,000 &= 0.5x^2 - 800 \\ 64,000 &= 0.5x^2 - 800 \\ 64,800 &= 0.5x^2 \\ 129,600 &= x^2 \\ x &= \pm \sqrt{129,600} = 360 \text{ or } \cancel{>360}\end{aligned}$$

The plane travels 360 mph in still air.

- 31.** Let x represent the speed at which Jean runs. Then $(x + 8)$ is the speed at which she rides.

	Distance (mi)	Rate (mph)	Time (hr)
Running	6	x	$\frac{6}{x}$
Riding	24	$x + 8$	$\frac{24}{x + 8}$

$$\begin{aligned}\frac{6}{x} + \frac{24}{x + 8} &= 2.25 \\ \frac{24}{x} + \frac{96}{x + 8} &= 9 \\ x(x + 8) \left(\frac{24}{x} + \frac{96}{x + 8} \right) &= x(x + 8)(9) \\ 24(x + 8) + 96x &= 9(x^2 + 8x) \\ 24x + 192 + 96x &= 9x^2 + 72x \\ 120x + 192 &= 9x^2 + 72x \\ 0 &= 9x^2 - 48x - 192 \\ 0 &= 3x^2 - 16x - 64 \\ 0 &= (3x + 8)(x - 8) \\ 3x + 8 = 0 &\quad \text{or} \quad x - 8 = 0 \\ 3x &= -8 &\quad x &= 8 \\ x &= \cancel{-\frac{8}{3}}\end{aligned}$$

$$x + 8 = 8 + 8 = 16$$

Jean runs 8 mph and rides 16 mph.

- 32.** Let x represent the speed at which Barbara drives in clear weather. Then $(x - 20)$ is the speed at which she drives during the thunderstorm.

	Distance (mi)	Rate (mph)	Time (hr)
Clear weather	50	x	$\frac{50}{x}$
Thunderstorm	15	$x - 20$	$\frac{15}{x - 20}$

$$\frac{50}{x} + \frac{15}{x - 20} = 1.5$$

$$\frac{100}{x} + \frac{30}{x - 20} = 3$$

$$x(x - 20) \left(\frac{100}{x} + \frac{30}{x - 20} \right) = x(x - 20)(3)$$

$$100(x - 20) + 30x = 3(x^2 - 20x)$$

$$100x - 2000 + 30x = 3x^2 - 60x$$

$$130x - 2000 = 3x^2 - 60x$$

$$0 = 3x^2 - 190x + 2000$$

$$0 = (3x - 40)(x - 50)$$

$$3x - 40 = 0 \quad \text{or} \quad x - 50 = 0$$

$$3x = 40 \quad x = 50$$

$$x = \frac{40}{3} = 13\frac{1}{3}$$

$$x - 20 = 13\frac{1}{3} - 20 = -6\frac{2}{3}$$

$$x - 20 = 50 - 20 = 30$$

Barbara drives 30 mph in the thunderstorm and 50 mph in nice weather.

$$|p| = 6$$

33. a. $p = 6$ or $p = -6$

$$\{6, -6\}$$

$$|p| = 0$$

b. $p = 0$

$$\{0\}$$

c. $|p| = -6$

$\{ \}$; Since an absolute value cannot be negative, there is no solution.

$$|w| = 2$$

34. a. $w = 2$ or $w = -2$

$$\{2, -2\}$$

$$|w| = 0$$

b. $w = 0$

$$\{0\}$$

c. $|w| = -2$

$\{ \}$; Since an absolute value cannot be negative, there is no solution.

$$\begin{aligned}
 &|x-3|=4 \\
 &x-3=4 \quad \text{or} \quad x-3=-4 \\
 \mathbf{35. a.} \quad &x=7 \qquad \qquad x=-1 \\
 &\{7, -1\} \\
 &|x-3|=0 \\
 &x-3=0 \\
 \mathbf{b.} \quad &x=3 \\
 &\{3\} \\
 \mathbf{c.} \quad &|x-3|=-7 \\
 &\{\}; \text{ Since an absolute value cannot be negative, there is no solution.} \\
 &|m+1|=5 \\
 &m+1=5 \quad \text{or} \quad m+1=-5 \\
 \mathbf{36. a.} \quad &m=4 \qquad \qquad m=-6 \\
 &\{4, -6\} \\
 &|m+1|=0 \\
 &m+1=0 \\
 \mathbf{b.} \quad &m=-1 \\
 &\{-1\} \\
 \mathbf{c.} \quad &|m+1|=-1 \\
 &\{\}; \text{ Since an absolute value cannot be negative, there is no solution.} \\
 &2|3x-4|+7=9 \\
 &2|3x-4|=9-7=2 \\
 &|3x-4|=1 \\
 &3x-4=1 \quad \text{or} \quad 3x-4=-1 \\
 &x=\frac{5}{3} \qquad \qquad x=1 \\
 \mathbf{37.} \quad &\left\{\frac{5}{3}, 1\right\} \\
 &4|2t+7|+2=22 \\
 &4|2t+7|=20 \\
 &|2t+7|=5 \\
 &2t+7=5 \quad \text{or} \quad 2t+7=-5 \\
 \mathbf{38.} \quad &t=-1 \qquad \qquad t=-6 \\
 &\{-1, -6\}
 \end{aligned}$$

$$\begin{aligned}
 &-3=-|c-7|+1 \\
 &-4=-|c-7| \\
 &4=|c-7| \\
 &c-7=4 \quad \text{or} \quad c-7=-4 \\
 \mathbf{39.} \quad &c=11 \qquad \qquad c=3 \\
 &\{11, 3\} \\
 &-4=-|z+8|-3 \\
 &-1=-|z+8| \\
 &1=|z+8| \\
 &z+8=1 \quad \text{or} \quad z+8=-1 \\
 \mathbf{40.} \quad &z=-7 \qquad \qquad z=-9 \\
 &\{-7, -9\} \\
 &2=8+|11y+4| \\
 \mathbf{41.} \quad &-6=|11y+4| \\
 &\{\}; \text{ Since an absolute value cannot be negative, there is no solution.} \\
 &6=7+|9z-3| \\
 \mathbf{42.} \quad &-1=|9z-3| \\
 &\{\}; \text{ Since an absolute value cannot be negative, there is no solution.} \\
 &\left|4-\frac{1}{2}w\right|-\frac{1}{3}=\frac{1}{2} \\
 &\left|4-\frac{1}{2}w\right|=\frac{1}{2}+\frac{1}{3}=\frac{5}{6} \\
 \mathbf{43.} \quad &4-\frac{1}{2}w=\frac{5}{6} \quad \text{or} \quad 4-\frac{1}{2}w=-\frac{5}{6} \\
 &-\frac{1}{2}w=-\frac{19}{6} \qquad \qquad -\frac{1}{2}w=-\frac{29}{6} \\
 &w=\frac{19}{3} \qquad \qquad w=\frac{29}{3} \\
 &\left\{\frac{19}{3}, \frac{29}{3}\right\} \\
 &\left|2-\frac{1}{3}p\right|-\frac{7}{6}=\frac{1}{2} \\
 &\left|2-\frac{1}{3}p\right|=\frac{1}{2}+\frac{7}{6}=\frac{10}{6} \\
 \mathbf{44.} \quad &
 \end{aligned}$$

$$2 - \frac{1}{3}p = \frac{5}{3} \quad \text{or} \quad 2 - \frac{1}{3}p = -\frac{5}{3}$$

$$-\frac{1}{3}p = -\frac{1}{3} \quad -\frac{1}{3}p = -\frac{11}{3}$$

$$p = 1 \quad p = 11$$

$$\{1, 11\}$$

$$|3y + 5| = |y + 1|$$

$$3y + 5 = y + 1 \quad \text{or} \quad 3y + 5 = -(y + 1)$$

$$2y = -4$$

$$4y = -6$$

$$y = -2$$

$$y = -\frac{3}{2}$$

45.

$$\left\{-2, -\frac{3}{2}\right\}$$

$$|2a - 3| = |a + 2|$$

$$2a - 3 = a + 2 \quad \text{or} \quad 2a - 3 = -(a + 2)$$

$$a = 5$$

$$a = \frac{1}{3}$$

46.

$$\left\{5, \frac{1}{3}\right\}$$

$$|4 - x| = |2x + 1|$$

$$4 - x = 2x + 1 \quad \text{or} \quad 4 - x = -(2x + 1)$$

$$-3x = -3$$

$$x = -5$$

47. $x = 1$

$$\{-5, 1\}$$

$$|3 - 2x| = |x + 5|$$

$$3 - 2x = x + 5 \quad \text{or} \quad 3 - 2x = -(x + 5)$$

$$-3x = 2$$

$$-x = -8$$

$$x = -\frac{2}{3}$$

$$x = 8$$

48.

$$\left\{-\frac{2}{3}, 8\right\}$$

$$\left|\frac{1}{4}w\right| = |4w|$$

$$\frac{1}{4}w = 4w \quad \text{or} \quad \frac{1}{4}w = -4w$$

$$-\frac{15}{4}w = 0$$

$$-\frac{17}{4}w = 0$$

49. $w = 0$

$$w = 0$$

$$\{0\}$$

$$|3z| = \left|\frac{1}{3}z\right|$$

$$3z = \frac{1}{3}z \quad \text{or} \quad 3z = -\frac{1}{3}z$$

$$\frac{8}{3}z = 0$$

$$\frac{10}{3}z = 0$$

50. $z = 0$

$$z = 0$$

$$\{0\}$$

$$|x + 4| = |x - 7|$$

$$x + 4 = x - 7 \quad \text{or} \quad x + 4 = -(x - 7)$$

$$4 \neq -7$$

$$2x = 3$$

$$x = \frac{3}{2}$$

51.

$$\left\{\frac{3}{2}\right\}$$

$$|k - 3| = |k + 3|$$

$$-3 \neq 3 \quad \text{or} \quad k - 3 = -(k + 3)$$

$$2k = 0$$

$$k = 0$$

52.

$$\{0\}$$

$$53. \quad |2p - 1| = |1 - 2p|$$

$$2p - 1 = 1 - 2p \quad \text{or} \quad 2p - 1 = -(1 - 2p)$$

$$4p = 2$$

$$-1 = -1$$

$$p = \frac{1}{2}$$

The solution set is $\square$.

$$|4d - 3| = |3 - 4d|$$

54.

$$4d - 3 = 3 - 4d \quad \text{or} \quad 4d - 3 = -(3 - 4d)$$

$$8d = 6$$

$$-3 = -3$$

$$d = \frac{3}{4}$$

The solution set is $\square$.

$$\begin{aligned}\sqrt{2x-4} &= 6 \\ (\sqrt{2x-4})^2 &= (6)^2 \\ 2x-4 &= 36 \\ 2x &= 40 \\ x &= 20\end{aligned}$$

55. {20}

$$\begin{aligned}\sqrt{3x+1} &= 11 \\ (\sqrt{3x+1})^2 &= (11)^2 \\ 3x+1 &= 121 \\ 3x &= 120 \\ x &= 40\end{aligned}$$

56. {40}

$$\begin{aligned}1 &= 3 + \sqrt{2x+7} \\ -2 &= \sqrt{2x+7} \\ 4 &= 2x+7 \\ -3 &= 2x \\ x &= -\frac{3}{2}\end{aligned}$$

57.

$$\begin{aligned}\text{Check: } x &= -\frac{3}{2} \\ 1 &= 3 + \sqrt{2x+7} \\ 1 &= 3 + \sqrt{2\left(-\frac{3}{2}\right)+7} \\ 1 &= 3+2 \\ 1 &= 5 \text{ false} \\ \{ \} &\end{aligned}$$

$$\begin{aligned}6 &= 9 + \sqrt{5-3x} \\ -3 &= \sqrt{5-3x} \\ 9 &= 5-3x \\ 4 &= -3x \\ x &= -\frac{4}{3}\end{aligned}$$

58.

$$\text{Check: } x = -\frac{4}{3}$$

$$\begin{aligned}6 &= 9 + \sqrt{5-3x} \\ 6 &= 9 + \sqrt{5-3\left(-\frac{4}{3}\right)} \\ 6 &= 9+3 \\ 6 &= 12 \text{ false} \\ \{ \} &\end{aligned}$$

$$\begin{aligned}\sqrt{7x+8} &= x+2 \\ (\sqrt{7x+8})^2 &= (x+2)^2 \\ 7x+8 &= x^2+4x+4 \\ 0 &= x^2-3x-4 \\ 0 &= (x+1)(x-4) \\ x &= -1 \text{ or } x = 4\end{aligned}$$

59.

$$\begin{aligned}\text{Check: } x &= -1 \\ \sqrt{7x+8} &= x+2 \\ \sqrt{7(-1)+8} &= 1 \\ \sqrt{1} &= 1 \\ 1 &= 1 \checkmark \text{ true}\end{aligned}$$

$$\begin{aligned}\text{Check: } x &= 4 \\ \sqrt{7x+8} &= x+2 \\ \sqrt{7(4)+8} &= 4+2 \\ \sqrt{36} &= 6 \\ 6 &= 6 \checkmark \text{ true}\end{aligned}$$

$$\begin{aligned}\{ -1, 4 \} & \\ \sqrt{9x+19} &= x+3 \\ (\sqrt{9x+19})^2 &= (x+3)^2\end{aligned}$$

$$\begin{aligned}9x+19 &= x^2+6x+9 \\ 0 &= x^2-3x-10 \\ 0 &= (x+2)(x-5) \\ x &= -2 \text{ or } x = 5\end{aligned}$$

60.

$$\begin{aligned}\text{Check: } x &= -2 \\ \sqrt{9x+19} &= x+3 \\ \sqrt{9(-2)+19} &= -2+3 \\ \sqrt{1} &= 1 \\ 1 &= 1 \checkmark \text{ true}\end{aligned}$$

Check: $x = 5$

$$\sqrt{9x+19} = x+3$$

$$\sqrt{9(5)+19} = 5+3$$

$$\sqrt{64} = 8$$

$$8 = 8 \checkmark \text{ true}$$

$$\{-2, 5\}$$

$$\sqrt{m+18}+2 = m$$

$$\sqrt{m+18} = m-2$$

$$(\sqrt{m+18})^2 = (m-2)^2$$

$$m+18 = m^2 - 4m + 4$$

$$0 = m^2 - 5m - 14$$

$$0 = (m+2)(m-7)$$

$$\mathbf{61.} \quad m = -2 \text{ or } m = 7$$

Check: $m = -2$

$$\sqrt{m+18}+2 = m$$

$$\sqrt{(-2)+18}+2 = (-2)$$

$$\sqrt{16}+2 = -2$$

$$4+2 = -2$$

$$6 = -2 \text{ false}$$

Check: $m = 7$

$$\sqrt{m+18}+2 = m$$

$$\sqrt{(7)+18}+2 = (7)$$

$$\sqrt{25}+2 = 7$$

$$5+2 = 7$$

$$7 = 7 \checkmark \text{ true}$$

 $\{7\}$; The value -2 does not check.

$$\sqrt{2n+29}+3 = n$$

$$\sqrt{2n+29} = n-3$$

$$(\sqrt{2n+29})^2 = (n-3)^2$$

$$2n+29 = n^2 - 6n + 9$$

$$0 = n^2 - 8n - 20$$

$$0 = (n+2)(n-10)$$

$$\mathbf{62.} \quad n = -2 \text{ or } n = 10$$

Check: $n = -2$

$$\sqrt{2n+29}+3 = n$$

$$\sqrt{2(-2)+29}+3 = -2$$

$$\sqrt{25}+3 = -2$$

$$5+3 = -2$$

$$8 = -2 \text{ false}$$

Check: $n = 10$

$$\sqrt{2n+29}+3 = n$$

$$\sqrt{2(10)+29}+3 = 10$$

$$\sqrt{49}+3 = 10$$

$$7+3 = 10$$

$$10 = 10 \checkmark \text{ true}$$

 $\{10\}$; The value -2 does not check.

$$-4\sqrt[3]{2x-5}+6 = 10$$

$$-4\sqrt[3]{2x-5} = 4$$

$$\sqrt[3]{2x-5} = -1$$

$$(\sqrt[3]{2x-5})^3 = (-1)^3$$

$$2x-5 = -1$$

$$2x = 4$$

$$x = 2$$

63.

$$\{2\}$$

$$64. -3\sqrt[5]{4x-1}+2=8$$

$$-3\sqrt[5]{4x-1}=6$$

$$\sqrt[5]{4x-1}=-2$$

$$\left(\sqrt[5]{4x-1}\right)^5=(-2)^5$$

$$4x-1=-32$$

$$4x=-31$$

$$x=-\frac{31}{4}$$

$$\left\{-\frac{31}{4}\right\}$$

$$\sqrt[4]{5y-3}-\sqrt[4]{2y+1}=0$$

$$\sqrt[4]{5y-3}=-\sqrt[4]{2y+1}$$

$$\left(\sqrt[4]{5y-3}\right)^4=\left(-\sqrt[4]{2y+1}\right)^4$$

$$5y-3=2y+1$$

$$3y=4$$

$$y=\frac{4}{3}$$

65.

$$\text{Check: } y=\frac{4}{3}$$

$$\sqrt[4]{5y-3}-\sqrt[4]{2y+1}=0$$

$$\sqrt[4]{5\left(\frac{4}{3}\right)-3}-\sqrt[4]{2\left(\frac{4}{3}\right)+1}=0$$

$$\sqrt[4]{\frac{20}{3}-\frac{9}{3}}-\sqrt[4]{\frac{8}{3}+\frac{3}{3}}=0$$

$$\sqrt[4]{\frac{11}{3}}-\sqrt[4]{\frac{11}{3}}=0$$

$$0=0 \checkmark \text{ true}$$

$$\left\{\frac{4}{3}\right\}$$

$$\sqrt[6]{y+7}-\sqrt[6]{4y+5}=0$$

$$\sqrt[6]{y+7}=\sqrt[6]{4y+5}$$

$$\left(\sqrt[6]{y+7}\right)^6=\left(\sqrt[6]{4y+5}\right)^6$$

$$y+7=4y+5$$

$$2=3y$$

$$\frac{2}{3}=y$$

66.

$$\text{Check: } y=\frac{2}{3}$$

$$\sqrt[6]{y+7}-\sqrt[6]{4y+5}=0$$

$$\sqrt[6]{\left(\frac{2}{3}\right)+7}-\sqrt[6]{4\left(\frac{2}{3}\right)+5}=0$$

$$\sqrt[6]{\frac{2}{3}+\frac{21}{3}}-\sqrt[6]{\frac{8}{3}+\frac{15}{3}}=0$$

$$\sqrt[6]{\frac{23}{3}}-\sqrt[6]{\frac{23}{3}}=0$$

$$0=0 \checkmark \text{ true}$$

$$\left\{\frac{2}{3}\right\}$$

$$67. \sqrt{8-p}-\sqrt{p+5}=1$$

$$\sqrt{8-p}=1+\sqrt{p+5}$$

$$\left(\sqrt{8-p}\right)^2=\left(1+\sqrt{p+5}\right)^2$$

$$8-p=1+2\sqrt{p+5}+p+5$$

$$-2\sqrt{p+5}=2p-2$$

$$\sqrt{p+5}=-p+1$$

$$\left(\sqrt{p+5}\right)^2=\left(-p+1\right)^2$$

$$p+5=p^2-2p+1$$

$$0=p^2-3p-4$$

$$0=(p+1)(p-4)$$

$$p=-1 \quad \text{or} \quad p=4$$

$$\text{Check: } p=-1$$

$$\sqrt{8-p}-\sqrt{p+5}=1$$

$$\sqrt{8-(-1)}-\sqrt{(-1)+5}=1$$

$$\sqrt{9}-\sqrt{4}=1$$

$$3-2=1 \checkmark \text{ true}$$

$$\text{Check: } p=4$$

$$\sqrt{8-p}-\sqrt{p+5}=1$$

$$\sqrt{8-(4)}-\sqrt{(4)+5}=1$$

$$\sqrt{4}-\sqrt{9}=1$$

$$2-3=1$$

$$-1=1 \text{ false}$$

$\{-1\}$; The value 4 does not check.

$$68. \sqrt{d+4} - \sqrt{6+2d} = -1$$

$$\sqrt{d+4} = \sqrt{6+2d} - 1$$

$$(\sqrt{d+4})^2 = (\sqrt{6+2d} - 1)^2$$

$$d+4 = 6+2d - 2\sqrt{6+2d} + 1$$

$$2\sqrt{6+2d} = d+3$$

$$(2\sqrt{6+2d})^2 = (d+3)^2$$

$$4(6+2d) = d^2 + 6d + 9$$

$$24 + 8d = d^2 + 6d + 9$$

$$0 = d^2 - 2d - 15$$

$$0 = (d+3)(d-5)$$

$$d = -3 \quad \text{or} \quad d = 5$$

$$\text{Check: } d = -3$$

$$\sqrt{d+4} - \sqrt{6+2d} = -1$$

$$\sqrt{(-3)+4} - \sqrt{6+2(-3)} = -1$$

$$\sqrt{1} - \sqrt{0} = -1$$

$$1 = -1 \text{ false}$$

$$\text{Check: } d = 5$$

$$\sqrt{d+4} - \sqrt{6+2d} = -1$$

$$\sqrt{(5)+4} - \sqrt{6+2(5)} = -1$$

$$\sqrt{9} - \sqrt{16} = -1$$

$$3 - 4 = -1$$

$$-1 = -1 \checkmark \text{ true}$$

$\{5\}$; The value -3 does not check.

$$69. \quad 3 - \sqrt{y+3} = \sqrt{2-y}$$

$$(3 - \sqrt{y+3})^2 = (\sqrt{2-y})^2$$

$$9 - 6\sqrt{y+3} + y + 3 = 2 - y$$

$$-6\sqrt{y+3} = -10 - 2y$$

$$3\sqrt{y+3} = 5 + y$$

$$(3\sqrt{y+3})^2 = (5+y)^2$$

$$9y + 27 = 25 + 10y + y^2$$

$$0 = y^2 + y - 2$$

$$0 = (y+2)(y-1)$$

$$y = -2 \quad \text{or} \quad y = 1$$

$$\text{Check: } y = -2$$

$$3 - \sqrt{y+3} = \sqrt{2-y}$$

$$3 - \sqrt{(-2)+3} = \sqrt{2-(-2)}$$

$$3 - \sqrt{1} = \sqrt{4}$$

$$3 - 1 = 2$$

$$2 = 2 \checkmark \text{ true}$$

$$\text{Check: } y = 1$$

$$3 - \sqrt{y+3} = \sqrt{2-y}$$

$$3 - \sqrt{(1)+3} = \sqrt{2-(1)}$$

$$3 - \sqrt{4} = \sqrt{1}$$

$$3 - 2 = 1$$

$$1 = 1 \checkmark \text{ true}$$

$$\{-2, 1\}$$

$$70. \quad \sqrt{k-2} = \sqrt{2k+3} - 2$$

$$(\sqrt{k-2})^2 = (\sqrt{2k+3} - 2)^2$$

$$k - 2 = 2k + 3 - 4\sqrt{2k+3} + 4$$

$$4\sqrt{2k+3} = k + 9$$

$$(4\sqrt{2k+3})^2 = (k+9)^2$$

$$16(2k+3) = k^2 + 18k + 81$$

$$32k + 48 = k^2 + 18k + 81$$

$$0 = k^2 - 14k + 33$$

$$0 = (k-3)(k-11)$$

$$k = 3 \quad \text{or} \quad k = 11$$

Check: $k = 3$

$$\sqrt{k-2} = \sqrt{2k+3}-2$$

$$\sqrt{(3)-2} = \sqrt{2(3)+3}-2$$

$$\sqrt{1} = \sqrt{9}-2$$

$$1 = 3-2$$

$$1 = 1 \checkmark \text{ true}$$

Check: $k = 11$

$$\sqrt{k-2} = \sqrt{2k+3}-2$$

$$\sqrt{(11)-2} = \sqrt{2(11)+3}-2$$

$$\sqrt{9} = \sqrt{25}-2$$

$$3 = 5-2$$

$$3 = 3 \checkmark \text{ true}$$

$$\{3, 11\}$$

$$\mathbf{71.} \quad 2(x+5)^{\frac{2}{3}} = 18$$

$$2\sqrt[3]{(x+5)^2} = 18$$

$$\sqrt[3]{(x+5)^2} = 9$$

$$\left[\sqrt[3]{(x+5)^2}\right]^3 = (9)^3$$

$$(x+5)^2 = 729$$

$$x+5 = \pm\sqrt{729}$$

$$x+5 = \pm 27$$

$$x = -5+27 \quad \text{or} \quad x = -5-27$$

$$x = 22$$

$$x = -32$$

Check: $x = 22$

$$2\sqrt[3]{(x+5)^2} = 18$$

$$2\sqrt[3]{(22+5)^2} = 18$$

$$2\sqrt[3]{729} = 18$$

$$18 = 18 \checkmark \text{ true}$$

Check: $x = -32$

$$2\sqrt[3]{(x+5)^2} = 18$$

$$2\sqrt[3]{(-32+5)^2} = 18$$

$$2\sqrt[3]{729} = 18$$

$$18 = 18 \checkmark \text{ true}$$

$$\{-32, 22\}$$

$$\mathbf{72.} \quad 3(x-6)^{\frac{2}{3}} = 48$$

$$3\sqrt[3]{(x-6)^2} = 48$$

$$\sqrt[3]{(x-6)^2} = 16$$

$$\left[\sqrt[3]{(x-6)^2}\right]^3 = (16)^3$$

$$(x-6)^2 = 4096$$

$$x-6 = \pm\sqrt{4096}$$

$$x-6 = \pm 64$$

$$x = 6+64 \quad \text{or} \quad x = 6-64$$

$$x = 70$$

$$x = -58$$

Check: $x = -58$

$$3\sqrt[3]{(x-6)^2} = 48$$

$$3\sqrt[3]{(-58-6)^2} = 48$$

$$3\sqrt[3]{4096} = 48$$

$$48 = 48 \checkmark \text{ true}$$

Check: $x = 70$

$$3\sqrt[3]{(x-6)^2} = 48$$

$$3\sqrt[3]{(70-6)^2} = 48$$

$$3\sqrt[3]{4096} = 48$$

$$48 = 48 \checkmark \text{ true}$$

$$\{-58, 70\}$$

$$\mathbf{73.} \quad (3x+1)^{\frac{3}{2}} + 2 = 66$$

$$\sqrt{(3x+1)^3} + 2 = 66$$

$$\sqrt{(3x+1)^3} = 64$$

$$\left[\sqrt{(3x+1)^3}\right]^2 = (64)^2$$

$$(3x+1)^3 = 4096$$

$$3x+1 = \sqrt[3]{4096}$$

$$3x+1 = 16$$

$$x = 5$$

Check: $x = 5$

$$\begin{aligned}\sqrt{(3x+1)^3} + 2 &= 66 \\ \sqrt{(15+1)^3} + 2 &= 66 \\ 64 + 2 &= 66 \\ 66 &= 66 \checkmark \text{ true} \\ \{5\}\end{aligned}$$

$$\begin{aligned}74. \quad (2x-1)^{\frac{3}{2}} - 3 &= 122 \\ \sqrt{(2x-1)^3} - 3 &= 122 \\ \sqrt{(2x-1)^3} &= 125 \\ \left[\sqrt{(2x-1)^3}\right]^2 &= (125)^2 \\ (2x-1)^3 &= 15625 \\ 2x-1 &= \sqrt[3]{15625} \\ 2x-1 &= 25 \\ x &= 13\end{aligned}$$

$$\begin{aligned}\text{Check: } x &= 13 \\ \sqrt{(2x-1)^3} - 3 &= 122 \\ \sqrt{(25)^3} - 3 &= 122 \\ 125 - 3 &= 122 \\ 122 &= 122 \checkmark \text{ true} \\ \{122\}\end{aligned}$$

$$\begin{aligned}m^{\frac{3}{4}} &= 5 \\ (m^{\frac{3}{4}})^{\frac{4}{3}} &= (5)^{\frac{4}{3}} \\ 75. \quad m &= 5^{\frac{4}{3}} \\ \{5^{\frac{4}{3}}\} \text{ or } \{5\sqrt[3]{5}\} \\ n^{\frac{5}{6}} &= 3\end{aligned}$$

$$\begin{aligned}(n^{\frac{5}{6}})^{\frac{6}{5}} &= (3)^{\frac{6}{5}} \\ 76. \quad n &= 3^{\frac{6}{5}} \\ \{3^{\frac{6}{5}}\} \text{ or } \{3\sqrt[5]{3}\}\end{aligned}$$

$$\begin{aligned}77. \quad 2p^{\frac{4}{5}} &= \frac{1}{8} \\ p^{\frac{4}{5}} &= \frac{1}{16} \\ (p^{\frac{4}{5}})^{\frac{5}{4}} &= \pm \left(\frac{1}{16}\right)^{\frac{5}{4}} \\ p &= \pm \left(\sqrt[4]{\frac{1}{16}}\right)^5 \\ &= \pm \left(\frac{1}{2}\right)^5 = \pm \frac{1}{32} \\ \left\{\pm \frac{1}{32}\right\}\end{aligned}$$

$$\begin{aligned}78. \quad 5t^{\frac{2}{3}} &= \frac{1}{5} \\ t^{\frac{2}{3}} &= \frac{1}{25} \\ (t^{\frac{2}{3}})^{\frac{3}{2}} &= \pm \left(\frac{1}{25}\right)^{\frac{3}{2}} \\ t &= \pm \left(\sqrt{\frac{1}{25}}\right)^3 \\ &= \pm \left(\frac{1}{5}\right)^3 = \pm \frac{1}{125} \\ \left\{\pm \frac{1}{125}\right\}\end{aligned}$$

$$\begin{aligned}(2v+7)^{\frac{1}{3}} - (v-3)^{\frac{1}{3}} &= 0 \\ (2v+7)^{\frac{1}{3}} &= (v-3)^{\frac{1}{3}} \\ \left[(2v+7)^{\frac{1}{3}}\right]^3 &= \left[(v-3)^{\frac{1}{3}}\right]^3 \\ 2v+7 &= v-3 \\ v &= -10\end{aligned}$$

$$\begin{aligned}79. \quad \{-10\}\end{aligned}$$

$$\begin{aligned}
 (5u-6)^{1/5} - (3u+1)^{1/5} &= 0 \\
 (5u-6)^{1/5} &= (3u+1)^{1/5} \\
 \left[(5u-6)^{1/5}\right]^5 &= \left[(3u+1)^{1/5}\right]^5 \\
 5u-6 &= 3u+1 \\
 2u &= 7 \\
 u &= \frac{7}{2}
 \end{aligned}$$

80.

$$\left\{\frac{7}{2}\right\}$$

81. Let $u = 2x + 5$.

$$\begin{aligned}
 (2x+5)^2 - 7(2x+5) - 30 &= 0 \\
 u^2 - 7u - 30 &= 0 \\
 (u+3)(u-10) &= 0 \\
 u &= 10 \\
 u = -3 \quad 2x+5 &= 10 \\
 2x+5 &= -3 \\
 x &= -4 \quad \text{or} \quad x = \frac{5}{2}
 \end{aligned}$$

$$\left\{-4, \frac{5}{2}\right\}$$

82. Let $u = 3x - 7$.

$$\begin{aligned}
 (3x-7)^2 - 6(3x-7) - 16 &= 0 \\
 u^2 - 6u - 16 &= 0 \\
 (u+2)(u-8) &= 0 \\
 u &= -2 \\
 3x-7 &= -2 \quad u = 8 \\
 x &= \frac{5}{3} \quad 3x-7 = 8 \\
 &\quad \text{or} \quad x = 5
 \end{aligned}$$

$$\left\{\frac{5}{3}, 5\right\}$$

83. Let $u = x^2 + 2x$.

$$\begin{aligned}
 (x^2+2x)^2 - 18(x^2+2x) &= -45 \\
 u^2 - 18u + 45 &= 0 \\
 (u-15)(u-3) &= 0
 \end{aligned}$$

$$\begin{aligned}
 u = 15 \quad \text{or} \quad u = 3 \\
 x^2 + 2x = 15 \quad \text{or} \quad x^2 + 2x = 3 \\
 x^2 + 2x - 15 = 0 \quad \text{or} \quad x^2 + 2x - 3 = 0 \\
 (x-3)(x+5) = 0 \quad \text{or} \quad (x+3)(x-1) = 0 \\
 x = 3, -5 \quad \text{or} \quad x = -3, 1 \\
 \{-5, -3, 1, 3\}
 \end{aligned}$$

84. Let $u = x^2 + 3x$.

$$\begin{aligned}
 (x^2+3x)^2 - 14(x^2+3x) &= -40 \\
 u^2 - 14u + 40 &= 0 \\
 (u-10)(u-4) &= 0 \\
 u = 10 \quad \text{or} \quad u = 4 \\
 x^2 + 3x = 10 \quad \text{or} \quad x^2 + 3x = 4 \\
 x^2 + 3x - 10 = 0 \quad \text{or} \quad x^2 + 3x - 4 = 0 \\
 (x-2)(x+5) = 0 \quad \text{or} \quad (x+4)(x-1) = 0 \\
 x = 2, -5 \quad \text{or} \quad x = -4, 1 \\
 \{-5, -4, 1, 2\}
 \end{aligned}$$

Let $u = x^2 + 2$.

$$\begin{aligned}
 (x^2+2)^2 + (x^2+2) - 42 &= 0 \\
 u^2 + u - 42 &= 0
 \end{aligned}$$

$$(u+7)(u-6) = 0$$

85.

$$\begin{aligned}
 u = -7 \quad \text{or} \quad u = 6 \\
 x^2 + 2 = -7 \quad \text{or} \quad x^2 + 2 = 6 \\
 x^2 = -9 \quad \text{or} \quad x^2 = 4 \\
 x = \pm 3i \quad \text{or} \quad x = \pm 2 \\
 \{\pm 3i, \pm 2\}
 \end{aligned}$$

86. Let $u = y^2 - 3$.

$$\begin{aligned}
 (y^2-3)^2 - 9(y^2-3) - 52 &= 0 \\
 u^2 - 9u - 52 &= 0 \\
 (u+4)(u-13) &= 0 \\
 u = -4 \quad \text{or} \quad u = 13 \\
 y^2 - 3 = -4 \quad \text{or} \quad y^2 - 3 = 13 \\
 y^2 = -1 \quad \text{or} \quad y^2 = 16 \\
 y = \pm i \quad \text{or} \quad y = \pm 4 \\
 \{\pm i, \pm 4\}
 \end{aligned}$$

87. Let $u = \frac{1}{a}$.

$$\left(-\frac{2}{a^2}\right) + \left(\frac{4}{a}\right) + 1 = 0$$

$$-2\left(\frac{1}{a}\right)^2 + 4\left(\frac{1}{a}\right) + 1 = 0$$

$$-2u^2 + 4u + 1 = 0$$

$$2u^2 - 4u - 1 = 0$$

$$u = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 2 \cdot (-1)}}{4}$$

$$u = 1 \pm \frac{\sqrt{6}}{2}$$

$$u = 1 + \frac{\sqrt{6}}{2} \text{ or } u = 1 - \frac{\sqrt{6}}{2}$$

$$u = 1 + \frac{\sqrt{6}}{2} \text{ or } u = 1 - \frac{\sqrt{6}}{2}$$

$$\frac{1}{a} = 1 + \frac{\sqrt{6}}{2} \text{ or } \frac{1}{a} = 1 - \frac{\sqrt{6}}{2}$$

$$a = \frac{1}{1 + \frac{\sqrt{6}}{2}} \text{ or } a = \frac{1}{1 - \frac{\sqrt{6}}{2}}$$

$$a = \frac{2}{2 + \sqrt{6}} \text{ or } a = \frac{2}{2 - \sqrt{6}}$$

$$a = -2 + \sqrt{6} \text{ or } a = -2 - \sqrt{6}$$

$$\{-2 \pm \sqrt{6}\}$$

88. Let $u = \frac{2}{x}$.

$$\left(-\frac{4}{x^2}\right) - \left(\frac{4}{x}\right) + 1 = 0$$

$$-\left(\frac{2}{x}\right)^2 - 2\left(\frac{2}{x}\right) + 1 = 0$$

$$-u^2 - 2u + 1 = 0$$

$$u^2 + 2u - 1 = 0$$

$$u = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-1)}}{2}$$

$$u = -1 \pm \sqrt{2}$$

$$u = -1 + \sqrt{2} \text{ or } u = -1 - \sqrt{2}$$

$$u = -1 + \sqrt{2} \text{ or } u = -1 - \sqrt{2}$$

$$\frac{2}{a} = -1 + \sqrt{2} \text{ or } \frac{2}{a} = -1 - \sqrt{2}$$

$$a = \frac{2}{-1 + \sqrt{2}} \text{ or } a = \frac{2}{-1 - \sqrt{2}}$$

$$a = 2 + 2\sqrt{2} \text{ or } a = 2 - 2\sqrt{2}$$

$$\{2 \pm 2\sqrt{2}\}$$

89. Let $u = \frac{1}{n+2}$.

$$\frac{2}{(n+2)^2} - \frac{3}{n+2} = 5$$

$$2\left(\frac{1}{n+2}\right)^2 - 3\left(\frac{1}{n+2}\right) - 5 = 0$$

$$2u^2 - 3u - 5 = 0$$

$$(u+1)(2u-5) = 0$$

$$u = -1 \text{ or } u = \frac{5}{2}$$

$$\frac{1}{n+2} = -1 \text{ or } \frac{1}{n+2} = \frac{5}{2}$$

$$n+2 = -1 \text{ or } n+2 = \frac{2}{5}$$

$$n = -3 \text{ or } n = -\frac{8}{5}$$

$$\left\{-3, -\frac{8}{5}\right\}$$

90. Let $u = \frac{1}{m-3}$.

$$\frac{3}{(m-3)^2} - \frac{7}{m-3} = -4$$

$$3\left(\frac{1}{m-3}\right)^2 - 7\left(\frac{1}{m-3}\right) + 4 = 0$$

$$3u^2 - 7u + 4 = 0$$

$$(u-1)(3u-4) = 0$$

$$\begin{aligned}
 u = 1 & \quad \text{or} \quad u = \frac{4}{3} \\
 \frac{1}{m-3} = 1 & \quad \text{or} \quad \frac{1}{m-3} = \frac{4}{3} \\
 m-3 = 1 & \quad \text{or} \quad m-3 = \frac{3}{4} \\
 m = 4 & \quad \text{or} \quad m = \frac{15}{4} \\
 \left\{ \frac{15}{4}, 4 \right\}
 \end{aligned}$$

91. Let $u = m - \frac{10}{m}$.

$$\begin{aligned}
 \left(m - \frac{10}{m}\right)^2 - 6\left(m - \frac{10}{m}\right) - 27 &= 0 \\
 u^2 - 6u - 27 &= 0 \\
 (u-9)(u+3) &= 0 \\
 u = 9 &\text{ or } u = -3 \\
 u = 9 & \\
 m - \frac{10}{m} &= 9 \\
 m^2 - 9m - 10 &= 0 \\
 (m+1)(m-10) &= 0 \\
 m = -1, 10 & \\
 u = -3 & \\
 m - \frac{10}{m} &= -3 \\
 m^2 + 3m - 10 &= 0 \\
 (m-5)(m+2) &= 0 \\
 m = 5, -2 & \\
 \{-5, -1, 2, 10\} &
 \end{aligned}$$

92. Let $u = x + \frac{6}{x}$.

$$\begin{aligned}
 \left(x + \frac{6}{x}\right)^2 - 12\left(x + \frac{6}{x}\right) + 35 &= 0 \\
 u^2 - 12u + 35 &= 0 \\
 (u-5)(u-7) &= 0 \\
 u = 5 &\text{ or } u = 7 \\
 u = 5 & \\
 x + \frac{6}{x} &= 5 \\
 x^2 - 5x + 6 &= 0 \\
 (x-2)(x-3) &= 0 \\
 x = 2, 3 & \\
 u = 7 & \\
 x + \frac{6}{x} &= 7 \\
 x^2 - 7x + 6 &= 0 \\
 (x-1)(x-6) &= 0 \\
 x = 1, 6 & \\
 \{1, 2, 3, 6\} &
 \end{aligned}$$

93. Let $u = 2 + \frac{3}{t}$.

$$\begin{aligned}
 \left(2 + \frac{3}{t}\right)^2 - \left(2 + \frac{3}{t}\right) &= 12 \\
 \left(2 + \frac{3}{t}\right)^2 - \left(2 + \frac{3}{t}\right) - 12 &= 0 \\
 u^2 - u - 2 &= 0 \\
 (u+3)(u-4) &= 0 \\
 u = -3 &\text{ or } u = 4 \\
 2 + \frac{3}{t} = -3 &\text{ or } 2 + \frac{3}{t} = 4 \\
 \frac{3}{t} = -5 &\text{ or } \frac{3}{t} = 2 \\
 t = -\frac{3}{5} &\text{ or } t = \frac{3}{2} \\
 \left\{ -\frac{3}{5}, \frac{3}{2} \right\} &
 \end{aligned}$$

94. Let $u = \frac{5}{y} + 3$.

$$\left(\frac{5}{y}+3\right)^2+6\left(\frac{5}{y}+3\right)=-8$$

$$\left(\frac{5}{y}+3\right)^2+6\left(\frac{5}{y}+3\right)+8=0$$

$$u^2+6u+8=0$$

$$(u+2)(u+4)=0$$

$$u=-2 \quad \text{or} \quad u=-4$$

$$\frac{5}{y}+3=-2 \quad \text{or} \quad \frac{5}{y}+3=-4$$

$$\frac{5}{y}=-5 \quad \text{or} \quad \frac{5}{y}=-7$$

$$y=-1 \quad \text{or} \quad y=-\frac{5}{7}$$

$$\left\{-1, -\frac{5}{7}\right\}$$

95. Let $u=c^{1/5}$.

$$5c^{2/5}-11c^{1/5}+2=0$$

$$5u^2-11u+2=0$$

$$(5u-1)(u-2)=0$$

$$u=\frac{1}{5} \quad \text{or} \quad u=2$$

$$c^{1/5}=\frac{1}{5} \quad \text{or} \quad c^{1/5}=2$$

$$\left(c^{1/5}\right)^5=\left(\frac{1}{5}\right)^5 \quad \text{or} \quad \left(c^{1/5}\right)^5=(2)^5$$

$$c=\frac{1}{3125} \quad \text{or} \quad c=32$$

$$\left\{\frac{1}{3125}, 32\right\}$$

96. Let $u=d^{1/3}$.

$$3d^{2/3}-d^{1/3}-4=0$$

$$3u^2-u-4=0$$

$$(3u-4)(u+1)=0$$

$$u=\frac{4}{3} \quad \text{or} \quad u=-1$$

$$d^{1/3}=\frac{4}{3} \quad \text{or} \quad d^{1/3}=-1$$

$$\left(d^{1/3}\right)^3=\left(\frac{4}{3}\right)^3 \quad \text{or} \quad \left(d^{1/3}\right)^3=(-1)^3$$

$$d=\frac{64}{27} \quad \text{or} \quad d=-1$$

$$\left\{\frac{64}{27}, -1\right\}$$

97. Let $u=y^{1/4}$.

$$y^{1/2}-y^{1/4}-6=0$$

$$u^2-u-6=0$$

$$(u-3)(u+2)=0$$

$$u=3 \quad \text{or} \quad u=-2$$

$$y^{1/4}=3 \quad \text{or} \quad y^{1/4}=-2$$

$$\left(y^{1/4}\right)^4=(3)^4$$

$$y=81$$

$$\{81\}$$

98. Let $u=n^{1/4}$.

$$n^{1/2}+6n^{1/4}-16=0$$

$$u^2+6u-16=0$$

$$(u+8)(u-2)=0$$

$$u=2 \quad \text{or} \quad u=-8$$

$$n^{1/4}=2 \quad \text{or} \quad n^{1/4}=-8$$

$$\left(n^{1/4}\right)^4=(2)^4$$

$$n=16$$

$$\{16\}$$

99. Let $u=y^{-2}$.

$$9y^{-4} - 10y^{-2} + 1 = 0$$

$$9u^2 - 10u + 1 = 0$$

$$(9u-1)(u-1) = 0$$

$$u = 1 \quad \text{or} \quad u = \frac{1}{9}$$

$$y^{-2} = 1 \quad \text{or} \quad y^{-2} = \frac{1}{9}$$

$$y^2 = 1 \quad y^2 = 9$$

$$y = 1, -1 \quad y = -3, 3$$

$$\{-3, -1, 1, 3\}$$

100. Let $u = x^{-2}$.

$$100x^{-4} - 29x^{-2} + 1 = 0$$

$$100u^2 - 29u + 1 = 0$$

$$(25u-1)(4u-1) = 0$$

$$u = \frac{1}{4} \quad \text{or} \quad u = \frac{1}{25}$$

$$x^{-2} = \frac{1}{4} \quad \text{or} \quad x^{-2} = \frac{1}{25}$$

$$x^2 = 4 \quad x^2 = 25$$

$$x = -2, 2 \quad x = -5, 5$$

$$\{-5, -2, 2, 5\}$$

Let $u = \sqrt{t}$.

$$4t - 25\sqrt{t} = 0$$

$$4u^2 - 25u = 0$$

101. $u(4u-25) = 0$

$$u = \frac{25}{4}$$

$$\sqrt{t} = \frac{25}{4}$$

$$u = 0 \quad \left(\sqrt{t}\right)^2 = \left(\frac{25}{4}\right)^2$$

$$\sqrt{t} = 0 \quad t = \frac{625}{16}$$

$$t = 0 \quad \text{or}$$

$$\left\{0, \frac{625}{16}\right\}$$

102. Let $u = \sqrt{m}$.

$$9m - 16\sqrt{m} = 0$$

$$9u^2 - 16u = 0$$

$$u(9u-16) = 0$$

$$u = \frac{16}{9}$$

$$\sqrt{m} = \frac{16}{9}$$

$$\left(\sqrt{m}\right)^2 = \left(\frac{16}{9}\right)^2$$

$$u = 0$$

$$\sqrt{m} = 0$$

$$m = 0 \quad \text{or}$$

$$m = \frac{256}{81}$$

$$\left\{0, \frac{256}{81}\right\}$$

$$x^2(x^2+5) = 7$$

103. $x^4 + 5x^2 - 7 = 0$

Let $u = x^2$.

$$u^2 + 5u - 7 = 0$$

$$u = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{53}}{2}$$

$$x^2 = \frac{-5 \pm \sqrt{53}}{2}$$

$$x = \pm \sqrt{\frac{-5 \pm \sqrt{53}}{2}}$$

$$\left\{\pm \sqrt{\frac{-5 \pm \sqrt{53}}{2}}\right\}$$

$$x^2(x^2-2) = x^2+13$$

$$x^4 - 2x^2 = x^2 + 13$$

104. $x^4 - 3x^2 - 13 = 0$

Let $u = x^2$.

$$u^2 - 3u - 13 = 0$$

$$\begin{aligned}
 u &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-13)}}{2(1)} \\
 &= \frac{3 \pm \sqrt{61}}{2} \\
 x^2 &= \frac{3 \pm \sqrt{61}}{2} \\
 x &= \pm \sqrt{\frac{3 \pm \sqrt{61}}{2}} \\
 &\left\{ \pm \sqrt{\frac{3 \pm \sqrt{61}}{2}} \right\}
 \end{aligned}$$

105. Let $u = k^{-1}$.

$$30k^{-2} - 23k^{-1} + 2 = 0$$

$$30u^2 - 23u + 2 = 0$$

$$(10u-1)(3u-2) = 0$$

$$u = \frac{1}{10} \quad \text{or} \quad u = \frac{2}{3}$$

$$k^{-1} = \frac{1}{10} \quad \text{or} \quad k^{-1} = \frac{2}{3}$$

$$k = 10 \quad \text{or} \quad k = \frac{3}{2}$$

$$\left\{ \frac{3}{2}, 10 \right\}$$

106. Let $u = q^{-1}$.

$$3q^{-2} + 16q^{-1} + 5 = 0$$

$$3u^2 + 16u + 5 = 0$$

$$(3u+1)(u+5) = 0$$

$$u = -\frac{1}{3} \quad \text{or} \quad u = -5$$

$$q^{-1} = -\frac{1}{3} \quad \text{or} \quad q^{-1} = -5$$

$$q = -3 \quad \text{or} \quad q = -\frac{1}{5}$$

$$\left\{ -\frac{1}{5}, -3 \right\}$$

107. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$$fpq\left(\frac{1}{f}\right) = fpq\left(\frac{1}{p} + \frac{1}{q}\right)$$

$$pq = fq + fp$$

$$pq - fp = fq$$

$$p(q-f) = fq$$

$$p = \frac{fq}{q-f}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

108.

$$RR_1R_2R_3\left(\frac{1}{R}\right) = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$R_1R_2R_3 = RR_2R_3 + RR_1R_3 + RR_1R_2$$

$$R_1R_2R_3 - RR_2R_3 - RR_1R_3 = RR_1R_2$$

$$R_3(R_1R_2 - RR_2 - RR_1) = RR_1R_2$$

$$R_3 = \frac{RR_1R_2}{R_1R_2 - RR_2 - RR_1}$$

$$E = kT^4$$

$$\frac{E}{k} = T^4$$

$$\sqrt[4]{\frac{E}{k}} = \sqrt[4]{T^4}$$

$$T = \sqrt[4]{\frac{E}{k}}$$

109.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{3V}{4\pi} = r^3$$

$$\sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{r^3}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

110.

$$\begin{aligned}
 a &= \frac{kF}{m} \\
 m(a) &= m\left(\frac{kF}{m}\right) \\
 ma &= kF \\
 m &= \frac{kF}{a} \\
 \mathbf{111.} \quad V &= \frac{k}{P} \\
 P(V) &= P\left(\frac{k}{P}\right) \\
 PV &= k \\
 P &= \frac{k}{V} \\
 \mathbf{112.} \quad 16 + \sqrt{x^2 - y^2} &= z \\
 \mathbf{113.} \quad \sqrt{x^2 - y^2} &= z - 16 \\
 \left(\sqrt{x^2 - y^2}\right)^2 &= (z - 16)^2 \\
 x^2 - y^2 &= (z - 16)^2 \\
 x^2 &= (z - 16)^2 + y^2 \\
 x &= \pm \sqrt{(z - 16)^2 + y^2} \\
 \mathbf{114.} \quad 4 + \sqrt{x^2 + y^2} &= z \\
 \sqrt{x^2 + y^2} &= z - 4 \\
 \left(\sqrt{x^2 + y^2}\right)^2 &= (z - 4)^2 \\
 x^2 + y^2 &= (z - 4)^2 \\
 y^2 &= (z - 4)^2 - x^2 \\
 y &= \pm \sqrt{(z - 4)^2 - x^2} \\
 \frac{P_1 V_1}{T_1} &= \frac{P_2 V_2}{T_2} \\
 T_1 T_2 \left(\frac{P_1 V_1}{T_1}\right) &= T_1 T_2 \left(\frac{P_2 V_2}{T_2}\right) \\
 T_2 P_1 V_1 &= T_1 P_2 V_2 \\
 \frac{P_1 V_1 T_2}{P_2 V_2} &= T_1 \\
 \mathbf{115.}
 \end{aligned}$$

$$\begin{aligned}
 \frac{t_1}{S_1 V_1} &= \frac{t_2}{S_2 V_2} \\
 S_1 V_1 S_2 V_2 \left(\frac{t_1}{S_1 V_1}\right) &= S_1 V_1 S_2 V_2 \left(\frac{t_2}{S_2 V_2}\right) \\
 t_1 S_2 V_2 &= t_2 S_1 V_1 \\
 V_2 &= \frac{t_2 S_1 V_1}{t_1 S_2} \\
 \mathbf{116.} \quad T &= 2\pi \sqrt{\frac{L}{g}} \\
 \mathbf{117.} \quad (T)^2 &= \left(2\pi \sqrt{\frac{L}{g}}\right)^2 \\
 T^2 &= \frac{4\pi^2 L}{g} \\
 g(T^2) &= g\left(\frac{4\pi^2 L}{g}\right) \\
 gT^2 &= 4\pi^2 L \\
 g &= \frac{4\pi^2 L}{T^2} \\
 t &= \sqrt{\frac{2s}{g}} \\
 \mathbf{118.} \quad (t)^2 &= \left(\sqrt{\frac{2s}{g}}\right)^2 \\
 t^2 &= \frac{2s}{g} \\
 t^2 \left(\frac{g}{2}\right) &= \left(\frac{2s}{g}\right) \left(\frac{g}{2}\right) \\
 \frac{t^2 g}{2} &= s \\
 y + 4\sqrt{y} &= 21 \\
 4\sqrt{y} &= 21 - y \\
 \left(4\sqrt{y}\right)^2 &= (21 - y)^2 \\
 16y &= 441 - 42y + y^2 \\
 0 &= 441 - 58y + y^2 \\
 0 &= (y - 9)(y - 49) \\
 y &= 9 \quad \text{or} \quad y = 49 \\
 \mathbf{119. a.} \quad \text{Check: } y &= 9
 \end{aligned}$$

$$\begin{aligned}
 y + 4\sqrt{y} &= 21 \\
 (9) + 4\sqrt{(9)} &= 21 \\
 9 + 12 &= 21 \\
 21 &= 21 \checkmark \text{ true}
 \end{aligned}$$

Check: $y = 49$

$$\begin{aligned}
 y + 4\sqrt{y} &= 21 \\
 (49) + 4\sqrt{(49)} &= 21 \\
 49 + 28 &= 21 \\
 77 &= 21 \text{ false}
 \end{aligned}$$

$\{9\}$

b. Let $u = \sqrt{y}$.

$$\begin{aligned}
 y + 4\sqrt{y} &= 21 \\
 y + 4\sqrt{y} - 21 &= 0 \\
 u^2 + 4u - 21 &= 0 \\
 (u + 7)(u - 3) &= 0 \\
 u = -7 \quad \text{or} \quad u = 3 \\
 \sqrt{y} = -7 \quad \text{or} \quad \sqrt{y} = 3 \\
 (\sqrt{y})^2 = (-7)^2 \quad \text{or} \quad (\sqrt{y})^2 = (3)^2 \\
 y = 49 \quad \text{or} \quad y = 9 \\
 \{9\} &; \text{ See checks in part (a).}
 \end{aligned}$$

120. a.

$$w = 4 \quad \text{or} \quad w = 25$$

Check: $w = 4$

$$\begin{aligned}
 w - 3\sqrt{w} &= 10 \\
 (4) - 3\sqrt{(4)} &= 10 \\
 4 - 6 &= 10 \\
 -2 &= 10 \text{ false}
 \end{aligned}$$

Check: $w = 25$

$$\begin{aligned}
 w - 3\sqrt{w} &= 10 \\
 (25) - 3\sqrt{(25)} &= 10 \\
 25 - 15 &= 10 \\
 10 &= 10 \checkmark \text{ true}
 \end{aligned}$$

$\{25\}$

b. Let $u = \sqrt{w}$.

$$\begin{aligned}
 w - 3\sqrt{w} &= 10 \\
 w - 3\sqrt{w} - 10 &= 0 \\
 u^2 - 3u - 10 &= 0 \\
 (u + 2)(u - 5) &= 0 \\
 u = -2 \quad \text{or} \quad u = 5 \\
 \sqrt{w} = -2 \quad \text{or} \quad \sqrt{w} = 5 \\
 (\sqrt{w})^2 = (-2)^2 \quad \text{or} \quad (\sqrt{w})^2 = (5)^2 \\
 w = 4 \quad \text{or} \quad w = 25 \\
 \{25\} &; \text{ See checks in part (a).}
 \end{aligned}$$

121.

$$\begin{aligned}
 \sqrt{x + \sqrt{x + 2}} &= 3 \\
 (\sqrt{x + \sqrt{x + 2}})^2 &= (3)^2 \\
 x + \sqrt{x + 2} &= 9 \\
 \sqrt{x + 2} &= 9 - x \\
 (\sqrt{x + 2})^2 &= (9 - x)^2 \\
 x + 2 &= x^2 - 18x + 81 \\
 x^2 - 19x + 79 &= 0 \\
 x &= \frac{19 \pm \sqrt{(-19)^2 - 4 \cdot 1 \cdot 79}}{2} \\
 x &= \frac{19 \pm 3\sqrt{5}}{2} \\
 x &= \frac{19 - 3\sqrt{5}}{2} \quad \text{or} \quad x = \frac{19 + 3\sqrt{5}}{2} \\
 x &= \frac{19 - 3\sqrt{5}}{2} \\
 \text{Check:} &
 \end{aligned}$$

$$\begin{aligned} \sqrt{x + \sqrt{x+2}} &= 3 \\ \sqrt{\frac{19-3\sqrt{5}}{2}} + \sqrt{\frac{19-3\sqrt{5}}{2} + 2} &= 3 \\ \sqrt{\frac{19-3\sqrt{5}}{2}} + \sqrt{\frac{23-3\sqrt{5}}{2}} &= 3 \\ \left(\sqrt{\frac{19-3\sqrt{5}}{2}} + \sqrt{\frac{23-3\sqrt{5}}{2}} \right)^2 &= (3)^2 \\ \sqrt{\frac{23-3\sqrt{5}}{2}} &= 9 - \left(\frac{19-3\sqrt{5}}{2} \right) \\ \sqrt{\frac{23-3\sqrt{5}}{2}} &= \frac{-1+3\sqrt{5}}{2} \\ \frac{23-3\sqrt{5}}{2} &= \frac{46-6\sqrt{5}}{4} \\ \frac{23-3\sqrt{5}}{2} &= \frac{23-3\sqrt{5}}{2} \quad \checkmark \text{ true} \\ \text{Check: } x &= \frac{19+3\sqrt{5}}{2} \end{aligned}$$

$$\begin{aligned} \sqrt{x + \sqrt{x+2}} &= 3 \\ \sqrt{\frac{19+3\sqrt{5}}{2}} + \sqrt{\frac{19+3\sqrt{5}}{2} + 2} &= 3 \\ \sqrt{\frac{19+3\sqrt{5}}{2}} + \sqrt{\frac{23+3\sqrt{5}}{2}} &= 3 \\ \left(\sqrt{\frac{19+3\sqrt{5}}{2}} + \sqrt{\frac{23+3\sqrt{5}}{2}} \right)^2 &= (3)^2 \\ \sqrt{\frac{23+3\sqrt{5}}{2}} &= 9 - \left(\frac{19+3\sqrt{5}}{2} \right) \\ \sqrt{\frac{23+3\sqrt{5}}{2}} &= \frac{-1-3\sqrt{5}}{2} \quad \text{false} \\ \left\{ \frac{19-3\sqrt{5}}{2} \right\} \end{aligned}$$

$$\begin{aligned} \sqrt{1 + \sqrt{x + \sqrt{x+1}}} &= 2 \\ \left(\sqrt{1 + \sqrt{x + \sqrt{x+1}}} \right)^2 &= (2)^2 \\ 1 + \sqrt{x + \sqrt{x+1}} &= 4 \\ x + \sqrt{x+1} &= 9 \\ \text{122. } (\sqrt{x+1})^2 &= (9-x)^2 \\ x+1 &= x^2 - 18x + 81 \\ x^2 - 19x + 80 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{19 \pm \sqrt{(-19)^2 - 4 \cdot 1 \cdot 80}}{2} \\ x &= \frac{19 \pm \sqrt{41}}{2} \\ x &= \frac{19 - \sqrt{41}}{2} \quad \text{or} \quad x = \frac{19 + \sqrt{41}}{2} \\ \text{Check: } x &= \frac{19 - \sqrt{41}}{2} \end{aligned}$$

$$\begin{aligned} \sqrt{1 + \sqrt{x + \sqrt{x+1}}} &= 2 \\ \sqrt{1 + \sqrt{\frac{19 - \sqrt{41}}{2} + \sqrt{\frac{19 - \sqrt{41}}{2} + 1}}} &= 2 \\ \sqrt{\frac{19 - \sqrt{41}}{2}} + \sqrt{\frac{21 - \sqrt{41}}{2}} &= 3 \\ \left(\sqrt{\frac{19 - \sqrt{41}}{2}} + \sqrt{\frac{21 - \sqrt{41}}{2}} \right)^2 &= (3)^2 \\ \sqrt{\frac{21 - \sqrt{41}}{2}} &= 9 - \left(\frac{19 - \sqrt{41}}{2} \right) \\ \sqrt{\frac{21 - \sqrt{41}}{2}} &= \frac{-1 + \sqrt{41}}{2} \\ \frac{21 - \sqrt{41}}{2} &= \frac{42 - 2\sqrt{41}}{4} \\ \frac{21 - \sqrt{41}}{2} &= \frac{21 - \sqrt{41}}{2} \quad \checkmark \text{ true} \\ \text{Check: } x &= \frac{19 + \sqrt{41}}{2} \end{aligned}$$

$$\sqrt{1+\sqrt{x+\sqrt{x+1}}}=2$$

$$\sqrt{1+\sqrt{\frac{19+\sqrt{41}}{2}}+\sqrt{\frac{19+\sqrt{41}}{2}}+1}=2$$

$$\sqrt{\frac{19+\sqrt{41}}{2}}+\sqrt{\frac{21+\sqrt{41}}{2}}=3$$

$$\left(\sqrt{\frac{19+\sqrt{41}}{2}}+\sqrt{\frac{21+\sqrt{41}}{2}}\right)^2=(3)^2$$

$$\sqrt{\frac{21+\sqrt{41}}{2}}=9-\left(\frac{19+\sqrt{41}}{2}\right)$$

$$\sqrt{\frac{21-\sqrt{41}}{2}}=\frac{-1-\sqrt{41}}{2} \text{ false}$$

$$\left\{\frac{19-\sqrt{41}}{2}\right\}$$

$$123. \quad r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$(6)^3 = \left(\sqrt[3]{\frac{3V}{4\pi}}\right)^3$$

$$216 = \frac{3V}{4\pi}$$

$$V = 288\pi \text{ in.}^3$$

$$124. \quad d = \frac{49}{40}\sqrt{h}$$

$$(24.5)^2 = \left(\frac{49}{40}\sqrt{h}\right)^2$$

$$600.25 = \frac{2401}{1600}h$$

$$400 = h$$

$$400 \text{ ft}$$

$$P = 48^{1/5}$$

$$P = 48(2)^{1/5}$$

$$125.a. \quad \approx 55\%$$

$$P = 48^{1/5}$$

$$75 = 48^{1/5}$$

$$\frac{25}{16} = t^{1/5}$$

$$\left(\frac{25}{16}\right)^5 = (t^{1/5})^5$$

$$b. \quad t \approx 9.3 \text{ hr}$$

$$h = 16(t+4)^{1/3}$$

$$h = 16[(14)+4]^{1/3}$$

$$h = 16(18)^{1/3}$$

$$126. a. \quad \approx 42 \text{ in.}$$

$$b. \quad h = 16(t+4)^{1/3}$$

$$60 = 16(t+4)^{1/3}$$

$$\frac{15}{4} = (t+4)^{1/3}$$

$$\left(\frac{15}{4}\right)^3 = [(t+4)^{1/3}]^3$$

$$t+4 = \left(\frac{15}{4}\right)^3$$

$$t = \left(\frac{15}{4}\right)^3 - 4$$

$$\approx 49 \text{ days}$$

$$v = \sqrt{2gh} = \sqrt{19.6h}$$

$$v = \sqrt{19.6(10)} = 14 \text{ m/sec}$$

$$127. a.$$

$$b.$$

$$v = \sqrt{19.6h}$$

$$26.8 = \sqrt{19.6h}$$

$$26.8 = \sqrt{19.6} \cdot \sqrt{h}$$

$$\frac{26.8}{\sqrt{19.6}} = \sqrt{h}$$

$$\left(\frac{26.8}{\sqrt{19.6}}\right)^2 = (\sqrt{h})^2$$

$$h = \frac{26.8^2}{19.6} \approx 36.6 \text{ m}$$

128. a.

$$r = 1 - \left(\frac{V}{C} \right)^{1/n}$$

$$r = 1 - \left(\frac{12,000}{18,000} \right)^{1/3}$$

$$r = 1 - \left(\frac{2}{3} \right)^{1/3}$$

≈ 0.126 or 12.6% per year

b.

$$r = 1 - \left(\frac{V}{C} \right)^{1/n}$$

$$0.15 = 1 - \left(\frac{11,000}{C} \right)^{1/5}$$

$$\left(\frac{11,000}{C} \right)^{1/5} = 0.85$$

$$\left[\left(\frac{11,000}{C} \right)^{1/5} \right]^5 = (0.85)^5$$

$$\frac{11,000}{C} = (0.85)^5$$

$$C = \frac{11,000}{(0.85)^5}$$

$$\approx \$24,800$$

129. a. $|x - 4| = 6$ or equivalently

$$|4 - x| = 6$$

b. $|x - 4| = 6$

$$(x - 4) = 6 \quad \text{or} \quad (x - 4) = -6$$

$$x = 10 \quad \quad \quad x = -2$$

$$\{-2, 10\}$$

130. a. $|x - 3| = 8$ or equivalently

$$|3 - x| = 8$$

b. $|x - 3| = 8$

$$(x - 3) = 8 \quad \text{or} \quad (x - 3) = -8$$

$$x = 11 \quad \quad \quad x = -5$$

$$\{-5, 11\}$$

131. An equation is in quadratic form if, after a suitable substitution, the equation can be written in the form $au^2 + bu + c = 0$, where u is a variable expression.

132. When solving a radical equation, if both sides of the equation are raised to an even power, then the potential solutions must be checked. This is

because some or all of the solutions may be extraneous solutions.

133. Let t represent the time Joan takes to

fill 100 orders by herself. Then $(t + 1)$ is the time it takes Henry to fill 100 orders.

$$\frac{1 \text{ job}}{t \text{ hr}} + \frac{1 \text{ job}}{(t + 1) \text{ hr}} = \frac{1 \text{ job}}{3 \text{ hr}}$$

$$\frac{1}{t} + \frac{1}{(t + 1)} = \frac{1}{3}$$

$$3(t + 1) \left(\frac{1}{t} + \frac{1}{t + 1} \right) = 3(t + 1) \left(\frac{1}{3} \right)$$

$$3(t + 1) + 3 = t(t + 1)$$

$$3t + 3 + 3 = t^2 + t$$

$$0 = t^2 - 5t - 3$$

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{37}}{2}$$

$$\approx 5.5 \text{ or } \cancel{0.5}$$

$$t + 1 = 5.5 + 1 = 6.5$$

It would take Joan approximately 5.5 hr working alone, and it would take Henry approximately 6.5 hr.

134. Let t represent the time it takes Antonio to complete one bathroom.

Then $(t + 4)$ is the time it takes Jeremy to complete one bathroom.

$$\frac{1 \text{ job}}{t \text{ hr}} + \frac{1 \text{ job}}{(t + 4) \text{ hr}} = \frac{1 \text{ job}}{8 \text{ hr}}$$

$$\frac{1}{t} + \frac{1}{(t + 4)} = \frac{1}{8}$$

$$8(t + 4) \left(\frac{1}{t} + \frac{1}{t + 4} \right) = 8(t + 4) \left(\frac{1}{8} \right)$$

$$8(t + 4) + 8 = t(t + 4)$$

$$8t + 32 + 8 = t^2 + 4t$$

$$0 = t^2 - 12t - 32$$

$$t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-32)}}{2(1)}$$

$$= \frac{12 \pm \sqrt{272}}{2}$$

$$\approx 14.2 \text{ or } \cancel{>22}$$

$$t + 4 = 14.2 + 4 = 18.2$$

It would take Antonio approximately 14.2 hr working alone, and it would take Jeremy approximately 18.2 hr.

135. Let x represent the distance along the

shoreline as shown in the figure.

	Distance	Rat	Time
Row	$\sqrt{400^2 + x^2}$	2.5	$\frac{\sqrt{400^2 + x^2}}{2.5}$
Walk	$800 - x$	5	$\frac{800 - x}{5}$

$$\begin{aligned} \frac{\sqrt{400^2 + x^2}}{2.5} + \frac{800 - x}{5} &= 300 \\ 5 \left(\frac{\sqrt{400^2 + x^2}}{2.5} + \frac{800 - x}{5} \right) &= 5(300) \\ 2\sqrt{400^2 + x^2} + 800 - x &= 1500 \\ 2\sqrt{400^2 + x^2} &= x + 700 \\ (x + 700)^2 &= (2\sqrt{400^2 + x^2})^2 \\ x^2 + 1400x + 490,000 &= 4(400^2 + x^2) \\ x^2 + 1400x + 490,000 &= 4x^2 + 640,000 \\ 3x^2 - 1400x + 150,000 &= 0 \\ (3x - 500)(x - 300) &= 0 \end{aligned}$$

Section 1.7 Linear Inequalities and Compound Inequalities

1. inequality
2. intersection
3. $a < x < b$

4. union

$$x = \frac{500}{3} \text{ or } x = 300$$

Pam can row to a point $166\frac{2}{3}$ ft down the beach or to a point 300 ft down the beach to be home in 5 min.

136. Let x represent the distance along the

shoreline as shown in the figure.

	Distance	Rat	Time
Boat	$\sqrt{48^2 + x^2}$	20	$\frac{\sqrt{48^2 + x^2}}{20}$
Car	$96 - x$	60	$\frac{96 - x}{60}$

$$\begin{aligned} \frac{\sqrt{48^2 + x^2}}{20} + \frac{96 - x}{60} &= 4 \\ 60 \left(\frac{\sqrt{48^2 + x^2}}{20} + \frac{96 - x}{60} \right) &= 60(4) \\ 3\sqrt{48^2 + x^2} + 96 - x &= 240 \\ 3\sqrt{48^2 + x^2} &= x + 144 \\ (3\sqrt{48^2 + x^2})^2 &= (x + 144)^2 \\ x^2 + 288x + 20,736 &= 9(48^2 + x^2) \\ x^2 + 288x + 20,736 &= 9x^2 + 20,736 \\ 8x^2 - 288x &= 0 \\ 8x(x - 36) &= 0 \\ x = 0 \text{ or } x &= 36 \end{aligned}$$

The marina is 36 mi up the coast.

5. $-k; k$
6. $-k; >$
7. $\square$

8. $\{ \}$

$$-2x - 5 > 17$$

$$-2x > 22$$

9. $x < -11$

$$\{x | x < -11\}; (-\infty, -11)$$



$$-8 + 1 < 17$$

$$-8 < 16$$

10. $t > -2$

$$\{t | t > -2\}; (-2, \infty)$$



$$-3 \leq -\frac{4}{3}w + 1$$

$$-4 \leq -\frac{4}{3}w$$

11. $3 \geq w$ or $w \leq 3$

$$\{w | w \leq 3\}; (-\infty, 3]$$



$$8 \geq -\frac{5}{2}y - 2$$

$$10 \geq -\frac{5}{2}y$$

12. $-4 \leq y$ or $y \geq -4$

$$\{y | y \geq -4\}; [-4, \infty)$$



-4

13. $-1.2 + 0.6a \leq 0.4a + 0.5$

$$0.2a \leq 1.7$$

$$a \leq \frac{1.7}{0.2}$$

$$a \leq 8.5$$

$$\{a | a \leq 8.5\}; (-\infty, 8.5]$$



8.5

$$-0.7 + 0.3x \leq 0.9x - 0.4$$

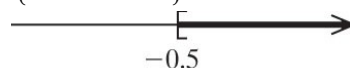
$$-0.6x \leq 0.3$$

$$x \geq -\frac{0.3}{0.6}$$

$$x \geq -0.5$$

14.

$$\{x | x \geq -0.5\}; [-0.5, \infty)$$



-0.5

$$-5 > 6(c - 4) + 7$$

$$-5 > 6c - 24 + 7$$

$$-5 > 6c - 17$$

$$12 > 6c$$

15. $2 > c$ or $c < 2$

$$\{c | c < 2\}; (-\infty, 2)$$



2

$$-14 < 3(m - 7) + 7$$

$$-14 < 3m - 21 + 7$$

$$-14 < 3m - 14$$

$$0 < 3m$$

16. $0 < m$ or $m > 0$

$$\{m | m > 0\}; (0, \infty)$$



0

$$\frac{4+x}{2} - \frac{x-3}{5} < -\frac{x}{10}$$

17.

$$10\left(\frac{4+x}{2} - \frac{x-3}{5}\right) < 10\left(-\frac{x}{10}\right)$$

$$5(4+x) - 2(x-3) < -x$$

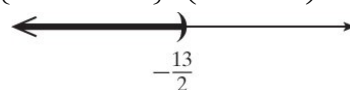
$$20 + 5x - 2x + 6 < -x$$

$$4x < -26$$

$$x < -\frac{26}{4}$$

$$x < -\frac{13}{2}$$

$$\left\{x \mid x < -\frac{13}{2}\right\}; \left(-\infty, -\frac{13}{2}\right)$$



$-\frac{13}{2}$

$$\begin{aligned}
 18. \quad & \frac{y+3}{4} - \frac{3y+1}{6} > -\frac{1}{12} \\
 & 12\left(\frac{y+3}{4} - \frac{3y+1}{6}\right) > 12\left(-\frac{1}{12}\right) \\
 & 3(y+3) - 2(3y+1) > -1 \\
 & 3y+9-6y-2 > -1 \\
 & -3y > -8 \\
 & y < \frac{8}{3} \\
 & \left\{y \mid y < \frac{8}{3}\right\}; \left(-\infty, \frac{8}{3}\right)
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{1}{3}(x+4) - \frac{5}{6}(x-3) \geq \frac{1}{2}x+1 \\
 & 6\left[\frac{1}{3}(x+4) - \frac{5}{6}(x-3)\right] \geq 6\left(\frac{1}{2}x+1\right) \\
 & 2(x+4) - 5(x-3) \geq 3x+6 \\
 & 2x+8-5x+15 \geq 3x+6 \\
 & -6x \geq -17 \\
 & x \leq \frac{17}{6} \\
 & \left\{x \mid x \leq \frac{17}{6}\right\}; \left(-\infty, \frac{17}{6}\right]
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{1}{2}(t-6) - \frac{4}{3}(t+2) \geq -\frac{3}{4}t-2 \\
 & 12\left[\frac{1}{2}(t-6) - \frac{4}{3}(t+2)\right] \geq 12\left(-\frac{3}{4}t-2\right) \\
 & 6(t-6) - 16(t+2) \geq -9t-24 \\
 & 6t-36-16t-32 \geq -9t-24 \\
 & -t \geq 44 \\
 & t \leq -44 \\
 & \{t \mid t \leq -44\}; (-\infty, -44]
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & 5(7-x) + 2x < 6x-2-9x \\
 & 35-5x+2x < 6x-2-9x \\
 & 35 < -2 \\
 & \{ \}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & 2(3x+1) - 4x > 2(x+8) - 5 \\
 & 6x+2-4x > 2x+16-5 \\
 & 2 > 9 \\
 & \{ \}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & 5-3[2-4(x-2)] \geq \\
 & 6\{2-[4-(x-3)]\} \\
 & 5-3[2-4x+8] \geq 6\{2-[4-x+3]\} \\
 & 5-3[-4x+10] \geq 6\{2-[-x+7]\} \\
 & 5+12x-30 \geq 6\{2+x-7\} \\
 & 12x-25 \geq 6\{x-5\} \\
 & 12x-25 \geq 6x-30 \\
 & 6x \geq -5 \\
 & x \geq -\frac{5}{6} \\
 & \left\{x \mid x \geq -\frac{5}{6}\right\}; \left[-\frac{5}{6}, \infty\right)
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & 8-[6-10(x-1)] \geq \\
 & 2\{1-3[2-(x+4)]\} \\
 & 8-[6-10x+10] \geq 2\{1-3[2-x-4]\} \\
 & 8-[-10x+16] \geq 2\{1-3[-x-2]\} \\
 & 8+10x-16 \geq 2\{1+3x+6\} \\
 & 10x-8 \geq 2\{3x+7\} \\
 & 10x-8 \geq 6x+14 \\
 & 4x \geq 22 \\
 & x \geq \frac{11}{2}
 \end{aligned}$$

$$\left\{x \mid x \geq \frac{11}{2}\right\}; \left[\frac{11}{2}, \infty\right)$$

25. $4 - 3k > -2(k + 3) - k$
 $4 - 3k > -2k - 6 - k$
 $4 > -6$
 $\square; (-\infty, \infty)$

26. $2x - 9 < 6(x - 1) - 4x$
 $2x - 9 < 6x - 6 - 4x$
 $-9 < -6$
 $\square; (-\infty, \infty)$

27. a. $x < 4$ and $x \geq -2$
 $[-2, 4)$

b. $x < 4$ or $x \geq -2$
 $(-\infty, \infty)$

28. a. $y \leq -2$ and $y > -5$
 $(-5, -2]$

b. $y \leq -2$ or $y > -5$
 $(-\infty, \infty)$

29. a. $m \leq 5$ or $\frac{1}{3}m < -2$
 $(-\infty, 5]$

b. $m \leq 5$ and $\frac{1}{3}m < -2$
 $(-\infty, -6)$

$n - 6 > 1$ or $\frac{3}{4}n \geq 6$

30. a. $n > 7$ or $n \geq 8$
 $(7, \infty)$

b. $n > 7$ and $\frac{3}{4}n \geq 8$
 $[8, \infty)$

$-\frac{2}{3}y > -12$ and $2.08 \geq 0.65y$

31. a. $y > 18$ and $y \leq 3.2$
 $(-\infty, 3.2]$

b. $y > 18$ or $y \leq 3.2$
 $(-\infty, 18)$

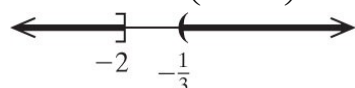
$-\frac{4}{5}m < 8$ and $0.85 \leq 0.34m$

32. a. $m > -10$ and $m \geq 2.5$
 $[2.5, \infty)$

b. $m > -10$ or $m \geq 2.5$
 $(-10, \infty)$

$$\begin{aligned}
 3(x-2)+2 &\leq x-8 \quad \text{or} \quad 4(x+1)+2 > -2x+4 \\
 3x-6+2 &\leq x-8 \quad \text{or} \quad 4x+4+2 > -2x+4 \\
 2x &\leq -4 \quad \text{or} \quad 6x > -2 \\
 x &\leq -2 \quad \text{or} \quad x > -\frac{1}{3}
 \end{aligned}$$

33. a.

$$\left(-\infty, -2\right] \cup \left(-\frac{1}{3}, \infty\right)$$


$$\begin{aligned}
 3(x-2)+2 &\leq x-8 \quad \text{and} \quad 4(x+1)+2 > -2x+4 \\
 x &\leq -2 \quad \text{and} \quad x > -\frac{1}{3}
 \end{aligned}$$

b.

$$\{ \}$$

$$\begin{aligned}
 5(t-4)+2 &> 3(t+1)-3 \quad \text{or} \quad 2t-6 > 3(t-4)-2 \\
 5t-20+2 &> 3t+3-3 \quad \text{or} \quad 2t-6 > 3t-12-2 \\
 2t &> 18 \quad \text{or} \quad -t > -8 \\
 t &> 9 \quad \text{or} \quad t < 8
 \end{aligned}$$

34. a.

$$\left(-\infty, -8\right) \cup \left(9, \infty\right)$$


$$\begin{aligned}
 5(t-4)+2 &> 3(t+1)-3 \quad \text{and} \quad 2t-6 > 3(t-4)-2 \\
 t &> 9 \quad \text{and} \quad t < 8
 \end{aligned}$$

b.

$$\{ \}$$

35. $-2.8 < y \text{ and } y \leq 5$

36. $-\frac{1}{2} \leq z \text{ and } z < 2.4$

$$-3 < -2x + 1 \leq 9$$

$$-4 < -2x \leq 8$$

$$2 > x \geq -4 \text{ or } -4 \leq x < 2$$

37.

$$\left[-4, 2\right)$$


$$\begin{aligned}
 -6 &\leq -3x+9 < 0 \\
 -15 &\leq -3x < -9
 \end{aligned}$$

38. $5 \geq x > 3 \text{ or } 3 < x \leq 5$

$$\left(3, 5\right]$$



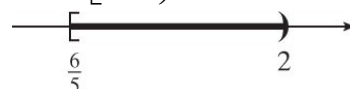
$$1 \leq \frac{5x-4}{2} < 3$$

$$2 \leq 5x-4 < 6$$

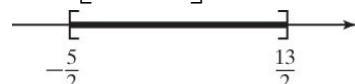
$$6 \leq 5x < 10$$

$$\frac{6}{5} \leq x < 2$$

39.

$$\left[\frac{6}{5}, 2\right)$$


$$\begin{aligned}
 -2 &\leq \frac{4x-1}{3} \leq 5 \\
 -6 &\leq 4x-1 \leq 15 \\
 -5 &\leq 4x \leq 16 \\
 -\frac{5}{4} &\leq x \leq 4 \\
 \mathbf{40.} \quad &\left[-\frac{5}{4}, 4\right]
 \end{aligned}$$


$$\begin{aligned}
 -2 &\leq \frac{-2x+1}{-3} \leq 4 \\
 6 &\geq -2x+1 \geq -12 \\
 5 &\geq -2x \geq -13 \\
 -\frac{5}{2} &\leq x \leq \frac{13}{2} \\
 \mathbf{41.} \quad &\left[-\frac{5}{2}, \frac{13}{2}\right]
 \end{aligned}$$


$$\begin{aligned}
 -4 &< \frac{-5x-2}{-2} < 4 \\
 -4 &< \frac{5x+2}{2} < 4 \\
 -8 &< 5x+2 < 8 \\
 -10 &< 5x < 6 \\
 -2 &< x < \frac{6}{5} \\
 \mathbf{42.} \quad &\left(-2, \frac{6}{5}\right)
 \end{aligned}$$


$$\begin{aligned}
 |x| &= 7 \\
 \mathbf{43. a.} \quad &x = 7 \text{ or } x = -7 \\
 &\{7, -7\} \\
 |x| &< 7 \\
 \mathbf{b.} \quad &-7 < x < 7 \\
 &(-7, 7)
 \end{aligned}$$

$$\begin{aligned}
 |x| &> 7 \\
 \mathbf{c.} \quad &x < -7 \text{ or } x > 7 \\
 &(-\infty, -7) \cup (7, \infty) \\
 |y| &= 8 \\
 \mathbf{44. a.} \quad &y = 8 \text{ or } y = -8 \\
 &\{8, -8\} \\
 |y| &< 8 \\
 \mathbf{b.} \quad &-8 < y < 8 \\
 &(-8, 8) \\
 |y| &> 8 \\
 \mathbf{c.} \quad &y < -8 \text{ or } y > 8 \\
 &(-\infty, -8) \cup (8, \infty) \\
 |a+9| + 2 &= 6 \\
 |a+9| &= 4 \\
 a+9 &= 4 \quad \text{or} \quad a+9 = -4 \\
 \mathbf{45. a.} \quad &a = -5 \quad \text{or} \quad a = -13 \\
 &\{-13, -5\} \\
 |a+9| + 2 &\leq 6 \\
 |a+9| &\leq 4 \\
 -4 &\leq a+9 \leq 4 \\
 \mathbf{b.} \quad &-13 \leq a \leq -5 \\
 &[-13, -5] \\
 |a+9| + 2 &\geq 6 \\
 |a+9| &\geq 4 \\
 a+9 &\leq -4 \quad \text{or} \quad a+9 \geq 4 \\
 \mathbf{c.} \quad &a \leq -13 \quad \text{or} \quad a \geq -5 \\
 &(-\infty, -13] \cup [-5, \infty) \\
 |b+1| - 4 &= 1 \\
 |b+1| &= 5 \\
 b+1 &= 5 \quad \text{or} \quad b+1 = -5 \\
 \mathbf{46. a.} \quad &b = 4 \quad \text{or} \quad b = -6 \\
 &\{-6, 4\}
 \end{aligned}$$

$$\begin{aligned}
 &|b+1|-4 \leq 1 \\
 &|b+1| \leq 5 \\
 &-5 \leq b+1 \leq 5 \\
 \text{b.} \quad &-6 \leq b \leq 4 \\
 &[-6, 4] \\
 &|b+1|-4 \geq 1 \\
 &|b+1| \geq 5 \\
 &b+1 \leq -5 \quad \text{or} \quad b+1 \geq 5 \\
 \text{c.} \quad &b \leq -6 \quad \text{or} \quad b \geq 4 \\
 &(-\infty, -6] \cup [4, \infty) \\
 47. \quad &3|4-x|-2 < 16 \\
 &3|4-x| < 18 \\
 &|4-x| < 6 \\
 &-6 < 4-x < 6 \\
 &-10 < -x < 2 \\
 &10 > x > -2 \quad \text{or} \quad -2 < x < 10 \\
 &(-2, 10) \\
 &2|7-y|+1 < 17 \\
 &2|7-y| < 16 \\
 &|7-y| < 8 \\
 &-8 < 7-y < 8 \\
 &-15 < -y < 1 \\
 48. \quad &15 > y > -1 \quad \text{or} \quad -1 < y < 15 \\
 &(-1, 15) \\
 &2|x+3|-4 \geq 6 \\
 &2|x+3| \geq 10 \\
 &|x+3| \geq 5 \\
 &x+3 \leq -5 \quad \text{or} \quad x+3 \geq 5 \\
 49. \quad &x \leq -8 \quad \text{or} \quad x \geq 2 \\
 &(-\infty, -8] \cup [2, \infty)
 \end{aligned}$$

$$\begin{aligned}
 &5|x+1|-9 \geq -4 \\
 &5|x+1| \geq 5 \\
 &|x+1| \geq 1 \\
 &x+1 \leq -1 \quad \text{or} \quad x+1 \geq 1 \\
 50. \quad &x \leq -2 \quad \text{or} \quad x \geq 0 \\
 &(-\infty, -2] \cup [0, \infty) \\
 &|4w-5|+6 \leq 2 \\
 51. \quad &|4w-5| \leq -4 \\
 &\{ \} \\
 &|2x+7|+5 < 1 \\
 52. \quad &|2x+7| < -4 \\
 &\{ \} \\
 &|5-p|+13 > 6 \\
 53. \quad &|5-p| > -7 \\
 &\square; (-\infty, \infty) \\
 &\text{An absolute value of any real number is greater than or equal to zero. Therefore, it is also greater than every negative number. This inequality is true for all real numbers, } p. \\
 &|12-7x|+5 \geq 4 \\
 54. \quad &|12-7x| \geq -1 \\
 &\square; (-\infty, \infty) \\
 &\text{An absolute value of any real number is greater than or equal to zero. Therefore, it is also greater than every negative number. This inequality is true for all real numbers, } x. \\
 &-11 \leq 5-|2p+4| \\
 &-16 \leq -|2p+4| \\
 &16 \geq |2p+4| \\
 &|2p+4| \leq 16 \\
 &-16 \leq 2p+4 \leq 16 \\
 &-20 \leq 2p \leq 12 \\
 55. \quad &-10 \leq p \leq 6 \\
 &[-10, 6]
 \end{aligned}$$

$$\begin{aligned}
 & -18 \leq 6 - |3z + 3| \\
 & -24 \leq -|3z + 3| \\
 & 24 \geq |3z + 3| \\
 & |3z + 3| \leq 24 \\
 & -24 \leq 3z + 3 \leq 24 \\
 & -27 \leq 3z \leq 21 \\
 & -9 \leq z \leq 7 \\
 \mathbf{56.} \quad & [-9, 7] \\
 & 10 < |-5c - 4| + 2 \\
 & 8 < |-5c - 4| \\
 & |-5c - 4| > 8 \\
 & -5c - 4 < -8 \quad \text{or} \quad -5c - 4 > 8 \\
 & -5c < -4 \quad \text{or} \quad -5c > 12 \\
 & c > \frac{4}{5} \quad \text{or} \quad c < -\frac{12}{5} \\
 \mathbf{57.} \quad & \left(-\infty, -\frac{12}{5}\right) \cup \left(\frac{4}{5}, \infty\right) \\
 & 15 < |-2d - 3| + 6 \\
 & 9 < |-2d - 3| \\
 & |-2d - 3| > 9 \\
 & -2d - 3 < -9 \quad \text{or} \quad -2d - 3 > 9 \\
 & -2d < -6 \quad \text{or} \quad -2d > 12 \\
 & d > 3 \quad \text{or} \quad d < -6 \\
 \mathbf{58.} \quad & (-\infty, -6) \cup (3, \infty) \\
 & \left|\frac{y+3}{6}\right| < 2 \\
 & -2 < \frac{y+3}{6} < 2 \\
 & -12 < y+3 < 12 \\
 \mathbf{59.} \quad & -15 < y < 9 \\
 & (-15, 9)
 \end{aligned}$$

$$\begin{aligned}
 & \left|\frac{m-4}{2}\right| < 14 \\
 & -14 < \frac{m-4}{2} < 14 \\
 & -28 < m-4 < 28 \\
 \mathbf{60.} \quad & -24 < m < 32 \\
 & (-24, 32) \\
 \mathbf{61. a.} \quad & |x| = -9 \\
 & \{ \} \\
 \mathbf{b.} \quad & |x| < -9 \\
 & \{ \} \\
 \mathbf{c.} \quad & |x| > -9 \\
 & \square; \{-\infty, \infty\} \\
 \mathbf{62. a.} \quad & |y| = -2 \\
 & \{ \} \\
 \mathbf{b.} \quad & |y| < -2 \\
 & \{ \} \\
 \mathbf{c.} \quad & |y| > -2 \\
 & \square; \{-\infty, \infty\} \\
 \mathbf{63. a.} \quad & 18 = 4 - |y - 7| \\
 & 14 = -|y - 7| \\
 & -14 = |y - 7| \\
 & \{ \} \\
 \mathbf{b.} \quad & 18 \leq 4 - |y - 7| \\
 & 14 \leq -|y - 7| \\
 & -14 \geq |y - 7| \\
 & \{ \} \\
 \mathbf{c.} \quad & 18 \geq 4 - |y - 7| \\
 & 14 \geq -|y - 7| \\
 & -14 \leq |y - 7| \\
 & \square; \{-\infty, \infty\}
 \end{aligned}$$

64. a. $15 = 2 - |p - 3|$

$$13 = -|p - 3|$$

$$-13 = |p - 3|$$

$$\{ \}$$

b. $15 \leq 2 - |p - 3|$

$$13 \leq -|p - 3|$$

$$-13 \geq |p - 3|$$

$$\{ \}$$

c. $15 \geq 2 - |p - 3|$

$$13 \geq -|p - 3|$$

$$-13 \leq |p - 3|$$

$$\square; \{-\infty, \infty\}$$

$$|z| = 0$$

65. a. $z = 0$ or $z = -0$

$$\{ \}$$

b. $|z| < 0$

$$\{ \}$$

$$|z| \leq 0$$

c. $z = 0$

$$\{0\}$$

d. $|z| > 0$

$$\{z \mid z < 0 \text{ or } z > 0\};$$

$$(-\infty, 0) \cup (0, \infty)$$

e. $|z| \geq 0$

$$\square; (-\infty, \infty)$$

$$|2w| = 0$$

$$2w = 0 \text{ or } 2w = -0$$

66. a. $w = 0$ or $w = -0$

$$\{0\}$$

b. $|2w| < 0$

$$\{ \}$$

$$|2w| \leq 0$$

$$2w = 0$$

c. $w = 0$

$$\{0\}$$

$$|2w| > 0$$

$$2w < -0 \text{ or } 2w > 0$$

d. $w < 0$ or $w > 0$

$$\{w \mid w < 0 \text{ or } w > 0\};$$

$$(-\infty, 0) \cup (0, \infty)$$

e. $|2w| \geq 0$

$$\square; (-\infty, \infty)$$

$$|k + 4| = 0$$

$$k + 4 = 0 \text{ or } k + 4 = -0$$

67. a. $k = -4$ or $k = -4$

$$\{-4\}$$

b. $|k + 4| < 0$

$$\{ \}$$

$$|k + 4| \leq 0$$

$$k + 4 = 0$$

c. $k = -4$

$$\{-4\}$$

$$|k + 4| > 0$$

$$k + 4 < -0 \text{ or } k + 4 > 0$$

d. $k < -4$ or $k > -4$

$$\{k \mid k < -4 \text{ or } k > -4\};$$

$$(-\infty, -4) \cup (-4, \infty)$$

e. $|k + 4| \geq 0$

$$\square; (-\infty, \infty)$$

$$|c - 3| = 0$$

$$c - 3 = 0 \text{ or } c - 3 = -0$$

68. a. $c = 3$ or $c = 3$

$$\{3\}$$

b. $|c - 3| < 0$

$$\{ \}$$

$$|c-3| \leq 0$$

$$c-3=0$$

c. $c=3$

$$\{3\}$$

$$|c-3| > 0$$

$$c-3 < -0 \quad \text{or} \quad c-3 > 0$$

d. $c < 3 \quad \text{or} \quad c > 3$

$$\{c \mid c < 3 \text{ or } c > 3\};$$

$$(-\infty, 3) \cup (3, \infty)$$

e. $|c-3| \geq 0$

$$\square; (-\infty, \infty)$$

69. $12.0 \leq x \leq 15.2$ g/dL

70. $18 < a < 25$ yr

71. $90 \leq d \leq 110$ yd

75. Let x be the score of Rita in the final exam.

$$80 \leq \left(\frac{78+82+90+80+75}{5} \times 60\% \right) + (85 \times 10\%) + (x \times 30\%) \leq 90$$

$$80 \leq \left(81 \times \frac{60}{100} \right) + \left(85 \times \frac{10}{100} \right) + \left(x \times \frac{30}{100} \right) \leq 90$$

$$80 \leq (48.6 + 8.5 + 0.3x) \leq 90$$

$$22.9 \leq 0.3x \leq 32.9$$

$$76.33 \leq x \leq 109.66$$

$$77 \leq x \leq 100$$

Because 100 is the highest score that can be earned on the final exam and only whole-number scores are given.

76. Let x be the score of Trent in the final exam.

$$70 \leq \left(\frac{66+84+72}{3} \times 50\% \right) + (60 \times 20\%) + (85 \times 10\%) + (x \times 20\%) \leq 100$$

$$70 \leq \left(74 \times \frac{50}{100} \right) + \left(60 \times \frac{20}{100} \right) + \left(85 \times \frac{10}{100} \right) + \left(x \times \frac{20}{100} \right) \leq 100$$

$$70 \leq (37 + 12 + 8.5 + 0.2x) \leq 100$$

$$12.5 \leq 0.2x \leq 42.5$$

$$62.5 \leq x \leq 212.5$$

$$63 \leq x \leq 100$$

Because 100 is the highest score that can be earned on the final exam and that whole-number scores are given.

77. Let t represent the time it takes for the car to be more than 16 miles ahead of the truck.

72. $220 \leq s \leq 410$ mph

$$\frac{88+92+100+80+90+2.5x}{7.5} \geq 92$$

$$\frac{450+2.5x}{7.5} \geq 92$$

$$450+2.5x \geq 690$$

$$2.5x \geq 240$$

$$x \geq 96$$

73.

Marilee needs to score at least 96 on the final exam.

$$\frac{36+36.9+37.1+37.4+x}{5} \geq 37$$

$$\frac{147.4+x}{5} \geq 37$$

$$147.4+x \geq 185$$

$$x \geq 37.6$$

74.

The child needs a score of at least 37.6.

$$50t > 40t + 16$$

$$10t > 16$$

$$t > 1.6 \bigcup_{i=1}^n X_i$$

It will take more than 1.6 hr or 1 hr 36 min.

- 78.** Let t represent the time it takes for a tutor to make over \$500 more than a student working in the library.

$$16.25t > 10.75t + 500$$

$$5.5t > 500$$

$$t > 91$$

It would take 91 or more hours.

- 79.** Let l represent the length of the garden. Then the perimeter of the

garden is $(2l + 200)$.

$$200 + 2l \leq 800$$

$$2l \leq 600$$

$$l \leq 300$$

The length must be 300 ft or less.

- 80.** Let x represent the length of the shortest side. Then the lengths of the

other two sides are $(x + 1)$ and $(x + 2)$.

$$x + (x + 1) + (x + 2) \leq 24$$

$$3x + 3 \leq 24$$

$$3x \leq 21$$

$$x \leq 7$$

The shortest side may be 2 ft, 3 ft, 4 ft,

5 ft, 6 ft, or 7 ft.

- 81.** Let x represent the average scores that would produce a nonnegative handicap of 72 or less.

$$0 \leq 0.9(220 - x) \leq 72$$

$$0 \leq 220 - x \leq 80$$

$$-220 \leq -x \leq -140$$

$$220 \geq x \geq 140 \text{ or } 140 \leq x \leq 220$$

An average score in league play between 140 and 220, inclusive, would produce a handicap of 72 or less.

$$C = \frac{5}{9}(F - 32)$$

$$36.5 \leq \frac{5}{9}(F - 32) \leq 37.5$$

$$65.7 \leq F - 32 \leq 67.5$$

$$\mathbf{82.} \quad 97.7 \leq F \leq 99.5$$

Normal body temperature is between 97.7°F and 99.5°F , inclusive.

- 83. a.** Let s represent the amount of sales.

$$25,000 + 0.1s > 30,000 + 0.08s$$

$$0.02s > 5000$$

$$s > 250,000$$

b. Job A

- 84.** Let x represent the number of nights.

$$40 + 1.18x(169) < 1.14x(179)$$

$$40 + 199.42x < 204.06x$$

$$40 < 4.64x$$

$$8.62 < x \text{ or } x > 8.62$$

After 8 nights (9 or more), Hotel B will be less expensive.

- 85. a.** $|v - 16| < 0.01$ or $|16 - v| < 0.01$

$$|v - 16| < 0.01$$

$$-0.01 < v - 16 < 0.01$$

$$\mathbf{b.} \quad 15.99 < v < 16.01$$

$$(15.99, 16.01)$$

86. a.

$$|t - 60| < 0.2 \text{ or } |60 - t| < 0.2$$

$$|t - 60| < 0.2$$

$$-0.2 < t - 60 < 0.2$$

$$\mathbf{b.} \quad 59.8 < t < 60.2$$

$$(59.8, 60.2)$$

- 87. a.** $|x - 4| > 1$ or $|4 - x| > 1$

$$|x - 4| > 1$$

$$x - 4 < -1 \text{ or } x - 4 > 1$$

$$\mathbf{b.} \quad x < 3 \text{ or } x > 5$$

$$(-\infty, 3) \cup (5, \infty)$$

- 88. a.** $|y - 10| > 2$ or $|10 - y| > 2$
 $|y - 10| > 2$
 $y - 10 < -2$ or $y - 10 > 2$
b. $y < 8$ or $y > 12$
 $(-\infty, 8) \cup (12, \infty)$
- 89. a.** $|t - 36.5| \leq 1.5$ or $|36.5 - t| \leq 1.5$
 $|t - 36.5| \leq 1.5$
 $-1.5 \leq t - 36.5 \leq 1.5$
b. $35 \leq t \leq 38$
 $[35, 38]$; If the refrigerator is set to 36.5°F , the actual temperature would be between 35°F and 38°F , inclusive.
- 90. a.** $|x - 16| \leq 0.5$ or $|16 - x| \leq 0.5$
 $|x - 16| \leq 0.5$
 $-0.5 \leq x - 16 \leq 0.5$
b. $15.5 \leq x \leq 16.5$
 $[15.5, 16.5]$; The boxes of cereal vary in weight between 15.5 oz and 16.5 oz, inclusive.
- 91. a.** $|x - 0.51| \leq 0.03$ or $|0.51 - x| \leq 0.03$
 $|x - 0.51| \leq 0.03$
 $-0.03 \leq x - 0.51 \leq 0.03$
b. $0.48 \leq x \leq 0.54$
 $[0.48, 0.54]$; The candidate is expected to receive between 48% of the vote and 54% of the vote, inclusive.
- 92. a.** $|x - 34| \leq 3$ or $|34 - x| \leq 3$
 $|x - 34| \leq 3$
 $-3 \leq x - 34 \leq 3$
b. $31 \leq x \leq 37$
 $[31, 37]$; The motorist was traveling between 31 mph and 37 mph, inclusive. The motorist should receive a ticket because even at the

lower end of the interval, the speed of 31 mph still exceeds the posted speed limit.

- $x - 2 \geq 0$
- 93. a.** $x \geq 2$
 $\{x | x \geq 2\}$
 $2 - x \geq 0$
b. $2 \geq x$ or $x \leq 2$
 $\{x | x \leq 2\}$
 $x - 6 \geq 0$
- 94. a.** $x \geq 6$
 $\{x | x \geq 6\}$
 $6 - x \geq 0$
b. $6 \geq x$ or $x \leq 6$
 $\{x | x \leq 6\}$
 $x + 4 \geq 0$
- 95. a.** $x \geq -4$
 $\{x | x \geq -4\}$
b. $\square$
 $x + 7 \geq 0$
- 96. a.** $x \geq -7$
 $\{x | x \geq -7\}$
b. $\square$
 $2x - 9 \geq 0$
 $2x \geq 9$
 $x \geq \frac{9}{2}$
- 97. a.** $\left\{x \mid x \geq \frac{9}{2}\right\}$
 $2x - 9 \geq 0$
 $2x \geq 9$
 $x \geq \frac{9}{2}$
b. $\left\{x \mid x \geq \frac{9}{2}\right\}$

- $3x - 7 \geq 0$
 $3x \geq 7$
 $x \geq \frac{7}{3}$
- 98. a.** $\left\{ x \mid x \geq \frac{7}{3} \right\}$
 $3x - 7 \geq 0$
 $3x \geq 7$
 $x \geq \frac{7}{3}$
- b.** $\left\{ x \mid x \geq \frac{7}{3} \right\}$
- 99.** $cd > a$ False
- 100.** $ab < c$ True
- 101.** If $a > c$, then $ad < cd$. True
- 102.** If $a < c$, then $ab < bc$. False
 $-3 \leq x \leq 7$
 $-3 - 2 \leq x - 2 \leq 7 - 2$
 $-5 \leq x \leq 5$
- 103.** $|x - 2| \leq 5$
 $2 < x < 6$
 $2 - 4 < x - 4 < 6 - 4$
 $-2 < x < 2$
- 104.** $|x - 4| < 2$
 $x < 4$ or $x > 10$
 $x - 7 < 4 - 7$ or $x - 7 > 10 - 7$
 $x - 7 < -3$ or $x - 7 > 3$
- 105.** $|x - 7| > 3$
 $x < -1$ or $x > 11$
 $x - 5 < -1 - 5$ or $x - 5 > 11 - 5$
 $x - 5 < -6$ or $x - 5 > 6$
- 106.** $|x - 5| > 6$
- 107.** The steps are the same with the following exception. If both sides of an inequality are multiplied or divided by a negative real number, then the direction of the inequality sign must be reversed.

108. The statement $8 < x < 2$ is equivalent to $8 < x$ and $x < 2$. No real number is greater than 8 and simultaneously less than 2.

109. The inequality $|x - 3| \leq 0$ will be true only for values of x for which $x - 3 = 0$ (the absolute value will never be less than 0). The solution set is $\{3\}$. The inequality $|x - 3| > 0$ is true for all values of x excluding 3. The solution set is $\{x \mid x < 3 \text{ or } x > 3\}$.

110. Taking the square root of both sides of the equation $x^2 = 4$ results in $\sqrt{x^2} = \sqrt{4}$ or equivalently $|x| = 2$. The solution set for each equation is $\{2, -2\}$, indicating that the equations are equivalent.

$$\begin{aligned}
 |x| + x &< 11 \\
 x + x &< 11 \quad \text{or} \quad -x + x < 11 \\
 2x &< 11 \quad \text{or} \quad 0 < 11
 \end{aligned}$$

$$\begin{aligned}
 x &< \frac{11}{2} \\
 \text{111.} \quad &\left(-\infty, \frac{11}{2}\right) \\
 |x| - x &> 10 \\
 x - x &> -10 \quad \text{or} \quad -x - x > 10 \\
 0 &> -10 \quad \text{or} \quad -2x > 10 \\
 &x < -5
 \end{aligned}$$

$$\begin{aligned}
 \text{112.} \quad &\left(-\infty, -5\right) \\
 1 &< |x| < 9 \\
 1 &< x < 9 \quad \text{or} \quad 1 < -x < 9 \\
 &-1 > x > -9
 \end{aligned}$$

$$\begin{aligned}
 \text{113.} \quad &-9 < x < -1 \\
 &\left(-9, -1\right) < \left(1, 9\right)
 \end{aligned}$$

$$2 < |y| < 11$$

$$2 < y < 11 \quad \text{or} \quad 2 < -y < 11$$

$$\text{or} \quad -2 > y > -11$$

$$\mathbf{114.} \quad \text{or} \quad -11 < y < -2$$

$$(-11, -2) < (2, 11)$$

$$5 \leq |2x + 1| \leq 7$$

$$5 \leq 2x + 1 \leq 7 \quad \text{or} \quad 5 \leq -2x - 1 \leq 7$$

$$4 \leq 2x \leq 6 \quad \text{or} \quad 6 \leq -2x \leq 8$$

$$2 \leq x \leq 3 \quad \text{or} \quad -3 \geq x \geq -4$$

$$\mathbf{115.} \quad \text{or} \quad -4 \leq x \leq -3$$

$$(-4, -3) < (2, 3)$$

$$\mathbf{116.} \quad 7 \leq |3x - 5| \leq 13$$

$$\begin{aligned}
 7 \leq 3x - 5 \leq 13 & \text{ or } 7 \leq -3x + 5 \leq 13 \\
 12 \leq 3x \leq 18 & \text{ or } 2 \leq -3x \leq 8 \\
 4 \leq x \leq 6 & \text{ or } -\frac{2}{3} \geq x \geq -\frac{8}{3} \\
 & \text{ or } -\frac{8}{3} \leq x \leq -\frac{2}{3} \\
 \left(-\frac{8}{3}, -\frac{2}{3}\right) & \subset (4, 6)
 \end{aligned}$$

$$\begin{aligned}
 |p - \hat{p}| & < z \sqrt{\frac{\hat{p}\hat{q}}{n}} \\
 -z \sqrt{\frac{\hat{p}\hat{q}}{n}} & < p - \hat{p} < z \sqrt{\frac{\hat{p}\hat{q}}{n}} \\
 \hat{p} - z \sqrt{\frac{\hat{p}\hat{q}}{n}} & < p < \hat{p} + z \sqrt{\frac{\hat{p}\hat{q}}{n}} \\
 \mathbf{117.} \quad |\mu - \bar{x}| & < \frac{z\sigma}{\sqrt{n}}
 \end{aligned}$$

$$\begin{aligned}
 -\frac{z\sigma}{\sqrt{n}} & < \mu - \bar{x} < \frac{z\sigma}{\sqrt{n}} \\
 \bar{x} - \frac{z\sigma}{\sqrt{n}} & < \mu < \bar{x} + \frac{z\sigma}{\sqrt{n}} \\
 \mathbf{118.}
 \end{aligned}$$

Problem Recognition Exercises: Recognizing and Solving Equations and Inequalities

1. a. Equation in quadratic form and a polynomial equation

b. Let $u = x^2 - 5$

$$\begin{aligned}
 (x^2 - 5)^2 - 5(x^2 - 5) + 4 &= 0 \\
 u^2 - 5u + 4 &= 0 \\
 (u - 4)(u - 1) &= 0 \\
 u = 4 & \text{ or } u = 1 \\
 x^2 - 5 = 4 & \text{ or } x^2 - 5 = 1 \\
 x^2 = 9 & \text{ or } x^2 = 6 \\
 x = \pm 3 & \text{ or } x = \pm \sqrt{6} \\
 \{\pm 3, \pm \sqrt{6}\}
 \end{aligned}$$

2. a. Absolute value inequality

$$\begin{aligned}
 2 &\leq |3x - 1| - 6 \\
 8 &\leq |3x - 1| \\
 |3x - 1| &\geq 8 \\
 3x - 1 &\leq -8 \quad \text{or} \quad 3x - 1 \geq 8 \\
 3x &\leq -7 \quad \text{or} \quad 3x \geq 9 \\
 t &\leq -\frac{7}{3} \quad \text{or} \quad t \geq 3
 \end{aligned}$$

b.

$$\left(-\infty, -\frac{7}{3}\right] \cup \left[3, \infty\right)$$

5. a. Rational equation

3. a. Radical equation

$$\begin{aligned}
 \sqrt[3]{2y - 5} - 4 &= -1 \\
 \sqrt[3]{2y - 5} &= 3 \\
 (\sqrt[3]{2y - 5})^3 &= (3)^3 \\
 2y - 5 &= 27 \\
 2y &= 32 \\
 y &= 16
 \end{aligned}$$

b.

$$\{16\}$$

4. a. Absolute value equation

$$\begin{aligned}
 -9|3z - 7| + 1 &= 4 \\
 -9|3z - 7| &= 3
 \end{aligned}$$

$$\begin{aligned}
 |3z - 7| &= -\frac{1}{3} \\
 \mathbf{b.} \quad \{ & \}
 \end{aligned}$$

$$\begin{aligned}\frac{2}{w-3} + \frac{5}{w+1} &= 1 \\ (w-3)(w+1)\left(\frac{2}{w-3} + \frac{5}{w+1}\right) &= (w-3)(w+1)(1) \\ 2(w+1) + 5(w-3) &= (w-3)(w+1) \\ 2w + 2 + 5w - 15 &= w^2 - 2w - 3 \\ 7w - 13 &= w^2 - 2w - 3\end{aligned}$$

b. $0 = w^2 - 9w + 10$

$$w = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(10)}}{2(1)} = \frac{9 \pm \sqrt{41}}{2}$$

$$\left\{ \frac{9 \pm \sqrt{41}}{2} \right\}$$

6. a. Polynomial equation

$$48x^3 + 80x^2 - 3x - 5 = 0$$

$$48x^3 - 3x + 80x^2 - 5 = 0$$

$$3x(16x^2 - 1) + 5(16x^2 - 1) = 0$$

$$(3x + 5)(16x^2 - 1) = 0$$

b. $(3x + 5)(4x + 1)(4x - 1) = 0$

$$x = -\frac{5}{3} \text{ or } x = -\frac{1}{4} \text{ or } x = \frac{1}{4}$$

$$\left\{ -\frac{5}{3}, \pm \frac{1}{4} \right\}$$

7. a. Compound inequality

$$-2(m+2) < -m+5 \text{ and } 6 \geq m+3$$

$$-2m-4 < -m+5 \text{ and } 3 \geq m$$

$$-m < 9 \text{ and } m \leq 3$$

b. $m > -9$

$$(-9, 3]$$

8. a. Compound inequality

$$6 \leq -2c + 8 \text{ or } \frac{1}{3}c - 2 < 2$$

$$-2 \leq -2c \text{ or } \frac{1}{3}c < 4$$

b. $1 \geq c \text{ or } c < 12$

$$(-\infty, 12)$$

9. a. Quadratic equation

$$(2p+1)(p+5) = 2p+40$$

$$2p^2 + 11p + 5 = 2p + 40$$

$$2p^2 + 9p - 35 = 0$$

b. $(2p-5)(p+7) = 0$

$$p = \frac{5}{2} \text{ or } p = -7$$

$$\left\{ \frac{5}{2}, -7 \right\}$$

10. a. Linear equation

$$2x(x-4) + 7 =$$

b. $2x^2 - 3[x+5-(2+x)]$

$$2x^2 - 8x + 7 = 2x^2 - 3[x+5-2-x]$$

$$-8x + 7 = -3[3]$$

$$-8x + 7 = -9$$

$$-8x = -16$$

$$x = 2$$

$$\{2\}$$

11. a. Linear inequality

$$\begin{aligned}\frac{a-4}{2} - \frac{3a+1}{4} &\leq -\frac{a}{8} \\ 8\left(\frac{a-4}{2} - \frac{3a+1}{4}\right) &\leq 8\left(-\frac{a}{8}\right) \\ 4(a-4) - 2(3a+1) &\leq -a \\ 4a - 16 - 6a - 2 &\leq -a \\ -a &\leq 18\end{aligned}$$

b. $a \geq -18$

$$[-18, \infty)$$

12. a. Quadratic equation

$$3x^2 + 11 = 4$$

$$3x^2 = -7$$

$$x^2 = -\frac{7}{3}$$

b.

$$\begin{aligned}x &= \pm \sqrt{-\frac{7}{3}} \\ &= \pm i \frac{\sqrt{7}}{\sqrt{3}} \\ &= \pm i \frac{\sqrt{7} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} \\ &= \pm \frac{\sqrt{21}}{3} i\end{aligned}$$

$$\left\{ \pm \frac{\sqrt{21}}{3} i \right\}$$

13. a. Compound inequality

$$-1 \leq \frac{6-x}{-5} \leq 7$$

$$-1 \leq \frac{x-6}{5} \leq 7$$

$$-5 \leq x - 6 \leq 35$$

b. $1 \leq x \leq 41$

$$[1, 41]$$

14. a. Radical equation

b. $5 = \sqrt{5+2n} + \sqrt{2+n}$

$$5 - \sqrt{2+n} = \sqrt{5+2n}$$

$$(5 - \sqrt{2+n})^2 = (\sqrt{5+2n})^2$$

$$25 - 10\sqrt{2+n} + 2 + n = 5 + 2n$$

$$-10\sqrt{2+n} = n - 22$$

$$(10\sqrt{2+n})^2 = (22-n)^2$$

$$100(2+n) = 484 - 44n + n^2$$

$$200 + 100n = 484 - 44n + n^2$$

$$n^2 - 144n + 284 = 0$$

$$(n-142)(n-2) = 0$$

$$n = 142 \text{ or } n = 2$$

Check: $n = 142$

$$5 = \sqrt{5+2n} + \sqrt{2+n}$$

$$5 = \sqrt{5+2(142)} + \sqrt{2+(142)}$$

$$5 = \sqrt{289} + \sqrt{144}$$

$$5 = 17 + 12$$

$$5 = 29 \text{ false}$$

Check: $n = 2$

$$5 = \sqrt{5+2n} + \sqrt{2+n}$$

$$5 = \sqrt{5+2(2)} + \sqrt{2+(2)}$$

$$5 = \sqrt{9} + \sqrt{4}$$

$$5 = 3 + 2$$

$$5 = 5 \checkmark \text{ true}$$

$\{2\}$; The value 142 does not check.

15. a. Absolute value equation

$$|4x-5| = |3x-2|$$

b.

$$4x-5 = 3x-2 \quad \text{or} \quad 4x-5 = -(3x-2)$$

$$x = 3 \quad \text{or} \quad 7x = 7$$

$$x = 1$$

$$\{1, 3\}$$

16. a. Rational equation

$$\frac{1}{d} - \frac{1}{2d-1} + \frac{2d}{2d-1} = 0$$

$$\frac{1}{d} + \frac{2d-1}{2d-1} = 0$$

$$\frac{1}{d} + 1 = 0$$

$$\frac{1}{d} = -1$$

$$d = -1$$

b.

$$\{-1\}$$

17. a. Absolute value inequality

$$-|x+4|+8>3$$

$$-|x+4|>-5$$

$$|x+4|<5$$

$$-5<x+4<5$$

b. $-9<x<1$

$$(-9, 1)$$

18. a. Radical equation and an equation in

quadratic form

b. Let $u = \sqrt{y}$

$$y - 4\sqrt{y} - 12 = 0$$

$$u^2 - 4u - 12 = 0$$

$$(u+2)(u-6) = 0$$

$$u = -2 \quad \text{or} \quad u = 6$$

$$\sqrt{y} = -2 \quad \text{or} \quad \sqrt{y} = 6$$

$$(\sqrt{y})^2 = (-2)^2 \quad \text{or} \quad (\sqrt{y})^2 = (6)^2$$

$$y = 4 \quad \text{or} \quad y = 36$$

Check: $y = 4$

$$y - 4\sqrt{y} - 12 = 0$$

$$(4) - 4\sqrt{(4)} - 12 = 0$$

$$4 - 8 - 12 = 0$$

$$-16 = 0 \text{ false}$$

Check: $y = 36$

$$y - 4\sqrt{y} - 12 = 0$$

$$(36) - 4\sqrt{(36)} - 12 = 0$$

$$36 - 24 - 12 = 0$$

$$0 = 0 \checkmark \text{ true}$$

$\{36\}$; The value 4 does not check.

19. a. Radical equation

$$c^{2/3} = 16$$

$$(c^{2/3})^{3/2} = \pm(16)^{3/2}$$

b. $c = \pm 64$

$$\{\pm 64\}$$

20. a. Absolute value inequality

$$2|z-14|+8>4$$

$$2|z-14|>-4$$

b. $|z-14|>-2$

$$(-\infty, \infty)$$

Equations and Inequalities for Calculus

$$\frac{2x}{25} + \frac{2y}{9}y' = 0$$

$$\frac{2y}{9}y' = -\frac{2x}{25}$$

$$y' = -\frac{9x}{25y}$$

1.

$$2xy^3 + 3x^2y^2y' - y' = 1$$

$$3x^2y^2y' - y' = 1 - 2xy^3$$

$$y'(3x^2y^2 - 1) = 1 - 2xy^3$$

$$y' = \frac{1 - 2xy^3}{3x^2y^2 - 1}$$

2.

$$3. \quad 3y^2y' + 6xy + 3x^2y' = 2y^2 + 4xyy'$$

$$3y^2y' - 4xyy' + 3x^2y' = 2y^2 - 6xy$$

$$y'(3y^2 - 4xy + 3x^2) = 2y(y - 3x)$$

$$y' = \frac{2y(y - 3x)}{(3y^2 - 4xy + 3x^2)}$$

$$4. \quad 3(x + y)^2 + 3(x + y)^2 y' - 3y^2 y' = 3x^2$$

$$3(x + y)^2 y' - 3y^2 y' = 3x^2 - 3(x + y)^2$$

$$y' \{ (x + y)^2 - y^2 \} = x^2 - (x + y)^2$$

$$y'(x^2 + 2xy) = -(y^2 + 2xy)$$

$$y' = -\frac{(y^2 + 2xy)}{(x^2 + 2xy)}$$

$$y' = -\frac{y(y + 2x)}{x(x + 2y)}$$

$$5. \quad 2x\sqrt{2x-3} + x^2 \left(\frac{1}{2} \right) \frac{1}{\sqrt{2x-3}} (2)$$

$$= 2x\sqrt{2x-3} + \frac{x^2}{\sqrt{2x-3}}$$

$$= \frac{2x(2x-3) + x^2}{\sqrt{2x-3}}$$

$$= \frac{x}{\sqrt{2x-3}} \{ 2(2x-3) + x \}$$

$$= \frac{x(5x-6)}{\sqrt{2x-3}}$$

$$2x(2x-7)^{\frac{1}{2}} - x^2 \left(\frac{1}{2} \right) (2x-7)^{-\frac{1}{2}} (2)$$

$$6. \quad \frac{\left[(2x-7)^{\frac{1}{2}} \right]^2}{\left[(2x-7)^{\frac{1}{2}} \right]^2}$$

$$= \frac{x(2x-7)^{-\frac{1}{2}} \{ 2(2x-7) - x \}}{\left[(2x-7)^{\frac{1}{2}} \right]^2}$$

$$= \frac{x(2x-7)^{-\frac{1}{2}} \{ 3x-14 \}}{\left[(2x-7)^{\frac{1}{2}} \right]^2}$$

$$= \frac{x(3x-14)}{(2x-7)^{\frac{3}{2}}}$$

$$(1)(x^2-9)^{\frac{1}{2}} - x \left(\frac{1}{2} \right) (x^2-9)^{-\frac{1}{2}} (2x)$$

$$\frac{\left[(x^2-9)^{\frac{1}{2}} \right]^2}{\left[(x^2-9)^{\frac{1}{2}} \right]^2}$$

7.

$$(x^2-9)^{-\frac{1}{2}} \left\{ (x^2-9) - x \left(\frac{1}{2} \right) (2x) \right\}$$

$$= \frac{\left[(x^2-9)^{\frac{1}{2}} \right]^2}{\left[(x^2-9)^{\frac{1}{2}} \right]^2}$$

$$= \frac{(x^2-9)^{-\frac{1}{2}} \{ (x^2-9) - x^2 \}}{\left[(x^2-9)^{\frac{1}{2}} \right]^2}$$

$$= \frac{(x^2-9)^{-\frac{1}{2}} (-9)}{\left[(x^2-9)^{\frac{1}{2}} \right]^2}$$

$$= -\frac{9}{(x^2-9)^{\frac{3}{2}}}$$

$$\frac{4x(4x-5) - 2x^2(4)}{(4x-5)^2}$$

$$= \frac{4x \{ (4x-5) - 2x \}}{(4x-5)^2}$$

$$= \frac{4x(2x-5)}{(4x-5)^2}$$

8. a.

$$\frac{4x(2x-5)}{(4x-5)^2} = 0$$

$$4x = 0 \quad \text{or} \quad 2x - 5 = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{5}{2}$$

b.

$$\frac{4x(2x-5)}{(4x-5)^2}$$

c.undefined for $(4x-5)^2 = 0$

$$4x - 5 = 0$$

$$x = \frac{5}{4}$$

$$\frac{-6x(6x+1) - (-3x^2)(6)}{(6x+1)^2}$$

9. a.

$$= -\frac{6x\{(6x+1) - 3x\}}{(6x+1)^2}$$

$$= -\frac{6x(3x+1)}{(6x+1)^2}$$

$$-\frac{6x(3x+1)}{(6x+1)^2} = 0$$

$$6x = 0 \quad \text{or} \quad 3x + 1 = 0$$

$$x = 0 \quad x = -\frac{1}{3}$$

b.

$$-\frac{6x(3x+1)}{(6x+1)^2}$$

c.undefined for $(6x+1)^2 = 0$

$$6x + 1 = 0$$

$$x = -\frac{1}{6}$$

10. a.

$$\sqrt{4-x^2} - x\left(\frac{1}{2}\right)\frac{1}{\sqrt{4-x^2}}(2x)$$

$$= \frac{1}{\sqrt{4-x^2}}\{(4-x^2) - x^2\}$$

$$= \frac{2(2-x^2)}{\sqrt{4-x^2}}$$

$$\frac{2(2-x^2)}{\sqrt{4-x^2}} = 0$$

$$2-x^2 = 0$$

$$\mathbf{b.} \quad x = \pm\sqrt{2}$$

$$\frac{2(2-x^2)}{\sqrt{4-x^2}}$$

c.undefined for $(4-x^2)^2 = 0$

$$4-x^2 = 0$$

$$x = \pm\sqrt{2}$$

$$\mathbf{11.} \quad \left|\frac{x+1}{2}\right| < 1$$

$$\frac{x+1}{2} < 1 \quad -\left(\frac{x+1}{2}\right) < 1$$

$$x+1 < 2 \quad -(x+1) < 2$$

$$x < 1 \quad \text{or} \quad -x < 3$$

$$(-3, 1) \quad x > -3$$

$$\mathbf{12.} \quad \left|-\frac{x}{2}\right| < 1$$

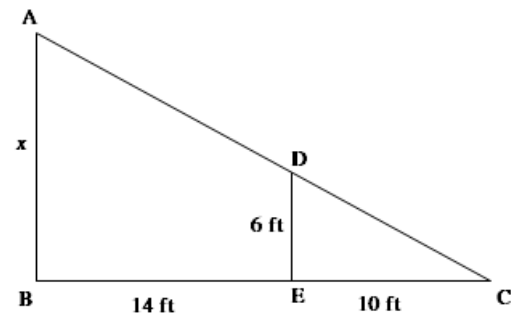
$$-\frac{x}{2} < 1 \quad -\left(-\frac{x}{2}\right) < 1$$

$$-x < 2 \quad x < 2$$

$$x > -2 \quad \text{or} \quad x < 2$$

$$(-2, 2)$$

13. Let x be the height of the lamppost. In two similar triangles, the ratio of corresponding sides is equal.



Hence,

$$\frac{x}{6} = \frac{(10+14)}{10}$$

$$\frac{x}{6} = \frac{24}{10}$$

$$x = \frac{144}{10}$$

$$x = 14.4 \text{ ft}$$

The height of the lamppost is 14.4 ft.

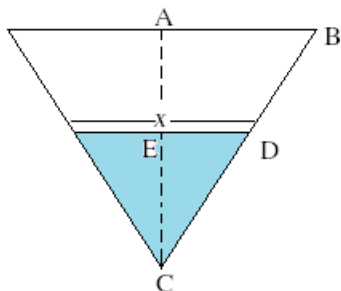
- 14.** The perpendicular of an equivalent triangle divide its base in two halves.

$$AB = \frac{1}{2} \text{ m}$$

Therefore

The height of the equivalent triangle is calculated using Pythagorean theorem

as $\frac{\sqrt{3}}{2}$ m
 ΔABC and ΔEDC are
 similar triangles.



$$\frac{x}{1} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$x = \frac{1}{\sqrt{3}}$$

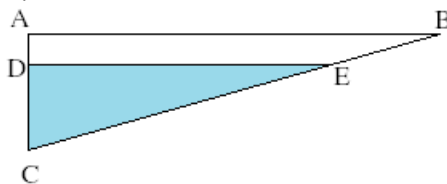
$$V = Ah$$

$$V = \frac{1}{2}bh(1)$$

$$V = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{2}\right)(3)$$

$$V = \frac{\sqrt{3}}{4} \text{ m}^3$$

15. Let DE be x ft. ΔABC is similar to ΔDCE .



$$\frac{x}{4} = \frac{50}{5}$$

$$x = 40 \text{ ft}$$

$$V = Ah$$

$$V = \frac{1}{2}bx(20)$$

$$V = \frac{1}{2}(4)(40)(20)$$

$$V = 1600 \text{ ft}^3$$

Chapter 1 Review Exercises

$$\frac{3}{x^2 - 4} + \frac{4}{2x - 7} = \frac{2}{3}$$

$$\frac{3}{(x+2)(x-2)} + \frac{4}{2x-7} = \frac{2}{3}$$

1. $x \neq 2, x \neq -2, x \neq \frac{7}{2}$

2. $-8(t-4)+7=4[t-3(1-t)]+6$

$$-8t+32+7=4(t-3+3t)+6$$

$$-8t+39=4(4t-3)+6$$

$$-8t+39=16t-12+6$$

$$-8t+39=16t-6$$

$$45=24t$$

$$t = \frac{45}{24} = \frac{15}{8}$$

$$\frac{4}{5}x - \frac{2}{3} = \frac{7}{10}x - 2$$

$$30\left(\frac{4}{5}x - \frac{2}{3}\right) = 30\left(\frac{7}{10}x - 2\right)$$

$$24x - 20 = 21x - 60$$

$$3x = -40$$

$$x = -\frac{40}{3}$$

3.

$$\left\{-\frac{40}{3}\right\}$$

$$\begin{aligned}\frac{m+2}{3} - \frac{m-4}{4} &= \frac{m+1}{6} - 1 \\ 12\left(\frac{m+2}{3} - \frac{m-4}{4}\right) &= 12\left(\frac{m+1}{6} - 1\right) \\ 4(m+2) - 3(m-4) &= 2(m+1) - 12 \\ 4m+8-3m+12 &= 2m+2-12 \\ m+20 &= 2m-10 \\ 30 &= m\end{aligned}$$

4.

$$\{30\}$$

$$\begin{aligned}x-5+2(x-4) &= 3(x+1)-5 \\ x-5+2x-8 &= 3x+3-5 \\ 3x-13 &= 3x-2 \\ -13 &= -2\end{aligned}$$

5.

$$\{ \}$$

$$\begin{aligned}0.2x+1.6 &= x-0.8(x-2) \\ 0.2x+1.6 &= x-0.8x+1.6 \\ 0.2x+1.6 &= 0.2x+1.6\end{aligned}$$

6.

$$0=0$$

□

9.

$$\begin{aligned}\frac{1}{m-1} &= \frac{5m}{m^2+3m-4} - \frac{3}{m+4} \\ \frac{1}{m-1} &= \frac{5m}{(m-1)(m+4)} - \frac{3}{m+4} \\ (m-1)(m+4)\left(\frac{1}{m-1}\right) &= \left[\frac{(m-1)(m+4)}{(m-1)(m+4)} - \frac{3}{m+4}\right] \\ 1(m+4) &= 5m-3(m-1) \\ m+4 &= 5m-3m+3 \\ 1 &= m\end{aligned}$$

 $\{ \}$; The value 1 does not check.

$$\begin{aligned}4x-3y &= 6 \\ -3y &= -4x+6 \\ y &= \frac{4}{3}x-2\end{aligned}$$

10.

$$\begin{aligned}(y-4)^2 &= (y+3)^2 \\ y^2-8y+16 &= y^2+6y+9 \\ 7 &= 14y \\ \frac{7}{14} &= y \\ \frac{1}{2} &= y\end{aligned}$$

7.

$$\left\{\frac{1}{2}\right\}$$

8.

$$\begin{aligned}\frac{x+3}{5x}+2 &= \frac{x-4}{x} \\ 5x\left(\frac{x+3}{5x}+2\right) &= 5x\left(\frac{x-4}{x}\right) \\ x+3+10x &= 5x-20 \\ 11x+3 &= 5x-20 \\ 6x &= -23 \\ x &= -\frac{23}{6}\end{aligned}$$

$$\left\{-\frac{23}{6}\right\}$$

$$11. \quad 2t_a - t_1 = t_2 \text{ or } t_2 = 2t_a - t_1$$

$$\begin{aligned}
 4x + 6y &= ax + c \\
 4x - ax &= c - 6y \\
 x(4 - a) &= c - 6y \\
 x &= \frac{c - 6y}{4 - a} \text{ or } x = \frac{6y - c}{a - 4}
 \end{aligned}$$

12.

$$\begin{aligned}
 A &= \frac{1}{41}c + \frac{1}{36}h \\
 11 &= \frac{1}{41}c + \frac{1}{36}(288) \\
 11 &= \frac{1}{41}c + 8 \\
 3 &= \frac{1}{41}c
 \end{aligned}$$

13. $123 = c$ or $c = 123$ mi

- 14.** Let x represent the amount invested in the international fund. Then, $(12,000 - x)$ is the amount invested in the real estate fund.

	International	Real Estate Fund	Total
Principal	x	$12,000 - x$	
Interest ($I =$	$x(0.082)(1)$	$(12,000 - x)(0.015)(1)$	749.50

$$x(0.082) + (12,000 - x)(0.015) = 749.50$$

$$0.082x + 180 - 0.015x = 749.50$$

$$0.067x + 180 = 749.50$$

$$0.067x = 569.50$$

$$x = 8500$$

$$12,000 - x = 12,000 - 8500$$

$$= 3500$$

Shawna invested \$8500 in the international fund and \$3500 in the real estate fund.

- 15.** Let x represent the amount invested in the 10-yr Treasury note. Then, $(x + 4000)$ is the amount invested in the 15-yr bond.

	10-yr Note	15-yr Bond	Total
Principal	x	$x + 4000$	
Interest ($I =$	$x(0.035)(10)$	$(x + 4000)(0.041)(15)$	10,180

$$x(0.035)(10) + (x + 4000)(0.041)(15) = 10,180$$

$$0.35x + 0.615x + 2460 = 10,180$$

$$0.965x + 2460 = 10,180$$

$$0.965x = 7720$$

$$x = 8000$$

$$x + 4000 = 8000 + 4000$$

$$= 12,000$$

Cassandra invested \$8000 in the Treasury note and \$12,000 in the bond.

- 16.** Let x represent the amount of the 20% acid solution (in cubic centimetres). 100 cc is the amount of the 60% acid solution. Therefore, $(x + 100)$ is the amount of the resulting 25% acid solution.

	20%	60%	25%
--	------------	------------	------------

Amount of	x	100	$x + 100$
Pure Acid	$0.2x$	$0.6(100)$	$0.25(x + 100)$

$$0.2x + 0.6(100) = 0.25(x + 100)$$

$$0.2x + 60 = 0.25x + 25$$

$$35 = 0.05x$$

$$700 = x$$

700 cc of 20% acid solution should be mixed with the 60% acid solution.

- 17.** Let x represent the amount of the pure sand (in cubic feet). 250 ft² is the amount of the concrete mix that is 50% sand. Therefore, $(x + 250)$ is the amount of the resulting 70% sand mixture.

	100%	50%	70% Sand
Amount of	x	250	$x + 250$
Pure Sand	x	$0.5(250)$	$0.7(x + 250)$

$$x + 0.5(250) = 0.7(x + 250)$$

$$x + 125 = 0.7x + 175$$

$$0.3x = 50$$

$$x = 166\frac{2}{3}$$

$166\frac{2}{3}$ ft² of sand should be mixed with the 50% sand mixture.

- 18.** Let x represent the distance from Kevin's place of work to his home.

	Distan	Rat	Tim
To Work	x	45	$\frac{x}{45}$
To Home	x	30	$\frac{x}{30}$

$$\frac{x}{45} + \frac{x}{30} = \frac{50}{60}$$

$$180\left(\frac{x}{45} + \frac{x}{30}\right) = 180\left(\frac{50}{60}\right)$$

$$4x + 6x = 150$$

$$10x = 150$$

$$x = 15$$

The distance is 15 mi.

- 19.** Let x represent the speed of the boat traveling north. Then, $(x + 6)$ is the speed of the boat traveling south.

	Distan	Rat	Tim
--	---------------	------------	------------

Northbound	$3x$	x	3
Southbound	$3(x + 6)$	$x + 6$	3

$$3x + 3(x + 6) = 66$$

$$3x + 3x + 18 = 66$$

$$6x + 18 = 66$$

$$6x = 48$$

$$x = 8$$

$$x + 6 = 8 + 6 = 14$$

The northbound boat travels 8 mph and the southbound boat travels 14 mph.

20. a. $C_A = 300 + 4x$

b. $C_A = 360 + 2x$

$$C_A = C_B$$

$$300 + 4x = 360 + 2x$$

$$2x = 60$$

c. $x = 30$

If Monique takes 30 classes during the year, the cost for each gym will be the same.

21. a. $C = 5x$

$$C = 80$$

$$5x = 80$$

b. $x = 16$

The dancer will save money on the 17th dance during a 3-month period.

22. Let t represent the time it takes Petra and Dawn to typeset the 150-page manuscript (which is equivalent to three 50-page manuscripts) if they work together.

23. Let t represent the time it takes the second pump to drain the pond by itself.

$$\frac{1 \text{ job}}{22 \text{ hr}} + \frac{1 \text{ job}}{t \text{ hr}} = \frac{1 \text{ job}}{10 \text{ hr}}$$

$$110t \left(\frac{1}{22} + \frac{1}{t} \right) = 110t \left(\frac{1}{10} \right)$$

$$5t + 110 = 11t$$

$$110 = 6t$$

$$t = \frac{55}{3} \approx 18.3 \text{ hr}$$

24. Let x represent the number of female officers. Then, $(x + 60)$ represents the number of male officers.

$$\frac{x + 60}{x} = \frac{10}{7}$$

$$7x \left(\frac{x + 60}{x} \right) = 7x \left(\frac{10}{7} \right)$$

$$7(x + 60) = 10x$$

$$7x + 420 = 10x$$

$$420 = 3x$$

$$140 = x$$

$$x + 60 = 140 + 60 = 200$$

There are 140 females and 200 males.

25. Let x represent the number of turtles in

the pond.

$$\frac{12}{x} = \frac{3}{36}$$

$$3x = 432$$

$$x = 144$$

There are approximately 144 turtles in the pond.

26. $-\sqrt{-169} = -i\sqrt{169} = -13i$

27. $\sqrt{-12} = i\sqrt{12} = 2i\sqrt{3}$

$$\sqrt{-16} \cdot \sqrt{-4} = i\sqrt{16} \cdot i\sqrt{4}$$

$$= 4i \cdot 2i$$

$$= 8i^2$$

$$= 8(-1)$$

$$= -8$$

28.

29. a. Real part: 3; Imaginary part: -7

b. Real part: 0; Imaginary part: **2**

$$\begin{aligned}
 i^{35} &= i^{32} \cdot i^3 \\
 &= (1) \cdot i^3 \\
 \mathbf{30. a.} \quad &= -i \\
 \mathbf{b.} \quad i^{56} &= 1 \\
 i^{62} &= i^{60} \cdot i^2 \\
 &= (1) \cdot i^2 \\
 \mathbf{c.} \quad &= -1 \\
 i^{17} &= i^{16} \cdot i^1 \\
 &= (1) \cdot i^1 \\
 \mathbf{d.} \quad &= i \\
 \mathbf{e.} \quad i^{-5} &= i^{-8} \cdot i^3 = (1) \cdot i^3 = -i \\
 \mathbf{31.} \quad &\left(\frac{2}{3} + \frac{3}{5}i\right) - \left(\frac{1}{6} + \frac{2}{5}i\right) \\
 &= \left(\frac{20}{30} + \frac{18}{30}i\right) - \left(\frac{5}{30} + \frac{12}{30}i\right) \\
 &= \left(\frac{20}{30} - \frac{5}{30}\right) + \left(\frac{18}{30} - \frac{12}{30}\right)i \\
 &= \frac{15}{30} + \frac{6}{30}i = \frac{1}{2} + \frac{1}{5}i \\
 3i(7+2i) &= 21i + 6i^2 \\
 &= 21i + 6(-1) \\
 &= -6 + 21i \\
 \mathbf{32.} \quad \sqrt{-5}(\sqrt{11} + \sqrt{-3}) &= i\sqrt{5}(\sqrt{11} + i\sqrt{3}) \\
 &= i\sqrt{55} + i^2\sqrt{15} \\
 &= i\sqrt{55} + (-1)\sqrt{15} \\
 &= -\sqrt{15} + i\sqrt{55} \\
 \mathbf{33.} \quad & \\
 \mathbf{34.} \quad (4-7i)(5+i) &= 20 + 4i - 35i - 7i^2 \\
 &= 20 - 31i - 7(-1) \\
 &= 20 - 31i + 7 \\
 &= 27 - 31i \\
 (4-6i)^2 &= (4)^2 - 2(4)(6i) + (6i)^2 \\
 &= 16 - 48i + 36i^2 \\
 &= 16 - 48i + 36(-1) \\
 &= 16 - 48i - 36 \\
 \mathbf{35.} \quad &= -20 - 48i
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{36.} \quad &(2 + \sqrt{-2})(4 + \sqrt{-2}) \\
 &= (2 + i\sqrt{2})(4 + i\sqrt{2}) \\
 &= 8 + 2i\sqrt{2} + 4i\sqrt{2} + 2i^2 \\
 &= 8 + 6i\sqrt{2} + 2(-1) \\
 &= 6 + 6i\sqrt{2} \\
 (8-3i)(8+3i) &= (8)^2 + (3)^2 \\
 &= 64 + 9 \\
 \mathbf{37.} \quad &= 73 \\
 \mathbf{38.} \quad \frac{4+3i}{3-i} &= \frac{(4+3i)(3+i)}{(3-i)(3+i)} \\
 &= \frac{12+4i+9i+3i^2}{3^2+1^2} \\
 &= \frac{12+13i-3}{9+1} \\
 &= \frac{9+13i}{10} \\
 &= \frac{9}{10} + \frac{13}{10}i \\
 \mathbf{39.} \quad (6-\sqrt{5}i)^{-1} &= \frac{1}{6-\sqrt{5}i} \\
 &= \frac{1(6+\sqrt{5}i)}{(6-\sqrt{5}i)(6+\sqrt{5}i)} \\
 &= \frac{6+\sqrt{5}i}{6^2+5} \\
 &= \frac{6+\sqrt{5}i}{36+5} \\
 &= \frac{6+\sqrt{5}i}{41} \\
 &= \frac{6}{41} + \frac{\sqrt{5}}{41}i \\
 \frac{7}{4i} &= \frac{7 \cdot i}{4i \cdot i} \\
 &= \frac{7i}{4i^2} \\
 &= \frac{7i}{4(-1)} \\
 &= -\frac{7}{4}i \\
 \mathbf{40.} \quad &
 \end{aligned}$$

$$\begin{aligned}
 3y^2 - 4y &= 8 - 6y \\
 3y^2 + 2y - 8 &= 0 \\
 (3y - 4)(y + 2) &= 0 \\
 3y - 4 &= 0 \quad \text{or} \quad y + 2 = 0 \\
 3y &= 4 & y &= -2 \\
 y &= \frac{4}{3}
 \end{aligned}$$

41.

$$\begin{aligned}
 &\left\{ \frac{4}{3}, -2 \right\} \\
 (2v + 3)^2 - 1 &= 6 \\
 (2v + 3)^2 &= 7 \\
 2v + 3 &= \pm\sqrt{7} \\
 2v &= -3 \pm \sqrt{7} \\
 v &= \frac{-3 \pm \sqrt{7}}{2}
 \end{aligned}$$

42.

$$\begin{aligned}
 &\left\{ \frac{-3 \pm \sqrt{7}}{2} \right\} \\
 10t^2 + 1210 &= 0 \\
 \frac{10t^2}{10} + \frac{1210}{10} &= \frac{0}{10} \\
 t^2 + 121 &= 0 \\
 t^2 &= -121
 \end{aligned}$$

43.

$$\begin{aligned}
 t &= \pm\sqrt{-121} = \pm 11i \\
 &\{\pm 11i\}
 \end{aligned}$$

$$2d(d - 3) = 1 + 4d$$

$$2d^2 - 6d = 1 + 4d$$

$$\begin{aligned}
 \mathbf{44.} \quad 2d^2 - 10d - 1 &= 0 \\
 a &= 2, b = -10, c = -1
 \end{aligned}$$

$$\begin{aligned}
 d &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(-1)}}{2(2)} \\
 &= \frac{10 \pm \sqrt{100 + 8}}{4} = \frac{10 \pm \sqrt{108}}{4} \\
 &= \frac{10 \pm 6\sqrt{3}}{4} = \frac{5 \pm 3\sqrt{3}}{2}
 \end{aligned}$$

$$\left\{ \frac{5 \pm 3\sqrt{3}}{2} \right\}$$

$$x^2 - 5 = (x + 2)(x - 4)$$

$$x^2 - 5 = x^2 - 4x + 2x - 8$$

$$x^2 - 5 = x^2 - 2x - 8$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

45.

$$\left\{ -\frac{3}{2} \right\}$$

46.

$$\begin{aligned}
 \frac{1}{5}x^2 - \frac{2}{3} &= \frac{7}{15}x \\
 15\left(\frac{1}{5}x^2 - \frac{2}{3}\right) &= 15\left(\frac{7}{15}x\right) \\
 3x^2 - 10 &= 7x
 \end{aligned}$$

$$3x^2 - 7x - 10 = 0$$

$$(3x - 10)(x + 1) = 0$$

$$3x - 10 = 0 \quad \text{or} \quad x + 1 = 0$$

$$3x = 10 \quad x = -1$$

$$x = \frac{10}{3}$$

$$\left\{ \frac{10}{3}, -1 \right\}$$

47.

$$\begin{aligned}
 x^2 + 18x + n &= x^2 + 18x + \left[\frac{1}{2}(18) \right]^2 \\
 &= x^2 + 18x + (9)^2 \\
 &= x^2 + 18x + 81 = (x + 9)^2 \\
 n &= 81; (x + 9)^2
 \end{aligned}$$

$$48. \quad x^2 + \frac{2}{7}x + n$$

$$\begin{aligned} &= x^2 + \frac{2}{7}x + \left[\frac{1}{2} \left(\frac{2}{7} \right) \right]^2 \\ &= x^2 + \frac{2}{7}x + \left(\frac{1}{7} \right)^2 \\ &= x^2 + \frac{2}{7}x + \frac{1}{49} = \left(x + \frac{1}{7} \right)^2 \end{aligned}$$

$$n = \frac{1}{49}; \left(x + \frac{1}{7} \right)^2$$

$$x^2 - 10x = -9$$

$$x^2 - 10x + 9 = 0$$

$$(x-1)(x-9) = 0$$

$$x-1=0 \quad \text{or} \quad x-9=0$$

$$x=1 \quad \quad \quad x=9$$

49. a.

$$\{1, 9\}$$

b.

$$x^2 - 10x = -9$$

$$x^2 - 10x + \left[\frac{1}{2}(-10) \right]^2 = -9 + \left[\frac{1}{2}(-10) \right]^2$$

$$x^2 - 10x + 25 = -9 + 25$$

$$(x-5)^2 = 16$$

$$x-5 = \pm\sqrt{16}$$

$$x = 5 \pm 4$$

$$x = 5-4 \quad \text{or} \quad x = 5+4$$

$$x = 1 \quad \quad \quad x = 9$$

$$\{1, 9\}$$

$$x^2 - 10x = -9$$

c. $x^2 - 10x + 9 = 0$

$$a = 1, b = -10, c = 9$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(9)}}{2(1)} \\ &= \frac{10 \pm \sqrt{100 - 36}}{2} = \frac{10 \pm \sqrt{64}}{2} = \frac{10 \pm 8}{2} \\ &= 5 \pm 4 = 1 \text{ or } 9 \\ &\{1, 9\} \end{aligned}$$

$$2x^2 - 3x - 5 = 0$$

$$(2x-5)(x+1) = 0$$

$$2x-5=0 \quad \text{or} \quad x+1=0$$

$$2x=5 \quad \quad \quad x=-1$$

$$x = \frac{5}{2}$$

50. a.

$$\left\{ \frac{5}{2}, -1 \right\}$$

b.

$$2x^2 - 3x - 5 = 0$$

$$\frac{2x^2}{2} - \frac{3x}{2} - \frac{5}{2} = \frac{0}{2}$$

$$x^2 - \frac{3}{2}x = \frac{5}{2}$$

$$x^2 - \frac{3}{2}x + \left[\frac{1}{2} \left(-\frac{3}{2} \right) \right]^2 = \frac{5}{2} + \left[\frac{1}{2} \left(-\frac{3}{2} \right) \right]^2$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{40}{16} + \frac{9}{16}$$

$$\left(x - \frac{3}{4} \right)^2 = \frac{49}{16}$$

$$x - \frac{3}{4} = \pm\sqrt{\frac{49}{16}}$$

$$x = \frac{3}{4} \pm \frac{7}{4}$$

$$x = \frac{3}{4} + \frac{7}{4} \quad \text{or} \quad x = \frac{3}{4} - \frac{7}{4}$$

$$x = \frac{10}{4} \quad \quad \quad x = \frac{-4}{4}$$

$$x = \frac{5}{2} \quad \quad \quad x = -1$$

$$\left\{ \frac{5}{2}, -1 \right\}$$

$$\begin{aligned}
 \text{c. } 2x^2 - 3x - 5 &= 0 \\
 a &= 2, b = -3, c = -5 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)} \\
 &= \frac{3 \pm \sqrt{9 + 40}}{4} = \frac{3 \pm \sqrt{49}}{4} = \frac{3 \pm 7}{4} \\
 x &= \frac{3}{4} + \frac{7}{4} \quad \text{or} \quad x = \frac{3}{4} - \frac{7}{4} \\
 x &= \frac{10}{4} \quad \quad \quad x = \frac{-4}{4} \\
 x &= \frac{5}{2} \quad \quad \quad x = -1 \\
 &\left\{ \frac{5}{2}, -1 \right\}
 \end{aligned}$$

51. False

52. True

53. a. $4x^2 - 20x + 25 = 0$

$$\begin{aligned}
 b^2 - 4ac &= (-20)^2 - 4(4)(25) \\
 &= 400 - 400 \\
 &= 0
 \end{aligned}$$

b. The discriminant is 0; there is one real solution.

$$-2y^2 = 5y - 1$$

54. a. $-2y^2 - 5y + 1 = 0$

$$\begin{aligned}
 b^2 - 4ac &= (-5)^2 - 4(-2)(1) \\
 &= 25 + 8 \\
 &= 33
 \end{aligned}$$

b. $33 > 0$; there are two real solutions.

$$5t(t+1) = 4t - 11$$

$$5t^2 + 5t = 4t - 11$$

55. a. $5t^2 + t + 11 = 0$

$$\begin{aligned}
 b^2 - 4ac &= (1)^2 - 4(5)(11) \\
 &= 1 - 220 \\
 &= -219
 \end{aligned}$$

b. $-219 < 0$; there are two nonreal solutions.

$$H = kl^2Rt$$

$$\frac{H}{kRt} = \frac{kl^2Rt}{kRt}$$

$$\frac{H}{kRt} = l^2$$

$$l = \sqrt{\frac{H}{kRt}} \quad \text{or} \quad l = \frac{\sqrt{HkRt}}{kRt}$$

56.

57.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(y-k)^2 = r^2 - (x-h)^2$$

$$y-k = \pm \sqrt{r^2 - (x-h)^2}$$

$$y = k \pm \sqrt{r^2 - (x-h)^2}$$

$$s = a_0t^2 + v_0t + s_0$$

$$a_0t^2 + v_0t + s_0 = s$$

58. $a_0t^2 + v_0t + s_0 - s = 0$

$$a = a_0, b = v_0, c = s_0 - s$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(v_0) \pm \sqrt{(v_0)^2 - 4(a_0)(s_0 - s)}}{2(a_0)}$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4a_0(s_0 - s)}}{2a_0}$$

59. Let x represent the height of the triangular plot. Then $x+5$ is the base of the triangular plot.

$$A = \frac{1}{2}bh$$

$$52 = \frac{1}{2}(x+5)x$$

$$104 = x^2 + 5x$$

$$x^2 + 5x - 104 = 0$$

$$(x-8)(x+13) = 0$$

$$x-8=0 \quad \text{or} \quad x+13=0$$

$$x=8 \quad \quad \quad \cancel{x=-13}$$

$$x+5=8+5=13$$

60. Let x and $(x-2)$ represent the width and length of the finished tablecloth.

Then $(x + 0.5)$ and $(x - 1.5)$ are the width and length of the cloth.

$$A = lw$$

$$19.25 = (x + 0.5)(x - 1.5)$$

$$19.25 = x^2 - 1.5x + 0.5x - 0.75$$

$$0 = x^2 - x - 20$$

$$0 = (x - 5)(x + 4)$$

$$x - 5 = 0 \quad \text{or} \quad x + 4 = 0$$

$$x = 5 \quad \cancel{x = -4}$$

$$x + 0.5 = 5 + 0.5 = 5.5$$

$$x - 1.5 = 5 - 1.5 = 3.5$$

The cloth is 3.5 ft by 5.5 ft.

- 61.** Let x and $(1.6x)$ represent the width and length of the screen.

$$a^2 + b^2 = c^2$$

$$(x)^2 + (1.6x)^2 = (50)^2$$

$$x^2 + 2.56x^2 = 2500$$

$$3.56x^2 = 2500$$

$$x^2 = \frac{2500}{3.56}$$

$$x = \pm \sqrt{\frac{2500}{3.56}} \approx \pm 26.5$$

$$1.6x = 1.6(26.5) = 42.4$$

The width is 26.5 in. and the length is 42.4 in.

- 62.** Let x and $(x + 2.7)$ represent the width and length of the screen.

$$a^2 + b^2 = c^2$$

$$(x)^2 + (x + 2.7)^2 = (7)^2$$

$$x^2 + x^2 + 5.4x + 7.29 = 49$$

$$2x^2 + 5.4x - 41.71 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(5.4) \pm \sqrt{(5.4)^2 - 4(2)(-41.71)}}{2(2)} \\ &= \frac{-5.4 \pm \sqrt{362.84}}{4} \\ &\approx 3.4 \text{ or } \cancel{>6.1} \end{aligned}$$

$$x + 2.7 = 3.4 + 2.7 = 6.1$$

The length is 6.1 in. and the width is 3.4 in.

$$\begin{aligned} d &= 0.048v^2 + 2.2v \\ &= 0.048(50)^2 + 2.2(50) \\ &= 0.048(2500) + 110 \\ &= 120 + 110 \end{aligned}$$

63. a. $= 230 \text{ ft}$

$$\begin{aligned} d &= 0.048v^2 + 2.2v \\ 390 &= 0.048v^2 + 2.2v \end{aligned}$$

b. $d = 0.048v^2 + 2.2v - 390$

$$\begin{aligned} v &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(2.2) \pm \sqrt{(2.2)^2 - 4(0.048)(-390)}}{2(0.048)} \\ &= \frac{-2.2 \pm \sqrt{79.72}}{0.096} \\ &\approx 70 \text{ or } \cancel{>116} \end{aligned}$$

The car was traveling 70 mph.

$$\begin{aligned} s &= -\frac{1}{2}gt^2 + v_0t + s_0 \\ s &= -\frac{1}{2}(32)t^2 + (200)t + 2 \end{aligned}$$

64. a. $s = -16t^2 + 200t + 2$

b. $s = -16t^2 + 200t + 2$

$$\begin{aligned}
 80 &= -16t^2 + 200t + 2 \\
 -40 &= 8t^2 - 100t - 1 \\
 0 &= 8t^2 - 100t + 39 \\
 t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-100) \pm \sqrt{(-100)^2 - 4(8)(39)}}{2(8)} \\
 &= \frac{100 \pm \sqrt{8752}}{16} \\
 &\approx 0.4 \text{ or } 12.1
 \end{aligned}$$

The mortar will be at an 80-ft height 0.4 sec after launch.

$$\begin{aligned}
 4x^3 - 6x^2 - 20x + 30 &= 0 \\
 4x^3 - 20x - 6x^2 + 30 &= 0 \\
 4x(x^2 - 5) - 6(x^2 - 5) &= 0 \\
 (x^2 - 5)(4x - 6) &= 0 \\
 (x^2 - 5)(2x - 3) &= 0
 \end{aligned}$$

65.

$$x^2 = 5 \quad \text{or} \quad 2x - 3 = 0$$

$$x = \pm\sqrt{5} \quad \text{or} \quad x = \frac{3}{2}$$

$$\left\{ \pm\sqrt{5}, \frac{3}{2} \right\}$$

$$3x^2(x^2 + 2) = 20 - x^2$$

$$3x^4 + 6x^2 = 20 - x^2$$

66. $3x^4 + 7x^2 - 20 = 0$

Let $u = x^2$.

$$3u^2 + 7u - 20 = 0$$

$$(3u - 5)(u + 4) = 0$$

$$u = \frac{5}{3} \quad \text{or} \quad u = -4$$

$$x^2 = \frac{5}{3} \quad \text{or} \quad x^2 = -4$$

$$x = \pm\sqrt{\frac{5}{3}} \quad \text{or} \quad x = \pm\sqrt{-4}$$

$$x = \pm\frac{\sqrt{15}}{3} \quad \text{or} \quad x = \pm 2i$$

67. $\sqrt{k+7} - \sqrt{3-k} = 2$

$$\sqrt{k+7} = \sqrt{3-k} + 2$$

$$(\sqrt{k+7})^2 = (\sqrt{3-k} + 2)^2$$

$$k+7 = 3-k+4\sqrt{3-k}+4$$

$$2k = 4\sqrt{3-k}$$

$$(2k)^2 = (4\sqrt{3-k})^2$$

$$4k^2 = 16(3-k)$$

$$4k^2 = 48 - 16k$$

$$4k^2 + 16k - 48 = 0$$

$$k^2 + 4k - 12 = 0$$

$$(k-2)(k+6) = 0$$

$$k = 2 \quad \text{or} \quad k = -6$$

Check: $k = 2$

$$\sqrt{k+7} - \sqrt{3-k} = 2$$

$$\sqrt{(2)+7} - \sqrt{3-(2)} = 2$$

$$\sqrt{9} - \sqrt{1} = 2$$

$$3 - 1 = 2$$

$$2 = 2 \checkmark \text{ true}$$

Check: $k = -6$

$$\sqrt{k+7} - \sqrt{3-k} = 2$$

$$\sqrt{(-6)+7} - \sqrt{3-(-6)} = 2$$

$$\sqrt{1} - \sqrt{9} = 2$$

$$1 - 3 = 2$$

$$-2 = 2 \text{ false}$$

$\{2\}$; The value -6 does not check.

$$\frac{n}{3n+2} + 1 = \frac{4}{n-2}$$

68.

$$(3n+2)(n-2)\left(\frac{n}{3n+2} + 1\right) = (3n+2)(n-2)\left(\frac{4}{n-2}\right)$$

$$n(n-2) + 1(3n+2)(n-2) = 4(3n+2)$$

$$n^2 - 2n + 3n^2 - 4n - 4 = 12n + 8$$

$$4n^2 - 18n - 12 = 0$$

$$2n^2 - 9n - 6 = 0$$

$$n = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-6)}}{2(2)} = \frac{9 \pm \sqrt{129}}{4}$$

$$\left\{ \frac{9 \pm \sqrt{129}}{4} \right\}$$

69. Let $u = v^{-1}$.

$$11v^{-2} + 23v^{-1} + 2 = 0$$

$$11u^2 + 23u + 2 = 0$$

$$(11u+1)(u+2) = 0$$

$$u = -\frac{1}{11} \quad \text{or} \quad u = -2$$

$$v^{-1} = -\frac{1}{11} \quad \text{or} \quad v^{-1} = -2$$

$$v = -11 \quad \text{or} \quad v = -\frac{1}{2}$$

$$\left\{ -\frac{1}{2}, -11 \right\}$$

$$\sqrt[3]{4-x} - \sqrt[3]{2x+1} = 0$$

$$\sqrt[3]{4-x} = \sqrt[3]{2x+1}$$

$$\left(\sqrt[3]{4-x}\right)^3 = \left(\sqrt[3]{2x+1}\right)^3$$

$$4-x = 2x+1$$

$$3 = 3x$$

$$1 = x$$

70.

$$\{1\}$$

$$-2\sqrt{3m+4} - 3 = 5$$

$$-2\sqrt{3m+4} = 8$$

71.

$$\sqrt{3m+4} = -4$$

$$\{ \}$$

72. $\sqrt{51-14x} + 4 = x - 2$

$$\sqrt{51-14x} = x - 6$$

$$\left(\sqrt{51-14x}\right)^2 = (x-6)^2$$

$$51-14x = x^2 - 12x + 36$$

$$0 = x^2 + 2x - 15$$

$$0 = (x+5)(x-3)$$

$$x = -5 \quad \text{or} \quad x = 3$$

Check: $x = -5$

$$\sqrt{51-14x} + 4 = x - 2$$

$$\sqrt{51-14(-5)} + 4 = (-5) - 2$$

$$\sqrt{51-70} = -11$$

$$\sqrt{-19} = -11 \text{ false}$$

Check: $x = 3$

$$\sqrt{51-14x} + 4 = x - 2$$

$$\sqrt{51-14(3)} + 4 = (3) - 2$$

$$\sqrt{51-42} = -3$$

$$\sqrt{9} = -3$$

$$3 = -3 \text{ false}$$

$\{ \}$; The values -5 and 3 do not check.

73.

$$(x-11)^{2/3} = 9$$

$$\left((x-11)^{2/3}\right)^{3/2} = \pm(9)^{3/2}$$

$$x-11 = \pm 27$$

$$x = 27+11 \quad \text{or} \quad x = -27+11$$

$$x = 38 \quad \quad \quad x = -16$$

$$\{-16, 38\}$$

$$(2x+1)^{3/4} = 27$$

$$\left((2x+1)^{3/4}\right)^{4/3} = (27)^{4/3}$$

$$2x+1 = 81$$

$$x = \frac{81-1}{2}$$

$$x = 40$$

74.

$$\{40\}$$

75.

$$-2|3y-10| + 4 = -6$$

$$-2|3y-10| = -10$$

$$|3y-10| = 5$$

$$3y-10 = 5 \quad \text{or} \quad 3y-10 = -5$$

$$y = 5 \quad \text{or} \quad y = \frac{5}{3}$$

$$\left\{ \frac{5}{3}, 5 \right\}$$

$$|6-w|+7=2$$

$$76. \quad |6-w|=-5$$

$$\{ \}$$

$$77. \quad |p-4|=|2p-3|$$

$$(p-4)=(2p-3) \quad \text{or} \quad (p-4)=-(2p-3)$$

$$-p=1 \qquad 3p=7$$

$$p=-1 \qquad p=\frac{7}{3}$$

$$\left\{-1, \frac{7}{3}\right\}$$

$$10w^{2/3}=\frac{1}{10}$$

$$10w^{2/3}=10^{-1}$$

$$w^{2/3}=10^{-2}$$

$$(w^{2/3})^{3/2}=\pm(10^{-2})^{3/2}$$

$$w=\pm 10^{-3}=\pm \frac{1}{1000}$$

$$78. \quad \left\{\pm \frac{1}{1000}\right\}$$

$$79. \quad \text{Let } u=d^{1/3}.$$

$$6d^{2/3}-7d^{1/3}-3=0$$

$$6u^2-7u-3=0$$

$$(3u+1)(2u-3)=0$$

$$u=-\frac{1}{3} \quad \text{or} \quad u=\frac{3}{2}$$

$$d^{1/3}=-\frac{1}{3} \quad \text{or} \quad d^{1/3}=\frac{3}{2}$$

$$(d^{1/3})^3=\left(-\frac{1}{3}\right)^3 \quad \text{or} \quad (d^{1/3})^3=\left(\frac{3}{2}\right)^3$$

$$d=-\frac{1}{27} \quad \text{or} \quad d=\frac{27}{8}$$

$$82. \quad \frac{4v}{5v-25}-\frac{10}{v-5}=v+\frac{4}{5}$$

$$\left\{-\frac{1}{27}, \frac{27}{8}\right\}$$

$$80. \quad \text{Let } v=2u^2-1.$$

$$(2u^2-1)^2-10(2u^2-1)+9=0$$

$$v^2-10v+9=0$$

$$(v-1)(v-9)=0$$

$$v=1 \quad \text{or} \quad v=9$$

$$2u^2-1=1 \quad \text{or} \quad 2u^2-1=9$$

$$2u^2=2 \quad \text{or} \quad 2u^2=10$$

$$u^2=1 \quad \text{or} \quad u^2=5$$

$$u=\pm 1 \quad \text{or} \quad u=\pm\sqrt{5}$$

$$\{\pm\sqrt{5}, \pm 1\}$$

$$81. \quad \text{Let } u=\frac{4}{w}+1.$$

$$2\left(\frac{4}{w}+1\right)^2-10\left(\frac{4}{w}+1\right)=0$$

$$2u^2-10u=0$$

$$2u(u-5)=0$$

$$u=0 \quad \text{or} \quad u=5$$

$$\frac{4}{w}+1=0 \quad \text{or} \quad \frac{4}{w}+1=5$$

$$\frac{4}{w}=-1 \quad \text{or} \quad \frac{4}{w}=4$$

$$w=-4 \quad \text{or} \quad w=1$$

$$\{1, -4\}$$

$$\begin{aligned}\frac{4v}{5(v-5)} - \frac{10}{v-5} &= v + \frac{4}{5} \\ 5(v-5)\left(\frac{4v}{5(v-5)} - \frac{10}{v-5}\right) &= 5(v-5)\left(v + \frac{4}{5}\right) \\ 4v - 5(10) &= (v-5)(5v+4) \\ 4v - 50 &= 5v^2 + 4v - 25v - 20 \\ -5v^2 + 25v - 30 &= 0 \\ v^2 - 5v + 6 &= 0 \\ (v-2)(v-3) &= 0 \\ v &= 2 \text{ or } v = 3\end{aligned}$$

$$83. \quad m = \frac{1}{2}\sqrt{2a^2 + 2b^2 - 2c^2}$$

$$\begin{aligned}2m &= \sqrt{2a^2 + 2b^2 - 2c^2} \\ (2m)^2 &= \left(\sqrt{2a^2 + 2b^2 - 2c^2}\right)^2\end{aligned}$$

$$4m^2 = 2a^2 + 2b^2 - 2c^2$$

$$2m^2 = a^2 + b^2 - c^2$$

$$a^2 = 2m^2 - b^2 + c^2$$

$$a = \pm\sqrt{2m^2 - b^2 + c^2}$$

$$84. \quad \frac{1}{a} = \frac{1}{b} + \frac{1}{c}$$

$$abc\left(\frac{1}{a}\right) = abc\left(\frac{1}{b} + \frac{1}{c}\right)$$

$$bc = ac + ab$$

$$bc - ab = ac$$

$$b(c-a) = ac$$

$$b = \frac{ac}{c-a} \text{ or } b = -\frac{ac}{a-c}$$

$$\frac{a_1 t_1}{v_1} = \frac{a_2 t_2}{v_2}$$

$$v_1 v_2 \left(\frac{a_1 t_1}{v_1}\right) = v_1 v_2 \left(\frac{a_2 t_2}{v_2}\right)$$

$$v_2 a_1 t_1 = v_1 a_2 t_2$$

$$v_2 = \frac{v_1 a_2 t_2}{a_1 t_1}$$

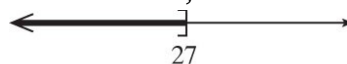
85.

$$-4 \leq -\frac{2}{3}p + 14$$

$$-18 \leq -\frac{2}{3}p$$

$$86. \quad 27 \geq p \text{ or } p \leq 27$$

$$\{p \mid p \leq 27\}; (-\infty, 27]$$



$$-0.6 + 0.2x < 0.8x - 1.8$$

$$1.2 < 0.6x$$

$$87. \quad 2 < x \text{ or } x > 2$$

$$\{x \mid x > 2\}; (2, \infty)$$



$$\frac{2+y}{3} - \frac{y-1}{4} < \frac{y}{6}$$

$$12\left(\frac{2+y}{3} - \frac{y-1}{4}\right) < 12\left(\frac{y}{6}\right)$$

$$4(2+y) - 3(y-1) < 2y$$

$$8 + 4y - 3y + 3 < 2y$$

$$88. \quad 11 < y \text{ or } y > 11$$

$$\{y \mid y > 11\}; (11, \infty)$$



89.

$$9 - [5 - 4(t - 1)] \geq 3\{2 - [5 - (t + 2)]\}$$

$$9 - [5 - 4t + 4] \geq 3\{2 - [5 - t - 2]\}$$

$$9 - [-4t + 9] \geq 3\{2 - [-t + 3]\}$$

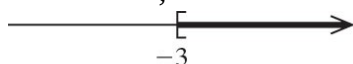
$$9 + 4t - 9 \geq 3\{2 + t - 3\}$$

$$4t \geq 3\{t - 1\}$$

$$4t \geq 3t - 3$$

$$t \geq -3$$

$$\{t \mid t \geq -3\}; [-3, \infty)$$



$$x - 12 \geq 0$$

90. a. $x \geq 12$

$$\{x \mid x \geq 12\}$$

$$12 - x \geq 0$$

b. $12 \geq x$ or $x \leq 12$

$$\{x \mid x \leq 12\}$$

$$5x + 7 \geq 0$$

$$5x \geq -7$$

$$x \geq -\frac{7}{5}$$

91. a.

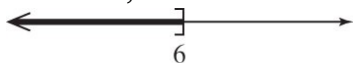
$$\left\{x \mid x \geq -\frac{7}{5}\right\}$$

b. $\square$

$$t + 2 \leq 8 \quad \text{or} \quad \frac{1}{3}t < -4$$

92. a. $t \leq 6$ or $t < -12$

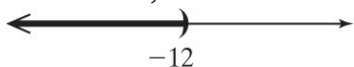
$$(-\infty, 6];$$



$$t + 2 \leq 8 \quad \text{and} \quad \frac{1}{3}t < -4$$

b. $t \leq 6$ and $t < -12$

$$(-\infty, -12);$$

**93. a.**

$$-2(x - 1) + 4 < x + 3 \quad \text{or} \quad 5(x + 2) - 3 \leq 4x + 1$$

$$-2x + 2 + 4 < x + 3 \quad \text{or} \quad 5x + 10 - 3 \leq 4x + 1$$

$$-3x < -3 \quad \text{or} \quad x \leq -6$$

$$x > 1 \quad \text{or} \quad x \leq -6$$

$$(-\infty, -6] \cup (1, \infty);$$



$$-2(x - 1) + 4 < x + 3$$

$$\text{and } 5(x + 2) - 3 \leq 4x + 1$$

b. $x > 1$ and $x \leq -6$

$$\{ \}$$

$$-11 \leq -4x - 1 \leq 7$$

$$-10 \leq -4x \leq 8$$

$$\frac{10}{4} \geq x \geq -2$$

$$-2 \leq x \leq \frac{5}{2}$$

94.

$$\left[-2, \frac{5}{2}\right];$$



$$0 < \frac{-3x + 9}{-4} < 6$$

$$0 < \frac{3x - 9}{4} < 6$$

$$0 < 3x - 9 < 24$$

$$9 < 3x < 33$$

95. $3 < x < 11$

$$(3, 11);$$

**96.** $29,000 < a \leq 31,000$ **97.** Let x represent the September rainfall.

$$\frac{8.54 + 5.79 + 8.63 + x}{4} > 7.83$$

$$\frac{22.96 + x}{4} > 7.83$$

$$22.96 + x > 31.32$$

$$x > 8.36$$

More than 8.36 in. is needed.

98. Let p represent the price of sod.

$$400 + 2000t \leq 850$$

$$2000t \leq 450$$

$$t \leq 0.225$$

She can afford sod that is \$0.225/ft² or less.

$$|w + 2| + 1 = 6$$

$$|w + 2| = 5$$

$$w + 2 = 5 \quad \text{or} \quad w + 2 = -5$$

99. a. $w = 3 \quad \text{or} \quad w = -7$

$$\{-7, 3\}$$

$$|w + 2| + 1 < 6$$

$$|w + 2| < 5$$

$$-5 < w + 2 < 5$$

b. $-7 < w < 3$

$$(-7, 3)$$

$$|w + 2| + 1 \geq 6$$

$$|w + 2| \geq 5$$

$$w + 2 \leq -5 \quad \text{or} \quad w + 2 \geq 5$$

c. $w \leq -7 \quad \text{or} \quad w \geq 3$

$$(-\infty, -7] \cup [3, \infty)$$

$$3 = |7x + 1| + 4$$

100. a. $-1 = |7x + 1|$

$$\{ \}$$

$$3 < |7x + 1| + 4$$

b. $-1 < |7x + 1|$

$$(-\infty, \infty)$$

$$3 \geq |7x + 1| + 4$$

c. $-1 \geq |7x + 1|$

$$\{ \}$$

$$|y + 5| - 3 = -3$$

$$|y + 5| = 0$$

$$y + 5 = 0$$

101. a. $y = -5$

$$\{-5\}$$

$$|y + 5| - 3 < -3$$

b. $|y + 5| < 0$

$$\{ \}$$

$$|y + 5| - 3 \leq -3$$

$$|y + 5| \leq 0$$

$$y + 5 = 0$$

$$y = -5$$

c.

$$\{-5\}$$

d.

$$|y + 5| - 3 > -3$$

$$|y + 5| > 0$$

$$y + 5 < 0 \quad \text{or} \quad y + 5 > 0$$

$$y < -5 \quad \text{or} \quad y > -5$$

$$(-\infty, -5) \cup (-5, \infty)$$

$$|y + 5| - 3 \geq -3$$

$$|y + 5| \geq 0$$

e.

$$(-\infty, \infty)$$

102. a.

$$|x - 1| = |3x + 5|$$

$$x - 1 = 3x + 5 \quad \text{or} \quad x - 1 = -3x - 5$$

$$-2x = 6 \quad \text{or} \quad 4x = -4$$

$$x = -3 \quad \text{or} \quad x = -1$$

$$\{-3, -1\}$$

b.

$$|x - 1| = |x + 5|$$

$$x - 1 = x + 5 \quad \text{or} \quad x - 1 = -x - 5$$

$$-1 = 5 \quad \text{or} \quad 2x = -4$$

$$\text{or} \quad x = -2$$

$$\{-2\}$$

c.

$$\begin{aligned}
 |x-1| &= |1-x| \\
 x-1 &= 1-x \quad \text{or} \quad x-1 = -1+x \\
 2x &= x \quad \text{or} \quad 0=0 \\
 x &= 1 \\
 (-\infty, \infty)
 \end{aligned}$$

103.

$$\begin{aligned}
 4|x+2|-10 &\geq -6 \\
 4|x+2| &\geq 4 \\
 |x+2| &\geq 1 \\
 x+2 &\leq -1 \quad \text{or} \quad x+2 \geq 1 \\
 x &\leq -3 \quad \text{or} \quad x \geq -1 \\
 (-\infty, -3] \cup [-1, \infty) \\
 |0.5x-8| &< 0.01 \\
 -0.01 &< 0.5x-8 < 0.01 \\
 7.99 &< 0.5x < 8.01
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{104.} \quad 15.98 &< x < 16.02 \\
 (15.98, 16.02)
 \end{aligned}$$

$$\begin{aligned}
 -9 &\leq 4-|2k-1| \\
 -13 &\leq -|2k-1| \\
 13 &\geq |2k-1| \\
 |2k-1| &\leq 13 \\
 -13 &\leq 2k-1 \leq 13 \\
 -12 &\leq 2k \leq 14 \\
 -6 &\leq k \leq 7
 \end{aligned}$$

105.

$$[-6, 7]$$

$$\mathbf{106. a.} \quad |x-3| \leq 0.5 \quad \text{or} \quad |3-x| \leq 0.5$$

$$\begin{aligned}
 |x-3| &\leq 0.5 \\
 -0.5 &\leq (x-3) \leq 0.5 \\
 2.5 &\leq x \leq 3.5 \\
 [2.5, 3.5]
 \end{aligned}$$

b.**107. a.**

$$\begin{aligned}
 |t-(-2)| &> 0.01 \\
 |t+2| &> 0.01 \quad \text{or} \quad |-2-t| > 0.01
 \end{aligned}$$

b.

$$\begin{aligned}
 |t+2| &> 0.01 \\
 t+2 &> 0.01 \quad \text{or} \quad -(t+2) > 0.01 \\
 t &> -1.99 \quad \text{or} \quad t < -2.01 \\
 (-\infty, -2.01) \cup (-1.99, \infty)
 \end{aligned}$$

Chapter 1 Test

$$\begin{aligned}
 \sqrt{-25} \cdot \sqrt{-4} &= 5i \cdot 2i \\
 &= 10i^2 \\
 &= 10(-1) \\
 &= -10
 \end{aligned}$$

1.

$$\begin{aligned}
 i^{89} &= i^{88} \cdot i^1 \\
 &= (1) \cdot i^1
 \end{aligned}$$

2. a.

$$\begin{aligned}
 &= i \\
 i^{46} &= i^{44} \cdot i^2 \\
 &= (1) \cdot i^2
 \end{aligned}$$

b.

$$= -1$$

$$\begin{aligned}
 i^{35} &= i^{32} \cdot i^3 \\
 &= (1) \cdot i^3 \\
 &= -i
 \end{aligned}$$

c.

$$\mathbf{d.} \quad i^{120} = 1$$

$$\begin{aligned}
 i^{-11} &= i^{-12} \cdot i^1 \\
 &= (1) \cdot i^1
 \end{aligned}$$

e.

$$\begin{aligned}
 (4-7i)(6+2i) &= 24+8i-42i-14i^2 \\
 &= 24-34i-14(-1) \\
 &= 24-34i+14 \\
 &= 38-34i
 \end{aligned}$$

3.

$$\begin{aligned}
 (3-5i)^2 &= (3)^2 - 2(3)(5i) + (5i)^2 \\
 &= 9 - 30i + 25i^2 \\
 &= 9 - 30i + 25(-1) \\
 &= -16 - 30i
 \end{aligned}$$

4.

$$\frac{4+3i}{2-5i} = \frac{(4+3i)(2+5i)}{(2-5i)(2+5i)}$$

5.

$$\begin{aligned}
 &= \frac{8+20i+6i+15i^2}{2^2+5^2} \\
 &= \frac{8+26i+15(-1)}{4+25} \\
 &= \frac{-7+26i}{29} \\
 &= -\frac{7}{29} + \frac{26}{29}i
 \end{aligned}$$

$$\begin{aligned}
 b^2 - 4ac &= (-4)^2 - 4(2)(7) \\
 &= 16 - 56
 \end{aligned}$$

6. a.

$$= -40$$

b. Because $-40 < 0$, there are two non-real solutions.

$$x^2 + 25 = 10x$$

$$\mathbf{7. a.} \quad x^2 - 10x + 25 = 0$$

$$\begin{aligned}
 b^2 - 4ac &= (-10)^2 - 4(1)(25) \\
 &= 100 - 100 \\
 &= 0
 \end{aligned}$$

b. Because the discriminant is 0, there is one real solution.

$$3x(x+4) = 2x-2$$

$$3x^2 + 12x = 2x - 2$$

$$\mathbf{8. a.} \quad 3x^2 + 10x + 2 = 0$$

$$\begin{aligned}
 b^2 - 4ac &= (10)^2 - 4(3)(2) \\
 &= 100 - 24 \\
 &= 76
 \end{aligned}$$

b. Because $76 > 0$, there are two real solutions.

$$\begin{aligned}
 3y + 2[5(y-4) - 2] \\
 = 5y + 6(7+y) - 3
 \end{aligned}$$

9.

$$3y + 2(5y - 20 - 2) = 5y + 42 + 6y - 3$$

$$3y + 2(5y - 22) = 11y + 39$$

$$3y + 10y - 44 = 11y + 39$$

$$13y - 44 = 11y + 39$$

$$2y = 83$$

$$y = \frac{83}{2}$$

$$\left\{ \frac{83}{2} \right\}$$

$$\frac{2+t}{6} - \frac{3-1}{4} = 1 - \frac{2t-5}{3}$$

$$12\left(\frac{2+t}{6} - \frac{3-1}{4}\right) = 12\left(1 - \frac{2t-5}{3}\right)$$

$$2(2+t) - 3(3-1) = 12 - 4(2t-5)$$

$$4+2t-9+3=12-8t+20$$

$$-7t+7=-8t+32$$

$$t = 25$$

10.

$$\{25\}$$

$$0.4(w+1) + 0.8$$

$$= 0.1w + 0.3(4+w)$$

11.

$$0.4w + 0.4 + 0.8 = 0.1w + 1.2 + 0.3w$$

$$0.4w + 1.2 = 0.4w + 1.2$$

$$0 = 0$$

□

$$\mathbf{12.} \quad \frac{-11}{2x^2 + x - 15} - \frac{2}{2x - 5} = \frac{1}{x + 3}$$

$$\begin{aligned} (2x-5)(x+3) \left[\frac{-11}{(2x-5)(x+3)} - \frac{2}{2x-5} \right] &= (2x-5)(x+3) \left(\frac{1}{x+3} \right) \\ -11 - 2(x+3) &= 1(2x-5) \\ -11 - 2x - 6 &= 2x - 5 \\ -2x - 17 &= 2x - 5 \\ -12 &= 4x \\ -3 &= x \end{aligned}$$

$\{ \}$; The value -3 does not check.

$$(3x-4)^2 - 2 = 11$$

$$(3x-4)^2 = 13$$

$$3x-4 = \pm\sqrt{13}$$

$$3x = 4 \pm \sqrt{13}$$

$$x = \frac{4 \pm \sqrt{13}}{3}$$

13.

$$\left\{ \frac{4 \pm \sqrt{13}}{3} \right\}$$

$$y^2 + 10y = 4$$

$$y^2 + 10y + \left[\frac{1}{2}(10) \right]^2 = 4 + \left[\frac{1}{2}(10) \right]^2$$

$$y^2 + 10y + 25 = 4 + 25$$

$$(y+5)^2 = 29$$

$$y+5 = \pm\sqrt{29}$$

$$y = -5 \pm \sqrt{29}$$

14.

$$\left\{ -5 \pm \sqrt{29} \right\}$$

15.

$$6t(2t+1) = 5-5t$$

$$12t^2 + 11t - 5 = 0$$

$$(3t-1)(4t+5) = 0$$

$$3t-1=0 \quad \text{or} \quad 4t+5=0$$

$$3t=1 \quad 4t=-5$$

$$t = \frac{1}{3} \quad t = -\frac{5}{4}$$

$$\left\{ \frac{1}{3}, -\frac{5}{4} \right\}$$

$$\frac{3x^2}{4} - x = -\frac{1}{2}$$

$$4 \left(\frac{3x^2}{4} - x \right) = 4 \left(-\frac{1}{2} \right)$$

$$3x^2 - 4x = -2$$

$$\mathbf{16.} \quad 3x^2 - 4x + 2 = 0$$

$$a=3, b=-4, c=2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{4 \pm \sqrt{16-24}}{6}$$

$$= \frac{4 \pm \sqrt{-8}}{6} = \frac{4 \pm 2i\sqrt{2}}{6}$$

$$= \frac{2 \pm i\sqrt{2}}{3}$$

$$\left\{ \frac{2 \pm i\sqrt{2}}{3} \right\}$$

$$12y^3 + 24y^2 = 3y + 6$$

$$12y^3 + 24y^2 - 3y - 6 = 0$$

$$12y^3 - 3y + 24y^2 - 6 = 0$$

$$3y(4y^2 - 1) + 6(4y^2 - 1) = 0$$

$$y(4y^2 - 1) + 2(4y^2 - 1) = 0$$

$$(4y^2 - 1)(y + 2) = 0$$

$$\mathbf{17.} \quad (2y + 1)(2y - 1)(y + 2) = 0$$

$$2y + 1 = 0 \quad \text{or} \quad 2y - 1 = 0 \quad \text{or} \quad y + 2 = 0$$

$$2y = -1 \quad 2y = 1 \quad y = -2$$

$$y = -\frac{1}{2} \quad y = \frac{1}{2}$$

$$\left\{ \pm \frac{1}{2}, -2 \right\}$$

18.

$$(2y - 3)^{1/3} - (4y + 5)^{1/3} = 0$$

$$(2y - 3)^{1/3} = (4y + 5)^{1/3}$$

$$\left[(2y - 3)^{1/3} \right]^3 = \left[(4y + 5)^{1/3} \right]^3$$

$$2y - 3 = 4y + 5$$

$$-8 = 2y$$

$$-4 = y$$

$$\{-4\}$$

$$\mathbf{19.} \quad \sqrt{2d} = 1 - \sqrt{d + 7}$$

$$(\sqrt{2d})^2 = (1 - \sqrt{d + 7})^2$$

$$2d = 1 - 2\sqrt{d + 7} + d + 7$$

$$2\sqrt{d + 7} = 8 - d$$

$$(2\sqrt{d + 7})^2 = (8 - d)^2$$

$$\frac{c}{c + 6} - 4 = \frac{72}{c^2 - 36}$$

$$\mathbf{20.} \quad \frac{c}{c + 6} - 4 = \frac{72}{(c - 6)(c + 6)}$$

$$4(d + 7) = 64 - 16d + d^2$$

$$4d + 28 = d^2 - 16d + 64$$

$$0 = d^2 - 20d + 36$$

$$0 = (d - 2)(d - 18)$$

$$d = 2 \text{ or } d = 18$$

$$\text{Check: } d = 2$$

$$\sqrt{2d} = 1 - \sqrt{d + 7}$$

$$\sqrt{2(2)} = 1 - \sqrt{(2) + 7}$$

$$\sqrt{4} = 1 - \sqrt{9}$$

$$2 = 1 - 3$$

$$2 = -2 \text{ false}$$

$$\text{Check: } d = 18$$

$$\sqrt{2d} = 1 - \sqrt{d + 7}$$

$$\sqrt{2(18)} = 1 - \sqrt{(18) + 7}$$

$$\sqrt{36} = 1 - \sqrt{25}$$

$$6 = 1 - 5$$

$$6 = -4 \text{ false}$$

$\{ \}$; The values 2 and 18 do not check.

$$(c-6)(c+6)\left(\frac{c}{c+6}-4\right)=(c-6)(c+6)\left[\frac{72}{(c-6)(c+6)}\right]$$

$$c(c-6)-4(c-6)(c+6)=72$$

$$c^2-6c-4c^2+144=72$$

$$-3c^2-6c+72=0$$

$$c^2+2c-24=0$$

$$(c+6)(c-4)=0$$

$$\cancel{c=-6} \quad \text{or} \quad c=4$$

$\{4\}$; The value -6 does not check.

$$w^{4/5}-11=0$$

$$w^{4/5}=11$$

$$\left(w^{4/5}\right)^{5/4}=\pm(11)^{5/4}$$

$$\mathbf{21.} \quad w=\pm 11^{5/4}$$

$$\{\pm 11^{5/4}\}$$

$$\mathbf{22.} \quad \text{Let } u=5-\frac{2}{k}.$$

$$\left(5-\frac{2}{k}\right)^2-6\left(5-\frac{2}{k}\right)-27=0$$

$$u^2-6u-27=0$$

$$(u+3)(u-9)=0$$

$$u=-3 \quad \text{or} \quad u=9$$

$$5-\frac{2}{k}=-3 \quad \text{or} \quad 5-\frac{2}{k}=9$$

$$-\frac{2}{k}=-8 \quad \text{or} \quad -\frac{2}{k}=4$$

$$-2=-8k \quad \text{or} \quad -2=4k$$

$$k=\frac{1}{4} \quad \text{or} \quad k=-\frac{1}{2}$$

$$\left\{\frac{1}{4}, -\frac{1}{2}\right\}$$

$$-2=|x-3|-6$$

$$4=|x-3|$$

$$x-3=4 \quad \text{or} \quad x-3=-4$$

$$\mathbf{23.} \quad x=7 \quad \text{or} \quad x=-1$$

$$\{-1, 7\}$$

24.

$$|2v+5|=|2v-1|$$

$$2v+5=2v-1 \quad \text{or} \quad 2v+5=-2v+1$$

$$5=-1 \quad \text{or} \quad 4v=-4$$

$$\text{or} \quad v=-1$$

$$\{-1\}$$

$$aP-4=Pt+2$$

$$aP-Pt=6$$

$$P(a-t)=6$$

$$P=\frac{6}{a-t} \quad \text{or} \quad P=-\frac{6}{t-a}$$

25.

$$\sqrt{a^2-b^2}=c$$

$$\left(\sqrt{a^2-b^2}\right)^2=(c)^2$$

$$a^2-b^2=c^2$$

$$-b^2=-a^2+c^2$$

$$b^2=a^2-c^2$$

$$b=\sqrt{a^2-c^2}$$

26.

$$-16t^2+v_0t+2=0$$

$$\mathbf{27.} \quad 16t^2-v_0t-2=0$$

$$a=16, b=-v_0, c=-2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-v_0) \pm \sqrt{(-v_0)^2 - 4(16)(-2)}}{2(16)}$$

$$t = \frac{v_0 \pm \sqrt{v_0^2 + 128}}{32}$$

$$a^2 + b^2 + c^2 = 49$$

$$c^2 = 49 - a^2 - b^2$$

$$28. \quad c = \sqrt{49 - a^2 - b^2}$$

$$-2 \leq \frac{4-x}{3} \leq 6$$

$$-6 \leq 4-x \leq 18$$

$$-10 \leq -x \leq 14$$

$$29. \quad 10 \geq x \geq -14 \text{ or } -14 \leq x \leq 10$$

$$[-14, 10]$$

$$-\frac{4}{3}y < -24 \quad \text{or} \quad y + 7 \leq 2y - 3$$

$$30. \quad y > 18 \quad \text{or} \quad 10 \leq y$$

$$[10, \infty)$$

$$3(x-5) + 1 \leq 4(x+2) + 6$$

$$31. \quad \text{and } 0.3x - 1.6 > 0.2$$

$$3x - 15 + 1 \leq 4x + 8 + 6 \quad \text{and} \quad 0.3x > 1.8$$

$$0 \leq x \quad \text{and} \quad x > 6$$

$$(6, \infty)$$

$$2 < -1 + |4w - 3|$$

$$3 < |4w - 3|$$

$$|4w - 3| > 3$$

$$4w - 3 < -3 \quad \text{or} \quad 4w - 3 > 3$$

$$4w < 0 \quad \text{or} \quad 4w > 6$$

$$w < 0 \quad \text{or} \quad w > \frac{3}{2}$$

$$32.$$

$$\left(-\infty, 0\right) \cup \left(\frac{3}{2}, \infty\right)$$

$$-|8-v| \geq -6$$

$$|8-v| \leq 6$$

$$-6 \leq 8-v \leq 6$$

$$-14 \leq -v \leq -2$$

$$14 \geq v \geq 2$$

$$33. \quad 2 \leq v \leq 14$$

$$[2, 14]$$

$$|7x+4|+11=2$$

$$34. \text{ a. } |7x+4|=-9$$

$$\{ \}$$

$$|7x+4|+11 < 2$$

$$\text{b. } |7x+4| < -9$$

$$\{ \}$$

$$|7x+4|+11 > 2$$

$$\text{c. } |7x+4| > -9$$

$$\square$$

$$|x-13|+4=4$$

$$|x-13|=0$$

$$x-13=0$$

$$35. \text{ a. } x=13$$

$$\{13\}$$

$$|x-13|+4 < 4$$

$$\text{b. } |x-13| < 0$$

$$\{ \}$$

$$|x-13|+4 \leq 4$$

$$|x-13| \leq 0$$

$$x-13=0$$

$$\text{c. } x=13$$

$$\{13\}$$

$$|x-13|+4 > 4$$

$$|x-13| > 0$$

$$x-13 < -0 \quad \text{or} \quad x-13 > 0$$

$$\text{d. } x < 13 \quad \text{or} \quad x > 13$$

$$\left(-\infty, 13\right) \cup \left(13, \infty\right)$$

$$\begin{aligned}
 |x - 13| + 4 &\geq 4 \\
 |x - 13| &\geq 0 \\
 x - 13 &\leq -0 \quad \text{or} \quad x - 13 \geq 0 \\
 \text{e.} \quad x &\leq 13 \quad \text{or} \quad x \geq 13 \\
 &(-\infty, \infty)
 \end{aligned}$$

- 36.** Let x represent the amount of 80% antifreeze solution (in gallons) to be mixed. 2 gal is the amount of the 50% antifreeze solution to be mixed.

Therefore, $(x + 2)$ is the amount of the resulting 60% antifreeze solution.

	80% Sol.	50% Sol.	60% Sol.
Amount of Sol.	x	2	$x + 2$
Pure Anti-freeze	$0.8(x)$	$0.5(2)$	$0.6(x + 2)$

$$\begin{aligned}
 0.8(x) + 0.5(2) &= 0.6(x + 2) \\
 0.8x + 1 &= 0.6x + 1.2 \\
 0.2x &= 0.2 \\
 x &= 1
 \end{aligned}$$

1 gal of 80% antifreeze should be used.

- 37.** Let x represent the speed of the plane flying to Seattle. Then, $(x + 60)$ is the speed of the plane flying to New York City.

	Distance	Rate	Time
Seattle Flight	$2.3x$	x	2.3
New York Flight	$3.3(x + 60)$	$x + 60$	3.3

$$\begin{aligned}
 2.3x + 3.3(x + 60) &= 2662 \\
 2.3x + 3.3x + 198 &= 2662 \\
 5.6x + 198 &= 2662 \\
 5.6x &= 2464 \\
 x &= 440 \\
 x + 60 &= 440 + 60 \\
 &= 500
 \end{aligned}$$

The plane flying to Seattle flies 440 mph, and the plane flying to New York flies 500 mph.

- 38.** Let t represent the time it would take the second hose to fill the pool if it worked alone.

$$\begin{aligned}
 \frac{1 \text{ job}}{3 \text{ hr}} + \frac{1 \text{ job}}{t \text{ hr}} &= \frac{1 \text{ job}}{1.2 \text{ hr}} \\
 6t \left(\frac{1}{3} + \frac{1}{t} \right) &= 6t \left(\frac{1}{1.2} \right) \\
 2t + 6 &= 5t \\
 6 &= 3t \\
 t &= 2
 \end{aligned}$$

The second hose can fill the pool in 2 hr.

- 39.** Let x represent the patient's LDL cholesterol level. The HDL cholesterol level is 70 mg/dL, and the total cholesterol is $(x + 70)$.

$$\begin{aligned}
 \frac{x + 70}{70} &= 3.8 \\
 70 \left(\frac{x + 70}{70} \right) &= 70(3.8) \\
 x + 70 &= 266 \\
 x &= 196 \\
 x + 70 &= 196 + 70 \\
 &= 266
 \end{aligned}$$

The LDL level is 196 mg/dL and the total cholesterol is 266 mg/dL.

- 40.** Let x represent the base of the triangular portions and $(x + 7)$ represent the height.

$$\begin{aligned}
 A &= 2\left(\frac{1}{2}bh\right) + lw \\
 276 &= 2\left[\frac{1}{2}(x)(x+7)\right] + 18(x+7) \\
 276 &= x^2 + 7x + 18x + 126 \\
 0 &= x^2 + 25x - 150 \\
 0 &= (x+30)(x-5) \\
 \cancel{x+30} &\quad \text{or} \quad x=5 \\
 x+7 &= 5+7 \\
 &= 12
 \end{aligned}$$

The base of the triangular portions is 5 ft and the height is 12 ft.

$$\begin{aligned}
 s &= -\frac{1}{2}gt^2 + v_0t + s_0 \\
 &= -\frac{1}{2}(32)t^2 + (60)t + 2 \\
 \mathbf{41. a.} \quad &= -16t^2 + 60t + 2 \\
 52 &= -16t^2 + 60t + 2 \\
 0 &= -16t^2 + 60t - 50 \\
 \mathbf{b.} \quad 0 &= 8t^2 - 30t + 25 \\
 t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-30) \pm \sqrt{(-30)^2 - 4(8)(25)}}{2(8)} \\
 &= \frac{30 \pm \sqrt{100}}{16} \\
 &= \frac{30 \pm 10}{16}
 \end{aligned}$$

$$t = \frac{40}{16} = \frac{5}{2} = 2.5 \quad \text{or} \quad t = \frac{20}{16} = \frac{5}{4} = 1.25$$

The ball will be at a height of 52 ft at times 1.25 sec and 2.5 sec after being kicked.

42. Let s represent the score on the sixth round.

$$\begin{aligned}
 \frac{92+88+85+90+89+s}{6} &< 88 \\
 \frac{444+s}{6} &< 88 \\
 444+s &< 528 \\
 s &< 84
 \end{aligned}$$

The golfer would need to score less than 84.

$$\begin{aligned}
 \mathbf{43.} \quad r &= \sqrt{\frac{3V}{\pi h}} \\
 9 &= \sqrt{\frac{3 \times 54\pi}{\pi h}} \\
 (9)^2 &= \left(\sqrt{\frac{3 \times 54\pi}{\pi h}}\right)^2 \\
 81 &= \frac{162}{h} \\
 h &= \frac{162}{81} \\
 h &= 2 \text{ in.}
 \end{aligned}$$

Chapter 1 Cumulative Review Exercises

$$\begin{aligned}
 \mathbf{1.} \quad &\left[(5x+3)^2 - (5x-3)^2\right]^2 \\
 &= \left[25x^2 + 30x + 9 - (25x^2 - 30x + 9)\right]^2 \\
 &= (25x^2 + 30x + 9 - 25x^2 + 30x - 9)^2 \\
 &= (60x)^2 = 3600x^2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2.} \quad &(4\sqrt{3} + 2\sqrt{2})(4\sqrt{3} - 2\sqrt{2}) \\
 &= (4\sqrt{3})^2 - (2\sqrt{2})^2 \\
 &= 16(3) - 4(2) \\
 &= 48 - 8 = 40
 \end{aligned}$$

$$\begin{aligned}
 \text{3. } \frac{3x^2 - x - 4}{4x^2 - 8x - 12} \div \frac{3x - 4}{6x^2 - 54} &= \frac{3x^2 - x - 4}{4(x^2 - 2x - 3)} \cdot \frac{6(x^2 - 9)}{3x - 4} \\
 &= \frac{(3x - 4)(x + 1)}{4(x - 3)(x + 1)} \cdot \frac{6(x + 3)(x - 3)}{3x - 4} \\
 &= \frac{\cancel{(3x - 4)}(x + 1)}{\cancel{4}(x - 3)\cancel{(x + 1)}} \cdot \frac{\cancel{6}^3(x + 3)\cancel{(x - 3)}}{\cancel{3x - 4}} \\
 &= \frac{3(x + 3)}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{4. } \frac{6}{x+2} - \frac{5}{x-2} + \frac{x}{x^2 - 4} &= \frac{6}{x+2} - \frac{5}{x-2} + \frac{x}{(x+2)(x-2)} \\
 &= \left(\frac{x-2}{x-2}\right)\left(\frac{6}{x+2}\right) - \left(\frac{x+2}{x+2}\right)\left(\frac{5}{x-2}\right) + \frac{x}{(x+2)(x-2)} \\
 &= \frac{6(x-2) - 5(x+2) + x}{(x+2)(x-2)} \\
 &= \frac{6x - 12 - 5x - 10 + x}{(x+2)(x-2)} \\
 &= \frac{2x - 22}{(x+2)(x-2)} \\
 &= \frac{\frac{1}{5x} - \frac{3}{5}}{\frac{2}{x} + \frac{1}{5}} = \frac{5x \cdot \left(\frac{1}{5x} - \frac{3}{5}\right)}{5x \cdot \left(\frac{2}{x} + \frac{1}{5}\right)} \\
 &= \frac{1 - 3x}{10 + x}
 \end{aligned}$$

5.

11. Let x represent the amount

borrowed at 4%. Then, $(8000 - x)$ is the amount borrowed at 5%.

	4%	5%	T
Pri	x	$8000 - x$	
Inte	$x(0.04)$	$(8000 - x)(0.05)$	38

$$\begin{aligned}
 \frac{2}{\sqrt{7} + \sqrt{3}} &= \frac{2(\sqrt{7} - \sqrt{3})}{(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})} \\
 &= \frac{2(\sqrt{7} - \sqrt{3})}{(\sqrt{7})^2 - (\sqrt{3})^2} \\
 &= \frac{2(\sqrt{7} - \sqrt{3})}{7 - 3} \\
 &= \frac{2(\sqrt{7} - \sqrt{3})}{4} \\
 &= \frac{\sqrt{7} - \sqrt{3}}{2}
 \end{aligned}$$

6.

$$\begin{aligned}
 \sqrt[3]{81y^5z^2w^{12}} &= \sqrt[3]{27y^3w^{12} \cdot 3y^2z^2} \\
 &= 3yw^4\sqrt[3]{3y^2z^2}
 \end{aligned}$$

7.

$$\text{8. a. } |4\pi - 11| \text{ or } |11 - 4\pi|$$

$$\text{b. } |4\pi - 11| = 4\pi - 11$$

$$\text{9. } 4x^3 - 32y^6$$

$$\begin{aligned}
 &= 4(x^3 - 8y^6) \\
 &= 4\left[x^3 - (2y^2)^3\right] \\
 &= 4\left[(x - 2y^2)(x^2 + 2xy^2 + 4y^4)\right]
 \end{aligned}$$

$$\begin{aligned}
 \frac{3 - 7i}{2 + 5i} &= \frac{(3 - 7i)(2 - 5i)}{(2 + 5i)(2 - 5i)} \\
 &= \frac{6 - 15i - 14i + 35i^2}{4 + 25} \\
 &= \frac{6 - 29i + 35(-1)}{29} \\
 &= \frac{-29 - 29i}{29} \\
 &= -1 - i
 \end{aligned}$$

10.

$$\begin{aligned}
 x(0.04) + (8000 - x)(0.05) &= 380 \\
 0.04x + 400 - 0.05x &= 380 \\
 -0.01x + 400 &= 380 \\
 -0.01x &= -20 \\
 x &= 2000
 \end{aligned}$$

$$8000 - x = 8000 - 2000$$

$$= 6000$$

$$(4x - 1)^2 + 3 = 6$$

$$(4x - 1)^2 = 3$$

$$4x - 1 = \pm\sqrt{3}$$

$$4x = 1 \pm \sqrt{3}$$

$$x = \frac{1 \pm \sqrt{3}}{4}$$

12.

$$\left\{ \frac{1 \pm \sqrt{3}}{4} \right\}$$

$$2x(x - 4) = 2x + 5$$

$$2x^2 - 8x = 2x + 5$$

13. $2x^2 - 10x - 5 = 0$

$$a = 2, b = -10, c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{10 \pm \sqrt{100 + 40}}{4}$$

$$= \frac{10 \pm \sqrt{140}}{4}$$

$$= \frac{10 \pm 2\sqrt{35}}{4} = \frac{5 \pm \sqrt{35}}{2}$$

$$\left\{ \frac{5 \pm \sqrt{35}}{2} \right\}$$

14. Let $u = \frac{x}{3} + 1$.

$$2\left(\frac{x}{3} + 1\right)^2 + 5\left(\frac{x}{3} + 1\right) - 12 = 0$$

$$2u^2 + 5u - 12 = 0$$

$$(2u - 3)(u + 4) = 0$$

Stephan borrowed \$6000 at 5% and \$2000 at 4%.

$$u = \frac{3}{2} \quad \text{or} \quad u = -4$$

$$\frac{x}{3} + 1 = \frac{3}{2} \quad \text{or} \quad \frac{x}{3} + 1 = -4$$

$$\frac{x}{3} = \frac{1}{2} \quad \text{or} \quad \frac{x}{3} = -5$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -15$$

$$\left\{ \frac{3}{2}, -15 \right\}$$

15. $\sqrt{x+4} - 2 = x$

$$\sqrt{x+4} = x + 2$$

$$(\sqrt{x+4})^2 = (x+2)^2$$

$$x + 4 = x^2 + 4x + 4$$

$$x^2 + 3x = 0$$

$$x(x + 3) = 0$$

$$x = 0 \quad \text{or} \quad x = -3$$

Check: $x = 0$

$$\sqrt{x+4} - 2 = x$$

$$\sqrt{(0)+4} - 2 = (0)$$

$$2 - 2 = 0$$

$$0 = 0 \checkmark \text{ true}$$

Check: $x = -3$

$$\sqrt{x+4} - 2 = x$$

$$\sqrt{(-3)+4} - 2 = (-3)$$

$$1 - 2 = -3$$

$$-1 = -3 \text{ false}$$

$$\{0\}$$

$$-|5-x| + 6 = 4$$

$$-|5-x| = -2$$

$$|5-x| = 2$$

$$5-x = -2 \quad \text{or} \quad 5-x = 2$$

$$-x = -7 \quad \text{or} \quad -x = -3$$

$$x = 7 \quad \text{or} \quad x = 3$$

16.

$$\{3, 7\}$$

$$x - 9 = \frac{72}{x - 8}$$

$$(x - 8)(x - 9) = (x - 8)\left(\frac{72}{x - 8}\right)$$

$$x^2 - 17x + 72 = 72$$

$$x^2 - 17x = 0$$

$$x(x - 17) = 0$$

$$\mathbf{17.} \quad x = 0 \quad \text{or} \quad x = 17$$

$$\{0, 17\}$$

$$\mathbf{18. a.} \quad A \cup B = \square$$

$$\mathbf{b.} \quad A \cap B = \{x \mid 4 \leq x < 11\}$$

$$\mathbf{c.} \quad A \cup C = \{x \mid x < 11\}$$

$$\mathbf{d.} \quad A \cap C = \{x \mid x < 2\}$$

$$\mathbf{e.} \quad B \cup C = \{x \mid x < 2 \text{ or } x \geq 4\}$$

$$\mathbf{f.} \quad B \cap C = \{ \}$$

$$|2x - 11| + 1 \leq 12$$

$$|2x - 11| \leq 11$$

$$-11 \leq 2x - 11 \leq 11$$

$$0 \leq 2x \leq 22$$

$$\mathbf{19.} \quad 0 \leq x \leq 11$$

$$[0, 11]$$

$$-\frac{3}{5}y < 15$$

$$\mathbf{20.} \quad y > -25$$

$$(-25, \infty)$$