Section 2.8

Check Point Exercises

1. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(2-(-1))^2+(3-(-3))^2}$

$=\sqrt{3^2+6^2}$

$=\sqrt{9+36}$

$=\sqrt{45}$

$=3\sqrt{5}$

≈ 6.71

2. $\left(\frac{1+7}{2},\frac{2+(-3)}{2}\right)=\left(\frac{8}{2},\frac{-1}{2}\right)=\left(4,-\frac{1}{2}\right)$

3. $h=0,\ k=0,\ r=4;$

$(x-0)^2+(y-0)^2=4^2$

$x^2+y^2=16$

4. $h=0,\ k=-6,\ r=10;$

$(x-0)^2+[y-(-6)]^2=10^2$

$(x-0)^2+(y+6)^2=100$

$x^2+(y+6)^2=100$

5. **a.** $(x+3)^2+(y-1)^2=4$

$[x-(-3)]^2+(y-1)^2=2^2$

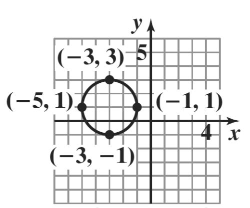
So in the standard form of the circle's equation $(x-h)^2+(y-k)^2=r^2$,

we have $h=-3,\ k=1,\ r=2.$

center: $(h,\ k)=(-3,\ 1)$

radius: $r=2$

b.



$(x+3)^2+(y-1)^2=4$

c. domain: $[-5,-1]$

range: $[-1,3]$

6. $x^2+y^2+4x-4y-1=0$

$x^2+y^2+4x-4y-1=0$

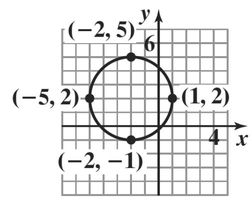
$(x^2+4x\quad)+(y^2-4y\quad)=0$

$(x^2+4x+4)+(y^2+4y+4)=1+4+4$

$(x+2)^2+(y-2)^2=9$

$[x-(-x)]^2+(y-2)^2=3^2$

So in the standard form of the circle's equation $(x-h)^2+(y-k)^2=r^2$, we have $h=-2,\ k=2,\ r=3$.



$x^2+y^2+4x-4y-1=0$

Concept and Vocabulary Check 2.8

1. $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

2. $\frac{x_1+x_2}{2}$; $\frac{y_1+y_2}{2}$

3. circle; center; radius

4. $(x-h)^2+(y-k)^2=r^2$

5. general

6. 4; 16

Exercise Set 2.8

1. $d=\sqrt{(14-2)^2+(8-3)^2}$

$=\sqrt{12^2+5^2}$

$=\sqrt{144+25}$

$=\sqrt{169}$

$=13$

2. $d=\sqrt{(8-5)^2+(5-1)^2}$
$=\sqrt{3^2+4^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

3. $d=\sqrt{\left(-6-4\right)^2+\left(3-(-1)\right)^2}$
$=\sqrt{\left(-10\right)^2+\left(4\right)^2}$
$=\sqrt{100+16}$
$=\sqrt{116}$
$=2\sqrt{29}$
≈ 10.77

4. $d=\sqrt{\left(-1-2\right)^2+\left(5-(-3)\right)^2}$
$=\sqrt{\left(-3\right)^2+\left(8\right)^2}$
$=\sqrt{9+64}$
$=\sqrt{73}$
≈ 8.54

5. $d=\sqrt{(-3-0)^2+(4-0)^2}$
$=\sqrt{3^2+4^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

6. $d=\sqrt{(3-0)^2+\left(-4-0\right)^2}$
$=\sqrt{3^2+\left(-4\right)^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

7. $d=\sqrt{[3-(-2)]^2+[-4-(-6)]^2}$
$=\sqrt{5^2+2^2}$
$=\sqrt{25+4}$
$=\sqrt{29}$
≈ 5.39

8. $d=\sqrt{[2-(-4)]^2+[-3-(-1)]^2}$
$=\sqrt{6^2+\left(-2\right)^2}$
$=\sqrt{36+4}$
$=\sqrt{40}$
$=2\sqrt{10}$
≈ 6.32

9. $d=\sqrt{(4-0)^2+[1-(-3)]^2}$
$=\sqrt{4^2+4^2}$
$=\sqrt{16+16}$
$=\sqrt{32}$
$=4\sqrt{2}$
≈ 5.66

10. $d=\sqrt{\left(4-0\right)^2+[3-\left(-2\right)]^2}$
$=\sqrt{4^2+[3+2]^2}$
$=\sqrt{16+5^2}$
$=\sqrt{16+25}$
$=\sqrt{41}$
≈ 6.40

11. $d=\sqrt{(-.5-3.5)^2+(6.2-8.2)^2}$
$=\sqrt{(-4)^2+(-2)^2}$
$=\sqrt{16+4}$
$=\sqrt{20}$
$=2\sqrt{5}$
≈ 4.47

12. $d=\sqrt{(1.6-2.6)^2+\left(-5.7-1.3\right)^2}$
$=\sqrt{\left(-1\right)^2+\left(-7\right)^2}$
$=\sqrt{1+49}$
$=\sqrt{50}$
$=5\sqrt{2}$
≈ 7.07

13. $$\begin{aligned} d &= \sqrt{(\sqrt{5}-0)^2+[0-(-\sqrt{3})]^2} \\ &= \sqrt{(\sqrt{5})^2+(\sqrt{3})^2} \\ &= \sqrt{5+3} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \\ &\approx 2.83 \end{aligned}$$

14. $$\begin{aligned} d &= \sqrt{\left(\sqrt{7}-0\right)^2+\left[0-\left(-\sqrt{2}\right)\right]^2} \\ &= \sqrt{\left(\sqrt{7}\right)^2+\left[-\sqrt{2}\right]^2} \\ &= \sqrt{7+2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

15. $$\begin{aligned} d &= \sqrt{(-\sqrt{3}-3\sqrt{3})^2+(4\sqrt{5}-\sqrt{5})^2} \\ &= \sqrt{(-4\sqrt{3})^2+(3\sqrt{5})^2} \\ &= \sqrt{16(3)+9(5)} \\ &= \sqrt{48+45} \\ &= \sqrt{93} \\ &\approx 9.64 \end{aligned}$$

16. $$\begin{aligned} d &= \sqrt{\left(-\sqrt{3}-2\sqrt{3}\right)^2+\left(5\sqrt{6}-\sqrt{6}\right)^2} \\ &= \sqrt{\left(-3\sqrt{3}\right)^2+\left(4\sqrt{6}\right)^2} \\ &= \sqrt{9\cdot 3+16\cdot 6} \\ &= \sqrt{27+96} \\ &= \sqrt{123} \\ &\approx 11.09 \end{aligned}$$

17. $$\begin{aligned} d &= \sqrt{\left(\frac{1}{3}-\frac{7}{3}\right)^2+\left(\frac{6}{5}-\frac{1}{5}\right)^2} \\ &= \sqrt{(-2)^2+1^2} \\ &= \sqrt{4+1} \\ &= \sqrt{5} \\ &\approx 2.24 \end{aligned}$$

18. $$\begin{aligned} d &= \sqrt{\left[\frac{3}{4}-\left(-\frac{1}{4}\right)\right]^2+\left[\frac{6}{7}-\left(-\frac{1}{7}\right)\right]^2} \\ &= \sqrt{\left(\frac{3}{4}+\frac{1}{4}\right)^2+\left[\frac{6}{7}+\frac{1}{7}\right]^2} \\ &= \sqrt{1^2+1^2} \\ &= \sqrt{2} \\ &\approx 1.41 \end{aligned}$$

19. $$\left(\frac{6+2}{2},\frac{8+4}{2}\right)=\left(\frac{8}{2},\frac{12}{2}\right)=(4,6)$$

20. $$\left(\frac{10+2}{2},\frac{4+6}{2}\right)=\left(\frac{12}{2},\frac{10}{2}\right)=(6,5)$$

21. $$\begin{aligned} &\left(\frac{-2+(-6)}{2},\frac{-8+(-2)}{2}\right) \\ &=\left(\frac{-8}{2},\frac{-10}{2}\right)=(-4,-5) \end{aligned}$$

22. $$\begin{aligned} &\left(\frac{-4+(-1)}{2},\frac{-7+(-3)}{2}\right)=\left(\frac{-5}{2},\frac{-10}{2}\right) \\ &=\left(\frac{-5}{2},-5\right) \end{aligned}$$

23. $$\begin{aligned} &\left(\frac{-3+6}{2},\frac{-4+(-8)}{2}\right) \\ &=\left(\frac{3}{2},\frac{-12}{2}\right)=\left(\frac{3}{2},-6\right) \end{aligned}$$

24. $$\left(\frac{-2+(-8)}{2}\frac{-1+6}{2}\right)=\left(\frac{-10}{2},\frac{5}{2}\right)=\left(-5,\frac{5}{2}\right)$$

25. $$\begin{aligned} &\left(\frac{\frac{-7}{2}+\left(-\frac{5}{2}\right)}{2},\frac{\frac{3}{2}+\left(-\frac{11}{2}\right)}{2}\right) \\ &=\left(\frac{\frac{-12}{2}}{2},\frac{\frac{-8}{2}}{2}\right)=\left(-\frac{6}{2},\frac{-4}{2}\right)=(-3,-2) \end{aligned}$$

26. $\left(\dfrac{-\frac{2}{5}+\left(-\frac{2}{5}\right)}{2},\dfrac{\frac{7}{15}+\left(-\frac{4}{15}\right)}{2}\right)=\left(\dfrac{-\frac{4}{5}}{2},\dfrac{\frac{3}{15}}{2}\right)$

$=\left(-\frac{4}{5}\cdot\frac{1}{2},\frac{3}{15}\cdot\frac{1}{2}\right)=\left(-\frac{2}{5},\frac{1}{10}\right)$

27. $\left(\dfrac{8+(-6)}{2},\dfrac{3\sqrt{5}+7\sqrt{5}}{2}\right)$

$=\left(\dfrac{2}{2},\dfrac{10\sqrt{5}}{2}\right)=\left(1,5\sqrt{5}\right)$

28. $\left(\dfrac{7\sqrt{3}+3\sqrt{3}}{2},\dfrac{-6+(-2)}{2}\right)=\left(\dfrac{10\sqrt{3}}{2},\dfrac{-8}{2}\right)$

$=\left(5\sqrt{3},-4\right)$

29. $\left(\dfrac{\sqrt{18}+\sqrt{2}}{2},\dfrac{-4+4}{2}\right)$

$=\left(\dfrac{3\sqrt{2}+\sqrt{2}}{2},\dfrac{0}{2}\right)=\left(\dfrac{4\sqrt{2}}{2},0\right)=(2\sqrt{2},0)$

30. $\left(\dfrac{\sqrt{50}+\sqrt{2}}{2},\dfrac{-6+6}{2}\right)=\left(\dfrac{5\sqrt{2}+\sqrt{2}}{2},\dfrac{0}{2}\right)$

$=\left(\dfrac{6\sqrt{2}}{2},0\right)=\left(3\sqrt{2},0\right)$

31. $(x-0)^2+(y-0)^2=7^2$

$x^2+y^2=49$

32. $(x-0)^2+(y-0)^2=8^2$

$x^2+y^2=64$

33. $(x-3)^2+(y-2)^2=5^2$

$(x-3)^2+(y-2)^2=25$

34. $(x-2)^2+[y-(-1)]^2=4^2$

$(x-2)^2+(y+1)^2=16$

35. $[x-(-1)]^2+(y-4)^2=2^2$

$(x+1)^2+(y-4)^2=4$

36. $[x-(-3)]^2+(y-5)^2=3^2$

$(x+3)^2+(y-5)^2=9$

37. $[x-(-3)]^2+[y-(-1)]^2=\left(\sqrt{3}\right)^2$

$(x+3)^2+(y+1)^2=3$

38. $[x-(-5)]^2+[y-(-3)]^2=\left(\sqrt{5}\right)^2$

$(x+5)^2+(y+3)^2=5$

39. $[x-(-4)]^2+(y-0)^2=10^2$

$(x+4)^2+(y-0)^2=100$

40. $[x-(-2)]^2+(y-0)^2=6^2$

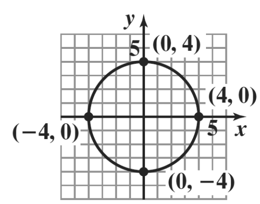
$(x+2)^2+y^2=36$

41. $x^2+y^2=16$

$(x-0)^2+(y-0)^2=y^2$

$h=0,\ k=0,\ r=4;$

center = (0, 0); radius = 4



$x^2+y^2=16$

domain: $[-4,4]$

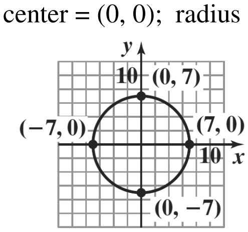
range: $[-4,4]$

42. $x^2+y^2=49$

$(x-0)^2+(y-0)^2=7^2$

$h=0,k=0,r=7;$

center = (0, 0); radius = 7



$x^2+y^2=49$

domain: $[-7,7]$

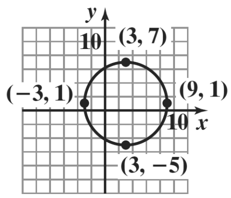
range: $[-7,7]$

43. $(x-3)^2+(y-1)^2=36$

$(x-3)^2+(y-1)^2=6^2$

$h=3,\ k=1,\ r=6;$

center = (3, 1); radius = 6



$(x-3)^2+(y-1)^2=36$

domain: $[-3,9]$

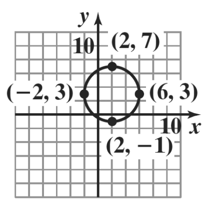
range: $[-5,7]$

44. $(x-2)^2+(y-3)^2=16$

$(x-2)^2+(y-3)^2=4^2$

$h=2, k=3, r=4;$

center = (2, 3); radius = 4



$(x-2)^2+(y-3)^2=16$

domain: $[-2,6]$

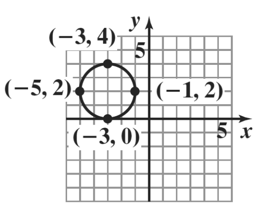
range: $[-1,7]$

45. $(x+3)^2+(y-2)^2=4$

$[x-(-3)]^2+(y-2)^2=2^2$

$h=-3,\ k=2,\ r=2$

center = (–3, 2); radius = 2



$(x+3)^2+(y-2)^2=4$

domain: $[-5,-1]$

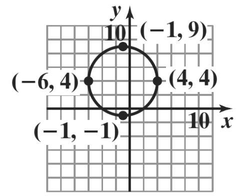
range: $[0,4]$

46. $(x+1)^2+(y-4)^2=25$

$[x-(-1)]^2+(y-4)^2=5^2$

$h=-1, k=4, r=5;$

center = (–1, 4); radius = 5



$(x+1)^2+(y-4)^2=25$

domain: $[-6,4]$

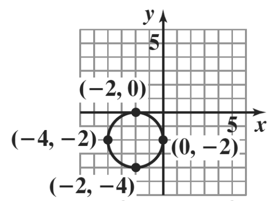
range: $[-1,9]$

47. $(x+2)^2+(y+2)^2=4$

$[x-(-2)]^2+[y-(-2)]^2=2^2$

$h=-2,\ k=-2,\ r=2$

center = (–2, –2); radius = 2



$(x+2)^2+(y+2)^2=4$

domain: $[-4,0]$

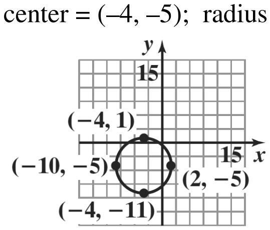
range: $[-4,0]$

48. $(x+4)^2+(y+5)^2=36$

$[x-(-4)]^2+[y-(-5)]^2=6^2$

$h=-4, k=-5, r=6;$

center = (–4, –5); radius = 6



$(x+4)^2+(y+5)^2=36$

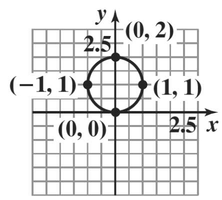
domain: $[-10,2]$

range: $[-11,1]$

49. $x^2 + (y-1)^2 = 1$

$h = 0, k = 1, r = 1;$

center = (0, 1); radius = 1



$x^2 + (y-1)^2 = 1$

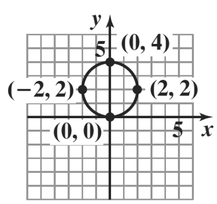
domain: $[-1,1]$

range: $[0,2]$

50. $x^2 + (y-2)^2 = 4$

$h = 0, k = 2, r = 2;$

center = (0,2); radius = 2



$x^2 + (y-2)^2 = 4$

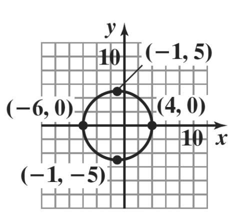
domain: $[-2,2]$

range: $[0,4]$

51. $(x+1)^2 + y^2 = 25$

$h = -1, k = 0, r = 5;$

center = (–1,0); radius = 5



$(x+1)^2 + y^2 = 25$

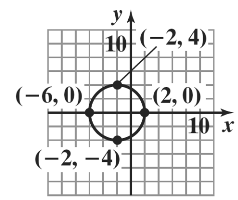
domain: $[-6,4]$

range: $[-5,5]$

52. $(x+2)^2 + y^2 = 16$

$h = -2, k = 0, r = 4;$

center = (–2,0); radius = 4



$(x+2)^2 + y^2 = 16$

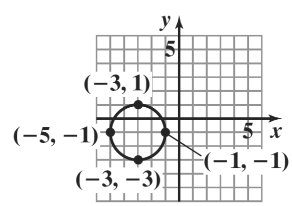
domain: $[-6,2]$

range: $[-4,4]$

53.

$$x^2 + y^2 + 6x + 2y + 6 = 0$$
$$(x^2 + 6x) + (y^2 + 2y) = -6$$
$$(x^2 + 6x + 9) + (y^2 + 2y + 1) = 9 + 1 - 6$$
$$(x+3)^2 + (y+1)^2 = 4$$
$$[x-(-3)]^2 + [9-(-1)]^2 = 2^2$$

center = (–3, –1); radius = 2

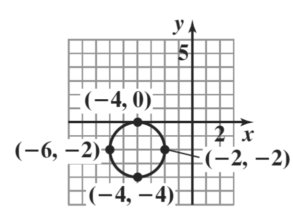


$x^2 + y^2 + 6x + 2y + 6 = 0$

54. $x^2 + y^2 + 8x + 4y + 16 = 0$

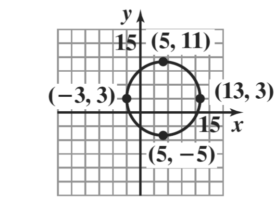
$$(x^2 + 8x) + (y^2 + 4y) = -16$$
$$(x^2 + 8x + 16) + (y^2 + 4y + 4) = 20 - 16$$
$$(x+4)^2 + (y+2)^2 = 4$$
$$[x-(-4)]^2 + [y-(-2)]^2 = 2^2$$

center = (–4, –2); radius = 2



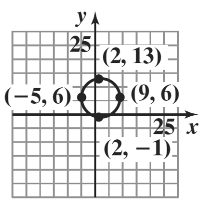
$x^2 + y^2 + 8x + 4y + 16 = 0$

55.
$$x^2 + y^2 - 10x - 6y - 30 = 0$$
$$\left(x^2 - 10x\right) + \left(y^2 - 6y\right) = 30$$
$$\left(x^2 - 10x + 25\right) + \left(y^2 - 6y + 9\right) = 25 + 9 + 30$$
$$\left(x-5\right)^2 + \left(y-3\right)^2 = 64$$
$$(x-5)^2 + (y-3)^2 = 8^2$$
center = (5, 3); radius = 8



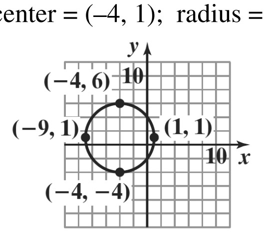
$x^2 + y^2 - 10x - 6y - 30 = 0$

56.
$$x^2 + y^2 - 4x - 12y - 9 = 0$$
$$\left(x^2 - 4x\right) + \left(y^2 - 12y\right) = 9$$
$$\left(x^2 - 4x + 4\right) + \left(y^2 - 12y + 36\right) = 4 + 36 + 9$$
$$\left(x-2\right)^2 + \left(y-6\right)^2 = 49$$
$$(x-2)^2 + (y-6)^2 = 7^2$$
center = (2, 6); radius = 7



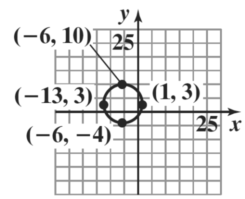
$x^2 + y^2 - 4x - 12y - 9 = 0$

57.
$$x^2 + y^2 + 8x - 2y - 8 = 0$$
$$\left(x^2 + 8x\right) + \left(y^2 - 2y\right) = 8$$
$$\left(x^2 + 8x + 16\right) + \left(y^2 - 2y + 1\right) = 16 + 1 + 8$$
$$\left(x+4\right)^2 + \left(y-1\right)^2 = 25$$
$$\left[x - (-4)\right]^2 + (y-1)^2 = 5^2$$
center = (–4, 1); radius = 5



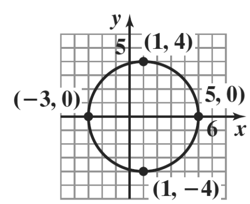
$x^2 + y^2 + 8x - 2y - 8 = 0$

58.
$$x^2 + y^2 + 12x - 6y - 4 = 0$$
$$\left(x^2 + 12x\right) + \left(y^2 - 6y\right) = 4$$
$$\left(x^2 + 12x + 36\right) + \left(y^2 - 6y + 9\right) = 36 + 9 + 4$$
$$\left[x - (-6)\right]^2 + \left(y-3\right)^2 = 7^2$$
center = (–6, 3); radius = 7



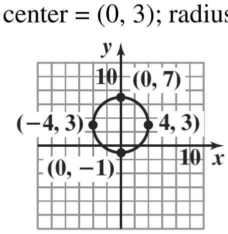
$x^2 + y^2 + 12x - 6y - 4 = 0$

59.
$$x^2 - 2x + y^2 - 15 = 0$$
$$\left(x^2 - 2x\right) + y^2 = 15$$
$$\left(x^2 - 2x + 1\right) + \left(y-0\right)^2 = 1 + 0 + 15$$
$$\left(x-1\right)^2 + \left(y-0\right)^2 = 16$$
$$\left(x-1\right)^2 + \left(y-0\right)^2 = 4^2$$
center = (1, 0); radius = 4



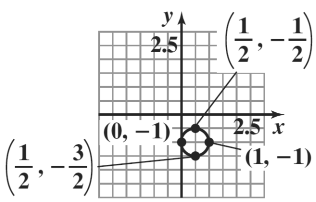
$x^2 - 2x + y^2 - 15 = 0$

60.
$$x^2 + y^2 - 6y - 7 = 0$$
$$x^2 + \left(y^2 - 6y\right) = 7$$
$$\left(x-0\right)^2 = \left(y^2 - 6y + 9\right) = 0 + 9 + 7$$
$$\left(x-0\right)^2 + \left(y-3\right)^2 = 16$$
$$(x-0)^2 + (y-3)^2 = 4^2$$
center = (0, 3); radius = 4



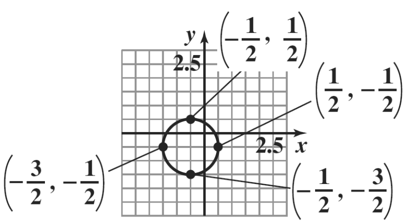
$x^2 + y^2 - 6y - 7 = 0$

61.
$$x^2+y^2-x+2y+1=0$$
$$x^2-x\quad+y^2+2y\quad=-1$$
$$x^2-x+\frac{1}{4}+y^2+2y+1=-1+\frac{1}{4}+1$$
$$\left(x-\frac{1}{2}\right)^2+(y+1)^2=\frac{1}{4}$$
center = $\left(\frac{1}{2},-1\right)$; radius = $\frac{1}{2}$



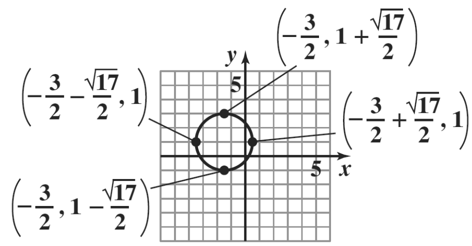
$x^2+y^2-x+2y+1=0$

62.
$$x^2+y^2+x+y-\frac{1}{2}=0$$
$$x^2+x\quad+y^2+y\quad=\frac{1}{2}$$
$$x^2+x+\frac{1}{4}+y^2+y+\frac{1}{4}=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$
$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=1$$
center = $\left(\frac{1}{2},\frac{1}{2}\right)$; radius = 1



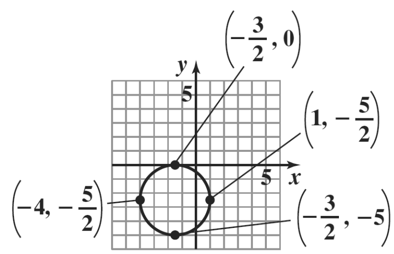
$x^2+y^2+x+y-\frac{1}{2}=0$

63.
$$x^2+y^2+3x-2y-1=0$$
$$x^2+3x\quad+y^2-2y\quad=1$$
$$x^2+3x+\frac{9}{4}+y^2-2y+1=1+\frac{9}{4}+1$$
$$\left(x+\frac{3}{2}\right)^2+(y-1)^2=\frac{17}{4}$$
center = $\left(-\frac{3}{2},1\right)$; radius = $\frac{\sqrt{17}}{2}$



$x^2+y^2+3x-2y-1=0$

64.
$$x^2+y^2+3x+5y+\frac{9}{4}=0$$
$$x^2+3x\quad+y^2+5y\quad=-\frac{9}{4}$$
$$x^2+3x+\frac{9}{4}+y^2+5y+\frac{25}{4}=-\frac{9}{4}+\frac{9}{4}+\frac{25}{4}$$
$$\left(x+\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{25}{4}$$
center = $\left(-\frac{3}{2},-\frac{5}{2}\right)$; radius = $\frac{5}{2}$



$x^2+y^2+3x+5y+\frac{9}{4}=0$

65. **a.** Since the line segment passes through the center, the center is the midpoint of the segment.
$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{3+7}{2},\frac{9+11}{2}\right)=\left(\frac{10}{2},\frac{20}{2}\right)$$
$$=(5,10)$$
The center is $(5,10)$.

b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,9)$, we get:

$$d = \sqrt{(5-3)^2 + (10-9)^2}$$
$$= \sqrt{2^2 + 1^2} = \sqrt{4+1}$$
$$= \sqrt{5}$$

The radius is $\sqrt{5}$ units.

c. $(x-5)^2 + (y-10)^2 = (\sqrt{5})^2$

$(x-5)^2 + (y-10)^2 = 5$

66. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{3+5}{2}, \frac{6+4}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right)$$
$$= (4,5)$$

The center is $(4,5)$.

b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,6)$, we get:

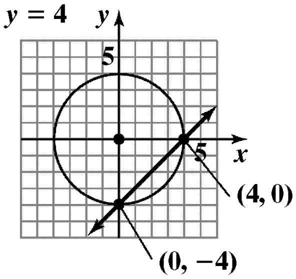
$$d = \sqrt{(4-3)^2 + (5-6)^2}$$
$$= \sqrt{1^2 + (-1)^2} = \sqrt{1+1}$$
$$= \sqrt{2}$$

The radius is $\sqrt{2}$ units.

c. $(x-4)^2 + (y-5)^2 = (\sqrt{2})^2$

$(x-4)^2 + (y-5)^2 = 2$

67. $x^2 + y^2 = 16$
$x - y = 4$



Intersection points: $(0,-4)$ and $(4,0)$

Check $(0,-4)$:

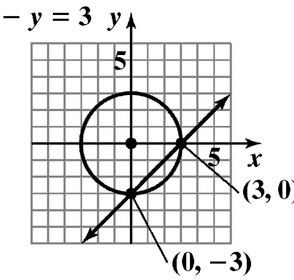
$0^2 + (-4)^2 = 16 \qquad 0-(-4) = 4$
$16 = 16$ true $\qquad 4 = 4$ true

Check $(4,0)$:

$4^2 + 0^2 = 16 \qquad 4 - 0 = 4$
$16 = 16$ true $\qquad 4 = 4$ true

The solution set is $\{(0,-4),(4,0)\}$.

68. $x^2 + y^2 = 9$
$x - y = 3$



Intersection points: $(0,-3)$ and $(3,0)$

Check $(0,-3)$:

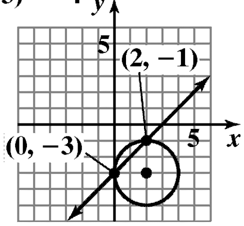
$0^2 + (-3)^2 = 9 \qquad 0-(-3) = 3$
$9 = 9$ true $\qquad 3 = 3$ true

Check $(3,0)$:

$3^2 + 0^2 = 9 \qquad 3 - 0 = 3$
$9 = 9$ true $\qquad 3 = 3$ true

The solution set is $\{(0,-3),(3,0)\}$.

69. $(x-2)^2+(y+3)^2=4$
$y = x - 3$



Intersection points: $(0,-3)$ and $(2,-1)$

Check $(0,-3)$:

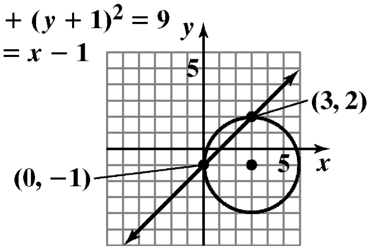
$(0-2)^2+(-3+3)^2=9 \qquad -3=0-3$
$(-2)^2+0^2=4 \qquad -3=-3$ true
$4=4$
true

Check $(2,-1)$:

$(2-2)^2+(-1+3)^2=4 \qquad -1=2-3$
$0^2+2^2=4 \qquad -1=-1$ true
$4=4$
true

The solution set is $\{(0,-3),(2,-1)\}$.

70. $(x-3)^2+(y+1)^2=9$
$y = x - 1$



Intersection points: $(0,-1)$ and $(3,2)$

Check $(0,-1)$:

$(0-3)^2+(-1+1)^2=9 \qquad -1=0-1$
$(-3)^2+0^2=9 \qquad -1=-1$ true
$9=9$
true

Check $(3,2)$:

$(3-3)^2+(2+1)^2=9 \qquad 2=3-1$
$0^2+3^2=9 \qquad 2=2$ true
$9=9$
true

The solution set is $\{(0,-1),(3,2)\}$.

71. $d=\sqrt{(8495-4422)^2+(8720-1241)^2}\cdot\sqrt{0.1}$
$d=\sqrt{72,524,770}\cdot\sqrt{0.1}$
$d\approx 2693$
The distance between Boston and San Francisco is about 2693 miles.

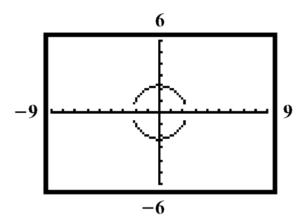
72. $d=\sqrt{(8936-8448)^2+(3542-2625)^2}\cdot\sqrt{0.1}$
$d=\sqrt{1,079,033}\cdot\sqrt{0.1}$
$d\approx 328$
The distance between New Orleans and Houston is about 328 miles.

73. If we place L.A. at the origin, then we want the equation of a circle with center at $(-2.4,-2.7)$ and radius 30.
$(x-(-2.4))^2+(y-(-2.7))^2=30^2$
$(x+2.4)^2+(y+2.7)^2=900$

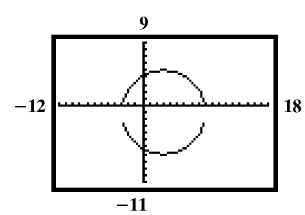
74. $C(0, 68+14)=(0, 82)$
$(x-0)^2+(y-82)^2=68^2$
$x^2+(y-82)^2=4624$

75. – 82. Answers will vary.

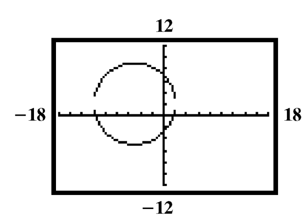
83.



84.



85.



86. makes sense

87. makes sense

88. does not make sense; Explanations will vary. Sample explanation: Since $r^2 = -4$ this is not the equation of a circle.

89. makes sense

90. false; Changes to make the statement true will vary. A sample change is: The equation would be $x^2 + y^2 = 256.$

91. false; Changes to make the statement true will vary. A sample change is: The center is at (3, –5).

92. false; Changes to make the statement true will vary. A sample change is: This is not an equation for a circle.

93. false; Changes to make the statement true will vary. A sample change is: Since $r^2 = -36$ this is not the equation of a circle.

94. The distance for A to B:

$$\begin{aligned}\overline{AB} &= \sqrt{(3-1)^2 + [3+d-(1+d)]^2}\\ &= \sqrt{2^2+2^2}\\ &= \sqrt{4+4}\\ &= \sqrt{8}\\ &= 2\sqrt{2}\end{aligned}$$

The distance from B to C:

$$\begin{aligned}\overline{BC} &= \sqrt{(6-3)^2 + [3+d-(6+d)]^2}\\ &= \sqrt{3^2+(-3)^2}\\ &= \sqrt{9+9}\\ &= \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

The distance for A to C:

$$\begin{aligned}\overline{AC} &= \sqrt{(6-1)^2 + [6+d-(1+d)]^2}\\ &= \sqrt{5^2+5^2}\\ &= \sqrt{25+25}\\ &= \sqrt{50}\\ &= 5\sqrt{2}\end{aligned}$$

$$\begin{aligned}\overline{AB}+\overline{BC} &= \overline{AC}\\ 2\sqrt{2}+3\sqrt{2} &= 5\sqrt{2}\\ 5\sqrt{2} &= 5\sqrt{2}\end{aligned}$$

95. a. d_1 is distance from (x_1, x_2) to midpoint

$$d_1 = \sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2}$$

$$d_1 = \sqrt{\left(\frac{x_1+x_2-2x_1}{2}\right)^2 + \left(\frac{y_1+y_2-2y_1}{2}\right)^2}$$

$$d_1 = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$

$$d_1 = \sqrt{\frac{x_2-2x_1x_2+x_1{}^2}{4} + \frac{y_2{}^2-2y_2y_1+y_1{}^2}{4}}$$

$$d_1 = \sqrt{\frac{1}{4}\left(x_2-2x_1x_2+x_1+y_2{}^2-2y_2y_1+y_1{}^2\right)}$$

$$d_1 = \frac{1}{2}\sqrt{x_2-2x_1x_2+x_1+y_2{}^2-2y_2y_1+y_1{}^2}$$

d_2 is distance from midpoint to (x_2, y_2)

$$d_2 = \sqrt{\left(\frac{x_1+x_2}{2} - x_2\right)^2 + \left(\frac{y_1+y_2}{2} - y_2\right)^2}$$

$$d_2 = \sqrt{\left(\frac{x_1+x_2-2x_2}{2}\right)^2 + \left(\frac{y_1+y_2-2y_2}{2}\right)^2}$$

$$d_2 = \sqrt{\left(\frac{x_1-x_2}{2}\right)^2 + \left(\frac{y_1-y_2}{2}\right)^2}$$

$$d_2 = \sqrt{\frac{x_1{}^2-2x_1x_2+x_2{}^2}{4} + \frac{y_1{}^2-2y_2y_1+y_2{}^2}{4}}$$

$$d_2 = \sqrt{\frac{1}{4}\left(x_1{}^2-2x_1x_2+x_2{}^2+y_1{}^2-2y_2y_1+y_2{}^2\right)}$$

$$d_2 = \frac{1}{2}\sqrt{x_1{}^2-2x_1x_2+x_2{}^2+y_1{}^2-2y_2y_1+y_2{}^2}$$

$$d_1 = d_2$$

b. d_3 is the distance from (x_1, y_1) to $(x_2 y_2)$

$$d_3 = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d_3 = \sqrt{x_2{}^2-2x_1x_2+x_1{}^2+y_2{}^2-2y_2y_1+y_1{}^2}$$

$d_1 + d_2 = d_3$ because $\frac{1}{2}\sqrt{a}+\frac{1}{2}\sqrt{a}=\sqrt{a}$

96. Both circles have center (2, –3). The smaller circle has radius 5 and the larger circle has radius 6. The smaller circle is inside of the larger circle. The area between them is given by

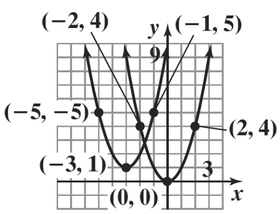
$$\begin{aligned}\pi(6)^2 - \pi(5)^2 &= 36\pi - 25\pi\\ &= 11\pi\\ &\approx 34.56 \text{ square units.}\end{aligned}$$

97. The circle is centered at (0,0). The slope of the radius with endpoints (0,0) and (3,–4) is $m = -\frac{-4-0}{3-0} = -\frac{4}{3}$. The line perpendicular to the radius has slope $\frac{3}{4}$. The tangent line has slope $\frac{3}{4}$ and passes through (3,–4), so its equation is:
$y+4 = \frac{3}{4}(x-3)$.

98.
$$0 = -2(x-3)^2 + 8$$
$$2(x-3)^2 = 8$$
$$(x-3)^2 = 4$$
$$x-3 = \pm\sqrt{4}$$
$$x = 3 \pm 2$$
$$x = 1, 5$$

99.
$$-x^2 - 2x + 1 = 0$$
$$x^2 + 2x - 1 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{8}}{2}$$
$$= \frac{2 \pm 2\sqrt{2}}{2}$$
$$= 1 \pm \sqrt{2}$$
The solution set is $\{1 \pm \sqrt{2}\}$.

100. The graph of g is the graph of f shifted 1 unit up and 3 units to the left.



Chapter 2 Review Exercises

1. function
domain: {2, 3, 5}
range: {7}

2. function
domain: {1, 2, 13}
range: {10, 500, π}

3. not a function
domain: {12, 14}
range: {13, 15, 19}

4.
$$2x + y = 8$$
$$y = -2x + 8$$
Since only one value of y can be obtained for each value of x, y is a function of x.

5.
$$3x^2 + y = 14$$
$$y = -3x^2 + 14$$
Since only one value of y can be obtained for each value of x, y is a function of x.

6.
$$2x + y^2 = 6$$
$$y^2 = -2x + 6$$
$$y = \pm\sqrt{-2x+6}$$
Since more than one value of y can be obtained from some values of x, y is not a function of x.

7. $f(x) = 5 - 7x$

a. $f(4) = 5 - 7(4) = -23$

b.
$$f(x+3) = 5 - 7(x+3)$$
$$= 5 - 7x - 21$$
$$= -7x - 16$$

c. $f(-x) = 5 - 7(-x) = 5 + 7x$

8. $g(x) = 3x^2 - 5x + 2$

a. $g(0) = 3(0)^2 - 5(0) + 2 = 2$

b.
$$g(-2) = 3(-2)^2 - 5(-2) + 2$$
$$= 12 + 10 + 2$$
$$= 24$$

c. $g(x-1) = 3(x-1)^2 - 5(x-1) + 2$
$= 3(x^2 - 2x + 1) - 5x + 5 + 2$
$= 3x^2 - 11x + 10$

d. $g(-x) = 3(-x)^2 - 5(-x) + 2$
$= 3x^2 + 5x + 2$

9. **a.** $g(13) = \sqrt{13-4} = \sqrt{9} = 3$

b. $g(0) = 4 - 0 = 4$

c. $g(-3) = 4 - (-3) = 7$

10. **a.** $f(-2) = \dfrac{(-2)^2 - 1}{-2-1} = \dfrac{3}{-3} = -1$

b. $f(1) = 12$

c. $f(2) = \dfrac{2^2 - 1}{2-1} = \dfrac{3}{1} = 3$

11. The vertical line test shows that this is not the graph of a function.

12. The vertical line test shows that this is the graph of a function.

13. The vertical line test shows that this is the graph of a function.

14. The vertical line test shows that this is not the graph of a function.

15. The vertical line test shows that this is not the graph of a function.

16. The vertical line test shows that this is the graph of a function.

17. **a.** domain: $[-3, 5)$

b. range: $[-5, 0]$

c. x-intercept: -3

d. y-intercept: -2

e. increasing: $(-2, 0)$ or $(3, 5)$
decreasing: $(-3, -2)$ or $(0, 3)$

f. $f(-2) = -3$ and $f(3) = -5$

18. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, 3]$

c. x-intercepts: -2 and 3

d. y-intercept: 3

e. increasing: $(-\infty, 0)$
decreasing: $(0, \infty)$

f. $f(-2) = 0$ and $f(6) = -3$

19. **a.** domain: $(-\infty, \infty)$

b. range: $[-2, 2]$

c. x-intercept: 0

d. y-intercept: 0

e. increasing: $(-2, 2)$
constant: $(-\infty, -2)$ or $(2, \infty)$

f. $f(-9) = -2$ and $f(14) = 2$

20. **a.** 0, relative maximum −2

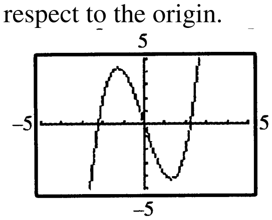
b. −2, 3, relative minimum −3, –5

21. **a.** 0, relative maximum 3

b. none

22. $f(x) = x^3 - 5x$
$f(-x) = (-x)^3 - 5(-x)$
$= -x^3 + 5x$
$= -f(x)$

The function is odd. The function is symmetric with respect to the origin.



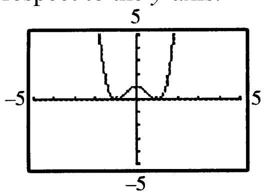
23. $f(x) = x^4 - 2x^2 + 1$

$f(-x) = (-x)^4 - 2(-x)^2 + 1$

$= x^4 - 2x^2 + 1$

$= f(x)$

The function is even. The function is symmetric with respect to the y-axis.



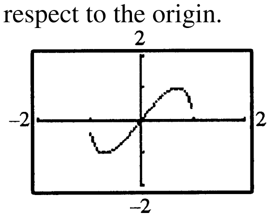
24. $f(x) = 2x\sqrt{1-x^2}$

$f(-x) = 2(-x)\sqrt{1-(-x)^2}$

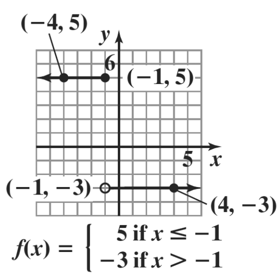
$= -2x\sqrt{1-x^2}$

$= -f(x)$

The function is odd. The function is symmetric with respect to the origin.

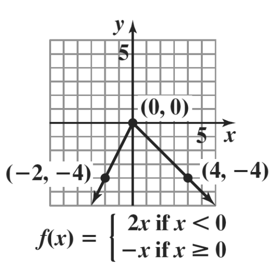


25. **a.**



b. range: $\{-3, 5\}$

26. **a.**



b. range: $\{y \mid y \le 0\}$

27. $\dfrac{8(x+h)-11-(8x-11)}{h}$

$= \dfrac{8x+8h-11-8x+11}{h}$

$= \dfrac{8h}{8}$

$= 8$

28. $\dfrac{-2(x+h)^2+(x+h)+10-\left(-2x^2+x+10\right)}{h}$

$= \dfrac{-2\left(x^2+2xh+h^2\right)+x+h+10+2x^2-x-10}{h}$

$= \dfrac{-2x^2-4xh-2h^2+x+h+10+2x^2-x-10}{h}$

$= \dfrac{-4xh-2h^2+h}{h}$

$= \dfrac{h\left(-4x-2h+1\right)}{h}$

$-4x-2h+1$

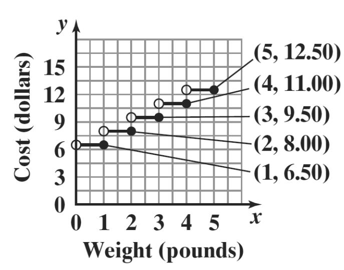
29. **a.** Yes, the eagle's height is a function of time since the graph passes the vertical line test.

b. Decreasing: (3, 12)
The eagle descended.

c. Constant: (0, 3) or (12, 17)
The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds.

d. Increasing: (17, 30)
The eagle was ascending.

30.



31. $m = \dfrac{1-2}{5-3} = \dfrac{-1}{2} = -\dfrac{1}{2}$; falls

32. $m = \dfrac{-4-(-2)}{-3-(-1)} = \dfrac{-2}{-2} = 1$; rises

33. $m = \dfrac{\frac{1}{4} - \frac{1}{4}}{6-(-3)} = \dfrac{0}{9} = 0;$ horizontal

34. $m = \dfrac{10-5}{-2-(-2)} = \dfrac{5}{0}$ undefined; vertical

35. point-slope form: $y - 2 = -6(x + 3)$
slope-intercept form: $y = -6x - 16$

36. $m = \dfrac{2-6}{-1-1} = \dfrac{-4}{-2} = 2$
point-slope form: $y - 6 = 2(x - 1)$
or $y - 2 = 2(x + 1)$
slope-intercept form: $y = 2x + 4$

37. $3x + y - 9 = 0$
$y = -3x + 9$
$m = -3$
point-slope form:
$y + 7 = -3(x - 4)$
slope-intercept form:
$y = -3x + 12 - 7$
$y = -3x + 5$

38. perpendicular to $y = \dfrac{1}{3}x + 4$
$m = -3$
point-slope form:
$y - 6 = -3(x + 3)$
slope-intercept form:
$y = -3x - 9 + 6$
$y = -3x - 3$

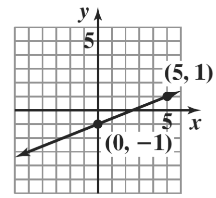
39. Write $6x - y - 4 = 0$ in slope intercept form.

$$6x - y - 4 = 0$$
$$-y = -6x + 4$$
$$y = 6x - 4$$

The slope of the perpendicular line is 6, thus the slope of the desired line is $m = -\dfrac{1}{6}$.

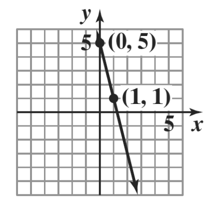
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\tfrac{1}{6}\left(x - (-12)\right)$$
$$y + 1 = -\tfrac{1}{6}(x + 12)$$
$$y + 1 = -\tfrac{1}{6}x - 2$$
$$6y + 6 = -x - 12$$
$$x + 6y + 18 = 0$$

40. slope: $\dfrac{2}{5}$; y-intercept: -1



$y = \dfrac{2}{5}x - 1$

41. slope: -4; y-intercept: 5

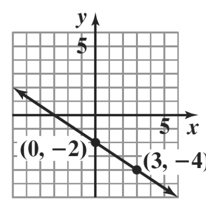


$f(x) = -4x + 5$

42. $2x + 3y + 6 = 0$

$$3y = -2x - 6$$
$$y = -\frac{2}{3}x - 2$$

slope: $-\dfrac{2}{3}$; y-intercept: -2

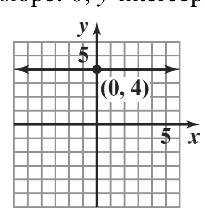


$2x + 3y + 6 = 0$

43. $2y - 8 = 0$

$$2y = 8$$
$$y = 4$$

slope: 0; y-intercept: 4



$2y - 8 = 0$

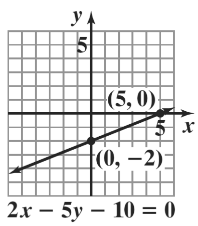
44. $2x-5y-10=0$

Find x-intercept:

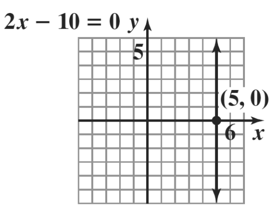
$$\begin{aligned}2x-5(0)-10&=0\\2x-10&=0\\2x&=10\\x&=5\end{aligned}$$

Find y-intercept:

$$\begin{aligned}2(0)-5y-10&=0\\-5y-10&=0\\-5y&=10\\y&=-2\end{aligned}$$



45.
$$\begin{aligned}2x-10&=0\\2x&=10\\x&=5\end{aligned}$$



46. **a.** $m=\dfrac{11-2.3}{90-15}=\dfrac{8.7}{75}=0.116$

$$y-y_1=m(x-x_1)$$
$$y-11=0.116(x-90)$$

or

$$y-2.3=0.116(x-15)$$

b.
$$\begin{aligned}y-11&=0.116(x-90)\\y-11&=0.116x-10.44\\y&=0.116x+0.56\\f(x)&=0.116x+0.56\end{aligned}$$

c. According to the graph, France has about 5 deaths per 100,000 persons.

d.
$$\begin{aligned}f(x)&=0.116x+0.56\\f(32)&=0.116(32)+0.56\\&=4.272\\&\approx 4.3\end{aligned}$$

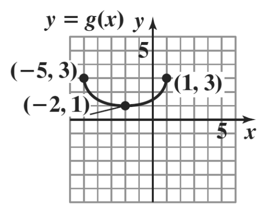
According to the function, France has about 4.3 deaths per 100,000 persons.
This underestimates the value in the graph by 0.7 deaths per 100,000 persons.
The line passes below the point for France.

47. **a.** $m=\dfrac{52-64}{2010-1985}=\dfrac{-12}{25}=-0.48$

b. For each year from 1985 through 2010, the percentage of U.S. college freshmen rating their emotional health high or above average decreased by <u>0.48</u>. The rate of change was <u>–0.48%</u> per <u>year</u>.

48. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{[9^2-4(9)]-[4^2-4\cdot 5]}{9-5}=10$

49.



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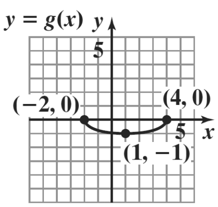

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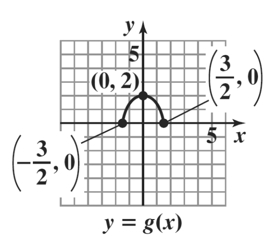

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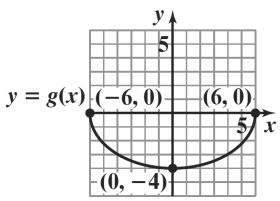

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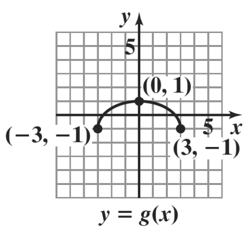

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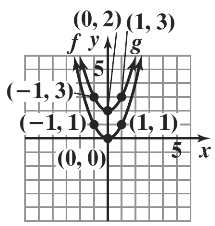

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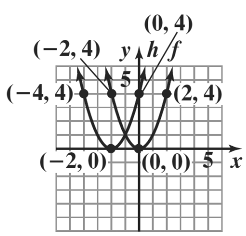

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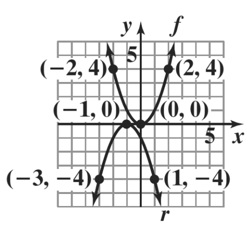

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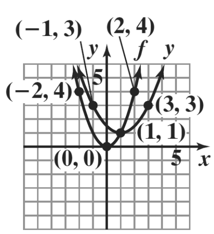

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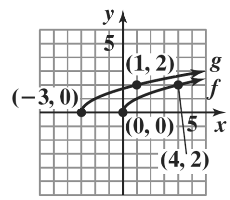

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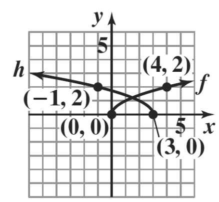

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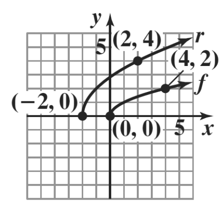

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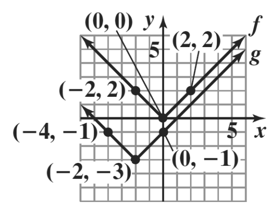

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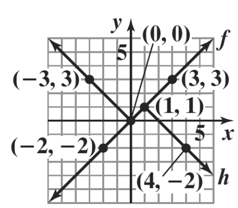

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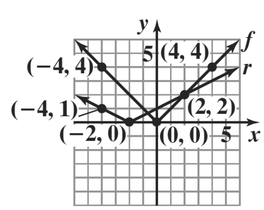

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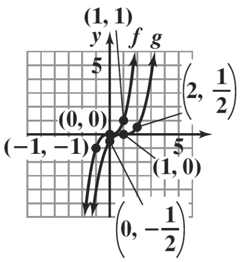

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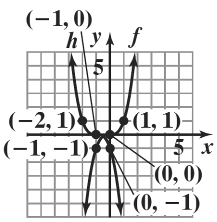

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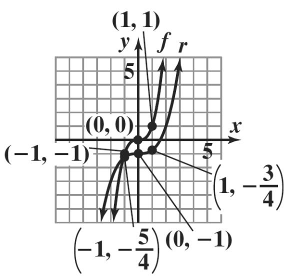

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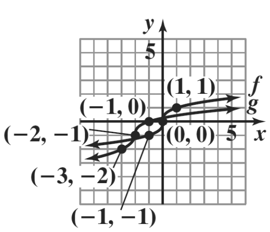

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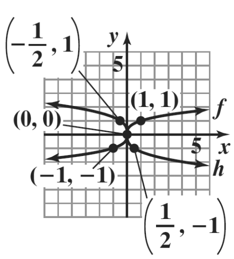

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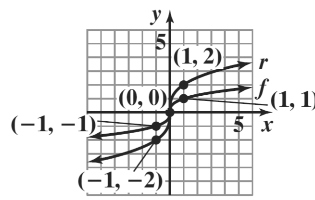


70. domain: $(-\infty, \infty)$

71. The denominator is zero when $x = 7$. The domain is $(-\infty, 7) \cup (7, \infty)$.

72. The expressions under each radical must not be negative.
$8 - 2x \geq 0$
$-2x \geq -8$
$x \leq 4$
domain: $(-\infty, 4]$.

73. The denominator is zero when $x = -7$ or $x = 3$.
domain: $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$

74. The expressions under each radical must not be negative. The denominator is zero when $x = 5$.
$x - 2 \geq 0$
$x \geq 2$
domain: $[2, 5) \cup (5, \infty)$

75. The expressions under each radical must not be negative.
$x - 1 \geq 0$ and $x + 5 \geq 0$
$x \geq 1$ $\quad$ $x \geq -5$
domain: $[1, \infty)$

76. $f(x) = 3x - 1$; $g(x) = x - 5$
$(f + g)(x) = 4x - 6$
domain: $(-\infty, \infty)$
$(f - g)(x) = (3x - 1) - (x - 5) = 2x + 4$
domain: $(-\infty, \infty)$
$(fg)(x) = (3x - 1)(x - 5) = 3x^2 - 16x + 5$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{3x - 1}{x - 5}$
domain: $(-\infty, 5) \cup (5, \infty)$

77. $f(x)=x^2+x+1;\ g(x)=x^2-1$

$(f+g)(x)=2x^2+x$

domain: $(-\infty,\infty)$

$(f-g)(x)=(x^2+x+1)-(x^2-1)=x+2$

domain: $(-\infty,\infty)$

$$\begin{aligned}(fg)(x)&=(x^2+x+1)(x^2-1)\\&=x^4+x^3-x-1\end{aligned}$$

$$\left(\frac{f}{g}\right)(x)=\frac{x^2+x+1}{x^2-1}$$

domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

78. $f(x)=\sqrt{x+7};\ g(x)=\sqrt{x-2}$

$(f+g)(x)=\sqrt{x+7}+\sqrt{x-2}$

domain: $[2,\infty)$

$(f-g)(x)=\sqrt{x+7}-\sqrt{x-2}$

domain: $[2,\infty)$

$$\begin{aligned}(fg)(x)&=\sqrt{x+7}\cdot\sqrt{x-2}\\&=\sqrt{x^2+5x-14}\end{aligned}$$

domain: $[2,\infty)$

$$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+7}}{\sqrt{x-2}}$$

domain: $(2,\infty)$

79. $f(x)=x^2+3;\ g(x)=4x-1$

a. $$\begin{aligned}(f\circ g)(x)&=(4x-1)^2+3\\&=16x^2-8x+4\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(x)&=4(x^2+3)-1\\&=4x^2+11\end{aligned}$$

c. $(f\circ g)(3)=16(3)^2-8(3)+4=124$

80. $f(x)=\sqrt{x};\quad g(x)=x+1$

a. $(f\circ g)(x)=\sqrt{x+1}$

b. $(g\circ f)(x)=\sqrt{x}+1$

c. $(f\circ g)(3)=\sqrt{3+1}=\sqrt{4}=2$

81. a. $$\begin{aligned}(f\circ g)(x)&=f\left(\frac{1}{x}\right)\\&=\frac{\frac{1}{x}+1}{\frac{1}{x}-2}=\frac{\left(\frac{1}{x}+1\right)x}{\left(\frac{1}{x}-2\right)x}=\frac{1+x}{1-2x}\end{aligned}$$

b. $x\neq 0$ $\qquad 1-2x\neq 0$

$x\neq\frac{1}{2}$

$(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

82. a. $(f\circ g)(x)=f(x+3)=\sqrt{x+3-1}=\sqrt{x+2}$

b. $x+2\geq 0$

$x\geq -2$ $\qquad [-2,\infty)$

83. $f(x)=x^4\qquad g(x)=x^2+2x-1$

84. $f(x)=\sqrt[3]{x}\qquad g(x)=7x+4$

85. $f(x)=\frac{3}{5}x+\frac{1}{2};\ g(x)=\frac{5}{3}x-2$

$$\begin{aligned}f(g(x))&=\frac{3}{5}\left(\frac{5}{3}x-2\right)+\frac{1}{2}\\&=x-\frac{6}{5}+\frac{1}{2}\\&=x-\frac{7}{10}\end{aligned}$$

$$\begin{aligned}g(f(x))&=\frac{5}{3}\left(\frac{3}{5}x+\frac{1}{2}\right)-2\\&=x+\frac{5}{6}-2\\&=x-\frac{7}{6}\end{aligned}$$

f and *g* are not inverses of each other.

86. $f(x) = 2 - 5x;\ g(x) = \dfrac{2-x}{5}$

$$\begin{aligned} f(g(x)) &= 2 - 5\left(\frac{2-x}{5}\right) \\ &= 2 - (2 - x) \\ &= x \end{aligned}$$

$$g(f(x)) = \frac{2-(2-5x)}{5} = \frac{5x}{5} = x$$

f and *g* are inverses of each other.

87. **a.**

$$\begin{aligned} f(x) &= 4x - 3 \\ y &= 4x - 3 \\ x &= 4y - 3 \\ y &= \frac{x+3}{4} \\ f^{-1}(x) &= \frac{x+3}{4} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= 4\left(\frac{x+3}{4}\right) - 3 \\ &= x + 3 - 3 \\ &= x \end{aligned}$$

$$f^{-1}(f(x)) = \frac{(4x-3)+3}{4} = \frac{4x}{4} = x$$

88. **a.**

$$\begin{aligned} f(x) &= 8x^3 + 1 \\ y &= 8x^3 + 1 \\ x &= 8y^3 + 1 \\ x - 1 &= 8y^3 \\ \frac{x-1}{8} &= y^3 \\ \sqrt[3]{\frac{x-1}{8}} &= y \\ \frac{\sqrt[3]{x-1}}{2} &= y \\ f^{-1}(x) &= \frac{\sqrt[3]{x-1}}{2} \end{aligned}$$

b.

$$\begin{aligned} f\left(f^{-1}(x)\right) &= 8\left(\frac{\sqrt[3]{x-1}}{2}\right)^3 + 1 \\ &= 8\left(\frac{x-1}{8}\right) + 1 \\ &= x - 1 + 1 \\ &= x \end{aligned}$$

$$\begin{aligned} f^{-1}\left(f(x)\right) &= \frac{\sqrt[3]{\left(8x^3+1\right)-1}}{2} \\ &= \frac{\sqrt[3]{8x^3}}{2} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

89. **a.**

$$\begin{aligned} f(x) &= \frac{2}{x} + 5 \\ y &= \frac{2}{x} + 5 \\ x &= \frac{2}{y} + 5 \\ xy &= 2 + 5y \\ xy - 5y &= 2 \\ y(x-5) &= 2 \\ y &= \frac{2}{x-5} \\ f^{-1}(x) &= \frac{2}{x-5} \end{aligned}$$

b.

$$\begin{aligned} f\left(f^{-1}(x)\right) &= \frac{2}{\frac{2}{x-5}} + 5 \\ &= \frac{2(x-5)}{2} + 5 \\ &= x - 5 + 5 \\ &= x \end{aligned}$$

$$\begin{aligned} f^{-1}\left(f(x)\right) &= \frac{2}{\frac{2}{x} + 5 - 5} \\ &= \frac{2}{\frac{2}{x}} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

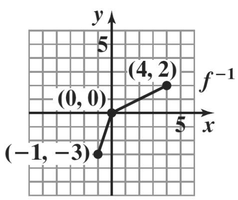
90. The inverse function exists.

91. The inverse function does not exist since it does not pass the horizontal line test.

92. The inverse function exists.

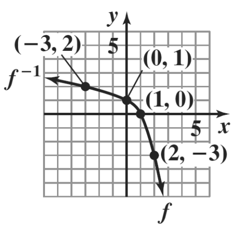
93. The inverse function does not exist since it does not pass the horizontal line test.

94.



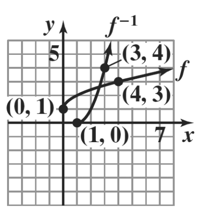
95.

$$\begin{aligned} f(x) &= 1-x^2 \\ y &= 1-x^2 \\ x &= 1-y^2 \\ y^2 &= 1-x \\ y &= \sqrt{1-x} \\ f^{-1}(x) &= \sqrt{1-x} \end{aligned}$$



96.

$$\begin{aligned} f(x) &= \sqrt{x}+1 \\ y &= \sqrt{x}+1 \\ x &= \sqrt{y}+1 \\ x-1 &= \sqrt{y} \\ (x-1)^2 &= y \\ f^{-1}(x) &= (x-1)^2, \quad x \ge 1 \end{aligned}$$



97.

$$\begin{aligned} d &= \sqrt{[3-(-2)]^2+[9-(-3)]^2} \\ &= \sqrt{5^2+12^2} \\ &= \sqrt{25+144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

98.

$$\begin{aligned} d &= \sqrt{[-2-(-4)]^2+(5-3)^2} \\ &= \sqrt{2^2+2^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \\ &\approx 2.83 \end{aligned}$$

99. $\left(\frac{2+(-12)}{2}, \frac{6+4}{2}\right) = \left(\frac{-10}{2}, \frac{10}{2}\right) = (-5, 5)$

100. $\left(\frac{4+(-15)}{2}, \frac{-6+2}{2}\right) = \left(\frac{-11}{2}, \frac{-4}{2}\right) = \left(\frac{-11}{2}, -2\right)$

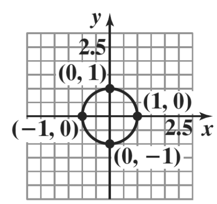
101. $x^2+y^2=3^2$

$x^2+y^2=9$

102. $(x-(-2))^2+(y-4)^2=6^2$

$(x+2)^2+(y-4)^2=36$

103. center: (0, 0); radius: 1

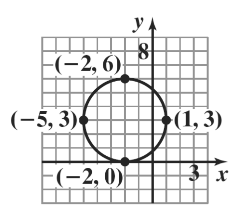


$x^2 + y^2 = 1$

domain: $[-1,1]$

range: $[-1,1]$

104. center: (–2, 3); radius: 3



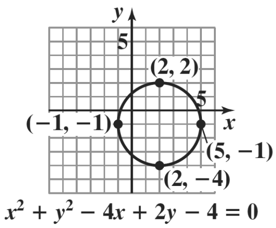
$(x + 2)^2 + (y - 3)^2 = 9$

domain: $[-5,1]$

range: $[0,6]$

105.
$$x^2 + y^2 - 4x + 2y - 4 = 0$$
$$x^2 - 4x \quad + y^2 + 2y \quad = 4$$
$$x^2 - 4x + 4 + y^2 + 2y + 1 = 4 + 4 + 1$$
$$(x-2)^2 + (y+1)^2 = 9$$

center: (2, –1); radius: 3



domain: $[-1, 5]$

range: $[-4, 2]$

Chapter 2 Test

1. (b), (c), and (d) are not functions.

2. **a.** $f(4) - f(-3) = 3 - (-2) = 5$

b. domain: (–5, 6]

c. range: [–4, 5]

d. increasing: (–1, 2)

e. decreasing: $(-5, -1)$ or $(2, 6)$

f. $2, f(2) = 5$

g. (–1, –4)

h. x-intercepts: –4, 1, and 5.

i. y-intercept: –3

3. **a.** –2, 2

b. –1, 1

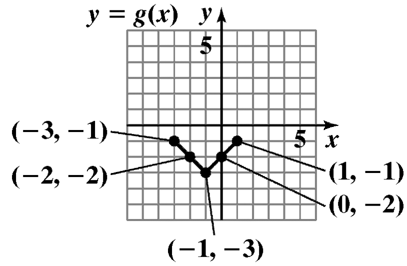
c. 0

d. even; $f(-x) = f(x)$

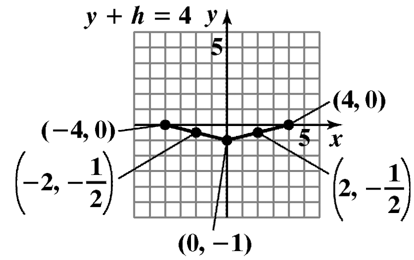
e. no; f fails the horizontal line test

f. $f(0)$ is a relative minimum.

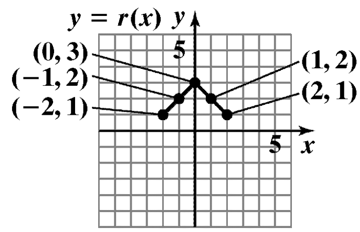
g.



h.

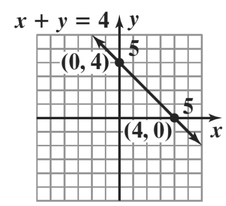


i.



j. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{-1-0}{1-(-2)} = -\dfrac{1}{3}$

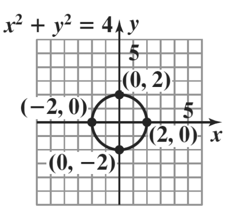
4.



domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

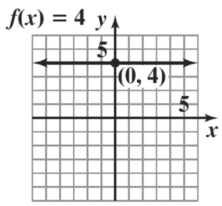
5.



domain: $[-2, 2]$

range: $[-2, 2]$

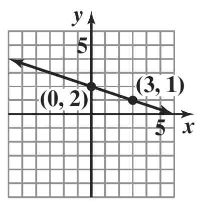
6.



domain: $(-\infty,\infty)$

range: $\{4\}$

7.

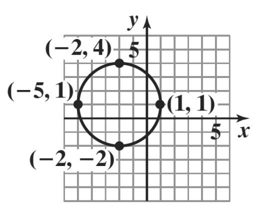


$f(x) = -\frac{1}{3}x + 2$

domain: $(-\infty,\infty)$

range: $(-\infty,\infty)$

8.

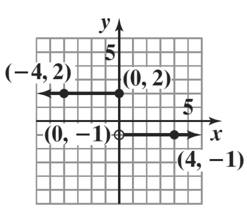


$(x + 2)^2 + (y - 1)^2 = 9$

domain: $[-5,1]$

range: $[-2,4]$

9.

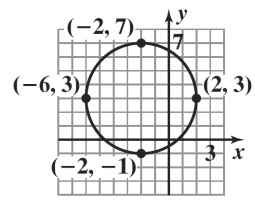


$f(x) = \begin{cases} 2 \text{ if } x \le 0 \\ -1 \text{ if } x > 0 \end{cases}$

domain: $(-\infty,\infty)$

range: $\{-1,2\}$

10.

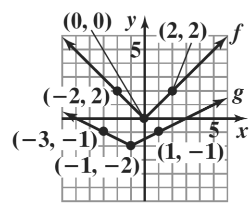


$x^2 + y^2 + 4x - 6y - 3 = 0$

domain: $[-6,2]$

range: $[-1,7]$

11.



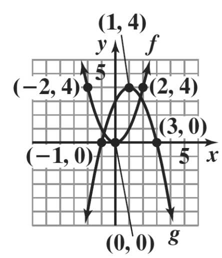
domain of *f*: $(-\infty,\infty)$

range of *f*: $[0,\infty)$

domain of *g*: $(-\infty,\infty)$

range of *g*: $[-2,\infty)$

12.



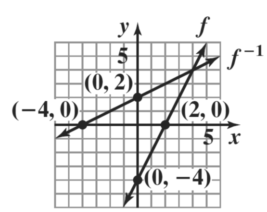
domain of *f*: $(-\infty,\infty)$

range of *f*: $[0,\infty)$

domain of *g*: $(-\infty,\infty)$

range of *g*: $(-\infty,4]$

13.



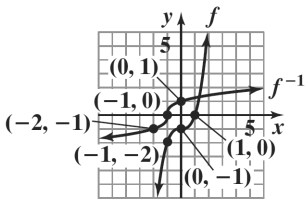
domain of *f*: $(-\infty,\infty)$

range of *f*: $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

14.



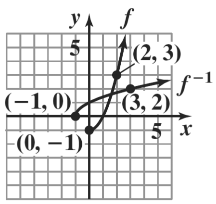
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

15.



domain of *f*: $[0, \infty)$

range of *f*: $[-1, \infty)$

domain of f^{-1}: $[-1, \infty)$

range of f^{-1}: $[0, \infty)$

16.
$$\begin{aligned} f(x) &= x^2 - x - 4 \\ f(x-1) &= (x-1)^2 - (x-1) - 4 \\ &= x^2 - 2x + 1 - x + 1 - 4 \\ &= x^2 - 3x - 2 \end{aligned}$$

17.
$$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 - (x+h) - 4 - \left(x^2 - x - 4\right)}{h} \\ &= \frac{x^2 + 2xh + h^2 - x - h - 4 - x^2 + x + 4}{h} \\ &= \frac{2xh + h^2 - h}{h} \\ &= \frac{h\left(2x + h - 1\right)}{h} \\ &= 2x + h - 1 \end{aligned}$$

18.
$$\begin{aligned} (g - f)(x) &= 2x - 6 - \left(x^2 - x - 4\right) \\ &= 2x - 6 - x^2 + x + 4 \\ &= -x^2 + 3x - 2 \end{aligned}$$

19.
$$\left(\frac{f}{g}\right)(x) = \frac{x^2 - x - 4}{2x - 6}$$

domain: $(-\infty, 3) \cup (3, \infty)$

20.
$$\begin{aligned} (f \circ g)(x) &= f\left(g(x)\right) \\ &= (2x-6)^2 - (2x-6) - 4 \\ &= 4x^2 - 24x + 36 - 2x + 6 - 4 \\ &= 4x^2 - 26x + 38 \end{aligned}$$

21.
$$\begin{aligned} (g \circ f)(x) &= g\left(f(x)\right) \\ &= 2\left(x^2 - x - 4\right) - 6 \\ &= 2x^2 - 2x - 8 - 6 \\ &= 2x^2 - 2x - 14 \end{aligned}$$

22.
$$\begin{aligned} g\left(f(-1)\right) &= 2\left((-1)^2 - (-1) - 4\right) - 6 \\ &= 2\left(1 + 1 - 4\right) - 6 \\ &= 2\left(-2\right) - 6 \\ &= -4 - 6 \\ &= -10 \end{aligned}$$

23.
$$\begin{aligned} f(x) &= x^2 - x - 4 \\ f(-x) &= (-x)^2 - (-x) - 4 \\ &= x^2 + x - 4 \end{aligned}$$

f is neither even nor odd.

24. $m = \dfrac{-8-1}{-1-2} = \dfrac{-9}{-3} = 3$

point-slope form: $y - 1 = 3(x - 2)$
or $y + 8 = 3(x + 1)$
slope-intercept form: $y = 3x - 5$

25. $y = -\dfrac{1}{4}x + 5$ so $m = 4$

point-slope form: $y - 6 = 4(x + 4)$
slope-intercept form: $y = 4x + 22$

26. Write $4x+2y-5=0$ in slope intercept form.

$$4x+2y-5=0$$
$$2y=-4x+5$$
$$y=-2x+\frac{5}{2}$$

The slope of the parallel line is –2, thus the slope of the desired line is $m=-2$.

$$y-y_1=m(x-x_1)$$
$$y-(-10)=-2\left(x-(-7)\right)$$
$$y+10=-2(x+7)$$
$$y+10=-2x-14$$
$$2x+y+24=0$$

27. **a.** Find slope: $m=\dfrac{5870-4571}{4-1}=\dfrac{1299}{3}=433$

point-slope form:

$$y-y_1=m\left(x-x_1\right)$$
$$y-4571=433\left(x-1\right)$$

b. slope-intercept form:

$$y-4571=433\left(x-1\right)$$
$$y-4571=433x-433$$
$$y=433x+4138$$
$$f(x)=433x+4138$$

c.
$$f(x)=433x+4138$$
$$=433(10)+4138$$
$$=8468$$

According to the model, 8468 fatalities will involve distracted driving in 2014.

28.
$$\frac{3(10)^2-5-[3(6)^2-5]}{10-6}$$
$$=\frac{205-103}{4}$$
$$=\frac{192}{4}$$
$$=48$$

29. $g(-1) = 3 - (-1) = 4$

$g(7)=\sqrt{7-3}=\sqrt{4}=2$

30. The denominator is zero when $x = 1$ or $x = -5$.

domain: $\left(-\infty,-5\right)\cup\left(-5,1\right)\cup\left(1,\infty\right)$

31. The expressions under each radical must not be negative.

$x+5\ge 0$ and $x-1\ge 0$

$x\ge -5$ $\quad$ $x\ge 1$

domain: $\left[1,\infty\right)$

32.
$$(f\circ g)(x)=\frac{7}{\frac{2}{x}-4}=\frac{7x}{2-4x}$$

$x\ne 0,\quad 2-4x\ne 0$

$$x\ne\frac{1}{2}$$

domain: $\left(-\infty,0\right)\cup\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

33. $f\left(x\right)=x^7 \qquad g\left(x\right)=2x+3$

34.
$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$d=\sqrt{(x_2-x_1)^2+\left(y_2-y_1\right)^2}$$
$$=\sqrt{(5-2)^2+\left(2-(-2)\right)^2}$$
$$=\sqrt{3^2+4^2}$$
$$=\sqrt{9+16}$$
$$=\sqrt{25}$$
$$=5$$
$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{2+5}{2},\frac{-2+2}{2}\right)$$
$$=\left(\frac{7}{2},0\right)$$

The length is 5 and the midpoint is $\left(\frac{7}{2},0\right)$ or $\left(3.5,0\right)$.

Cumulative Review Exercises (Chapters 1–2)

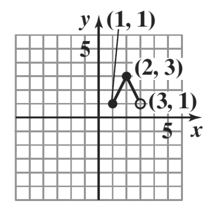
1. domain: $[0,2)$

range: $[0,2]$

2. $f(x)=1$ at $\frac{1}{2}$ and $\frac{3}{2}$.

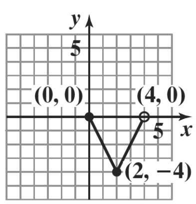
3. relative maximum: 2

4.



$g(x) = f(x-1)+1$

5.



$h(x) = -2f\left(\frac{1}{2}x\right)$

6.
$$(x+3)(x-4)=8$$
$$x^2-x-12=8$$
$$x^2-x-20=0$$
$$(x+4)(x-5)=0$$
$$x+4=0 \quad \text{or} \quad x-5=0$$
$$x=-4 \quad \text{or} \quad x=5$$

7.
$$3(4x-1)=4-6(x-3)$$
$$12x-3=4-6x+18$$
$$18x=25$$
$$x=\frac{25}{18}$$

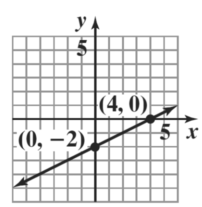
8.
$$\sqrt{x}+2=x$$
$$\sqrt{x}=x-2$$
$$(\sqrt{x})^2=(x-2)^2$$
$$x=x^2-4x+4$$
$$0=x^2-5x+4$$
$$0=(x-1)(x-4)$$
$$x-1=0 \quad \text{or} \quad x-4=0$$
$$x=1 \quad \text{or} \quad x=4$$

A check of the solutions shows that $x = 1$ is an extraneous solution.
The solution set is $\{4\}$.

9.
$$x^{2/3}-x^{1/3}-6=0$$
Let $u=x^{1/3}$. Then $u^2=x^{2/3}$.
$$u^2-u-6=0$$
$$(u+2)(u-3)=0$$
$$u=-2 \quad \text{or} \quad u=3$$
$$x^{1/3}=-2 \quad \text{or} \quad x^{1/3}=3$$
$$x=(-2)^3 \quad \text{or} \quad x=3^3$$
$$x=-8 \quad \text{or} \quad x=27$$

10.
$$\frac{x}{2}-3\le\frac{x}{4}+2$$
$$4\left(\frac{x}{2}-3\right)\le 4\left(\frac{x}{4}+2\right)$$
$$2x-12\le x+8$$
$$x\le 20$$
The solution set is $(-\infty,20]$.

11.

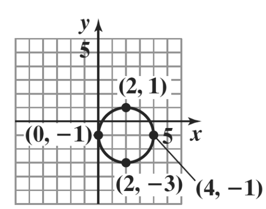


$3x-6y-12=0$

domain: $(-\infty,\infty)$

range: $(-\infty,\infty)$

12.

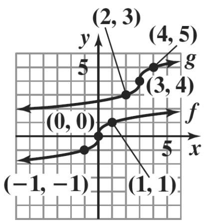


$(x-2)^2+(y+1)^2=4$

domain: $[0,4]$

range: $[-3,1]$

13.



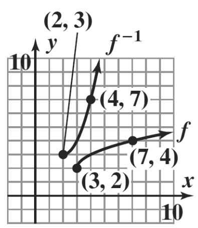
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of *g*: $(-\infty, \infty)$

range of *g*: $(-\infty, \infty)$

14.



domain of *f*: $[3, \infty)$

range of *f*: $[2, \infty)$

domain of f^{-1}: $[2, \infty)$

range of f^{-1}: $[3, \infty)$

15. $\dfrac{f(x+h)-f(x)}{h}$

$$= \frac{\left(4-(x+h)^2\right)-\left(4-x^2\right)}{h}$$

$$= \frac{4-(x^2+2xh+h^2)-\left(4-x^2\right)}{h}$$

$$= \frac{4-x^2-2xh-h^2-4+x^2}{h}$$

$$= \frac{-2xh-h^2}{h}$$

$$= \frac{h(-2x-h)}{h}$$

$$= -2x-h$$

16. $(f \circ g)(x) = f\left(g(x)\right)$

$(f \circ g)(x) = f\left(x+5\right)$

$$0 = 4-\left(x+5\right)^2$$

$$0 = 4-(x^2+10x+25)$$

$$0 = 4-x^2-10x-25$$

$$0 = -x^2-10x-21$$

$$0 = x^2+10x+21$$

$$0 = (x+7)(x+3)$$

The value of $(f \circ g)(x)$ will be 0 when $x = -3$ or $x = -7$.

17. $y = -\dfrac{1}{4}x + \dfrac{1}{3}$, so $m = 4$.

point-slope form: $y - 5 = 4(x + 2)$

slope-intercept form: $y = 4x + 13$

general form: $4x - y + 13 = 0$

18. $0.07x + 0.09(6000 - x) = 510$

$$0.07x + 540 - 0.09x = 510$$

$$-0.02x = -30$$

$$x = 1500$$

$$6000 - x = 4500$$

\$1500 was invested at 7% and \$4500 was invested at 9%.

19. $200 + 0.05x = .15x$

$$200 = 0.10x$$

$$2000 = x$$

For \$2000 in sales, the earnings will be the same.

20. width = w

length = $2w + 2$

$2(2w + 2) + 2w = 22$

$4w + 4 + 2w = 22$

$6w = 18$

$w = 3$

$2w + 2 = 8$

The garden is 3 feet by 8 feet.