Chapter 1

Introduction to Functions and Graphs

Section 1.1 Numbers, Data, and Problem Solving

Teaching Example 1 (page 3)

$\sqrt{64}$: natural number, integer, and rational number
-6: integer and rational number
$\sqrt{8}$: irrational number
9.1: rational number
$\frac{5}{8}$: rational number

Teaching Example 2 (page 4)

(a) $6+4\div 2=8$

(b) $8-4-2=2$

(c) $-6^2=-36$
$(-6)^2=36$

(d) $3+\frac{2-5}{4+5}=3-\frac{1}{3}=\frac{8}{3}$

Teaching Example 3 (page 4)

$636,900=6.369\times 10^5$

Teaching Example 4 (page 5)

(a) 800 nanovolts $=8\times 10^{-7}$ volts

(b) 11.39 watts $=1.139\times 10^1$ watts

Teaching Example 5 (page 5)

(a) $\left(4\times 10^2\right)\left(2\times 10^4\right)=8\times 10^6=8,000,000$

(b) $\left(6\times 10^{-4}\right)\left(2\times 10^{-3}\right)=12\times 10^{-7}=1.2\times 10^{-6}$
$=0.0000012$

(c) $\frac{6\times 10^{-4}}{2\times 10^{-3}}=3\times 10^{-1}=0.3$

Teaching Example 6 (page 5)

(a) $\left(\frac{4\times 10^5}{3\times 10^7}\right)\left(1.3\times 10^6\right)\approx 1.73\times 10^4$

(b) $\sqrt{500\pi}\left(\frac{10+91}{27}\right)^3\approx 2074.6\approx 2.07\times 10^3$

Teaching Example 7 (page 6)

(a) $\sqrt{21}+21^2\approx 445.583$

(b) $\frac{1.2+4.6}{5.4^2-\sqrt{3}}\approx 0.211$

(c) $\sqrt[3]{4}\approx 1.587$

(d) $\left|1.41-1.41^3\right|\approx\left|1.41-2.803\right|\approx\left|-1.393\right|\approx 1.393$

Teaching Example 8 (page 6)

Begin by changing 10,767 days to hours:
$10,767$ days $=10,767\times 24=258,408$ hours
Since the radius of the orbit of Saturn is about 887 million miles, the distance it travels in one orbit is
$C=2\pi r=2\pi\times 887,000,000\approx 5,573,000,000$ mi.
Saturn's orbital speed is approximately
$\frac{5,573,000,000}{258,408}\approx 21,567$ mi/hr.

Teaching Example 9 (page 7)

(a) $V=\pi(1.5)^2(4.5)=10.125\pi\approx 31.8\,\text{in}^3$

(b) $31.8\cdot 0.55=17.49$

Yes, the can could hold 16 fluid ounces.

Section 1.2 Visualizing and Graphing Data

Teaching Example 1 (page 12)

Maximum: 5; minimum: -4
mean $=\frac{5+(-4)+3+2}{4}=\frac{3}{2}=1.5$
To find the median, first arrange the numbers in order from least to greatest: -4, 2, 3, 5. Then find the mean of the middle two numbers, 2 and 3:
median $=\frac{2+3}{2}=\frac{5}{2}=2.5$

Teaching Example 2 (page 13)

(a) $S = \{\{60,1\},\{120,2\},\{150,2.5\}\}$

(b) $D = \{60,120,150\}$
$R = \{1,2,2.5\}$

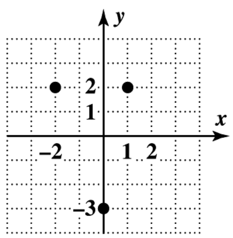
Teaching Example 3 (page 14)

(a) $D = \{-2,0,1\}$
$R = \{-3,2\}$

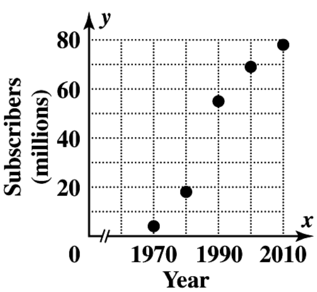
(b) x-minimum: −2; x-maximum: 1
y-minimum: −3; y-maximum: 2

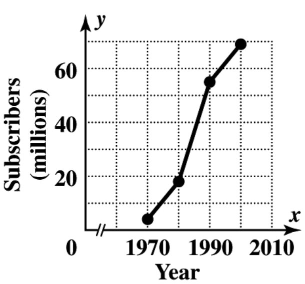
(c) x-minimum: −4; x-maximum: 4
y-minimum: −4; y-maximum: 4

(d)



Teaching Example 4 (page 15)



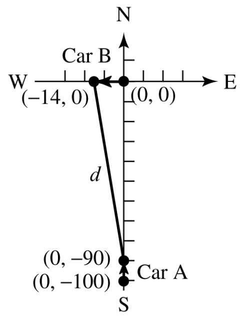


Teaching Example 5 (page 16)

$$\sqrt{(-5-7)^2+(3-(-2))^2} = \sqrt{144+25} = \sqrt{169} = 13$$

Teaching Example 6 (page 16)

If the initial coordinates of car B are (0, 0), then the initial coordinates of car A are (0, −100), because car A is 100 miles south of car B. After 12 minutes, or 0.2 hours, car A has traveled $0.2\times 50 = 10$ miles north, and so it is located 90 miles south of the initial location of car B. Thus its coordinates are (0, −90) at 12:12 AM. Car B traveled $0.2\times 70 = 14$ miles west, so its coordinates are (−14, 0) at 12:12 AM.



So, we must find the distance between (−14, 0) and (0, −90).

$$\sqrt{(-14-0)^2+(0-(-90))^2} = \sqrt{196+8100} = \sqrt{8296}$$
$$\approx 91.1$$

The two cars are about 91.1 miles apart at 12:12 AM.

Teaching Example 7 (page 17)

$$M = \left(\frac{5+3}{2}, \frac{-9+(-1)}{2}\right) = (4,-5)$$

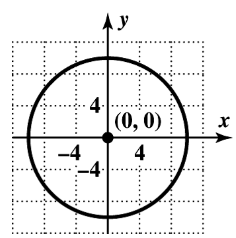
Teaching Example 8 (page 17)

$$M = \left(\frac{1960+1980}{2}, \frac{179+228}{2}\right) = (1970, 203.5)$$

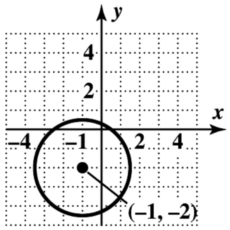
The midpoint formula estimates a population of 203.5 million in 1970, which is very close to the actual population of 203 million.

Teaching Example 9 (page 18)

(a) $x^2 + y^2 = 100 \Rightarrow (h,k) = (0,0)$
$r^2 = 100 \Rightarrow r = 10$
The center is (0, 0) and the radius is 10.



(b) $(x+1)^2 + (y+2)^2 = 6 \Rightarrow (h,k) = (-1,-2)$
$r^2 = 6 \Rightarrow r = \sqrt{6}$
The center is $(-1,-2)$ and the radius is $\sqrt{6}$



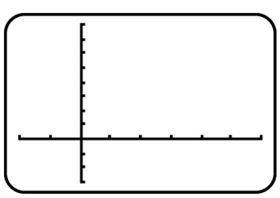
Teaching Example 10 (page 18)

The radius of the circle is $\sqrt{(2-(-2))^2 + (-3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5.$ Since the center is (2, −3), the equation of the circle is $(x-2)^2 + (y+3)^2 = 5^2$ or $(x-2)^2 + (y+3)^2 = 25.$

Teaching Example 11 (page 19)

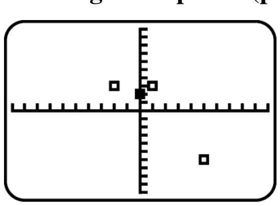
The diameter of the circle is $\sqrt{(-1-3)^2 + (2-(-2))^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2},$ so the radius of the circle is $\frac{4\sqrt{2}}{2} = 2\sqrt{2}.$ The center of the circle is $\left(\frac{-1+3}{2}, \frac{2+(-2)}{2}\right) = (1,0).$ The equation of the circle is $(x-1)^2 + (y-0)^2 = \left(2\sqrt{2}\right)^2$ or $(x-1)^2 + y^2 = 8.$

Teaching Example 12 (page 20)



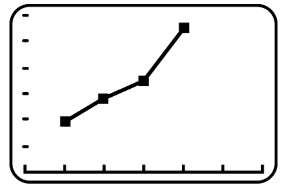
[−4, 12, 2] by [−30, 80, 10]

Teaching Example 13 (page 20)



[−10, 10, 1] by [−10, 10, 1]

Teaching Example 14 (page 21)



[1985, 2015, 5] by [0, 6, 1]

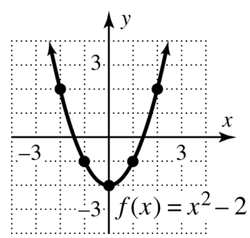
Section 1.3 Functions and Their Representations

Teaching Example 1 (page 29)

Start by creating a table of values:

x	$f(x) = x^2 - 2$
−2	2
−1	−1
0	−2
1	−1
2	2

Now plot the points and connect them with a smooth curve.



Teaching Example 2 (page 30)

$f = \{(1,2),(2,3),(3,4)\}$
$D = \{1,2,3\};\ R = \{2,3,4\}$

Teaching Example 3 (page 31)

(a) $f = \{(Apple, 95),(Google, 150),(Yahoo, 50)\}$

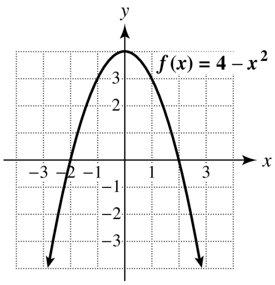
(b) $D = \{Apple, Google, Yahoo\};$
$R = \{50, 95, 150\}$

Teaching Example 4 (page 31)

(a)
$$f(2) = \frac{4}{2-2} = \frac{4}{0} = \text{undefined}$$
$$f(1) = \frac{4}{2-1} = \frac{4}{1} = 4$$
$$f(a+1) = \frac{4}{2-(a+1)} = \frac{4}{1-a}$$

(b) The formula $f(x) = \dfrac{4}{2-x}$ is undefined whenever $2-x$ equals 0. We exclude 2 from the domain. $\{x \mid x \neq 2\}$

Teaching Example 5 (page 32)

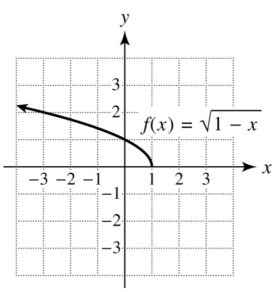


(a) $D = \textit{all real numbers}$
$R = \{y \mid y \leq 4\}$

(b) $f(-1) = 4 - (-1)^2 = 3$

(c) From the graph, we see that (−1,3) lies on the curve.

Teaching Example 6 (page 32)

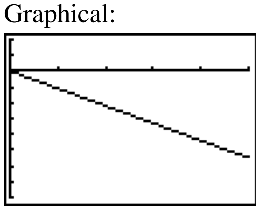


$f(2) = \sqrt{1-2} =$
$\sqrt{-1} = \textit{undefined}$

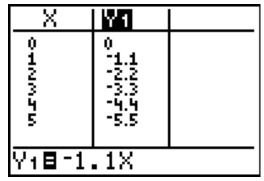
$D = \{x \mid x \leq 1\}$,
$R = \{y \mid y \geq 0\}$

Teaching Example 7 (page 34)

Verbal: Multiply the input x by −1.1 to obtain y, the temperature change.
Symbolic: $f(x) = -1.1x$
Graphical:



[0, 5, 1] by [−8, 2, 1]
Numerical:
On the Table Setup screen, set Indpnt to Ask, since the domain is restricted to [0, 5].



Teaching Example 8 (page 34)

(a) $f = \{(1,2),(2,2),(3,2)\}$
This is a function.

(b) $g = \{(2,1),(2,2),(2,3)\}$
This is not a function because the input value 2 has two output values.

Teaching Example 9 (page 35)

If the graph of f is a nonvertical line, then f is a function.

Teaching Example 10 (page 37)

(a) $4y = x \Rightarrow y = \dfrac{x}{4}$
This is a function.

(b) $x^2 + (y-3)^2 = 4$
This is the graph of a circle, which is not a function.

Section 1.4 Types of Functions and Their Rates of Change

Teaching Example 1 (page 44)

$m = \dfrac{2-(-4)}{-1-3} = \dfrac{6}{-4} = -\dfrac{3}{2}$ The line falls $\dfrac{3}{2}$ units for each unit increase in x.

Teaching Example 2 (page 45)

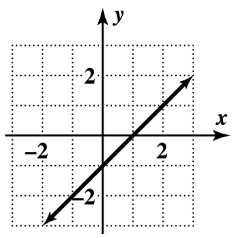
(a) $G(5) = 100 - 5(5) = 100 - 25 = 75$; after 5 minutes the tank contains 75 gallons of water.

(b) $m = -5$, water is leaving the tank at 5 gal/min

Teaching Example 3 (page 45)

$m = \dfrac{270-95}{2005-1985} = \dfrac{175}{20} = 8.75$
This means that advertising expenditures increased, on average, by \$8.75 billion per year from 1985 to 2005.

Teaching Example 4 (page 46)



The slope is $\frac{0-(-1)}{1-0}=1$. The y-intercept is -1, and the x-intercept is 1. $f(x)=x-1$. The zero of the function is 1.

Teaching Example 5 (page 48)

(a) Increasing when $x > 0$,

Decreasing when $x < 0$

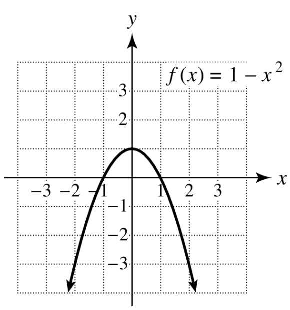
(b) Never increasing,

Decreasing for all real numbers

(c) Increasing when $x > 2$,

Decreasing when $x < 2$

Teaching Example 6 (page 50)



$f(x)$ is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$

Teaching Example 7 (page 51)

$f(2)=4$ and $f(4)=16$, so the average rate of change is $\frac{12-(-3)}{1-4}=\frac{15}{-3}=-5$

Teaching Example 8 (page 52)

$f(1)=12$ and $f(4)=-3$, so the average rate of change is $\frac{12-(-3)}{1-4}=\frac{15}{-3}=-5$

Teaching Example 9 (page 53)

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{-2(x+h)+5-(-2x+5)}{h}\\&=\frac{-2x-2h+5+2x-5}{h}\\&=\frac{-2h}{h}=-2\end{aligned}$$

Teaching Example 10 (page 54)

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{(x+h)^2-x^2}{h}=\frac{x^2+2xh+h^2-x^2}{h}\\&=\frac{2xh+h^2}{h}=2x+h\end{aligned}$$