Chapter 1

INTRODUCTION

Conceptual Questions

1. Knowledge of physics is important for a full understanding of many scientific disciplines, such as chemistry, biology, and geology. Furthermore, much of our current technology can only be understood with knowledge of the underlying laws of physics. In the search for more efficient and environmentally safe sources of energy, for example, physics is essential. Also, many people study physics for the sense of fulfillment that comes with learning about the world we inhabit.

2. Without precise definitions of words for scientific use, unambiguous communication of findings and ideas would be impossible.

3. Even when simplified models do not exactly match real conditions, they can still provide insight into the features of a physical system. Often a problem would become too complicated if one attempted to match the real conditions exactly, and an approximation can yield a result that is close enough to the exact one to still be useful.

4. After solving a problem, it is a good idea to check that the solution is reasonable and makes intuitive sense. Do the units work out correctly? In the symbolic version of the answer, before numbers are substituted, would the expression change in a reasonable way if each parameter were made larger? Smaller? Very much larger or smaller? It may also be useful to explore other possible methods of solution as a check on the validity of the first. A good student thinks of a framework of ideas and skills that she is constructing for herself. The problem solution may extend or strengthen this framework

5. Scientific notation eliminates the need to write many zeros in very large or small numbers, and to count them. Also, the number of significant digits is indicated unambiguously when a quantity is written this way.

6. In scientific notation the decimal point is often placed after the first (leftmost) numeral. The number of digits written equals the number of significant figures.

7. Not all of the significant digits are known definitely. The last (rightmost) digit, called the least significant digit, is an estimate and is less definitely known than the others.

8. It is important to write a quantity with the correct number of significant figures so that we can indicate how precisely a quantity is known and so that we do not mislead the reader by writing digits that are not at all known to be correct.

9. The kilogram, meter, and second are three of the base units used in the SI, the international system of units.

10. The international system SI uses a well-defined set of internationally agreed upon standard units and makes measurements in terms of these units, their combinations, and their powers of ten. The U.S. customary system contains units that are primarily of historical origin and are not based upon powers of ten. As a result of this international acceptance and of the ease of manipulation that comes from dealing with powers of ten, scientists around the world prefer to use SI.

11. Fathoms, kilometers, miles, and inches are units with the dimension length. Grams and kilograms are units with the dimension mass. Years, months, and seconds are units with the dimension time.

12. The first step toward successfully solving almost any physics problem is to thoroughly read the question and obtain a precise understanding of the scenario. The second step is to visualize the problem, often making a quick sketch to outline the details of the situation and the known parameters.

13. Trends in a set of data are often the most interesting aspect of the outcome of an experiment. Such trends are more apparent when data is plotted graphically rather than listed in numerical tables.

14. The statement gives a number for the speed of sound in air, but fails to indicate the units used for the measurement. Without units, the reader cannot relate the speed to one given in familiar units such as km/s.

Multiple-Choice Questions

1. (b) **2.** (b) **3.** (a) **4.** (c) **5.** (d) **6.** (d) **7.** (b) **8.** (d) **9.** (b) **10.** (c)

Problems

1. Strategy The new fence will be $100\% + 37\% = 137\%$ of the height of the old fence.

$$1.37 \times 1.8\text{ m} = \boxed{2.5\text{ m}}$$

Solution Find the height of the new fence.

Discussion. Long ago you were told that 37% *of* 1.8 is 0.37 *times* 1.8.

2. Strategy. Relate the surface area A to the radius r using $A = 4\pi r^2$.

Solution. Find the ratio of the new radius to the old.

$$A_1 = 4\pi r_1^2 \text{ and } A_2 = 4\pi r_2^2 = 2.0A_1 = 2.0(4\pi r_1^2).$$

The radius of the balloon $\boxed{\text{increases by a factor of 1.4}}$.

3. Strategy Relate the surface area A to the radius r using $A = 4\pi r^2$.
Solution Find the ratio of the new radius to the old.

$A_1 = 4\pi r_1^2$ and $A_2 = 4\pi r_2^2 = 1.160A_1 = 1.160(4\pi r_1^2)$.

$$4\pi r_2^2 = 1.160(4\pi r_1^2)$$
$$r_2^2 = 1.160 r_1^2$$
$$\left(\frac{r_2}{r_1}\right)^2 = 1.160$$
$$\frac{r_2}{r_1} = \sqrt{1.160} = 1.077$$

The radius of the balloon increases by $\boxed{7.7\%}$.

Discussion. Because the surface area is proportional to the *square* of the radius, the percentage change in radius is smaller than the percentage change in area—in fact, a bit less than half as large. The factor of 4π divides out and plays no part in determining the answer. The answer just comes from the proportionality of area to the square of the radius. The circumference also increases by 7.7%.

4. Strategy To find the factor by which Samantha's height increased, divide her new height by her old height. Subtract 1 from this value and multiply by 100 % to find the percentage increase.

Solution Find the factor.

$$\frac{1.65\text{ m}}{1.50\text{ m}} = \boxed{1.10}$$

Find the percentage.

$1.10 - 1 = 0.10$, so the percent increase is $\boxed{10\ \%}$.

5. Strategy To find the factor by which the metabolic rate of a 70 kg human exceeds that of a 5.0 kg cat, use a ratio.

$$\left(\frac{m_\text{h}}{m_\text{c}}\right)^{3/4} = \left(\frac{70}{5.0}\right)^{3/4} = \boxed{7.2}$$

Solution Find the factor:

Discussion. The proportionality could be written into an equation as (Metabolic rate) = K (Body mass)$^{3/4}$ where K is a proportionality constant (with very odd units). If we write down this equation for a human and again for a cat, and then divide the two, the K divides out and we obtain the quantity $(m_\text{h}/m_\text{c})^{3/4}$ that we evaluated. Get used to using your calculator to follow the order of operations without your having to re-enter any numbers. On my calculator I type 70 ÷ 5 = ^ 0.75 = .

6. Strategy Let X be the original value of the index. Follow what happens to it.

Solution Find the net percentage change in the index for the two days.

final value = (originalvalue) × (first day change factor) × (second day change factor) =

$$= X \times (1 + 0.0500) \times (1 - 0.0500) = 0.9975X$$

The net percentage change is $(0.9975 - 1) \times 100\% = -0.25\%$, or $\boxed{\text{down } 0.25\%}$.

The index starts higher on day 2 than on day 1, so the decrease on the second day is five percent of a larger number. This decrease therefore exceeds the increase on the previous day.

7. Strategy Recall that area has dimensions of length squared.

Solution Find the ratio of the area of the park as represented on the map to the area of the actual park.

$$\frac{\text{map length}}{\text{actual length}} = \frac{1}{10{,}000} = 10^{-4}, \text{ so } \frac{\text{map area}}{\text{actual area}} = (10^{-4})^2 = \boxed{10^{-8}}.$$

8. Strategy We use the given equation to form an equation of ratios comparing heat transferred and thickness.

Solution Represent the first trial, with 86.0 J going through a pad 3.40 cm thick, with the symbols

$Q_1/\Delta t_1 = \kappa_1 A_1\ \Delta T_1/d_1$. Now the new trial in this problem 8 is represented by $Q_8/\Delta t_8 = \kappa_8 A_8\ \Delta T_8/d_8$. Dividing the two equations gives

$$\frac{Q_8 \Delta t_1}{\Delta t_8 Q_1} = \frac{\kappa_8 A_8 \Delta T_8}{d_8} \frac{d_1}{\kappa_1 A_1 \Delta T_1}$$

We are given $\Delta t_1 = \Delta t_8$ and $\Delta T_1 = \Delta T_8$ and $A_1 = A_8$ and $\kappa_1 = \kappa_8$. Then $\frac{Q_8}{Q_1} = \frac{d_1}{d_8}$ so

$$Q_8 = Q_1 \frac{d_1}{d_8} = 86.0\text{ J}\,\frac{3.40\text{ cm}}{5.20\text{ cm}} = \boxed{56.2\text{ J}}$$

9. Strategy We use the given equation to form an equation of ratios—a proportion—comparing heat transferred, temperature difference, and thickness

Solution Represent the first trial, with a temperature difference of 37.0°C – 2.0°C = 35.0°C driving 86.0 J to go through a pad 3.40 cm thick, with the symbols $Q_1/\Delta t_1 = \kappa_1 A_1\ \Delta T_1/d_1$. Now the new trial in this problem 9 is represented by $Q_9/\Delta t_9 = \kappa_9 A_9\ \Delta T_9/d_9$. Dividing the two equations gives

$$\frac{Q_9 \Delta t_1}{\Delta t_9 Q_1} = \frac{\kappa_9 A_9 \Delta T_9}{d_9} \frac{d_1}{\kappa_1 A_1 \Delta T_1}$$

We are given $\Delta t_1 = \Delta t_9$ (same duration), $A_1 = A_9$ (same face area), and $\kappa_1 = \kappa_9$ (same material). Then the full proportion reduces to

$$\frac{Q_9}{Q_1} = \frac{d_1}{d_9} \frac{\Delta T_9}{\Delta T_1}$$

and

$$d_9 = d_1 \frac{Q_1 \Delta T_9}{Q_9 \Delta T_1} = 3.40\ \text{cm} \frac{86.0\ \text{J} \times 48.0°\text{C}}{47.0\ \text{J} \times 35.0°\text{C}} = \boxed{8.53\ \text{cm}}$$

Discussion. We could alternatively phrase the solution in terms of ratios (fractions or factors of change) and proportionalities (patterns of change). The original equation implies that the heat transferred is directly proportional to the temperature difference and inversely proportional to the thickness of the conductor. The conductor can equally well be called an insulator. Making the temperature difference 48 degrees instead of 35 degrees would by itself increase the heat flow by a factor of 48/35. To make the actual transfer of heat smaller instead of larger, the thickness of insulation would have to first be increased by this factor. And then to make the heat 47 J instead of 86 J, the insulation thickness would need to be further increased by the factor 86/47. Then the answer 3.40 cm (86/47)(48/35) = 8.53 cm has been assembled.

10. Strategy We use the given equation about heat transfer to form an equation of ratios—a proportion—comparing time and thickness.

Solution Represent the first trial, with a pad 3.40 cm thick, with the symbols $Q_1/\Delta t_1 = \kappa_1 A_1\ \Delta T_1/d_1$. Now the new trial in this problem 10 is represented by $Q_{10}/\Delta t_{10} = \kappa_{10} A_{10}\ \Delta T_{10}/d_{10}$. Dividing the two equations gives

$$\frac{Q_{10} \Delta t_1}{\Delta t_{10} Q_1} = \frac{\kappa_{10} A_{10} \Delta T_{10}}{d_{10}} \frac{d_1}{\kappa_1 A_1 \Delta T_1}$$

We are given $\Delta T_1 = \Delta T_{10}$ and $A_1 = A_{10}$ and $\kappa_1 = \kappa_{10}$ and $Q_1 = Q_{10}$. The unknown we can identify not as any single symbol but as the factor or ratio $\Delta t_{10}/\Delta t_1$. For it we have

$\Delta t_{10}/\Delta t_1 = d_{10}/d_1 = (4.10\ \text{cm})/(3.40\ \text{cm}) = \boxed{1.21}$.

11. Strategy The area of a rectangular poster is given by $A = \ell w.$ Let the original and final areas be $A_1 = \ell_1 w_1$ and $A_2 = \ell_2 w_2,$ respectively.

Solution Calculate the percentage reduction of the area.

$A_2 = \ell_2 w_2 = (0.800\ell_1)(0.800 w_1) = 0.640 \ell_1 w_1 = 0.640 A_1$

$$\frac{A_1 - A_2}{A_1} \times 100\% = \frac{A_1 - 0.640 A_1}{A_1} \times 100\% = \boxed{36.0\%}$$

Discussion. Twenty-percent increases in the two independent factors would contribute to a 44% increase in area, from 1.20 × 1.20 = 1.44. Twenty-percent decreases in length and width contribute together to a 36% decrease. Proportional reasoning is so profound that it applies to a triangular, round, star-shaped, or dragon-shaped poster, as long as the final shape is geometrically similar to the original and length and width are interpreted as two perpendicular maximum distances across the poster.

12. **Strategy** The volume of the rectangular room is given by $V = \ell wh.$ Let the original and final volumes be $V_1 = \ell_1 w_1 h_1$ and $V_2 = \ell_2 w_2 h_2,$ respectively.
Solution Find the factor by which the volume of the room increased.

$$\frac{V_2}{V_1} = \frac{\ell_2 w_2 h_2}{\ell_1 w_1 h_1} = \frac{(1.50\ell_1)(2.00w_1)(1.20h_1)}{\ell_1 w_1 h_1} = \boxed{3.60}$$

13. **Strategy** Assuming that the cross section of the artery is a circle, we use the area of a circle, $A = \pi r^2.$

Solution $A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2 = \pi(2.0r_1)^2 = 4.0\pi r_1^2.$ Form a proportion. $\dfrac{A_2}{A_1} = \dfrac{4.0\pi r_1^2}{\pi r_1^2} = 4.0$

The cross-sectional area of the artery increases by a factor of $\boxed{4.0}$.

Discussion. In everyday life, people say "increases proportionally" to mean that an effect changes by the same factor as the cause. Here you are asked to consider proportionality-to-the-square and, in other situations, inverse proportionality, proportionality to the square root, and other possibilities. To keep on feeling a lot (four times?) better, the patient needs to exercise and reduce risk factors.

14. **(a)** **Strategy** The diameter of the xylem vessel is one six-hundredth of the magnified image.

Solution Find the diameter of the vessel. $d_{\text{actual}} = \dfrac{d_{\text{magnified}}}{600} = \dfrac{3.0\text{ cm}}{600} = \boxed{5.0 \times 10^{-3}\text{cm}}$

(b) **Strategy** The area of the cross section is given by $A = \pi r^2 = \pi(d/2)^2 = (1/4)\pi d^2.$
Solution Find by what factor the cross-sectional area has been increased in the micrograph.

$$\frac{A_{\text{magnified}}}{A_{\text{actual}}} = \frac{\frac{1}{4}\pi d_{\text{magnified}}{}^2}{\frac{1}{4}\pi d_{\text{actual}}{}^2} = \left(\frac{3.0\text{ cm}}{5.0 \times 10^{-3}\text{cm}}\right)^2 = 600^2 = \boxed{360{,}000}.$$

15. **Strategy** Use a proportion.

Solution Find Jupiter's orbital period. $T^2 \propto R^3,$ so $\dfrac{T_{\text{J}}^2}{T_{\text{E}}^2} = \dfrac{R_{\text{J}}^3}{R_{\text{E}}^3} = 5.19^3.$ Thus, $T_{\text{J}} = 5.19^{3/2}T_{\text{E}} = \boxed{11.8\text{ yr}}.$

Discussion. People since the ancient Babylonians have watched Jupiter step majestically every year from one constellation into the next of twelve lying along the ecliptic. (You should too.) In Chapter 5 we will show that Kepler's third law is a logical consequence of Newton's law of gravitation and Newton's second law of motion. Science does not necessarily answer "why" questions, but that derivation and this problem give reasons behind the motion of Jupiter in the sky.

16. **Strategy** Recall that each digit to the right of the decimal point is significant.
Solution Comparing the significant figures of each value, we have (a) 5, (b) 4, (c) 2, (d) 2, and (e) 3. From fewest to greatest we have $\boxed{\text{c = d, e, b, a}}$.

17. **(a)** **Strategy** To see what is going on, rewrite the numbers so that the power of 10 is the same for each. Then add and give the answer with the number of significant figures determined by the less precise of the two numbers.
Solution Perform the operation with the appropriate number of significant figures.

$$3.783 \times 10^6\text{ kg} + 1.25 \times 10^8\text{ kg} = 0.03783 \times 10^8\text{ kg} + 1.25 \times 10^8\text{ kg} = \boxed{1.29 \times 10^8\text{ kg}}$$

(b) **Strategy** Find the quotient and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Perform the operation with the appropriate number of significant figures.

$(3.783\times10^{6}\ \text{m})\div(3.0\times10^{-2}\ \text{s})=\boxed{1.3\times10^{8}\ \text{m/s}}$

Discussion. Notice also the units. A number of meters plus a number of meters is still a number of meters. A number of meters divided by a number of seconds is a number of m/s.

18. **(a)** **Strategy** Move the decimal point five places to the left and multiply by 10^{5}.

Solution Write the number in standard scientific notation. 170 000 kg = $\boxed{1.7\times10^{5}\ \text{kg}}$

(b) **Strategy** Move the decimal point 15 places to the right and multiply by 10^{-15}.

Solution Write the number in scientific notation. 0.000 000 000 000 003 8 m = $\boxed{3.8\times10^{-15}\text{m}}$

19. **(a)** **Strategy** Rewrite the numbers so that the power of 10 is the same for each. Then subtract and give the answer with the number of significant figures determined by the less precise of the two numbers.

Solution Perform the calculation using an appropriate number of significant figures.

$3.68\times10^{7}\ \text{g}-4.759\times10^{5}\ \text{g}=3.68\times10^{7}\ \text{g}-0.04759\times10^{7}\ \text{g}=\boxed{3.63\times10^{7}\ \text{g}}$

(b) **Strategy** Find the quotient and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Perform the calculation using an appropriate number of significant figures.

$$\frac{6.497\times10^{4}\ \text{m}^{2}}{5.1037\times10^{2}\ \text{m}}=\boxed{1.273\times10^{2}\ \text{m}}$$

20. **Strategy** Even without working out the results of each calculation, we can see that …

Solution …the answer to (a) has three significant digits, (b) has five significant digits, and (c) and (d) both have two significant digits. Then the ranking is from fewest significant figures $\boxed{\text{d = c, a, b}}$ to most significant figures.

Alternatively, we can proceed step by step, finding the result of each calculation by following the rules:

(a) Rewrite the numbers so that the power of 10 is the same for each. Then add and give the answer with the number of significant figures determined by the less precise of the two numbers.
Write your answer using the appropriate number of significant figures.

$6.85\times10^{-5}\ \text{m}+2.7\times10^{-7}\ \text{m}=6.85\times10^{-5}\ \text{m}+0.027\times10^{-5}\ \text{m}=6.88\times10^{-5}\ \text{m}$, with 3 significant digits

(b) Add and give the answer with the number of significant figures determined by the less precise of the two numbers.

$702.35\ \text{km}+1897.648\ \text{km}=2600.00\ \text{km}$, with 5 significant digits

(c) Multiply and give the answer with the number of significant figures determined by the quantity with the fewest significant figures.

$5.0\ \text{m}\times4.302\ \text{m}=22\ \text{m}^{2}$, with 2 significant digits

(d) Find the quotient and give the answer with the number of significant figures determined by the number with the fewest significant figures.

$(0.040/\pi)\ \text{m}=0.013\ \text{m}$, with 2 significant digits

Thus the ranking in order of better precision is again d = c, a, b.

21. Strategy Multiply and give the answer in scientific notation with the number of significant figures determined by the quantity with the fewest significant figures.

Solution Solve the problem.

$$(3.209\text{ m})\times(4.0\times10^{-3}\text{ m})\times(1.25\times10^{-8}\text{ m})=\boxed{1.6\times10^{-10}\text{ m}^3}$$

Discussion. Many calculations combine quantities having very different numbers of significant digits. In laboratory you see how some things are much easier to measure precisely than others. Suppose you are using your calculator to find (3.2×10^2 m)(10^{-3} m)(5.2×10^{-8} m). What keys would you hit? On my calculator it is 3 • 2 enterexponent 2 × 1 enterexponent changesign 3 × 5 • 2 enterexponent changesign 8 = . Make sure you can make sense of where the 1 comes from.

22. Strategy Follow the rules for identifying significant figures.

Solution We can proceed step by step, identifying the number of significant digits in each quantity:

(a) All three digits are significant, so 7.68 g has 3 significant figures.

(b) The first zero is not significant, since it is used only to place the decimal point. The digits 4 and 2 are significant, as is the final zero, so 0.420 kg has 3 significant figures.

(c) The first two zeros are not significant, since they are used only to place the decimal point. The digits 7 and 3 are significant, so 0.073 m has 2 significant figures.

(d) All three digits are significant, so $7.68\times10^{5}\text{ g}$ has 3 significant figures.

(e) The zero is significant, since it comes after the decimal point. The digits 4 and 2 are significant as well, so $4.20\times10^{3}\text{ kg}$ has 3 significant figures.

(f) Both 7 and 3 are significant, so $7.3\times10^{-2}\text{ m}$ has 2 significant figures.

(g) Both 2 and 3 are significant. The two zeros are significant as well, since they come after the decimal point, so $2.300\times10^{4}\text{ s}$ has 4 significant figures.

Alternatively, experience with measuring things can let you look just at the digits as they would be read from a meter, ignoring the units, the power of ten, and the position of the decimal point, to see that the ranking is from least precise $\boxed{\text{c = f, b = e = a = d, g}}$ to most significant digits.

23. Strategy Use the rules for determining significant figures and for writing numbers in scientific notation.

Solution (a) 0.00574 kg has three significant figures, 5, 7, and 4. The zeros are not significant, since they are used only to place the decimal point. To write this measurement in scientific notation, we move the decimal point three places to the right and multiply by 10^{-3}. We have 5.74×10^{-3} kg.

(b) 2 m has one significant figure, 2. This measurement is already written in scientific notation, or we could show it as 2×10^0 m.

(c) $0.450\times10^{-2}\text{ m}$ has three significant figures, 4, 5, and the 0 to the right of 5. The zero is significant, since it comes after the decimal point and is not used to place the decimal point. To write this measurement in scientific notation, we move the decimal point one place to the right and multiply by 10^{-1}.

(d) 45.0 kg has three significant figures, 4, 5, and 0. The zero is significant, since it comes after the decimal point and is not used to place the decimal point. To write this measurement in scientific notation, we move the decimal point one place to the left and multiply by 10^{1}.

(e) $10.09\times10^{4}\text{ s}$ has four significant figures, 1, 9, and the two zeros. The zeros are significant, since they are between two significant figures. To write this measurement in scientific notation, we move the decimal point one place to the left and multiply by 10^{1}.

(f) $0.09500\times10^{5}\text{ mL}$ has four significant figures, 9, 5, and the two zeros to the right of 5. The zeros are significant, since they come after the decimal point and are not used to place the decimal point. To write this measurement in scientific notation, we move the decimal point two places to the right and multiply by 10^{-2}.

The results of parts (a) through (f) are shown in the table below.

	Measurement	Significant Figures	Scientific Notation
(a)	0.00574 kg	3	5.74×10^{-3} kg
(b)	2 m	1	2 m
(c)	0.450×10^{-2} m	3	4.50×10^{-3} m
(d)	45.0 kg	3	4.50×10^{1} kg
(e)	10.09×10^{4} s	4	1.009×10^{5} s
(f)	0.09500×10^{5} mL	4	9.500×10^{3} mL

Discussion. It would be more natural to convert to scientific notation first and then count the significant digits.

24. Strategy Convert each length to meters. Then, rewrite the numbers so that the power of 10 is the same for each. Finally, add and give the answer with the number of significant figures determined by the less precise of the two numbers.

Solution Solve the problem.

$$3.08 \times 10^{-1}\text{ km} + 2.00 \times 10^{3}\text{ cm} = 3.08 \times 10^{2}\text{ m} + 2.00 \times 10^{1}\text{ m} = 3.08 \times 10^{2}\text{ m} + 0.200 \times 10^{2}\text{ m} = \boxed{3.28 \times 10^{2}\text{ m}}$$

25. Strategy Divide and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Solve the problem.

$$\frac{3.21\text{ m}}{7.00\text{ ms}} = \frac{3.21\text{ m}}{7.00 \times 10^{-3}\text{ s}} = \boxed{459\text{ m/s}}$$

Discussion. The SI standard is for a unit to contain never more than one prefix, with that prefix in the numerator if the unit is a fraction. With $\text{m} = \text{milli} = 10^{-3}$, we could say $1/\text{milli} = \text{kilo} = \text{k}$.

26. Strategy Recall that 1 kg = 1000 g and 100 cm = 1 m.

Solution Convert the density of body fat from g/cm^3 to kg/m^3.

$$0.9\text{ g/cm}^3 \times 1\text{ kg}/1000\text{ g} \times (100\text{ cm/m})^3 = \boxed{900\text{ kg/m}^3}$$

27. Strategy There are exactly 2.54 centimeters in one inch.

Solution Find the thickness of the cell membrane in inches.

$$7.0 \times 10^{-9}\text{ m} \times \frac{1\text{ inch}}{0.0254\text{ m}} = \boxed{2.8 \times 10^{-7}\text{ inches}}$$

Discussion. The thickness of plastic film or the diameter of a wire can be stated in mils, where one mil is a thousandth of an inch. But no one speaks of a microinch or a nanoinch.

28. Strategy Convert each length to scientific notation.

Solution In scientific notation, the lengths are:

(a) $1\ \mu\text{m} = 1 \times 10^{-6}$ m, (b) $1000\text{ nm} = 1 \times 10^{3} \times 10^{-9}\text{ m} = 1 \times 10^{-6}$ m,

(c) $100\,000\text{ pm} = 1 \times 10^{5} \times 10^{-12}\text{ m} = 1 \times 10^{-7}$ m, (d) $0.01\text{ cm} = 1 \times 10^{-2} \times 10^{-2}\text{ m} = 1 \times 10^{-4}$ m, and

(e) $0.000\,000\,000\,1\text{ km} = 1 \times 10^{-10} \times 10^{3}\text{ m} = 1 \times 10^{-7}$ m.

From smallest to greatest, we have $\boxed{\text{c} = \text{e} < \text{a} = \text{b} < \text{d}}$.

29. Strategy Convert each length to meters and each time to seconds. From Appendix B, 1 mi = 1609 m.

Solution In scientific notation, we have:

(a) $55\text{ mi/h}\times 1609\text{ m/mi}\times 1\text{ h/3600 s} = 25\text{ m/s}$, (b) $82\text{ km/h}\times 1\text{ h/3600 s}\times 1000\text{ m/km} = 23\text{ m/s}$,

(c) 33 m/s, (d) $3.0\text{ cm/ms}\times 1\text{ m/100 cm}\times 1000\text{ ms/s} = 30\text{ m/s}$, and

(e) $1.0\text{ mi/min}\times 1\text{ min/60 s}\times 1609\text{ m/mi} = 27\text{ m/s}$.

From smallest to greatest, we have $\boxed{b < a < e < d < c}$.

Discussion. In ranking-task exercises, it is good to see what you can say about particular pairs, before doing brute-force calculations to put everything onto the same basis. Among the quantities listed here, see if you can directly make sense of how one mile per minute is greater than 55 mi/h. Three centimeters per millisecond is greater than either because a millisecond is so short a time.

30. (a) Strategy There are approximately 3.785 liters per gallon from ~~the inside front cover~~Appendix B, and 128 fluid ounces per gallon.

Solution Find the number of fluid ounces in the bottle.

$$\frac{128\text{ fl oz}}{1\text{ gal}}\times\frac{1\text{ gal}}{3.785\text{ L}}\times 355\text{ mL}\times\frac{1\text{ L}}{10^3\text{ mL}} = \boxed{12.0\text{ fluid ounces}}$$

(b) Strategy From part (a), we have 355 mL = 12.0 fluid ounces.

Solution Find the number of milliliters in the drink.

$$16.0\text{ fl oz}\times\frac{355\text{ mL}}{12.0\text{ fl oz}} = \boxed{473\text{ mL}}$$

31. Strategy There are exactly 2.54 centimeters in an inch and 12 inches in one foot.

Solution Convert to meters. The length of the section of the bridge above the river is

$$1595.5\text{ ft}\times\frac{12\text{ in}}{1\text{ ft}}\times\frac{0.0254\text{ m}}{1\text{ in}} = \boxed{4.8631\times 10^2\text{ m}}$$

For the whole bridge,

$$6016\text{ ft}\times\frac{12\text{ in}}{1\text{ ft}}\times\frac{0.0254\text{ m}}{1\text{ in}} = \boxed{1.834\times 10^3\text{ m}}$$

32. Strategy For (a), convert milliliters to liters; then convert liters to cubic centimeters using the conversion $1\text{ L} = 10^3\text{ cm}^3$. For (b), convert cubic centimeters to cubic meters using the fact that 100 cm = 1 m.

Solution Convert each volume.

(a)
$$255\text{ mL}\times\frac{10^{-3}\text{ L}}{1\text{ mL}}\times\frac{10^3\text{ cm}}{1\text{ L}} = \boxed{255\text{ cm}^3}$$

(b)
$$255\text{ cm}^3\times\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3 = 255\text{ cm}^3\times\frac{1\text{ m}^3}{10^6\text{ cm}^3} = \boxed{2.55\times 10^{-4}\text{ m}}$$

33. Strategy For (a), we can convert meters per second to miles per hour using the conversion 1 mi/h = 0.4470 m/s. For (b), convert meters per second to centimeters per millisecond using the conversions 1 m = 100 cm and 1 s =

1000 ms.

Solution Convert each speed. **(a)**

$$80\ \text{m/s} \times \frac{1\ \text{mi/h}}{0.4470\ \text{m/s}} = \boxed{180\ \text{mi/h}}$$

(b)

$$80\ \text{m/s} \times \frac{10^2\ \text{cm}}{1\ \text{m}} \times \frac{1\ \text{s}}{10^3\ \text{ms}} = \boxed{8.0\ \text{cm/ms}}$$

Discussion. John Kennedy was President for a thousand days. Quoting the speed of the nerve impulse either in miles per hour or in meters per second should make it sound fast to you. Quoting it in centimeters per millisecond suggests that one bit of the cell responds quickly to a stimulus from an adjacent bit.

34. Strategy With the datum quoted to five significant digits, we use the exact conversions between miles and feet, feet and inches, inches and centimeters, and centimeters and kilometers.

Solution Find the length of the marathon race in miles.

$$42.195\ \text{km} \times \frac{100{,}000\ \text{cm}}{1\ \text{km}} \times \frac{1\ \text{in}}{2.54\ \text{cm}} \times \frac{1\ \text{ft}}{12\ \text{in}} \times \frac{1\ \text{mi}}{5280\ \text{ft}} = \boxed{26.219\ \text{mi}}$$

35. Strategy Calculate the change in the exchange rate and divide it by the original price to find the drop.

Solution Find the actual drop in the value of the dollar over the first year.

$$\frac{1.27 - 1.45}{1.45} = \frac{-0.18}{1.45} = -0.12$$

The actual drop is $\boxed{0.12 \text{ or } 12\%}$.

36. Strategy There are 1000 watts in one kilowatt and 100 centimeters in one meter.

Solution Convert $1.4\ \text{kW/m}^2$ to W/cm^2.

$$\frac{1.4\ \text{kW}}{1\ \text{m}^2} \times \frac{1000\ \text{W}}{1\ \text{kW}} \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = \boxed{0.14\ \text{W/cm}^2}$$

37. Strategy Convert the radius to centimeters; then use the conversions 1 L = 10^3 cm^3 and 60 s = 1 min.
Solution Find the volume rate of blood flow

$$\textit{volume rate of blood flow} = \pi r^2 v = \pi (1.2\ \text{cm})^2 (18\ \text{cm/s}) \times \frac{1\ \text{L}}{10^3\ \text{cm}^3} \times \frac{60\ \text{s}}{1\ \text{min}} = \boxed{4.9\ \text{L/min}}$$

Discussion. The given equation for the volume flow rate makes sense because the volume passing in one unit of time can be visualized as the volume of a cylinder with base area equal to the cross-sectional area of the vessel and length equal to the distance that one atom travels in a unit of time, representing its linear speed.

38. Strategy The distance traveled d is equal to the rate of travel v times the time of travel t. There are 1000 milliseconds in one second.

Solution Find the distance the molecule would move.

$$d = vt = \frac{459\ \text{m}}{1\ \text{s}} \times 7.00\ \text{ms} \times \frac{1\ \text{s}}{1000\ \text{ms}} = \boxed{3.21\ \text{m}}$$

39. Strategy There are 1000 meters in a kilometer and 1,000,000 millimeters in a kilometer.
Solution The volume can be visualized as that of a very thin, wide ribbon of plastic film that rolls out from a press in a factory, perhaps in a couple of days. We multiply its three perpendicular dimensions to find its volume, just as if it were a brick. We find the product and express the answer in the huge unit of km^3 with the appropriate number of significant figures.

$$(3.2\ \text{km}) \times (4.0\ \text{m}) \times (13.24 \times 10^{-3}\ \text{mm}) \times \frac{1\ \text{km}}{1000\ \text{m}} \times \frac{1\ \text{km}}{1{,}000{,}000\ \text{mm}} = \boxed{1.7 \times 10^{-10}\ \text{km}^3}$$

40. **(a)** **Strategy** There are 12 inches in one foot and 2.54 centimeters in one inch.
Solution Find the number of square centimeters in one square foot.

$$1\text{ ft}^2\times\left(\frac{12\text{ in}}{1\text{ ft}}\right)^2\times\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)^2=\boxed{929\text{ cm}^2}$$

(b) **Strategy** There are 100 centimeters in one meter.

Solution Find the number of square centimeters in one square meter.

$$1\text{ m}^2\times\left(\frac{100\text{ cm}}{1\text{ m}}\right)^2=\boxed{1\times10^4\text{cm}^2}$$

41. **(a)** **Strategy** There are 12 inches in one foot, 2.54 centimeters in one inch, and 60 seconds in one minute.
Solution Express the snail's speed in feet per second.

$$\frac{5.0\text{ cm}}{1\text{ min}}\times\frac{1\text{min}}{60\text{ s}}\times\frac{1\text{ in}}{2.54\text{ cm}}\times\frac{1\text{ ft}}{12\text{ in}}=\boxed{2.7\times10^{-3}\text{ ft/s}}$$

(b) **Strategy** There are 5280 feet in one mile, 12 inches in one foot, 2.54 centimeters in one inch, and 60 minutes in one hour.
Solution Express the snail's speed in miles per hour.

$$\frac{5.0\text{ cm}}{1\text{ min}}\times\frac{60\text{ min}}{1\text{ h}}\times\frac{1\text{ in}}{2.54\text{ cm}}\times\frac{1\text{ ft}}{12\text{ in}}\times\frac{1\text{ mi}}{5280\text{ ft}}=\boxed{1.9\times10^{-3}\text{ mi/h}}$$

Discussion. It is natural to use a small unit for a low speed. Continents can drift at a few cm/yr. Here after the conversions the speed is expressed as small numbers with more familiar, ordinary-size units.

42. **Strategy** A micrometer is 10^{-6} m and a millimeter is 10^{-3} m; therefore, a micrometer is $10^{-6}/10^{-3}=10^{-3}$ mm.

Solution Find the area in square millimeters.

$$150\ \mu\text{m}^2\times\left(\frac{10^{-3}\text{ mm}}{1\ \mu\text{m}}\right)^2=\boxed{1.5\times10^{-4}\text{ mm}^2}$$

43. **Strategy** Replace each quantity in $U=mgy$ with its SI base units.

Solution Find the combination of SI base units that are equivalent to joules.

$U=mgy$ implies $\text{J}=\text{kg}\times\text{m/s}^2\times\text{m}=\boxed{\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}}$

44. **(a)** **Strategy** Replace each quantity in ma and kx with its dimensions.

Solution Show that the dimensions of ma and kx are equivalent.

ma has dimensions $[\text{M}]\times\dfrac{[\text{L}]}{[\text{T}]^2}$ and kx has dimensions $\dfrac{[\text{M}]}{[\text{T}]^2}\times[\text{L}]=[\text{M}]\times\dfrac{[\text{L}]}{[\text{T}]^2}$.

Since $\boxed{[\text{M}][\text{L}][\text{T}]^{-2}=[\text{M}][\text{L}][\text{T}]^{-2}}$, the dimensions are equivalent.

(b) **Strategy** Use the results of part (a).

Solution Since $F_{net} = ma$ and $F = -kx,$ the dimensions of the force unit newton are $[\text{M}][\text{L}][\text{T}]^{-2}$ and its SI unit is $\boxed{\text{kg·m·s}^{-2}}$

45. Strategy Determine the SI unit of momentum using a process of unit analysis.

Solution Find the SI unit of momentum.

$K = \frac{p^2}{2m}$ has units of $\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}.$ Since the SI unit for m is kg, the SI unit for $p^2 = 2mK$ is $\frac{\text{kg}^2\cdot\text{m}^2}{\text{s}^2}.$ Taking the square root, we find that the SI unit for momentum is $\boxed{\text{kg}\cdot\text{m}\cdot\text{s}^{-1}}.$

Discussion. Kinetic energy and momentum can both be thought of as "measures of motion." It may seem remarkable to you that theories referring to quite different quantities can both quite correctly describe the motion of an object. Spoiler alert: The net effect of an external force acting on an object can be considered as the effect the force has over time, or the effect the force has over the distance through which it acts on the object. The unit of momentum is a newton·second, and the unit of energy is a newton·meter.

46. Strategy Replace each quantity in $T^2 = 4\pi^2 r^3 / (GM)$ with its dimensions.
Solution Show that the equation is dimensionally correct.

T^2 has dimensions $[\text{T}]^2$ and $\frac{4\pi^2 r^3}{GM}$ has dimensions $\frac{[\text{L}]^3}{\frac{[\text{L}]^3}{[\text{M}][\text{T}]^2}\times[\text{M}]} = \frac{[\text{L}]^3}{[\text{M}]}\times\frac{[\text{M}][\text{T}]^2}{[\text{L}]^3} = [\text{T}]^2.$

Since $\boxed{[\text{T}]^2 = [\text{T}]^2},$ the equation is dimensionally correct.

Discussion. It might seem more natural to you to write the test in terms of units, like this:

s^2 is supposed to be equal to $\frac{\text{m}^3}{\left(\text{m}^3/\text{kg}\cdot\text{s}^2\right)\text{kg}} = \frac{\text{m}^3}{\text{kg}}\frac{\text{kg}\cdot\text{s}^2}{\text{m}^3} = \text{s}^2$ and it is.

The test in terms of dimensions is stated in more general terms. Someone might insist on measuring the period of a planet in years or the comparative masses of astronomical objects as multiples of some large mass that is actually unknown, but those choices still leave the equation dimensionally correct.

47. (a) Strategy Replace each quantity (except for V) in $F_\text{B} = \rho g V$ with its dimensions.
Solution Find the dimensions of V.

$V = \frac{F_\text{B}}{\rho g}$ has dimensions $\frac{[\text{MLT}^{-2}]}{[\text{ML}^{-3}]\times[\text{LT}^{-2}]} = \boxed{[\text{L}^3]}.$

(b) Strategy and Solution Since velocity has dimensions $[\mathrm{LT}^{-1}]$ and volume has dimensions $[\mathrm{L}^3]$, the correct interpretation of V is that is represents $\boxed{\text{volume}}$.

48. (a) Strategy a has dimensions $\frac{[\mathrm{L}]}{[\mathrm{T}]^2}$; v has dimensions $\frac{[\mathrm{L}]}{[\mathrm{T}]}$; r has dimension [L]. We assume that a depends just on v and r.

Solution Let $a \propto v^p r^q$ where p and q are unknown exponents. The units in the equation, $\mathrm{m{\cdot}s^{-2}} = (\mathrm{m/s})^p \mathrm{m}^q$ must agree across the equality sign, so we have $\mathrm{m}^1 = m^{p+q}$ and $\mathrm{s}^{-2} = \mathrm{s}^{-p}$. Then $p = 2$ and $1 = 2 + q$ so $q = -1$. The equation must be $a = K\frac{v^2}{r}$, $\boxed{\text{where } K \text{ is a dimensionless constant}}$.

(b) Strategy Divide the new acceleration by the old, and use the fact that the new speed is 1.100 times the old.
Solution Find the percent increase in the radial acceleration.

$$\frac{a_2}{a_1} = \frac{K\frac{v_2^2}{r}}{K\frac{v_1^2}{r}} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{1.100v_1}{v_1}\right)^2 = 1.100^2 = 1.210$$

$1.210 - 1 = 0.210$, so the radial acceleration increases by $\boxed{21.0\%}$.

49. Strategy Approximate the distance from your eyes to a book held at your normal reading distance.

Solution The normal reading distance is about 30 to 40 cm, so the approximate distance from your eyes to a book you are reading is $\boxed{\text{30 to 40 cm}}$.

Discussion. If the problem explicitly asked us for an *order of magnitude* estimate, we would be undecided on a particular answer, and would state it as *between* 10^{-1} *m and* 10^0 *m.*

50. Strategy Estimate the diameter of a soccer ball. Then use $V = \frac{4}{3}\pi r^3$ to estimate its volume.

Solution Find the approximate volume of a soccer ball in cm^3.
The diameter of a soccer ball for teenagers and adults is about 22 cm. Then its radius is 11 cm and its volume is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(11\ \mathrm{cm})^3 = \boxed{5600\ \mathrm{cm}^3}$$

or on the order of 10^4 cm^3.

51. Strategy and Solution Model a lower leg, including the foot, as a cylinder 60 cm long and 10 cm in diameter. Then its volume is $\pi(0.6\ \mathrm{m})(0.05\ \mathrm{m})^2 = 0.005\ \mathrm{m}^3$. Its average density is similar to that of water, about 1 g/cm^3 or 1000 kg/m^3, so its mass is about $0.005(1000)\ \mathrm{kg} = 5\ \mathrm{kg}$. Similarly, the mass of an upper leg is about 7 kg, so an order of magnitude estimate of the mass of a person's leg is $\boxed{10\ \mathrm{kg}} = 10^1$ kg.

52. Strategy and Solution A normal heart rate is about 70 beats per minute and a person may live for about 70 years, so the heart beats about $\frac{70\ \text{beats}}{1\ \text{min}} \times \frac{70\ \text{yr}}{\text{lifetime}} \times \frac{5.26 \times 10^5\ \text{min}}{1\ \text{yr}} = 2.6 \times 10^9$ times per lifetime, or about $\boxed{3 \times 10^9\ \text{heartbeats}}$, on the order of a billion times.

53. Strategy One story is about 3 m high.

Solution Find the order of magnitude of the height in meters of a 40-story building.

$(3\text{ m})(40) \sim \boxed{100\text{ m}} = 10^2\text{ m}$

Discussion You have learned to "round off" to 6 all of the numbers that are closer to 6 than they are to 5 or to 7. This means, all of the numbers from 5.5 to 6.5. In that operation we think of changing numbers by addition or subtraction. On this linear scale: …, 4, 5, 6, 7, …, the numbers are equally spaced by addition. But these numbers: …, 10^1, 10^2, 10^3, 10^4, …, are equally spaced by multiplication. They form a logarithmic scale rather than a linear scale. By this standard, the numbers that are on the order of 10^2, rather than on the order of 10^1 or of 10^3, are those between $\sqrt{10}\times 10^1 \approx 31.6$ and $\sqrt{10}\times 10^2 \approx 316$. All the numbers between 31.6 and 316 are closer to 10^2 on a logarithmic scale than they are to any other integer power of ten. This is a wide range. Fewer than twenty of the tallest structures in the world qualify as being on the order of 10^3 m tall, and none of course are on the order of 10^4 m tall.

54. Strategy We choose to solve three subproblems: We estimate the volume of a human, estimate the volume of a cell, and use the ratio of the two volumes to find the number of cells.

Solution Model an adult human body as a cylinder about 2 m high and about 1 m in maximum circumference. The corresponding maximum radius is $1\text{ m}/2\pi \approx (1/6)$ m. For an average radius we take about 0.1 m. Then the body volume is the height times the cross-sectional area: $V = \pi r^2 h \approx \pi(0.1\text{ m})^2 2\text{ m} = 0.06\text{ m}^3$.

Some cells are roughly spherical, but we choose to model a cell as a cube, so that the volume is easy to find and we do not have to worry about empty spaces between cells. Its volume is on the order of $d^3 \sim (10\times 10^{-6}\text{m})^3 = 10^{-15}\text{m}^3$ for each cell.

Now the number of cells is on the order of $0.06\text{ m}^3/(10^{-15}\text{m}^3/\text{cell}) = 6\times 10^{13}\text{ cells} \sim \boxed{10^{14}\text{ cells}}$

Discussion As evidence that our body volume estimate is reasonable, we note that the density of the body is about the same as that of water, 1000 kg/m^3, so that volume 0.06 m^3 implies a mass of $(0.06\text{ m}^3)(1000\text{ kg/m}^3) = 60$ kg. This is a reasonable mass for a person, even though only a small minority of people are fully 2.0 m tall. A different line of reasoning might lead to an estimate on the order of 10^{13} cells. Still, we can be sure that the number of cells is large compared to the number of stars in a bright galaxy, hundreds of billions.

55. Strategy Use the two temperatures and their corresponding times to find the rate of temperature change with respect to time (the slope of the graph of temperature vs. time). Then, write the linear equation for the temperature with respect to time and find the temperature at 3:35 P.M.

Solution Find the rate of temperature change.

$$m = \frac{\Delta T}{\Delta t} = \frac{101.0°\text{F} - 97.0°\text{F}}{4.0\text{ h}} = 1.0\,°\text{F/h}$$

Use the slope-intercept form of a graph of temperature vs. time to find the temperature at 3:35 P.M.

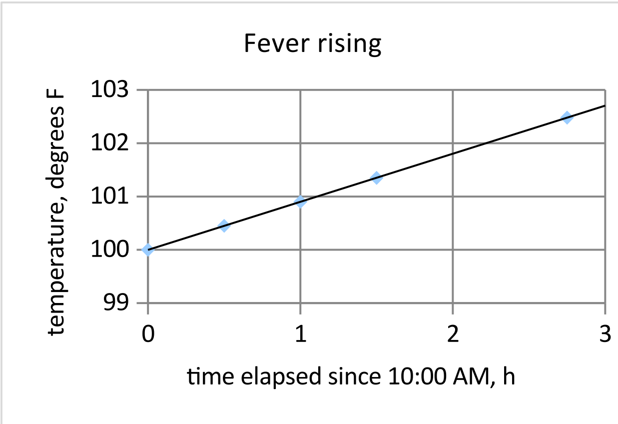
$T = mt + T_0 = (1.0\ °\text{F/h})(3.5\text{ h}) + 101.0°\text{F} = \boxed{104.5°\text{F}}$

56. Strategy The plot of temperature versus elapsed time is shown. We use the graph to answer the questions.

(a) By inspection of the graph, it appears that the temperature at noon was $\boxed{101.8°\text{F}}$.

(b) Estimate the slope of the line.

$$m = \frac{102.7°\text{F} - 100.0°\text{F}}{3.00\text{ h} - 0} = \frac{2.7°\text{F}}{3\text{ h}} = \boxed{0.90\,°\text{F/h}}$$



(c) After the next twelve hours, the temperature would, according to the trend, be approximately $T = (0.87\,°\text{F/h})(12\text{ h}) + 102.5°\text{F} = 113°\text{F}.$

$\boxed{\text{The patient would be dead before the temperature reached so high a value. So, the answer is no.}}$

57. Strategy Put the equation that describes the line in slope-intercept form, $y = mx + b$, with v replacing y and t replacing x. Here $at = v - v_0$ becomes $v = at + v_0$

Solution

(a) v is the dependent variable and t is the independent variable, so $\boxed{a}$ is the slope of the line.

(b) The slope-intercept form is $y = mx + b$. Find the vertical-axis intercept.

$v \leftrightarrow y,\ t \leftrightarrow x,\ a \leftrightarrow m,$ so $v_0 \leftrightarrow b.$

Thus, $\boxed{+v_0}$ is the vertical-axis intercept of the line.

Discussion. It is remarkable how much a graph shows. A graph shows the data as individual points, and shows a model describing all of them as a line. The most important thing about a graph is its shape. In this case the straightness of a line fitting the data points convincingly indicates that the acceleration is constant. The scatter of the points away from the best-fit line can give an estimate of the experimental uncertainty. Plus, as the problem notes, the slope and vertical-axis intercept of the line give values for the acceleration and original velocity. You will study soon enough that the area under a v-versus-t graph line gives the displacement (change in position) of the object.

58. (a) Strategy Modeled on $y = mx + b$, the equation of the speed versus time graph is given by $v = a\,t + v_0$, where $a = 6.0\ \text{m/s}^2$ and $v_0 = 3.0\ \text{m/s}.$

Solution Find the change in speed.

$$\begin{array}{l} v_2 = at_2 + v_0 \\ \underline{-(v_1 = at_1 + v_0)} \\ v_2 - v_1 = a(t_2 - t_1) \\ v_2 - v_1 = (6.0\ \text{m/s}^2)(6.0\ \text{s} - 4.0\ \text{s}) = \boxed{12\ \text{m/s}} \end{array}$$

(b) Strategy Use the equation found in part (a).

Solution Find the speed when the elapsed time is equal to 5.0 seconds.

$$v = (6.0\ \text{m/s}^2)(5.0\ \text{s}) + 3.0\ \text{m/s} = \boxed{33\ \text{m/s}}$$

59. (a) Strategy Refer to the graph in the statement of the problem. Use the definition of the slope of a line and the fact that the vertical axis intercept is the position (x) value corresponding to $t = 0$.

Solution Compute the slope.

$$\frac{\Delta x}{\Delta t} = \frac{17.0\ \text{km} - 3.0\ \text{km}}{9.0\ \text{h} - 0.0\ \text{h}} = \boxed{1.6\ \text{km/h}}.$$

When $t = 0$, $x = 3.0$ km; therefore, the vertical axis intercept is $\boxed{3.0\ \text{km}}$.

(b) Strategy and Solution The physical significance of the slope of the graph is that it represents the $\boxed{\text{speed}}$ of the object. The physical significance of the vertical axis intercept is that it represents the $\boxed{\text{starting position}}$ of the object (its x coordinate at elapsed time zero).

60. Strategy To determine values for c and A_0 from the experimental data, graph A versus B^3.

Solution To graph A versus B^3, graph $\boxed{A \text{ on the vertical axis and } B^3 \text{ on the horizontal axis}}$. Then the slope is the experimental value of c and the vertical-axis intercept is the experimental value for A_0. Both can then be compared with theoretical or separately-measured values for c and A_0.

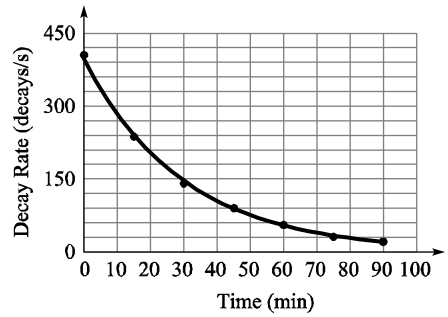
61. Strategy Use the slope-intercept form, $y = mx + b$.

Solution Since x is on the vertical axis, it corresponds to y. Since t^4 is on the horizontal axis, it corresponds to x (in $y = mx + b$). So, the equation for x as a function of t is $\boxed{x = (25\ \text{m/s}^4)t^4 + 3\ \text{m}}$.

Discussion. A graph of x versus t from the same data would curve strongly upward as a "quartic parabola." Your eye can judge the straightness of a straight line and how well it fits points around it. But a quartic parabola is much less familiar. That is why we make the particular choice to use the fourth power of every t value as the horizontal-axis coordinate of a data point.

62. (a) Strategy Plot the decay rate on the vertical axis and the time on the horizontal axis.

Solution The plot is shown.

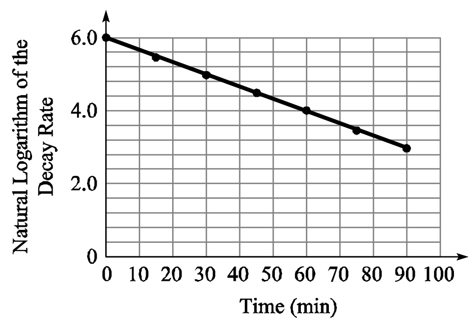


(b) Strategy Plot the natural logarithm of the decay rate on the vertical axis and the time on the horizontal axis.

Solution. We find the natural logarithm of each decay rate. The first is $\ln(405) = 6.00$. The others are shown:

Time (min)	0	15	30	45	60	75	90
ln(decay rate in Bq)	6.00	5.47	4.94	4.50	4.00	3.47	2.94

The plot is shown.



Presentation of the data in this form—with the natural logarithm of the decay rate—is useful because $\boxed{\text{the graph is linear, demonstrating that the decay rate decreases exponentially}}$. That is, the data points fit a function like $R = R_0 e^{-\lambda t}$ where R_0 and λ are constants. In this case $\ln(R) = -\lambda t + \ln(R_0)$, which has the form $y = mx + b$.

63. Strategy Use graphing rules 3, 5, and 7 under Graphing Data in Section 1.9 Graphs.

Solution

(a) $\boxed{\text{To obtain a linear graph, the students should plot } v \text{ versus } r^2\text{, where } v \text{ is the dependent variable and } r^2 \text{ is the independent variable.}}$

(b) $\boxed{\text{The students should measure the slope of the best-fit line obtained from the graph of the data; set the value of the slope equal to } 2g(\rho-\rho_\text{f})/(9\eta)\text{; and solve for}}$ η.

64. (a) Strategy Make an order-of-magnitude estimate. Assume 8 seconds per breath.

Solution Estimate the number of breaths you take in one year.

$$\textit{breaths per year} = \frac{1 \text{ breath}}{8 \text{ s}} \times \frac{3.156\times10^7 \text{ s}}{1 \text{ yr}} = 4\times10^6 \text{ breaths/yr} \sim \boxed{10^7 \text{ breaths/yr}}$$

We choose to write the order of magnitude as 10^7 breaths instead of 10^6 because our answer differs from 10^7 by a factor of only 2.5 while differing from 10^6 by a factor of 4.

(b) Strategy Assume 0.5 L per breath.

Solution Estimate the volume of air you breathe in during one year.

$$\textit{volume} = 4\times10^6 \text{ breaths} \times \frac{0.5 \text{ L}}{1 \text{ breath}} = 2\times10^6 \text{ L} \times \frac{10^{-3} \text{ m}^3}{1 \text{ L}} = 2000 \text{ m}^3 \sim \boxed{10^3 \text{ m}^3}$$

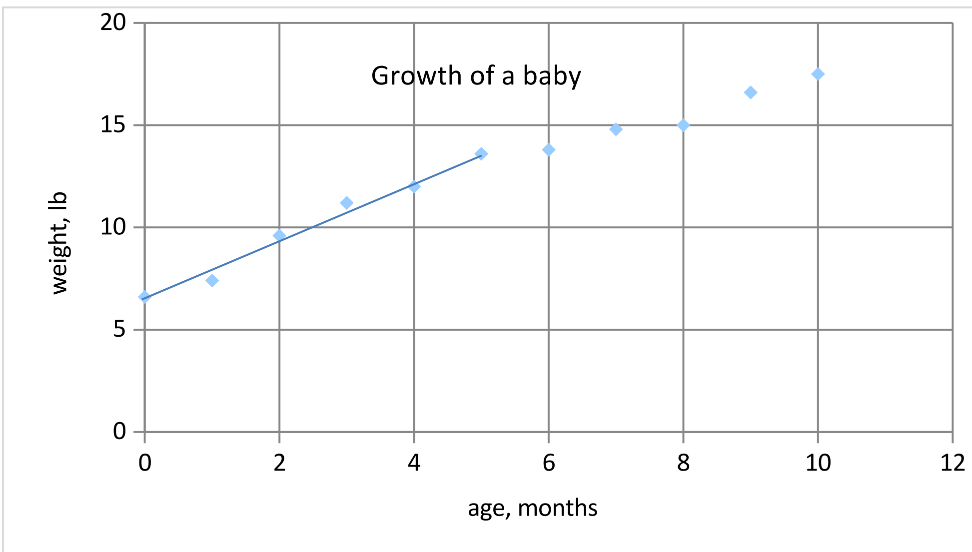
65. Strategy Replace v, r, ω, and m with their dimensions. Then use dimensional analysis to determine how v depends upon some or all of the other quantities.

Solution v, r, ω, and m have dimensions $\frac{[\text{L}]}{[\text{T}]}$, [L], $\frac{1}{[\text{T}]}$, and [M], respectively. Let $v = r^e \omega^f m^g$, where e, f, and g are unknown exponents. Then $\frac{[\text{L}]}{[\text{T}]} = [\text{L}]^e \frac{1}{[\text{T}]^f} [\text{M}]^g$ The lefthand side is $[\text{L}]^1[\text{T}]^{-1}[\text{M}]^0$ so we require $e = 1$, $-f = -1$, and $g = 0$. Thus $f = 1$. We have $v = r^1\omega^1$ and v does not depend upon m. That is, v is directly proportional to r and directly proportional to ω, as in $v = Kr\omega$ where K is a dimensionless constant. Stated with a different symbol, v is proportional to the product of r and ω, as $\boxed{v \propto \omega r}$.

Discussion. The argument could be stated in terms of units. We need a combination of m and s^{-1} and kg that will equal m/s, and the only combination is $(\text{m})(\text{s}^{-1})$.

66. (a) Strategy. Prepare the graph of the baby's weight as a function of time.

Solution.



(b) Strategy Find the slope of the average-gain line shown, between age 0 and age 5 months. We interpret the average monthly gain to be the average of the separate increases for each of the five month-long intervals. In this case the values of weight at all the intermediate points cancel out, and we can find the average gain by using the straight line connecting the first point (at age zero) to the last point.

Solution Find the slope.

$$m = \frac{13.6 \text{ lb} - 6.6 \text{ lb}}{5.0 \text{ mo} - 0.0 \text{ mo}} = \frac{7.0 \text{ lb}}{5.0 \text{ mo}} = \boxed{1.4 \text{ lb/mo}}$$

(c) Strategy Find the slope of the average-gain line between age 5 and age 10 months.

Solution Find the slope.

$$m = \frac{17.5 \text{ lb} - 13.6 \text{ lb}}{10.0 \text{ mo} - 5.0 \text{ mo}} = \frac{3.9 \text{ lb}}{5.0 \text{ mo}} = \boxed{0.78 \text{ lb/mo}}$$

(d) Strategy Write a linear equation for the weight of the baby as a function of time. The slope is that found in part (b), 1.4 lb/mo. The intercept is the weight of the baby at birth.

Solution Find the extrapolated weight of the child at age 12 years.

$$W = (1.4 \text{ lb/mo})(144 \text{ mo}) + 6.6 \text{ lb} = \boxed{210 \text{ lb}}$$

67. Strategy (Answers will vary.) We use San Francisco, California, for the city. The population of San Francisco is approximately 750,000. Assume that there is one automobile for every two residents of San Francisco, that an average automobile needs three repairs or services per year, and that the average shop can service 10 automobiles per day.

Solution Estimate the number of automobile repair shops in San Francisco.

If an automobile needs three repairs or services per year, then it needs $\frac{3\text{ repairs}}{\text{auto}\cdot\text{yr}}\times\frac{1\text{ yr}}{365\text{ d}}\approx\frac{0.01\text{ repairs}}{\text{auto}\cdot\text{d}}$.

If there is one auto for every two residents, then there are $\frac{1\text{ auto}}{2\text{ residents}}\times 750,000\text{ residents}\approx 4\times 10^5\text{ autos}$.

If a shop requires one day to service 10 autos, then the number of shops-days per repair is

$1\text{ shop}\times\frac{1\text{ d}}{10\text{ repairs}}=\frac{0.1\text{ shop}\cdot\text{d}}{\text{repair}}$.

The estimated number of auto shops is $4\times 10^5\text{ autos}\times\frac{0.01\text{ repairs}}{\text{auto}\cdot\text{d}}\times\frac{0.1\text{ shop}\cdot\text{d}}{\text{repair}}=\boxed{400\text{ shops}}$.

Checking the internet, we find that there are approximately 300 automobile repair and service shops in San Francisco. Our estimate has the right order of magnitude.

68. Strategy For parts (a) through (d), perform the calculations.

Solution (a) $186.300+0.0030=\boxed{186.303}$ **(b)** $186.300-0.0030=\boxed{186.297}$

(c) $186.300\times 0.0030=\boxed{0.56}$ **(d)** $186.300/0.0030=\boxed{62,000}$

(e) Strategy For cases (a) and (b), the percent error is given by $\frac{0.0030}{\textit{Actual Value}}\times 100\%$.

Solution Find the percent error. Case (a): $\frac{0.0030}{186.303}\times 100\%=\boxed{0.0016\%}$

Case (b): $\frac{0.0030}{186.297}\times 100\%=\boxed{0.0016\%}$

For case (c), ignoring 0.0030 causes you to multiply by zero and get a zero result. For case (d), ignoring 0.0030 causes you to divide by zero and get an infinite answer.

(f) Strategy Make a rule about neglecting small values using the results obtained above.

Solution

You can neglect small values when they are added to or subtracted from sufficiently large values. The meaning of the term 'sufficiently large' is determined by the number of significant figures required.

69. Strategy There are between 10^2 and 10^3 hairs in a one-square-centimeter area of a typical human head. An order-of-magnitude estimate of the area of the average human scalp is between 10^2 and 10^3 square centimeters.

Solution Calculate the estimate.

$$10^{2.5}\text{ hairs}/\text{cm}^2 \times 10^{2.5}\text{ cm}^2 = \boxed{10^5\text{ hairs}}$$

Discussion. "On the order of 10^5" includes all the numbers between about 3×10^4 and about 3×10^5. Our answer applies to very many people, if not to the bald.

70. Strategy Use the metric prefixes n (10^{-9}), μ (10^{-6}), m (10^{-3}), or M (10^6).

Solution

(a) M (or mega) is equal to 10^6, so $6 \times 10^6\text{ m} = \boxed{6\text{ Mm}}$.

(b) There are approximately 3.28 feet in one meter, so $6\text{ ft} \times \dfrac{1\text{ m}}{3.28\text{ ft}} = \boxed{2\text{ m}}$.

(c) μ (or micro) is equal to 10^{-6}, so 2 × $10^{-6}\text{ m} = \boxed{2\ \mu\text{m}}$.

(d) n (or nano) is equal to 10^{-9}, so $3 \times 10^{-9}\text{ m} = \boxed{3\text{ nm}}$.

(e) n (or nano) is equal to 10^{-9}, so $3 \times 10^{-10}\text{ m} = \boxed{0.3\text{ nm}} = 300\text{ pm}$

71. Strategy The volume of the spherical virus is given by $V_{\text{virus}} = (4/3)\pi r_{\text{virus}}^3$. The volume of viral particles is one billionth the volume of the saliva.

Solution Calculate the number of viruses that have landed on you.

$$\text{number of viral particles} = \frac{10^{-9}V_{\text{saliva}}}{V_{\text{virus}}} = \frac{0.010\text{ cm}^3}{10^9\left(\frac{4}{3}\pi\right)\left(\frac{85\text{ nm}}{2}\right)^3\left(\frac{10^{-7}\text{ cm}}{1\text{ nm}}\right)^3} \sim \boxed{10^4\text{ viruses}}$$

72. Strategy The circumference of a viroid is approximately 300 times 0.35 nm. The diameter is given by $C = \pi d$, or $d = C/\pi$.

Solution Find the diameter of the viroid in the required units.

(a) $$d = \frac{300(0.35\text{ nm})}{\pi} \times \frac{10^{-9}\text{ m}}{1\text{ nm}} = \boxed{3.3 \times 10^{-8}\text{ m}}$$

(b) $$d = \frac{300(0.35\text{ nm})}{\pi} \times \frac{10^{-3}\ \mu\text{m}}{1\text{ nm}} = \boxed{3.3 \times 10^{-2}\ \mu\text{m}}$$

(c) $$d = \frac{300(0.35\text{ nm})}{\pi} \times \frac{10^{-7}\text{ cm}}{1\text{ nm}} \times \frac{1\text{ in}}{2.54\text{ cm}} = \boxed{1.3 \times 10^{-6}\text{ in}}$$

73. (a) Strategy There are 3.28 feet in one meter.

Solution Find the length in meters of the largest recorded blue whale.

$$1.10 \times 10^2\text{ ft} \times \frac{1\text{ m}}{3.28\text{ ft}} = \boxed{33.5\text{ m}}$$

(b) Strategy Divide the length of the largest recorded blue whale by the length of a double-decker London bus.

Solution Find the length of the blue whale in double-decker-bus lengths.

$$1.10\times10^2\text{ ft}\times\frac{1\text{ m}}{3.28\text{ ft}}\times\frac{1\text{ bus length}}{8.0\text{ m}}=\boxed{4.2\text{ bus lengths}}$$

Discussion. The picture of the whale riding on busses in the problem statement is quite accurate. In elementary education you may have learned a rule such as "In converting from larger to smaller (units), multiply (by the conversion factor). In converting from smaller to larger, divide." In this problem, to get from feet to meters, we did indeed divide by 3.28. But please use instead the method of writing down the given quantity and multiplying it by a conversion fraction. The fraction is set up with numerator and denominator equal to each other and so arranged as to divide out the unit no longer desired and give instead the unit for the answer. Our method leaves behind a written record of what you have done.

74. Strategy The volume of the blue whale can be found by dividing the mass of the whale by its average density.
Solution Find the volume of the blue whale in cubic meters. From $\rho = m/V$ we have

$$V=\frac{m}{\rho}=\frac{1.9\times10^5\text{kg}}{0.85\ \text{g/cm}^3}\times\frac{1000\text{ g}}{1\text{ kg}}\times\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3=\boxed{2.2\times10^2\ \text{m}^3}$$

75. Strategy Modeling the capillaries as completely filled with blood, the total volume of blood is given by the cross-sectional area of the blood vessel times the length.

Solution Estimate the total volume of blood in the human body.

$$V=\pi r^2 l=\pi\left(4\times10^{-6}\ \text{m}\right)^2\left(10^8\ \text{m}\right)=0.005\text{ m}^3=\boxed{5\text{ L}}$$

In reality, blood flow through the capillaries is regulated, so they are not always full of blood. On the other hand, we've neglected the additional blood found in the larger vessels (arteries, arterioles, veins, and venules).

76. Strategy The shape of a sheet of paper (when not deformed) is a rectangular prism (like a brick). The volume of a rectangular prism is equal to the product of its length, width, and height (or thickness).

Solution Find the volume of a sheet of paper in cubic meters.

$$27.95\text{ cm}\times8.5\text{ in}\times0.10\text{ mm}\times\frac{1\text{ m}}{100\text{ cm}}\times\frac{0.0254\text{ m}}{1\text{ in}}\times\frac{1\text{ m}}{1000\text{ mm}}=\boxed{6.0\times10^{-6}\ \text{m}^3}$$

77. Strategy If v is the speed of the molecule, then $v\propto\sqrt{T}$ where T is the temperature.

Solution Form a proportion.

$$\frac{v_{\text{cold}}}{v_{\text{warm}}}=\frac{\sqrt{T_{\text{cold}}}}{\sqrt{T_{\text{warm}}}}$$

Find v_{cold}.

$$v_{\text{cold}}=v_{\text{warm}}\sqrt{\frac{T_{\text{cold}}}{T_{\text{warm}}}}=(475\ \text{m/s})\sqrt{\frac{250.0\text{ K}}{300.0\text{ K}}}=\boxed{434\ \text{m/s}}$$

Discussion. When we study thermodynamics, we will do a lot with the idea that a thermometer is like a speedometer for molecules. Observe that the factor of change in temperature is 250.0/300.0 = 0.8333, but the factor of change in speed is $(0.8333)^{1/2}=0.9129$. When molecular speed is proportional to the square root of absolute temperature, the absolute temperature is proportional to the square of speed. Then an 8.7% reduction in speed goes with a 17% drop in temperature.

78. Strategy Use dimensional analysis to convert from furlongs per fortnight to the required units.

Solution (a) Convert to $\mu\text{m/s}$.

$$\frac{1\text{ furlong}}{1\text{ fortnight}}\times\frac{220\text{ yd}}{1\text{ furlong}}\times\frac{1\text{ fortnight}}{14\text{ days}}\times\frac{1\text{ day}}{86{,}400\text{ s}}\times\frac{3\text{ ft}}{1\text{ yd}}\times\frac{1\text{ m}}{3.28\text{ ft}}\times\frac{1{,}000{,}000\ \mu\text{m}}{1\text{ m}}=\boxed{166\ \mu\text{m/s}}$$

(b) Convert to km/day. $\frac{1\text{ furlong}}{1\text{ fortnight}}\times\frac{220\text{ yd}}{1\text{ furlong}}\times\frac{1\text{ fortnight}}{14\text{ days}}\times\frac{3\text{ ft}}{1\text{ yd}}\times\frac{1\text{ m}}{3.28\text{ ft}}\times\frac{1\text{ km}}{1000\text{ m}}=\boxed{0.0144\text{ km/day}}$

79. Strategy There are 2.54 cm in one inch and 3600 seconds in one hour.

Solution Find the conversion factor for changing meters per second to miles per hour. The conversion equation is

$$1\ \frac{\text{m}}{\text{s}}=\frac{1\text{ m}}{1\text{ s}}\times\frac{100\text{ cm}}{1\text{ m}}\times\frac{1\text{ in}}{2.54\text{ cm}}\times\frac{1\text{ ft}}{12\text{ in}}\times\frac{1\text{ mi}}{5280\text{ ft}}\times\frac{3600\text{ s}}{1\text{ h}}=\boxed{2.24\text{ mi/h}=1\text{ m/s}}$$

or the conversion factor is $\boxed{(2.24\text{ mi/h})/(1\text{ m/s})=1}$

So, for a quick, rough conversion, multiply by 2.

80. Strategy There are $\$1{,}000{,}000/\$20=50{,}000$ twenty-dollar bills in \$1,000,000. The mass of a twenty-dollar bill is about 1 gram, or 10^{-3} kilograms.

Solution Estimate the mass of the bills as $50{,}000\text{ bills}\times10^{-3}\text{ kg/bill}=50\text{ kg, or }\boxed{\sim 100\text{ kg}}.$

81. Strategy The SI base unit for mass is kg. Replace each quantity in $W=mg$ with its SI base units.

Solution The SI unit for weight is $\text{kg}\cdot\frac{\text{m}}{\text{s}^2}=\boxed{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}$

Discussion. In everyday life one meets combination units such as kilometers per hour, person-hours, and even dollars per person-hour. No doubt gloomy research has been done on how few people understand quantities involving these units. We may be glossing over the difficulty of forming a mental picture of a kilogram-meter-per-second-squared, when we call it a newton and feel one newton as the gravitational force on a particular apple. Repeated practice is the standard method for helping to form a mental picture. Get your practice by writing down units faithfully with the calculations you do and the answers you get in this course.

82. Strategy It is given that $T^2\propto r^3.$ Divide the period of Mars by that of Venus.

Solution Compare the period of Mars to that of Venus.

$$\frac{T_{\text{Mars}}^2}{T_{\text{Venus}}^2}=\frac{r_{\text{Mars}}^3}{r_{\text{Venus}}^3},\text{ so }T_{\text{Mars}}^2=\left(\frac{r_{\text{Mars}}}{r_{\text{Venus}}}\right)^3T_{\text{Venus}}^2,\text{ or }T_{\text{Mars}}\approx\left(\frac{2r_{\text{Venus}}}{r_{\text{Venus}}}\right)^{3/2}T_{\text{Venus}}=\boxed{2^{3/2}T_{\text{Venus}}\approx 2.8T_{\text{Venus}}}.$$

83. Strategy \$59,000,000,000 has a precision of 1 billion dollars; \$100 would affect the balance only if her wealth were quoted with a precision of 100 dollars, so her net worth is the same to two significant figure.

Solution Find the net worth. $\$59{,}000{,}000{,}000-\$100=\boxed{\$59{,}000{,}000{,}000}$

84. Strategy Solution methods will vary, but the order of magnitude answer should not. One example follows: In a car on the interstate highway you can drive a thousand kilometers in two days, but that does not get you around the circumference of the Earth by a very large angle. The circumference of the Earth is about forty thousand kilometers. The radius of the Earth is on the order of 10^7 m. The area of a sphere is $4\pi r^2,$ or on the order of

$10^1 \cdot r^2.$ The average depth of the oceans is about 4×10^3 m. The oceans cover about seven tenths of the Earth's surface.

Solution Calculate an order-of-magnitude estimate of the volume of water contained in Earth's oceans.

The surface area of the Earth is on the order of $10^1 \cdot (10^7\text{ m})^2 = 10^{15}\text{m}^2;$ therefore, the volume of water in the oceans is about $\text{area} \times \text{depth} \sim 0.7(10^{15}\text{m}^2)(4 \times 10^3\text{m}) \approx 3 \times 10^{18}\text{m}^3 : \boxed{10^{18}\text{m}^3}.$

85. (a) Strategy There are 7.0 leagues in one step and 4.8 kilometers in one league.

Solution Find your speed in kilometers per hour.

$$\frac{120\text{ steps}}{1\text{ min}} \times \frac{7.0\text{ leagues}}{1\text{ step}} \times \frac{4.8\text{ km}}{1\text{ league}} \times \frac{60\text{ min}}{1\text{ h}} = \boxed{2.4 \times 10^5\text{ km/h}}$$

(b) Strategy The circumference of the earth is approximately 40,000 km. The time it takes to march around the Earth is found by dividing the distance by the speed.

Solution Find the time of travel.

$$40{,}000\text{ km} \times \frac{1\text{ h}}{2.4 \times 10^5\text{km}} \times \frac{60\text{ min}}{1\text{ h}} = \boxed{10\text{ min}}$$

Discussion. You could use the radius of the Earth as listed in Appendix B to find its circumference. But we happen to know that a meter was originally defined to make the circumference of the planet (around meridians of longitude) forty thousand kilometers.

86. Strategy Use conversion factors from Appendix B.

Solution (a)

$$\frac{12.5\text{ US gal}}{1} \times \frac{3.785\text{ L}}{\text{US gal}} \times \frac{10^3\text{ mL}}{\text{L}} \times \frac{0.06102\text{ in}^3}{\text{mL}} = \boxed{2890\text{ in}^3}$$

(b)

$$\frac{2887\text{ in}^3}{1} \times \left(\frac{1\text{ cubit}}{18\text{ in}}\right)^3 = \boxed{0.495\text{ cubic cubits}}$$

87. Strategy The weight is proportional to the planet mass and inversely proportional to the square of the radius, so $W \propto m/r^2.$ Thus, for Earth and Jupiter, we have $W_\text{E} = K\, m_\text{E}/r_\text{E}^2$ and $W_\text{J} = K\, m_\text{J}/r_\text{J}^2.$ with some constant K.

Solution Form a proportion.

$$\frac{W_\text{J}}{W_\text{E}} = \frac{m_\text{J}/r_\text{J}^2}{m_\text{E}/r_\text{E}^2} = \frac{m_\text{J}}{m_\text{E}}\left(\frac{r_\text{E}}{r_\text{J}}\right)^2 = \frac{320m_\text{E}}{m_\text{E}}\left(\frac{r_\text{E}}{11r_\text{E}}\right)^2 = \frac{320}{121}$$

On Jupiter, the apple would weigh $\frac{320}{121}(1.0\text{ N}) = \boxed{2.6\text{ N}}.$

88. Strategy Replace each quantity in $v = K\lambda^p g^q$ by its units. Then, use the relationships between p and q to determine their values.

Solution Find the values of p and q.

In units, $\frac{\text{m}}{\text{s}} = \text{m}^p \cdot \frac{\text{m}^q}{\text{s}^{2q}} = \frac{\text{m}^{p+q}}{\text{s}^{2q}}.$

So, we have the following restrictions on p and q: $p + q = 1$ and $2q = 1$.

Solve for q and p.

$$p+q=1$$

$$2q=1 \qquad p+\frac{1}{2}=1$$

$$q=\boxed{\frac{1}{2}} \qquad p=\boxed{\frac{1}{2}}$$

Thus, $v=K\lambda^{1/2}g^{1/2}=K\sqrt{\lambda g}.$

89. Strategy Since there are about 3×10^8 people in the U.S., a reasonable estimate of the number of automobiles is 1.5×10^8. There are 365 days per year. A reasonable estimate for the average volume of gasoline used per day per car is greater than 1 gal, but less than 10 gal; for an order-of-magnitude estimate, let's guess 2 gallons per day.

Solution Calculate the estimate.

$$1.5\times10^8\text{ cars}\times365\text{ days}\times2\,\frac{\text{gal}}{\text{car}\cdot\text{day}}\sim\boxed{10^{11}\text{ gal}}$$

Discussion. Not everyone uses the tilde symbol $\sim$ for "is on the order of," but we find it convenient. Another problem solution might begin with different data. But an order of magnitude is such a wide target that another problem solution should arrive at the same answer.

90. Strategy Use the fact that $R_B=1.42R_A.$

Solution Calculate the ratio of P_B to $P_A.$

$$\frac{P_B}{P_A}=\frac{\frac{V^2}{R_B}}{\frac{V^2}{R_A}}=\frac{R_A}{R_B}=\frac{R_A}{1.42R_A}=\frac{1}{1.42}=\boxed{0.704}$$

91. (a) Strategy Inspect the units of G, c, and h and generate three simultaneous equations from how the units (or dimensions) must agree across the equality sign when these constants are used to compute a time.

Solution Do not assume that p, q, and r are integers, but they are dimensionless numbers in $t_{Planck}=c^pG^qh^r$, requiring in terms of units $\text{s}^1=\left(\frac{\text{m}}{\text{s}}\right)^p\left(\frac{\text{m}^3}{\text{kg}^1\text{s}^2}\right)^q\left(\frac{\text{kg}^1\text{m}^2}{\text{s}^1}\right)^r$ Then the base units must separately appear to the same power on both sides of the equation. For seconds, $1=-p-2q-r.$ For meters, $0=p+3q+2r.$ For kilograms, $0=0-q+r.$ Substitute $r=q$ from the last equation into the first two to get down to two equations in two unknowns: $1=-p-3q$ and $0=p+5q.$ Now substitute $p=-5q$ from the second equation into the first: $1=+5q-3q$. Then $q=+1/2.$ If you tried to solve simultaneous equations with determinants, you would have to do much more work to find the other two unknowns. The method of substitution makes all the answers after the first come out easily, just as when the cat has kittens. From above, $p=-5q=-5/2$ and $r=q=+1/2.$ We find that the only combination of G, c, and h that has the dimensions of time is $\boxed{\sqrt{\frac{hG}{c^5}}}$.

(b) Strategy Substitute the values of the constants into the expression found in part (a).

Solution Find the time in seconds.

$$\sqrt{\frac{hG}{c^5}} == \sqrt{\frac{\left(6.6\times 10^{-34}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}\right)\left(6.7\times 10^{-11}\ \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)}{\left(3.0\times 10^{8}\ \frac{\text{m}}{\text{s}}\right)^5}} = \boxed{1.3\times 10^{-43}\,\text{s}}$$

92. Strategy The dimensions of L, g, and m are length, length per time squared, and mass, respectively. The period has units of time, so T cannot depend upon m. (There are no other quantities with units of mass with which to cancel the units of m.) Use a combination of L and g.

Solution In $T = L^p g^q$ we require for the units to agree $\text{s} = \text{m}^p\,(\text{m/s}^2)^q$ so separately $1 = -2q$ and $0 = p + q$. Then $q = -1/2$ and $p = -q = +½$. The square root of L/g does indeed have dimensions of time, so

$\boxed{T = C\sqrt{\frac{L}{g}},\ \text{where } C \text{ is a constant of proportionality}}$.

93. Strategy The dimensions of *k* and *m* are mass per time squared and mass, respectively. Dividing either quantity by the other will eliminate the mass dimension, so the equation giving frequency must involve the quotient of *m* and *k*.

Solution The square root of k/m has dimensions of inverse time, which is correct for frequency.

So, $f = C\sqrt{k/m}.$ where *C* is some dimensionless constant. We set up a proportion to find *k*.

$$f_1 = C\sqrt{\frac{k}{m_1}}, \text{ so } f_1^2 = C^2\frac{k}{m_1}, \text{ or } k = \frac{m_1 f_1^2}{C^2}.$$

Find the frequency of the chair with the 75-kg astronaut.

$$f_2 = C\sqrt{\frac{k}{m_2}} = C\sqrt{\frac{m_1 f_1^2}{C^2 m_2}} = f_1\sqrt{\frac{m_1}{m_2}} = (0.50\text{ s}^{-1})\sqrt{\frac{62\text{ kg} + 10.0\text{ kg}}{75\text{ kg} + 10.0\text{ kg}}} = \boxed{0.46\text{ s}^{-1}}$$

Discussion. We will study this bouncing of a mass on a spring in the chapter on vibration. Here we see that dimensional analysis and proportional reasoning are more powerful than you might imagine.

94. Strategy Approach it as a unit-conversion problem.

Solution (a) The length of the road section, expressed as a multiple of the distance between reflectors, is

$$2.20\text{ mile}\left(\frac{5280\text{ ft}}{1\text{ mile}}\right)\frac{1\text{ yd}}{3\text{ ft}}\left(\frac{1\text{ space between reflectors}}{17.6\text{ yd}}\right) = 220\text{ spaces}$$

. We may need an extra reflector to furnish both ends, so we requisition $\boxed{\text{221 reflectors}}$.

(b) As in part (a), 3.54 km = $3.54\text{ km}\left(\frac{1000}{\text{k}}\right)\frac{1\text{ space between}}{16\text{ m}} = 221\text{ spaces between,}$ so we order $\boxed{\text{222 reflectors}}$ in case adjoining road sections have none.

Within the strict limit of doing routine calculations that people have thought of in advance, U.S. customary units (or "former British" units) are fine. The reflectors are just 1/100 mile apart. In real life $\boxed{\text{SI units}}$ are better. Professionals relied on reasoning from routine to build the *Titanic*. Noah thought about what he was doing while building the ark.

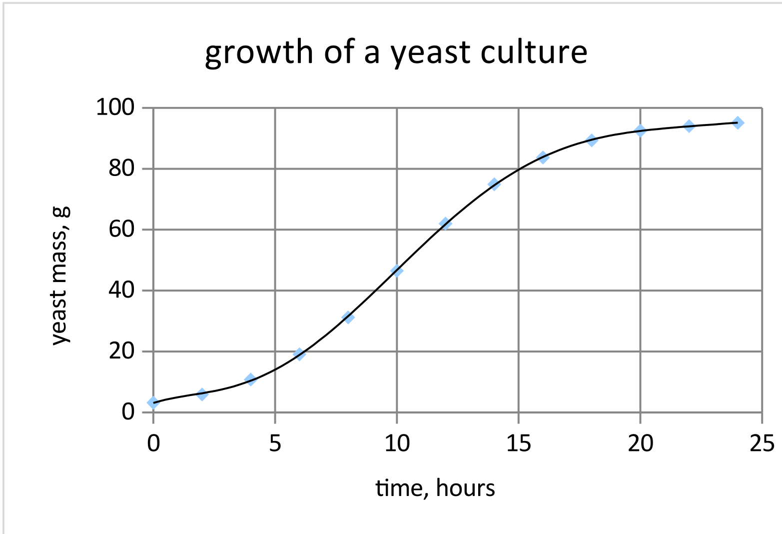
95. Strategy and solution. The doctor prescribed ¾ milliliter for each dose. The pharmacist printed ¾ teaspoon for each dose, which is larger by $\boxed{\text{4.9 times}}$. The most common unit-conversion mistake comes from ignoring the units.

96. Strategy. Approach it as a unit-conversion problem.

Solution. The captain (pilot) intended to dive from 1450 m to 1500 ft(1 m/3.28 ft) = 457 m, a downward vertical distance of 1450 m – 457 m = $\boxed{\text{990 m}}$. The first officer (copilot) made the most common unit-conversion mistake: ignoring the units.

97. **(a)** **Strategy.** Plot the data on a graph with mass on the vertical axis and time on the horizontal axis. Then draw a best-fit smooth curve.

Solution.



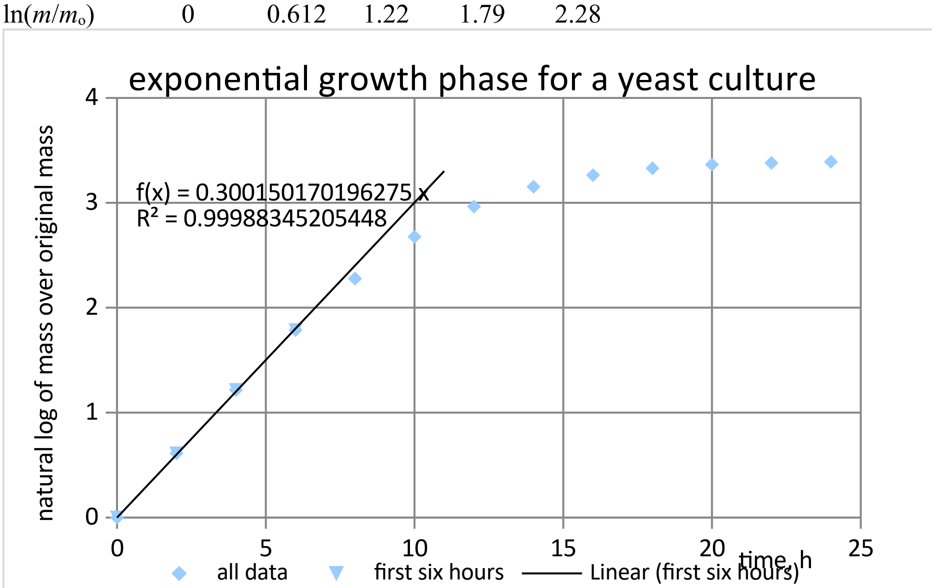
(b) **Strategy** Estimate by eye the value of the total mass that the graph appears to be approaching asymptotically.

Solution The graph appears to be approaching asymptotically a maximum value close to 100 g, so the carrying capacity is $\boxed{\text{about 100 g}}$.

(c) **Strategy.** Plort the data on a graph with the natural logarithm of m/m_o on the vertical axis and time on the horizontal axis. Draw a straight line fitting the first few points and find its slope to estimate the intrinsic growth rate.

Solution. We find for the starting point $m/m_o = 1$ and $\ln(m/m_o) = 0$. For the first nonzero point $m/m_o = 5.9/3.2 = 1.84$ and $\ln(m/m_o) = 0.612$. We compute similarly

Time, h	0	2	4	6	8
$\ln(m/m_o)$	0	0.612	1.22	1.79	2.28



From these we plot the graph. Only the zeroth point and the next three lie close to a single straight line, so we do not consider the point for 8 hours or the later points as we fit the straight trendline. From the plot of $\ln \frac{m}{m_0}$ vs. t, the slope r appears to be $r = \frac{1.83 - 0.0}{6.0\text{ s} - 0.0\text{ h}} = \frac{1.83}{6.0\text{ h}} = \boxed{0.3\text{ h}^{-1}}$.

Discussion. Many politicians and economists behave as if our national motto was "In Growth we trust." They put their faith in promoting "steady growth" by some nice percentage per year. They try to sell the idea to other societies, not understanding that this exponential increase is always unsustainable. An equation like $y = At^2$ or $y = Bt^5$ does not describe exponential growth—those equations are associated with some constant factor of increase in y whenever t increases by a constant factor. In the proportionality described by $y = Bt^5$, for example, y increases by 32 times whenever t doubles. The exponential function $m = m_0 e^{rt}$ is infinitely more sinister. It describes m doubling again and again, without limit, whenever t changes by a certain fixed step. From $\ln 2 = 0.693$, the doubling time T_d for the yeast cells with unlimited resources is given by $rT_d = 0.693$ so $T_d = 0.693/(0.30\text{ h}^{-1}) = 2.3$ hours. As time regularly and inexorably ticks on as 2.3 h, 4.6 h, 6.9 h, 9.2 h, … the population of yeast cells counts up in the pattern 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, … . In every doubling time the population uses up as much resources as they have in all previous history. The numbers tabulated in the problem start their increase at "only" 30% per hour or 0.5 % per minute. But the numbers show that this exponential growth could not continue even as long as six or eight hours. Again, "steady growth" is exponential increase and is unsustainable.