1.1: PROBLEM DEFINITION

Find: List three common units for each variable:
a. Volume flow rate (Q), mass flow rate ($\dot{m}$), and pressure (p).
b. Force, energy, power.
c. Viscosity, surface tension.

PLAN

Use Table F.1 to find common units

SOLUTION

a. Volume flow rate, mass flow rate, and pressure.

- Volume flow rate, $\mathrm{m^3/s}$, $\mathrm{ft^3/s}$ or cfs, cfm or $\mathrm{ft^3/m}$.
- Mass flow rate. kg/s, lbm/s, slug/s.
- Pressure. Pa, bar, psi or $\mathrm{lbf/in^2}$.

b. Force, energy, power.

- Force, lbf, N, dyne.
- Energy, J, ft·lbf, Btu.
- Power. W, Btu/s, ft·lbf/s.

c. Viscosity.

- Viscosity, Pa·s, kg/(m·s), poise.

1.2: PROBLEM DEFINITION

Situation: The hydrostatic equation has three common forms:

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 = \text{ constant}$$

$$p_z = p_1 + \gamma z_1 = p_2 + \gamma z_2 = \text{ constant}$$

$$\Delta p = -\gamma \Delta z$$

Find: For each variable in these equations, list the name, symbol, and primary dimensions of each variable.

PLAN

Look up variables in Table A.6. Organize results using a table.

SOLUTION

Name	**Symbol**	**Primary dimensions**
pressure	p	M/LT^2
specific weight	γ	M/L^2T^2
elevation	z	L
piezometric pressure	p_z	M/LT^2
change in pressure	Δp	M/LT^2
change in elevation	Δz	L

1.3: PROBLEM DEFINITION

Situation:

Five units are specified.

Find:

Primary dimensions for each given unit: kWh, poise, slug, cfm, CSt.

PLAN

1. Find each primary dimension by using Table F.1.
2. Organize results using a table.

SOLUTION

Unit	**Associated Dimension**	**Associated Primary Dimensions**
kWh	Energy	ML^2/T^2
poise	Viscosity	$M/(L \cdot T)$
slug	Mass	M
cfm	Volume Flow Rate	L^3/T
cSt	Kinematic viscosity	L/T^2

1.4: PROBLEM DEFINITION

Situation:

The hydrostatic equation is

$$\frac{p}{\gamma} + z = C$$

p is pressure, γ is specific weight, z is elevation and C is a constant.

Find:

Prove that the hydrostatic equation is dimensionally homogeneous.

PLAN

Show that each term has the same primary dimensions. Thus, show that the primary dimensions of p/γ equal the primary dimensions of z. Find primary dimensions using Table F.1.

SOLUTION

1. Primary dimensions of p/γ:

$$\left[\frac{p}{\gamma}\right] = \frac{[p]}{[\gamma]} = \left(\frac{M}{LT^2}\right)\left(\frac{L^2T^2}{M}\right) = L$$

2. Primary dimensions of z :

$$[z] = L$$

3. Dimensional homogeneity. Since the primary dimensions of each term is length, the equation is dimensionally homogeneous. Note that the constant C in the equation will also have the same primary dimension.

1.5: PROBLEM DEFINITION

Situation:

Four terms are given in the problem statement.

Find: Primary dimensions of each term.

a) $\rho V^2/\sigma$ (kinetic pressure).
b) T (torque).
c) P (power).
d) $\rho V^2 L/\sigma$ (Weber number).

SOLUTION

a. Kinetic pressure:

$$\left[\frac{\rho V^2}{2}\right] = [\rho]\,[V]^2 = \left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)^2 = \frac{M}{L \cdot T^2}$$

b. Torque.

$$[\text{Torque}] = [\text{Force}]\,[\text{Distance}] = \left(\frac{ML}{T^2}\right)(M) = \frac{M^2 \cdot L}{T^2}$$

c. Power (from Table F.1).

$$[P] = \frac{M \cdot L^2}{T^3}$$

d. Weber Number:

$$\left[\frac{\rho V^2 L}{\sigma}\right] = \frac{[\rho]\,[V]^2\,[L]}{[\sigma]} = \frac{(M/L^3)\,(L/T)^2\,(L)}{(M/T^2)} = []$$

Thus, this is a dimensionless group

1.6: PROBLEM DEFINITION

Situation:

The power provided by a centrifugal pump is given by:

$$P = \dot{m}gh$$

Find:

Prove that the above equation is dimensionally homogenous.

PLAN

1. Look up primary dimensions of P and $\dot{m}$ using Table F.1.
2. Show that the primary dimensions of P are the same as the primary dimensions of $\dot{m}gh$.

SOLUTION

1. Primary dimensions:

$$\begin{aligned}
[P] &= \frac{M \cdot L^2}{T^3} \\
[\dot{m}] &= \frac{M}{T} \\
[g] &= \frac{L}{T^2} \\
[h] &= L
\end{aligned}$$

2. Primary dimensions of $\dot{m}gh$:

$$[\dot{m}gh] = [\dot{m}]\,[g]\,[h] = \left(\frac{M}{T}\right)\left(\frac{L}{T^2}\right)(L) = \frac{M \cdot L^2}{T^3}$$

Since $[\dot{m}gh] = [P]$, $\boxed{\text{The power equation is dimensionally homogenous.}}$

1.7: PROBLEM DEFINITION

Situation:

Two terms are specified.

a. $\int \rho V^2 dA$.

b. $\frac{d}{dt}\int_{\forall} \rho V d\forall$.

Find:

Primary dimensions for each term.

PLAN

1. To find primary dimensions for term a, use the idea that an integral is defined using a sum.
2. To find primary dimensions for term b, use the idea that a derivative is defined using a ratio.

SOLUTION

Term a:

$$\left[\int \rho V^2 dA\right] = [\rho]\left[V^2\right][A] = \left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)^2\left(L^2\right) = \boxed{\frac{ML}{T^2}}$$

Term b:

$$\left[\frac{d}{dt}\int_{\forall} \rho V d\forall\right] = \frac{\left[\int \rho V d\forall\right]}{[t]} = \frac{[\rho]\,[V]\,[\forall]}{[t]} = \frac{\left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)\left(L^3\right)}{T} = \boxed{\frac{ML}{T^2}}$$

Problem 1.8

No solution provided.

$\boxed{\text{1.9: PROBLEM DEFINITION}}$

Apply the grid method.

Situation:

Density of ideal gas is given by:

$$\rho = \frac{p}{RT}$$

$p = 35$ psi, $R = 1716\,\text{ft-lbf/slug-}^\circ\text{R}$.
$T = 100\,^\circ\text{F} = 560\ ^\circ\text{R}$.

Find:

Calculate density (in lbm/ft^3).

$\boxed{\text{PLAN}}$

Follow the process given in the text. Look up conversion ratios in Table F.1.

$\boxed{\text{SOLUTION}}$

(note: unit cancellations not shown).

$$\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \left(\frac{35\,\text{lbf}}{\text{in}^2}\right)\left(\frac{12\,\text{in}}{\text{ft}}\right)^2\left(\frac{\text{slug}\cdot{}^\text{o}\,\text{R}}{1716\,\text{ft}\cdot\text{lbf}}\right)\left(\frac{1.0}{560\ ^\text{o}\text{R}}\right)\left(\frac{32.17\ \text{lbm}}{1.0\,\text{slug}}\right)
\end{aligned}$$

$$\boxed{\rho = 0.169\ \text{lbm/ft}^3}$$

1.10: PROBLEM DEFINITION

Apply the grid method.

Situation:

Wind is hitting a window of building.
$\Delta p = \frac{\rho V^2}{2}$.
$\rho = 1.2\,\mathrm{kg/\,m^3}, \quad V = 60$ mph.

Find:

a. Express the answer in pascals.
b. Express the answer in pounds force per square inch (psi).
c. Express the answer in inches of water column (inch H_20).

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a)

Pascals.

$$\begin{aligned} \Delta p &= \frac{\rho V^2}{2} \\ &= \frac{1}{2}\left(\frac{1.2\,\mathrm{kg}}{\mathrm{m}^3}\right)\left(\frac{60\ \mathrm{mph}}{1.0}\right)^2\left(\frac{1.0\,\mathrm{m/\,s}}{2.237\ \mathrm{mph}}\right)^2\left(\frac{\mathrm{Pa}\cdot\ \mathrm{m}\cdot\ \mathrm{s}^2}{\mathrm{kg}}\right) \end{aligned}$$

$$\boxed{\Delta p = 432\,\mathrm{Pa}}$$

b)

Pounds per square inch.

$$\Delta p = 432\,\mathrm{Pa}\left(\frac{1.450\times 10^{-4}\ \mathrm{psi}}{\mathrm{Pa}}\right)$$

$$\boxed{\Delta p = 0.062\,6\ \mathrm{psi}}$$

c)

Inches of water column

$$\Delta p = 432\,\mathrm{Pa}\left(\frac{0.004019\ \text{in-H20}}{\mathrm{Pa}}\right)$$

$$\boxed{\Delta p = 1.74\ \text{in-H20}}$$

1.11: PROBLEM DEFINITION

Apply the grid method.

Situation:

Force is given by $F = ma$.

a) $m = 10\,\text{kg}$, $a = 10\,\text{m/s}^2$.

b) $m = 10\,\text{lb}$, $a = 10\,\text{ft/s}^2$.

c) $m = 10\,\text{slug}$, $a = 10\,\text{ft/s}^2$.

Find:

Calculate force.

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a)

Force in newtons for $m = 10\,\text{kg}$ and $a = 10\,\text{m/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10\,\text{kg})\left(10\frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right) \end{aligned}$$

$$\boxed{F = 100\,\text{N}}$$

b)

Force in lbf for $m = 10$ lbm and $a = 10\,\text{ft/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10\text{ lbm})\left(10\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{\text{lbf}\cdot\text{s}^2}{32.2\text{ lbm}\cdot\text{ft}}\right) \end{aligned}$$

$$\boxed{F = 3.11\,\text{lbf}}$$

c)

Force in newtons for $m = 10$ slug and acceleration is $a = 10\,\text{ft/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10\,\text{slug})\left(10\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right)\left(\frac{4.448\,\text{N}}{\text{lbf}}\right) \end{aligned}$$

$$\boxed{F = 445\,\text{N}}$$

1.12: PROBLEM DEFINITION

Apply the grid method.

Situation:

A cyclist is travelling along a road.
$P = FV$.
$V = 24\,\text{mi/h}$, $F = 5\,\text{lbf}$.

Find:

a) Find power in watts.
b) Find the energy in food calories to ride for 1 hour.

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a)

Power

$$\begin{aligned} P &= FV \\ &= (5\,\text{lbf})\left(\frac{4.448\,\text{N}}{\text{lbf}}\right)(24\text{ mph})\left(\frac{1.0\,\text{m/s}}{2.237\text{ mph}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right) \end{aligned}$$

$$\boxed{P = 239\,\text{W}}$$

b)

Energy

$$\begin{aligned} \Delta E &= P\Delta t \\ &= \left(\frac{239\,\text{J}}{\text{s}}\right)(1\,\text{h})\left(\frac{3600\,\text{s}}{\text{h}}\right)\left(\frac{1.0\text{ calorie (nutritional)}}{4187\,\text{J}}\right) \end{aligned}$$

$$\boxed{\Delta E = 205\text{ calories}}$$

1.13: PROBLEM DEFINITION

Apply the grid method.

Situation:

A pump operates for one year.

$P = 20\,\text{hp}$.

The pump operates for 20 hours/day.

Electricity costs \$0.10/kWh.

Find:

The cost (U.S. dollars) of operating the pump for one year.

PLAN

1. Find energy consumed using $E = Pt$, where P is power and t is time.
2. Find cost using $C = E \times (\$0.1/\text{kWh})$.

SOLUTION

1. Energy Consumed

$$\begin{aligned} E &= Pt \\ &= (20\,\text{hp})\left(\frac{\text{W}}{1.341 \times 10^{-3}\,\text{hp}}\right)\left(\frac{20\,\text{h}}{\text{d}}\right)\left(\frac{365\,\text{d}}{\text{y}}\right) \\ &= 1.09 \times 10^{8}\,\text{W} \cdot \text{h}\left(\frac{\text{kWh}}{1000\,\text{W} \cdot \text{h}}\right) \end{aligned}$$

$$\boxed{E = 1.09 \times 10^{5}\ \text{kWh}}$$

2. Cost

$$\begin{aligned} C &= E(\$0.1/\text{kWh}) \\ &= \left(1.09 \times 10^{8}\ \text{kWh}\right)\left(\frac{\$0.10}{\text{kWh}}\right) \end{aligned}$$

$$\boxed{C = \$10,900}$$