Solutions Manual

The Design of High-Efficiency Turbomachinery and Gas Turbines

Second Edition

David Gordon Wilson • Theodosios Korakianitis

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Special Projects Manager: Barbara A. Murray
Production Editor: Michele Wells
Supplement Cover Manager: Paul Gourhan
Supplement Cover Designer: Liz Nemeth
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PREFACE

This preface is taken from the solutions manual to the first edition. The manual is similar. There are new problems in most chapters, and solutions to these have been added. The treatment of, particularly, compressible-flow analysis has been changed, along with small changes in nomenclature and in the distinction between stagnation and total conditions. The new problems and the existing problems all come from the senior author, so that he/I am to blame for the idiosyncracies of the presentation of the problems and of the solutions. Here, then, is the lightly edited first-edition preface.

Dave Wilson, MIT Cambridge, MA April 1999

Every author presumably believes that her/his work is markedly different – one hopes better – than that of others. A textbook takes so many years produce that it is easy for an author to become insulated from other texts. She/he then makes comparisons with work of the more-or-less distant past. I may be firmly in this category. I have tried to avoid what seems to me to be deficiencies in the academic problems I once had to solve, and to make these problems a little nearer those that I have frequently faced in an engineering career. There are three respects in which some of these problems differ from what might be regarded as the "conventional" academic problem:

(i) frequently there is no one "correct response;
(ii) there are sometimes provided more data, and sometimes less, than are necessary to arrive at a solution;
(iii) the problem statement is often placed first, with the data following, rather than the usual academic practice of first giving much mysterious information, with the question to be answered stated at the end.

Because the text and these problems are aimed at designers, I force students to make choices. Young people who have been brought up on the analysis of problems for which there is normally only one solution, often only one way to arrive at it, and exactly the right amount of information needed, frequently become unsettled and unhappy at the prospect of having to choose velocity-diagram parameters, or all the choices involved in a simple heat-exchanger design, for instances. I make no apology for these types of problems when I have managed to introduce them, even though they are less tidy, and the solutions perhaps less satisfying, than the tight one-answer type of problem. (They also take a little more grading effort on the part of instructors.)

The problems are also non-uniform in difficulty and in the time taken for solving them. Real-life problems also come varied. These problems were devised for two graduate courses at MIT. In each course there were normally eight homework assignments, each of which could take from four to twelve hours. The most time-consuming problems are from this group. Each course also had three quizzes lasting about 100 minutes. The quizzes started with some simple questions that required no calculation and only a sentence or two for a response, taking five or ten minutes. The last quiz problems normally involve calculation and could take about 30 minutes. Use of textbooks was allowed in all cases. The problems are in all three categories: ten-minute, thirty-minute and four-hour challenges.

Three chapters (the brief history, starting and control system principles, and mechanical design consideration) in the main text have no problems because their topics were treated in a non-quantitative manner, suitable for background information. Some chapters, for instance chapters 6 and 12, have rather few problems, because we take the treatment to only an elementary level for which the inclusion of many problems would not be justifiable.

The solutions have been written out mainly by hand at many different times and in different places in the world. The quality is not consistently high, but I hope that the solutions are legible and understandable.

David Gordon Wilson

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Problems

This first chapter has dealt principally with definitions, and with some discussion of the characteristics and capabilities of turbomachinery and gas turbines. Answering questions on definitions does not engender a love for the subject matter. Accordingly, the following questions probe background knowledge. Few readers will have more than a few of the data asked for in the first question. Its purpose is to stimulate a survey of the engineering-society papers in the library, and perhaps the advertisements in some of the engineering magazines. Some of the other questions ask for opinions. Again, the purpose is to stimulate thought and perhaps some reading. There are not necessarily "correct" answers to these questions. August committees of eminent scientists and engineers have been frequently totally wrong when they have tried to forecast the future.

1.1. Complete as much of table P1.1 as possible, from your own knowledge or from library study. In general, every entry will be for a different machine, although in some cases there will be relationships between two figures. For instance, the highest-power steam turbine may not operate at the highest pressure used for steam turbines but may well have the largest low-pressure flow volume. Use numbered references to footnotes to identify your sources.

Table P1.1.

Give the maximum known value of	Steam turbine in generating station	Boiler-feed pump in generating station	Gas expander in gas-turbine engine	Compressor in gas-turbine engine	Axial compressor, any duty per single casing
Power (MW)	1500 (1750?)(a)	50 (a)	340(b)	200	200
Pressure (MPa)	25 (a)	33	3.5	4.0	4.0
Temperature (deg C)	565 (a)	40	1550(c)	845	845
Pressure ratio (max/min)	900	1,000	75(b)	75(b)	75(b)
Temperature ratio (max/min)	2.7	—	7.0	3.6	3.6
Number of stages	32	8	11	24(d)	81 (g)
Low-pressure flow volume (m^3/s)	30,000	1.4	800	410	7650(f)
Mass flow (kg/s)	1400	1400	500	500	7700(f)
Efficiency (percent, which?)	90 polytropic	90 hydraulic	92 polytropic	93 ? polytropic (d)	93 polytropic

Give the maximum known value of	Centrifugal compressor for any duty, per casing	Water pump for pumped-storage system	Pelton water turbine (high-head impulse)	Francis water turbine (medium-head inflow)	Kaplan water turbine (low-head axial-flow)
Power (MW)	39	250	1400 (h)	1000 (j)	170
Pressure (MPa)	50	10	18 (1800 m)	7.3 (734 m)	0.75 (75 m)
Temperature (deg C)	350	—	—	—	—
Pressure ratio (max/min)	500	100	180	73	7.5
Temperature ratio (max/min)	2.0	—	—	—	—
Number of stages	8	2	1	1	1
Low-pressure flow volume (m^3/s)	75	2.5	17	100	200
Mass flow (kg/s)	410	25,000	17,000	100,000	200,000
Efficiency (percent, which?)	87 (polytropic, t-s)	92 (hydraulic)	90 (hydraulic)	93 (hydraulic)	94 (hydraulic)

Problem 1.1, notes for table

a. The largest steam-turbine generating units are at least 1500 MW, possibly 1750 MW. Until the energy "crises" of the 1970s, unit sizes of steam turbines were steadily increasing. The sharply increased energy price reduced demand, and many units on order were canceled. In the mid-1980s combined cycles became popular (gas turbines supplying the exhaust heat to steam generators feeding steam turbines), and these have become preferred over "straight" steam-turbine power plants. Large combined-cycle units are producing over 400 MW in the late 1990s, with efficiencies approaching 60 percent (see chapter 3).

b. The highest compressor pressure ratio I know of is 75:1, in the Brown-Boveri L-GT 10/13 used in an compressed-air storage plant. The air mass flow is 410 kg/s, giving a low-pressure flow volume of about 400 cu m/s. It is an axial-radial compressor of 25 stages. The turbine-expander exhaust volume at 400° C or above would be approximately 760 cu m/s; the turbine has eleven stages. The highest pressure ratio for a gas-turbine engine is probably 40.5 for the GE-36.

c. The highest turbine-inlet temperature for an operating (rather than a research) engine is about 1825 K, possibly higher, for military jets. This gives a temperature ratio with the compressor-inlet temperature, when operating at altitude, approaching 7:1.

d. The highest compressor efficiency, polytropic total-to-static to the diffuser outlet, a low-blade-speed 24-stage axial compressor of 4:1 pressure ratio for the BTH gas turbine in the Shell tanker "Auris", was stated to be almost 0.94 by the designer, C. L. G. Worn. However, in published records I have not been able to find confirmation of an efficiency higher than 0.91. I believe that modifications were made after these publications were produced.

e. Axial-flow turbines should be capable of reaching levels of efficiency at least equal to those of axial compressors of the same size and pressure ratio. However, I cannot quote cases to verify this assumption.

f. Some of the largest gas-moving turbomachines are for research wind tunnels. The large-scale wind tunnel at Moffet Field, NASA-Ames Research Center with a 24.4m by 36.6m section has six single-stage axial fans each of 12.2-m diameter, each driven by a 17-MW electric motor at 180 RPM.

g. Parsons designed and built some axial-flow compressors with as many as 81 stages (see the historical chapter)

h. The highest rating for a Pelton turbine is 1400 MW, a Sulzer-Escher-Wyss unit for Colombia.

j. The highest head used for a Francis turbine is 734 m. These are Sulzer 180-MW units for the Hausling power station in Austria. The highest power rating of a Francis turbine is 1000 MW

For comparison of sizes, the world's biggest Diesel engine is the Sulzer 12RTA84, 42 MW, twelve-cylinder two-stroke running at 95 RPM, weighing 1750 tonnes. The smallest Diesel engines are for model aircraft, and give outputs of under 100 W.

Another comparison outside the scope of the book is the largest controllable-pitch propeller, with a power rating of 20.5 MW (Sulzer).

Problem 1.2 Gas-turbine engines have not reached the power-output levels of the largest steam turbines. Why?

Gas turbines are increasing in output, but the largest (at under 300 MW) is still small compared with the largest steam turbines at over 1500 MW. Gas turbines may reach 1500 MW, especially in combined-cycle form. However, the trend at the end of the 1990s is towards more-distributed power rather than fewer very large generating stations. Two reasons for this trend are the result of gas-turbine characteristics. One is that high efficiencies can be produced by relatively small gas turbines, whereas only the largest steam plants can incorporate all the devices and processes needed to attain high efficiencies. A second reason is that gas turbines require far fewer supervisory personnel than do steam turbines: gas turbines can even run unattended for long periods. The large quantities of natural gas currently available, and the consequent low prices, have greatly helped the conversion of much electrical generation to gas-turbine or combined-cycle systems.

Problem 1.3 Estimate the design power output of the smallest gas-turbine engine produced in the last decade. Why aren't smaller engines made?

There are very small model-aircraft jet engines made, but these are not, perhaps, in the spirit of the question. Small shaft-power gas turbines are in the 20-30-kW range (Solar T-206, 21 kW, or Capstone, a recuperated engine with rotating parts from turbochargers, at 24 kW). The component and overall efficiencies worsen as engines are made smaller, because of increased relative tip clearances, of lower Reynolds numbers, of increased relative roughness, and of the impracticality of providing blade cooling for very small blades. At some power level it is more attractive or more cost-effective to use piston engines even for applications that might seem to be suited for gas turbines.

Problem 1.4 Give your opinion of the two most-promising new applications for gas-turbine engines in the next twenty years, and give reasons for your opinion.

Crystal balls are unreliable. I believe that gas-turbine engines have sufficient advantages over Diesel engines for trucks and buses that they are likely to be used in these applications. Nowadays, many, perhaps most, transitions of this scope are driven by government regulations or taxes, and these could produce a significant incentive. The same is true to a lesser extent for private automobiles, the engines of which have been dominated by the requirements of regulation for twenty-five years.

Problem 1.5 Why is the maximum temperature of steam turbines so much lower than the current turbine-inlet temperature of gas-turbine engines?

Steam-turbine maximum temperatures are limited at present by the corrosive nature of high-temperature steam on superheater tubes. The limit is presently about 566° C, and there is no immediate prospect that I know of (given the increasing prices of high-chromium-cobalt-nickel steels) for this "limit" to be exceeded. The efficiency of steam plants has been decreasing in recent years because of the energy requirements imposed by environmental-pollution limits. Improved methods of removing sulfur should reduce energy costs and allow the thermal efficiency to rise slightly again, but there seems little prospect of steam-plant efficiencies reaching 50 percent while remaining economically competitive.

Problem 1.6 What do you think are the two principal problems preventing gas-turbine engines from having a much wider application?

Up to close to the present time, gas-turbine engines have poorer design-point and much poorer off-design-point efficiencies than Diesel engines, while usually costing more per kilowatt of design power. The situation is changing: simple-cycle engines can have thermal efficiencies of 40%, similar to that of large Diesels, and regenerative engines can have a considerably higher efficiency. If ceramic regenerators and ceramic turbines are developed to be reliable low-cost components, two substantial blocks to the greater use of gas turbines would be removed.

Problem 1.7 Do you think that gas-turbine engines will be used in outboard motor boats by 2010? Why, or why not?

It is most unlikely. Although the higher power levels used in outboard "motors" (strictly they are not "motors" but "engines") are in the range where gas-turbine engines can be competitive (particularly where lightness and compactness are important, as in this application) the salt-spray environment near the air intake in seagoing boats is hostile to turbine-engine survival. An inboard location where the air intake can be situated as remote from spray as possible is much more favorable.

Problem 1.8 For which of the applications in the list below do you think that the gas-turbine engine, as a prime mover, would be:

a. suitable now, and, if so, how would it be used, or in what form?

b. suitable after certain developments have been successfully completed, and if so, which?

c. Unsuitable, and if so, why?

(The list is not repeated here).

When ceramics are developed to enable reliable high-temperature regenerators and turbines to be incorporated, gas turbines should be suitable for highway trucks, city buses, long-distance interurban buses, and automobiles. The automobile application is the least likely of these because of the huge changeover costs, but much, pro and con, depend on governmental regulation.

Governmental regulation also seems likely to play an increasing role in the pollution from motorcycles, snowmobiles, and lawn mowers. The uncontrolled level of emissions is very high. Gas-turbine engines are possible for all three applications, although they would suffer from the effects of small size (as do piston engines) and any applications would be near the limit of what would be possible. They would be likely to be low-pressure-ratio regenerative cycles. The exploitation of geothermal energy and sea-water thermal gradient with depth are unlikely to use Brayton cycles because the temperature differences are too small to give good measurable efficiencies. Solar energy, where mirrors are used to concentrate the radiation on to a target, and nuclear energy from high-temperature reactions are suitable applications for gas-turbine engines.

Problem 1.9 Discuss any present applications of, and future prospects for, vapor-cycle engines using fluids other than water.

Vapor-cycle engines have been frequently proposed as "bottoming" (heat-rejection) cycles for other heat engines (in the way that steam cycles act as bottoming cycles for gas-turbine engines in combined-cycle plants.) ThermoElectron of Waltham MA built and sold vapor-cycle engines for the exhaust systems of Diesel trucks. (These lost their economic worth when price of fuel dropped after the so-called "energy crises" of the 1970s and 1980s.) A major proposal was the Ocean Thermal Energy Cycle (OTEC) in which a fluid, probably ammonia, would be boiled by the warm surface waters of the Gulf of Mexico (for example), the vapor would be expanded through a large single-stage axial turbine, and the exhaust vapor would be condensed by heat exchange with deeper cooler water. This cycle also fell victim to low energy prices.

2.1 Does the stagnation temperature of the working fluid rise or fall in passing through a gas-turbine expander? Why?

The stagnation temperature falls. The steady-flow energy equation is:

$$\frac{\dot{Q}_{in} + \dot{W}_{in} - \dot{Q}_{ex} - \dot{W}_{ex}}{\dot{m}} = \Delta_1^2(\Delta h_o) + \Delta_1^2\left(\frac{gz}{g_c}\right)$$

The flow is normally adiabatic, and $\Delta\left(\frac{gz}{g_c}\right)$ is negligible compared with other changes in a gaseous flow.

Therefore, for positive work, Δh_o must be negative, and ΔT_o

2.2. Does the temperature of the water rise or fall in passing through a water turbine? Why?

The total temperature rises. Transfer of useful work produces no change of water temperature in hydraulic machines. However, dissipation processes convert kinetic energy to thermal energy. These and other frictional losses lead to the increase in temperature.

Total, or stagnation temperature in an incompressible fluid is the same as static temperature: there is no mechanism coupling pressure and temperature when moving water is brought isentropically to rest.

(2) 2.3. By how much, and in which direction, does the temperature of water change when it falls over a waterfall 50 m high? What is the mechanism for this change?

The temperature rises. The steady-flow energy equation is

$$\frac{\dot{Q}-\dot{W}}{\dot{m}} = 0 = \frac{\Delta p_{st}}{\rho} + \Delta u_{st} + \frac{\Delta C^2}{2g_c} + \Delta z \frac{g}{g_c}$$

There is no power extraction, so $\dot{W} = 0$.
There should be negligible heat transfer, so $\dot{Q} \to 0$
We'll assume a stream leaving the waterfall of similar size, and therefore velocity, as the entering stream.
The static pressure will depend on the atmospheric pressure and depth. The atmospheric pressure will differ only through the effects of height, and the difference will be negligible for 50m and for standard air density. So for a streamline at any given depth below the surface, $\Delta p_{st} = 0$

$$\therefore \Delta u_{st} = \text{Specific heat} \times \Delta T = -\Delta z \frac{g}{g_c}$$

$$\Delta T = +\frac{50\text{m} \times 9.81\text{ m/s}^2}{4186.2\text{ J/kg}^\circ\text{K}} = \underline{0.117^\circ\text{K}}$$

(water specific heat = 4186.2 J/kg °K)

As in P 2.2, ~~total~~ stagnation temperature for a liquid is the same as static temperature.

2. 4. Sketch in your qualitative estimates of the variation of stagnation and static enthalpy and pressure through the intercooled compressor shown in figure P2.4. Some end points are shown. (Four lines are required.)

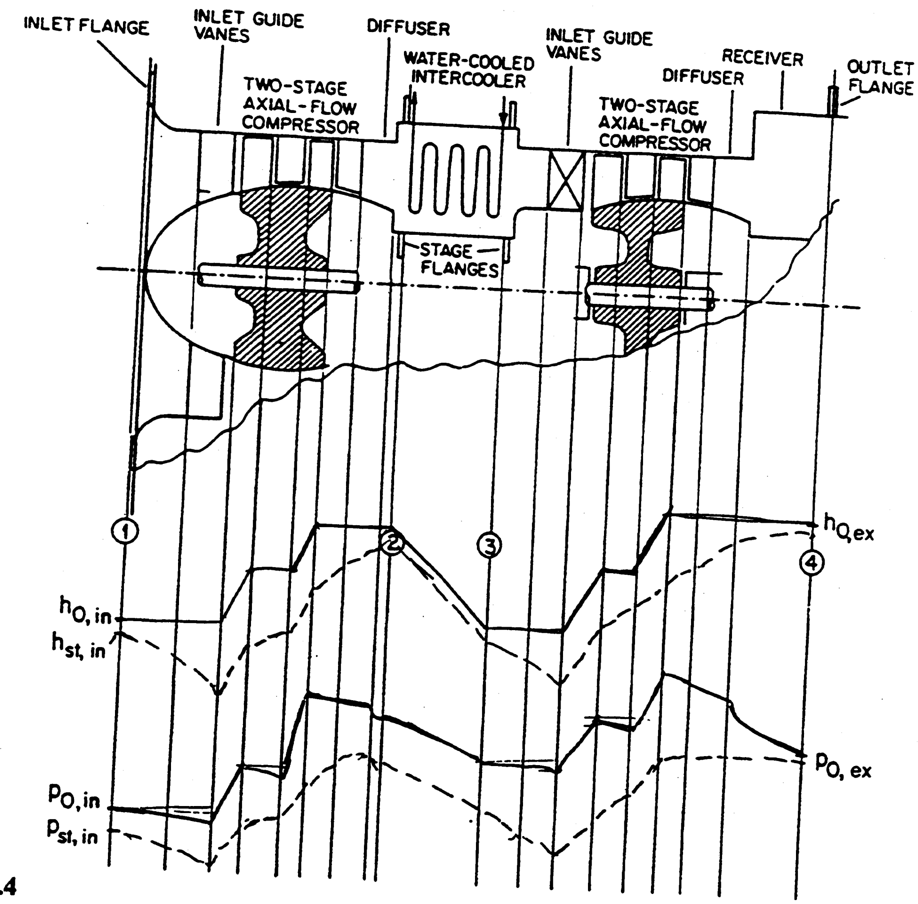

Figure P2.4

Total enthalpy is constant in places where fluid friction causes the total pressure to drop, such as in ducts, diffusers, and sudden enlargements. In a sudden enlargement, the static pressure is more likely to be constant than is the total pressure (whereas the total enthalpy is precisely constant). All the foregoing are adiabatic. In the intercooler, the total enthalpy falls because of heat transfer out of the gas, while the total pressure falls because of fluid friction.

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2.5. For the subsonic axial-flow air expander specified, calculate the stagnation and static pressures and temperatures and the Mach number at the rotor-inlet plane (figure P2.5). Also find the rotor rotational speed and the nozzle-inlet blade height for constant-outer-diameter blading. Sketch the form of the complete turbine expansion on an enthalpy-entropy chart. The axial velocity will be constant at this design point.

Mass flow, $\dot{m} = 2$ kg/s
Nozzle-inlet stagnation temperature, $T_{0,ni} = 400$ °C.
Absolute nozzle-inlet stagnation pressure, $p_{0,ni} = 3$ bars$(= 3 \times 10^5$ N/m$^2)$
Mean diameter, $d_m = 0.25$ m.
Blade height at rotor entry, $l = 0.1 d_m (= 1/2[d_s - d_h])$.
Flow angle at nozzle exit, $\alpha_1 = 70°$ to axial direction.
Drop in stagnation pressure in nozzle, $\Delta p_0 = 0.05$ bar
Rotor peripheral speed at mean diameter, $u_m = 0.5\times$ (component of nozzle outlet velocity, $C_{\theta,1}$).

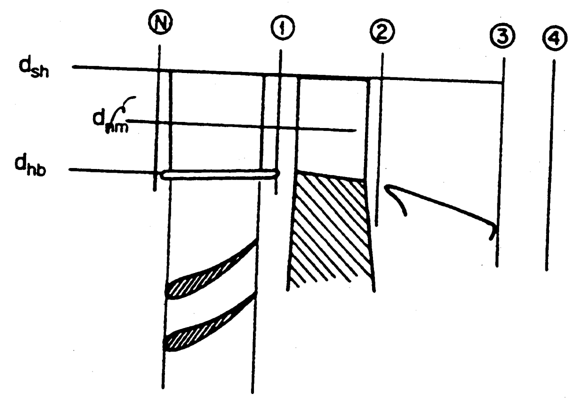


Figure P2.5

SUMMARY

I calculated this in ~~two~~ three ways. One was using compressible-flow relations (CFR) with a constant value of γ of 1.4. The ~~other~~ second used the steady-flow energy equation (SFEE) and mean specific heats ($\bar{C}_p$). The results show close agreement. (See table on the next page). I will give my calculations for the mean-Cp and compressible flow methods only.

A third way uses figures 2.9 & 2.10, and equations 2.61 & 2.62.

P 2·5 cont. (2)

RESULTS

	CFR	SFEE, $\bar{C}_p$
Nozzle-exit total pressure, bars	2.95	2.95
" " static " "	2.62	2.617
" " total temp., K	673	673
" " static " "	650.6	651.7
" " Mach number	0.415	0.421
" inlet blade height, mm	22.88	23.4
Rotor rotational speed, rev./min	7530	7638

CALCULATIONS

Geometry $d_m = 0.25 m \equiv \frac{d_s + d_h}{2}$

$l_1 = 0.025 m \equiv \frac{d_s - d_h}{2}$

$\therefore d_{st} = 0.275 m$

$d_{hb} = 0.225 m$

$l_1 = 0.025 m$

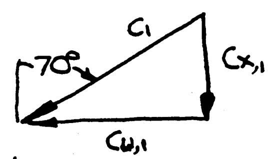
Annulus area, $A_{an,1} = \frac{\pi}{4}\left[0.275^2 - 0.225^2\right] = 0.01963 m^2$

Flow area $A = A_{an,1} \cos 70^\circ = 0.006716 m^2$

Continuity $\dot{m} = \rho_{st,1} A C_1$

$\rho_{st,1} C_1 = \frac{\dot{m}}{A} = 297.82 \frac{kg}{m^2 s}$

Iterations In the table on the next page, we find $\rho_{st,1}$ for various values of C_1, the nozzle-exit velocity.



The last column, for $C_1 = 212.8 m/s$, gives a close-enough match of $\rho_{st,1} C_1$:

P 2.5, cont. (3)

TABLE GIVING SUCCESSIVE TRIALS

C_1	m/s	200	218	213.5	212.8
$C_1^2/(2g_c\bar{C}_p)$	deg K	18.76	22.31	21.40	21.255
$T_{st,1}$	K	654.24	650.69	651.60	651.74
$T_m \equiv (T_{o,1}+T_{st,1})/2$	K	663.62	661.84	662.30	662.37
$\bar{C}_p$ at $\bar{T}_m$, J/kg °K		1065.63	1065.10	1065.20	1065.21
$[(\bar{C}_p/R)-1]$		2.7135	2.7116	2.7120	2.7121
$\frac{\rho_{st,1}}{\rho_{o,1}} = \left(\frac{T_{st,1}}{T_{o,1}}\right)^{(\frac{\bar{C}_p}{R}-1)}$		0.9261	0.9126	0.9161	0.9166
ρ_{st1}	kg/m³	1.4147	1.3948	1.3993	1.4002
$\rho_{st,1}C_1$	kg/m²s	282.94	303.90	298.76	297.96

ROTATIONAL SPEED

$$C_{y,1} = C_1 \sin 70° = 199.97 \text{ m/s}$$
$$u_m = 0.5\, C_{y,1} = 99.983 \text{ m/s}$$
$$N = u_m \times \left(\frac{60}{2\pi}\right) \times \left(\frac{2}{d_m}\right) = 7638.2 \text{ rev/min}$$

Nozzle-inlet blade height

$$C_{x,1} = C_1 \cos 70° = 72.782 \text{ m/s}$$

Guess $\bar{C}_p = 1066.2$ J/kg°K

$$\frac{C_{x1}^2}{2g\bar{C}_p} = 2.48\ °K$$
$$\therefore T_{st,ni} = 670.52 \text{ K}$$
$$T_m = \frac{T_o + T_{st,ni}}{2} = 671.76 \text{ K}$$
$$\bar{C}_p - 1067.4 \text{ J/kg°K (second iteration)}$$
$$\frac{\bar{C}_p}{R} - 1 = 2.7197$$
$$\left(\frac{\rho_{st}}{\rho_o}\right)_{ni} = \ldots \left(\frac{670.52}{673}\right)^{2.7197} = 0.9900$$
$$\rho_{st,ni} = 0.990 \cdot \frac{3 \times 10^5}{286.96 \cdot 673} = 1.5379 \text{ kg/m}^3$$

P.2.5 cont (4)

Annulus area at nozzle inlet

$(A_{an,ni})$ $$A_{aN} = \frac{\dot{m}}{\rho_{st,Ni} C_{x_1}} = 0.01787 m^2 \qquad \left(\frac{?}{.3735 \times 76.782}\right)$$

$$= \frac{\pi}{4}\left(d_{s}^2 - d_{hN,i}^2\right) \qquad d_s = 0.275 m$$

$(d_{hb,ni})$ $d_{hN} = 0.2299 m$

Nozzle blade height at inlet

$$= \frac{0.275 - 0.2299}{2}$$

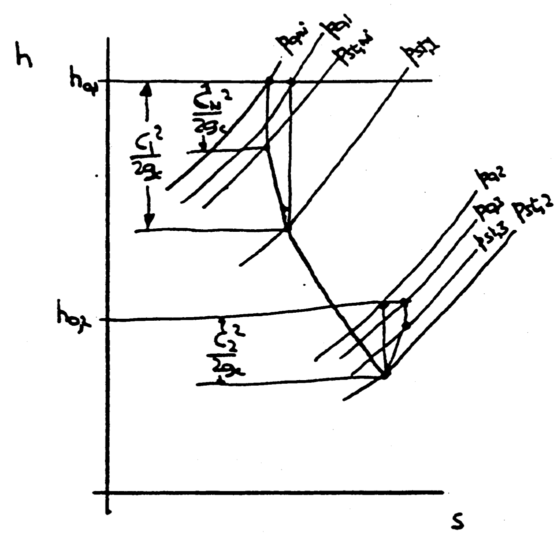
$$= 0.0225 m = \underline{22.5 mm}$$

· Static pressure at nozzle exit

$$\frac{p_{st,1}}{p_{o,1}} = \left[\frac{T_{st,1}}{T_{o1}}\right]^{\frac{\bar{c}_p}{R}} = 0.88696$$

$p_{o1} = 2.95$ bar

$\therefore \underline{p_{st,1} = 2.617 \text{ bar}}$



PROBLEM 2.5 (p.5, added 1989)

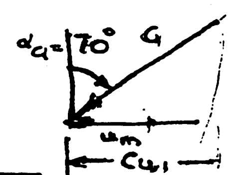
An alternative method of solution for the flow velocity (C_1) using the following curves, introduced in a later version of the text, is as follows.

At nozzle exit, $\dfrac{\dot{m}\sqrt{RT_0}}{Ap_0} = \dfrac{2\sqrt{286.96 \times 673}}{0.006716 \times 2.95 \times 10^5} = 0.4436$

Guessing (Cp/R) = 3.71, one can read M_1 = ~~0.418~~ 0.420 approx. from figure "2.9" (new text). In figure "2.10", $C/\sqrt{g_c R T_0} = 0.48$ $\therefore C_1 = 212.3$ m/s

Another iteration on (Cp/R) and some larger-scale graphs would give a more-accurate solution. But the iterative method given on p.3 is obvious.

David Gordon Wilson, Cambridge, MA June 1989



$\sin 70° = C_{u1}/C_1$

$C_{u1} = C_1 \sin 70°$

$= 199.5$ m/s

$u_m = 99.75$ m/s

$= r_m \omega$

$\omega = \dfrac{99.75}{0.125}$

$= \dfrac{2\pi N}{60}$

$N = \dfrac{60 \times 99.75}{0.125 \times 2\pi}$

$= 7{,}620$ RPM

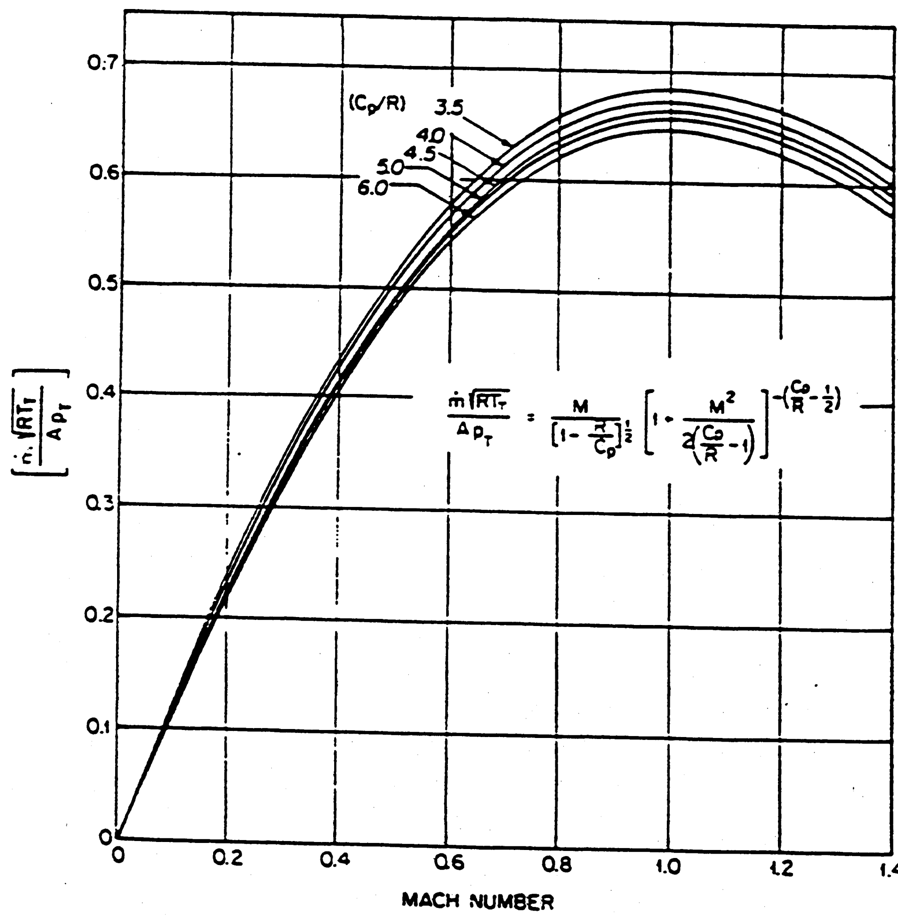


Figure 2.9 Universal flow-function plot.

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2.6. Complete the table of efficiencies, table P2.6, for the turbine expander of problem 2.5, for inlet conditions 1,500 K and 8×10^5 N/m^2. The turbine stagnation enthalpy drop is twice the mean blade speed squared ($\Delta h_0 = 2u_m^2$) and the mean blade speed is 500 m/s. The flow velocity at rotor exit is axial and is 0.728 of the mean blade speed, and it is reduced by 50 percent in the diffuser. The rise in static pressure in the diffuser is 75 percent of the value it would have if the diffuser flow were isentropic. Also calculate the power, in kW, delivered by the blading. (Do not count the "external" losses—bearing friction and so forth—here or in the efficiencies.) The inlet conditions to be used for the efficiencies are the stagnation conditions at plane N.

Table P2.6

Efficiency	Outlet plane 2	3
$\eta_{s.tt}$	0.95	0.929
$\eta_{s.ts}$	0.846	0.902
$\eta_{p.tt}$	0.941	0.916
$\eta_{p.ts}$	0.817	0.884

RESULTS

CALCULATIONS

(See fig P 2.5) (Also see h-s chart in response to P2.5)

Given $T_{0N} = 1500K$

$p_{0N} = 8 \times 10^5$ N/m^2

$d_m = 0.25$ m

$u_m = 500$ m/s

$C_x = 0.728 u_m = 364$ m/s

Temperature drop.

$\Delta h_0 = h_{01} - h_{02} = 2u_m^2$

If one has a program to interpolate enthalpies it should be used here. If not, using mean specific heats may be as accurate, and easier, than interpolating enthalpies.

$T_{01} - T_{02} = 2u_m^2/\bar{C}_{pe}$ Guess $\bar{C}_{pe} = 1040$ J/kg°K

$= 480.77$ °K

$\bar{T}_0 = T_{01} - 480.77/2 = 1259.5$ K for which $\bar{C}_{pe} = 1183$ J/kg°K

P2.6 cont. (2)

Two more iterations lead to $\bar{C}_{pe} = 1196.0$ J/kg°K

$T_{02} = 1081.9$ K Power, $\dot{W} = \dot{m}\bar{C}_{pe}(T_{01} - T_{02}) =$ **1.0 MW**

Total pressure at exit, p_{02}

This is obtained from the given efficiency η_{stt2}

$$\eta_{s,tt,2} = 0.95 = \frac{T_{01} - T_{02}}{T_{01} - T_{02s}}$$

$$T_{02s} = T_{01} - \frac{\Delta T_{012}}{0.95} = 1059.9\text{ K}$$

$$p_{02} = p_{01}\left(\frac{T_{02s}}{T_{01}}\right)^{(\bar{C}_{pe}/R)} = 1.881 \times 10^5\ \text{N/m}^2$$

Static pressure at exit, p_{st2}

$$T_{02} - T_{st2} = \frac{C_{x2}^2}{2g_c\bar{C}_{pe}} = \frac{364^2}{2 \times 1150}$$

using 1150 as a first guess for a local $\bar{C}_{pe}$.

A second iteration leads to $\bar{C}_{pe} = 1151.4$ J/kg°K

and to $T_{st2} = 1024.4$ K

$$\frac{p_{st2}}{p_{02}} = \left(\frac{T_{st2}}{T_{02}}\right)^{\left(\frac{\bar{C}_{pe}}{R}\right)} \qquad p_{st2} = 151{,}065\ \text{N/m}^2$$

Diffuser-outlet pressures p_{03} & p_{st3}

we first calculate the isentropic outlet state. We will use the above values of $\bar{C}_{pe}$ as approximations.

$$\frac{p_{st,3s}}{p_{st,2}} = \left(\frac{T_{st,3s}}{T_{st,2}}\right)^{\frac{\bar{C}_{pe}}{R}} = \left[\frac{T_{02} - \frac{C_{x3}^2}{2g_c\bar{C}_{pe}}}{T_{st2}}\right]^{\frac{\bar{C}_{pe}}{R}}$$

where $C_3 = C_{x3} = 182.0$ m/s

$\therefore\ p_{st,3s} = 178{,}240$ N/m²

$(p_{st,3s} - p_{st2}) = 27{,}175$ "

$(p_{st3} - p_{st2}) = 20{,}381$ " $(= 0.75(p_{st,3s} - p_{st2}))$

$p_{st,3} = 171{,}446$ "

P 2·6, cont (3)

$$\frac{p_{o3}}{p_{st3}} = \left[\frac{T_{o3}}{T_{st3}}\right]^{\frac{\bar{C}_{pe}}{R}} = \left[\frac{1}{1-\frac{C_3^2}{2g_c \bar{C}_{pe} T_{o3}}}\right]^{\frac{\bar{C}_{pe}}{R}} \quad \text{where } T_{o3}=T_{o2}$$

$$p_{o3} = 180{,}905 \ N/m^2$$

All pressures required to enable the efficiencies to be calculated are now found. Each is designated as p_{o2}' in the following equations.

$$\eta_{se} = \frac{\bar{C}_{pe}(T_{o1}-T_{o2})}{\bar{C}_{pe}(T_{o1}-T_{o2,s})} = \frac{(T_{o1}-T_{o2})/T_{o1}}{\left[1-\left(\frac{p_{o2}'}{p_{o1}}\right)^{R/\bar{C}_{pe}}\right]}$$

$$\eta_{pe} = \frac{\bar{C}_{pe}}{R}\,\frac{\ln(T_{o1}/T_{o2})}{\ln(p_{o1}/p_{o2}')}$$

The values obtained are given in the "RESULTS" table.

2.7. Given that an axial-compressor stage is a row of rotor blades which act as diffusers, followed by a row of stator blades which also act as diffusers, explain (from first-law and second-law considerations) why a high Mach number is necessary if the compressor is to give a high pressure ratio.

High Mach numbers are necessary for a compressor stage to produce a relatively high pressure rise because, if the stage were isentropic, with the flow in each row also being isentropic, the pressure ratio between (relative) total pressure and static pressure would be given by the relation :

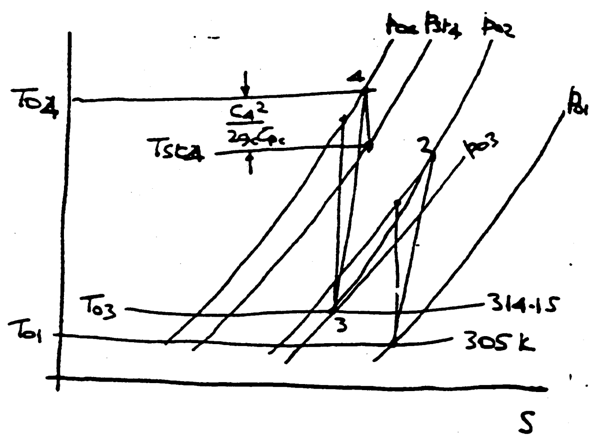
$$\frac{p_0}{p_{st}} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}} = \left[1 + \frac{M^2}{2\left(\frac{c_p}{R} - 1\right)}\right]^{\left(\frac{c_p}{R}\right)}$$

When the aim is to achieve a high pressure ratio per stage, therefore, the Mach number is chosen so that, for subsonic compressors, it is as high as possible without incurring the steep rise in losses that occurs with subsonic blades near unity Mach number. A maximum relative Mach number between 0.9 and 0.95 is generally used. So-called "transonic" blades use higher relative Mach numbers, perhaps 1.6 at the rotor tips, with very carefully shaped blades, accepting an increase in losses as the cost of a substantial increase in stage pressure ratio and a decrease in overall diameter.

2.8. Calculate the isentropic stagnation-to-static internal efficiency of an intercooled air compressor (rather similar to that shown in problem 2.4) from inlet flange to outlet flange. Comment on your findings. Supppose that instead of the actual arrangement shown, two centrifugal stages are used, one upstream and one downstream of the intercooler. Each stage has a stagnation-to-stagnation flange-to-flange pressure ratio of 4 : 1, and each has a stagnation-to-stagnation polytropic efficiency of 88 percent for the same planes (1 to 2 and 3 to 4). The mean velocity at the outlet flange of each stage is 30 m/s. The intercooler has a stagnation-pressure loss of 6 percent of the stagnation pressure of the flow entering it, and it lowers the stagnation temperature to 1.03 times the first-stage-inlet stagnation temperature of 305 K. Use a constant value of $\gamma = 1.4$. Draw a temperature-entropy diagram.

RESULT:

$\eta_{sc,ts,1-4} = 1.012$



APPROACH:

Calculate the actual internal work per stage from the pressure ratio, internal temp-eratures, and poly-tropic efficiency. Then calculate the static pressure at the outlet flange and the ideal isentropic work needed to attain this pressure

CALCULATIONS

If $\gamma = 1.4$, $C_{pc} = R\left(\frac{\gamma}{\gamma-1}\right) = 1004.4$ J/kg °K

$$\frac{T_{04}}{T_{03}} = \frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}\frac{1}{\eta_{pc}}} = 4^{\frac{0.4}{1.4\times0.88}} = 1.5685$$

$(T_{02}-T_{01}) = 305 \times 0.5685 = 173.38$ °K

$(T_{04}-T_{03}) = 1.03 \times 305 \times 0.5685 = 178.58$ °K

Total $\Delta T_0 = 351.96$ °K

" $\Delta h_0 = 353{,}497$ J/kg

---

* Alternatively, use $\bar{C}_{pc} = 1009$ J/kg °K, approximately the mean specific heat during compression.

Then $\frac{T_{T,4}}{T_{T,3}} = \frac{T_{T,2}}{T_{T,1}} = \left(\frac{p_{T,2}}{p_{T,1}}\right)^{\frac{R}{C_p}\cdot\frac{1}{\eta}} = 1.5652$ writing $T_{T,4}$ for T_{04} etc.

$(T_{T,2}-T_{T,1}) = 172.39$ °K

$(T_{T,4}-T_{T,3}) = 177.56$ °K

$\Sigma\Delta T_T = 349.95$ °K

$\Delta h_T = 353{,}103$ J/kg   (other revised values are given in the LH column on p.2)

Alternative calculation

(Using $\bar{C}_{pc} = 1009\,J/kg\,°K$

$(\bar{C}p/R) = 3.5162$

14.992 ok

$T_{st,4} = 491.71\,K$

2.1598

353.74

$\eta_{sc,ts} = 1.0108$)

P 2.8, cont (2)

Outlet static pressure, p_{st4}

$$\left(\frac{p_{st4}}{p_{01}}\right) = \left(\frac{p_{st4}}{p_{04}}\right)\left(\frac{p_{04}}{p_{03}}\right)\left(\frac{p_{03}}{p_{02}}\right)\left(\frac{p_{02}}{p_{01}}\right)$$

$$= \left(\frac{T_{st4}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}} \times 4 \times 0.94 \times 4$$

$$= \left[1 - \frac{30^2}{2 \times 1004.4 \times 492.7}\right]^{3.5} \times 15.04 = 14.992$$

where $T_{04} = 1.03 \times 305 + (T_{04} - T_{03}) = 492.73\,K$

$$\frac{T_{04s}}{T_{01}} = 14.992^{\frac{0.4}{1.4}} = 2.1675$$

$T_{04s} - T_{01} = 305 \times 1.1675 = 356.09\,°K$

$$\eta_{sc,ts,14} = \frac{356.09}{351.96} = 1.012$$

The isentropic efficiency is just over 100% because intercooling reduces the second-stage work enough for the total work to be less than the isentropic work.

The use of the isothermal efficiency would be more appropriate.

$$\eta_{it,ts,14} = RT_{01} \ln\left(\frac{p_{02}'}{p_{01}}\right) / \Sigma \Delta T_0 \times C_{pc}$$

$$= \frac{286.96 \times 305 \times \ln 14.992}{351.96 \times 1004.4}$$

$$= 0.670$$

2.9. Using figure P2.9, calculate the rotor rotational speed, the static temperature and pressure at nozzle exit, and the (absolute) Mach number at that point, of a radial-inward-flow turbine expander of nozzle-outlet diameter (surface 1) of 250 mm. As an approximation, take this as the rotor-inlet diameter also. The nozzle-exit direction of the flow is 75 degrees from the radial direction, and the axial height of the blade passage is 10 percent of the rotor diameter. The turbine nozzles are supplied with 2 kg/s of air at 2 bars stagnation pressure and 125 °C stagnation temperature. The rotor peripheral speed is 90 percent of the tangential velocity of the air at nozzle exit. The stagnation-pressure losses of the flow through the nozzles are small enough to be neglected. Sketch the nozzle expansion on a temperature-entropy diagram.

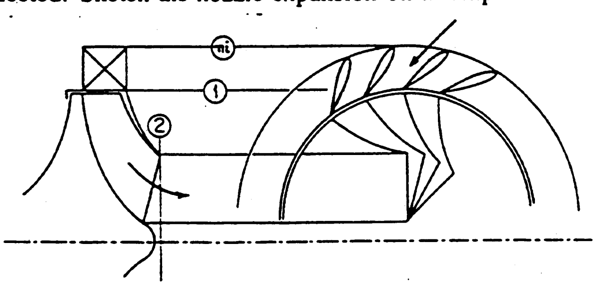
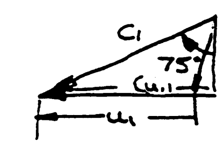
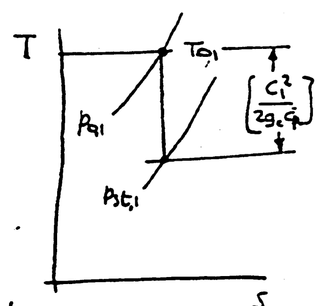


Figure P2.9





APPROACH

The principal part of this problem is the calculation of the velocity necessary through the nozzle throat to pass the required mass flow. This could be solved fairly directly by compressible-flow functions, but only at discrete values of γ not likely to correspond to the actual mean value. We solve the problem here using the steady-flow energy equation, iterating on the mean specific heat and the throat velocity.

P 2.9, cont. (2)

CALCULATIONS

Flow area $A = \pi d_1 b_1 \cos 75^\circ$

$= \pi \times 0.25 \times 0.025 \times \cos 75^\circ$

$= 0.00508 \ m^2$

Continuity $\dot{m} = \rho_{st,1} C_1 A$

$$= \rho_{01} C_1 A \left[\frac{\rho_{st,1}}{\rho_{01}}\right]$$

$$= \frac{p_{01}}{R T_{01}} C_1 A \left[\frac{T_{01} - \frac{C_1^2}{2 g_c C_{pc}}}{T_{01}}\right]^{\left(\frac{C_{pc}}{R} - 1\right)}$$

$$\frac{\dot{m} R T_{01}}{A p_{01}} = C_1 \left[1 - \frac{C_1^2}{2 g_c C_{pc} T_{01}}\right]^{\left(\frac{C_{pc}}{R} - 1\right)}$$

Mean specific heat

Guess $\left(\frac{\rho_{st,1}}{\rho_{01}}\right) = 0.7$

Then $C_1 = 321$ m/s

$(T_{01} - T_{st,1}) = \frac{C_1^2}{2 g_c C_{pc}}$

Guess $\bar{C}_{pc} = 1015$ J/kg°K . Then $T_m = 372.6$ K

$\bar{C}_{pc} = 1011.5$ J/kg°K (second iteration)

The above equation becomes

$$224.74 = C_1 \left[1 - \frac{C_1^2}{2 \times 1011.5 \times 398}\right]^{\frac{1011.5}{286.96} - 1}$$

I solved this with a short calculator program

from which $C_1 = 309.32$ m/s

$\therefore C_{u1} = C_1 \sin 75^\circ = 298.78$ m/s

$u_1 = 0.9 C_{\theta 1} = 268.9$ m/s $= \frac{2\pi N}{60} \frac{d_1}{2}$

$N = \frac{60 u}{0.25\pi} = 20{,}543$ rev/min

P 2.9, cont (3)

Static temperature

$$T_{st,1} = T_{0,1} - \frac{C_1^2}{2 g_c c_{pc}}$$

$$= 125\,C - \frac{309.32^2}{2 \times 1011.5} = \underline{77.7\,C\ (350.7K)}$$

Static pressure

$$\frac{p_{st,1}}{p_{0,1}} = \left(\frac{T_{st,1}}{T_{0,1}}\right)^{\frac{c_p}{R}} = \left(\frac{350.7}{398}\right)^{\frac{1011.5}{286.96}} = 0.6402$$

$$\underline{p_{st,1} = 0.6402 \times 2 = 1.2805 \text{ bars}}$$

Mach number

$$\text{Sonic velocity } a_{st,1} = \sqrt{g_c \gamma R T_{st,1}}$$

$$= \sqrt{\frac{g_c T_{st,1}}{\left(\frac{1}{R} - \frac{1}{c_p}\right)}}$$

$$= 374.83 \text{ m/s}$$

$$\underline{\text{Mach no}} \equiv \frac{C_1}{a_{st,1}} = \frac{309.32}{374.83} = \underline{0.825}$$

Alternative method using the revised flow-function plots 2.9 & 2.10, substituting for fig 2.10 of the original text.

$$\frac{\dot{m}\sqrt{RT_T}}{A\,p_T} = \frac{2\sqrt{286.96 \cdot 398}}{0.00508 \cdot 2.10^5} = 0.6633$$

At an estimated static temperature of 350K, $(c_p/R) = 3.51$

From the revised fig 2.9, $M \approx 0.825$

" " " 2.10, $\dfrac{C_1}{\sqrt{g_c R T_{T,1}}} \approx 0.91$

$\therefore C_1 = 307.5$ m/s

$C_{\theta 1} = C_1 \sin 75^\circ = 297$ m/s

$u = 0.9 \times C_{\theta 1} = 267.3 \text{ m/s} = \left(\frac{2\pi N}{60}\right)\left(\frac{d}{2}\right)$

$\therefore N = \dfrac{604}{0.25\pi} = \underline{20{,}420 \text{ rpm.}}$

2.10. Draw lines approximating the changes of stagnation and static enthalpy and pressure through the compressor test rig shown diagrammatically below (figure P2.10). The centrifugal compressor is driven by a motor and by the energy-recovery turbine, which expands the flow back to atmospheric pressure (as static pressure). A throttle valve is used to produce different back pressures on the compressor.

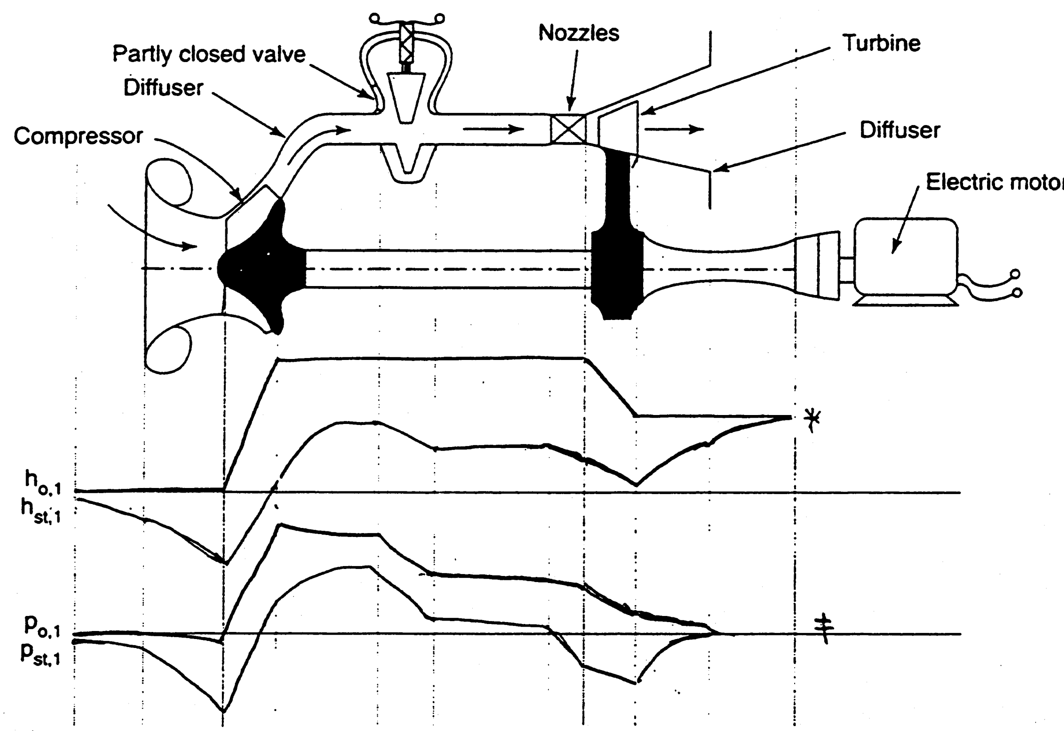


Figure P2.10. Stagnation and static pressure and enthalpy through a compressor test rig

* There is an increase of stagnation & static enthalpy across the unit

‡ The outlet static pressure equals the inlet stagnation pressure.

2.11. Figure P2.11 shows a block diagram of the compressor test rig of problem 2.10. Your task is to draw control volumes around each of the three components, and then one around the three as an assembly, excluding the motor. Calculate and discuss the external energy flows and changes of enthalpy using the steady-flow energy equation. Do this for two different cases. In the first case, the throttle valve is wide open, so that there is no loss of stagnation pressure between the outlet of the compressor diffuser and the inlet of the turbine bell mouth. In the second case, the throttle reduces the turbine expansion ratio from 10 to 5 to 1.

In both cases the compressor takes in 25 kg/s of air at 288 K and compresses it through a stagnation-to-static pressure ratio (to diffuser exit) of 10 : 1 and a polytropic efficiency (same conditions) of 0.80. The turbine takes the flow from the throttle valve and expands it to atmospheric pressure with a stagnation-to-static polytropic efficiency of 0.90. A mean specific heat of 1020 J/(kg · K) and a gas constant of 286.96 J/(kg · K) may be used throughout.

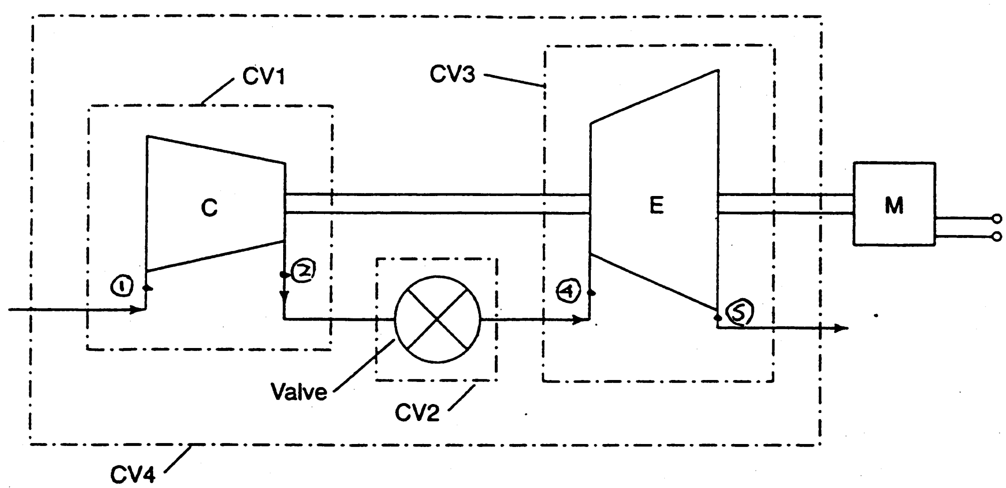


Figure P2.11. Block diagram of compressor test rig

Control-volume 1, compressor

Calculate temperature and enthalpy changes

$$\frac{T_{0,2}}{T_{0,1}} = \left(\frac{p_{0,2}}{p_{0,1}}\right)^{\frac{R}{c_{pc}} \cdot \frac{1}{\eta_{p.c}}} = 10^{\frac{286.96}{1020} \cdot \frac{1}{0.8}} = 2.2473$$

$$(T_{0,2} - T_{0,1}) = T_{0,1}\left(\frac{T_{0,2}}{T_{0,1}} - 1\right) = 359.2\ °K$$

$T_{0,2} = T_{0,4} = 647.21$ K

SFEE (eqn 2.16):

$$\frac{\dot{Q}_{in} + \dot{W}_{in} - \dot{Q}_{ex} - \dot{W}_{ex}}{\dot{m}} = \Delta h_0 = \frac{\dot{W}_{in}}{\dot{m}}$$

$$\dot{W}_{in} = \dot{m}\, c_{p,c}(T_{0,2} - T_{0,1})$$

$$\dot{W}_{in} = 25 \cdot 1020 \cdot 359.2 = 9.160 \text{ MW}$$

CV2, throttle valve

$\dot{Q} = 0 = \dot{W}$ $\therefore \Delta h_0 = 0$

$\therefore T_{0,2} = T_{0,4}$

CV3, turbine expander

Case ① $\dfrac{T_{0,4}}{T_{0,5}} = 10^{\frac{286.96}{1020} \times 0.9} = 2.05396$

Case ② $\dfrac{T_{0,4}}{T_{0,5}} = 5^{\frac{286.96}{1020} \times 0.9} = 1.65383$

	Case ①	Case ②
$\dfrac{T_{0,4}}{T_{0,5}} = T_{0,4}\left[1 - \dfrac{1}{(T_{0,4}/T_{0,5})}\right]$	$= 332.11\ °K$	$255.87\ °K$
$T_{0,5} =$	315.10 K	391.33 K
From SFEE $+\dot{W}_{ex} = \dot{m}\, c_p (T_{0,4} - T_{0,5})$	$= 8.469$ MW	6.525 MW