Problems

Chapter 1

- 1.1 Derive the primary dimension for the MLT system of units for the following:
 - (a) Specific weight
 - (b) Power
 - (c) Flow rate
 - (d) Energy
- 1.2 Show that the equation for the drag force given by

$$F_D = \frac{1}{2} C_D \rho v^2 A$$

is dimensionally homogenous (i.e., the dimension on the left-hand side is the same as the dimension on the right-hand side)

- 1.3 Show that the following equations (a) Bernoulli equation and (b) normal stress in cylindrical coordinate systems, are dimensionally homogenous.
 - (a)

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2$$

(b)

$$\begin{split} \sigma_{rr} &= -p + 2\mu \frac{\partial v_r}{\partial r} \\ \sigma_{\theta\theta} &= -p + 2\mu \left(\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta} + \frac{v_r}{r} \right) \\ \sigma_{zz} &= -p + 2\mu \frac{\partial v_z}{\partial z} \end{split}$$

where s is stress, m is dynamic viscosity, v is velocity and r is radius.

1.4 If *l* is length, *V* is velocity and *n* is kinematic viscosity, which of the following combinations give dimensionless quantities?

$$\frac{Vl^2}{v}$$

(b)
$$\frac{Vl}{v}$$

$$\frac{V}{I}$$

(d)
$$\frac{v}{Vl}$$

1.5 If a pressure loss in a pipe can be expressed by the equation given below, where Δp is pressure loss, V is velocity, l is pipe length and V is flow velocity. Determine the primary dimensions of the constant "A"?

$$\Delta p = A \left(\frac{L}{\sqrt{D}}\right) (V^2)$$

1.6 Determine the dimensional and dimensionless and specific speed for a centrifugal pump with the following design point parameters.

Dimensional

$$N_s \coloneqq rac{1400 \; m{rpm \cdot igl(2800 \; m{gpm}igr)}^{rac{1}{2}}}{igl(90 \; m{ft}igr)^{rac{3}{4}}} = igl(2.535 \cdot 10^3igr) \; m{rpm \cdot rac{igl(m{gpm}igr)^{rac{1}{2}}}{m{ft}^{rac{3}{4}}}$$

Dimensionless

$$N_s \coloneqq rac{N \cdot Q^{rac{1}{2}}}{\left(g \cdot H
ight)^{rac{3}{4}}}$$

$$N_s \coloneqq rac{146.608 \cdot s^{-1} \cdot \left(6.238 \, rac{ft^3}{s}
ight)^{rac{1}{2}}}{\left(32.174 \, rac{ft}{s^2} \cdot 90 \, ft
ight)^{rac{3}{4}}} = 0.928$$

Chapter 2

Problem 2. 1

A pump is operating at a head of 32 m and discharges 3 m³/s of water when rotating at 1200 rpm. Its impeller diameter is 1.2m. A second pump, which is geometrically similar to the first one having an impeller diameter of 1m is operating at 800 rpm. Determine the head and discharge of the second pump. Verify that the specific speeds of the 2 pumps are same.

$$\begin{split} Q_1 &\coloneqq 3 \; \frac{m^3}{s} & N_1 \coloneqq 1200 \; rpm & N_2 \coloneqq 800 \; rpm & D_1 \coloneqq 1 \; m \\ D_2 &\coloneqq 1.2 \; m & H_1 \coloneqq 32 \; m \\ \\ Q_2 &\coloneqq \frac{Q_1 \cdot N_2 \cdot D_2^{\; 3}}{N_1 \cdot D_1^{\; 3}} = 3.456 \; \frac{m^3}{s} \\ H_2 &\coloneqq \frac{\left(N_2 \cdot D_2\right)^2 \cdot H_1}{\left(N_1 \cdot D_1\right)^2} = 20.48 \; m \\ N_{s1} &\coloneqq \frac{N_1 \cdot \sqrt[2]{Q_1}}{\frac{3}{4}} = 2.919 & \text{N1 is in rad/s} \\ &\frac{\left(g \cdot H_1\right)}{\left(g \cdot H_2\right)^2} & \text{N2 is in rad/s} \end{split}$$

Problem 2.2

Calculate the dimensionless specific speed of a turbine with the following parameters $Q=0.15\,\mathrm{m}^3/\mathrm{s}$, $H=20\mathrm{m}$, $N=900\mathrm{rpm}$.

$$Q \coloneqq 0.15 \frac{m^3}{s} \qquad H \coloneqq 20 \ m \qquad N \coloneqq 900 \ rpm \qquad \rho \coloneqq 1000 \frac{kg}{m^3}$$

$$Ns \coloneqq \frac{N \cdot \sqrt[2]{Q}}{\sqrt[3]{4}} = 0.696 \qquad P \coloneqq Q \cdot \rho \cdot g \cdot H = (2.942 \cdot 10^4) \ W$$

$$(g \cdot H)$$

$$Ns \coloneqq \frac{N \cdot \sqrt[2]{P}}{\rho^{\frac{1}{2}} (g \cdot H)^{\frac{5}{4}}} = 0.696$$

Problem 2.3

A turbine operates at a head of 30m, and 260 rpm has a flow rate of 7 m³/s. If the same turbine operates at a head of 15m calculate the speed, flow rate, and brake horsepower of the turbine.

$$H1 := 30 \ m \qquad N1 := 260 \ rpm \qquad Q1 := 7 \ \frac{m^3}{s}$$

$$H1 = 30 \ m \qquad D1 := D2$$

$$H2 := 15 \ m$$

$$\frac{H1}{N1 \cdot D1^3} = \frac{H2}{N2 \cdot D2^3}$$

$$N2 := \sqrt[2]{\frac{H2 \cdot N1^2}{H1}} = 183.848 \ rpm$$

$$\frac{Q1}{N1 \cdot D1^3} = \frac{Q2}{N2 \cdot D2^3}$$

 $Q2 := \left(\frac{N2 \cdot Q1}{N1}\right) = 4.95 \frac{m^3}{s}$