SOLUTIONS MANUAL

Design of MACHINE ELEMENTS

EIGHTH EDITION

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Preface

We are pleased to provide this solutions manual, to accompany the book *Design of Machine Elements*, Eighth Edition, by M. F. Spotts, T. E. Shoup, and L. E. Hornberger. We hope that you will find it to be a useful tool to augment your teaching. Those who have used the previous edition of the text will discover that some of the problems in the text are from the previous verasion and some are entirely new to this edition. Every effort has been made to provide solutions that are error free and show sufficient detail to explain the solution methodology. In some cases, problem solutions call for values to be read from graphs and charts in the book. The accuracy of this process can sometimes be rather low, owing to the small size of the graphs in the text. Thus, your students answers may be slightly different from those presented here, and this reality should be taken into account when evaluating your students work. In some cases, problem solutions call for the use of the spreadsheet modules supplied with the text. These often produce better accuracy than solutions that rely on the use of charts or graphs.

If you discover other errors or have helpful suggestions about the problems and their solutions, we would welcome your comments. You may send your thoughts in writing or by e-mail to the addresses shown below.

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PROBLEMS-Chapter 1

1. The lower ends of the two hangers in Fig. 1-41 were at the same elevation before the loads were applied. The horizontal member is of uniform cross section. Find the value of length *l* if the horizontal member (assumed to be rigid) is horizontal after all loads are acting. *Ans.* $l = 38.3$ in.

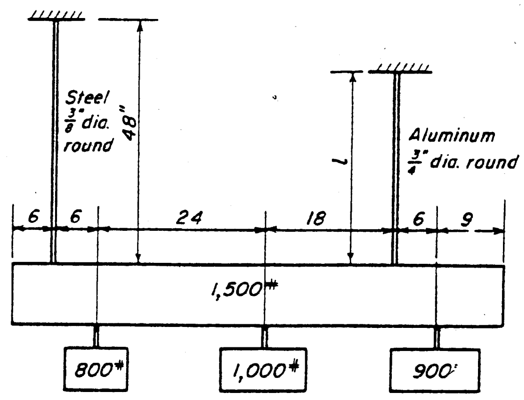
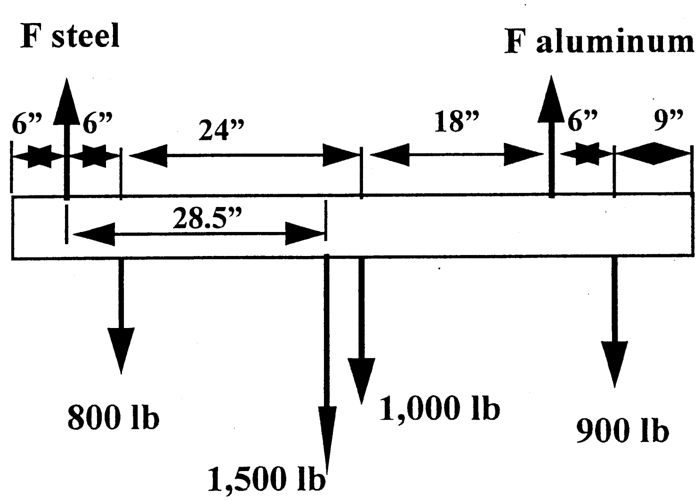


Figure 1-41 Problem 1.

Solution: To solve this problem we will need to draw a free body diagram as shown below:



The moment at the left rod:

$$48F_a = 1{,}500x28.5 + 800x6 + 1{,}000x30 + 900x54$$

$$F_a = \frac{126{,}150}{48} = 2{,}628 \text{ lbf}$$

The moment at the right rod:

$$48F_s = 1{,}500x19.5 + 1{,}000x18 + 800x42 - 900x6$$

$$F_s = \frac{75{,}450}{48} = 1{,}572 \text{ lbf.}$$

For the steel rod:

$$A_s = \frac{\pi}{4}x0.375^2 = 0.11045 \text{ in}^2$$

By table 2-3, for steel E = 30 x 10^6 psi and for aluminum E = 10 x 10^6 psi.

For the aluminum rod:

$$A_a = \frac{\pi}{4}x0.75^2 = 0.4418 \text{ in}^2$$

The deflection of the steel rod by Eq. (4) will be:

$$\delta = \frac{Pl}{AE} = \frac{1{,}572x48}{0.11045x30{,}000{,}000} = 0.02277 \text{ in}$$

We will require the same deflection in the aluminum rod. This will allow us to solve for the length using Eq. (4) once again:

$$l = \frac{\delta AE}{P} = \frac{0.2277x0.4418x10{,}000{,}000}{2{,}628} = 38.3 \text{ in.}$$

2. The bottom member in Fig. 1-42 is of uniform cross section and can be assumed to be rigid. Find the value of the distance x if the lower member is to be horizontal. *Ans.* x = 564 mm.

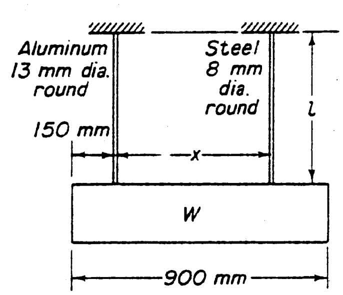
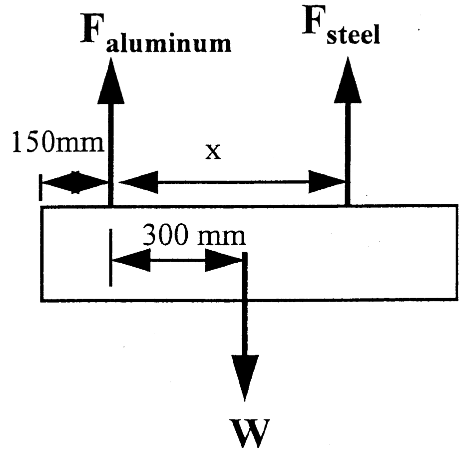


Figure 1-42 Problem 2.

Solution: We must look at a free body diagram of the member:



Summing moments about the left support gives:

$$F_s x = 300W$$

Summing moments about the right support gives:

$$F_a x = (x - 300)W$$

We will require the deflections in the two rods to be the same by equation (4):

$$\delta_a = \delta_s = \frac{F_s l}{A_s E_s} = \frac{F_a l}{A_a E_a}$$

By Table 2-3A we know E_s = 206,900 MPa, and E_a = 69,000 Mpa.

We know that the cross sectional area of a round member will be $\pi d^2/4$.

Substituting the values for the forces, areas and elastic moduli into the deflection equality allows gives:

$$\frac{300Wl}{x(\pi 8^2 / 4)206{,}900} = \frac{(x - 300)Wl}{x(\pi 13^2 / 4)69{,}000}$$

The term $4Wl/(\pi x)$ can be cancelled out from each side. The result is a single equation in "x" of the form:

$264 = (x - 300)$

Thus: *Ans.* $x = 564$ mm.

3. The bottom member in Fig. 1-43 is of uniform cross section and can be assumed to be rigid. Its hinge is frictionless. Find the number of degrees of rotation of the lower member. *Ans.* $\varphi = 0.137°$.

Solution: Summing moments about the pivot point will give the force in the bar:

$$15P = 1{,}800x21$$

Solving this gives $P = 2{,}520$ lbf.

From Table 2-3 for brass $E = 15 \times 10^6$ psi.

The area of the cross section of the round bar will be: $A = \dfrac{\pi d^2}{4} = \dfrac{\pi 0.5^2}{4} = 0.19635 \text{ in}^2$

The length extension of the bar will be determined by Equation (4):

$$\delta = \frac{Pl}{AE} = \frac{2{,}520x42}{0.19635x15{,}000{,}000} = 0.03594 \text{ in.}$$

For relatively small deflections, the relationship between rod extension and the angular movement of the horizontal member will be:

Ans: $\varphi = \dfrac{\delta}{r} = \dfrac{0.03594}{15} = 0.002396 \text{ radians} = 0.002396\dfrac{180}{\pi} = 0.1373 \text{ degrees}$

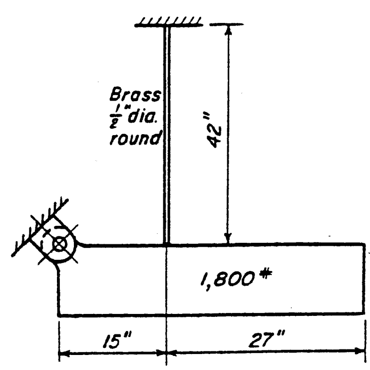


Figure 1-43 Problem 3.

4. The bottom member in Fig. 1-44 is of uniform cross section. Its hinge is frictionless. The rods are of steel. Find the distance point A drops upon attachment of the weight. *Ans.* $\delta = 0.12$ mm.

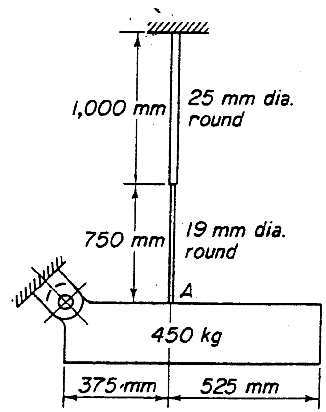


Figure 1-44 Problem 4.

Solution: The load associated with the mass of the horizontal bar will be:

$$Load = 450 \text{ kg x } 9.807 \text{ m / sec}^2 = 4{,}413 \text{ N}$$

We can summ moments about the pivot point to determine for force in the rod:

$375P = 4{,}413 \ x \ 450$ $\qquad\qquad P = 5{,}295 \ N$

From Table 2-3 we know E = 206,900 Mpa.

For the upper rod:

$$A = \frac{\pi d^2}{4} = \frac{\pi 25^2}{4} = 490.9 \text{ mm}^2$$

$$\delta = \frac{Pl}{AE} = \frac{5{,}295x1{,}000}{490.9x206{,}900} = 0.052 \text{ mm}$$

For the lower rod:

$$A = \frac{\pi d^2}{4} = \frac{\pi 19^2}{4} = 283.5 \text{ mm}^2$$

$$\delta = \frac{Pl}{AE} = \frac{5{,}295x750}{283.5x206{,}900} = 0.068 \text{ mm}$$

Thus the total deflection will be the sum of the deflections of each part:

Ans: $\qquad$ *total deflection = 0.52+ 0.068 = 0.120 mm*

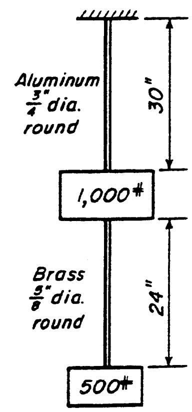


Figure 1-45 Problem 5.

5. In Fig. 1-45, find the drop of the 500-lb weight. *Ans.* $\delta = 0.0126$ in.

Solution: Table 2-3 gives $E_a = 10 \times 10^6$ psi and $E_b = 15 \times 10^6$ psi.

The cross section areas of the two rods will be:

$$A_a = \frac{\pi d^2}{4} = \frac{\pi x 0.75^2}{4} = 0.4418 \text{ in}^2$$

$$A_b = \frac{\pi d^2}{4} = \frac{\pi x 0.625^2}{4} = 0.3068 \text{ in}^2$$

The upper (aluminum) rod sees a weight of 1,500 lbf and will deflect an amount:

$$\delta = \frac{Pl}{AE} = \frac{1{,}500x30}{0.4418x10{,}000{,}000} = 0.01019 \text{ in}$$

The lower (brass) rod sees a weight of 500 lbf and stretches an amount:

$$\delta = \frac{Pl}{AE} = \frac{500x24}{0.3068x15{,}000{,}000} = 0.00261 \text{ in}$$

The total deflection of the lower weight will be the sum of the deflections of the two rods:

Ans: $\delta = 0.01019 + 0.00261 = 0.01280$ in

6. In Fig. 1-46 the lower member is of uniform cross section and can be assumed to be rigid. Find the angular rotation of the lower member in degrees. Ans. $\varphi = 0.029°$.

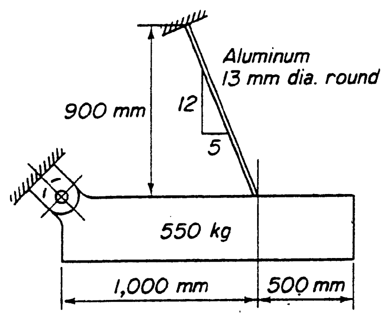
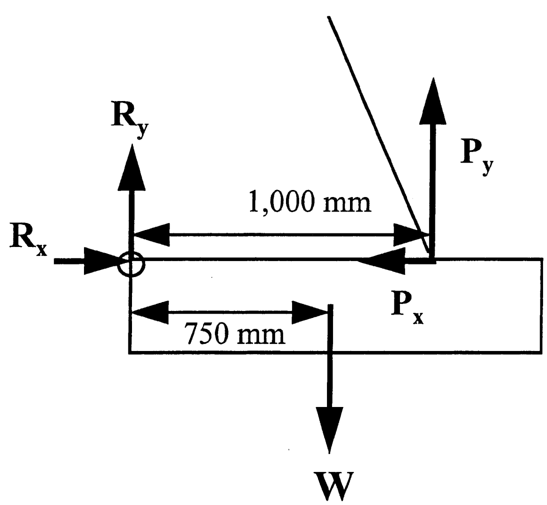


Figure 1-46 Problem 6

Solution: The aluminum bar makes a 5-12-13 right triangle. From Table 2-3a we know that E = 69,000 Mpa. From the geometry of the bar we know that its length can be determined from the relationship:

$$\frac{l}{900} = \frac{13}{12} \qquad \text{thus} \qquad l = 975\ mm$$

We will look at a free body diagram of the members:



The weight of the lower member will be:

$$550 \text{ kg} \times 9.807 = 5{,}393 \text{ N}$$

Taking moments about the pivot point on the left edge of the member gives:

$$(750)(5{,}393) = P_y 1{,}000$$

Thus: $P_y = 975$ mm

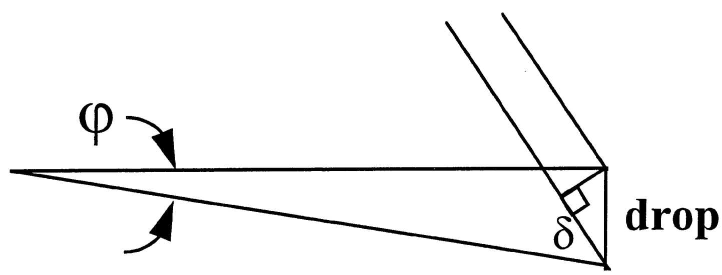
The total axial force on the alunimum rod is:

$$P = \frac{13}{12} P_y = 4{,}382 \text{ N}$$

The axial deflection of the aluminum rod will be:

$$\delta = \frac{Pl}{AE} = \frac{4{,}382x975}{\frac{\pi(13^2)}{4}x69{,}000} = 0.467 \text{ mm}$$

The relationship between this axial deflection and the actual drop of the horizontal member can be determined by looking at the geometry of the bar and the rod:



Thus the total drop will be:

$$drop = 0.467x\frac{13}{12} = 0.505 \text{ mm}$$

Thus the angle of rotation will be:

Ans: $\varphi = \frac{0.505}{1{,}000} = 0.000505 \text{ rad } = 0.029 \text{ degrees}$

7. In Fig.1-47 the lower member is of uniform cross section and can be assumed to be rigid. Find the change in elevation of the left end because of the stretch of the rods. *Ans.* $Drop = 0.0352$ in.

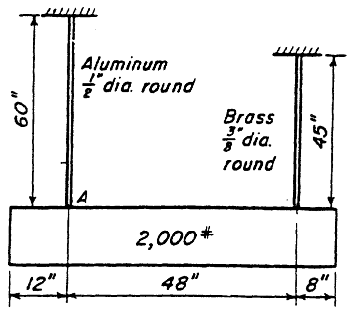
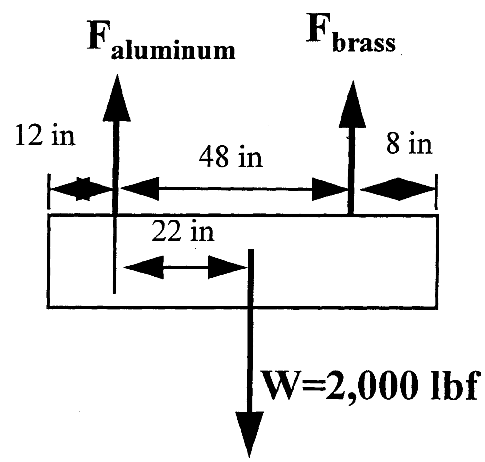


Figure 1-47 Problem 7

Solution: From Table 2-3a E_b = 15,000,000 psi, E_a = 10,000,000 psi.

If we look at a free body diagram of the lower member we see:



Summing moments about the left rod gives:

$$48F_b = 22x2{,}000 \qquad \text{thus} \qquad F_b = 917 \text{ lbf}$$

Summing moments about the right rod gives:

$$48F_a = 26x2{,}000 \qquad \text{thus} \qquad F_a = 1{,}083 \text{ lbf}$$

The deflection at each of the rods will be determined by equation (4):

$$\delta_b = \frac{F_b l_b}{A_b E_b} = \frac{(917)45}{\frac{\pi 0.375^2}{4}(15{,}000{,}000)} = 0.0249 \text{ in}$$

$$\delta_a = \frac{F_a l_a}{A_a E_a} = \frac{(1{,}083)60}{\frac{\pi 0.5^2}{4}(10{,}000{,}000)} = 0.0331 \text{ in}$$

The angular deflection of the platform will be:

$$angle = \frac{0.0331 - 0.0249}{48} = 0.000171 \text{ radians}$$

Thus the drop of the left end will be the deflection of the aluminum rod plus the additional deflection caused by the angle of the platform and the 12 inch length:

Ans: $drop = 0.0331 + 12x0.000171 = 0.0352$ in

8. The members in Fig. 1-48 have a neat fit at the time of assembly. Find the force caused by an increase in temperature of 50°C. Supports are immovable. *Ans.* P = 96,720 N.

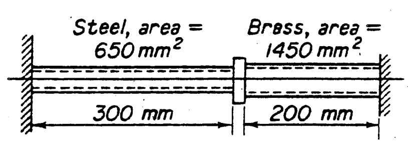


Figure 1-48 Problem 8.

Solution: From Table 2-3a we can find the coefficients of thermal expansion to be:

α_{steel} = 0.000 0117 mm/mm°C
α^{brass} = 0.000 0184 mm/mm°C

The lengthening due to the 50 °C temperature rise will be:

$$\delta_{steel} = 0.0000117x300x50 = 0.1775 \text{ mm}$$

$$\delta_{brass} = 0.0000184x200x50 = 0.1840 \text{ mm}$$

Thus the total lengthening will be: $\delta = 0.1775 + 0.1840 = 0.3615$ mm

Both members will see the same axial force "P." This force will be the amount needed to compress the two members enough to fit the 500 mm length. Thus the shortening due to this force will be:

$$\delta_{steel} = \frac{P l_s}{A_s E_s} = \frac{300P}{650x206{,}900} = 0.000002231P$$

$$\delta_{brass} = \frac{Pl_b}{A_{bs}E_{bs}} = \frac{200P}{1{,}450x103{,}400} = 0.000001334P$$

So the total shortening will be: $\delta = 0.000002231P + 0.000001334P = 0.000003565P$

If we equate this shortening to the total lengthening, we can determine the required load:

Ans: $0.000003565P = 0.3615$ thus $P = 101{,}402$ N

9. After being drawn up snug, the nut in Fig. 1-49 is given one-quarter additional turn. Threads on both bolt and nut are hardened. Find the force in the pipe and bolt. *Ans.* $F = 13{,}750$ lb.

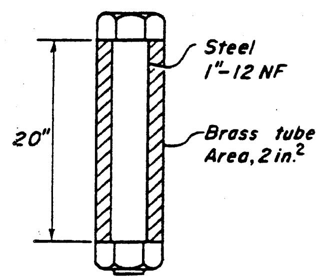


Figure 1-49 Problem 9.

Solution: From Table 2-3 we can find $E_{steel} = 30{,}000{,}000$ psi and $E_{brass} = 15{,}000{,}000$ psi.

The deflection of the tube plus the deflection of the bolt will be:

$$\delta_{tube} + \delta_{bolt} = \frac{1}{4}x\frac{1}{12} = 0.020833 \text{ in.}$$

Since the combination is subjected to the same load, we can determine the deflection of each from equation (4):

Bolt, $$\delta_s = \frac{Pl}{A_sE_s} = \frac{20P}{0.7854x30{,}000{,}000} = 0.000002231P$$

Tube, $$\delta_b = \frac{Pl}{A_bE_b} = \frac{20P}{2x15{,}000{,}000} = 0.000001334P$$

Thus the total deflection will be: $\delta = \delta_s + \delta_b = 0.000002231P + 0.000001334P = 0.0000015155P$

Equating this to the total deflection found previously allows us to determine the load:

Ans: $0.0000015155P = 0.020833$ gives $P = 13{,}750$ lbf

10. The bars in Fig. 1-50 are fitted top and bottom to immovable supports. The bars are of the same material and have the same cross section. Find the force in each bar. *Ans.* Top, 57,140 N; bottom, 42,860 N.

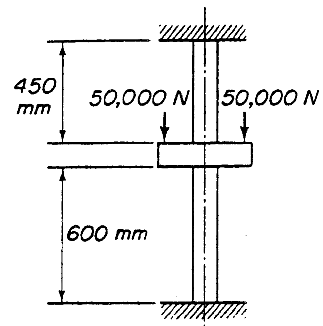


Figure 1-50 Problem 10.

Solution: Let P_t be the force in the top bar and (100,000 - P_t) the force in the bottom bar. The deflection of the top bar should equal the deflection in the bottom bar. Thus:

$$\delta = \frac{P_t 450}{AE} = \frac{(100{,}000 - P_t)600}{AE}$$

Solving this equation gives: $P_t = 57{,}140$ N

The load in the bottom bar will be 100,000 - P_t = 42,860 N

11. In Fig. 1-51 the outer bars are symmetrically placed with respect to the center bar. The top member is rigid and located symmetrically on the supports. Find the load carried by each of the supports. Modulus for the bars is 2,000,000 psi. *Ans.* Center, 5,161.4 lb; outer, 2,419.3 lb.

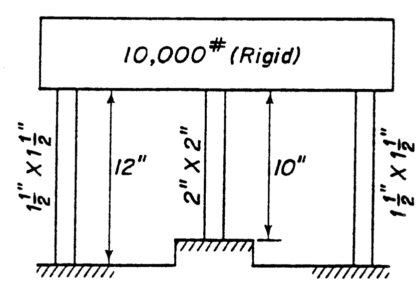


Figure 51 Problem 11

Solution: Let F_o = Force in each outer bar

F_c = Force in center bar.

The deflection of the outer bars will be:

$$\delta_o = \frac{F_o L_o}{A_o E} = \frac{F_o 12}{(1.5x1.5)x2x10^6} = \frac{F_o}{375{,}000}$$

The deflection of the center bars will be:

$$\delta_c = \frac{F_c L_c}{A_c E} = \frac{F_c 10}{(2x2)x2x10^6} = \frac{F_c}{800{,}000}$$

Since the deflection of center bar should equal the deflection of the outer bars, so:

$$\delta_o = \delta_c$$

$$\delta_c = \frac{F_o}{375{,}000} = \delta_c = \frac{F_c}{800{,}000}$$

So $$F_c = \frac{800{,}000}{375{,}000} F_o = 2.133 F_o$$

We know from equilibrium that:

$$2F_o + F_c = 10{,}000$$

Substitute the value of $F_c = 2.133F_o$ into the above equation, we get:

$$2F_o + 2.133F_o = 10{,}000$$
$$4.133F_o = 10{,}000$$
$$F_o = \frac{10{,}000}{4.133} = 2{,}419.6 lbs$$

Solve for F_c:

$$F_c = 2.133F_o = 2.133x2{,}419.6 = 5{,}160.9 lbs$$

Ans: $F_o = 2{,}419.6 lbs$ $\quad$ $F_c = 5{,}160.9 lbs$

12. In Fig.1-52, find the defection at the load. *Ans.* $y = 23.36$ mm.

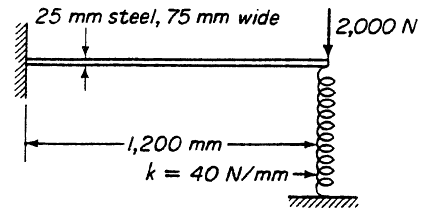


Figure 52 Problem 12

Solution: Calculate the moment of Inertia of the beam:

$$I = \frac{bh^3}{12} = \frac{75x25^3}{12} = 97{,}656.25mm^4$$

Let P_1 = load carried by the spring.
2000 - P_1 = load carried by the beam.

So the deflection at the tip of the beam will be:

$$y_b = \frac{Pl^3}{3EI} = \frac{P_1 1200^3}{3x206{,}900x97{,}656.25} = 0.0285P_1$$

And the deflection of the spring will be:

$$y_s = \frac{F}{k} \text{ where k = spring constant.}$$

$$y_s = \frac{2{,}000 - P_1}{k} = \frac{2{,}000 - P_1}{40} = 50 - 0.025P_1$$

We know that the deflection of the beam must equal the deflection of the spring, so:

$$y_b = y_s$$
$$0.0285P_1 = 50 - 0.025P_1$$
$$P_1 = 934.59N$$

So the deflection at the tip of the beam will be:

$$y = 0.0285P_1 = 0.0285x93y \ = 26.64mm$$

Ans: $y \ = 26.64mm$

13. Because of an error in fabrication, the center strut in Fig. 1-53 was made 0.005 in. shorter than the other two. The members on top and bottom can be considered rigid. Bars are made of the same material and have equal cross sections. $E = 2{,}000{,}000$ psi. Find the load carried by each bar.

Ans. Outer, 14,667 lb; inner, 10,666 lb.

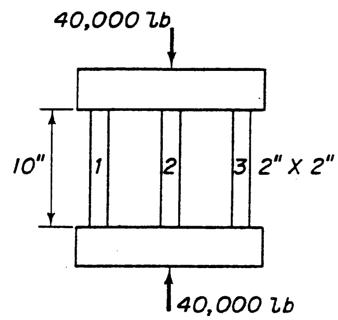


Figure 53 Problem 13

Solution:

Let δ_1 = deflection in outer bar.
Since center bar is 0.005 in shorter than the outer bars,
so $\delta_1 - 0.005$ = deflection in center bar.
Let P_1 = force in outer bars,
And $P_2 = 40{,}000 - 2P_1$ = force in center bar.
The deflection of outer bars is:

$$\delta_1 = \frac{P_1 l}{AE}$$

and the deflection in the center bar is:

$$\delta_1 - 0.005 = \frac{P_1 l}{AE} - 0.005 = \frac{(40{,}000 - 2P_1)l}{AE}$$

Expand the above equation, we have:

$$3P_1 l = 0.005AE + 40{,}000l$$

$$P_1 = \frac{0.005AE + 40{,}000l}{3l} = \frac{0.005*(2x2)*2{,}000{,}000 + 40{,}000*10}{3*10} = 14{,}667lbs$$

So force in the center bar is:

$$P_2 = 40{,}000 - 2P_1 = 40{,}000 - 2*14{,}667 = 10{,}666lbs$$

Ans: $P_1 = 14{,}667lbs$ $\quad P_2 = 10{,}666lbs$

14. In Fig. 1-54 the three bars are of the same material and have equal cross sections and lengths. Find the force in each bar. *Ans.* P_1 = 200,000 N; P_2 = 500,000 N; P_3 = 700,000 N.

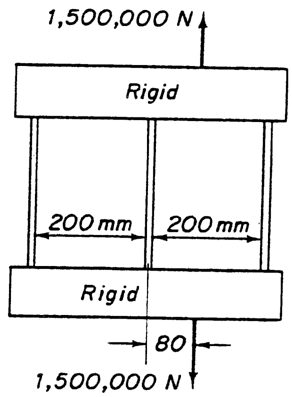


Figure 54 Problem 14

Solution:

Let P_1 = load in bar 1

P_2 = load in bar 2

P_3 = load in bar 3

Due to equilibrium condition,

Summation of forces in all 3 bars should equal to the external force.

$$P_1 + P_2 + P_3 = 1{,}500{,}000 \qquad (a)$$

Summation of moment at applied load location should equal to zero.

$$280P_1 + 80P_2 - 120P_3 = 0$$

Or $$7P_1 + 2P_2 - 3P_3 = 0 \qquad (b)$$

Since 3 bars always stay attached to the rigid blocks at both ends, the deflection of the bars will have the following relationship:

$$\delta_1 - \delta_2 = \delta_2 - \delta_3$$

Or $$\delta_1 - 2\delta_2 + \delta_3 = 0$$

This equation can be written as:

$$\frac{P_1 l}{AE} - 2\frac{P_2 l}{AE} + \frac{P_3 l}{AE} = 0$$

Or $$P_1 - 2P_2 + P_3 = 0 \qquad (c)$$

From equation (a) and (c), we have

$$P_1 + P_3 = 1{,}500{,}000 - P_2 = 2P_2$$

$$P_2 = \frac{1{,}500{,}000}{3} = 500{,}000 lbs$$

Insert P_2 into equation (a), we have

$$P_1 + P_3 = 1{,}500{,}000 - 500{,}000$$

So $$P_1 = 1{,}000{,}000 - P_3 \qquad (d)$$

Substitute P_1 and P_2 into (b):

$$7(1{,}000{,}000 - P_3) + 2 * 500{,}000 - 3P_3 = 0$$

$$P_3 = \frac{7{,}000{,}000 + 1{,}000{,}000}{10} = 800{,}000 lbs$$

Substitute P_3 into equation (d),

$$P_1 = 1{,}000{,}000 - 800{,}000 = 200{,}000 lbs$$

Ans:

$P_1 = 500{,}000 lbs$

$P_2 = 200{,}000 lbs$

$P_3 = 800{,}000 lbs$

15. Find the distance x in Fig.1-55 that causes the 1,000-lb rigid weight to remain level if the lower ends of the hanger are at the same elevation before the weight is applied.

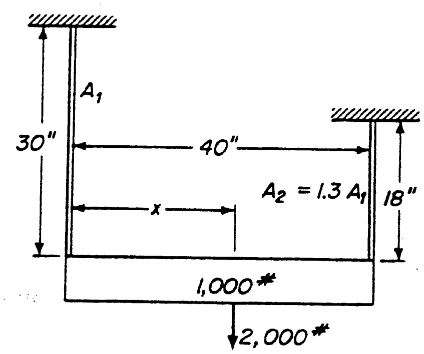


Figure 55 Problem 15

Solution:

Let P_1 = force in left bar caused by 2000lbs.

P_2 = force in right bar caused by 2000lbs.

So $$P_1 = \frac{2000(40 - x)}{40} = 2000 - 50x$$

$$P_2 = \frac{2000x}{40} = 50x$$

Total force in bars are:

$$P_1 = \frac{1000}{2} + 2000 - 50x = 500 + 2000 - 50x$$

$$P_2 = \frac{1000}{2} + 50x = 500 + 50x$$

Let δ_1 = deflection in left bar.

δ_2 = deflection in right bar.

$$\delta_1 = \frac{(500 + 2000 - 50x)l_1}{A_1 E_1}$$

$$\delta_2 = \frac{(500 + 50x)l_2}{A_2 E_2}$$

We know that $\delta_1 = \delta_2$, $A_2 = 1.3\ A_1$, and $E_1 = 2E_2$

So, $$\delta_1 = \frac{(500 + 2000 - 50x)l_1}{A_1 E_1} = \delta_2 = \frac{(500 + 50x)l_2}{A_2 E_2}$$

$$\frac{(500 + 2000 - 50x)30}{A_1 E_1} = \frac{(500 + 50x)18}{1.3 A_1 (0.5) E_1}$$

75,000-1,500x = 13,486+1385x

2,885x = 61,154

Ans: x = 21.2 inches

16. Find the force in each bar in Fig. I-53. $E_1 = 2E_2$. The 2,300-kg mass can be considered rigid.
Ans. F_1 = 7,200 N; F_2 = 6,480 N.

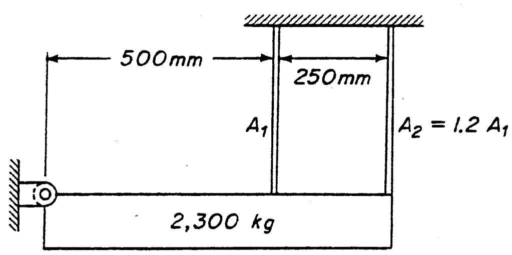
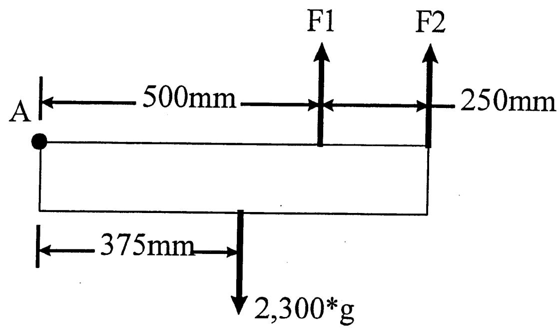


Figure 56 Problem 16

Solution:



Let δ_1 = deflection in left bar.

δ_2 = deflection in right bar.

Since the block is rigid, we know:

$$\frac{\delta_1}{500} = \frac{\delta_2}{750}$$

$$\delta_2 = \frac{750}{500}\delta_1 = 1.5\delta_1$$

$$\delta_1 = \frac{F_1 l_1}{A_1 E_1}$$

$$\delta_2 = \frac{F_2 l_2}{A_2 E_2}$$

$$\frac{F_2 l_2}{A_2 E_2} = 1.5\frac{F_1 l_1}{A_1 E_1}$$

We know:

$$A_2 = 1.2A_1$$

$$E_1 = 2E_2$$

So we have:

$$\frac{F_2 l_2}{1.2 A_1 E_2} = 1.5\frac{F_1 l_1}{A_1 2E_2}$$

$$F_2 = 0.9F_1$$

Calculate load of the block:

$$W = 2{,}300g = 2{,}300 * 9.8066 = 22{,}555N$$

Summation of moment about point "A":

$$500F_1 + 750F_2 - 375 * 22{,}555 = 0$$

$$500F_1 + 750 * 0.9F_1 - 375 * 22{,}555 = 0$$

$$F_1 = 7{,}198N$$

$$F_2 = 0.9F_1 = 0.9 * 7{,}198$$

$$F_2 = 6{,}479N$$

Ans:

$$F_1 = 7{,}198N$$
$$F_2 = 6{,}479N$$

17. The rigid beam in Fig. 1-57 was level before the load was applied. Find the force in each hanger.
Ans. Steel, 10,210 lb; aluminum, 7,660 lb.

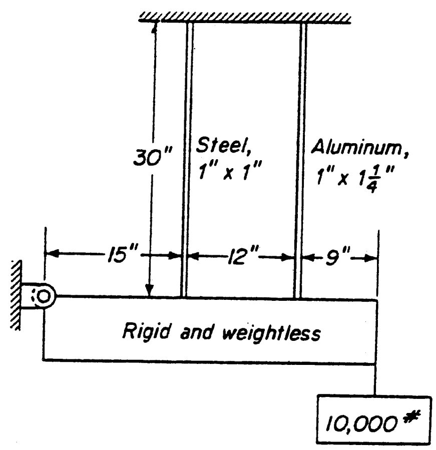


Figure 57 Problem 17.

Solution:
If the rigid beam moves down 1",
Al rod would stretch:

$$\frac{\delta_{Al}}{1} = \frac{27}{36}$$

So, $$\delta_{AL} = \frac{27}{36} = \frac{3}{4} = 0.75$$

The force in the Aluminum bar that moves the beam down 1" is:

$$P_{Al} = \frac{\delta_{Al} AE}{l} = \frac{0.75 * 1.25 * 10^6}{30} = 312{,}500 lbs$$

Steel rod would stretch:

$$\frac{\delta_{St}}{1} = \frac{15}{36}$$

So, $$\delta_{St} = \frac{15}{36} = \frac{5}{12}$$

The force in the Steel bar that moves the beam down 1" is:

$$P_{St} = \frac{\delta_{St} AE}{l} = \frac{5}{12} * \frac{1 * 30 * 10^6}{30} = 416{,}667 lbs$$

The forces at 1" weight,

$$P_{Al} * \delta_{Al} = 312{,}500 * \frac{3}{4} = 234{,}375 in * lbs$$

$$P_{St} * \delta_{St} = 416{,}667 * \frac{5}{12} = 173{,}611 in * lbs$$

Total Force at 1" weight is:

$$P_{Al} + P_{St} = 234{,}375 + 173{,}611 = 407{,}986 in * lbs$$

The actual deflection can be calculated as following:

$$\delta = \frac{10{,}000}{407{,}986} = 0.02451 in$$

Force in Aluminum beam that create a deflection of 0.02451 in,

$$F_{Al} = \frac{P_{Al} * \delta_{Al} * \delta}{A_{Al}} = \frac{234{,}375 * 0.02451}{1.25} = 4{,}595 lbs$$

Force in Steel beam that create an deflection of 0.02451 in,

$$F_{St} = \frac{P_{St} * \delta_{St} * \delta}{A_{St}} = \frac{407{,}986 * 0.02451}{1} = 10{,}213 lbs$$

Ans: $F_{Al} = 4{,}595 lbs$

$F_{St} = 10{,}213 lbs$

18. The bars in Fig. I-58 are of the same material and have equal cross-sectional areas. There is no stress in the bars before the load is applied. Find the load carried by each bar.

Ans. Outer, 19,350 N; inner, 32,980 N.

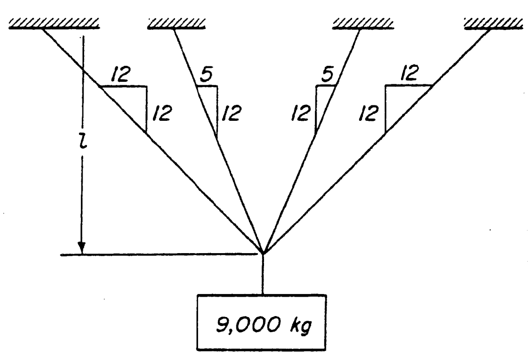
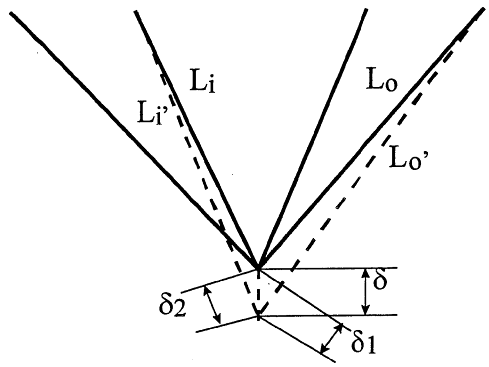


Figure 58 Problem 18.

Solution:



Let,

δ = vertical deflection

δ_1 = deflection in the outer bar

δ_2 = deflection in the inner bar

P_1 = Force in outer bar

P_2 = Force in inner bar

We know,

$$(L_o^{'})^2 = l^2 + (l+\delta)^2$$

$$(L_o + \delta_1)^2 = l^2 + (l+\delta)^2$$

$$L_o = 1.414l$$

$$(1.414l + \delta_1)^2 = l^2 + (l+\delta)^2$$

Assume that $\delta^2 = \delta_1^2$

We get $\delta_1 = 0.707\delta$

Apply the same approach:

$$(L_i^{'})^2 = [l\tan(22.62)]^2 + (l+\delta)^2$$

$$(L_i + \delta_2)^2 = 0.1736l^2 + (l+\delta)^2$$

$$L_i = \frac{l}{\cos(22.62)} = 1.083l$$

$$(1.0833l + \delta_2)^2 = 0.17361l^2 + (l+\delta)^2$$

$$1.1735l^2 + 2.167l\delta_2 + \delta_2^{\,2} = 0.1736l^2 + l^2 + 2l\delta + \delta^2$$

Assume that $\delta^2 = \delta_2^{\,2}$

$$\delta_2 = \frac{2}{2.167}\delta = 0.923\delta$$

Calculate forces in each bar

$$P_1 = \frac{\delta_1 AE}{L_o} = \frac{0.707\delta AE}{1.414l} = 0.5\frac{\delta AE}{l}$$

Vertical component is:

$$P_{1_{ver}} = (0.5\frac{\delta AE}{l}) * \cos(45) = 0.3536\frac{\delta AE}{l}$$

Calculate forces in inner bar

$$P_2 = \frac{\delta_2 AE}{L_i} = \frac{0.923\delta AE}{1.0833l} = 0.852\frac{\delta AE}{l}$$

Vertical component is:

$$P_{2ver} = (0.852\frac{\delta AE}{l}) * \cos(22.62) = 0.786\frac{\delta AE}{l}$$

Total vertical component is:

$$P_{1ver} + P_{2ver} = 900 * g = 0.5 * 9000 * 9.81 = 44{,}130$$

$$0.3536\frac{\delta AE}{l} + 0.786\frac{\delta AE}{l} = 44{,}130$$

$$\delta = \frac{44{,}130}{1.14}\frac{l}{AE} = 38{,}707\frac{l}{AE}$$

Find P_1 and P_2

$$P_1 = 0.5 * 38{,}707\frac{l}{AE}\frac{AE}{l} = 19{,}355N$$

$$P_2 = 0.852 * 38{,}707\frac{l}{AE}\frac{AE}{l} = 32{,}978N$$

Ans:

$$P_1 = 19{,}355N$$
$$P_2 = 32{,}978N$$

19. The bars in Fig.1-59 have the same cross-sectional area. There is no stress in the bars before the load is applied. Each bar is 0.5 in. square.

(a) Find the force in each bar.

(b) Find the force in each bar if the temperature drops 100oF.

Ans. (a) Steel, 8,340 lb, brass, 5,560 lb;
(b) Steel, 8,100 lb, brass, 5,970 lb.

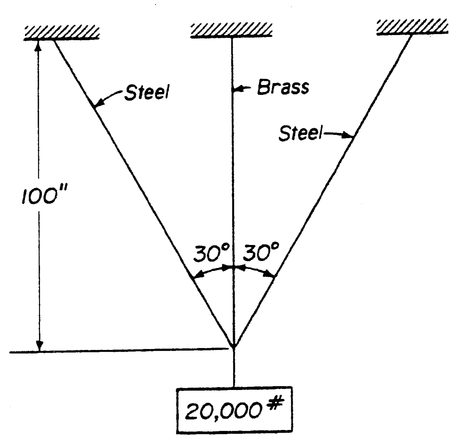
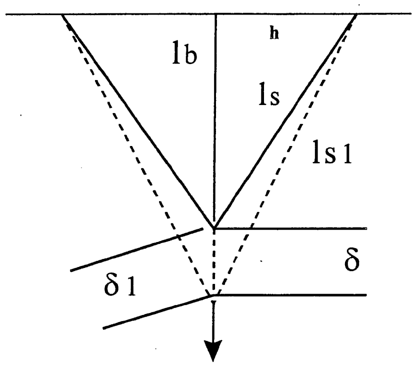


Figure 59 Problem 19

Solution:



a. Let,

δ = vertical deflection

δ_1 = deflection in the bar

We know,

$$(l_{s1})^2 = h^2 + (l_b + \delta)^2$$

$$(l_s + \delta_1)^2 = h^2 + (l_b + \delta)^2 = l_s^2(\tan 30)^2 + l_b^2 + 2l_b\delta + \delta^2$$

$$l_s^2 + 2l_s\delta_1 + \delta_1^2 = l_s^2(\tan 30)^2 + l_b^2 + 2l_b\delta + \delta^2$$

$$l_b = l_s \cos 30$$

$$l_s^2 + 2l_s\delta_1 + \delta_1^2 = l_s^2(\tan 30)^2 + l_s^2(\cos 30)^2 + 2l_s\delta\cos 30 + \delta^2$$

Assume that $\delta^2 = \delta_1^2$

We get

$$100^2 + 2*100\delta_1 = 100^2(\tan 30)^2 + 100^2(\cos 30)^2 + 2*100\delta\cos 30$$

$$\delta_1 \approx \delta\cos 30 = .866\delta$$

For steel bar:

$$P_S = \frac{\delta_S AE}{l_s} = \frac{0.866\delta AE}{l_s} = \frac{.0.866\delta A 30{,}000{,}000}{100/\cos 30} = 225{,}000\delta A$$

The vertical component of force in steel bar is:

$$P_{S_v} = P_s * \cos 30 = 225{,}000\delta A * \cos 30 = 0.866 * 225{,}00\delta A = 194{,}850\delta A$$

For brass bar:

$$P_b = \frac{\delta_b AE}{l_b} = \frac{\delta AE}{l_s} = \frac{\delta A 15{,}000{,}000}{100} = 150{,}000\delta A$$

Summation of forces in vertical direction:

$$2P_{S_v} + P_b = 20{,}000$$

$$2 * 194{,}850 A\delta + 150{,}000 A\delta = 20{,}000$$

$$A\delta = \frac{20{,}000}{539{,}700} = 0.03706$$

Use this to find forces in the bars:

$$P_s = 225{,}000\delta A = 225{,}000 * 0.03706 = 8{,}338 lbs$$

$$P_b = 150{,}000 A\delta = 150{,}000 * 0.03706 = 5{,}559 lbs$$