

Chapter 1

1-1 $d_1 + d_2 = \sqrt{2(100)} + \sqrt{2(800)} = \underline{54.14 \text{ miles}}$

1-2 Using (1-1),

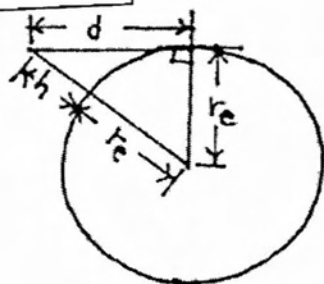
$$d_1 = \sqrt{2h_1} = \sqrt{2(1000)} \quad , \quad d_2 = \sqrt{2h_2}$$

$$d_1 + d_2 = 55 \text{ miles} \Rightarrow \sqrt{2(1000)} + \sqrt{2h_2} = 55 \text{ miles}$$

$$\Rightarrow h_2 = \frac{(55 - \sqrt{2(1000)})^2}{2} = \underline{52.8 \text{ ft}}$$



1-3



$$(h + r_e)^2 = d^2 + r_e^2$$

$$\Rightarrow h^2 + 2hr_e = d^2 \text{ where } h \ll r_e$$

$$\Rightarrow d^2 \approx 2hr_e \text{ and } r_e = \frac{4}{3}(3960 \text{ miles}) = 5280 \text{ miles}$$

Let h = antenna height in feet, and d in miles.

$$\Rightarrow d^2 (\text{miles})^2 = 2h (\text{feet}) \left(\frac{1 \text{ mile}}{5280 \text{ feet}} \right) (5280 \text{ miles})$$

$$\Rightarrow d^2 = 2h \text{ or } \underline{d = \sqrt{2h}}$$

1-4

$$2d = 30 \text{ miles} = 2\sqrt{2h} \Rightarrow 4(2h) = (30)^2$$

$$\Rightarrow h = (30)^2 / 8 = \underline{112.5 \text{ ft}}$$

1-5

$$d_1 = 2h = \sqrt{2(100)} = 14.14 \text{ m} \quad d_2 = 2h = \sqrt{2(4)} = 2.83 \text{ miles}$$

$$\text{Total radius of coverage} = d_1 + d_2 = 14.14 + 2.83 = \underline{16.97 \text{ miles}}$$

1-6

$$\{m_i\} = \{-1.0, 0.0, 3.0, 4.0\} \quad i=1, 4; \quad P_1=P_2=0.2; \quad P_3=P_4=0.3$$

$$\Rightarrow I_1 = I_2 = \frac{-\ln(0.2)}{\ln 2} = 2.322 \text{ bits}, \quad I_3 = I_4 = \frac{-\ln(0.3)}{\ln 2} = 1.737 \text{ bits}$$

$$H = \sum_{j=1}^M P_j I_j = 2[(0.2)(2.322) + (0.3)(1.737)] = \underline{1.971 \text{ bits}}$$

1-7

$$I = \log_2(x) \Rightarrow 2^I = x \quad \text{Thus, } \log_2(2^I) = \log_2(x)$$

$$\Rightarrow I \log_2(2) = \log_2(x) \Rightarrow I = \frac{\log_2(x)}{\log_2(2)} = \log_2(x)$$

1-8

$$p_j = p = \frac{1}{M}; \quad j=1, m$$

$$Eq. (1-3) \Rightarrow H = \sum_{j=1}^M p_j I_j = \sum_{j=1}^M p_j \log_2 \left(\frac{1}{p_j} \right)$$

or

$$H = \sum_{j=1}^M p \log_2 \left(\frac{1}{p} \right) = M \left(\frac{1}{M} \right) \log_2 \left(\frac{1}{p} \right) = \log_2 \left(\frac{1}{p} \right)$$

1-9

Let p_1 = prob. of sending a binary 1

p_2 = prob. of sending a binary 0 = $1 - p_1$

$$(a) \quad H = \sum_{i=1}^2 p_i I_i = p_1 \log_2 \left(\frac{1}{p_1} \right) + (1 - p_1) \log_2 \left(\frac{1}{1 - p_1} \right)$$

$$H = \frac{1}{\ln 2} \left[-p_1 \ln(p_1) - (1 - p_1) \ln(1 - p_1) \right]$$

$$\frac{\partial H}{\partial p_1} = 0 \Rightarrow -(\ln p_1 + 1) - ((-1) \ln(1 - p_1) + \frac{1 - p_1}{1 - p_1} (-1)) = 0$$

$$\Rightarrow -\ln p_1 - 1 + \ln(1 - p_1) + 1 = 0$$

$$\text{or } \ln \left(\frac{1 - p_1}{p_1} \right) = 0 = \ln 1$$

$$\text{Thus } \frac{1}{p_1} - 1 = 1 \Rightarrow \underline{p_1 = \frac{1}{2} = p_2}$$

$$(b) \quad H_{\max} = \frac{1}{2} \log_2 2 + \left(1 - \frac{1}{2} \right) \log_2 2 = \underline{\underline{1 \text{ bit}}}$$

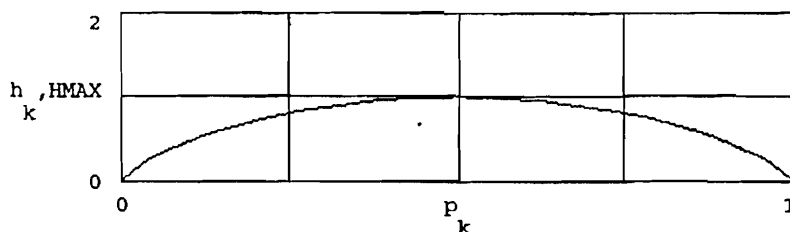
Math CAD Solution

LET p = The probability for sending a binary 1, then the probability for sending a binary 0 is $(1 - p)$. From the entropy formula for $H(p)$, we can draw the figure of $H(p)$, and from this figure, we can find the maximum entropy and the p .

$$H(p) = (p \ln(p) + (1 - p) \ln(1 - p)) / (-\ln(2))$$

$$k \equiv 0 \dots 50 \quad p_k := \frac{k}{50} \quad H_{\max} := 1$$

$$h_k := \frac{-1}{\ln(2)} \left[p_k \ln[p_k] + [1 - p_k] \cdot \ln[1 - p_k] \right]$$



From the above figure, we know the maximum entropy is 1 where the probability for sending 1 or 0 is 0.5.

1-10

$$P_1 = 0.25 ; P_2 = P_3 = 0.15 ;$$

$$P_4 = P_5 = P_6 = P_7 = P_8 = P_9 = 0.07$$

$$0.25 + 2(0.15) + 6(0.07) = 0.97$$

$$\therefore P_{10} = 0.03 \quad \text{since} \quad \sum_{j=1}^{10} P_j = 1.0$$

$$H = \sum_{j=1}^m P_j \log_2 \left(\frac{1}{P_j} \right) = \left[\frac{-1}{\ln 2} \right] \sum_{j=1}^{10} P_j \ln P_j$$

$$= \left[\frac{-1}{\ln 2} \right] \left[.25 \ln .25 + (2) .15 \ln .15 \right. \\ \left. + (6) .07 \ln .07 + .03 \ln .03 \right]$$

$$H = \underline{\underline{3.084 \text{ bits}}}$$

1-11

$$(a.) P_1 = 0.3 ; P_2 = 0.7$$

$$H = \frac{-\sum_{j=1}^2 P_j \ln P_j}{\ln 2} = \frac{0.3 \ln 0.3 + 0.7 \ln 0.7}{-\ln 2}$$

$$H = \underline{\underline{0.881 \text{ bits}}}$$

$$(b.) H_{\max} \text{ for } P_j = \frac{1}{m} = \frac{1}{2} = 0.5 \quad j=1, m$$

$$H_{\max} = \frac{-2(0.5) \ln 0.5}{\ln 2} = \underline{\underline{1 \text{ bit} = H_{\max}}}$$

1-12

$$M = 10 \quad P_j = \frac{1}{10} \quad j=1, 10 \quad R = \frac{H}{T} = \underline{\underline{3 \frac{b}{s}}}$$

$$H = \frac{-10(.1) \ln .1}{\ln 2} = 3.322 \text{ bits}$$

$$T = \frac{H}{R} = \frac{3.322 \text{ bits}}{3 \text{ bits/sec}} = \underline{\underline{1.11 \text{ sec.} = T}}$$

1-13

12 binary digits/word $M=2^{12}$; $P_j = (\frac{1}{2})^{13}$, $j=1, \frac{M}{2}$; $P_j = 3(\frac{1}{2})^{13}$, $j=\frac{M}{2}+1, M$

$$H = \frac{\sum_{j=1}^M P_j \ln P_j}{-\ln 2}$$

$$= \frac{\frac{2^{12}}{2} \left[\left(\frac{1}{2}\right)^{13} \ln \left(\frac{1}{2}\right)^{13} + 3 \left(\frac{1}{2}\right)^{13} \ln 3 \left(\frac{1}{2}\right)^{13} \right]}{-\ln 2}$$

$$\Rightarrow \underline{H = 11.810 \text{ bits}}$$

$$\text{Check: } \sum_{j=1}^M P_j = 2^{11} \left[\left(\frac{1}{2}\right)^{13} + 3 \left(\frac{1}{2}\right)^{13} \right]$$

$$= 2^{11} (4) \left(\frac{1}{2}\right)^{13} = 1.0$$

1-14

$$B = 300 \text{ Hz} \quad \left(\frac{S}{N}\right)_{dB} = 30 \text{ dB}$$

$$C = B \log_2 \left[1 + \left(\frac{S}{N}\right) \right] = \frac{B}{\ln 2} \ln \left[1 + \frac{S}{N} \right] \quad \& \quad \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \left(\frac{S}{N}\right)$$

$$\Rightarrow \frac{S}{N} = 10^{\frac{(S/N)_{dB}}{10}} = 10^3 = 1000$$

$$C = \frac{300}{\ln 2} \ln [1001] = \underline{2.99 \times 10^3 \text{ bits/sec}}$$

1-15

(a) chars := 110 Number of characters available

$$b := \text{ceil} \left[\frac{\log(\text{chars})}{\log(2)} \right] \quad \text{Number of bits required to represent a character}$$

$$\Longrightarrow b = 7 \quad \text{bits}$$

(b) B := 3200 Hz Channel bandwidth
SNRdB := 20 dB Signal to noise ratio

$$\frac{\text{SNRdB}}{10}$$

$$\text{SNR} := 10 \quad \Longrightarrow \quad \text{SNR} = 100 \quad (\text{Absolute power ratio})$$

$$C := B \cdot \left[\frac{\log(1 + \text{SNR})}{\log(2)} \right] \quad \Longrightarrow \quad C = 2.131 \cdot 10^4 \quad \text{Channel capacity (bits/sec)}$$

$$C := \frac{C}{b} \quad \Longrightarrow \quad C = 3.044 \cdot 10^3 \quad \text{Channel capacity (chars/sec)}$$

(c) Assuming equally likely characters,
information content of each character is:

$$P := \frac{1}{\text{chars}} \quad \text{Probability of each character}$$

$$I := \frac{\log \left[\frac{1}{P} \right]}{\log(2)} \quad \Longrightarrow \quad I = 6.781 \quad \text{bits}$$

1-16

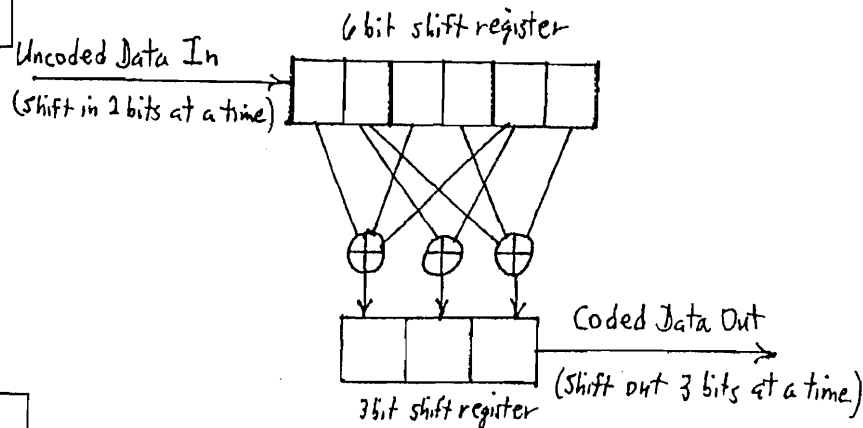
$$R = B \log_2 \left(1 + \frac{S}{N} \right) = B \frac{\log_{10} (1 + 10^{4.5/10})}{\log_{10} (2)} = B (14.949)$$

(a) $B = 1200 - 300 = 900 \Rightarrow R = (900)(14.949) = \underline{13.45 \text{ kbits/s}}$

(b) $B = 3200 - 1500 = 1700 \Rightarrow R = (1700)(14.949) = \underline{25.41 \text{ kbits/s}}$

(c) $B = 3200 - 300 = 2900 \Rightarrow R = (2900)(14.949) = \underline{43.35 \text{ kbits/s}}$

1-17



1-18

```
x := 1 x := 0 x := 1 x := 1 x := 1 Input vector
0 1 2 3 4
ga := 1 ga := 0 ga := 0 ga := 1 Gain vector, mod2 adder
0 1 2 3
gb := 1 gb := 1 gb := 1 gb := 1 Gain vector, mod2 adder
0 1 2 3
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```
k := 0 .. length(ga) - 1
v := 0 k := 0 .. length(x) - 1 v := x
k k+length(ga)-1 k
k := length(x) + length(ga) - 1 .. length(x) + 2 length(ga) - 3
v := 0 i := 0 .. length(v) - length(ga)
k j := 0 .. length(ga) - 1
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$$sa_i := \sum_j [ga_{\text{length}(ga)-j-1} v_{j+i}]$$

$$sa_i := \text{mod}[sa_i, 2]$$

$$sb_i := \sum_j [gb_{\text{length}(gb)-j-1} v_{j+i}]$$

$$sb_i := \text{mod}[sb_i, 2] \quad s_{2i} := sa_i \quad s_{2i+1} := sb_i$$

$$i := 0 \dots 2 \text{ length}(x) - 1$$

$$\text{out}_i := s_i$$

For $x^T = (1 \ 0 \ 1 \ 1 \ 1)$
====> $\text{out}^T = (1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 1)$