Complete Solutions Manual

Discovering Mathematics
A Quantitative Reasoning Approach

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Chapter 1: Introduction to Problem Solving

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3. One example is 5, which is an odd number.

EXERCISE SET 1.1

1. 64. The numbers are the squares of consecutive integers. $8^2 = 64$.

2. 35. Subtract 1 less than the integer subtracted from the previous integer.

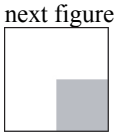
3. $\frac{13}{15}$. Add 2 to the numerator and denominator.

4. $\frac{6}{7}$. Add 1 to the numerator and denominator.

5. –13. Use the pattern of adding 5, then subtracting 10 to obtain the next pair of numbers.

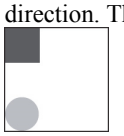
6. 21. Subtract 8 from 5, add 12 to –3, subtract 16 from 9, add 20 to –7, etc., increasing the amount by 4 each time while alternating between adding and subtracting.

7. Each image has a smaller square inside a larger square. The smaller square moves to a new position in a counterclockwise direction. The next figure is shown.

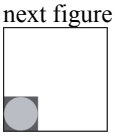


8. Each image has a smaller square inside a larger square. The smaller square moves to a new position in a counterclockwise direction. The next figure is shown.

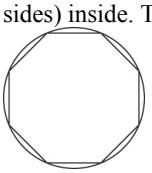
9. Each image has a smaller square and a circle inside a larger square. The smaller square moves to a new position in a corner in a counterclockwise direction. The circle moves to a new position in a counterclockwise direction. The next figure is shown.



10. Each image has a smaller square and a circle inside a larger square. The smaller square moves to a new position in a corner in a counterclockwise direction. The circle moves to a new position in a clockwise direction. The next figure is shown.



11. Each figure is a circle with a polygon, alternating positions inside and outside. The first figure is a triangle (3 sides) with a circle inside. The second is a circle with a square (4 sides) inside. The next figure is shown.



12. Each figure is a polygon within a polygon, alternating blue and yellow interiors. The first figure is a 7-sided figure inside an 8-sided figure. The second figure is a 6-sided figure inside a 7-sided figure. The next figure is shown.

13. The amount is decreasing by $1000 per month. Thus, the amount in year 6 will be $5000.

14. a. Greater since the temperature is increasing.
    b. No. Fall and winter would come and the temperature would decrease.

15. More than. The increase in average annual salary increases from $7763 to $8927 to $10,266. Thus the increase from 20 years to 25 years will be more than $10,266, giving an average annual salary for a teacher having 25 years of experience of greater than $78,705 + $10,266 = $88,971, which is greater than $88,000.

16. Fewer. From the bar graph, it appears that as the price of cell phones increases, the number of cell phones sold decreases. Assuming the same trend, if the price of the cell phone is $700, the company will sell fewer than 333,000 cell phones.

17. For every 10 seconds, the distance is increasing by 300 feet. Therefore, after 70 seconds, the athlete will run a distance of 2100 feet.

18. For every 2 hours, the distance is decreasing by 100 miles. Therefore, after 6 hours, the trip will have 200 miles remaining.

19. From the table, as the depth increases, the temperature decreases. Since the last entry of the table is 40 m and 11°C, then for 45 m, the temperature should be less than 11°C.

20. Each year the car depreciates by a smaller amount: \$5336, \$5191, \$2930, and \$2878. The value of the car at year 5 is \$11,743. After year 6, the value of the car will be less than \$11,000.

21. From the table, the distance (in feet) is equal to the time (in seconds) squared. So at 7 seconds the distance is 49 feet, and at 8 seconds the distance is 64 feet.

22. The pattern from 0 seconds to 7 seconds, repeats again starting at 8 seconds. So at 14 seconds, the distance will be the same as at 6 seconds, which is 0 inches.

23. Answers will vary. For instance, $\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$.

24. Answers will vary. For instance, $5-(-3)=8$.

25. A diamond shape can have four equal sides and is not a square.

26. Answers will vary. For instance, 20.

27. Whales are mammals and do not have legs.

28. A penguin is a bird that does not fly.

29. a. Deductive
    b. Inductive

30. a. Inductive
    b. Deductive

31. Place 4 coins in the left balance pan, 4 coins in the right balance pan, and set 4 coins aside. There are three possibilities. The right side goes down, the left side goes down, or the scale balances (the heavier coin is set aside).
    Take the 4 coins that include the heavier coin and place 2 on the left balance pan and 2 on the right balance pan.
    From the 2 coins on the heavier pan, place one coin on each balance pan to determine the heavier coin.

32. Label the coins 1, 2, 3, … 8. Set coins 5, 6, 7, and 8 aside.
    Weighing 1: Place coins 1 and 2 on one balance and coins 3 and 4 on the other.
    If the scale is balanced, the heavier or lighter coin is 5, 6, 7, or 8. Coins 1, 2, 3, or 4 are of equal weight.
    If the scale is unbalanced, the heavier or lighter coin is 1, 2, 3, or 4. Coins 5, 6, 7, or 8 are of equal weight.
    Weighing 2: From the four coins that contain the heavier or lighter coin, choose two coins, placing one coin on each balance, and set the other two coins aside. Suppose that coins 5, 6, 7, or 8 are not of equal weight. Choose coins 5 and 6 and place one on each side of the balance.
    If the scale is balanced, then coins 5 and 6 are of equal weight and the coin that is heavier or lighter is either coin 7 or 8.
    If the scale is unbalanced, then coins 7 and 8 are of equal weight and the coin that is heavier or lighter is either coin 5 or 6.
    Weighing 3: Suppose the coin that is heavier or lighter is either coin 7 or 8. Place coin 7 on one balance and coin 1 (or any one of coins 2 to 6, since they are of equal weight), on the other balance.
    If the scale is balanced, then coin 8 is the coin that is heavier or lighter.
    If the scale is unbalanced, then coin 7 is the coin that is heavier or lighter.

33. Label each stack of coins as 1, 2, 3, … 10. From each stack select the same amount of coins as the labels, so select 1 coin from stack 1, 2 coins from stack 2, and so on to 10 coins from stack 10. Weigh the selected coins. Since the counterfeit coins each weigh 0.1 g more, the multiple of 0.1 g over the weight expected will determine which stack contains the counterfeit coins.

34. Label each stack of coins as 0, 1, 2, 3, … 10. Note that there are 11 stacks. From each stack select the same amount of coins as the labels, so select 0 coins from stack 0, 1 coin from stack 1, 2 coins from stack 2, and so on to 10 coins from stack 10. Weigh the selected coins. Since the counterfeit coins each weigh 0.1 g more, the multiple of 0.1g over the weight expected will determine which stack contains the counterfeit coins. Note that if the weight is not more than expected, the counterfeit coins are in stack 0.

35.

	Util	Auto	Tech	Oil
A	X1	X1	✓	X3
T	X1	X1	X3	✓
M	✓	X2	X3	X3
J	X2	✓	X2	X2

Maria: the utility stock; Jose: the automotive stock; Anita: the technology stock; Tony: the oil stock

36.

	Soup	Entree	Salad	Dessert
C	X2	✓	X3	X2
S	X2	X3	✓	X2
O	X1	X2	X2	✓
G	✓	X2	X2	X2

Changs: entrée; Steinbergs: salad; Ontkeans: dessert; Gonzaleses: soup

37.

	Coin	Stamp	Comic	Baseball
A	X3	✓	X3	X4
C	X2	X4	X1	✓
P	✓	X3	X3	X2
S	X2	X3	✓	X3

Atlanta: stamps; Chicago: baseball cards; Philadelphia: coins; San Diego: comic books

38. a. Yes. Change the color of Iowa to yellow and Oklahoma to blue.
    b. No. One possible explanation: Oklahoma, Arkansas, and Louisiana must each have a different color than the color of Texas and they cannot all be the same color. Thus, the map cannot be colored using only two colors.

THINK ABOUT IT 1.2

1. a. Estimate
   b. Estimate
   c. Exact
   d. Estimate
   e. Exact
   f. Exact

2. $5.32\times10^{-6} = 0.00000532$
   The answer is b, greater than 0 but less than 1.

3. True

4. False; $10^{-3} = 0.001 = \frac{1}{1000}$

5. Yes

EXERCISE SET 1.2

1. 1487 miles $\approx$ 1500 miles
   $\frac{1500 \text{ miles}}{3 \text{ days}} = 500$ miles per day

2. 11 children $\approx$ 10 children; 3 slices $\approx$ 4 slices
   $10 \text{ children} \times \frac{4 \text{ slices}}{1 \text{ child}} = 40 \text{ slices}$
   $\frac{40 \text{ slices}}{8 \text{ slices/pizza}} = 5$ pizzas
   4 pizzas is conservative; 5 pizzas should be plenty.

3. $2(25) + 30 = 80$
   So, 25°C $\approx$ 80°F.

4. 238 square feet $\approx$ 240 square feet
   \$0.28 $\approx$ \$0.30
   $240 \times \$0.30 = \$72$
   The cost will be approximately \$72.

5. $400 \times 20 = 8000$ square feet

6. Since 1 gallon of paint covers 350 square feet, and the room has 400 square feet of wall space, including windows and doors, you should purchase 2 gallons of paint.

7. Answers will vary depending on the grid. Using a 3 by 5 grid, we count 30 cars. Multiply 30 by the number of sections, 15, to get 450. So our estimate is 450 cars.

8. Answers will vary depending on the grid. Using a 3 by 5 grid, we count 28 flowers. Multiply 24 by the number of sections, 15, to get 360. So our estimate is 360 flowers.

9. Answers will vary depending on the grid. Using a 3 by 5 grid, we count 18 cadets. Multiply 15 by the number of sections, 15, to get 225. So our estimate is 225 cadets.

10. Answers will vary depending on the grid. Using a 4 by 5 grid, we count 9 marchers. Multiply 9 by the number of sections, 20, to get 180. So our estimate is 180 marchers.

11. $\frac{6,400,000 \text{ square feet}}{4 \text{ square feet/(1 person)}} = 1,600,000$ people
    or 1.6 million people

12. 68.3 square miles $\approx$ 68 square miles
    $68 \text{ sq mi} \times \frac{10,000 \text{ people}}{1 \text{ sq mi}} = 680,000$ people
    Approximately 680,000 people

13. If a 10-foot pole casts a shadow of 18 feet, and a telephone pole has a shadow of 72 feet, then the telephone pole will be more than half as high as its shadow is long. The telephone pole has height
    $72 \times \frac{10}{18} = 40$ feet

14. If a 8-foot pole casts a shadow of 6 feet, and a building has a shadow of 94 feet, then the building will be taller than its shadow. The building has height
$$94 \times \frac{8}{6} \approx 125 \text{ feet}$$

15. $0.0000000000089 \text{ meter} = 8.9 \times 10^{-12} \text{ meter}$

16. $30,900,000,000,000,000 \text{ m} = 3.09 \times 10^{16} \text{ m}$

17. $35,000,000,000,000 \text{ cells} = 3.5 \times 10^{13} \text{ cells}$

18. $0.000000076 \text{ meter} = 7.6 \times 10^{-8} \text{ meter}$

19. Yes. The higher the advanced degree, the higher the bar of the lifetime earnings is.

20. Yes

21. Yes. The Dogs category is more than half of the pie graph.

22. Yes

23. a. The graph crosses \$1 trillion (or \$1000 billion) in years 2013 to 2014.
    b. The graph crosses \$1.2 trillion in year 2016.

24. The year 2020 had the largest difference in nominal wage and real wage.

THINK ABOUT IT, SECTION 1.3

1. Understand the problem, devise a plan, carry out the plan, review the solution

2. No. We would need to know the speed at which she ran.

EXERCISE SET 1.3

1. Let g be the number of first grade girls, and let b be the number of first grade boys. Then $b + g = 364$ and $g = b + 26$. Solving gives $g = 195$, so there are 195 girls.

2. Let a be the length in feet of the shorter ladder and b be the length in feet of the longer ladder. Then $a + b = 31.5$ and $a + 6.5 = b$. Solving gives $a = 12.5$ and $b = 19$, so the ladders are 12.5 feet and 19 feet long.

3. There are 36 1 × 1 squares, 25 2 × 2 squares, 16 3 × 3 squares, 9 4 × 4 squares, 4 5 × 5 squares and 1 6 × 6 square in the figure, making a total of 91 squares.

4. The first decimal digit, like all the odd decimal digits, is a zero, and the second decimal digit, like all the even decimal digits, is a 9. Since 44 is even, the 44^{th} decimal digit is a 9.

5. Solving:
x = cost of the shirt
$x - 30$ = cost of the tie
$$\begin{aligned} (x-30)+x &= 50 \\ 2x-30 &= 50 \\ 2x &= 80 \\ x &= 40 \end{aligned}$$
The shirt costs \$40.

6. Using the results of example 3, 12 teams play each of 11 teams for a total of (12 × 11) ÷ 2 = 66 games. Since each team plays each of the teams twice 2 × 66 = 132 total games.

7. There are 14 different routes to get to Fourth Avenue and Gateway Boulevard and 4 different routes to get to Second Avenue and Crest Boulevard. Adding gives that there are 18 different routes altogether.

8. a. There are 2 different routes from point A to the Starbucks and 2 different routes from the Starbucks to point B. Multiplying gives a total of 4 different routes.
   b. There is only one direct route to the Subway. There are 3 different routes from the Subway to point B, so there are 3 different routes altogether.
   c. Since there is only one direct route to the Subway, starting the count there will not change the number of routes. There is only one direct route from the Subway to Starbucks, and there are 2 different routes from Starbucks to point B, so there are 2 different routes altogether.

9. Try solving a simpler problem to find a pattern. If the test had only 2 questions, there would be 4 ways. If the test had 3 questions, there would be 8 ways. Further experimentation shows that for an n question test, there are 2^n ways to answer. Letting
$n = 12$, there are $2^{12} = 4096$ ways

10. The frog gains 2 feet for each leap. By the 7^{th} leap, he has gained 14 feet. On the 8^{th} leap, he moves up 3 feet to 17 feet, escaping the well.

11. 8 people shake hands with 7 other people. Multiply 8 and 7 and divide by 2 to eliminate repetitions to obtain 28 handshakes.

12. There are $\frac{24\times 23}{2} = 276$ ways to join the points. 276 line segments

13. Let p be the number of pigs and let d be the number of ducks. Then $p + d = 35$ and $4p + 2d = 98$. Solving gives $d = 21$ and $p = 14$, so there are 21 ducks and 14 pigs.

14. Carla arrives home first because she spends more time running than Allison. Running half the time means she will cover more than half the distance in the time that Allison who is walking half the distance.

15.

Dimes	Nickels	Pennies
0	0	25
0	1	20
0	2	15
0	3	10
0	4	5
0	5	0
1	0	15
1	1	10
1	2	5
1	3	0
2	0	5
2	1	0

There are 12 ways.

16. Area of the room is $12 \times 15 = 180$ square feet. Each square of carpet has an area 9 square feet. Divide 180 by 9 to get 20 squares.

17. The units digits of powers of 4 form the sequence 4, 6, 4, 6,…. Even powers end in 6. Therefore, the units digit of 4^{300} is 6.

18. The units digits of powers of 2 form the sequence 2, 4, 8, 6, 2, 4, 8, 6.… Divide 725 by 4 to obtain a remainder of 1 which corresponds to 2. Therefore the units digit of 2^{725} is 2.

19. The units digits of powers of 3 form the sequence 3, 9, 7, 1, 3, 9, 7, 1, …. Divide 412 by 4 to obtain the remainder 0, which corresponds to 1. Therefore the units digit of 3^{412} is 1.

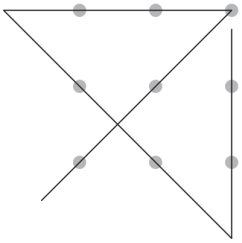
20. The units digits of powers of 7 are 7, 9, 3, 1, 7, 9, 3, 1, ... Divide 146 by 4 to obtain the remainder 2, which corresponds to 9. Therefore the units digit of 7^{146} is 9.

21. Draw a simpler picture:

Note that the first page of the first volume is the second dot on the line, and the last page of the third volume is the seventh dot on the line. This is because when books sit on a shelf, the first pages are on the right side of the book and their last pages are on the left side.

$\frac{1}{8}+\frac{1}{8}+1+\frac{1}{8}+\frac{1}{8}=1\frac{1}{2}$ inches.

22. Yes, it is possible. One possible way is shown below.



23. If n is the number of columns of tile on the floor, then there are n blue tiles on the diagonal that starts in the upper left hand corner. Similarly, there are n blue tiles in the diagonal that starts in the upper right hand corner. The two diagonals have one tile in common, so the actual total number of blue tiles is $2n - 1$. Since $2n - 1 = 101$, we can solve to find $n = 51$. The total number of tiles is n^2. Substituting the value for n yields 2601.

24. Trying several values for the number of voters, n:
If $n = 10$, the number of votes must be between 9.4 and 10.
If $n = 15$, the number of votes must be between 14.1 and 15.
If $n = 16$, the number of votes must be between 15.04 and 16.
If $n = 17$, the number of votes must be between 15.98 and 17. It is possible to have 16 votes for the candidate.
Thus, the least possible number of votes cast is 17.

25. Let b be the number of boys in the family and g be the number of girls. The first two statements imply that the speaker is a girl. Thus, $g - 1 = b + 2$. Solving for b, $b = g - 3$. To answer the last question, we must omit the youngest brother, so $b - 1 = g - 4$. There are four more sisters than brothers.

26. Let b be the number of boys in the family and g be the number of girls. If each girl has as many brothers as sisters, $b = g - 1$. If each boy has twice as many sisters as brothers, $g = 2(b - 1)$. Substituting for b in the second equation, we get $g = 2(g - 2)$. Solving, $g = 4$ and $b = 3$. Thus, there are 7 children.

27. The bacteria population doubles every day, so on the 11th day there are half as many bacteria as on the 12th day.

28. If you take 22 pennies, you have 22 pennies.

29. Let x be the score that Dana needs on the fourth exam:
$$\frac{82+91+76+x}{4}=85$$
$$249+x=340$$
$$x=91$$

30. a. Let A, B, C, and D represent the four people with weights 80, 100, 150, and 170 pounds, respectively. A and B make the first trip across. A comes back alone. C crosses the river and B comes back alone. C is now on the opposite bank. Repeat this procedure to get D to the opposite bank. Then one more trip will get both A and B to the opposite bank.

    b. The minimum number of crossings is 9.

31. a. Place four coins on the left balance pan and the other 4 coins on the right balance pan. The pan that is the higher contains the fake coin. Take the four coins from the higher pan and use the balance scale to compare the weight of two of these coins to the weight of the other two coins. The pan that is the higher contains the fake coin. Take the two coins from the higher pan and use the balance scale to compare the weights. The pan that is the higher contains the fake coin. This procedure enables you to determine the fake coin in 3 weighings.

    b. Place 3 of the coins on one of the balance pans and 3 coins on the other balance pan. If the pans balance, then the fake coin is one of the two remaining coins. You can put each one of these coins on a balance pan and the higher pan contains the fake coin. If the 3 coins on the left do not balance with the 3 coins on the right, then the higher pan contains the fake coin. Pick any 2 of these 3 coins and use the balance scale to compare their weights. If these 2 coins do not balance, then the higher pan contains the fake coin. If these two coins balance, then the 3rd coin (the one that you did not place on the balance pan) is the fake. In either case this procedure enables you to determine the fake coin in 2 weighings.

32. Fill the 5-gallon jug. Pour from it into the 3-gallon jug, until the 3-gallon jug is full. Now empty the 3-gallon jug. There are 2 gallons in the 5-gallon jug. Pour these 2 gallons into the 3-gallon jug. Fill the 5-gallon jug and pour from it into the 3- gallon jug. At this point there are 2 gallons in the 3-gallon jug, so only 1 more gallon will fit. Thus, when the 3-gallon jug is full, there will be 4 gallons left in the 5-gallon jug.

33. The correct answer is a., 1600. Sally likes perfect squares.

34. The correct answer is c., 21:00. If it were two hours later (23:00 – 1 hour before midnight), it would be half as long as if it were an hour later (22:00 – 2 hours before midnight).

35. The correct answer is d., 64. The numbers are all perfect cubes. The missing number is the cube of 4.

36. The correct answer is b., No. The other 800 elephants can be any mix of all blue and pink and green stripes.

THINK ABOUT IT, SECTION 1.4

1. 25

2. Assuming Jeremiah divides the land as required by the rules of fair division, this is a fair division of the land. The size of a piece does not matter: the value of the piece is the important fact.

3. Craig. The divider must divide an asset so that each piece has the same value.

4. $P_1 : S_1,\ S_3;\quad P_2 : S_1,\ S_3;\quad P_3 : S_1,\ S_2, S_3$

EXERCISE SET 1.4

1. a. \$9 + \$6 + \$3 = \$18

    b. One-third of the cheese, one half of the pepperoni, and none of the green pepper is $\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{3}+0\cdot\frac{1}{3}=\frac{5}{18}$, which is a lot less than half of the pizza.
    No. She does not receive a fair share.

2. a. No. As Abigail values the pizza, the half that contains two-thirds of the cheese and one third of the pepperoni has value $\$4\cdot\frac{2}{3}+\$6\cdot\frac{1}{3}\approx\$4.67$. The other half has one-third of the cheese and two-thirds of the pepperoni and has value $\$4\cdot\frac{1}{3}+\$6\cdot\frac{2}{3}\approx\$5.33$. Because the two halves do not have the same value, the cut

does not satisfy criterion 1.

b. Yes.
As Abigail values the pizza, the part that is one-half cheese and one-half pepperoni has value $\$4 \cdot \frac{1}{2} + \$6 \cdot \frac{1}{2} = \$5,$ and the other part has the same value.
Because the two parts have the same value, the cut does satisfy criterion 1.

3. Answers may vary. The easiest solution is to cut the cake so that each piece is one-half strawberry and one-half chocolate.

4. There are multiple solutions to this exercise. Here is one solution.
Charlotte considers A or B a fair share. Brandon considers B or C a fair share. Because A is not contested between Charlotte and Brandon, Charlotte gets A. Then Brandon can receive B and Ione gets C.

5. Yes. Since Amanda values the chocolate icing at \$12 and the strawberry icing also at \$12, dividing the cake into three equal pieces, regardless of icing, causes each piece to be a fair share.

6. P_1 receives S_3, P_2 receives S_1, and P_3 receives S_2

7. Answers may vary. Here is one possibility.

	S_1	S_2	S_3	Bids
Caroline	45%	25%	30%	S_1
Jasmine	40%	30%	30%	S_1
Jean	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	S_1, S_2, S_3

Because S_1 is contested among the choosers, set it aside. Now randomly give Jean S_2 or S_3. Assume she receives S_2. Now combine the remaining shares and use the divider-chooser method to distribute the remaining two shares. Randomly choose Caroline or Jasmine to be the divider. The other partner is the chooser. We will assume Caroline is the divider and divides the shares as T_1 and T_2. Assume Jasmine chooses T_1. Then Caroline gets T_2, Jasmine gets T_1, and Jean gets S_2.

8. Answers may vary. Here is one possibility.

	S_1	S_2	S_3	Bids
P_1	30%	30%	40%	S_3
P_2	25%	30%	45%	S_3
P_3	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	S_1, S_2, S_3

Because S_3 is contested among the choosers, set it aside. Now randomly give P_3 S_1 or S_2. Assume P_3 receives S_2. Now combine the remaining shares and use the divider-chooser method to distribute the remaining two shares. Randomly choose P_1 or P_2 to be the divider. The other partner is the chooser. We will assume P_2 is the divider and divides the shares as T_1 and T_2. Assume P_1 chooses T_1. Then P_2 gets T_2
Thus, P_2 gets T_2, P_1 gets T_1, and P_3 gets S_2.

9. Answers may vary. Here is one possibility.

	S_1	S_2	S_3	S_4	Bids
A	25%	25%	25%	25%	S_1, S_2, S_3, S_4
B	30%	10%	30%	30%	S_1, S_3, S_4
C	20%	35%	25%	20%	S_2, S_3
D	30%	30%	20%	20%	S_1, S_2

S_4 is uncontested among the choosers. Award S_4 to B. Combine S_1, S_2, and S_3. Then choose one of the remaining players to be the divider. We choose A. A divides the shares into three shares: T_1, T_2, and T_3. Everyone now submits bids for those shares. *This is where solutions may vary. It will depend on how you assign the bids. This is just one possibility.*

	T_1	T_2	T_3	Bids
A	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	T_1, T_2, T_3
C	30%	30%	40%	T_3
D	30%	35%	35%	T_2, T_3

T_1 is uncontested, so award that to A. C only wants T_3 so award T_3 to C. That leaves D with T_2. The final result: A gets T_1; B gets S_4; C gets T_3; D gets T_2.

10. Answers may vary. Here is one possibility.

	S_1	S_2	S_3	S_4	Bids
Jonas	25%	25%	25%	25%	S_1, S_2, S_3, S_4
Anne	25%	35%	20%	20%	S_1, S_2
Gavin	20%	35%	25%	20%	S_2, S_3
Alisa	20%	10%	30%	40%	S_3, S_4

Among the choosers, S_1 and S_4 are uncontested. Give S_1 to Anne and S_4 to Alisa. For Gavin, randomly choose between S_2 and S_3, say, by tossing a coin. For instance, suppose Gavin gets S_2 and then S_3 goes to Jonas.

11. Answers may vary. Here is one possibility.

	S_1	S_2	S_3	S_4	Bids
A	25%	25%	25%	25%	S_1, S_2, S_3, S_4
R	15%	30%	20%	35%	S_2, S_4
M	20%	45%	15%	20%	S_2
C	30%	35%	15%	20%	S_1, S_2

Michel only values S_2, so he receives S_2. Because S_2 went to Michel, award S_4 to Richard. Then S_1 goes to Carlos. Ariana gets S_3.

12. Answers may vary. Here is one possibility.

	S_1	S_2	S_3	S_4	Bids
Kyle	25%	25%	25%	25%	S_1, S_2, S_3, S_4
Cody	30%	30%	20%	20%	S_1, S_2
Colin	15%	30%	20%	35%	S_2, S_4
Marian	20%	40%	20%	20%	S_2

All siblings contest S_2 so put it aside. Shares S_1 and S_4 are uncontested by the choosers. Award S_1 to Cody and S_4 to Colin. Now combine the remaining two shares and use the divider-chooser method for two players to divide the remaining shares. We select Kyle to divide the shares as T_1 and T_2. Marian chooses T_2 and Kyle receives T_1. Cody receives S_1 and Colin receives S_4.

13. Answers may vary. This is one possibility.

	S_1	S_2	S_3	S_4	Bids
J	25%	25%	25%	25%	S_1, S_2, S_3, S_4
Y	35%	20%	10%	35%	S_1, S_4
M	15%	40%	35%	10%	S_2, S_3
D	25%	35%	10%	30%	S_1, S_2, S_4

S_3 is uncontested among the choosers. Award S_3 to Jorge. Now choose one of the remaining investors to be the divider. We choose Yolanda. She divides the shares into three shares: T_1, T_2, and T_3. Everyone now submits bids for those shares. *This is where solutions may vary. It will depend on how you assign the bids. This is just one possibility.*

	T_1	T_2	T_3	Bids
Y	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	T_1, T_2, T_3
M	30%	30%	40%	T_3
D	30%	35%	35%	T_2, T_3

T_2 is uncontested among the choosers. Award T_2 to Daniel. Now use the divider-chooser method for two people, Yolanda and Maria, to divide the remaining two shares: call them U_1 and U_2. We will assume Maria divides and Yolanda chooses U_2. The final result: Jorge gets S_3, Yolanda gets U_2, Maria gets U_1, and Daniel gets T_2.

14. Answers may vary. This is one possibility.

	S_1	S_2	S_3	S_4	Bids
G	25%	25%	25%	25%	S_1, S_2, S_3, S_4
M	15%	35%	30%	20%	S_2, S_3
C	40%	30%	5%	25%	S_1, S_2, S_4
B	20%	30%	20%	30%	S_2, S_4

S_3 is uncontested among the choosers. Award S_3 to Georgiana. Now choose one of the remaining players to be the divider. We choose Macon. He divides the shares into three shares: T_1, T_2, and T_3. Everyone now submits bids for those shares. *This is where solutions may vary. It will depend on how you assign the bids. This is just one possibility.*

	T_1	T_2	T_3	Bids
M	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	T_1, T_2, T_3
C	30%	40%	30%	T_2
B	30%	35%	35%	T_2, T_3

T_3 is uncontested among the choosers. Award T_3 to Barbara. Now use the divider-chooser method for two people, Macon and Clarisse, to divide the remaining two shares: call them U_1 and U_2. We will assume Macon divides and Clarisse chooses U_2. The final result: Georgiana gets S_3, Macon gets U_2, Clarisse gets U_1, and Barbara gets T_3.

15. Assign initial winners:

	Chong	Riana	Winner
Boat	15	5	Chong
House	45	65	Riana
Condo	40	30	Chong
Total:	55	65	

Riana is the initial winner and Chong is the initial loser.

Adjusted winner procedure:

	Chong	Riana	Winner	Ratio
Boat	15	5	Chong	
House	45	65	Riana	$\frac{65}{45} \approx 1.4$
Condo	40	30	Chong	
Total:	55	65		

The sharing procedure for the house:

$$15+45(1-x)+40=65x$$
$$15+45-45x+40=65x$$
$$100-45x=65x$$
$$100=110x$$
$$0.909\approx x$$

For the house, 90.9% will go to Riana, and 9.1% will go to Chong.

Distribute the estate: Chong gets the boat, condo and 9.1% from the sale of the house. Riana gets 90.9% of the sale of the house.

16. Assign initial winners:

	Therese	Geraldine	Winner
Oak Table	10	8	Therese
Cedar Chest	12	16	Geraldine
Chandelier	20	25	Geraldine
Tapestry	25	25	tie
Sculpture	33	26	Therese
Total:	43	67	

Geraldine is the initial winner and Therese is the initial loser.

The sharing procedure:

$$16+25+25(1-x)=10+25x+33$$
$$16+25+25-25x=10+25x+33$$
$$66-25x=43+25x$$
$$23=50x$$
$$0.46=x$$

For the tapestry, 46% will go to Geraldine, and 54% will go to Therese.

Distribute the estate: Therese gets the oak table, sculpture and 54% from the sale of the tapestry. Geraldine gets the cedar chest, chandelier, and 46% of the sale of the tapestry.

17. Assign initial winners:

	Elliot	Francine	Winner
Residence	20	25	Francine
Summer Home	10	10	tie
Airplane	18	25	Francine
Sailboat	20	15	Elliot
Desk	32	25	Elliot
Total:	52	50	

Elliot is the initial winner and Francine is the initial loser.

The sharing procedure:

$$10(1-x)+20+32=25+10x+25$$
$$10-10x+20+32=25+10x+25$$
$$62-10x=50+10x$$
$$12=20x$$
$$0.60=x$$

For the summer home, 60% will go to Francine, and 40% will go to Elliot.

Distribute the estate: Francine gets the residence, airplane, and 60% from the sale of the summer home. Elliot gets the sailboat, desk, and 40% of the sale of the summer home.

18. Determine highest bidder.

	Office	House
Samuel	$255,000	$327,000
Ashton	$276,000	$300,000
Carmelo	$250,000	$275,000
Highest bidder	Ashton	Samuel

Samuel: $\$255,000+\$327,000=\$582,000$

Fair share: $\frac{1}{3}(\$582,000)=\$194,000$

Ashton: $\$276,000+\$300,000=\$576,000$

Fair share: $\frac{1}{3}(\$576,000)=\$192,000$

Carmelo: $\$250,000+\$275,000=\$525,000$

Fair share: $\frac{1}{3}(\$525,000)=\$175,000$

Difference between fair share and item won.
Samuel: $194,000 – $327,000 = –$133,000
Ashton: $192,000 – $276,000 = –$84,000
Carmelo: $175,000 – $0 = $175,000

Estate surplus:
$133,000 + $84,000 – $175,000 = $42,000

Equal amount = $\frac{1}{3}(\$42,000)=\$14,000$

Amounts owed to/from estate:
Samuel: –$133,000 + $14,000 = –$119,000
Ashton: –$84,000 + $14,000 = –$70,000
Carmelo: $175,000 + $14,000 = $189,000

Samuel gets the house and pays $119,000 into the estate. Ashton gets the office building and pays $70,000 into the estate. Carmelo receives $189,000 from the estate.

19. Determine highest bidder.

	M. Suites	Real Estate
Amelia	$510,000	$480,000
Justine	$624,000	$540,000
Colby	$540,000	$545,000
Highest bidder	Justine	Colby

Amelia: $\$510,000+\$480,000=\$990,000$

Fair share: $\frac{1}{3}(\$990,000)=\$330,000$

Justine: $\$624,000+\$540,000=\$1,164,000$

Fair share: $\frac{1}{3}(\$1,164,000)=\$388,000$

Colby: $\$540,000+\$545,000=\$1,085,000$

Fair share: $\frac{1}{3}(\$1,085,000)=\$361,667$

Difference between fair share and item won.
Amelia: $330,000 – $0 = $330,000
Justine: $388,000 – $624,000 = –$236,000
Colby: $361,667 – $545,000 = –$183,333

Estate surplus:
–$330,000 + $236,000 + $183,333 = $89,333

Equal amount = $\frac{1}{3}(\$89,333) = \$29,777$

Amounts owed to/from estate:
Amelia: $330,000 + $29,777 = $359,777
Justine: –$236,000 + $29,777 = –$206,223
Colby: –$183,333 + $29,777 = –$153,556

Amelia receives $359,777 from the estate. Justine gets the medical suites and pays $206,223 into the estate. Colby gets the real estate and pays $153,556 into the estate.

20. Determine highest bidder.

	Lois	Juliette	Melody	High
Condo	$177,000	$222,000	$195,000	Juliette
Car	$18,000	$9000	$12,000	Lois
Yacht	$51,000	$66,000	$75,000	Melody
Ring	$12,000	$12,600	$9000	Juliette

Lois: $177,000 + $18,000 + $51,000 + $12,000 = $258,000

Fair share: $\frac{1}{3}(\$258,000) = \$86,000$

Juliette: $222,000 +$9000 +$66,000 +$12,600 = $309,600

Fair share: $\frac{1}{3}(\$309,600) = \$103,200$

Melody: $195,000 +$12,000 +$75,000 +$9000 = $291,000

Fair share: $\frac{1}{3}(\$291,000) = \$97,000$

Difference between fair share and item won.
Lois: $86,000 – $18,000 = $68,000
Juliette: $103,200 – $222,000 – $12,600 = –$131,400
Melody: $97,000 – $75,000 = $22,000

Estate surplus:
–$68,000 + $131,400 – $22,000 = $41,400

Equal amount = $\frac{1}{3}(\$41,400) = \$13,800$

Amounts owed to/from estate:
Lois: $68,000 + $13,800 = $81,800
Juliette: –$131,400 + $13,800 = –$117,600
Melody: $22,000 + $13,800 = $35,800

Lois gets the car and receives $81,800 from the estate. Juliette gets the condo, the emerald ring, and pays $117,600 into the estate. Melody gets the yacht and receives $35,800 from the estate.

21. Determine highest bidder. (in thousands)

	Leonard	Taylor	Madison	High
Home	$600	$520	$475	Leonard
SUV	$12	$10	$14	Madison
Boat	$58	$70	$50	Taylor
Paint	$110	$120	$121	Madison

Leonard: $600 + $12 + $58 + $110 = $780

Fair share: $\frac{1}{3}(\$780) = \$260$

Taylor: $520 + $10 + $70 + $120 = $720

Fair share: $\frac{1}{3}(\$720) = \$240$

Madison: $475 + $14 + $50 + $121 = $660

Fair share: $\frac{1}{3}(\$660) = \$220$

Difference between fair share and item won.
Leonard: $260 – $600 = –$340
Taylor: $240 – $70 = $170
Madison: $220 – $14 – $121 = $85

Estate surplus:
$340 – $170 – $85 = $85

Equal amount = $\frac{1}{3}(\$85) = \$28.333$

Amounts owed to/from estate:
Leonard: –$340 + $28.333 = –$311.667
Taylor: $170 + $28.333 = $198.333
Madison: $85 + $28.333 = $113.333

Leonard gets the home and pays $311,667 into the estate. Taylor gets the boat and receives $198,333 from the estate. Madison gets the SUV and painting and receives $113,333 from the estate.

22. Determine who gets the room.

	Jacquelyn	Maria	Petra	Room to
1	$700	$600	$800	Petra
2	$600	$900	$700	Maria
3	$800	$600	$600	Jacquelyn

	Jacquelyn	Maria	Petra
Total Valuation	$2100	$2100	$2100
Fair share	$700	$700	$700
Value of bd won	$800	$900	$800
Owed to rent	–$100	–$200	–$100
Receives from rent	0	0	0
Share of surplus	$133	$133	$133
Disposition of money	$33	–$67	$33

Rent surplus:
$100 + $200 + $100 = $400

Equal amount = $\frac{1}{3}(\$400) = \$133$

Rent paid:
Jacquelyn: $700 – $33 = $667
Maria: $700 + $67 = $767
Petra: $700 – $33 = $667

Jacquelyn gets bedroom 1 and pays $667. Maria gets bedroom 2 and pays $766. (She saves the extra dollar.)
Petra gets bedroom 3 and pays $667.

23. Determine who gets the room.

	Clara	Manuel	Charles	Beatrice	Room to
1	\$500	\$800	\$500	\$600	Manuel
2	\$600	\$400	\$500	\$700	Beatrice
3	\$750	\$700	\$900	\$800	Charles
4	\$650	\$600	\$600	\$400	Clara

	Clara	Manuel	Charles	Beatrice
Total Valuation	\$2500	\$2500	\$2500	\$2500
Fair share	\$625	\$625	\$625	\$625
Value of bd won	\$650	\$800	\$900	\$700
Owed to rent	–\$25	–\$175	–\$275	–\$75
Receives from rent	0	0	0	0
Share of surplus	\$137.5	\$137.5	\$137.5	\$137.5
Disposition of money	\$112.5	–\$37.5	–\$137.5	\$62.5

Rent surplus:
\$25 + \$175 + \$275 + \$75 = \$550

Equal amount = $\frac{1}{4}(\$550) = \$137.50$

Rent paid:
Clara: \$625 – \$112.50 = \$512.50
Manuel: \$625 + \$37.50 = \$662.50
Charles: \$625 + \$137.50 = \$762.50
Beatrice: \$625 – \$62.50 = \$562.50

Clara gets bedroom 4 and pays \$512.50.
Manuel gets bedroom 1 and pays \$662.50.
Charles gets bedroom 3 and pays \$762.50.
Beatrice gets bedroom 2 and pays \$562.50.

24. Determine winner:

	Sebastian	Annabel	Winner
Ferrari	\$240,000	\$250,000	Annabel

$$\frac{n-1}{n} = \frac{2-1}{2} = \frac{1}{2}$$

$$\frac{1}{2}(250,000) = 125,000$$

Escrow account contributions:

Sebastian: $\frac{1}{2}(-240,000) = -120,000$

Annabel: 125,000
Total in escrow: –120,000 + 125,000 = 5000
Divide 5000 by 2 = 2500.
Final net amount:
Sebastian: 120,000 + 2500 = 122,500
Annabel: Ferrari – 125,000 + 2500
= Ferrari – 122,500
Verify the division is fair:

Sebastian: $\frac{122,500}{240,000} \approx 0.51$

Annabel: $\frac{127,500}{250,000} = 0.51$

The division is fair.

Annabel receives Ferrari and pays \$122,500.
Sebastian receives \$122,500.

25. Determine winner:

	Lenore	Raven	Winner
Chalet	\$500,000	\$600,000	Raven

$$\frac{n-1}{n} = \frac{2-1}{2} = \frac{1}{2}$$

$$\frac{1}{2}(600,000) = 300,000$$

Escrow account contributions:

Lenore: $\frac{1}{2}(-500,000) = -250,000$

Raven: 300,000
Total in escrow: –250,000 + 300,000 = 50,000
Divide 50,000 by 2 = 25,000.
Final net amount:
Lenore: 250,000 + 25,000 = 275,000
Raven: chalet – 300,000 + 25,000
= chalet – 275,000
Verify the division is fair:

Lenore: $\frac{275,000}{500,000} = 0.55$

Raven: $\frac{325,000}{600,000} \approx 0.54$

The division is fair.

Raven receives chalet and pays \$275,000.
Lenore receives \$275,000.

26. Determine winner:
Item 1 is diamond pendant, Item 2 is estate.

	Megan	Hudson	Jeremiah	Winner
1	\$24,000	\$30,000	\$21,000	Hudson
2	\$360,000	\$300,000	\$330,000	Megan

$$\frac{n-1}{n} = \frac{3-1}{3} = \frac{2}{3}$$

Item 1: diamond pendant
Escrow account contributions:

Megan receives: $\frac{1}{3}(24,000) = 8000$

Hudson owes: $\frac{2}{3}(30,000) = 20,000$

Jeremiah receives: $\frac{1}{3}(21,000) = 7000$

Money left in the joint account:
–8000 + 20,000 – 7000 = 5000
Divide 5000 by 3 = 1667.
Final accounting for the diamond pendant:
Megan: 8000 + 1667 = \$9667
Hudson: pendant – 20,000 + 1667
= pendant – \$18,333
Jeremiah: 7000 + 1667 = \$8667

Item 2: estate (country house)
Escrow account contributions:

Megan owes: $\frac{2}{3}(360,000) = 240,000$

Hudson receives: $\frac{1}{3}(300,000) = 100,000$

Jeremiah receives: $\frac{1}{3}(330,000) = 110,000$

Money left in the joint account:
240,000 – 100,000 – 110,000 = 30,000
Divide 30,000 by 3 = 10,000.

Final accounting for the estate:
Megan: estate – 240,000 + 10,000
= estate – $230,000
Hudson: 100,000 + 10,000 = $110,000
Jeremiah: 110,000 + 10,000 = $120,000

Disposition of pendant, estate, and money:
Megan: 9667 + estate – $230,000
= estate – $220,333
Hudson: pendant – $18,333 + $110,000
= pendant + $91,667
Jeremiah: $8667 + $120,000 = $128,667

Thus, Megan receives the estate and pays $220,333. Hudson receives the pendant and receives $91,667. Jeremiah receives $128,667.

27. Determine winner:
Item 1 is sailboat. Item 2 is RV. Item 3 is Cabin.

	Marshall	Jordan	Jan	Winner
1	$75,000	$60,000	$63,000	Marshall
2	$66,000	$63,000	$57,000	Marshall
3	$87,000	$90,000	$75,000	Jordan

$\frac{n-1}{n} = \frac{3-1}{3} = \frac{2}{3}$

Item 1: sailboat
Escrow account contributions:

Marshall owes: $\frac{2}{3}(75,000) = 50,000$

Jordan receives: $\frac{1}{3}(60,000) = 20,000$

Jan receives: $\frac{1}{3}(63,000) = 21,000$

Money left in the joint account:
50,000 – 20,000 – 21,000 = 9000
Divide 9000 by 3 = 3000.

Final accounting for the sailboat:
Marshall: sailboat – 50,000 + 3000
= sailboat – $47,000
Jordan: 20,000 + 3000 = $23,000
Jan: 21,000 + 3000 = $24,000

Item 2: RV
Escrow account contributions:

Marshall owes: $\frac{2}{3}(66,000) = 44,000$

Jordan receives: $\frac{1}{3}(63,000) = 21,000$

Jan receives: $\frac{1}{3}(57,000) = 19,000$

Money left in the joint account:
44,000 – 21,000 – 19,000 = 4000
Divide 4000 by 3 = 1333.

Final accounting for the RV:
Marshall: RV – 44,000 + 1333
= RV – $42,667
Jordan: 21,000 + 1333 = $22,333
Jan: 19,000 + 1333 = $20,333

Item 3: cabin
Escrow account contributions:

Marshall receives: $\frac{1}{3}(87,000) = 29,000$

Jordan owes: $\frac{2}{3}(90,000) = 60,000$

Jan receives: $\frac{1}{3}(75,000) = 25,000$

Money left in the joint account:
–29,000 + 60,000 – 25,000 = 6000
Divide 6000 by 3 = 2000.

Final accounting for the cabin:
Marshall: 29,000 + 2000 = 31,000
Jordan: cabin – 60,000 + 2000
= cabin – $58,000
Jan: 25,000 + 2000 = $27,000

Disposition of sailboat, RV, cabin, and money:
Marshall:
sailboat – 47,000 + RV – 42,667 + 31,000
= sailboat + RV – $58,667
Jordan: 23,000 + 22,333 + cabin – 58,000
= cabin – $12,667
Jan: 24,000 + 20,333 + 27,000 = $71,333

Thus, Marshall receives the sailboat and RV and pays $58,667. Jordan receives the cabin and pays $12,667. Jan receives $71,333.

28. Determine winner:
Item 1 is condo. Item 2 is car. Item 3 is yacht. Item 4 is emerald ring. (Amounts in thousands)

	Lois	Juliette	Melody	Winner
1	$177	$222	$195	Juliette
2	$18	$9	$12	Lois
3	$51	$66	$75	Melody
4	$12	$12.6	$9	Juliette

$\frac{n-1}{n} = \frac{3-1}{3} = \frac{2}{3}$

Item 1: condo
Escrow account contributions:

Lois receives: $\frac{1}{3}(177) = 59$

Juliette owes: $\frac{2}{3}(222) = 148$

Melody receives: $\frac{1}{3}(195) = 65$

Money left in the joint account:
–59 + 148 – 65 = 24
Divide 24 by 3 = 8.

Final accounting for the condo:
Lois: 59 + 8 = $67
Juliette: condo – 148 + 8

= condo – $140
Melody: 65 + 8 = $73

Item 2: car
Escrow account contributions:

Lois owes: $\frac{2}{3}(18) = 12$

Juliette receives: $\frac{1}{3}(9) = 3$

Melody receives: $\frac{1}{3}(12) = 4$

Money left in the joint account:
12 – 3 – 4 = 5
Divide 5 by 3 = 1.667
Final accounting for the car:
Lois: car – 12 + 1.667
= car – $10.333
Juliette: 3 + 1.667 = $4.667
Melody: 4 + 1.667 = $5.667

Item 3: yacht
Escrow account contributions:

Lois receives: $\frac{1}{3}(51) = 17$

Juliette receives: $\frac{1}{3}(66) = 22$

Melody owes: $\frac{2}{3}(75) = 50$

Money left in the joint account:
–17 – 22 + 50 = 11
Divide 11 by 3 = 3.667
Final accounting for the yacht:
Lois: 17 + 3.667 = $20.667
Juliette: 22 + 3.667 = $25.667
Melody: yacht – 50 + 3.667
= yacht – $46.333

Item 4: emerald ring
Escrow account contributions:

Lois receives: $\frac{1}{3}(12) = 4$

Juliette owes: $\frac{2}{3}(12.6) = 8.4$

Melody receives: $\frac{1}{3}(9) = 3$

Money left in the joint account:
–4 + 8.4 – 3 = 1.4
Divide 1.4 by 3 = 0.467
Final accounting for the emerald ring:
Lois: 4 + 0.467 = $4.467
Juliette: ring – 8.4 + 0.467
= ring – $7.933
Melody: 3 + 0.467 = $3.467

Disposition of condo, car, yacht, emerald ring, and money:
Lois: 67 + car – 10.333 + 20.667 + 4.467
= car + $81.801
Juliette:
condo –140 +4.667 + 25.667 + ring – 7.933
= condo + ring – $117.599
Melody: 73 + 5.667 + yacht – 46.333 + 3.467
= yacht + 35.801

Thus, Lois receives the car and $81,801. Juliette receives the condo, the emerald ring and pays $117,599. Melody receives the yacht and $35,801.

There is no difference from Exercise 20.

29. Determine winner:
Item 1 is piano. Item 2 is organ. Item 3 is studio. Item 4 is condo. (Amounts in thousands)

	June	Melissa	Lloyd	Diane	Winner
1	$15	$10	$12	$10	June
2	$9	$12	$10	$9	Melissa
3	$195	$225	$175	$200	Melissa
4	$175	$250	$225	$200	Melissa

$$\frac{n-1}{n} = \frac{4-1}{4} = \frac{3}{4}$$

Item 1: piano
Escrow account contributions:

June owes: $\frac{3}{4}(15) = 11.25$

Melissa receives: $\frac{1}{4}(10) = 2.5$

Lloyd receives: $\frac{1}{4}(12) = 3$

Diane receives: $\frac{1}{4}(10) = 2.5$

Money left in the joint account:
11.25 – 2.5 – 3 – 2.5 = 3.25
Divide 3.25 by 4 = 0.8125
Final accounting for the piano:
June: piano – 11.25 + 0.8125
= piano – $10.4375
Melissa: 2.5 + 0.8125 = $3.3125
Lloyd: 3 + 0.8125 = $3.8125
Diane: 2.5 + 0.8125 = $3.3125

Item 2: organ
Escrow account contributions:

June receives: $\frac{1}{4}(9) = 2.25$

Melissa owes: $\frac{3}{4}(12) = 9$

Lloyd receives: $\frac{1}{4}(10) = 2.5$

Diane receives: $\frac{1}{4}(9) = 2.25$

Money left in the joint account:
–2.25 + 9 – 2.5 – 2.25 = 2
Divide 2 by 4 = 0.5
Final accounting for the organ:
June: 2.25 + 0.5 = $2.75
Melissa: organ – 9 + 0.5
= organ – $8.5
Lloyd: 2.5 + 0.5 = $3
Diane: 2.25 + 0.5 = $2.75

Item 3: studio

Escrow account contributions:

June receives: $\frac{1}{4}(195) = 48.75$

Melissa owes: $\frac{3}{4}(225) = 168.75$

Lloyd receives: $\frac{1}{4}(175) = 43.75$

Diane receives: $\frac{1}{4}(200) = 50$

Money left in the joint account:
–48.75 + 168.75 – 43.75 – 50 = 26.25
Divide 26.25 by 4 = 6.5625

Final accounting for the studio:
June: 48.75 + 6.5625 = $55.3125
Melissa: studio – 168.75 + 6.5625
= studio – $162.1875
Lloyd: 43.75 + 6.5625 = $50.3125
Diane: 50 + 6.5625 = $56.5625

Item 4: condo

Escrow account contributions:

June receives: $\frac{1}{4}(175) = 43.75$

Melissa owes: $\frac{3}{4}(250) = 187.5$

Lloyd receives: $\frac{1}{4}(225) = 56.25$

Diane receives: $\frac{1}{4}(200) = 50$

Money left in the joint account:
–43.75 + 187.5 – 56.25 – 50 = 37.5
Divide 37.5 by 4 = 9.375

Final accounting for the condo:
June: 43.75 + 9.375 = $53.125
Melissa: condo – 187.5 + 9.375
= studio – $178.125
Lloyd: 56.25 + 9.375= $65.625
Diane: 50 + 9.375= $59.375

Disposition of piano, organ, studio, condo, and money:
June:
piano – 10.4375 + 2.75 +55.3125 + 53.125
= piano + $100.75
Melissa: 3.3125 + organ – 8.5 + studio
– 162.1875 + condo – 178.125
= organ + studio + condo – $345.5
Lloyd: 3.8125 + 3 + 50.3125 + 65.625
= $122.75
Diane: 3.3125 + 2.75 + 56.5625 + 59.375
= $122

Thus, June receives the piano and $100,750. Melissa receives the organ, studio, and condo and pays $345,500. Lloyd receives $122,750. Diane receives $122,000.

CHAPTER 1 REVIEW EXERCISES

1. $\frac{1}{32}$ Multiply by $\frac{1}{2}$.

2. –64 Multiply by –2.

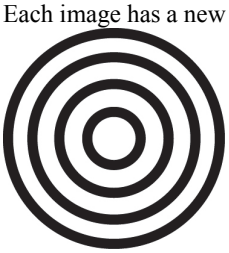
3. –2. Subtract 3, then add 2, then subtract 3, then add 2, etc.

4. 64. Multiply by 4, then divide by 2, then multiply by 4, then divide by 2, etc.

5. 1000
This is counting with binary numbers.

6. 1 6 15 20 15 6 1
This is Pascal's triangle.
Start the row with 1, then add the two numbers above to get 6 (1+5), 15 (5+10), 20 (10+10), 15 (10+5), 6 (5+1), and then end with 1.

7. Each image has a new circle.



8. Each image has 2, 4, 8 gray rectangles. The next image will have 16 gray rectangles. Each image has one less white rectangle than gray rectangles: 1, 3, 7. The next image will have 15 white rectangles.

9. Each image has a 6-sided figure. Figures alternate gray interior and white interior. The square and triangles move in counter-clockwise position.

10. Each image has a square. The black interior square moves in clockwise direction. The gray interior circle moves in counter-clockwise direction.

11. More than. From the graph, it appears that as the Number of Units Willing to Produce increases, the Cost Per Unit increases.

12. 10 cases. From the bar graph, it appears that as the Months increases, the number of cases decreases by 3.

13. More than. From the table, the time (in seconds) is increasing by 0.5 second. The distance (in meters) 0^2, 2^2, 4^2, ... , 16^2. So at 4.5 seconds the distance is 18^2, or 324 meters, which is more than 316 meters.

14. Less than. From the table, as the Depth (in meters) increases, the Percent of Light decreases. Since the last entry is 160 meters and 9% light, it is expected that for 180 meters, the percent of light will be less than 9%.

15. Answers may vary. One possibility is 0.5.

16. 0 does not have a reciprocal.

17. Answers may vary. One possibility is $-3(6) = -18$.

18. Answers may vary. One possibility is rectangle 1 has sides 10, 40, 10, and 40. The perimeter is 100 units; the area is 400 square units. Rectangle 2 has sides 20, 30, 20, and 30. The perimeter is 100 units; the area is 600 square units.

19.

	CS	Chem	Bus	Bio
M	X1	X3	X3	✓
C	X4	X2	✓	X2
R	✓	X2	X4	X2
E	X3	✓	X3	X3

Michael is a Biology major; Clarissa is a Business major; Reggie is a Computer Science major; Ellen is a Chemistry major.

20.

	Bank	Super	Service	Drug
D	X4	X3	X3	✓
P	X2	✓	X4	X2
T	✓	X3	X1	X4
G	X2	X3	✓	X2

Dodgers sponsored by drugstore; Pirates sponsored by supermarket; Tigers sponsored by bank; Giants sponsored by service station.

21. First weighing: Place 5 coins at each side of the scale. Exclude all the 5 coins in the side that goes up (the heavier coin is on the pan that went down). Now you have only 5 coins.
Second Weighing: Hold one coin in your hand and put 2 coins at each side of the scale. If the two sides balance, then the heavier one is the one you are holding in your hand. You are done. If one side is heavier than the other, one of the two coins is on that side is the heavier one. Now you have only 2 coins and a scale to find out which is the heavier by using the scale for a third time.

22. $500 \text{ bricks} \times \dfrac{5 \text{ pounds}}{1 \text{ brick}} = 2500 \text{ pounds}$
No, since 2500 > 2000 the truck cannot haul the pallet of bricks.

23. $1 \text{ inch} > 2.5 \text{ cm}$
$10(1 \text{ inch}) > 10(2.5 \text{ cm})$
$10 \text{ inches} > 25 \text{ cm}$
More than

24. $\dfrac{3000 \text{ words}}{300 \text{ words/minute}} = 10 \text{ minutes}$

25. Answers will vary depending on the grid. Using a 5 by 5 grid, we count 23 berries. Multiply 23 by the number of sections, 25, to get 575. So our estimate is 575 berries.

26. $72 \text{ inches} \cdot \dfrac{1}{8} = 9 \text{ inches}$

27. $41{,}320{,}000{,}000{,}000 = 4.132 \times 10^{13}$

28. $0.00000000012 = 1.2 \times 10^{-10}$

29. The difference, 7, is greatest between 2015 and 2016.

30. Physicians, since that is the largest section of the chart.

31. a. The largest decrease is between Years 0 and 1.

    b. The end of Year 4 is the beginning of Year 5, which is about \$14,000. The car's original value is \$35,000. The car's value is less than one-half of the original value.

32. If 11 is across from 35, then there are
$(35 - 11) - 1 = 23$ people
on each side of person 11 and person 35. So the total number of people is $2(23) + 2 = 48$ people.

33. Seth and Sarah cross the bridge, and Sarah returns. Time for round trip, 3 minutes.
Kevin and Asher cross the bridge, and Seth returns. Time for round trip is 12 minutes.
Total elapsed time is 15 minutes.
Now Sarah and Seth cross the bridge. Total elapsed time is 17 minutes.

34. Start with 9 red balls and 9 blue balls. Because there is an odd number of each color of ball, the last two balls selected from the box will be different colors, resulting in one red ball in the box.

35. No. A fair share is \$6. If piece 1 is one-third pepperoni and one-sixth cheese, then the value of the piece is

$$\frac{1}{3}(9)+\frac{1}{6}(3)=3+2=5<6.$$

36. Answers may vary. Here is one possibility.

	S_1	S_2	S_3	S_4	Bids
J	25%	25%	25%	25%	S_1, S_2, S_3, S_4
Cl	15%	35%	20%	30%	S_2, S_4
A	15%	45%	20%	20%	S_2
Ca	20%	35%	10%	35%	S_2, S_4

Jeffrey only values S_3, so he receives S_3.

Now combine the remaining shares and randomly select a new divider. Suppose Clara is chosen as the divider and separates the assets into shares that to her are equal. Alyssa and Cameron bid on the shares as shown in the following table. *This is where solutions may vary. It will depend on how you assign the bids. This is just one possibility.*

	T_1	T_2	T_3	Bids
Cl	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	T_1, T_2, T_3
A	25%	40%	35%	T_2, T_3
Ca	20%	35%	45%	T_2, T_3

Because T_2 and T_3 are contested and T_1 is uncontested by the choosers, award T_1 to Clara.

Now use the divider-chooser method for two people, Alyssa and Cameron, to divide the remaining two shares; call them U_1 and U_2. We will assume Alyssa divides and Cameron chooses U_2. The final result: Cameron gets U_2, Alyssa gets U_1, Clara gets T_1, and Jeffrey gets S_3.

37. Assign initial winners:

	Alana	Taft	Winner
Condo	40	50	Taft
Motorcycle	10	5	Alana
Piano	25	20	Alana
Painting	25	25	tie
Total:	35	50	

Taft is the initial winner and Alana is the initial loser.

The sharing procedure:

$$50+25(1-x)=10+25+25x$$
$$50+25-25x=10+25+25x$$
$$75-25x=35+25x$$
$$40=50x$$
$$0.80=x$$

For the painting, 80% will go to Alana, and 20% will go to Taft.

Distribute the estate: Alana gets the motorcycle, piano, and 80% from the sale of the painting. Taft gets the condo and 20% of the sale of the painting.

38. Determine highest bidder.

	House	Condo
Miguel	\$550,000	\$300,000
Cannon	\$450,000	\$350,000
Ralph	\$350,000	\$400,000
Highest bidder	Miguel	Ralph

Miguel: $\$550,000+\$300,000=\$850,000$

Fair share: $\frac{1}{3}(\$850,000)\approx\$283,333$

Cannon: $\$450,000+\$350,000=\$800,000$

Fair share: $\frac{1}{3}(\$800,000)=\$266,667$

Ralph: $\$350,000+\$400,000=\$750,000$

Fair share: $\frac{1}{3}(\$750,000)=\$250,000$

Difference between fair share and item won.
Miguel: \$283,333 – \$550,000 = –\$266,667
Cannon: \$266,667 – \$0 = \$266,667
Ralph: \$250,000 – \$400,000 = –\$150,000

Estate surplus:
\$266,667 – \$266,667 + \$150,000 = \$150,000

Equal amount = $\frac{1}{3}(\$150,000)=\$50,000$

Amounts owed to/from estate:
Miguel: –\$266,667 + \$50,000 = –\$216,667
Cannon: \$266,667 + \$50,000 = \$316,667
Ralph: –\$150,000 + \$50,000 = –\$100,000

Miguel gets the house and pays \$216,667 into the estate. Cannon receives \$316,667 from the estate. Ralph gets the condo and pays \$100,000 into the estate.

39. Determine who gets the room.

	Thomas	Michael	Claudia	Room to
Garden	\$1000	\$1100	\$1000	Michael
Skyline	\$900	\$900	\$1000	Claudia
No view	\$800	\$700	\$700	Thomas

	Thomas	Michael	Claudia
Total Valuation	$2700	$2700	$2700
Fair share	$900	$900	$900
Value of bd won	$800	$1100	$1000
Owed to rent		–$200	–$100
Receives from rent	$100	0	0
Share of surplus	$67	$67	$67
Disposition of money	$167	–$133	–$33

Rent surplus:
–$100 + $200 + $100 = $200

Equal amount = $\frac{1}{3}(\$200) = \$67$

Rent paid:
Thomas: $900 – $167 = $733
Michael: $900 + $133 = $1033
Claudia: $900 + $33 = $933

Thomas gets the no view and pays $733.
Michael gets the garden view and pays $1033.
Claudia gets the skyline view and pays $933.

40. Determine winner:
Item 1 is vase. Item 2 is violin. Item 3 is painting. (Amounts in thousands)

	Jocelyn	Harrison	Colby	Winner
1	$810	$900	$750	Harrison
2	$1200	$1500	$1200	Harrison
3	$1500	$1650	$1800	Colby

$$\frac{n-1}{n} = \frac{3-1}{3} = \frac{2}{3}$$

Item 1: vase
Escrow account contributions:

Jocelyn receives: $\frac{1}{3}(810) = 270$

Harrison owes: $\frac{2}{3}(900) = 600$

Colby receives: $\frac{1}{3}(750) = 250$

Money left in the joint account:
–270 + 600 – 250 = 80
Divide 80 by 3 = 26.666
Final accounting for the vase:
Jocelyn: 270 + 26.666 = $296.666
Harrison: vase – 600 + 26.666
= vase – $573.334
Colby: 250 + 26.666 = $276.666

Item 2: violin
Escrow account contributions:

Jocelyn receives: $\frac{1}{3}(1200) = 400$

Harrison owes: $\frac{2}{3}(1500) = 1000$

Colby receives: $\frac{1}{3}(1200) = 400$

Money left in the joint account:
–400 + 1000 – 400 = 200
Divide 200 by 3 = 66.666.
Final accounting for the violin:
Jocelyn: 400 + 66.666 = $466.666
Harrison: violin – 1000 + 66.666
= violin –$933.334
Colby: 400 + 66.666 = $466.666

Item 3: painting
Escrow account contributions:

Jocelyn receives: $\frac{1}{3}(1500) = 500$

Harrison receives: $\frac{1}{3}(1650) = 550$

Colby owes: $\frac{2}{3}(1800) = 1200$

Money left in the joint account:
–500 – 550 + 1200 = 150
Divide 150 by 3 = 50.
Final accounting for the painting:
Jocelyn: 500 + 50 = $550
Harrison: 550 + 50 = $600
Colby: painting – 1200 + 50
= painting – $1150

Disposition of vase, violin, painting, and money:
Jocelyn: 296.666 + 466.666 + 550
= $1313.332
Harrison:
vase – 573.334 + violin – 933.334 + 600
= vase + violin – $906.668
Colby:
276.666 + 466.666 + painting – 1150
= painting – $406.668

Thus, Jocelyn receives $1,313,332. Harrison receives the vase and violin and pays $906,668. Colby receives the painting and pays $406,668.

41. Determine winner:
Item 1 is Bentley. Item 2 is condo. Item 3 is harp. Item 4 is sculpture. (Amounts in thousands)

	Santoro	Ben	Bianca	Vivian	Winner
1	$150	$125	$120	$130	Santoro
2	$1000	$1200	$1500	$1300	Bianca
3	$12	$12	$8	$16	Vivian
4	$1500	$2000	$1800	$1600	Ben

$$\frac{n-1}{n} = \frac{4-1}{4} = \frac{3}{4}$$

Item 1: Bentley
Escrow account contributions:

Santoro owes: $\frac{3}{4}(150) = 112.5$

Ben receives: $\frac{1}{4}(125) = 31.25$

Bianca receives: $\frac{1}{4}(120) = 30$

Vivian receives: $\frac{1}{4}(130) = 32.5$

Money left in the joint account:
112.5 – 31.25 – 30 – 32.5 = 18.75
Divide 18.75 by 4 = 4.6875

Final accounting for the Bentley:
Santoro: Bentley – 112.5 + 4.6875
= Bentley – $107.8125
Ben: 31.25 + 4.6875 = $35.9375
Bianca: 30 + 4.6875 = $34.6875
Vivian: 32.5 + 4.6875 = $37.1875

Item 2: condo
Escrow account contributions:

Santoro receives: $\frac{1}{4}(1000) = 250$

Ben receives: $\frac{1}{4}(1200) = 300$

Bianca owes: $\frac{3}{4}(1500) = 1125$

Vivian receives: $\frac{1}{4}(1300) = 325$

Money left in the joint account:
–250 – 300 +1125 – 325 = 250
Divide 250 by 4 = 62.5

Final accounting for the condo:
Santoro: 250 + 62.5 = $312.5
Ben: 300 + 62.5 = $362.5
Bianca: condo – 1125 + 62.5
= condo – $1062.5
Vivian: 325 + 62.5 = $387.5

Item 3: harp
Escrow account contributions:

Santoro receives: $\frac{1}{4}(12) = 3$

Ben receives: $\frac{1}{4}(12) = 3$

Bianca receives: $\frac{1}{4}(8) = 2$

Vivian owes: $\frac{3}{4}(16) = 12$

Money left in the joint account:
–3 – 3 – 2 + 12 = 4
Divide 4 by 4 = 1

Final accounting for the harp:
Santoro: 3 + 1 = $4
Ben: 3 + 1 = $4
Bianca: 2 + 1 = $3
Vivian: harp – 12 + 1 = harp – $11

Item 4: sculpture
Escrow account contributions:

Santoro receives: $\frac{1}{4}(1500) = 375$

Ben owes: $\frac{3}{4}(2000) = 1500$

Bianca receives: $\frac{1}{4}(1800) = 450$

Vivian receives: $\frac{1}{4}(1600) = 400$

Money left in the joint account:
–375 + 1500 – 450 – 400 = 275
Divide 275 by 4 = 68.75

Final accounting for the sculpture:
Santoro: 375 + 68.75 = $443.75
Ben: sculpture – 1500 + 68.75
= sculpture – $1431.25
Bianca: 450 + 68.75 = $518.75
Vivian: 400 + 68.75 = $468.75

Disposition of Bentley, condo, harp, sculpture, and money:
Santoro: Bentley–107.8125+312.5 +4+443.75
= Bentley + $652.4375
Ben: 35.9375 +362.5 +4 +sculpture –1431.25
= sculpture – $1028.8125
Bianca:
34.6875 + condo – 1062.5 + 3 + 518.75
= condo – $506.0625
Vivian:
37.1875 + 387.5 + harp – 11 + 468.75
= harp + $882.4375

Thus, Santoro receives the Bentley and $652,437.50. Ben receives the sculpture and pays $1,028,812.50. Bianca receives the condo and pays $506,062.50. Vivian receives the harp and receives $882,437.50.

Chapter 2: Sets and Logic

THINK ABOUT IT, SECTION 2.1

1. a. True
   b. True
   c. True
   d. False
   e. False
   f. True
   g. True
   h. True

2. a. No
   b. Yes

3. A and B are disjoint—that is, they have no elements in common.

4. $(A')' = A$

5. False negative

6. Correct outcome

EXERCISE SET 2.1

1. {penny, nickel, dime, quarter, 50 cent piece}

2. {January, February, May, July}

3. {Mercury, Mars}

4. The negative integers greater than –6 are –5, –4, –3, –2, –1. Using the roster method, write the set as {–5, –4, –3, –2, –1}.

5. {0, 1, 2, 3, 4, 5, 6, 7}

6. Adding 4 to each side of the equation produces $x = 7$. {7} is the solution set.

7. Solving:
   $2x - 1 = -11$
   $2x = -10$
   $x = -5$
   So {–5} is the solution set.

8. Solving:
   $x + 4 < 1$
   $x < -3$
   But any number less than –3 is not a natural number so the solution set is empty, Ø.

9. Because b is an element of the given set, the statement is true.

10. False; 0 is not a natural number.

11. True

12. False; the empty set has no elements.

13. True

14. True

15. False; although $\{5\} \quad \{5\} \subset \{1,\ 2,\ 3,\ 4,\ 5\}$.

16. False; although $\{a, b, c\} = \{a, b, c\}$, $\{a,\ b,\ c\} \not\subset \{a,\ b,\ c\}$.

17. The complement of {2, 4, 6, 7} contains elements in U but not in the set: {0, 1, 3, 5, 8}.

18. $\{3, 6\}' = \{0, 1, 2, 4, 5, 7, 8\}$

19. $\varnothing' = U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

20. $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}' = U' = \varnothing$

21. {prime numbers}′ = {0, 1, 4, 6, 8}

22. {composite numbers}′ = {0, 1, 2, 3, 5, 7}

23. Ø, {a}, {z}, {a, z}

24. Ø, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}

25. Ø, {I}, {II}, {III}, {IV}, {I, II}, {I, III}, {I, IV}, {II, III}, {II, IV}, {III, IV}, {I, II, III}, {I, II, IV}, {I, III, IV}, {II, III, IV}, {I, II, III, IV}

26. Ø

27. Since there are 5 elements in the set {a, e, i, o, u}, there are $2^5 = 32$ subsets.

28. Since there are 7 elements in the set {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}, there are $2^7 = 128$ subsets.

29. $2^{12} = 4{,}096$ different versions

30. a. $2^{10} = 1{,}024$ types of omelets
    b. Solve $2^x > 4{,}000$ by guessing and checking.
    $2^{11} = 2{,}048$
    $2^{12} = 4{,}096$
    At least 12 ingredients must be available.

31. $A \cup B = \{2, 4, 6\} \cup \{1, 2, 5, 8\}$
$= \{1, 2, 4, 5, 6, 8\}$

32. $A \cap B = \{2, 4, 6\} \cap \{1, 2, 5, 8\} = \{2\}$

33. $A \cap B' = \{2, 4, 6\} \cap \{3, 4, 6, 7\} = \{4, 6\}$

34. $B \cap C' = \{1, 2, 5, 8\} \cap \{2, 4, 5, 6, 8\}$
$= \{2, 5, 8\}$

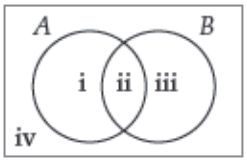
35. $(A \cup B)' = (\{1, 2, 4, 5, 6, 8\})' = \{3, 7\}$

36. $A' = \{1, 3, 5, 7, 8\}$.
$B' = \{3, 4, 6, 7\}$
$A' \cup B' = \{1, 3, 5, 7, 8,\} \cup \{3, 4, 6, 7\}$
$= \{1, 3, 4, 5, 6, 7, 8\}$.

37. $B \cap C = \{1, 2, 5, 8\} \cap \{1, 3, 7\} = \{1\}$.
$A \cap (B \cap C) = \{2, 4, 6\} \cap \{1\} = \emptyset$

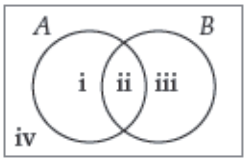
38. $C \cap C' = \{1, 3, 7\} \cap \{2, 4, 5, 6, 8\} = \emptyset$

39. True



$A \cap B$: ii
Since ii $\subseteq A$, then $A \cap B \subseteq A$ is true.

40. True

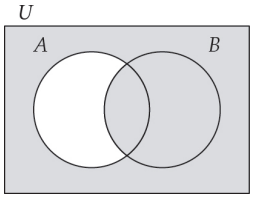


$A \cup B$: i, ii, iii
Since $A \subseteq$ i, ii, iii, then $A \subseteq A \cup B$ is true.

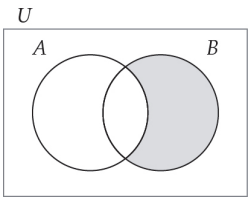
41. True
Since $A \subseteq U$, then $A \cap U = A$ is true.

42. False
Since $B \subseteq U$, then $B \cup U = U$.

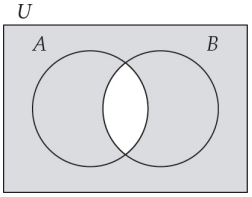
43. $A' \cup B$



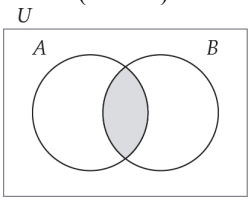
44. $A' \cap B$



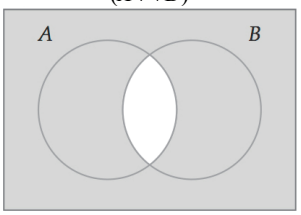
45. $A' \cup B'$



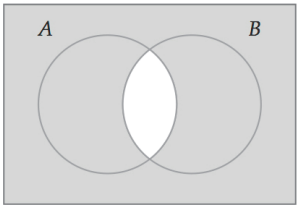
46. $(A' \cup B')'$



47. $(A \cap B)'$

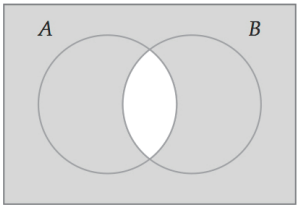


$A' \cup B'$

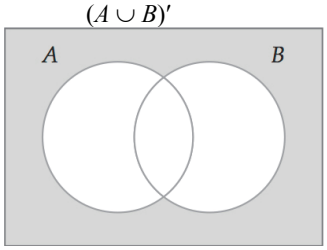


Therefore, $(A \cap B)' = A' \cup B'$.

48. $A' \cup B'$

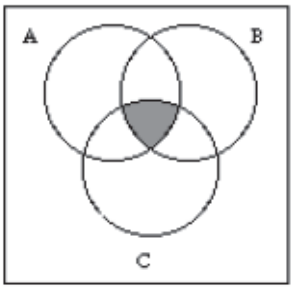


$(A \cup B)'$

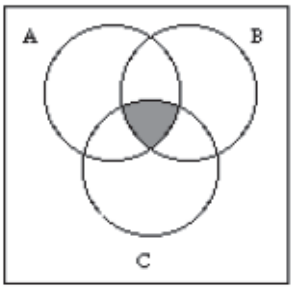


Because the sets $A' \cup B'$ and $(A \cup B)'$ are represented by different regions, $A' \cup B' \neq (A \cup B)'$.

49. $A \cap (B \cap C)$

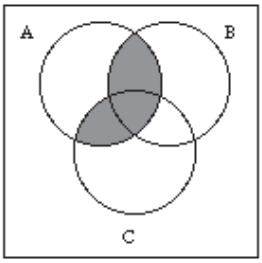


$(A \cap B) \cap C$

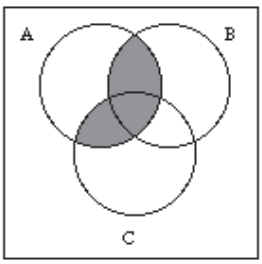


Therefore, $A \cap (B \cap C) = (A \cap B) \cap C$.

50. $A \cap (B \cup C)$



$(A \cap B) \cup (A \cap C)$



Therefore, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

51. a. S = {investors in stocks} and let B = {investors in bonds}. Since 75 had not invested in either stocks or bonds, $n(S \cup B) = 600 - 75 = 525$.

$$\begin{aligned} n(S \cup B) &= n(S) + n(B) - n(S \cap B) \\ 525 &= 380 + 325 - n(S \cap B) \\ 525 &= 705 - n(S \cap B) \\ -180 &= -n(S \cap B) \\ n(S \cap B) &= 180 \end{aligned}$$

$n(S \cap B) = 180$ represents the number of investors in both stocks and bonds.

b. $n(S \text{ only}) = 380 - 180 = 200$

52. a. Let S = {commuters taking the subway} and let B = {commuters taking the bus}. Since 120 commuters do not take either the subway or the bus, $n(S \cup B) = 1500 - 120 = 1380$.

$$\begin{aligned} n(S \cup B) &= n(S) + n(B) - n(S \cap B) \\ 1380 &= 1140 + 680 - n(S \cap B) \\ 1380 &= 1820 - n(S \cap B) \\ -440 &= -n(S \cap B) \\ n(S \cap B) &= 440 \end{aligned}$$

$n(S \cap B) = 440$ commuters take both the subway and the bus.

b. Using the formula:

$$\begin{aligned} n(S \text{ only}) &= n(S) - n(S \cap B) \\ &= 1140 - 440 = 700 \end{aligned}$$

53. a. 1785 + 1219 + 831 = 3835 people

b. 245 people

c. 755 + 700 + 275 = 1730 people

d. 1785+755+700+245+1219+275+831=5810
6000 – 5810 = 190 people

54. a. 305 + 290 + 390 = 985 people

b. 85 people

c. 110 + 135 + 150 = 395 people

d. 305+110+135+85+290+150+390 = 1465
2000 – 1465 = 535 people

55. a. 101 people

b. 124 + 82 + 65 + 51 + 48 = 370 people

c. 124 + 82 + 133 + 41 = 380 people

d. 124 + 82 + 101 + 66 = 373 people

e. 124 + 82 + 101 + 66 = 373 people

f. 124 + 82 + 65 + 101 + 66 + 51 + 41
= 530 people

56. a. 175 students

b. 180+162+190+110+86 = 728 students

c. 114 +175 +162 +126 +190 +110 + 86 = 963 students

d. 210 +175 = 385 students

e. 110 students

f. 210 + 175 + 180 + 162 = 727 students

57. a.

	File Is Infected	File Is Not Infected
Found	813	15
Did Not Find	27	42

b. 15

c. The virus software did not find 27 files that were infected.

58. a.

	Reduced BP	No BP Reduction
New Drug	750	250
Placebo	238	12

b. 250

c. After receiving a placebo, 238 had a decrease in blood pressure.

59. a. 5

b. There were 5 cases in which the test was positive but the child did not have the genetic marker.

c. 12

d. There were 12 cases in which the test was negative but the child had the genetic marker.

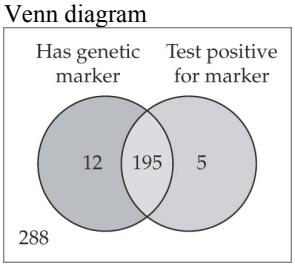
e. 195

f. There were 195 cases in which the test was positive and the child had the genetic marker.

g. 288

h. There were 288 cases in which the test was negative and the child did not have the genetic marker.

i. Venn diagram



60. a. 4

b. There were 4 tests in which the test was positive but the person does not have Rh–.

c. 21

d. There were 21 tests in which the test was negative but the person has Rh–.

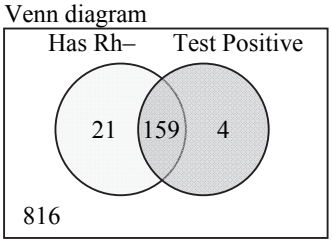
e. 159

f. There were 159 tests in which the test was positive and the person has Rh–.

g. 816

h. There were 816 tests in which the test was negative and the person does not have Rh–.

i. Venn diagram



THINK ABOUT IT, SECTION 2.2

1. All humans are mammals. No cat is a dog.

2. Some pets are small. Some pets are not dogs.

3. No. The negation is: "Some Ferraris are not red."

4. Yes. For instance, "If I like ice cream, all numbers are positive. I like ice cream. Therefore, all numbers are positive." is a valid argument but the conclusion is false.

EXERCISE SET 2.2

1. No lions are playful.

2. All dogs are friendly.

3. Some classic movies were not first produced in black and white.

4. Some people did not enjoy the dinner.

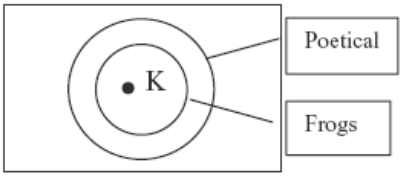
5. Some even numbers are odd numbers.

6. All actors are not rich.

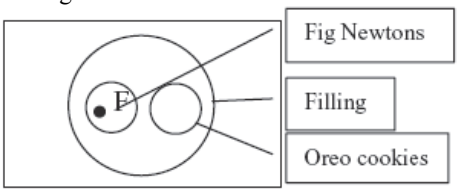
7. Some cars do not run on gasoline.

8. Some of the students took my advice.

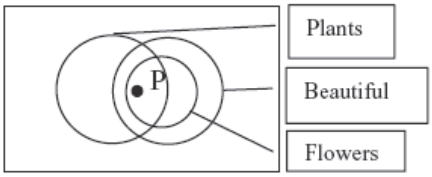
9. The argument is valid.



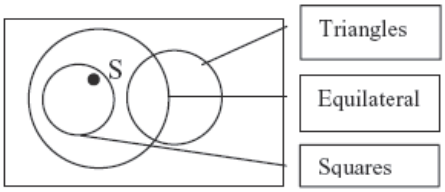
10. The argument is invalid.



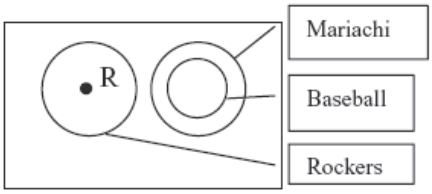
11. The argument is valid.



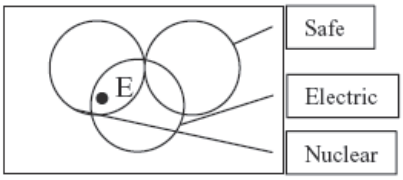
12. The argument is invalid.



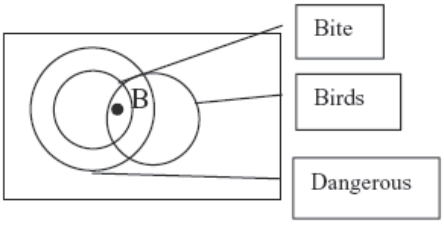
13. The argument is valid.



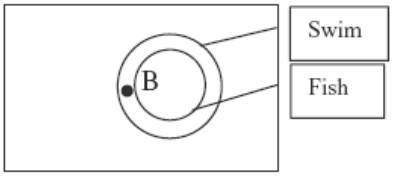
14. The argument is invalid.



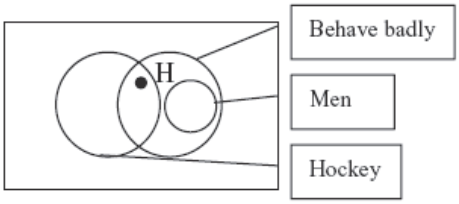
15. The argument is valid.



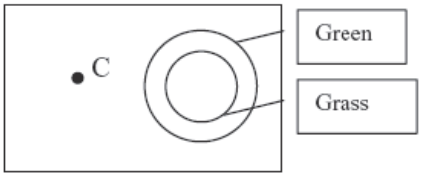
16. The argument is invalid.



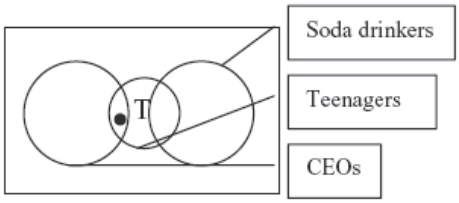
17. The argument is invalid.



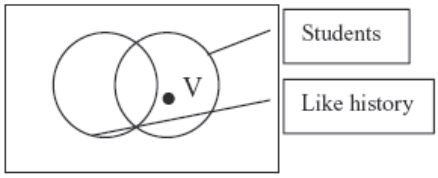
18. The argument is valid.



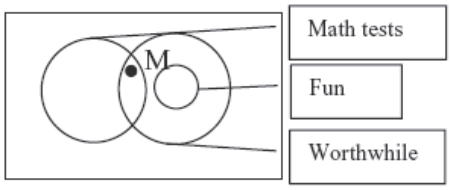
19. The argument is invalid.



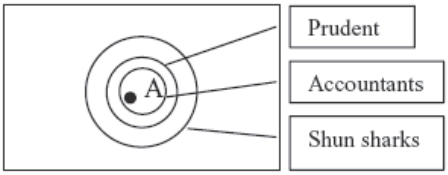
20. The argument is invalid.



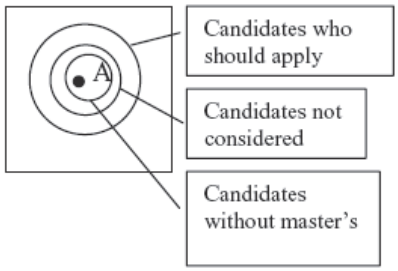
21. The argument is invalid.



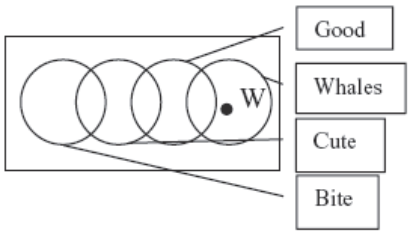
22. The argument is valid.



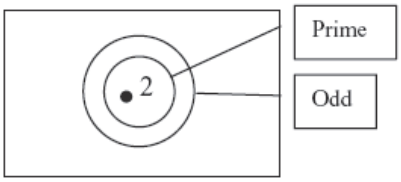
23. The argument is valid.



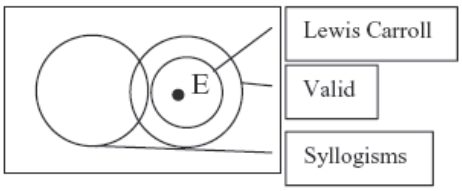
24. The argument is invalid.



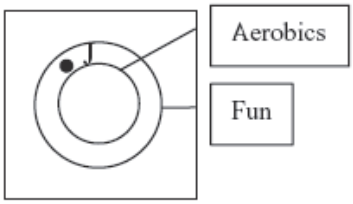
25. The argument is valid.



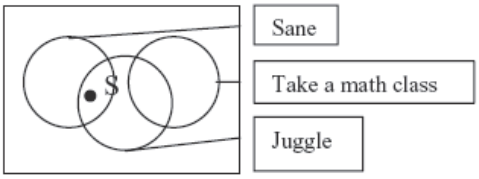
26. The argument is invalid.



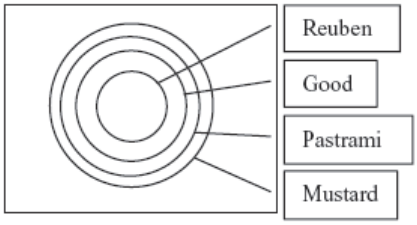
27. The argument is invalid.



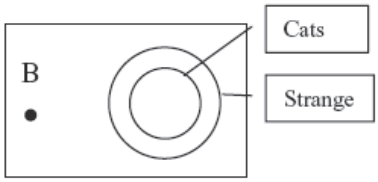
28. The argument is invalid.



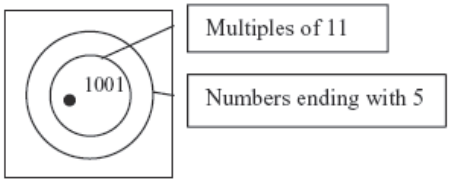
29. All Reuben sandwiches need mustard.



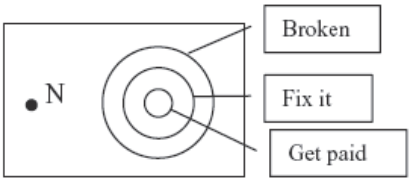
30. Boomer is not a cat.



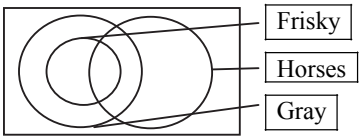
31. 1001 ends with a 5.



32. If it isn't broken, then I don't get paid.



33. Some horses are gray.



34. Label the statements

s: We like to ski.
v: We move to Vail.
h: We buy a house.
c: We buy a condo.

In symbolic form the argument is:

$s \rightarrow v$ Premise 1
$v \rightarrow \sim h$ Premise 2
$\sim c \rightarrow h$ Premise 3
$\therefore ?$

Applying the law of syllogism to Premise 1 and Premise 2 produces

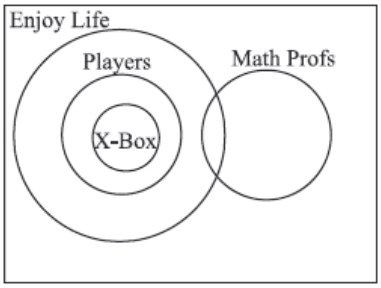
$s \rightarrow v$ Premise 1
$v \rightarrow \sim h$ Premise 2
$\therefore s \rightarrow \sim h$ Law of Syllogism

Premise 3 can be written as $\sim h \rightarrow c$ using the contrapositive form. Combining Premise 3 with the conclusion from above gives

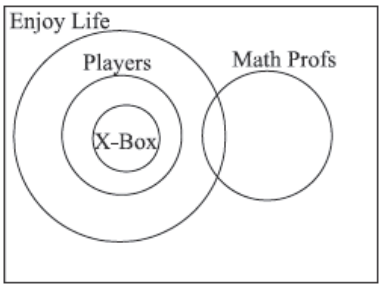
$s \rightarrow \sim h$
$\sim h \rightarrow c$
$\therefore s \rightarrow c$

If we like to ski, then we will buy a condo.

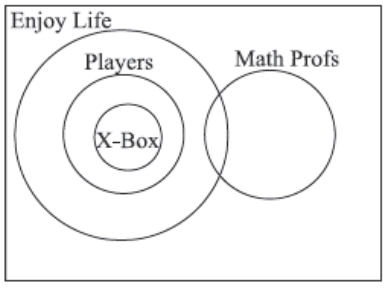
35. a. Invalid



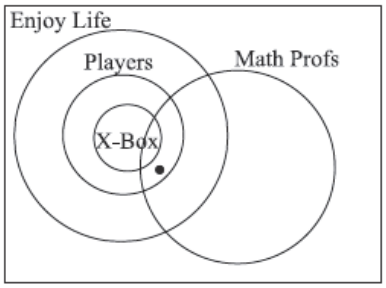
b. Invalid



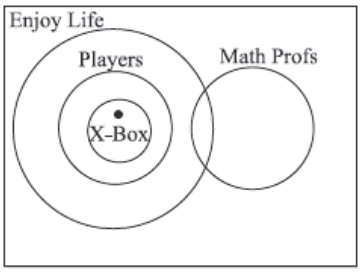
c. Invalid



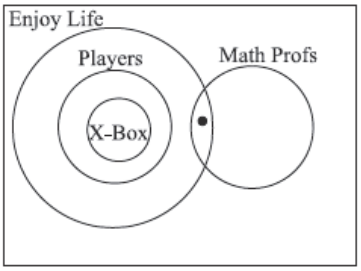
d. Invalid



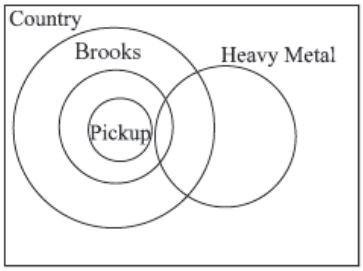
e. Valid



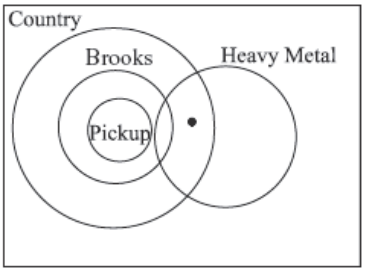
f. Valid



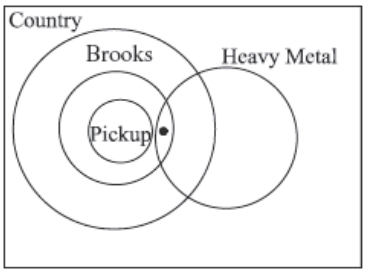
36. a. Invalid



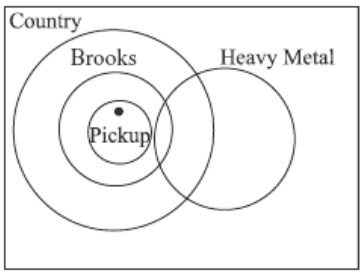
b. Valid



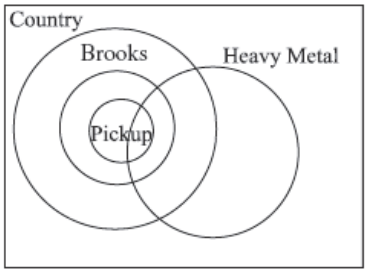
c. Valid



d. Valid



e. Invalid



THINK ABOUT IT, SECTION 2.3

1. Or
2. And
3. If–then
4. True
5. False
6. True

EXERCISE SET 2.3

1. The sentence is an opinion. It can be true for one person and false for another person. Since it can be both true and false, it is not a statement.

2. You may not know if Harvey Mudd College is in Oregon or not; however, you do know that it is either in Oregon or it is not in Oregon. The sentence is either true or false, and it is not both true and false, so it is a statement.

3. You may not know the area code for Storm Lake, Iowa; however, you do know that it is either 512 or it is not 512. The sentence is either true or it is false, and it is not both true and false, so it is a statement.

4. You may not know if January 1, 2024 will be on a Sunday or not; however, you do know that it is either on Sunday or not. The sentence is either true or false, and it is not both true and false, so it is a statement.

5. The sentence "Have a fun trip," is a command, so it is not a statement.

6. The sentence "Do you like to read?" is a question, not a declarative sentence. Thus it is not a statement.

7. The Giants did not lose the game.

8. The lunch was not served at noon.

9. The game did go into overtime.

10. The game was shown on television.

11. $w \rightarrow t$

12. $p \wedge s$

13. $d \rightarrow f$

14. $p \rightarrow t$

15. $m \vee c$

16. $p \rightarrow s$

17. I can play the piano and the violin.

18. I ride a bicycle or take the bus to work.

19. If the painting is a watercolor, then James will not buy it.

20. If you cannot count your money, then you don't have less than a million dollars.

21. If Carmella gets an A in mathematics, then she will major in physics or astronomy.

22. If the car does not get less than 25 miles per gallon and has a collision avoidance system, then Jordan will buy the car.

23. Let p represent the statement "It rained." Let q represent the statement "It snowed." In symbolic form, the original sentence is $\sim(p \vee q)$. One of De Morgan's laws states that this is

equivalent to $\sim p \wedge \sim q$. Thus an equivalent sentence is "It did not rain and it did not snow."

24. Let p represent the statement "I passed the test." Let q represent the statement "I completed the course." In symbolic form, the original sentence is $\sim p \wedge \sim q$. One of De Morgan's laws states that this is equivalent to $\sim(p \vee q)$. Thus an equivalent sentence is "It is not true that I passed the test or completed the course."

25. Let p represent the statement "She visited France." Let q represent the statement "She visited Italy." In symbolic form, the original sentence is $\sim p \wedge \sim q$. One of De Morgan's laws states that this is equivalent to $\sim(p \vee q)$. Thus an equivalent sentence is "She did not visit either France or Italy."

26. Let p represent the statement "I bought a new car." Let q represent the statement "I moved to Florida." In symbolic form, the original sentence is $\sim(p \wedge q)$. One of De Morgan's laws states that this is equivalent to $\sim p \vee \sim q$. Thus an equivalent sentence is "I did not buy a new car or I did not move to Florida."

27. *Antecedent:* I had the money.
Consequent: I would buy the painting.

28. *Antecedent:* Shelly goes on the trip.
Consequent: She will not be able to take part in the graduation ceremony.

29. *Antecedent:* She would go to the movies.
Consequent: Her friend would go.

30. *Antecedent:* I will fly to San Francisco.
Consequent: I can afford the airfare.

31. *Antecedent:* Cameron gets a raise.
Consequent: She will buy a new tablet computer.

32. *Antecedent:* She will get into medical school.
Consequent: Nuntiya passes biology.

33. False. The antecedent is true and the consequent is false.

34. True. A false antecedent yields a true statement whether the consequent is true or false.

35. True. A false antecedent yields a true statement whether the consequent is true or false.

36. True. A false antecedent yields a true statement whether the consequent is true or false.

37. False. The antecedent is true and the consequent is false.

38. True. A false antecedent yields a true statement whether the consequent is true or false.

39. Converse: If I quit this job, then I would be rich.
Inverse: If I were not rich, then I would not quit this job.
Contrapositive: If I did not quit this job, I would not be rich.

40. Converse: If we were able to take the class, we would have a car.
Inverse: If we did not have a car, then we would not be able to take the class.
Contrapositive: If we were not able to take the class, we would not have a car.

41. Converse: If we are not able to attend the party, she will not return soon.
Inverse: If she returns soon, then we will be able to attend the party.
Contrapositive: If we are able to attend the party, she will return soon.

42. Converse: If you need to move to Denver, you get the promotion.
Inverse: If you do not get the promotion, you will not need to move to Denver.
Contrapositive: If you do not need to move to Denver, you will not get the promotion.

43. Converse: If we will take the train, we will be able to take the entire family.
Inverse: If we do not take the entire family, we can't take the train.
Contrapositive: If we do not take the train, we can't take the entire family.

44. Converse: If she saves for the vacation, she will visit Kauai.
Inverse: If she does not visit Kauai, she does not save for the vacation.
Contrapositive: If she does not save for the vacation, she will not visit Kauai.

45. In symbolic form:
$h \rightarrow r$
$\underline{\sim h}$
$\therefore \sim r$

h	r	First premise $h \rightarrow r$	Second premise $\sim h$	Conclusion $\sim r$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

The premises are true in rows 3 and 4. Because the conclusion in row 3 is false and the premises are both true, we know the argument is invalid.