CHAPTER 2
FUNDAMENTALS OF LOGIC

Section 2.1

1. The sentences in parts (a), (c), (d), and (f) are statements.

2. The statements in parts (a), (c), and (f) are primitive statements.

3. Since $p \to q$ is false the truth value for p is 1 and that of q is 0. Consequently, the truth values for the given compound statements are

   (a) 0 (b) 0 (c) 1 (d) 0

4. (a) $r \to q$ (b) $q \to p$ (c) $(s \wedge r) \to q$

5. (a) If triangle ABC is equilateral, then it is isosceles.
   (b) If triangle ABC is not isosceles, then it is not equilateral.
   (c) Triangle ABC is equilateral if and only if it is equiangular.
   (d) Triangle ABC is isosceles but it is not equilateral.
   (e) If triangle ABC is equiangular, then it is isosceles.

6. (a) True (1) (b) False (0) (c) True (1)

7. (a) If Darci practices her serve daily then she will have a good chance of winning the tennis tournament.
   (c) If Mary is to be allowed on Larry's motorcycle, then she must wear her helmet.

8.

p	q	$p \vee q$	(a) $\neg(p \vee \neg q) \to \neg p$	$p \to q$	$q \to p$	(d) $(p \to q) \to (q \to p)$
0	0	0	1	1	1	1
0	1	1	1	1	0	0
1	0	1	1	0	1	1
1	1	1	1	1	1	1

p	q	$p \to q$	$p \wedge (p \to q)$	(e) $[p \wedge (p \to q)] \to q$	(f)	(g)
0	0	1	0	1	1	0
0	1	1	0	1	1	1
1	0	0	0	1	1	0
1	1	1	1	1	1	0

p	q	r	$q \to r$	(b) $p \to (q \to r)$	$p \to q$	(c) $(p \to q) \to r$	(h)
0	0	0	1	1	1	0	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	0	1
0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	0	0	1	0	1
1	1	1	1	1	1	1	1

9. Propositions (a), (e), (f), and (h) are tautologies.

10.

p	q	r	$\overbrace{p \to (q \to r)}^{s}$	$\overbrace{(p \to q) \to (p \to r)}^{t}$	$s \to t$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

11. (a) $2^5 = 32$ (b) 2^n

12. (a) $[(p \wedge q) \wedge r] \to (s \vee t)$ is false (0) when $(p \wedge q) \wedge r$ is true (1) and $s \vee t$ is false (0). Hence $p, q,$ and r must be true (1) while s and t must be false (0).

13. $p: 0;$ $r: 0;$ $s: 0$

14. (a) $n = 9$ (b) $n = 19$ (c) $n = 19$

15.

(a) $m = 3,\ n = 6$ (b) $m = 3,\ n = 9$ (c) $m = 18,\ n = 9$
(d) $m = 4,\ n = 9$ (e) $m = 4,\ n = 9$

16.

(a) $10^2 - 10 = 90$ (b) $20^2 - 20 = 380$
(c) $(10)(20) - 10 = 190$ (d) $(20)(10) - 10 = 190$

17. Consider the following possibilities:
(i) Suppose that either the first or the second statement is the true one. Then statements (3) and (4) are false — so their negations are true. And we find from (3) that Tyler did not eat the piece of pie — while from (4) we conclude that Tyler did eat the pie.
(ii) Now we'll suppose that statement (3) is the only true statement. So statements (3) and (4) no longer contradict each other. But now statement (2) is false, and we have Dawn

guilty (from statement (2)) and Tyler guilty (from statement (3)).
(iii) Finally, consider the last possibility — that is, statement (4) is the true one. Once again statements (3) and (4) do not contradict each other, and here we learn from statement (2) that Dawn is the vile culprit.

Section 2.2

1. (a)

(i)

p	q	r	$q \wedge r$	$p \to (q \wedge r)$	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

(ii)

p	q	r	$p \vee q$	$(p \vee q) \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

(iii)

p	q	r	$q \vee r$	$p \to (q \vee r)$	$p \to q$	$\neg r \to (p \to q)$
0	0	0	0	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	1	1	0	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

b)

$$\begin{aligned}
[p \to (q \vee r)] &\iff [\neg r \to (p \to q)] && \text{From part (iii) of part (a)}\\
&\iff [\neg r \to (\neg p \vee q)] && \text{By the 2nd Substitution Rule, and } (p \to q) \iff (\neg p \vee q)\\
&\iff [\neg(\neg p \vee q) \to \neg\neg r] && \text{By the 1st Substitution Rule, and } (s \to t) \iff (\neg t \to \neg s)\text{, for primitive statements } s, t\\
&\iff [(\neg\neg p \wedge \neg q) \to r] && \text{By DeMorgan's Law, Double Negation and the 2nd Substitution Rule}\\
&\iff [(p \wedge \neg q) \to r] && \text{By Double Negation and the 2nd Substitution Rule}
\end{aligned}$$

2.

p	q	$p \wedge q$	$p \vee (p \wedge q)$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

3. a) For a primitive statement s, $s \vee \neg s \iff T_0$. Replace each occurrence of s by $p \vee (q \wedge r)$ and the result follows by the 1st Substitution Rule.

b) For primitive statements s, t we have $(s \to t) \iff (\neg t \to \neg s)$. Replace each occurrence of s by $p \vee q$, and each occurrence of t by r, and the result is a consequence of the 1st Substitution Rule.

4. (1) $[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r] \iff (p \wedge q) \wedge (r \vee \neg r) \iff (p \wedge q) \wedge T_0 \iff p \wedge q$.
(2) $[(p \wedge q) \vee \neg q] \iff (p \vee \neg q) \wedge (q \vee \neg q) \iff (p \vee \neg q) \wedge T_0 \iff p \vee \neg q$.
Therefore, the given statement simplifies to $(p \vee \neg q) \to s$ or $(q \to p) \to s$

5. a) Kelsey placed her studies before her interest in cheerleading, but she (still) did not get a good education.

b) Norma is not doing her mathematics homework or Karen is not practicing her piano lesson.

c) Harold did pass his C++ course and he did finish his data structures project, but he did not graduate at the end of the semester.

6. (a) $\neg[p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)] \iff \neg p \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r) \iff (\neg q \wedge \neg r) \vee [\neg p \vee (p \wedge q \wedge \neg r)] \iff (\neg q \wedge \neg r) \vee [T_0 \wedge (\neg p \vee (q \wedge \neg r))] \iff (\neg q \wedge \neg r) \vee [\neg p \vee (q \wedge \neg r)] \iff \neg p \vee [(\neg q \vee q) \wedge \neg r] \iff \neg p \vee \neg r$.
(b) $\neg[(p \wedge q) \to r] \iff \neg[\neg(p \wedge q) \vee r] \iff (p \wedge q) \wedge \neg r$.
(c) $p \wedge (q \vee \neg r)$ (d) $\neg p \wedge \neg q \wedge \neg r$

7. a)

p	q	$(\neg p \vee q) \wedge (p \wedge (p \wedge q))$	$p \wedge q$
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

b) $(\neg p \wedge q) \vee (p \vee (p \vee q)) \iff p \vee q$

8. (a) $q \to p \iff \neg q \vee p$, so $(q \to p)^d \iff \neg q \wedge p$.
(b) $p \to (q \wedge r) \iff \neg p \vee (q \wedge r)$, so $[p \to (q \wedge r)]^d \iff \neg p \wedge (q \vee r)$.
(c) $p \leftrightarrow q \iff (p \to q) \wedge (q \to p) \iff (\neg p \vee q) \wedge (\neg q \vee p)$, so $(p \leftrightarrow q)^d \iff (\neg p \wedge q) \vee (\neg q \wedge p)$.
(d) $p \veebar q \iff (p \wedge \neg q) \vee (\neg p \wedge q)$, so $(p \veebar q)^d \iff (p \vee \neg q) \wedge (\neg p \vee q)$.

9. (a) If $0 + 0 = 0$, then $2 + 2 = 1$.
Let $p : 0 + 0 = 0$, $q : 1 + 1 = 1$.
(The implication: $p \to q$) – If $0 + 0 = 0$, then $1 + 1 = 1$. – False.
(The Converse of $p \to q$: $q \to p$) – If $1 + 1 = 1$, then $0 + 0 = 0$. – True
(The Inverse of $p \to q$: $\neg p \to \neg q$) – If $0 + 0 \neq 0$, then $1 + 1 \neq 1$. – True
(The Contrapositive of $p \to q$: $\neg q \to \neg p$) – If $1 + 1 \neq 1$, then $0 + 0 \neq 0$. – False

(b) If $-1 < 3$ and $3 + 7 = 10$, then $\sin(\frac{3\pi}{2}) = -1$. (TRUE)
Converse: If $\sin(\frac{3\pi}{2}) = -1$, then $-1 < 3$ and $3 + 7 = 10$. (TRUE)
Inverse: If $-1 \geq 3$ or $3 + 7 \neq 10$, then $\sin(\frac{3\pi}{2}) \neq -1$. (TRUE)
Contrapositive: If $\sin(\frac{3\pi}{2}) \neq -1$, then $-1 \geq 3$ or $3 + 7 \neq 10$.

10. (a) True (b) True (c) True

11. a) $(q \to r) \vee \neg p$ b) $(\neg q \vee r) \vee \neg p$

12.

p	q	$p \veebar q$	$p \wedge \neg q$	$\neg p \wedge q$	$(p \wedge \neg q) \vee (\neg p \wedge q)$	$\neg(p \leftrightarrow q)$
0	0	0	0	0	0	0
0	1	1	0	1	1	1
1	0	1	1	0	1	1
1	1	0	0	0	0	0

13.

p	q	r	$[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)]$	$[(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$
0	0	0	1	1
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

14.

(a)

p	q	$p \wedge q$	$q \rightarrow (p \wedge q)$	$p \rightarrow [q \rightarrow (p \wedge q)]$
0	0	0	1	1
0	1	0	0	1
1	0	0	1	1
1	1	1	1	1

(b) Replace each occurrence of p by $p \vee q$. Then we have the tautology $(p \vee q) \rightarrow [q \rightarrow [(p \vee q) \wedge q]]$ by the first substitution rule. Since $(p \vee q) \wedge q \iff q$, by the absorption laws, it follows that $(p \vee q) \rightarrow [q \rightarrow q] \iff T_0$.

(c)

p	q	$p \vee q$	$p \wedge q$	$q \rightarrow (p \wedge q)$	$(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$
0	0	0	0	1	1
0	1	1	0	0	0
1	0	1	0	1	1
1	1	1	1	1	1

So the given statement is not a tautology. If we try to apply the second substitution rule to the result in part (a) we would replace the first occurrence of p by $p \vee q$. But this does not result in a tautology because it is not a valid application of this substitution rule – for p is not logically equivalent to $p \vee q$.

15. (a) $\neg p \iff (p \uparrow p)$

(b) $p \vee q \iff \neg(\neg p \wedge \neg q) \iff (\neg p \uparrow \neg q) \iff (p \uparrow p) \uparrow (q \uparrow q)$

(c) $p \wedge q \iff \neg\neg(p \wedge q) \iff \neg(p \uparrow q) \iff (p \uparrow q) \uparrow (p \uparrow q)$

(d) $p \rightarrow q \iff \neg p \vee q \iff \neg(p \wedge \neg q) \iff (p \uparrow \neg q) \iff p \uparrow (q \uparrow q)$

(e) $p \leftrightarrow q \iff (p \rightarrow q) \wedge (q \rightarrow p) \iff t \wedge u \iff (t \uparrow u) \uparrow (t \uparrow u)$, where t stands for $p \uparrow (q \uparrow q)$ and u for $q \uparrow (p \uparrow p)$.

16. (a) $\neg p \iff (p \downarrow p)$

(b) $p \vee q \iff \neg\neg(p \vee q) \iff \neg(p \downarrow q) \iff (p \downarrow q) \downarrow (p \downarrow q)$

(c) $p \wedge q \iff \neg\neg p \wedge \neg\neg q \iff (\neg p \downarrow \neg q) \iff (p \downarrow p) \downarrow (q \downarrow q)$

(d) $p \rightarrow q \iff \neg p \vee q \iff (\neg p \downarrow q) \downarrow (\neg p \downarrow q) \iff [(p \downarrow p) \downarrow q] \downarrow [(p \downarrow p) \downarrow q]$

(e) $p \leftrightarrow q \iff (r \downarrow r) \downarrow (s \downarrow s)$ where r stands for $[(p \downarrow p) \downarrow q] \downarrow [(p \downarrow p) \downarrow q]$ and s for $[(q \downarrow q) \downarrow p] \downarrow [(q \downarrow q) \downarrow p]$

17.

p	q	$\neg(p \downarrow q)$	$(\neg p \uparrow \neg q)$	$\neg(p \uparrow q)$	$(\neg p \downarrow \neg q)$
0	0	0	0	0	0
0	1	1	1	0	0
1	0	1	1	0	0
1	1	1	1	1	1

18.

		Reasons
(a)	$[(p \vee q) \wedge (p \vee \neg q)] \vee q$	
$\Leftrightarrow$	$[p \vee (q \wedge \neg q)] \vee q$	Distributive Law of $\vee$ over $\wedge$
$\Leftrightarrow$	$(p \vee F_0) \vee q$	$q \wedge \neg q \Leftrightarrow F_0$ (Inverse Law)
$\Leftrightarrow$	$p \vee q$	$p \vee F_0 \Leftrightarrow p$ (Identity Law)

		Reasons
(b)	$(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)]$	
$\Leftrightarrow$	$(p \rightarrow q) \wedge \neg q$	Absorption Law (and the Commutative Law of $\vee$)
$\Leftrightarrow$	$(\neg p \vee q) \wedge \neg q$	$p \rightarrow q \Leftrightarrow \neg p \vee q$
$\Leftrightarrow$	$\neg q \wedge (\neg p \vee q)$	Commutative Law of $\wedge$
$\Leftrightarrow$	$(\neg q \wedge \neg p) \vee (\neg q \wedge q)$	Distributive Law of $\wedge$ over $\vee$
$\Leftrightarrow$	$(\neg q \wedge \neg p) \vee F_0$	Inverse Law
$\Leftrightarrow$	$\neg q \wedge \neg p$	Identity Law
$\Leftrightarrow$	$\neg(q \vee p)$	DeMorgan's Laws

19.

		Reasons
(a)	$p \vee [p \wedge (p \vee q)]$	
$\Leftrightarrow$	$p \vee p$	Absorption Law
$\Leftrightarrow$	p	Idempotent Law of $\vee$

		Reasons
(b)	$p \vee q \vee (\neg p \wedge \neg q \wedge r)$	
$\Leftrightarrow$	$(p \vee q) \vee [\neg(p \vee q) \wedge r]$	DeMorgan's Laws
$\Leftrightarrow$	$[(p \vee q) \vee \neg(p \vee q)] \wedge (p \vee q \vee r)$	Distributive Law of $\vee$ over $\wedge$
$\Leftrightarrow$	$T_0 \wedge (p \vee q \vee r)$	Inverse Law
$\Leftrightarrow$	$p \vee q \vee r$	Identity Law

		Reasons
(c)	$[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)]$	
$\Leftrightarrow$	$\neg(\neg p \vee \neg q) \vee (p \wedge q \wedge r)$	$s \rightarrow t \Leftrightarrow \neg s \vee t$
$\Leftrightarrow$	$(\neg\neg p \wedge \neg\neg q) \vee (p \wedge q \wedge r)$	DeMorgan's Laws
$\Leftrightarrow$	$(p \wedge q) \vee (p \wedge q \wedge r)$	Law of Double Negation
$\Leftrightarrow$	$p \wedge q$	Absorption Law

20. (a) $[p \wedge (\neg r \vee q \vee \neg q)] \vee [(r \vee t \vee \neg r) \wedge \neg q] \Longleftrightarrow [p \wedge (\neg r \vee T_0)] \vee [(T_0 \vee t) \wedge \neg q] \Longleftrightarrow (p \wedge T_0) \vee (T_0 \wedge \neg q) \Longleftrightarrow p \vee \neg q$

(b) $[p \vee (p \wedge q) \vee (p \wedge q \wedge r)] \wedge [(p \wedge r \wedge t) \vee t] \Longleftrightarrow p \wedge t$ by the Absorption Law.

Section 2.3

1. (a)

p	q	r	$p \to q$	$(p \vee q)$	$(p \vee q) \to r$
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	1	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	1	1	1

The validity of the argument follows from the results in the last row. (The first seven rows may be ignored.)

(b)

p	q	r	$(p \wedge q) \to r$	$\neg q$	$p \to \neg r$	$\neg p \vee \neg q$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	1	1
0	1	1	1	0	1	1
1	0	0	1	1	1	1
1	0	1	1	1	0	1
1	1	0	0	0	1	0
1	1	1	1	0	0	0

The validity of the argument follows from the results in rows 1, 2, and 5 of the table. The results in the other five rows may be ignored.

(c)

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$[p \vee (q \vee r)] \wedge \neg q$	$p \vee r$
0	0	0	0	0	0	0
0	0	1	1	1	1	1
0	1	0	1	1	0	0
0	1	1	1	1	0	1
1	0	0	0	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	0	1
1	1	1	1	1	0	1

Consider the last two columns of this truth table. Here we find that whenever the truth value of $[p \vee (q \vee r)] \wedge \neg q$ is 1 then the truth value of $p \vee r$ is also 1. Consequently,

$$[[p \vee (q \vee r)] \wedge \neg q] \Rightarrow p \vee r.$$

(The rows of the table that are crucial for assessing the validity of the argument are rows 2, 5, and 6. Rows 1, 3, 4, 7, and 8 may be ignored.)

2. (a)

p	q	r	$p \to q$	$q \to r$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

(b)

p	q	$p \to q$	$(p \to q) \wedge \neg q$	$[(p \to q) \wedge \neg q] \to \neg p$
0	0	1	1	1
0	1	1	0	1
1	0	0	0	1
1	1	1	0	1

(c)

p	q	$\neg p$	$p \vee q$	$(p \vee q) \wedge \neg p$	$[(p \vee q) \wedge \neg p] \to q$
0	0	1	0	0	1
0	1	1	1	1	1
1	0	0	1	0	1
1	1	0	1	0	1

(d)

p	q	r	$p \to r$	$q \to r$	$\overbrace{(p \vee q) \to r}^{s}$	$[(p \to r) \wedge (q \to r)] \to s$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	1	1	1	1
1	1	0	0	0	0	1
1	1	1	1	1	1	1

3. (a) If p has the truth value 0, then so does $p \wedge q$.
(b) When $p \vee q$ has the truth value 0, then the truth value of p (and that of q) is 0.
(c) If q has truth value 0, then the truth value of $[(p \vee q) \wedge \neg p]$ is 0, regardless of the truth value of p.
(d) The statement $q \vee s$ has truth value 0 only when each of q, s has truth value 0. Then

$(p \rightarrow q)$ has truth value 1 when p has truth value 0; $(r \rightarrow s)$ has truth value 1 when r has truth value 0. But then $(p \vee r)$ must have truth value 0, not 1.
(e) For $(\neg p \vee \neg r)$ the truth value is 0 when both p, r have truth value 1. This then forces q, s to have truth value 1, in order for $(p \rightarrow q)$, $(r \rightarrow s)$ to have truth value 1. However, this results in truth value 0 for $(\neg q \vee \neg s)$.

4. (a) Janice's daughter Angela will check Janice's spark plugs. (Modus Ponens)
(b) Brady did not solve the first problem correctly. (Modus Tollens)
(c) This is a **repeat-until** loop. (Modus Ponens)
(d) Tim watched television in the evening. (Modus Tollens)

5. (a) Rule of Conjunctive Simplification
(b) Invalid – attempt to argue by the converse
(c) Modus Tollens
(d) Rule of Disjunctive Syllogism
(e) Invalid – attempt to argue by the inverse

6. (a)

	Steps	**Reasons**
(1)	$q \wedge r$	Premise
(2)	q	Step (1) and the Rule of Conjunctive Simplification
(3)	$\therefore q \vee r$	Step (2) and the Rule of Disjunctive Amplification

Consequently, $(q \wedge r) \rightarrow (q \vee r)$ is a tautology, or $q \wedge r \Rightarrow q \vee r$.

(b) Consider the truth value assignments $p: 0$, $q: 1$, and $r: 0$. For these assignments $[p \wedge (q \wedge r)] \vee \neg[p \vee (q \wedge r)]$ has truth value 1, while $[p \wedge (q \vee r)] \vee \neg[p \vee (q \vee r)]$ has truth value 0. Therefore, $P \rightarrow P_1$ is *not* a tautology, or $P \not\Rightarrow P_1$.

7.

(1) & (2)	Premise
(3)	Steps (1), (2) and the Rule of Detachment
(4)	Premise
(5)	Step (4) and $(r \rightarrow \neg q) \Longleftrightarrow (\neg\neg q \rightarrow \neg r) \Longleftrightarrow (q \rightarrow \neg r)$
(6)	Steps (3), (5) and the Rule of Detachment
(7)	Premise
(8)	Steps (6), (7) and the Rule of Disjunctive Syllogism
(9)	Step (8) and the Rule of Disjunctive Amplification

8.

(1)	Premise
(2)	Step (1) and the Rule of Conjunctive Simplification
(3)	Premise
(4)	Steps (2), (3) and the Rule of Detachment

(5) Step (1) and the Rule of Conjunctive Simplification
(6) Steps (4), (5) and the Rule of Conjunction
(7) Premise
(8) Step (7) and $[r \rightarrow (s \vee t)] \Longleftrightarrow [\neg(s \vee t) \rightarrow \neg r]$
(9) Step (8) and DeMorgan's Laws
(10) Steps (6), (9) and the Rule of Detachment
(11) Premise
(12) Step (11) and $[(\neg p \vee q) \rightarrow r] \Longleftrightarrow [\neg r \rightarrow \neg(\neg p \vee q)]$
(13) Step (12) and DeMorgan's Laws and the Law of Double Negation
(14) Steps (10), (13) and the Rule of Detachment
(15) Step (14) and the Rule of Conjunctive Simplification

9. (a)

(1) Premise (The Negation of the Conclusion)
(2) Step (1) and $\neg(\neg q \rightarrow s) \Longleftrightarrow \neg(\neg\neg q \vee s) \Longleftrightarrow \neg(q \vee s) \Longleftrightarrow \neg q \wedge \neg s$
(3) Step (2) and the Rule of Conjunctive Simplification
(4) Premise
(5) Steps (3), (4) and the Rule of Disjunctive Syllogism
(6) Premise
(7) Step (2) and the Rule of Conjunctive Simplification
(8) Steps (6), (7) and Modus Tollens
(9) Premise
(10) Steps (8), (9) and the Rule of Disjunctive Syllogism
(11) Steps (5), (10) and the Rule of Conjunction
(12) Step (11) and the Method of Proof by Contradiction

(b)

(1)	$p \rightarrow q$	Premise
(2)	$\neg q \rightarrow \neg p$	Step (1) and $(p \rightarrow q) \Longleftrightarrow (\neg q \rightarrow \neg p)$
(3)	$p \vee r$	Premise
(4)	$\neg p \rightarrow r$	Step (3) and $(p \vee r) \Longleftrightarrow (\neg p \rightarrow r)$
(5)	$\neg q \rightarrow r$	Steps (2), (4) and the Law of the Syllogism
(6)	$\neg r \vee s$	Premise
(7)	$r \rightarrow s$	Step (6) and $(\neg r \vee s) \Longleftrightarrow (r \rightarrow s)$
(8)	$\therefore \neg q \rightarrow s$	Steps (5), (7) and the Law of the Syllogism

(c)

(1)	$\neg p \leftrightarrow q$	Premise
(2)	$(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$	Step (1) and $(\neg p \leftrightarrow q) \Longleftrightarrow [(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)]$

(3) $\neg p \to q$ — Step (2) and the Rule of Conjunctive Simplification
(4) $q \to r$ — Premise
(5) $\neg p \to r$ — Steps (3), (4) and the Law of the Syllogism
(6) $\neg r$ — Premise
(7) $\therefore p$ — Steps (5), (6) and Modus Tollens.

10. (a)

(1) $p \land \neg q$ — Premise
(2) p — Step (1) and the Rule of Conjunctive Simplification
(3) r — Premise
(4) $p \land r$ — Steps (2), (3) and the Rule of Conjunction
(5) $\therefore (p \land r) \lor q$ — Step (4) and the Rule of Disjunctive Amplification

(b)

(1) $p,\ p \to q$ — Premises
(2) q — Step (1) and the Rule of Detachment
(3) $\neg q \lor r$ — Premise
(4) $q \to r$ — Step (3) and $\neg q \lor r \iff (q \to r)$
(5) $\therefore r$ — Steps (2), (4) and the Rule of Detachment

(c)

(1) $p \to q,\ \neg q$ — Premises
(2) $\neg p$ — Step (1) and Modus Tollens
(3) $\neg r$ — Premise
(4) $\neg p \land \neg r$ — Steps (2), (3) and the Rule of Conjunction
(5) $\therefore \neg(p \lor r)$ — Step (4) and DeMorgan's Laws

(d)

(1) $r,\ r \to \neg q$ — Premises
(2) $\neg q$ — Step (1) and the Rule of Detachment
(3) $p \to q$ — Premise
(4) $\therefore \neg p$ — Steps (2), (3) and Modus Tollens

(e)

(1)	p	Premise
(2)	$\neg q \rightarrow \neg p$	Premise
(3)	$p \rightarrow q$	Step (2) and $(p \rightarrow q) \iff (\neg q \rightarrow \neg p)$
(4)	q	Steps (1), (3) and the Rule of Detachment
(5)	$p \wedge q$	Steps (1), (4) and the Rule of Conjunction
(6)	$p \rightarrow (q \rightarrow r)$	Premise
(7)	$(p \wedge q) \rightarrow r$	Step (6), and $[p \rightarrow (q \rightarrow r)] \iff [(p \wedge q) \rightarrow r]$
(8)	$\therefore r$	Steps (5), (7) and the Rule of Detachment

(f)

(1)	$p \wedge q$	Premise
(2)	p	Step (1) and the Rule of Conjunctive Simplification
(3)	$p \rightarrow (r \wedge q)$	Premise
(4)	$r \wedge q$	Steps (2), (3) and the Rule of Detachment
(5)	r	Step (4) and the Rule of Conjunctive Simplification
(6)	$r \rightarrow (s \vee t)$	Premise
(7)	$s \vee t$	Steps (5), (6) and the Rule of Detachment
(8)	$\neg s$	Premise
(9)	$\therefore t$	Steps (7), (8) and the Rule of Disjunctive Syllogism

(g)

(1)	$\neg s,\ p \vee s$	Premises
(2)	p	Step (1) and the Rule of Disjunctive Syllogism
(3)	$p \rightarrow (q \rightarrow r)$	Premise
(4)	$q \rightarrow r$	Steps (2), (3) and the Rule of Detachment
(5)	$t \rightarrow q$	Premise
(6)	$t \rightarrow r$	Steps (4), (5) and the Law of the Syllogism
(7)	$\therefore \neg r \rightarrow \neg t$	Step (6) and $(t \rightarrow r) \iff (\neg r \rightarrow \neg t)$

(h)

(1)	$\neg p \vee r$	Premise
(2)	$p \rightarrow r$	Step (1) and $(p \rightarrow r) \Leftrightarrow (\neg p \vee r)$
(3)	$\neg r$	Premise
(4)	$\neg p$	Steps (2), (3) and Modus Tollens
(5)	$p \vee q$	Premise
(6)	$\neg p \rightarrow q$	Step (5) and $(p \vee q) \Leftrightarrow (\neg\neg p \vee q) \Leftrightarrow (\neg p \rightarrow q)$
(7)	$\therefore q$	Steps (4), (6) and Modus Ponens

11. (a) $p:1$ $q:0$ $r:1$

(b) $p:0$ $q:0$ $r:0$ or 1

$p:0$ $q:1$ $r:1$

(c) $p,q,r:1$ $s:0$

(d) $p,q,r:1$ $s:0$

12. a) p: Rochelle gets the supervisor's position.

q: Rochelle works hard.

r: Rochelle gets a raise.

s: Rochelle buys a new car.

$$\begin{array}{l} (p \wedge q) \rightarrow r \\ r \rightarrow s \\ \underline{\neg s} \\ \therefore \neg p \vee \neg q \end{array}$$

(1)	$\neg s$	Premise
(2)	$r \rightarrow s$	Premise
(3)	$\neg r$	Steps (1), (2) and Modus Tollens
(4)	$(p \wedge q) \rightarrow r$	Premise
(5)	$\neg(p \wedge q)$	Steps (3), (4) and Modus Tollens
(6)	$\therefore \neg p \vee \neg q$	Step (5) and $\neg(p \wedge q) \iff \neg p \vee \neg q$.

b) p: Dominic goes to the racetrack.

q: Helen gets mad.

r: Ralph plays cards all night.

s: Carmela gets mad.

t: Veronica is notified.

$$\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ (q \vee s) \rightarrow t \\ \underline{\neg t} \\ \therefore \neg p \wedge \neg r \end{array}$$

(1)	$\neg t$	Premise
(2)	$(q \vee s) \rightarrow t$	Premise
(3)	$\neg(q \vee s)$	Steps (1), (2) and Modus Tollens
(4)	$\neg q \wedge \neg s$	Step (3) and $\neg(q \vee s) \iff \neg q \wedge \neg s$
(5)	$\neg q$	Step (4) and the Rule of Conjunctive Simplification
(6)	$p \rightarrow q$	Premise
(7)	$\neg p$	Steps (5), (6) and Modus Tollens
(8)	$\neg s$	Step (4) and the Rule of Conjunctive Simplification

(9)	$r \rightarrow s$	Premise
(10)	$\neg r$	Steps (8), (9) and Modus Tollens
(11)	$\therefore \neg p \wedge \neg r$	Steps (7), (10) and the Rule of Conjunction

c) p: There is a chance of rain.
q: Lois' red head scarf is missing.
r: Lois does not mow her lawn.
s: The temperature is over 80^o F.

$$\begin{array}{l} (p \vee q) \rightarrow r \\ s \rightarrow \neg p \\ s \wedge \neg q \\ \hline \therefore \neg r \end{array}$$

The following truth value assignments provide a counterexample to the validity of this argument:
$p: 0;\ q: 0;\ r: 1;\ s: 1$

13. (a)

p	q	r	$p \vee q$	$\neg p \vee r$	t: $(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$	$t \rightarrow (q \vee r)$
0	0	0	0	1	0	0	1
0	0	1	0	1	0	1	1
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	0	0	0	1
1	0	1	1	1	1	1	1
1	1	0	1	0	0	1	1
1	1	1	1	1	1	1	1

From the last column of the truth table it follows that $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$ is a tautology.

Alternately we can try to see if there are truth values that can be assigned to p, q, and r so that $(q \vee r)$ has truth value 0 while $(p \vee q)$, $(\neg p \vee r)$ both have truth value 1.

For $(q \vee r)$ to have truth value 0, it follows that $q: 0$ and $r: 0$. Consequently, for $(p \vee q)$ to have truth value 1, we have $p: 1$ since $q: 0$. Likewise, with $r: 0$ it follows that $\neg p: 1$ if $(\neg p \vee) r$ has truth value 1. But we cannot have $p: 1$ and $\neg p: 1$. So whenever $(p \vee q)$, $(\neg p \vee r)$ have truth value 1, we have $(q \vee r)$ with truth value 1 and it follows that $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$ is a tautology.

Finally we can also argue as follows:

Steps		Reasons
1.	$p \vee q$	1. Premise
2.	$q \vee p$	2. Step (1) and the Commutative Law of $\vee$
3.	$\neg(\neg q) \vee p$	3. Step (2) and the Law of Double Negation
4.	$\neg q \rightarrow p$	4. Step (3), $\neg q \rightarrow p \Leftrightarrow \neg(\neg q) \vee p$
5.	$\neg p \vee r$	5. Premise
6.	$p \rightarrow r$	6. Step (5), $p \rightarrow r \Leftrightarrow \neg p \vee r$
7.	$\neg q \rightarrow r$	7. Steps (4), (6), and the Law of the Syllogism
8.	$\therefore q \vee r$	8. Step (7), $\neg q \rightarrow r \Leftrightarrow q \vee r$

(b)

(i)

Steps		Reasons
1.	$p \vee (q \vee r)$	1. Premise
2.	$(p \vee q) \wedge (p \vee r)$	2. Step (1) and the Distribution Law of $\vee$ over $\wedge$
3.	$p \vee r$	3. Step (2) and the Rule of Conjunctive Simplification
4.	$p \rightarrow s$	4. Premise
5.	$\neg p \vee s$	5. Step (4), $p \rightarrow s \Leftrightarrow \neg p \vee s$
6.	$\therefore r \vee s$	6. Steps (3), (5), the Rule of Conjunction, and Resolution

(ii)

Steps		Reasons
1.	$p \leftrightarrow q$	1. Premise
2.	$(p \rightarrow q) \wedge (q \rightarrow p)$	2. $(p \leftrightarrow q) \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow p)]$
3.	$p \rightarrow q$	3. Step (2) and the rule of Conjunctive Simplification
4.	$\neg p \vee q$	4. Step (3), $p \rightarrow q \Leftrightarrow \neg p \vee q$
5.	p	5. Premise
6.	$p \vee q$	7. Step (5) and the Rule of Disjunctive Amplification
7.	$[(p \vee q) \wedge (\neg p \vee q)]$	7. Steps (6), (4), and the Rule of Conjunction
8.	$q \vee q$	8. Step (7) and Resolution
9.	$\therefore q$	9. Step (8) and the Idempotent Law of $\vee$.

(iii)

Steps		Reasons
1.	$p \vee q$	1. Premise
2.	$p \rightarrow r$	2. Premise
3.	$\neg p \vee r$	3. Step (2), $p \rightarrow r \Leftrightarrow \neg p \vee r$
4.	$[(p \vee q) \wedge (\neg p \vee r)]$	4. Steps (1), (3), and the Rule of Conjunction
5.	$q \vee r$	5. Step (4) and Resolution
6.	$r \rightarrow s$	6. Premise
7.	$\neg r \vee s$	7. Step (6), $r \rightarrow s \Leftrightarrow \neg r \vee s$
8.	$[(r \vee q) \wedge (\neg r \vee s)]$	8. Steps (5), (7), the Commutative Law of $\vee$, and the Rule of Conjunction
9.	$\therefore q \vee s$	9. Step (8) and Resolution

(iv)

	Steps		Reasons
1.	$\neg p \vee q \vee r$	1.	Premise
2.	$q \vee (\neg p \vee r)$	2.	Step (1) and the Commutative and Associative Laws of $\vee$
3.	$\neg q$	3.	Premise
4.	$\neg q \vee (\neg p \vee r)$	4.	Step (3) and the Rule of Disjunctive Amplification
5.	$[[q \vee (\neg \vee r)] \wedge [\neg q \vee (\neg p \vee r)]]$	5.	Steps (2), (4), and the Rule of Conjunction
6.	$(\neg p \vee r)$	6.	Step (5), Resolution, and the Idempotent Law of $\wedge$
7.	$\neg r$	7.	Premise
8.	$\neg r \vee \neg p$	8.	Step (7) and the Rule of Disjunctive Amplification
9.	$[(r \vee \neg p) \wedge (\neg r \vee \neg p)]$	9.	Steps (6), (8), the Commutative Law of $\vee$, and the Rule of Conjunction
10.	$\therefore \neg p$	10.	Step (9), Resolution, and the Idempotent Law of $\vee$

(v)	Steps		Reasons
1.	$\neg p \vee s$	1.	Premise
2.	$p \vee q \vee t$	2.	Premise
3.	$p \vee (q \vee t)$	3.	Step (2) and the Associative Law of $\vee$
4.	$[[p \vee (q \vee t)] \wedge (\neg p \vee s)]$	4.	Steps (3), (1), and the Rule of Conjunction
5.	$(q \vee t) \vee s$	5.	Step (4) and Resolution (and the First Substitution Rule)
6.	$q \vee (t \vee s)$	6.	Step (5) and the Associative Law of $\vee$
7.	$\neg q \vee r$	7.	Premise
8.	$[[q \vee (t \vee s)]$	8.	Steps (6), (7), and the Rule of Conjunction
9.	$(t \vee s) \vee r$	9.	Step (8) and Resolution (and the First Substitution Rule)
10.	$t \vee (s \vee r)$	10.	Step (9) and the Associative Law of $\vee$
11.	$\neg t \vee (s \wedge r)$	11.	Premise
12.	$(\neg t \vee s) \wedge (\neg t \vee r)$	12.	Step (11) and the Distributive Law of $\vee$ over $\wedge$
13.	$\neg t \vee s$	13.	Step (12) and the Rule of Conjunctive Simplification
14.	$[[t \vee (s \vee r)] \wedge (\neg t \vee s)]$	14.	Steps (10), (13), and the Rule of Conjunction
15.	$(s \vee r) \vee s$	15.	Step (14) and Resolution (and the First Substitution Rule)
16.	$\therefore r \vee s$	16.	Step (15) and the Commutative, Associative, and Idempotent Laws of $\vee$

(c) Consider the following assignments.

p: Jonathan has his driver's license.
q: Jonathan's new car is out of gas.
r: Jonathan likes to drive his new car.

Then the given argument can be written in symbolic form as

$$\begin{array}{l} \neg p \vee q \\ p \vee \neg r \\ \underline{\neg q \vee \neg r} \\ \therefore \neg r \end{array}$$

	Steps		Reasons
	Steps		Reasons
1.	$\neg p \vee q$	1.	Premise
2.	$p \vee \neg r$	2.	Premise
3.	$(p \vee \neg r) \wedge (\neg p \vee q)$	3.	Steps (2), (1), and the Rule of Conjunction
4.	$\neg r \vee q$	4.	Step (3) and Resolution
5.	$q \vee \neg r$	5.	Step (4) and the Commutative Law of $\vee$
6.	$\neg q \vee \neg r$	6.	Premise
7.	$(q \vee \neg r) \wedge (\neg q \vee \neg r)$	7.	Steps (5), (6), and the Rule of Conjunction
8.	$\neg r \vee \neg r$	8.	Step (7) and Resolution
9.	$\therefore \neg r$	9.	Step (8) and Idempotent Law of $\vee$

Section 2.4

1.

(a) False (b) False (c) False
(d) True (e) False (f) False

2. (a) (i) True (ii) True (iii) True (iv) True

(b) The only substitution for x that makes the open statement $[p(x) \wedge q(x)] \wedge r(x)$ into a true statement is $x = 2$.

3. Statements (a), (c), and (e) are true, while statements (b), (d), and (f) are false.

4. (a) Every polygon is a quadrilateral or a triangle (but not both). (True — for this universe.)
(b) Every isosceles triangle is equilateral. (False)
(c) There exists a triangle with an interior angle that exceeds 180°. (False)
(d) A triangle has all of its interior angles equal if and only if it is an equilateral triangle. (True)
(e) There exists a quadrilateral that is not a rectangle. (True)
(f) There exists a rectangle that is not a square. (True)
(g) If all the sides of a polygon are equal, then the polygon is an equilateral triangle. (False)
(h) No triangle has an interior angle that exceeds 180°. (True)
(i) A polygon (of three or four sides) is a square if and only if all of its interior angles are equal and all of its sides are equal. (False)

(j) A triangle has all interior angles equal if and only if all of its sides are equal. (True)

5.

(a)	$\exists x\ [m(x) \wedge c(x) \wedge j(x)]$	True
(b)	$\exists x\ [s(x) \wedge c(x) \wedge \neg m(x)]$	True
(c)	$\forall x\ [c(x) \rightarrow (m(x) \veebar p(x))]$	False
(d)	$\forall x\ [(g(x) \wedge c(x)) \rightarrow \neg p(x)]$,	True
	or $\forall x\ [(p(x) \wedge c(x)) \rightarrow \neg g(x)]$,	
	or $\forall x\ [(g(x) \wedge p(x)) \rightarrow \neg c(x)]$	
(e)	$\forall x\ [(c(x) \wedge s(x)) \rightarrow (p(x) \veebar e(x))]$,	True

6.

(a)	True	(b)	True	(c)	False
(d)	True	(e)	False	(f)	False

7. (a)

(i) $\exists x\ q(x)$
(ii) $\exists x\ [p(x) \wedge q(x)]$
(iii) $\forall x\ [q(x) \rightarrow \neg t(x)]$
(iv) $\forall x\ [q(x) \rightarrow \neg t(x)]$
(v) $\exists x\ [q(x) \wedge t(x)]$
(vi) $\forall x\ [(q(x) \wedge r(x)) \rightarrow s(x)]$

(b) Statements (i), (iv), (v), and (vi) are true. Statements (ii) and (iii) are false: $x = 10$ provides a counterexample for either statement.

(c)

(i) If x is a perfect square, then $x > 0$.
(ii) If x is divisible by 4, then x is even.
(iii) If x is divisible by 4, then x is not divisible by 5.
(iv) There exists an integer that is divisible by 4 but it is not a perfect square.

(d) (i) Let $x = 0$. (iii) Let $x = 20$.

8. (a) True (b) False: For $x = 1$, $q(x)$ is true while $p(x)$ is false.
(c) True (d) True (e) True (f) True
(g) True (h) False: For $x = -1$, $(p(x) \vee q(x))$ is true but $r(x)$ is false.

9.

(a)	(i)	True	(ii)	False –	Consider $x = 3$.
	(iii)	True	(iv)	True	
(b)	(i)	True	(ii)	False –	Consider $x = 3$.
	(iii)	True	(iv)	True	
(c)	(i)	True	(ii)	True	
	(iii)	True	(iv)	False –	For $x = 2$ or 5, the truth value of $p(x)$ is 1 while that of $r(x)$ is 0.

10. (a) $\forall m, n\ \ A[m, n] > 0$
(b) $\forall m, n\ \ 0 < A[m, n] \leq 70$

(c) $\exists m, n \;\; A[m, n] > 60$
(d) $\forall m \;\; [(1 \leq n < 19) \rightarrow (A[m, n] < A[m, n+1])]$
(e) $\forall n \;\; [(1 \leq m < 9) \rightarrow (A[m, n] < A[m+1, n])]$
(f) $\forall \; 1 \leq m, i \leq 3 \;\; \forall \; 1 \leq n, j \leq 20 \; [((m, n) \neq (i, j)) \rightarrow (A[m, n] \neq A[i, j])]$

11. (a) In this case the variable x is free while the variables y, z are bound.
(b) Here the variables x, y are bound; the variable z is free.

12. (a)

(i)	False	(ii)	True	(iii)	True
(iv)	False, if $x = 0$	(v)	False, if $x = 0$	(vi)	True

(vii) False — If $y = 0$ then $x \neq 0$; if $y \neq 0$, let $x = 2y$.
(viii) False — Let $x = 2$ and $y = -2$, for example.

(b) Statements (iv), (v), and (viii) are now true — because of the change in universe.

(c) (i) True (ii) True (iii) True
(iv) False — For any y consider $x = 2y$.

13. (a) $p(2,3) \wedge p(3,3) \wedge p(5,3)$
(b) $[p(2,2) \vee p(2,3) \vee p(2,5)] \vee [p(3,2) \vee p(3,3) \vee p(3,5)] \vee [p(5,2) \vee p(5,3) \vee p(5,5)]$
(c) $[p(2,2) \vee p(3,2) \vee p(5,2)] \wedge [p(2,3) \vee p(3,3) \vee p(5,3)] \wedge [p(2,5) \vee p(3,5) \vee p(5,5)]$

14. Statements (a), (b), (e), and (f) are logically equivalent and each may be expressed as $\forall n[q(n) \rightarrow p(n)]$. Statements (c), (g) are logically equivalent and each may be expressed as $\forall n[p(n) \rightarrow q(n)]$. Statement (d) is not logically equivalent to any of the other six statements.

15. a) The proposed negation is correct and is a true statement.
b) The proposed negation is wrong. A correct version of the negation is: For all rational numbers x, y, the sum $x + y$ is rational. This correct version of the negation is a true statement.
c) The proposed negation is correct — but false. The (original) statement is true.
d) The proposed negation is wrong. A correct version of the negation is: For all integers x, y, if x, y are both odd, then xy is even.
The (original) statement is true.

16. (a) Some student in Professor Lenhart's C++ class is not majoring in either computer science or mathematics.
(b) If a student is in Professor Lenhart's C++ class, then that student is not majoring in history.
or, No student majoring in history is in Professor Lenhart's C++ class.

17. a) There exists an integer n such that n is not divisible by 2 but n is even (that is, not odd).
b) There exist integers k, m, n such that $k - m$ and $m - n$ are odd, and $k - n$ is odd.

c) For some real number x, $x^2 > 16$ but $-4 \leq x \leq 4$ (that is, $-4 \leq x$ and $x \leq 4$).
d) There exists a real number x such that $|x - 3| < 7$ and either $x \leq -4$ or $x \geq 10$.

18. (a) $\forall x \ [\neg p(x) \land \neg q(x)]$
(b) $\exists x \ [\neg p(x) \lor q(x)]$
(c) $\exists x \ [p(x) \land \neg q(x)]$
(d) $\forall x \ [(p(x) \lor q(x)) \land \neg r(x)]$

19. (a) Statement: For all positive integers m, n, if $m > n$ then $m^2 > n^2$. (TRUE)
Converse: For all positive integers m, n, if $m^2 > n^2$ then $m > n$. (TRUE)
Inverse: For all positive integers m, n, if $m \leq n$ then $m^2 \leq n^2$. (TRUE)
Contrapositive: For all positive integers m, n, if $m^2 \leq n^2$ then $m \leq n$. (TRUE)
(b) Statement: For all integers a, b, if $a > b$ then $a^2 > b^2$. (FALSE — let $a = 1$ and $b = -2$.)
Converse: For all integers a, b, if $a^2 > b^2$ then $a > b$. (FALSE — let $a = -5$ and $b = 3$.)
Inverse: For all integers a, b, if $a \leq b$ then $a^2 \leq b^2$. (FALSE — let $a = -5$ and $b = 3$.)
Contrapositive: For all integers a, b, if $a^2 \leq b^2$ then $a \leq b$. (FALSE — let $a = 1$ and $b = -2$.)
(c) Statement: For all integers m, n, and p, if m divides n and n divides p then m divides p. (TRUE)
Converse: For all integers m and p, if m divides p, then for each integer n it follows that m divides n and n divides p. (FALSE — let $m = 1$, $n = 2$, and $p = 3$.)
Inverse: For all integers m, n, and p, if m does not divide n or n does not divide p, then m does not divide p. (False — let $m = 1$, $n = 2$, and $p = 3$.)
Contrapositive: For all integers m and p, if m does not divide p, then for each integer n it follows that m does not divide n or n does not divide p. (TRUE)
(d) Statement: $\forall x \ [(x > 3) \rightarrow (x^2 > 9)]$ (TRUE)
Converse: $\forall x \ [(x^2 > 9) \rightarrow (x > 3)]$ (FALSE — let $x = -5$.)
Inverse: $\forall x \ [(x \leq 3) \rightarrow (x^2 \leq 9)]$ (FALSE — let $x = -5$.)
Contrapositive: $\forall x \ [(x^2 \leq 9) \rightarrow (x \leq 3)]$ (TRUE)
(e) Statement: $\forall x \ [(x^2 + 4x - 21 > 0) \rightarrow [(x > 3) \lor (x < -7)]]$ (TRUE)
Converse: $\forall x \ [[(x > 3) \lor (x < -7)] \rightarrow (x^2 + 4x - 21 > 0)]$ (TRUE)
Inverse: $\forall x \ [(x^2 + 4x - 21 \leq 0) \rightarrow [(x \leq 3) \land (x \geq -7)]]$, or $\forall x \ [(x^2 + 4x - 21 \leq 0) \rightarrow (-7 \leq x \leq 3)]$ (TRUE)
Contrapositive: $\forall x \ [[(x \leq 3) \land (x \geq -7)] \rightarrow (x^2 + 4x - 21 \leq 0)]$, or $\forall x \ [(-7 \leq x \leq 3) \rightarrow (x^2 + 4x - 21 \leq 0)]$ (TRUE)

20. For each of the following answers it is possible to have the implication and its contrapositive interchanged. When this happens the corresponding converse and inverse must also be interchanged.

(a) Implication: If a positive integer is divisible by 21, then it is divisible by 7. (TRUE)
Converse: If a positive integer is divisible by 7, then it is divisible by 21. (FALSE — consider the positive integer 14.)

Inverse: If a positive integer is not divisible by 21, then it is not divisible by 7. (FALSE — consider the positive integer 14.)
Contrapositive: If a positive integer is not divisible by 7, then it is not divisible by 21. (TRUE)
(b) Implication: If a snake is a cobra, then it is dangerous.
Converse: If a snake is dangerous, then it is a cobra.
Inverse: If a snake is not a cobra, then it is not dangerous.
Contrapositive: If a snake is not dangerous, then it is not a cobra.
(c) Implication: For each complex number z, if z^2 is real then z is real. (FALSE — let $z = i$.)
Converse: For each complex number z, if z is real then z^2 is real. (TRUE)
Inverse: For each complex number z, if z^2 is not real then z is not real. (TRUE)
Contrapositive: For each complex number z, if z is not real then z^2 is not real. (FALSE — let $z = i$.)

21. (a) True (b) False (c) False (d) True (e) False

22. (a) True (b) False (c) True (d) True (e) True

23. (a) $\forall a\, \exists b\, [a + b = b + a = 0]$
(b) $\exists u\, \forall a\, [au = ua = a]$
(c) $\forall a \neq 0\, \exists b\, [ab = ba = 1]$
(d) The statement in part (b) remains true but the statement in part (c) is no longer true for this new universe.

24. (a) True (b) False (c) False (d) True

25. (a) $\exists x\, \exists y\, [(x > y) \wedge (x - y \leq 0)]$
(b) $\exists x\, \exists y\, [(x < y) \wedge \forall z [x \geq z \vee z \geq y]]$
(c) $\exists x\, \exists y\, [(|x| = |y|) \wedge (y \neq \pm x)]$

26. $\lim_{n \to \infty} r_n \neq L \Leftrightarrow \exists \epsilon > 0\, \forall k > 0\, \exists n\, [(n > k) \wedge |r_n - L| \geq \epsilon]$

Section 2.5

1. Although we may write $28 = 25 + 1 + 1 + 1 = 16 + 4 + 4 + 4$, there is no way to express 28 as the sum of at most three perfect squares.

2. Although $3 = 1 + 1 + 1$ and $5 = 4 + 1$, when we get to 7 there is a problem. We can write $7 = 4 + 1 + 1 + 1$, but we cannot write 7 as the sum of three or fewer perfect squares. [There is also a problem with the integers 15 and 23.]

3. Here we find that

$30 = 25 + 4 + 1$	$40 = 36 + 4$	$50 = 25 + 25$
$32 = 16 + 16$	$42 = 25 + 16 + 1$	$52 = 36 + 16$
$34 = 25 + 9$	$44 = 36 + 4 + 4$	$54 = 25 + 25 + 4$
$36 = 36$	$46 = 36 + 9 + 1$	$56 = 36 + 16 + 4$
$38 = 36 + 1 + 1$	$48 = 16 + 16 + 16$	$58 = 49 + 9$

4.

$4 = 2 + 2$	$16 = 13 + 3$	$28 = 23 + 5$
$6 = 3 + 3$	$18 = 13 + 5$	$30 = 17 + 13$
$8 = 3 + 5$	$20 = 17 + 3$	$32 = 19 + 13$
$10 = 5 + 5$	$22 = 17 + 5$	$34 = 17 + 17$
$12 = 7 + 5$	$24 = 17 + 7$	$36 = 19 + 17$
$14 = 7 + 7$	$26 = 19 + 7$	$38 = 19 + 19$

5. (a) The real number π is not an integer.
(b) Margaret is a librarian.
(c) All administrative directors know how to delegate authority.
(d) Quadrilateral $MNPQ$ is not equiangular.

6. (a) Valid — This argument follows from the Rule of Universal Specification and Modus Ponens.
(b) Invalid — Attempt to argue by the converse.
(c) Invalid — Attempt to argue by the inverse.

7. (a) When the statement $\exists x\ [p(x) \vee q(x)]$ is true, there is at least one element c in the prescribed universe where $p(c) \vee q(c)$ is true. Hence at least one of the statements $p(c), q(c)$ has the truth value 1, so at least one of the statements $\exists x\ p(x)$ and $\exists x\ q(x)$ is true. Therefore, it follows that $\exists x\ p(x) \vee \exists x\ q(x)$ is true, and $\exists x\ [p(x) \vee q(x)] \Longrightarrow \exists x\ p(x) \vee \exists x\ q(x)$. Conversely, if $\exists x\ p(x) \vee \exists x\ q(x)$ is true, then at least one of $p(a)$, $q(b)$ has truth value 1, for some a, b in the prescribed universe. Assume without loss of generality that it is $p(a)$. Then $p(a) \vee q(a)$ has truth value 1 so $\exists x\ [p(x) \vee q(x)]$ is a true statement, and $\exists x\ p(x) \vee \exists x\ q(x) \Longrightarrow \exists x\ [p(x) \vee q(x)]$.
(b) First consider when the statement $\forall x\ [p(x) \wedge q(x)]$ is true. This occurs when $p(a) \wedge q(a)$ is true for each a in the prescribed universe. Then $p(a)$ is true (as is $q(a)$) for all a in the universe, so the statements $\forall x\ p(x)$, $\forall x\ q(x)$ are true. Therefore, the statement $\forall x\ p(x) \wedge \forall x\ q(x)$ is true and $\forall x\ [p(x) \wedge q(x)] \Longrightarrow \forall x\ p(x) \wedge \forall x\ q(x)$. Conversely, suppose that $\forall x\ p(x) \wedge \forall x\ q(x)$ is a true statement. Then $\forall x\ p(x)$, $\forall x\ q(x)$ are both true. So now let c be any element in the prescribed universe. Then $p(c)$, $q(c)$, and $p(c) \wedge q(c)$ are all true. And, since c was chosen arbitrarily, it follows that the statement $\forall x\ [p(x) \wedge q(x)]$ is true, and $\forall x\ p(x) \wedge \forall x\ q(x) \Longrightarrow \forall x\ [p(x) \wedge q(x)]$.

8. (a) Suppose that the statement $\forall x\ p(x) \vee \forall x\ q(x)$ is true, and suppose without loss of generality that $\forall x\ p(x)$ is true. Then for each c in the given universe $p(c)$ is true, as is

$p(c) \vee q(c)$. Hence $\forall x\, [p(x) \vee q(x)]$ is true and $\forall x\, p(x) \vee \forall x\, q(x) \Longrightarrow \forall x\, [p(x) \vee q(x)]$.
(b) Let $p(x):\ x > 0$ and $q(x):\ x < 0$ for the universe of all nonzero integers. Then $\forall x\, p(x), \forall x\, q(x)$ are false, so $\forall x\, p(x) \vee \forall x\, q(x)$ is false, while $\forall x\, [p(x) \vee q(x)]$ is true.

9. (1) Premise
(2) Premise
(3) Step (1) and the Rule of Universal Specification
(4) Step (2) and the Rule of Universal Specification
(5) Step (4) and the Rule of Conjunctive Simplification
(6) Steps (5), (3), and Modus Ponens
(7) Step (6) and the Rule of Conjunctive Simplification
(8) Step (4) and the Rule of Conjunctive Simplification
(9) Steps (7), (8), and the Rule of Conjunction
(10) Step (9) and the Rule of Universal Generalization

10.
(4) Step (1) and the Rule of Universal Specification
(5) Steps (3), (4), and the Rule of Disjunctive Syllogism
(6) Premise
(7) Step (6) and the Rule of Universal Specification
(8) Step (7) and $\neg q(a) \vee r(a) \Leftrightarrow q(a) \rightarrow r(a)$
(9) Steps (5), (8), and Modus Ponens (or the Rule of Detachment)
(10) Premise
(11) Step (10) and the Rule of Universal Specification
(12) Step (11) and $s(a) \rightarrow \neg r(a) \Leftrightarrow \neg\neg r(a) \rightarrow \neg s(a) \Leftrightarrow r(a) \rightarrow \neg s(a)$
(13) Steps (9), (12), and Modus Ponens (or the Rule of Detachment)

11. Consider the open statements
$w(x)$: x works for the credit union
$\ell(x)$: x writes loan applications
$c(x)$: x knows COBOL
$q(x)$: x knows Excel
and let r represent Roxe and i represent Imogene.

In symoblic form the given argument is given as follows:

$$\begin{array}{l} \forall x\, [w(x) \rightarrow c(x)] \\ \forall x\, [(w(x) \wedge \ell(x)) \rightarrow q(x)] \\ w(r) \wedge \neg q(r) \\ \underline{q(i) \wedge \neg c(i)} \\ \therefore \neg \ell(r) \wedge \neg w(i) \end{array}$$

The steps (and reasons) needed to verify this argument can now be presented.

	Steps	Reasons
(1)	$\forall x\, [w(x) \to c(x)]$	Premise
(2)	$q(i) \wedge \neg c(i)$	Premise
(3)	$\neg c(i)$	Step (2) and the Rule of Conjunctive Simplification
(4)	$w(i) \to c(i)$	Step (1) and the Rule of Universal Specification
(5)	$\neg w(i)$	Steps (3), (4), and Modus Tollens
(6)	$\forall x\, [(w(x) \wedge \ell(x)) \to q(x)]$	Premise
(7)	$w(r) \wedge \neg q(r)$	Premise
(8)	$\neg q(r)$	Step (7) and the Rule of Conjunctive Simplification
(9)	$(w(r) \wedge \ell(r)) \to q(r)$	Step (6) and the Rule of Universal Specification
(10)	$\neg(w(r) \wedge \ell(r))$	Steps (8), (9), and Modus Tollens
(11)	$w(r)$	Step (7) and the Rule of Conjunctive Simplification
(12)	$\neg w(r) \vee \neg \ell(r)$	Step (10) and DeMorgan's Law
(13)	$\neg \ell(r)$	Steps (11), (12), and the Rule of Disjunctive Syllogism
(14)	$\therefore\ \neg \ell(r) \wedge \neg w(i)$	Steps (13), (5), and the Rule of Conjunction

12. (a) Proof: Since k, ℓ are both even we may write $k = 2c$ and $\ell = 2d$, where c, d are integers. This follows from Definition 2.8. Then the sum $k + \ell = 2c + 2d = 2(c + d)$ by the distributive law of multiplication over addition for integers. Consequently, by Definition 2.8, it follows from $k + \ell = 2(c + d)$, with $c + d$ an integer, that $k + \ell$ is even.
(b) Proof: As in part (a) we write $k = 2c$ and $\ell = 2d$ for integers c, d. Then — by the commutative and associative laws of multiplication for integers — the product $k\ell = (2c)(2d) = 2(2cd)$, where $2cd$ is an integer. With $(2c)(2d) = 2(2cd)$, and $2cd$ an integer, it now follows from Definition 2.8 that $k\ell$ is even.

13. (a) Contrapositive: For all integers k and ℓ, if k, ℓ are not both odd then $k\ell$ is not odd. — OR, For all integers k and ℓ, if at least one of k, ℓ is even then $k\ell$ is even.
Proof: Let us assume (without loss of generality) that k is even. Then $k = 2c$ for some integer c — because of Definition 2.8. Then $k\ell = (2c)\ell = 2(c\ell)$, by the associative law of multiplication for integers — and $c\ell$ is an integer. Consequently, $k\ell$ is even — once again, by Definition 2.8. [Note that this result does not require anything about the integer ℓ.]

(b) Contrapositive: For all integers k and ℓ, if k and ℓ are not both even or both odd then $k + \ell$ is odd. — OR, For all integers k and ℓ, if one of k, ℓ is odd and the other even then $k + \ell$ is odd.
Proof: Let us assume (without loss of generality) that k is even and ℓ is odd. Then it follows from Definition 2.8 that we may write $k = 2c$ and $\ell = 2d + 1$ for integers c and d. And now we find that $k + \ell = 2c + (2d + 1) = 2(c + d) + 1$, where $c + d$ is an integer — by the associative law of addition and the distributive law of multiplication over addition for integers. From Definition 2.8 we find that $k + \ell = 2(c + d) + 1$ implies that $k + \ell$ is odd.

14. Proof: Since n is odd we may write $n = 2a + 1$, where a is an integer — by Definition 2.8. Then $n^2 = (2a + 1)^2 = 4a^2 + 4a + 1 = 2(2a^2 + 2a) + 1$, where $2a^2 + 2a$ is an integer. So again by Definition 2.8 it follows that n^2 is odd.

15. Proof: Assume that for some integer n, n^2 is odd while n is not odd. Then n is even and we may write $n = 2a$, for some integer a — by Definition 2.8. Consequently, $n^2 = (2a)^2 = (2a)(2a) = (2 \cdot 2)(a \cdot a)$, by the commutative and associative laws of multiplication for integers. Hence, we may write $n^2 = 2(2a^2)$, with $2a^2$ an integer — and this means that n^2 is even. Thus we have arrived at a contradiction since we now have n^2 both odd (at the start) and even. This contradiction came about from the false assumption that n is not odd. Therefore, for every integer n, it follows that n^2 odd $\Rightarrow n$ odd.

16. Here we must prove two results — namely, (i) if n^2 is even, then n is even; and (ii) if n is even, then n^2 is even.
Proof (i): Using the method of contraposition, suppose that n is not even — that is, n is odd. Then $n = 2a+1$, for some integer a, and $n^2 = (2a+1)^2 = 4a^2+4a+1 = 2(2a^2+2a)+1$, where $2a^2 + 2a$ is an integer. Hence n^2 is odd (or, not even).
Proof (ii): If n is even then $n = 2c$ for some integer c. So $n^2 = (2c)^2 = (2c)(2c) = 2(c(2c)) = 2((c \cdot 2)c) = 2((2c)c) = 2(2c^2)$, by the associative and commutative laws of multiplication for integers. Since $2c^2$ is an integer, it follows that n^2 is even.

17. Proof:
(1) Since n is odd we have $n = 2a + 1$ for some integer a. Then $n + 11 = (2a + 1) + 11 = 2a + 12 = 2(a + 6)$, where $a + 6$ is an integer. So by Definition 2.8 it follows that $n + 11$ is even.
(2) If $n + 11$ is not even, then it is odd and we have $n + 11 = 2b + 1$, for some integer b. So $n = (2b + 1) - 11 = 2b - 10 = 2(b - 5)$, where $b - 5$ is an integer, and it follows from Definition 2.8 that n is even — that is, not odd.
(3) In this case we stay with the hypothesis — that n is odd — and also assume that $n + 11$ is not even — hence, odd. So we may write $n + 11 = 2b + 1$, for some integer b. This then implies that $n = 2(b - 5)$, for the integer $b - 5$. So by Definition 2.8 it follows that n is even. But with n both even (as shown) and odd (as in the hypothesis) we have arrived at a contradiction. So our assumption was wrong, and it now follows that $n + 11$ is even for every odd integer n.

18. Proof: [Here we provide a direct proof.] Since m, n are perfect squares, we may write $m = a^2$ and $n = b^2$, where a, b are (positive) integers. Then by the associative and commutative laws of multiplication for integers we find that $mn = (a^2)(b^2) = (aa)(bb) = ((aa)b)b = (a(ab))b = ((ab)a)b = (ab)(ab) = (ab)^2$, so mn is also a perfect square.

19. This result is not true, in general. For example, $m = 4 = 2^2$ and $n = 1 = 1^2$ are two positive integers that are perfect squares, but $m + n = 2^2 + 1^2 = 5$ is not a perfect square.

20. Let $m = 9 = 3^2$ and $n = 16 = 4^2$. Then $m + n = 25 = 5^2$, so the result is true.

21. Proof: We shall prove the given result by establishing the truth of its (logically equivalent) contrapositive.
Let us consider the negation of the conclusion — that is, $x < 50$ and $y < 50$. Then with $x < 50$ and $y < 50$ it follows that $x + y < 50 + 50 = 100$, and we have the negation of the hypothesis. The given result now follows by this indirect method of proof (by the contrapositive).

22. Proof: Since $4n + 7 = 4n + 6 + 1 = 2(2n + 3) + 1$, it follows from Definition 2.8 that $4n + 7$ is odd.

23. Proof: If n is odd, then $n = 2k + 1$ for some (particular) integer k. Then $7n + 8 = 7(2k + 1) + 8 = 14k + 7 + 8 = 14k + 15 = 14k + 14 + 1 = 2(7k + 7) + 1$. It then follows from Definition 2.8 that $7n + 8$ is odd.

To establish the converse, suppose that n is not odd. Then n is even, so we can write $n = 2t$, for some (particular) integer t. But then $7n + 8 = 7(2t) + 8 = 14t + 8 = 2(7t + 4)$, so it follows from Definition 2.8 that $7n+8$ is even – that is, $7n+8$ is not odd. Consequently, the converse follows by contraposition.

24. Proof: If n is even, then $n = 2k$ for some (particular) integer k. Then $31n + 12 = 31(2k) + 12 = 62k + 12 = 2(31k + 6)$, so it follows from Definition 2.8 that $31n + 12$ is even.

Conversely, suppose that n is not even. Then n is odd, so $n = 2t + 1$ for some (particular) integer t. Therefore, $31n + 12 = 31(2t + 1) + 12 = 62t + 31 + 12 = 62t + 43 = 2(31t + 21) + 1$, so from Definition 2.8 we have $31n + 12$ odd – hence, not even. Consequently, the converse follows by contraposition.

Supplementary Exercises

1.

p	q	r	s	$q \wedge r$	$\neg(s \vee r)$	$\overbrace{[(q \wedge r) \rightarrow \neg(s \vee r)]}^{t}$	$p \leftrightarrow t$
0	0	0	0	0	1	1	0
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	0	1	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	0	1
1	0	0	0	0	1	1	1
1	0	0	1	0	0	1	1
1	0	1	0	0	0	1	1
1	0	1	1	0	0	1	1
1	1	0	0	0	1	1	1
1	1	0	1	0	0	1	1
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

2. (a)

p	q	r	$p \rightarrow q$	$\neg p \rightarrow r$	$(p \rightarrow q) \wedge (\neg p \rightarrow r)$
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	1	1	1

(b) If p, then q, else r.

3. (a)

p	q	r	$q \leftrightarrow r$	$p \leftrightarrow (q \leftrightarrow r)$	$(p \leftrightarrow q)$	$(p \leftrightarrow q) \leftrightarrow r$
0	0	0	1	0	1	0
0	0	1	0	1	1	1
0	1	0	0	1	0	1
0	1	1	1	0	0	0
1	0	0	1	1	0	1
1	0	1	0	0	0	0
1	1	0	0	0	1	0
1	1	1	1	1	1	1

It follows from the results in columns 5 and 7 that $[p \leftrightarrow (q \leftrightarrow r)] \Leftrightarrow [(p \leftrightarrow q) \leftrightarrow r]$.

(b) The truth value assignments $p : 0;\ q : 0;\ r : 0$ result in the truth value 1 for $[p \rightarrow (q \rightarrow r)]$ and 0 for $[(p \rightarrow q) \rightarrow r]$. Consequently, these statements are not logically equivalent.

4. $p \leftrightarrow q \Longleftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \Longleftrightarrow (\neg p \vee q) \wedge (\neg q \vee p)$, so $\neg(p \leftrightarrow q) \Longleftrightarrow$ $\neg(\neg p \vee q) \vee \neg(\neg q \vee p) \Longleftrightarrow (p \wedge \neg q) \vee (q \wedge \neg p)$

5. Since $p \vee \neg q \Leftrightarrow \neg\neg p \vee \neg q \Leftrightarrow \neg p \rightarrow \neg q$, we can express the given statement as:

(1) If Kaylyn does not practice her piano lessons, then she cannot go to the movies.

But $p \vee \neg q \Leftrightarrow \neg q \vee p \Leftrightarrow q \rightarrow p$, so we can also express the given statement as:
(2) If Kaylyn is to go to the movies, then she will have to practice her piano lessons.

6. a) $p \rightarrow (q \wedge r)$
Converse: $(q \wedge r) \rightarrow p$
Inverse: $[\neg p \rightarrow \neg(q \wedge r)] \Leftrightarrow [\neg p \rightarrow (\neg q \vee \neg r)]$
Contrapositive: $[\neg(q \wedge r) \rightarrow \neg p] \Leftrightarrow [(\neg q \vee \neg r) \rightarrow \neg p]$

b) $(p \vee q) \rightarrow r$
Converse: $r \rightarrow (p \vee q)$
Inverse: $[\neg(p \vee q) \rightarrow \neg r] \Leftrightarrow [(\neg p \wedge \neg q) \rightarrow \neg r]$
Contrapositive: $[\neg r \rightarrow \neg(p \vee q)] \Leftrightarrow [\neg r \rightarrow (\neg p \wedge \neg q)]$

7.

(a) $(\neg p \vee \neg q) \wedge (F_0 \vee p) \wedge p$

(b) $(\neg p \vee \neg q) \wedge (F_0 \vee p) \wedge p$

$\Longleftrightarrow$	$(\neg p \vee \neg q) \wedge (p \wedge p)$	$F_0 \vee p \Longleftrightarrow p$
$\Longleftrightarrow$	$(\neg p \vee \neg q) \wedge p$	Idempotent Law of $\wedge$
$\Longleftrightarrow$	$p \wedge (\neg p \vee \neg q)$	Commutative Law of $\wedge$
$\Longleftrightarrow$	$(p \wedge \neg p) \vee (p \wedge \neg q)$	Distributive Law of $\wedge$ over $\vee$
$\Longleftrightarrow$	$F_0 \vee (p \wedge \neg q)$	$p \wedge \neg p \Longleftrightarrow F_0$
$\Longleftrightarrow$	$p \wedge \neg q$	F_0 is the identity for $\vee$.

8. (a) $(p \wedge \neg q) \vee (\neg r \wedge s)$
(b) Since $p \rightarrow (q \wedge \neg r \wedge s) \Leftrightarrow \neg p \vee (q \wedge \neg r \wedge s)$ it follows that $[p \rightarrow (q \wedge \neg r \wedge s)]^d \Leftrightarrow \neg p \wedge (q \vee \neg r \vee s)$.
(c) $[(p \wedge F_0) \vee (q \wedge T_0)] \wedge [r \vee s \vee F_0]$

9.

(a) contrapositive	(b) inverse	(c) contrapositive
(d) inverse	(e) converse	

10. Proof by Contradiction

(1)	$\neg(p \rightarrow s)$	Premise (Negation of Conclusion)
(2)	$p \wedge \neg s$	Step (1), $(p \rightarrow s) \Longleftrightarrow \neg p \vee s$, DeMorgan's Laws, and the Law of Double Negation
(3)	p	Step (2) and the Rule of Conjunctive Simplification
(4)	$p \rightarrow q$	Premise
(5)	q	Steps (3), (4), and the Rule of Detachment
(6)	r	Premise
(7)	$q \wedge r$	Steps (5), (6), and the Rule of Conjunction
(8)	$(q \wedge r) \rightarrow s$	Premise
(9)	s	Steps (7), (8), and the Rule of Detachment
(10)	$\neg s$	Step (2) and the Rule of Conjunctive Simplification
(11)	$s \wedge \neg s \ (\Longleftrightarrow F_0)$	Steps (9), (10), and the Rule of Conjunction
(12)	$\therefore p \rightarrow s$	Steps (1), (11), and the Method of Proof by Contradiction

Method 2

(1)	$(q \wedge r) \rightarrow s$	Premise
(2)	$r \rightarrow (q \rightarrow s)$	$r \rightarrow (q \rightarrow s) \Longleftrightarrow (q \wedge r) \rightarrow s$
(3)	r	Premise
(4)	$q \rightarrow s$	Steps (2), (3), and Modus Ponens
(5)	$p \rightarrow q$	Premise
(6)	$\therefore p \rightarrow s$	Steps (4), (5), and the Law of the Syllogism

Method 3

(1)	$(q \wedge r) \rightarrow s$	Premise
(2)	$\neg s \rightarrow \neg(q \wedge r)$	Step (1) and for primitive statements u, v $u \rightarrow v \Leftrightarrow \neg v \rightarrow \neg u$ – and the 1st Substitution Rule.
(3)	$s \vee \neg(q \wedge r)$	Step (2) and for primitive statements $u, v, u \rightarrow v \Longleftrightarrow \neg u \vee v$ – and the lst Substitution Rule. Also, $\neg\neg s \Longleftrightarrow s$.
(4)	$(s \vee \neg q) \vee \neg r$	Step (3), DeMorgan's Law, and the Associative Law of $\vee$
(5)	r	Premise
(6)	$s \vee \neg q$	Steps (4), (5), and the Law of Disjunctive Syllogism
(7)	$q \rightarrow s$	Step (6) and $s \vee \neg q \Longleftrightarrow \neg q \vee s \Longleftrightarrow q \rightarrow s$
(8)	$p \rightarrow q$	Premise
(9)	$\therefore p \rightarrow s$	Steps (7), (8), and the Law of the Syllogism

Method 4 (Here we assume p as an additional premise and obtain s as our conclusion.)

(1)	p	Premise (assumed)
(2)	$p \rightarrow q$	Premise
(3)	q	Steps (1), (2), and Modus Ponens
(4)	r	Premise
(5)	$q \wedge r$	Steps (3), (4), and the Rule of Conjunction
(6)	$(q \wedge r) \rightarrow s$	Premise
(7)	$\therefore s$	Steps (5), (6), and Modus Ponens

11. (a)

p	q	r	$p \veebar q$	$(p \veebar q) \veebar r$	$q \veebar r$	$p \veebar (q \veebar r)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	0	0	0
1	0	0	1	1	0	1
1	0	1	1	0	1	0
1	1	0	0	0	1	0
1	1	1	0	1	0	1

It follows from the results in columns 5 and 7 that $[(p \veebar q) \veebar r] \Longleftrightarrow [p \veebar (q \veebar r)]$.
(b) The given statements are not logically equivalent. The truth value assignments $p: 1;\ q: 0;\ r: 0$ provide a counterexample.

12. p : The temperature is cool on Friday.
q : Craig wears his suede jacket.
r : The pockets (of the suede jacket) are mended.

$$\begin{array}{l} p \to (r \to q) \\ \underline{p \wedge \neg r} \\ \therefore \neg q \end{array}$$

The argument is invalid. The truth value assignments $p : 1;\quad q : 1;\quad r : 0$ provide a counterexample.

13.

(a)	True	(b)	False	(c)	True	(d)	True
(e)	False	(f)	False	(g)	False	(h)	True

14. a) This statement is true. Note that $1 = 7(-2) + 5(3)$, so for each integer x, $x = 7(-2x) + 5(3x)$.
b) Since 2 divides both 4 and 6, it follows that 2 divides $4y + 6z$. Consequently, the result is false for each odd integer x. [Since $2 = 4(-1) + 6(1)$, the result is true for each even integer x.]

15. Suppose that the 62 squares in this 8×8 chessboard (with two opposite missing corners) can be covered with 31 dominos. We agree to place each domino on the board so that the blue part is on top of a blue square (and the white part is then necessarily above a white square). The given chessboard contains 30 blue squares and 32 white ones. Each domino covers one blue and one white square – for a total of 31 blue squares and 31 white ones. This contradiction tells us that we cannot cover this 62 square chessboard with the 31 dominos.

16. Suppose that the 60 squares in the 8×8 chessboard (with two squares – one blue and one white – removed from each of two opposite corners) can be covered with 15 of these T-shaped figures. When covering the chessboard we agree to place each T-shaped figure on the board so that the color of each square in the T-shaped figure matches the color of the chessboard square that it covers. Let n be the number of T-shaped figures with three blue squares (and one white one) used in the covering. The chessboard contains 30 blue squares, so it follows that

$$3n + 1 \cdot (15n - n) = 30.$$

Consequently, $2n = 15$ – so 15 is both odd and even. This contradiction tells us that we cannot cover the given chessboard with these T-shaped figures.