

# INSTRUCTOR'S SOLUTIONS MANUAL

## DISCRETE MATHEMATICS

FIFTH EDITION

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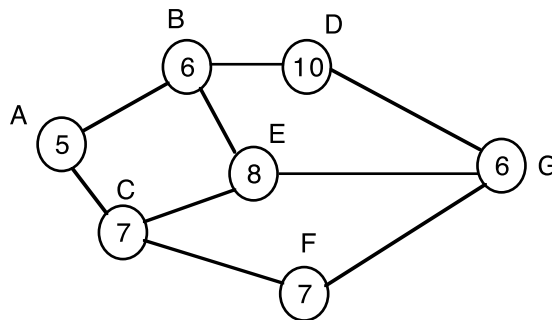
# Chapter 1

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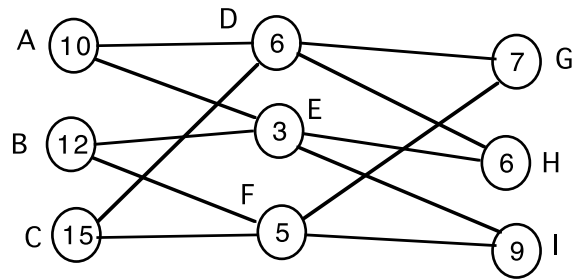
## An Introduction to Combinatorial Problems and Techniques

### 1.1 THE TIME TO COMPLETE A PROJECT

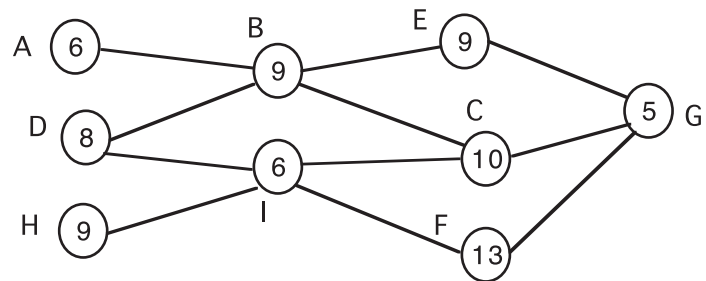
- 2. 31; A-B-E-G
- 4. 39; A-C-G-H
- 6. 16; B-D-F-H
- 8. 27; A-D-E-H
- 10. 27; A-B-D-G



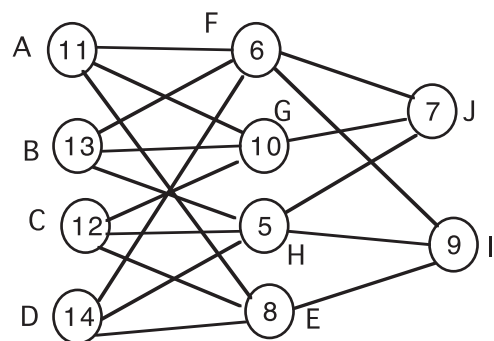
12. 29; C-F-I



14. 33; H-I-F-G



16. 31; D-E-I



18. 20 minutes

**1.2 A MATCHING PROBLEM**

- |                |                   |          |          |
|----------------|-------------------|----------|----------|
| 2. 720         | 4. 210            | 6. 84    | 8. 1680  |
| 10. 19,958,400 | 12. $\frac{1}{8}$ | 14. 5040 | 16. 126  |
| 18. 210        | 20. 119           | 22. 1320 | 24. 5040 |
| 26. 240        | 28. 1200          |          |          |

**1.3 A KNAPSACK PROBLEM**

- |                                                                                                                                                    |          |         |        |
|----------------------------------------------------------------------------------------------------------------------------------------------------|----------|---------|--------|
| 2. T                                                                                                                                               | 4. F     | 6. F    | 8. F   |
| 10. T                                                                                                                                              | 12. T    | 14. T   | 16. no |
| 18. yes; 32                                                                                                                                        |          |         |        |
| 20. $\emptyset$ , {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}; 16 |          |         |        |
| 22. 128                                                                                                                                            | 24. 1024 | 26. 256 | 28. 26 |
| 30. {1, 4, 6, 7, 8, 9, 10, 11, 12}                                                                                                                 |          |         |        |

**1.4 ALGORITHMS AND THEIR EFFICIENCY**

- |                                   |                         |
|-----------------------------------|-------------------------|
| 2. yes; 0                         | 4. no                   |
| 6. no                             | 8. -1, 9, 84; 3, 17, 84 |
| 10. -4, -4, 41, 95; 2, 11, 33, 95 | 12. 111000              |
| 14. 001010                        |                         |

16.

| $k$ | $j$ | $a_1$ | $a_2$ | $a_3$ |
|-----|-----|-------|-------|-------|
| 3   |     | 1     | 1     | 1     |
| 2   |     | 1     | 1     | 1     |
| 1   |     | 1     | 1     | 1     |
| 0   |     | 1     | 1     | 1     |

18.

| $k$ | $j$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|-----|-----|-------|-------|-------|-------|
| 4   |     | 1     | 1     | 1     | 0     |
|     |     | 1     | 1     | 1     | 1     |

20. The circled numbers in the table below indicated the items being compared.

| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $j$ | $k$ |
|-------|-------|-------|-------|-----|-----|
| 23    | 5     | 17    | 12    | 1   | 3   |
| 23    | 5     | 12    | 17    |     | 2   |
| 23    | 5     | 12    | 17    |     | 1   |
| 5     | 23    | 12    | 17    | 2   | 3   |
| 5     | 23    | 12    | 17    |     | 2   |
| 5     | 12    | 23    | 17    | 3   | 3   |
| 5     | 12    | 17    | 23    |     |     |

22. The circled numbers in the table below indicated the items being compared.

| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $j$ | $k$ |
|-------|-------|-------|-------|-------|-----|-----|
| 88    | 2     | 75    | 10    | 48    | 1   | 4   |
| 88    | 2     | 75    | 10    | 48    |     | 3   |
| 88    | 2     | 10    | 75    | 48    |     | 2   |
| 88    | 2     | 10    | 75    | 48    |     | 1   |
| 2     | 88    | 10    | 75    | 48    | 2   | 4   |
| 2     | 88    | 10    | 48    | 75    |     | 3   |
| 2     | 88    | 10    | 48    | 75    |     | 2   |
| 2     | 10    | 88    | 48    | 75    | 3   | 4   |
| 2     | 10    | 88    | 48    | 75    |     | 3   |
| 2     | 10    | 48    | 88    | 75    | 4   | 4   |
| 2     | 10    | 48    | 75    | 88    |     |     |

24. 6.5 years, 2.7 seconds

26.  $2.3 \times 10^{10}$  years, 12.5 seconds

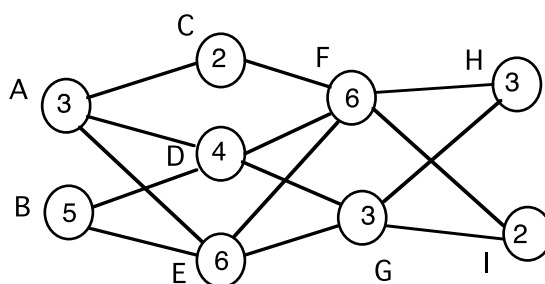
28.  $4n - 3$

30.  $3n - 2$

32.  $-4, -4, 41, 95$

**SUPPLEMENTARY EXERCISES**

2. 20; B-E-F-H



4. 336

6. 40

8. 14040

10. T

12. F

14. T

16. T

18. 16

20. no

22. yes; 0

24.  $-5, 7, 7, 88$

26.  $\emptyset, \{4\}, \{3\}, \{3, 4\}, \{2\}, \{2, 4\}, \{2, 3\}, \{2, 3, 4\}, \{1\}, \{1, 4\}, \{1, 3\}, \{1, 3, 4\}, \{1, 2\}, \{1, 2, 4\}, \{1, 2, 3\}, \{1, 2, 3, 4\}$

28.  $4.92 \times 10^8$  years

30. 4

32.  $4r - 3$

## Chapter 2

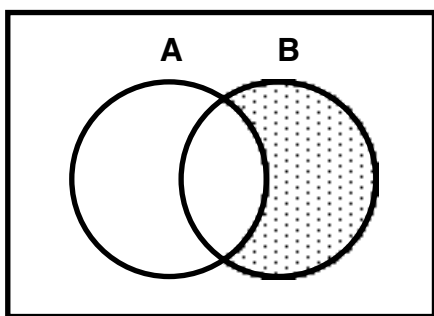
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# Sets, Relations, and Functions

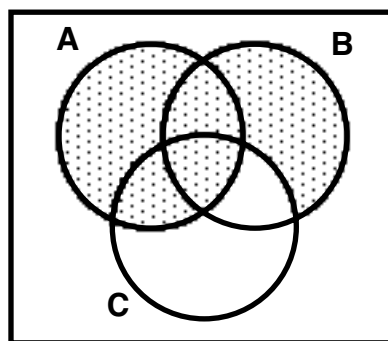
### 2.1 SET OPERATIONS

2.  $A \cup B = \{1, 2, 4, 5, 6, 7, 9\}$ ,  $A \cap B = \{1, 4, 6, 9\}$ ,  $A - B = \emptyset$ ,  $\overline{A} = \{2, 3, 5, 7, 8\}$ , and  $\overline{B} = \{3, 8\}$
4.  $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$ ,  $A \cap B = \{7, 9\}$ ,  $A - B = \{3, 4, 6, 8\}$ ,  $\overline{A} = \{1, 2, 5\}$ , and  $\overline{B} = \{1, 3, 4, 6, 8\}$
6.  $\{(3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$
8.  $\{(p, a), (p, c), (p, e), (q, a), (q, c), (q, e), (r, a), (r, c), (r, e), (s, a), (s, c), (s, e)\}$

10.



12.



14.  $A = \{1, 2\}$ ,  $B = \{1, 3\}$ , and  $C = \{1\}$
16.  $A = \{1, 2\}$ ,  $B = \{1, 3\}$ , and  $C = \{1\}$

2. reflexive and symmetric
4. reflexive, symmetric, and transitive
6. reflexive and symmetric
8. none
10. reflexive and symmetric
12. reflexive and transitive
14. The equivalence class of  $R$  containing ABCD consists of the string ABC and the strings of 4 letters having A as their first letter and C as their third letter. There are  $26^2 = 676$  distinct equivalence classes of  $R$ .
16. The equivalence class of  $R$  containing  $\{1, 2, 3\}$  is the set containing the following four elements of  $S$ :  $\{1, 3\}$ ,  $\{1, 2, 3\}$ ,  $\{1, 2, 3, 4\}$ , and  $\{1, 3, 4\}$ . There are 8 different equivalence classes of  $R$ , namely the sets consisting of the elements  $S$ ,  $S \cup \{2\}$ ,  $S \cup \{4\}$ , and  $S \cup \{2, 4\}$  for every  $S \subseteq \{1, 3, 5\}$ .
18. The equivalence class of  $R$  containing  $(5, 8)$  is the set

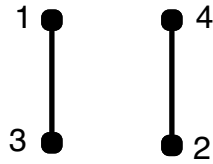
There are infinitely many distinct equivalence classes of  $R$ , namely, the sets of the form

20.  $\{(1, 1), (1, 3), (1, 6), (3, 1), (3, 3), (3, 6), (6, 1), (6, 3), (6, 6), (2, 2), (2, 5), (5, 2), (5, 5), (4, 4)\}$
24. The equivalence classes have the form  $E_1 \times E_2$ , where  $E_i$  is an equivalence class of  $R_i$ .
28. There are 5 partitions of a set with three elements.
32. Let  $S$  be a nonempty set and  $R$  an equivalence relation on  $S$ . Then there is a function  $f$  with domain  $S$  such that  $s_1 R s_2$  if and only if  $f(s_1) = f(s_2)$ .

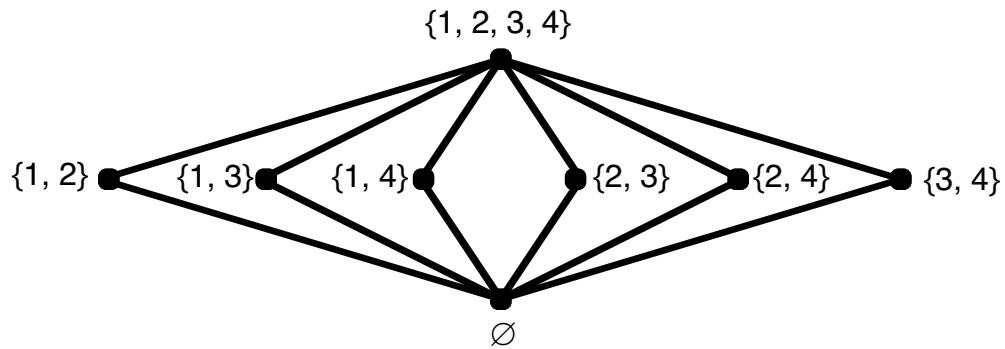
### 2.3 PARTIAL ORDERING RELATIONS

- 2. not antisymmetric
- 4. partial ordering
- 6. not antisymmetric
- 8. not antisymmetric

10.

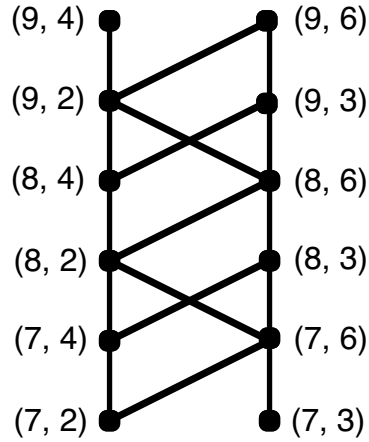


12.



- 14.  $R = \{(a, a), (b, b), (b, a), (c, c), (c, b), (c, a), (d, d), (d, a)\}$
- 16.  $R = \{(1, 1), (2, 2), (2, 1), (2, 4), (4, 4), (8, 8), (8, 4)\}$
- 18. The maximal elements are  $\{1\}$ ,  $\{2\}$ , and  $\{3\}$ ; the only minimal element is  $\{1, 2, 3\}$ .
- 20. The only minimal element is 0; there are no maximal elements.
- 22. One possible sequence is 1, 3, 2, 4.
- 24. One possible sequence is 1, 3, 2, 6, 4, 12.
- 26. Let  $S$  denote the set of residents of Illinois and  $R$  be defined so that  $x R y$  means that  $x$  is a sister of  $y$ .
- 28. Every prime integer is a minimal element; there are no maximal elements.
- 30.  $(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)$

32.



36. (a) Suppose that  $y$  is element of  $S$  such that  $x R y$  is false. If there is no element  $y_1$  in  $S$  such that  $y_1 R y$ , then  $y$  is a minimal element of  $S$ , contradicting that  $x$  is the unique minimal element of  $S$ . Thus there must be such an element  $y_1$ . If there is no element  $y_2$  in  $S$  such that  $y_2 R y_1$ , then  $y_1$  is a minimal element of  $S$ , another contradiction. So there must be such an element  $y_2$ . Because  $S$  is finite, continuing in this manner must produce a minimal element  $y_k$  of  $S$  different from  $x$ . Because  $x$  is the unique minimal element of  $S$ , the assumption that there is an element  $y$  of  $S$  such that  $x R y$  is false must be incorrect. Thus  $x R s$  is true for every  $s \in S$ .
- (b) Let  $Z$  denote the set of integers and  $S = Z \cup \{\emptyset\}$ . Let  $R$  be the relation defined on  $S$  by  $x R y$  if and only if one of the following holds: (i)  $x, y \in Z$  and  $x \leq y$ , or (ii)  $x = y = \emptyset$ . Then  $\emptyset$  is the unique minimal element in  $S$ , but  $\emptyset R z$  is false for every  $z \in Z$ .

40.  $2^n$ 42.  $2^n \cdot 3^{n(n-1)/2}$

## 2.4 FUNCTIONS

2. not a function with domain  $X$                       4. a function with domain  $X$
6. a function with domain  $X$                       8. a function with domain  $X$
10. a function with domain  $X$                       12. not a function with domain  $X$
14. 4                      16. 13                      18. 8                      20. 7
22. -1                      24. 6                      26. -2                      28. 10
30. 0.78                      32. 6.64                      34. 3.22                      36. -2.56
38.  $gf(x) = g(f(x)) = \sqrt{x^2 + 1}$  and  $fg(x) = f(g(x)) = x + 1$
40.  $gf(x) = \frac{1}{3x}$  and  $fg(x) = \frac{3}{x}$
42.  $gf(x) = 5(2^x) - 2^{2x}$  and  $fg(x) = 2^{5x-x^2}$
44.  $gf(x) = \frac{4x-1}{2-x}$  and  $fg(x) = \frac{3x-2}{3-x}$
46. one-to-one and onto                      48. neither one-to-one nor onto
50. one-to-one but not onto                      52. onto but not one-to-one
54.  $f^{-1}(x) = \frac{x+2}{3}$                       56.  $f^{-1}(x)$  does not exist.
58.  $f^{-1}(x)$  does not exist.                      60.  $f^{-1}(x) = \sqrt[3]{x+1}$
62. For  $Y = \{x \in X: x \neq 0\}$ , we have  $g^{-1}(x) = \frac{-1}{x}$ .
64. If  $n < m$ , there are no one-to-one functions; and if  $m \leq n$ , there are
- $$P(n, m) = n(n-1)(n-2) \cdots (n-m+1)$$
- one-to-one functions.
68. Let  $X = \{1\}$ ,  $Y = \{2, 3\}$ , and  $Z = 4$ . Define  $f: X \rightarrow Y$  by  $f(x) = 2$  for all  $x \in X$  and  $g: Y \rightarrow Z$  by  $g(y) = 4$  for all  $y \in Y$ .

**2.5 MATHEMATICAL INDUCTION**

2. 7, 9, 13, 21, 37, 69
4.  $x_n = \begin{cases} x & \text{if } n = 1 \\ x \cdot x^{n-1} & \text{if } n \geq 2 \end{cases}$
6. Let  $x_n$  denote the  $n$ th odd positive integer. Then  $x_n = \begin{cases} 1 & \text{if } n = 1 \\ x_{n-1} + 2 & \text{if } n \geq 2. \end{cases}$
8. If  $k = 1$ , the sets  $\{x_1, x_2, \dots, x_k\}$  and  $\{x_2, x_3, \dots, x_{k+1}\}$  are disjoint.
10. If  $k = 0$ , the induction hypothesis cannot be applied to  $a^{k-1}$ .
28.  $s_0 + s_1 + \dots + s_n = (n+1) \left( s_0 + \frac{nd}{2} \right)$

**2.6 APPLICATIONS**

- |                                                |          |
|------------------------------------------------|----------|
| 2. 56                                          | 4. 924   |
| 6. 120                                         | 8. 715   |
| 10. $n$                                        | 12. $r!$ |
| 14. 31                                         | 16. 4096 |
| 18. 128                                        | 20. 15   |
| 22. 36                                         | 24. 286  |
| 26. 126                                        | 28. 64   |
| 44. There are $\frac{n^2 + n + 2}{2}$ regions. |          |

# SUPPLEMENTARY EXERCISES

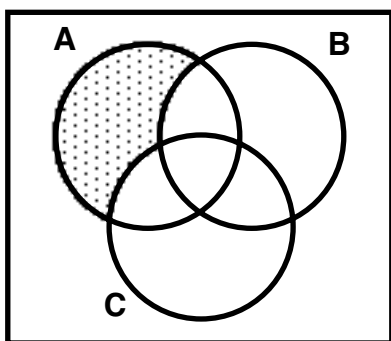
2.  $\{1, 2, 3, 4, 5\}$

4.  $\{1, 2, 4\}$ .

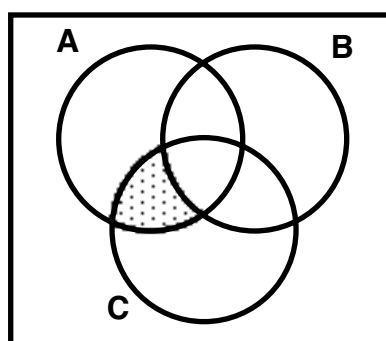
6.  $\{1, 2, 4, 5, 6\}$

8.  $\{2, 3\}$

10.



12.



14.  $gf(x) = 2(x^3 + 1) - 5 = 2x^3 - 3$  and  $fg(x) = (2x - 5)^3 + 1 = 8x^3 - 60x^2 + 150x - 124$

16. not a function with domain  $X$

18. not a function with domain  $X$

20. neither one-to-one nor onto

22. one-to-one and onto

24.  $f^{-1}$  does not exist.

26.  $f^{-1} = \sqrt[3]{x-5}$

28. 84

30. 210

32.  $\{1, 7\}, \{2, 6\}, \{3, 5\}, \{4\}, \{8\}$

34. the sets of positive and negative real numbers

36. 5

38. 6

40. (b)  $a = \pm 1$

42. reflexive only

48.  $x \vee x = x$  for all  $x \in S$ ,  $1 \vee y = y \vee 1 = y$  for all  $y \in S$ ,  $2 \vee 3 = 3 \vee 2 = 6$ ,  $2 \vee 4 = 4 \vee 2 = 4$ ,  $2 \vee 6 = 6 \vee 2 = 6$ ,  $3 \vee 6 = 6 \vee 3 = 6$ ,  $x \vee y$  is undefined in all other cases

52. Let  $S$  denote the set of all subsets of  $\{1, 2, 3\}$  that contain at most two elements, and let  $R$  be the relation “is a subset of.” If  $A = \{1\}$ ,  $B = \{2\}$ , and  $C = \{3\}$ , then  $A \vee B = \{1, 2\}$ ,  $B \vee C = \{2, 3\}$ , and  $A \vee C = \{1, 3\}$ , but  $(A \vee B) \vee C$  does not exist.
54.  $[n] = \{-n, n\}$
56. 
$$f(x) = \begin{cases} x & \text{if } x \equiv 0 \pmod{3} \\ x - 1 & \text{if } x \equiv 1 \pmod{3} \\ x - 2 & \text{if } x \equiv 2 \pmod{3} \end{cases}$$