Instructor's Solutions Manual

Discrete Mathematics

Fifth Edition

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Chapter 1

An Introduction to Combinatorial Problems and Techniques

1.1 THE TIME TO COMPLETE A PROJECT

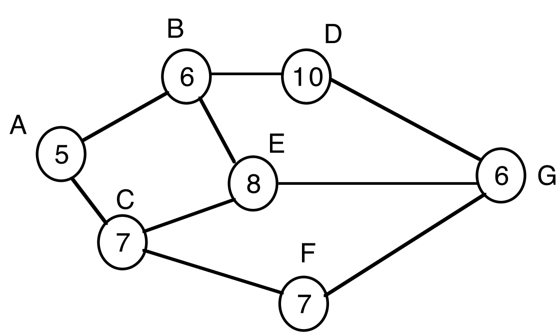
2. 31; A-B-E-G

4. 39; A-C-G-H

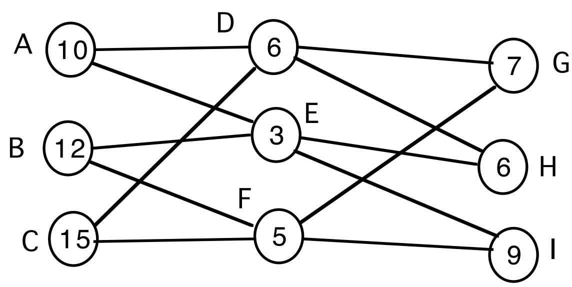
6. 16; B-D-F-H

8. 27; A-D-E-H

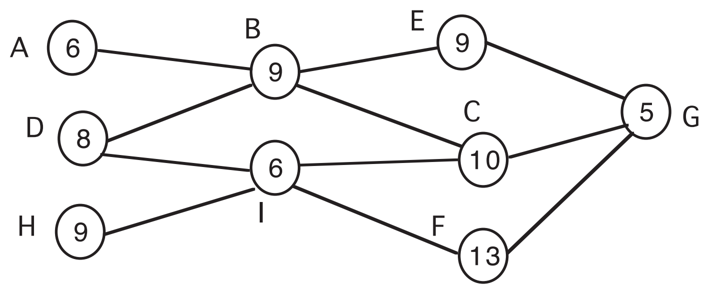
10. 27; A-B-D-G



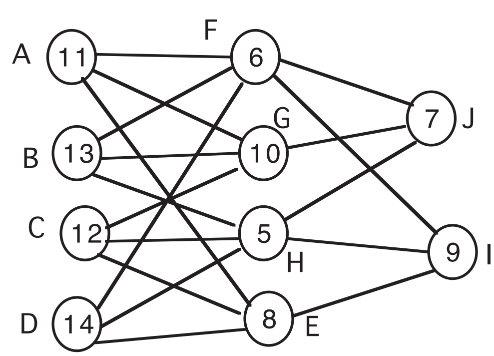
12. 29; C-F-I



14. 33; H-I-F-G



16. 31; D-E-I



18. 20 minutes

1.2 A MATCHING PROBLEM

2.	720	**4.**	210	**6.**	84	**8.**	1680
10.	19,958,400	**12.**	$\frac{1}{8}$	**14.**	5040	**16.**	126
18.	210	**20.**	119	**22.**	1320	**24.**	5040
26.	240	**28.**	1200				

1.3 A KNAPSACK PROBLEM

2.	T	**4.**	F	**6.**	F	**8.**	F
10.	T	**12.**	T	**14.**	T	**16.**	no

18. yes; 32

20. $\varnothing$, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{1,2\}$, $\{1,3\}$, $\{1,4\}$, $\{2,3\}$, $\{2,4\}$, $\{3,4\}$, $\{1,2,3\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$, $\{1,2,3,4\}$; 16

22.	128	**24.**	1024	**26.**	256	**28.**	26

30. $\{1,4,6,7,8,9,10,11,12\}$

1.4 ALGORITHMS AND THEIR EFFICIENCY

2.	yes; 0	**4.**	no
6.	no	**8.**	-1, 9, 84; 3, 17, 84
10.	-4, -4, 41, 95; 2, 11, 33, 95	**12.**	111000
14.	001010		

16.

k	j	a_1	a_2	a_3
3		1	1	1
2		1	1	1
1		1	1	1
0		1	1	1

18.

k	j	a_1	a_2	a_3	a_4
4		1	1	1	0
		1	1	1	1

20. The circled numbers in the table below indicated the items being compared.

a_1	a_2	a_3	a_4	j	k
23	5	17	12	1	3
23	5	12	17		2
23	5	12	17		1
5	23	12	17	2	3
5	23	12	17		2
5	12	23	17	3	3
5	12	17	23		

22. The circled numbers in the table below indicated the items being compared.

a_1	a_2	a_3	a_4	a_5	j	k
88	2	75	10	48	1	4
88	2	75	10	48		3
88	2	10	75	48		2
88	2	10	75	48		1
2	88	10	75	48	2	4
2	88	10	48	75		3
2	88	10	48	75		2
2	10	88	48	75	3	4
2	10	88	48	75		3
2	10	48	88	75	4	4
2	10	48	75	88		

24. 6.5 years, 2.7 seconds

26. 2.3×10^{10} years, 12.5 seconds

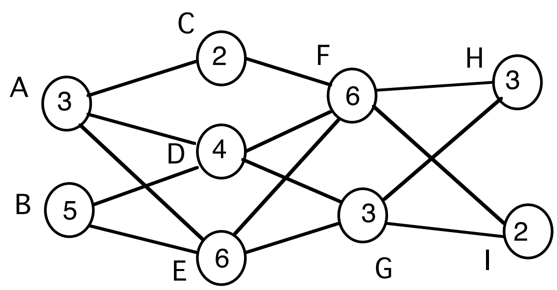
28. $4n - 3$

30. $3n - 2$

32. $-4, -4, 41, 95$

SUPPLEMENTARY EXERCISES

2. 20; B-E-F-H



4. 336

6. 40

8. 14040

10. T

12. F

14. T

16. T

18. 16

20. no

22. yes; 0

24. $-5, 7, 7, 88$

26. $\varnothing, \{4\}, \{3\}, \{3,4\}, \{2\}, \{2,4\}, \{2,3\}, \{2,3,4\}, \{1\}, \{1,4\}, \{1,3\}, \{1,3,4\}, \{1,2\}, \{1,2,4\}, \{1,2,3\}, \{1,2,3,4\}$

28. 4.92×10^8 years

30. 4

32. $4r - 3$

Chapter 2

Sets, Relations, and Functions

2.1 SET OPERATIONS

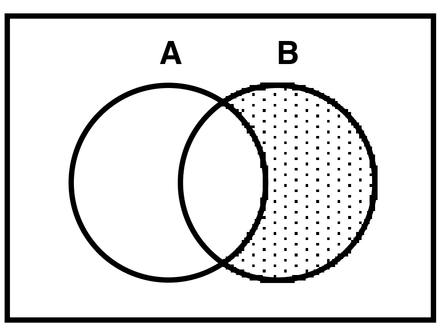
2. $A \cup B = \{1, 2, 4, 5, 6, 7, 9\}$, $A \cap B = \{1, 4, 6, 9\}$, $A - B = \varnothing$, $\overline{A} = \{2, 3, 5, 7, 8\}$, and $\overline{B} = \{3, 8\}$

4. $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$, $A \cap B = \{7, 9\}$, $A - B = \{3, 4, 6, 8\}$, $\overline{A} = \{1, 2, 5\}$, and $\overline{B} = \{1, 3, 4, 6, 8\}$

6. $\{(3,\ 1),\ (3,\ 2),\ (3,\ 3),\ (4,\ 1),\ (4,\ 2),\ (4,\ 3),\ (5,\ 1),\ (5,\ 2),\ (5,\ 3)\}$

8. $\{(p, a), (p, c), (p, e), (q, a), (q, c), (q, e), (r, a), (r, c), (r, e), (s, a), (s, c), (s, e)\}$

10.



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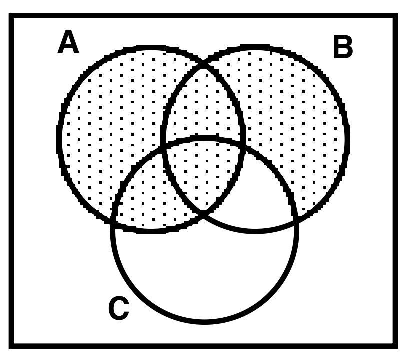


14. $A = \{1,\ 2\}$, $B = \{1,\ 3\}$, and $C = \{1\}$

16. $A = \{1,\ 2\}$, $B = \{1,\ 3\}$, and $C = \{1\}$

18. A

20. $\varnothing$

22. $A \cap B$

24. $A \cup \overline{B}$

26. The equality $A - B = B - A$ holds if and only if $A = B$.

28. The equality $A \cap B = A$ holds if and only if $A \subseteq B$.

2.2 EQUIVALENCE RELATIONS

2. reflexive and symmetric

4. reflexive, symmetric, and transitive

6. reflexive and symmetric

8. none

10. reflexive and symmetric

12. reflexive and transitive

14. The equivalence class of R containing ABCD consists of the string ABC and the strings of 4 letters having A as their first letter and C as their third letter. There are $26^2 = 676$ distinct equivalence classes of R.

16. The equivalence class of R containing $\{1,\ 2,\ 3\}$ is the set containing the following four elements of S: $\{1,\ 3\}$, $\{1,\ 2,\ 3\}$, $\{1,\ 2,\ 3,\ 4\}$, and $\{1,\ 3,\ 4\}$. There are 8 different equivalence classes of R, namely the sets consisting of the elements S, $S \cup \{2\}$, $S \cup \{4\}$, and $S \cup \{2, 4\}$ for every $S \subseteq \{1, 3, 5\}$.

18. The equivalence class of R containing $(5, 8)$ is the set

$$\{(x_1, x_2)\colon \text{each } x_i \text{ is an integer and } x_1 - x_2 = 5 - 8\}.$$

There are infinitely many distinct equivalence classes of R, namely, the sets of the form

$$\{(x_1, x_2)\colon \text{each } x_i \text{ is an integer and } x_1 - x_2 = k\},$$

where $k = 0, \pm 1, \pm 2, \pm 3, \ldots$.

20. $\{(1,1), (1,3), (1,6), (3,1), (3,3), (3,6), (6,1), (6,3), (6,6), (2,2), (2,5), (5,2), (5,5), (4,4)\}$

24. The equivalence classes have the form $E_1 \times E_2$, where E_i is an equivalence class of R_i.

28. There are 5 partitions of a set with three elements.

32. Let S be a nonempty set and R an equivalence relation on S. Then there is a function f with domain S such that $s_1\ R\ s_2$ if and only if $f(s_1) = f(s_2)$.

2.3 PARTIAL ORDERING RELATIONS

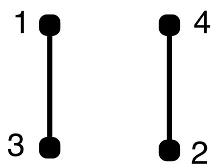
2. not antisymmetric

4. partial ordering

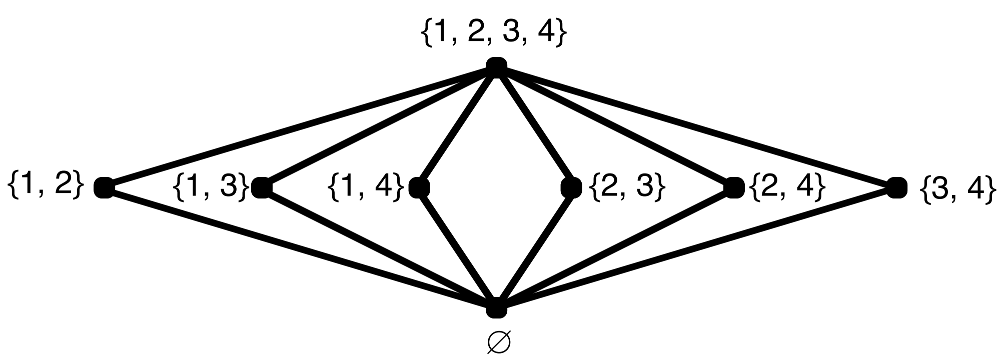
6. not antisymmetric

8. not antisymmetric

10.



12.



14. $R = \{(a,a),(b,b),(b,a),(c,c),(c,b),(c,a),(d,d),(d,a)\}$

16. $R = \{(1,1),(2,2),(2,1),(2,4),(4,4),(8,8),(8,4)\}$

18. The maximal elements are {1}, {2}, and {3}; the only minimal element is {1, 2, 3}.

20. The only minimal element is 0; there are no maximal elements.

22. One possible sequence is 1, 3, 2, 4.

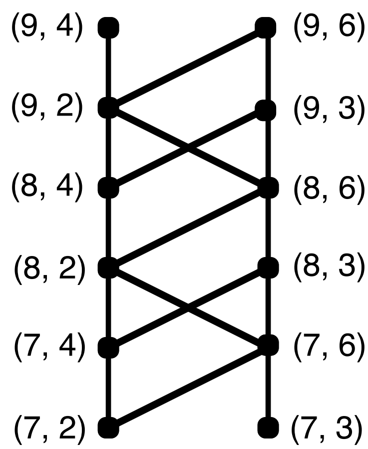
24. One possible sequence is 1, 3, 2, 6, 4, 12.

26. Let S denote the set of residents of Illinois and R be defined so that $x\ R\ y$ means that x is a sister of y.

28. Every prime integer is a minimal element; there are no maximal elements.

30. $(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4)$

32.



36. **(a)** Suppose that y is element of S such that $x\ R\ y$ is false. If there is no element y_1 in S such that $y_1\ R\ y$, then y is a minimal element of S, contradicting that x is the unique minimal element of S. Thus there must be such an element y_1. If there is no element y_2 in S such that $y_2\ R\ y_1$, then y_1 is a minimal element of S, another contradiction. So there must be such an element y_2. Because S is finite, continuing in this manner must produce a minimal element y_k of S different from x. Because x is the unique minimal element of S, the assumption that there is an element y of S such that $x\ R\ y$ is false must be incorrect. Thus $x\ R\ s$ is true for every $s \in S$.

(b) Let Z denote the set of integers and $S = Z \cup \{\varnothing\}$. Let R be the relation defined on S by $x\ R\ y$ if and only if one of the following holds: (i) $x, y \in Z$ and $x \leq y$, or (ii) $x = y = \varnothing$. Then $\varnothing$ is the unique minimal element in S, but $\varnothing\ R\ z$ is false for every $z \in Z$.

40. 2^n

42. $2^n \cdot 3^{n(n-1)/2}$

2.4 FUNCTIONS

2. not a function with domain X

4. a function with domain X

6. a function with domain X

8. a function with domain X

10. a function with domain X

12. not a function with domain X

14. 4

16. 13

18. 8

20. 7

22. -1

24. 6

26. -2

28. 10

30. 0.78

32. 6.64

34. 3.22

36. -2.56

38. $gf(x) = g(f(x)) = \sqrt{x^2+1}$ and $fg(x) = f(g(x)) = x+1$

40. $gf(x) = \dfrac{1}{3x}$ and $fg(x) = \dfrac{3}{x}$

42. $gf(x) = 5(2^x) - 2^{2x}$ and $fg(x) = 2^{5x-x^2}$

44. $gf(x) = \dfrac{4x-1}{2-x}$ and $fg(x) = \dfrac{3x-2}{3-x}$

46. one-to-one and onto

48. neither one-to-one nor onto

50. one-to-one but not onto

52. onto but not one-to-one

54. $f^{-1}(x) = \dfrac{x+2}{3}$

56. $f^{-1}(x)$ does not exist.

58. $f^{-1}(x)$ does not exist.

60. $f^{-1}(x) = \sqrt[3]{x+1}$

62. For $Y = \{x \in X\colon\ x \neq 0\}$, we have $g^{-1}(x) = \dfrac{-1}{x}$.

64. If $n < m$, there are no one-to-one functions; and if $m \leq n$, there are

$$P(n,m) = n(n-1)(n-2)\cdots(n-m+1)$$

one-to-one functions.

68. Let $X = \{1\}$, $Y = \{2,3\}$, and $Z = 4$. Define $f\colon\ X \to Y$ by$f(x) = 2$ for all $x \in X$ and $g\colon\ Y \to Z$ by $g(y) = 4$ for all $y \in Y$.

2.5 MATHEMATICAL INDUCTION

2. 7, 9, 13, 21, 37, 69

4. $x_n = \begin{cases} x & \text{if } n = 1 \\ x \cdot x^{n-1} & \text{if } n \geq 2 \end{cases}$

6. Let x_n denote the nth odd positive integer. Then $x_n = \begin{cases} 1 & \text{if } n = 1 \\ x_{n-1} + 2 & \text{if } n \geq 2. \end{cases}$

8. If $k = 1$, the sets $\{x_1, x_2, \ldots, x_k\}$ and $\{x_2, x_3, \ldots, x_{k+1}\}$ are disjoint.

10. If $k = 0$, the induction hypothesis cannot be applied to a^{k-1}.

28. $s_0 + s_1 + \cdots + s_n = (n+1)\left(s_0 + \frac{nd}{2}\right)$

2.6 APPLICATIONS

2. 56

4. 924

6. 120

8. 715

10. n

12. $r!$

14. 31

16. 4096

18. 128

20. 15

22. 36

24. 286

26. 126

28. 64

44. There are $\dfrac{n^2 + n + 2}{2}$ regions.

SUPPLEMENTARY EXERCISES

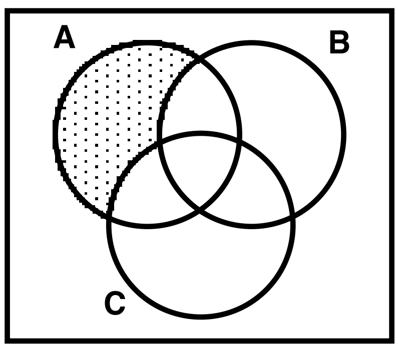
2. $\{1, 2, 3, 4, 5\}$

4. $\{1, 2, 4\}$.

6. $\{1, 2, 4, 5, 6\}$

8. $\{2, 3\}$

10.



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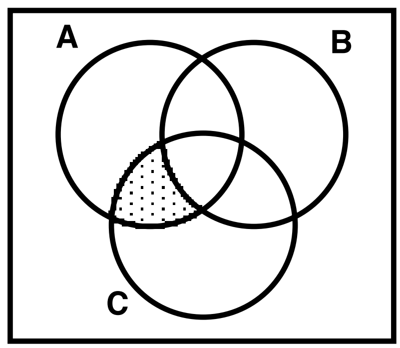

14. $gf(x) = 2(x^3 + 1) - 5 = 2x^3 - 3$ and $fg(x) = (2x - 5)^3 + 1 = 8x^3 - 60x^2 + 150x - 124$

16. not a function with domain X

18. not a function with domain X

20. neither one-to-one nor onto

22. one-to-one and onto

24. f^{-1} does not exist.

26. $f^{-1} = \sqrt[3]{x - 5}$

28. 84

30. 210

32. $\{1, 7\}$, $\{2, 6\}$, $\{3, 5\}$, $\{4\}$, $\{8\}$

34. the sets of positive and negative real numbers

36. 5

38. 6

40. **(b)** $a = \pm 1$

42. reflexive only

48. $x \vee x = x$ for all $x \in S$, $1 \vee y = y \vee 1 = y$ for all $y \in S$, $2 \vee 3 = 3 \vee 2 = 6$, $2 \vee 4 = 4 \vee 2 = 4$, $2 \vee 6 = 6 \vee 2 = 6$, $3 \vee 6 = 6 \vee 3 = 6$, $x \vee y$ is undefined in all other cases

52. Let S denote the set of all subsets of $\{1, 2, 3\}$ that contain at most two elements, and let R be the relation "is a subset of." If $A = \{1\}$, $B = \{2\}$, and $C = \{3\}$, then $A \vee B = \{1, 2\}$, $B \vee C = \{2, 3\}$, and $A \vee C = \{1, 3\}$, but $(A \vee B) \vee C$ does not exist.

54. $[n] = \{-n, n\}$

56. $f(x) = \begin{cases} x & \text{if } x \equiv 0 \pmod{3} \\ x - 1 & \text{if } x \equiv 1 \pmod{3} \\ x - 2 & \text{if } x \equiv 2 \pmod{3} \end{cases}$