Solutions Manual

Second Edition

Discrete-Time Control Systems

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A Pearson Education Company
Upper Saddle River, New Jersey 07458

10 9 8 7 6 5

ISBN 0-13-317190-6

Printed in the United States of America

Table of Contents

Preface v

Chapter 2 1

Chapter 3 18

Chapter 4 44

Chapter 5 74

Chapter 6 91

Chapter 7 111

Chapter 8 135

Preface

This solutions manual for Discrete-Time Control Systems, second edition, contains solutions to all B problems (unsolved problems in the text).

All the materials in the text may be covered in two quarters. In a semester course, the instructor will have some flexibility in choosing the subjects to be covered. If the student has an adequate background in the vector-matrix analysis, then by leaving approximately 10 percent of the text material to the student's self study, most of the important subjects of the text may be covered in one semester. In a quarter course, a good part of the first six chapters may be covered.

Katsuhiko Ogata

CHAPTER 2

B-2-1.

$$X(z) = \mathcal{Z}\left[\frac{1}{a}(1 - e^{-at})\right] = \frac{1}{a}\left(\frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-aT}z^{-1}}\right)$$

$$= \frac{1}{a}\frac{z^{-1}(1 - e^{-aT})}{(1 - z^{-1})(1 - e^{-aT}z^{-1})}$$

B-2-2.

Method 1: Noting that

$$\mathcal{Z}[k] = \frac{z^{-1}}{(1 - z^{-1})^2}, \qquad \mathcal{Z}[k^2] = \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$$

it can be expected that $\mathcal{Z}[k^3]$ will involve a term $(1 - z^{-1})^4$ in the denominator. Since

$$\mathcal{Z}[k^3] = \sum_{k=0}^{\infty} k^3 z^{-k} = z^{-1} + 2^3 z^{-2} + 3^3 z^{-3} + 4^3 z^{-4} + \cdots$$

$$= z^{-1} + 8z^{-2} + 27z^{-3} + 64z^{-4} + \cdots$$

and

$$(z^{-1} + 8z^{-2} + 27z^{-3} + 64z^{-4} + \cdots)(1 - z^{-1})^4$$

$$= (z^{-1} + 7z^{-2} + 19z^{-3} + 37z^{-4} + \cdots)(1 - z^{-1})^3$$

$$= (z^{-1} + 6z^{-2} + 12z^{-3} + 18z^{-4} + \cdots)(1 - z^{-1})^2$$

$$= (z^{-1} + 5z^{-2} + 6z^{-3} + 6z^{-4} + \cdots)(1 - z^{-1})$$

$$= z^{-1} + 4z^{-2} + z^{-3}$$

we find

$$\mathcal{Z}[k^3] = \frac{z^{-1} + 4z^{-2} + z^{-3}}{(1 - z^{-1})^4} = \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4}$$

Method 2:

$$\mathcal{Z}[k^3] = \mathcal{Z}[k \cdot k^2] = -z\frac{dX(z)}{dz}$$

where

$$X(z) = \mathcal{Z}[k^2] = \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$$

Since

$$\frac{dX(z)}{dz} = \frac{d}{dz}\left[\frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}\right] = -\frac{z^{-2} + 4z^{-3} + z^{-4}}{(1 - z^{-1})^4}$$

we have

$$\mathcal{Z}\left[k^3\right] = \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4}$$

B-2-3.

Method 1: Noting that

$$\mathcal{Z}\left[te^{-at}\right] = \frac{Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$$

we have

$$\mathcal{Z}\left[t^2e^{-at}\right] = \mathcal{Z}\left[-\frac{\partial}{\partial a} te^{-at}\right] = \frac{\partial}{\partial a}\left[\frac{-Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}\right]$$

$$= \frac{T^2e^{-aT}(1 + e^{-aT}z^{-1})z^{-1}}{(1 - e^{-aT}z^{-1})^3}$$

Method 2:

$$\mathcal{Z}\left[t^2e^{-at}\right] = \sum_{k=0}^{\infty} (kT)^2e^{-akT}z^{-k}$$

$$= T^2\left[(e^{aT}z)^{-1} + 4(e^{aT}z)^{-2} + 9(e^{aT}z)^{-3} + 16(e^{aT}z)^{-4} + \cdots\right]$$

Referring to Problem A-2-2, we have

$$\mathcal{Z}\left[t^2e^{-at}\right] = \frac{T^2e^{-aT}z^{-1}(1 + e^{-aT}z^{-1})}{(1 - e^{-aT}z^{-1})^3}$$

B-2-4.

$$\mathcal{Z}\left[x(k)\right] = \mathcal{Z}\left[9k(2^{k-1})\right] - \mathcal{Z}\left[2^k\right] + \mathcal{Z}\left[3\right]$$

$$= \frac{9z^{-1}}{(1 - 2z^{-1})^2} - \frac{1}{1 - 2z^{-1}} + \frac{3}{1 - z^{-1}}$$

$$= \frac{2 + z^{-2}}{(1 - 2z^{-1})^2(1 - z^{-1})}$$

B-2-5. Referring to Problem A-2-4, we have

$$\mathcal{Z}\left[\sum_{h=0}^{k} a^h\right] = \frac{1}{1 - z^{-1}} X(z)$$

where

$$X(z) = \mathcal{Z}\left[a^h\right] = \frac{1}{1 - az^{-1}}$$

Hence

$$\mathcal{Z}\left[\sum_{h=0}^{k} a^h\right] = \frac{1}{1 - z^{-1}} \cdot \frac{1}{1 - az^{-1}}$$

B-2-6.

$$\mathcal{Z}\left[ka^{k-1}\right] = \mathcal{Z}\left[\frac{\partial}{\partial a} a^k\right] = \frac{\partial}{\partial a}\mathcal{Z}\left[a^k\right] = \frac{\partial}{\partial a}\left(\frac{1}{1 - az^{-1}}\right)$$

$$= \frac{z^{-1}}{(1 - az^{-1})^2} = \frac{z}{(z - a)^2}$$

$$\mathcal{Z}\left[k(k - 1)a^{k-2}\right] = \mathcal{Z}\left[\frac{\partial}{\partial a}(ka^{k-1})\right] = \frac{\partial}{\partial a}\left[\frac{z}{(z - a)^2}\right]$$

$$= \frac{2z}{(z - a)^3} = \frac{(2!)z}{(z - a)^3}$$

$$\mathcal{Z}\left[k(k - 1) \cdots (k - h + 1)a^{k-h}\right]$$

$$= \mathcal{Z}\left[\frac{\partial}{\partial a} k(k - 1) \cdots (k - h + 2)a^{k-h+1}\right]$$

$$= \frac{\partial}{\partial a} X(z, a)$$

where

$$X(z, a) = \frac{(h - 1)!\, z}{(z - a)^h}$$

Hence

$$\mathcal{Z}\left[k(k - 1) \cdots (k - h + 1)a^{k-h}\right] = \frac{\partial}{\partial a} X(z, a)$$

$$= (h - 1)!\, zh(z - a)^{-h-1} = \frac{h!\, z}{(z - a)^{h+1}}$$

B-2-7. From Figure 2-8 we have

$x(0) = 0,\qquad x(1) = 0,\qquad x(2) = 0,\qquad x(3) = \frac{1}{3}$

$x(4) = \frac{2}{3},\qquad x(k) = 1\quad \text{for } k = 5, 6, 7, \ldots$

Then

$$X(z) = \sum_{k=0}^{\infty} x(k)\, z^{-k}$$

$$= \frac{1}{3} z^{-3} + \frac{2}{3} z^{-4} + z^{-5} + z^{-6} + z^{-7} + \cdots$$

$$= \frac{1}{3} (z^{-3} + 2z^{-4}) + \frac{z^{-5}}{1 - z^{-1}}$$

$$= \frac{1}{3} \frac{z^{-3} + z^{-4} + z^{-5}}{1 - z^{-1}}$$

B-2-8. By dividing both numerator and denominator by z^4, we have

$$X(z) = 5 + 4z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$$

This last equation is already in the form of a power series in z^{-1}. By inspection, we have

$$x(0) = 5$$
$$x(1) = 4$$
$$x(2) = 3$$
$$x(3) = 2$$
$$x(4) = 1$$
$$x(k) = 0 \qquad k \geqslant 5$$

Note that the given X(z) is the z transform of a signal of finite length.

B-2-9.

1. Partial-fraction-expansion method:

$$X(z) = \frac{z^{-1}(0.5 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2} = \frac{z(0.5z - 1)}{(z - 0.5)(z - 0.8)^2}$$

Hence,

$$\frac{X(z)}{z} = -\frac{8.3333}{z - 0.5} + \frac{8.3333}{z - 0.8} - \frac{2}{(z - 0.8)^2}$$

or

$$X(z) = -\frac{8.3333}{1 - 0.5z^{-1}} + \frac{8.3333}{1 - 0.8z^{-1}} - \frac{2z^{-1}}{(1 - 0.8z^{-1})^2}$$

Thus,

$$x(k) = -8.3333(0.5)^k + 8.3333(0.8)^k - 2k(0.8)^{k-1}, \qquad k = 0, 1, 2, \ldots$$

2. Computational solution with MATLAB: