

Chapter 1

The Facts to Be Explained

Note: Special icons in the margin identify problems requiring a computer or calculator .

■ Solutions to Problems

1. A ratio scale transforms absolute differences in the variable of interest to proportional differences. For instance, the GDP of Country X, whose GDP is 10 times greater than Country Y, will be the same distance apart as a Country Z whose GDP is 10 times smaller than Country Y's GDP, i.e., the distance between X, Y, and Z will be the same. On a common linear scale, the distance between X and Y would be 10 times greater than the distance between Y and Z. As a result, transforming Figure 1.1 into a ratio scale would convert the absolute differences in the height of marchers into proportional differences.
2. Let g be the rate of growth. The rule of 72 says that $72/g \approx 9$. So $g \approx 8\%$.
3. Using the rule of 72, we know that GDP per capita will double every $72/g$ years, where g is the annual growth rate of GDP per capita. Working backwards, if we start in the year 1900 with a GDP per capita of \$1,000, to reach \$4,000 by the year 1948, GDP per capita must have doubled twice. To see this, note that after doubling once, GDP per capita would be \$2,000 in some year, and doubling again, GDP per capita would be \$4,000, exactly the GDP per capita in year 1948. Using the fact that GDP doubled twice within 48 years and assuming a constant annual growth rate, we conclude that GDP per capita doubles every 24 years. Solving for the equation, $72/g = 24$, we get g , the annual growth rate, to be three percent per year.
4. Between-country inequality is the inequality associated with average incomes of different countries. Country A's average income is given by adding Alfred's Income and Doris's Income and then dividing by 2. This yields an average income of 2,500 for Country A. Similar calculations reveal that Country B's average income is 2,500. Because the average income for Country A is equal to that of Country B, there is no between-country inequality in this world.

Within-country inequality is the inequality associated with incomes of people in the same country. In Country A, Alfred earns 1,000 while Doris earns 4,000, making it an income disparity of 3,000. In Country B, the income disparity is 1,000. Therefore, we see within-country income inequality in both Country A and Country B. Because there is no between-country inequality, world inequality can be entirely attributed to within-country inequality.

5. We can solve for the average annual growth rate, g , by substituting the appropriate values into the equation:

$$(Y_{1900}) \times (1 + g)^{100} = Y_{2000}.$$

Letting $Y_{1900} = \$1,617$, $Y_{2000} = \$23,639$, and rearranging to solve for g , we get:

$$g = (\$23,639/\$1,617)^{(1/100)} - 1,$$

$$g \approx 0.0272.$$

Converting g into a percent, we conclude that the growth rate of income per capita in Japan over this period was approximately 2.72 percent per year.

To find the income per capita of Japan 100 years from now, in 2100, we solve

$$(Y_{2000}) \times (1 + g)^{100} = Y_{2100}.$$

Letting $Y_{2000} = \$23,639$ and $g = 0.0272$,

$$(\$23,639) \times (1 + 0.0272)^{100} = Y_{2100},$$

$$Y_{2100} = \$346,043.09.$$

That is, if Japan grew at the average growth rate of 2.72 percent per year, we would find the income per capita of Japan in 2100 to be about \$346,043.09.

6. In order to calculate the year in which income per capita in the United States was equal to income per capita in Sri Lanka, we need to find t , the number of years that passed between the year 2009 and the year U.S. income per capita equaled that of 2009 Sri Lanka income per capita. Equating income per capita of Sri Lanka in year 2009 to income per capita of the United States in year 2009 – t , we now write an equation for the United States as

$$(Y_{\text{U.S., } 2009-t}) \times (1 + g)^t = Y_{\text{U.S., } 2009}.$$

Since $Y_{\text{U.S., } 2009-t} = Y_{\text{Sri Lanka, } 2009} = \$4,034$, $Y_{\text{U.S., } 2009} = \$41,099$, and $g = 0.018$, we then substitute in these values and solve for t .

$$(\$4,034) \times (1 + 0.018)^t = \$41,099.$$

$$(1 + 0.018)^t = (\$41,099/\$4,034).$$

One can solve for t by simply trying out different values on a calculator. Alternatively, taking the natural log of both sides, and noting that $\ln(x^y) = y \ln(x)$, we get

$$t \ln(1 + 0.018) = \ln(\$41,099/\$4,034)$$

$$t = 130.11.$$

That is, 130.11 years ago, the income per capita of the United States equaled that of Sri Lanka's income in the year 2009. This year was roughly 2009 – t , i.e., the year 1879.

7. In order to calculate the year in which income per capita in China will overtake the income per capita in the United States, we first need to find t , the number of years it will take for the income per capita in the two countries to be equal. That is,

$$(Y_{\text{U.S., } 2009}) \times (1 + .018)^t = (Y_{\text{China, } 2009}) \times (1 + .079)^t.$$

Since $Y_{\text{U.S., 2009}} = \$41,099$, $Y_{\text{China, 2005}} = \$7,634$, we then substitute in these values and solve for t .

$$(1 + 0.079/1 + .018)^t = (\$41,099/\$7,634).$$

We can solve for t by trying out different values on a calculator. Alternatively, taking the Natural Log of both sides, and noting that $\ln(x^y) = y \ln(x)$, we get

$$\begin{aligned} t \ln(1.06) &= \ln(\$41,099/\$7,634) \\ t &= 28.89. \end{aligned}$$

That is, in 28.89 years, assuming they grow at the current growth rates, the income per capita of China will surpass that of the United States. This year will roughly be $2009 + t$, i.e., the year 2038.

■ Solutions to Appendix Problems

A.1. The number of people living on less than a dollar a day will be larger if we calculate it using market exchange rates instead of purchasing power exchange rates because market exchange rates only take into account the relative value of traded goods, which are relatively more expensive in poorer countries. Individuals in these countries will have low purchasing power for traded goods. By using the market exchange rate, we are assuming that traded goods and non-traded goods are the same price, and therefore individuals in poor countries will have low purchasing power for non-traded goods as well, which will make them appear poorer than they actually are.

A.2. a. The level of GDP per capita in each country, measured in its own currency is

$$(\text{CPUs per capita} \times \text{Price}) + (\text{IC per capita} \times \text{Price}) = \text{GDP per capita}.$$

Therefore, Richland's GDP per capita is 40 and Poorland's GDP per capita is 4.

- b. The market exchange rate is determined by the law of one price. As CPUs are the only traded good, the price of computers should be the same. Consequently, the exchange rate must be 2 Richland dollars to 1 Poorland dollar.
- c. To find the ratio of GDP per capita between Richland and Poorland, we must first convert GDP denominations into the same currency. In the analysis that follows, I choose to convert GDP denominations into Poorland dollars, but converting to Richland dollars is equally correct, similar, and will yield the same result. From Part (a), we convert Richland's GDP per capita, denominated in Richland dollars, into Poorland dollars by multiplying GDP per capita with the market exchange rate. Since from Part (b), we know 2 Richland dollars equals 1 Poorland dollar, we multiply $1/2$ to Richland's GDP per capita, yielding 20 Poorland dollars. Thus, the ratio of Richland GDP per capita to Poorland GDP per capita is 5:1.
- d. A natural basket to use is 3 computers and 1 ice cream. The cost of this basket in Richland is 10 Richland dollars. The cost of this basket in Poorland is 4 Poorland dollars. Equating the costs of baskets to be one price, the purchasing power parity exchange rate must be 10 Richland dollars: 4 Poorland dollars.
- e. To find the ratio of GDP per capita between Richland and Poorland, we must first convert GDP denominations into the same currency. In the analysis that follows, I choose to convert GDP denominations into Poorland dollars, but converting to Richland dollars is equally correct, similar, and will yield the same result. From Part (a), we convert Richland's GDP per capita, denominated in Richland dollars, into Poorland dollars by multiplying GDP per capita with the PPP exchange rate. Since from Part (d), we know 10 Richland dollars equals 4 Poorland dollars, we multiply $4/10$ to Richland's GDP per capita, yielding 16 Poorland dollars. Thus the ratio of Richland GDP per capita to Poorland GDP per capita is 4:1.