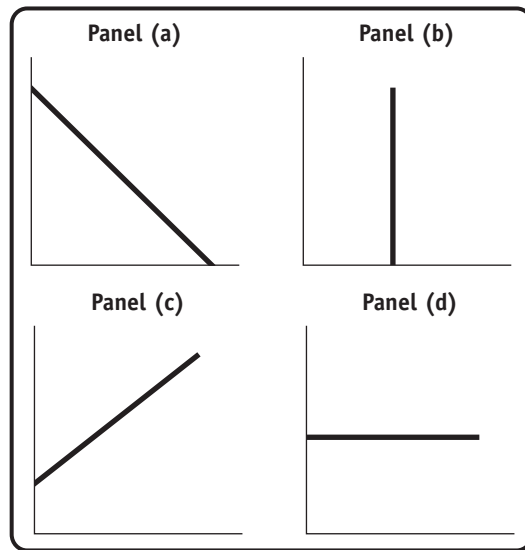


Appendix: Graphs in Economics

CHAPTER 2A

1. Study the four accompanying diagrams. Consider the following statements and indicate which diagram matches each statement. Which variable would appear on the horizontal and which on the vertical axis? In each of these statements, is the slope positive, negative, zero, or infinity?



- a. If the price of movies increases, fewer consumers go to see movies.
- b. More experienced workers typically have higher incomes than less experienced workers.
- c. Whatever the temperature outside, Americans consume the same number of hot dogs per day.
- d. Consumers buy more frozen yogurt when the price of ice cream goes up.
- e. Research finds no relationship between the number of diet books purchased and the number of pounds lost by the average dieter.
- f. Regardless of its price, Americans buy the same quantity of salt.

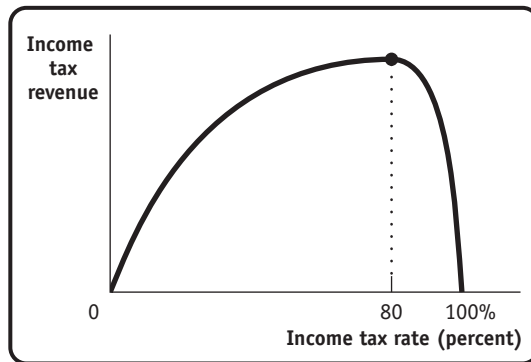
Solution

1. a. Panel (a) illustrates this relationship. The higher price of movies causes consumers to see fewer movies. The relationship is negative, and the slope is therefore negative. The price of movies is the independent variable, and the number of movies seen is the dependent variable. However, there is a convention in economics that if price is a variable, it is measured on the vertical axis. So the quantity of movies is measured on the horizontal axis.
- b. Panel (c) illustrates this relationship. Since it is likely that their greater experience causes firms to pay workers more, years of experience is the independent variable and would go on the horizontal axis and the resulting income, the dependent variable, on the vertical axis. The slope is positive.

- c. Panel (d) illustrates this relationship. With the temperature on the horizontal axis as the independent variable, and the consumption of hot dogs on the vertical axis as the dependent variable, we see there is no change in hot dog consumption whatever the temperature. The slope is zero.
 - d. Panel (c) illustrates this relationship. When the price of ice cream goes up, this would cause consumers to choose a close alternative, frozen yogurt. The price of ice cream is the independent variable and the consumption of frozen yogurt is the dependent variable. However, there is a convention in economics that if price is a variable, it is measured on the vertical axis. The quantity of frozen yogurt that consumers buy is on the horizontal axis. The slope is positive.
 - e. Panel (d) illustrates this relationship. The fact that there is no discernible relationship between the number of diet books purchased and the weight loss of the average dieter results in a horizontal curve. The slope is zero.
 - f. Panel (b) illustrates this relationship. Although price is the independent variable and salt consumption the dependent variable, by convention the price appears on the vertical axis and the quantity of salt on the horizontal axis. Since salt consumption does not change whatever the price, the curve is a vertical line; the slope is infinity.
2. During the Reagan administration, economist Arthur Laffer argued in favor of lowering income tax rates in order to increase tax revenues. Like most economists, he believed that at tax rates above a certain level, tax revenue would fall because high taxes would discourage some people from working and that people would refuse to work at all if they received no income after paying taxes. This relationship between tax rates and tax revenue is graphically summarized in what is widely known as the Laffer curve. Plot the Laffer curve relationship assuming that it has the shape of a nonlinear curve. The following questions will help you construct the graph.
- a. Which is the independent variable? Which is the dependent variable? On which axis do you therefore measure the income tax rate? On which axis do you measure income tax revenue?
 - b. What would tax revenue be at a 0% income tax rate?
 - c. The maximum possible income tax rate is 100%. What would tax revenue be at a 100% income tax rate?
 - d. Estimates now show that the maximum point on the Laffer curve is (approximately) at a tax rate of 80%. For tax rates less than 80%, how would you describe the relationship between the tax rate and tax revenue, and how is this relationship reflected in the slope? For tax rates higher than 80%, how would you describe the relationship between the tax rate and tax revenue, and how is this relationship reflected in the slope?

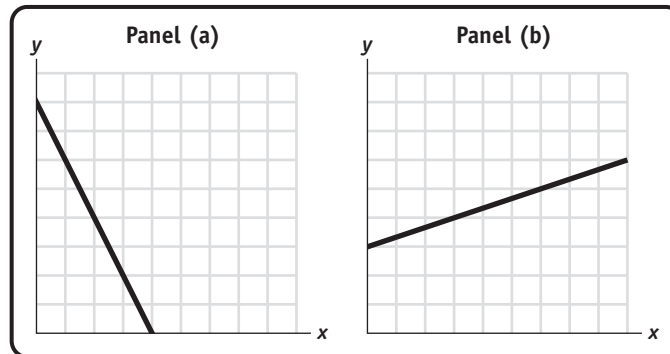
Solution

2. a. The income tax rate is the independent variable and so is measured on the horizontal axis. Income tax revenue is the dependent variable and so is measured on the vertical axis.



- b. If the income tax rate is 0% (there is no tax), tax revenue is obviously zero.
- c. If the income tax rate is 100% (all your income is taxed away), you will have zero income left after tax. Since people are unwilling to work if they receive no income after tax, no income will be earned. As a result, there is no income tax revenue.
- d. For tax rates less than 80%, tax rate and tax revenue are positively related and so the Laffer curve has a positive slope. For tax rates higher than 80%, the relationship between tax rate and tax revenue is negative and so the Laffer curve has a negative slope. The Laffer curve therefore looks like the accompanying diagram with a maximum point at a tax rate of 80%.

3. In the accompanying figures, the numbers on the axes have been lost. All you know is that the units shown on the vertical axis are the same as the units on the horizontal axis.



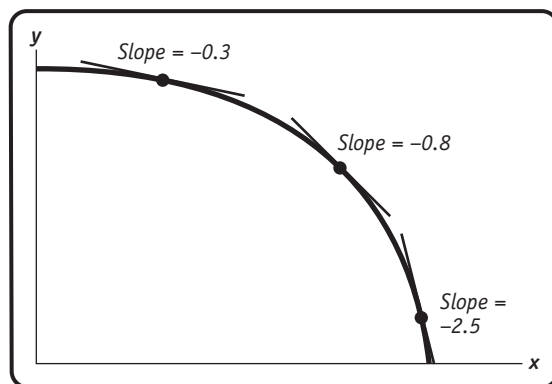
- a. In panel (a), what is the slope of the line? Show that the slope is constant along the line.
- b. In panel (b), what is the slope of the line? Show that the slope is constant along the line.

Solution

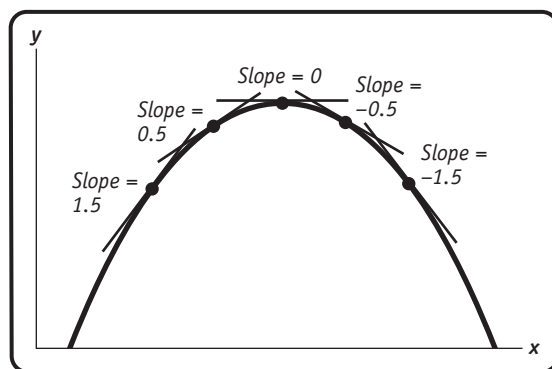
3. a. In panel (a), the slope is -2 . From any point on the line, moving one unit to the right along the horizontal axis requires moving down two units along the vertical axis in order to remain on the line. The slope is the “rise” (-2) over the “run” ($+1$); that is, the slope is $\frac{-2}{1} = -2$. The same is true starting at *any* point along the line, so the slope at every point is the same. The slope is constant.
- b. In panel (b), the slope is $\frac{1}{3}$. From any point on the line, moving three units to the right along the horizontal axis requires moving up one unit along the vertical axis in order to remain on the line. The slope is the “rise” ($+1$) over the “run” ($+3$); that is, the slope is $\frac{1}{3}$. The same is true starting at *any* point along the line, so the slope at every point is the same. The slope is constant.
4. Answer each of the following questions by drawing a schematic diagram.
- a. Taking measurements of the slope of a curve at three points farther and farther to the right along the horizontal axis, the slope of the curve changes from -0.3 , to -0.8 , to -2.5 , measured by the point method. Draw a schematic diagram of this curve. How would you describe the relationship illustrated in your diagram?
- b. Taking measurements of the slope of a curve at five points farther and farther to the right along the horizontal axis, the slope of the curve changes from 1.5 , to 0.5 , to 0 , to -0.5 , to -1.5 , measured by the point method. Draw a schematic diagram of this curve. Does it have a maximum or a minimum?

Solution

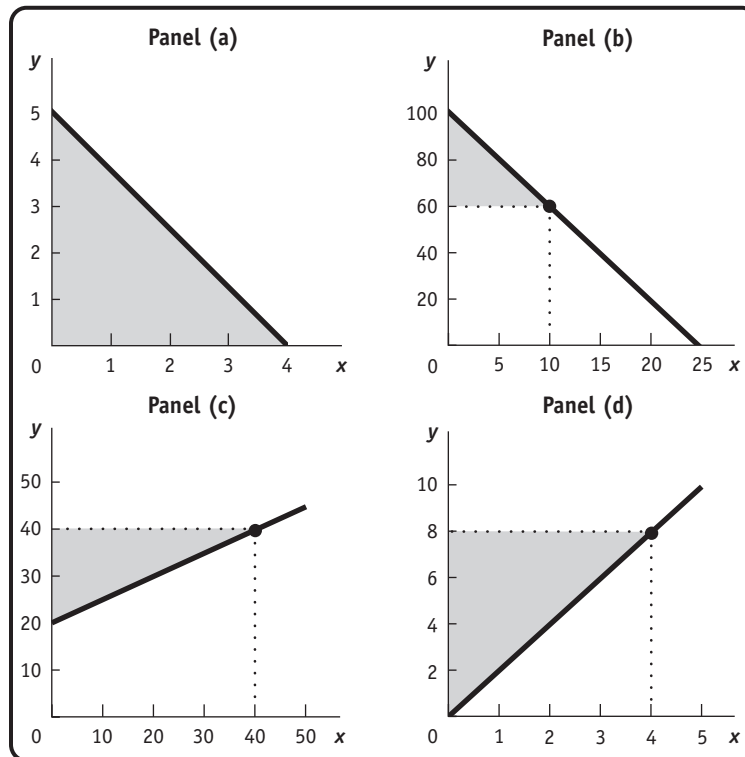
4. a. The accompanying diagram schematically shows this curve. The slope is negative throughout. That means that the curve is downward sloping. Because the absolute value of the slope is increasing, the curve becomes steeper. The slope is negative increasing.



- b. The accompanying diagram schematically shows this curve. The slope is positive decreasing at first. Then it becomes negative increasing. The curve therefore has a maximum just at the point where the slope is equal to zero.



5. For each of the accompanying diagrams, calculate the area of the shaded right triangle.



Solution

5. a. In panel (a), the height of the shaded triangle is $5 - 0 = 5$, and its base is $4 - 0 = 4$. The area of the triangle is $\frac{5 \times 4}{2} = 10$.
- b. In panel (b), the height of the shaded triangle is $100 - 60 = 40$, and its base is $10 - 0 = 10$. The area of the triangle is $\frac{40 \times 10}{2} = 200$.
- c. In panel (c), the height of the shaded triangle is $40 - 20 = 20$, and its base is $40 - 0 = 40$. The area of the triangle is $\frac{20 \times 40}{2} = 400$.
- d. In panel (d), the height of the shaded triangle is $8 - 0 = 8$, and its base is $4 - 0 = 4$. The area of the triangle is $\frac{8 \times 4}{2} = 16$.

6. The base of a right triangle is 10, and its area is 20. What is the height of this right triangle?

Solution

6. The area of a right triangle is calculated as

$$\frac{\text{Height} \times \text{Base}}{2} = \text{Area}$$

Substituting what we know from the question (base = 10 and area = 20), we get

$$\frac{\text{Height} \times 10}{2} = 20$$

Solving this for height, we find that the height of this right triangle is 4.

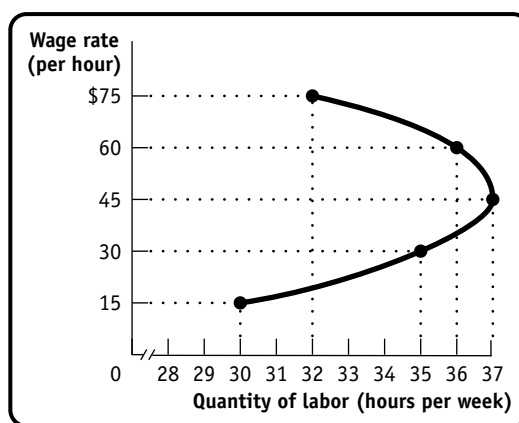
7. The accompanying table shows the relationship between workers' hours of work per week and their hourly wage rate. Apart from the fact that they receive a different hourly wage rate and work different hours, these five workers are otherwise identical.

Name	Quantity of labor (hours per week)	Wage rate (per hour)
Athena	30	\$15
Boris	35	30
Curt	37	45
Diego	36	60
Emily	32	75

- Which variable is the independent variable? Which is the dependent variable?
- Draw a scatter diagram illustrating this relationship. Draw a (nonlinear) curve that connects the points. Put the hourly wage rate on the vertical axis.
- As the wage rate increases from \$15 to \$30, how does the number of hours worked respond according to the relationship depicted here? What is the average slope of the curve between Athena's and Boris's data points using the arc method?
- As the wage rate increases from \$60 to \$75, how does the number of hours worked respond according to the relationship depicted here? What is the average slope of the curve between Diego's and Emily's data points using the arc method?

Solution

- If the wage rate is greater than your opportunity cost of time, you will choose to work. So the wage rate is the independent variable and the number of hours worked is the dependent variable.
- The accompanying diagram illustrates the relationship between the hourly wage rate and the number of hours worked. Since the hourly wage rate is the price paid for labor, economists place wages on the vertical axis—just as in the case of other types of prices.

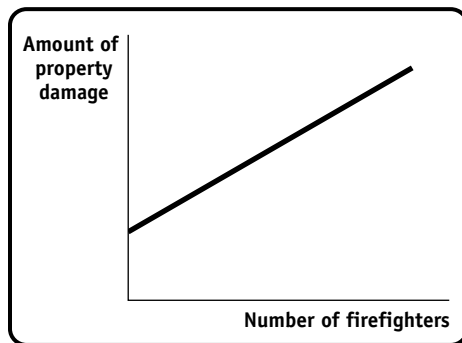


- As the wage rate increases from \$15 to \$30, the number of hours worked increases by 5. The average slope of the curve between the two points is therefore $\frac{15}{5} = 3$.
- As the wage rate increases from \$60 to \$75, the number of hours worked decreases by 4. The average slope of the curve between the two points is therefore $\frac{15}{-4} = -3.75$.

8. An insurance company has found that the severity of property damage in a fire is positively related to the number of firefighters arriving at the scene.
- Draw a diagram that depicts this finding with number of firefighters on the horizontal axis and amount of property damage on the vertical axis. What is the argument made by this diagram? Suppose you reverse what is measured on the two axes. What is the argument made then?
 - In order to reduce its payouts to policyholders, should the insurance company therefore ask the city to send fewer firefighters to any fire?

Solution

8. a. By drawing the diagram with number of firefighters on the horizontal axis and amount of property damage on the vertical axis, you are assuming that the number of firefighters is the independent variable and amount of property damage is the dependent variable. That graph is shown here. It makes the argument that as the number of firefighters on the scene increases, the amount of damage increases. You could also have drawn the graph with amount of property damage as the independent variable (on the horizontal axis) and the number of firefighters as the dependent variable (on the vertical axis). In this case the diagram implies that more and more firefighters come to the scene as the amount of property damage increases. (But be aware that any diagram shows only a relationship between two variables and does not imply causation.)



- b. The statement implies that there is a causal link between the number of firefighters and the amount of property damage, and this is likely not the case. It is instead likely that there is a third, omitted, variable that is related to both the number of firefighters and the amount of property damage. This variable is the severity of the fire: more severe fires cause both greater property damage and a greater number of firefighters to be sent to the fire.
9. The accompanying table illustrates annual salaries and income tax owed by five individuals. Apart from the fact that they receive different salaries and owe different amounts of income tax, these five individuals are otherwise identical.

Name	Annual salary	Annual income tax owed
Susan	\$22,000	\$3,304
Eduardo	63,000	14,317
John	3,000	454
Camila	94,000	23,927
Peter	37,000	7,020

- a. If you were to plot these points on a graph, what would be the average slope of the curve between the points for Eduardo's and Camila's salaries and taxes using the arc method? How would you interpret this value for slope?
- b. What is the average slope of the curve between the points for John's and Susan's salaries and taxes using the arc method? How would you interpret that value for slope?
- c. What happens to the slope as salary increases? What does this relationship imply about how the level of income taxes affects a person's incentive to earn a higher salary?

Solution

9. a. Annual salary is the independent variable and so is measured on the horizontal axis. Annual income tax owed is the dependent variable and so is measured on the vertical axis. As salary increases by \$31,000 from Eduardo's \$63,000 to Camila's \$94,000, income tax owed increases by \$9,610. That is, the slope of the curve is $\frac{9,610}{31,000} = 0.31$. The interpretation is that in this income bracket, each additional dollar of income implies a tax of \$0.31.
- b. As salary increases by \$19,000 from John's \$3,000 to Susan's \$22,000, income tax owed increases by \$2,850. That is, the slope of the curve is $\frac{2,850}{19,000} = 0.15$. The interpretation is that in this income bracket, each additional dollar of income implies a tax of \$0.15.
- c. The slope is positive increasing. This implies that the tax scheme is "progressive": the higher the annual salary, the greater the amount of income tax owed per dollar of income. Therefore, the incentive to earn more and more income becomes weaker and weaker, since more of the additional income earned is owed as income taxes.

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10. Studies have found a relationship between a country's yearly rate of economic growth and the yearly rate of increase in airborne pollutants. It is believed that a higher rate of economic growth allows a country's residents to have more cars and travel more, thereby releasing more airborne pollutants.
 - a. Which variable is the independent variable? Which is the dependent variable?
 - b. Suppose that in the country of Sudland, when the yearly rate of economic growth fell from 3.0% to 1.5%, the yearly rate of increase in airborne pollutants fell from 6% to 5%. What is the average slope of a nonlinear curve between these points using the arc method?
 - c. Now suppose that when the yearly rate of economic growth rose from 3.5% to 4.5%, the yearly rate of increase in airborne pollutants rose from 5.5% to 7.5%. What is the average slope of a nonlinear curve between these two points using the arc method?
 - d. How would you describe the relationship between the two variables here?

Solution

- 10.**
- a.** According to the question, economic growth causes the increase in airborne pollutants. That is, the growth rate is the independent variable and the rate of increase in airborne pollutants is the dependent variable. So the rate of increase in airborne pollutants is measured on the vertical axis and the growth rate is measured on the horizontal axis.
 - b.** The change in the growth rate is -1.5 . The change in the rate of increase in airborne pollutants is -1 . The slope is therefore $\frac{-1}{-1.5} = \frac{2}{3}$.
 - c.** The change in the growth rate is $+1$. The change in the rate of increase in airborne pollutants is $+2$. The slope is therefore $\frac{2}{1} = 2$.
 - d.** The slope is positive and, as can be seen from the answers to parts b and c, increasing.

