SOLUTIONS MANUAL FOR

MATHEMATICAL STATISTICS

by

Bickel and Doksum

CRC Press
Taylor & Francis Group

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CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed on acid-free paper
Version Date: 20150601

International Standard Book Number-13: 978-1-4987-2384-8 (Ancillary)

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http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

MATHEMATICAL STATISTICS
VOL I.

Bickel and Doksum

SOLUTIONS MANUAL

Solutions to Selected Problems

December 2014

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1 1.1 PROBLEMS

1.1.1. (a) Let X = pebble diameter. Then $Z = \log X$ has a $\mathcal{N}(\mu, \sigma^2)$ distribution. Since $X = e^Z$, the transformation theorem (Theorem $B.2.2$) with $k = 1$ tells us that X has density

$$f(x, \theta) = (x/\sigma)\varphi\left(\frac{\log x - \mu}{\sigma}\right), \quad x > 0, \quad \theta = (\mu, \sigma)$$

where φ is the $\mathcal{N}(0,1)$ density. Let $X_1, \ldots, X_n$ denote the n pebble diameters. We assume that they are independent. Then the probability law of the "data" $X_1, \ldots, X_n$ is $p(\mathbf{x}, \theta) = \Pi_{i=1}^n f(x_i, \theta)$ with $f(x, \theta)$ as given above.

Since there is no knowledge about the magnitude of μ, σ^2, the parameter space is

$$\Theta = \{(\mu, \sigma^2) : -\infty < \mu < \infty, \;\; \sigma^2 > 0\}.$$

The model is parametric.

(b) Let $X_1, \ldots, X_n$ denote the n measurements. The distribution of $\mathbf{X} = (X_1, \ldots, X_n)$ has density

$$p(\mathbf{x}, \theta) = \prod_{i=1}^n \frac{1}{\sigma}\varphi\left(\frac{x_i - \mu - 0.1}{\sigma}\right)$$

where φ is the $\mathcal{N}(0,1)$ density, σ is known, $\theta = \mu$, and the parameter space is $\Theta = \{\mu : -\infty < \mu < \infty\}$. The model is parametric.

(c) Let $\beta > 0$ stand for the unknown bias. Then

$$p(\mathbf{x}, \theta) = \prod_{i=1}^n \frac{1}{\sigma}\varphi\left(\frac{x_i - \mu - \beta}{\sigma}\right)$$

where $\theta = (\mu, \beta)$ and $\Theta = \{(\mu, \beta) : -\infty < \mu < \infty, \;\; \beta > 0\}$. In this model, since β is unknown, we cannot correct for the bias and we cannot determine μ even if $\sigma = 0$. The model is parametric.

(d) Let Y denote the number of eggs laid and X the number of eggs hatching. Then, from $A.6.3$, the conditional frequency function of X given $Y = y$ is binomial, $\mathcal{B}(y, p)$, i.e.,

$$p(x|y) = \binom{y}{x} p^x (1-p)^{y-x}, \;\; x = 0, 1, \ldots, y.$$

Since Y is Poisson, $\mathcal{P}(\lambda)$, we find using $(B.1.3)$ and $(A.13.9)$ that the joint frequency function of X and Y is

$$f(x, y, \theta) = \binom{y}{x} p^x (1-p)^{y-x} \frac{e^{-\lambda}\lambda^y}{y!}, \;\; x = 0, 1, \ldots, y$$

where y is a nonnegative integer and $\theta = (\lambda, p)$. For n insects, the observations are $(X_1, Y_1), \ldots, (X_n, Y_n)$ with probability law given by (assuming independence)

$$p(\mathbf{x}, \mathbf{y}, \theta) = \prod_{i=1}^n f(x_i, y_i, \theta)$$

where $f(x, y, \theta)$ is as given above. The parameter space is $\Theta = \{(\lambda, p) : \lambda > 0, 0 \le p \le 1\}$. The model is parametric.

1.1.2. (a) Unidentifiable: In the notation of 1.1.1 (c), above, let $\theta_1 = (1, 1)$ and $\theta_2 = (0, 2)$. Then $p(\mathbf{x}, \theta_1) = p(\mathbf{x}, \theta_2)$ and two different values of θ yield the same probability law.

(b) Identifiable: We must show, in the notation of 1.1.1 (d) above, that $p(\mathbf{x}, \mathbf{y}, \theta_1) = p(\mathbf{x}, \mathbf{y}, \theta_2)$, all $\mathbf{x}, \mathbf{y}$ implies $\theta_1 = \theta_2$. Write $\theta_1 = (\lambda_1, p_1)$ and $\theta_2 = (\lambda_2, p_2)$. When the x's and y's are all zero, $p(\mathbf{x}, \mathbf{y}, \theta_1) = p(\mathbf{x}, \mathbf{y}, \theta_2)$ reduces to $e^{-\lambda_1} = e^{-\lambda_2}$ so $\lambda_1 = \lambda_2$. When $x_1 = y_1 = 1$ and all the remaining x's and y's are zero, $p(\mathbf{x}, \mathbf{y}, \theta_1) = p(\mathbf{x}, \mathbf{y}, \theta_2)$ reduces to $p_1 = p_2$.

(c) Unidentifiable: See 1.1.1 (d) above. By $A.8.11$, X has frequency function

$$f(x, \theta) = \sum_{y=x}^{\infty} f(x, y, \theta) = \sum_{y=x}^{\infty} \binom{y}{x} p^x (1-p)^{y-x} \frac{e^{-\lambda}\lambda^y}{y!}.$$

In this expression, we set $\binom{y}{x} = \frac{y!}{x!(y-x)!}$, factor out the terms not depending on y, change variable from y to k where $k = y - x$, and use the fact that

$$\sum_{k=0}^{\infty} \frac{(\lambda - \lambda p)^k}{k!} = e^{(\lambda - \lambda p)}$$

to obtain

$$f(x, \theta) = \frac{e^{-\lambda p}(\lambda p)^x}{x!}, \quad x = 0, 1, \ldots .$$

The X has a Poisson, $\mathcal{P}(\lambda p)$, distribution and $\mathbf{X} = (X_1, \ldots, X_n)$ has frequency function

$$p(\mathbf{x}, \theta) = \prod_{i=1}^{n} f(x_i, \theta).$$

Now if $\theta_1 = (\lambda_1, p_1)$, $\theta_2 = (\lambda_2, p_2)$, $\theta_1 \neq \theta_2$, but $\lambda_1 p_1 = \lambda_2 p_2$, then $p(\mathbf{x}, \theta_1) = p(\mathbf{x}, \theta_2)$, and the model is not identifiable.

1.1.3. (a) Unidentifiable: Set $\theta_1 = (1, \ldots, 1, 2, \sigma^2)$ and $\theta_2 = (0, \ldots, 0, 3, \sigma^2)$, then $p(\mathbf{x}, \theta_1) = p(\mathbf{x}, \theta_2)$ where

$$p(\mathbf{x}, \theta) = \prod_{i=1}^{n} \frac{1}{\sigma} \varphi\left(\frac{x_i - (\alpha_i + \nu)}{\sigma}\right)$$

and φ is the $\mathcal{N}(0, 1)$ density.

(b) Identifiable: We must show that if $\theta' = (\alpha_1', \alpha_p', \nu', (\sigma')^2)$, then $p(\mathbf{x}, \theta) = p(\mathbf{x}, \theta')$ implies $\theta = \theta'$, where $p(\mathbf{x}, \theta)$ is given in 1.1.3(a) above. If the joint distributions are the same under θ and θ', so are the marginal distributions of each X_i (See $A.8.12$). Thus the variances and means must be equal, i.e., $(\sigma_1')^2 = \sigma^2$ and $\alpha_i' + \nu' = \alpha_i + \nu$ for each i. But this implies $\Sigma(\alpha_1' + \nu') = \Sigma(\alpha_i + \nu)$, which in turn implies $\nu' = \nu$ since $\Sigma\alpha_i' = \Sigma\alpha_i = 0$. Finally, the