Solutions Manual
Accompanying

Elements of Electromagnetics,

Third Edition

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CHAPTER 1

P. E. 1.1

(a) $\boldsymbol{A}+\boldsymbol{B}=(1,0,3)+(5,2,-6)=(6,2,-3)$

$$|\boldsymbol{A}+\boldsymbol{B}|=\sqrt{36+4+9}=\underline{\underline{7}}$$

(b) $5\boldsymbol{A}-\boldsymbol{B}=(5,0,15)-(5,2,-6)=\underline{\underline{(0,-2,21)}}$

(c) The component of $\boldsymbol{A}$ along $\mathbf{a}_y$ is $\mathbf{A}_y=\underline{\underline{0}}$

(d) $3\boldsymbol{A}+\boldsymbol{B}=(3,0,9)+(5,2,-6)=(8,2,3)$

A unit vector parallel to this vector is

$$\boldsymbol{a}_{11}=\frac{(8,2,3)}{\sqrt{64+4+9}}$$
$$=\pm\underline{\underline{(0.9117\boldsymbol{a}_x+0.2279\boldsymbol{a}_y+0.3419\boldsymbol{a}_z)}}$$

P. E. 1.2 (a) The distance vector

$$\boldsymbol{r}_{QR}=\boldsymbol{r}_R-\boldsymbol{r}_Q=(0,3,8)-(2,4,6)$$
$$=\underline{\underline{-2\boldsymbol{a}_x-\boldsymbol{a}_y+2\boldsymbol{a}_z}}$$

(b) The distance between Q and R is

$$|\boldsymbol{r}_{QR}|=\sqrt{4+1+4}=\underline{\underline{3}}$$

(c) Vector $\boldsymbol{r}_{QP}=\boldsymbol{r}_P-\boldsymbol{r}_Q=(1,-3,5)-(2,4,6)=(-1,-7,-1)$

$$\cos\theta_{PQR}=\frac{\boldsymbol{r}_{QR}\cdot\boldsymbol{r}_{QP}}{|\boldsymbol{r}_{QR}||\boldsymbol{r}_{QP}|}=\frac{7}{3\sqrt{51}}$$
$$\theta_{PQR}=\underline{\underline{70.93^\circ}}$$

(d) Area $=\frac{1}{2}|\boldsymbol{r}_{QR}\times\boldsymbol{r}_{QP}|=\frac{1}{2}|(15,-4,13)|$

$$=\underline{\underline{10.12}}$$

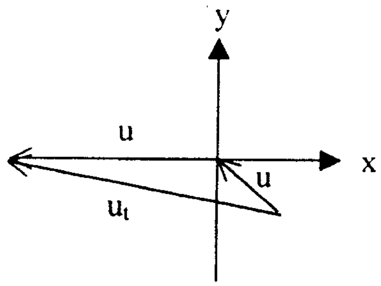
P. E. 1.3 Consider the figure shown below:

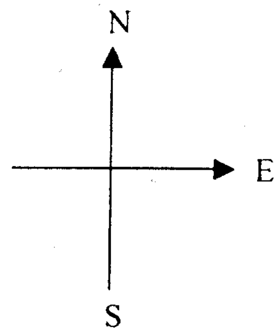
$$U_Z = U_P + U_W = -350\mathbf{a}_X + \frac{40}{\sqrt{2}}(-\mathbf{a}_X + \mathbf{a}_Y)$$

$$= -378\mathbf{a}_X + 28.28\mathbf{a}_Y$$

or

$$\mathbf{u} = 379.3\angle 175.72^\circ$$





P. E. 1.4

At point (1,0), $\mathbf{G} = \mathbf{a}_y$;
at point (0,1), $\mathbf{G} = -\mathbf{a}_x$;
at point (2,0), $\mathbf{G} = \mathbf{a}_y$;
at point (1,1), $\mathbf{G} = \dfrac{-\mathbf{a}_x + \mathbf{a}_y}{\sqrt{2}}$; and so on.

It is evident that **G** is a unit vector at each point. Thus the vector field **G** is as sketched in Fig.1.8.

P. E. 1.5

Using the dot product,

$$\cos\theta_{AB} = \frac{\mathbf{A} \bullet \mathbf{B}}{AB} = \frac{-13}{\sqrt{10}\sqrt{65}} = -\sqrt{\frac{13}{50}}$$

or using the cross product,

$$\sin\theta_{AB} = \frac{|\mathbf{A} \times \mathbf{B}|}{AB} = \sqrt{\frac{481}{650}}$$

Either way,

$$\underline{\underline{\theta_{AB} = 120.66^\circ}}$$

P. E. 1.6

(a) $\boldsymbol{E}_F = (\boldsymbol{E} \cdot \boldsymbol{a}_F)\boldsymbol{a}_F = \dfrac{(\boldsymbol{E} \cdot \boldsymbol{F})\boldsymbol{F}}{|\boldsymbol{F}|^2} = \dfrac{-10(4,-10,5)}{141}$

$$= \underline{\underline{-0.2837\boldsymbol{a}_x + 0.7092\boldsymbol{a}_y - 0.3546\boldsymbol{a}_z}}$$

(b) $\boldsymbol{E} \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ 0 & 3 & 4 \\ 4 & -10 & 5 \end{vmatrix} = (55,16,-12)$

$$\boldsymbol{a}_{E \times F} = \underline{\underline{\pm(0.9398, 0.2734, -0.205)}}$$

P. E. 1.7 $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = 0$ showing that $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ form the sides of a triangle.

$\boldsymbol{a} \cdot \boldsymbol{b} = 0$,
hence it is a right angle triangle.

$$\text{Area} = \frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b}| = \frac{1}{2}|\boldsymbol{b} \times \boldsymbol{c}| = \frac{1}{2}|\boldsymbol{c} \times \boldsymbol{a}|$$

$$\frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b}| = \frac{1}{2}\begin{vmatrix} 4 & 0 & -1 \\ 1 & 3 & 4 \end{vmatrix} = \frac{1}{2}|(3,-17,12)|$$

$$\text{Area} = \frac{1}{2}\sqrt{9 + 289 + 144} = \underline{\underline{10.51}}$$

P. E. 1.8

(a) $P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$$= \sqrt{25 + 4 + 64} = \underline{\underline{9.644}}$$

(b) $\boldsymbol{r}_P = \boldsymbol{r}_{P_1} + \lambda(\boldsymbol{r}_{P_2} - \boldsymbol{r}_{P_1})$

$$= (1,2,-3) + \lambda(-5,-2,8)$$

$$= \underline{\underline{(1 - 5\lambda, 2 - 2\lambda, -3 + 8\lambda)}}.$$

(c) The shortest distance is

$$d = P_1P_3 \sin\theta = |\boldsymbol{P}_1\boldsymbol{P}_3 \times \boldsymbol{a}_{P_1P_2}|$$

$$= \frac{1}{\sqrt{93}}\begin{vmatrix} 6 & -3 & 5 \\ -5 & -2 & 8 \end{vmatrix}$$

$$= \frac{1}{\sqrt{93}}|(-14,-73,-27)| = \underline{\underline{8.2}}$$

Prob.1.1

$$\boldsymbol{r} = (-3,2,2) - (2,4,4) = (-5,-2,-2)$$

$$\boldsymbol{a}_r = \frac{\boldsymbol{r}}{|\boldsymbol{r}|} = \frac{(-5,-2,-2)}{\sqrt{25+4+4}} = -0.8703\boldsymbol{a}_x - 0.3482\boldsymbol{a}_y - 0.3482\boldsymbol{a}_z$$

Prob. 1.2

(a) $\boldsymbol{A} + 2\boldsymbol{B} = (2,5,-3) + (6,-8,0) = \underline{\underline{8\boldsymbol{a}_x - 3\boldsymbol{a}_y - 3\boldsymbol{a}_z}}$

(b) $\boldsymbol{A} - 5\boldsymbol{C} = (2,5,-3) - (5,5,5) = (-3,0,-8)$

$|\boldsymbol{A} - 5\boldsymbol{C}| = \sqrt{9+0+64} = \underline{\underline{8.544}}$

(c) $k\boldsymbol{B} = 3k\boldsymbol{a}_x - 4k\boldsymbol{a}_y$

$|k\boldsymbol{B}| = \sqrt{9k^2 + 16k^2} = \pm 5k = 2$

$\Rightarrow \underline{\underline{k = \pm 0.4}}$

(d) $\boldsymbol{A} \cdot \boldsymbol{B} = (2,5,-3) \cdot (3,-4,0) = 6 - 20 + 0 = 14$

$$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} 2 & 5 & -3 \\ 3 & -4 & 0 \end{vmatrix} = (-12,-9,-23)$$

$$\frac{\boldsymbol{A} \times \boldsymbol{B}}{\boldsymbol{A} \cdot \boldsymbol{B}} = \left(\frac{12}{14}, \frac{9}{14}, \frac{23}{14}\right) = \underline{\underline{0.8571\boldsymbol{a}_x + 0.6428\boldsymbol{a}_y + 1.642\boldsymbol{a}_z}}$$

Prob. 1.3

(a) $\boldsymbol{A} - 2\boldsymbol{B} = (2,1,-3) - (0,2,-2) = (2,-1,-1)$

$\boldsymbol{A} - 2\boldsymbol{B} + \boldsymbol{C} = \underline{\underline{5\boldsymbol{a}_x + 4\boldsymbol{a}_y + 6\boldsymbol{a}_z}}$

(b) $\boldsymbol{A} + \boldsymbol{B} = (2,2,-4)$

$\boldsymbol{C} - 4(\boldsymbol{A} + \boldsymbol{B}) = (3,5,7) - (8,8,-16) = \underline{\underline{-5\boldsymbol{a}_x - 3\boldsymbol{a}_y + 23\boldsymbol{a}_z}}$

(c) $2\boldsymbol{A} - 3\boldsymbol{B} = (4,2,-6) - (0,3,-3) = (4,-1,-3)$

$|\boldsymbol{C}| = \sqrt{9+25+49} = 9.11$

$$\frac{2\boldsymbol{A} - 3\boldsymbol{B}}{|\boldsymbol{C}|} = \underline{\underline{0.439\boldsymbol{a}_x - 0.11\boldsymbol{a}_y - 0.3293\boldsymbol{a}_z}}$$

(d) $\boldsymbol{A}\cdot\boldsymbol{C} = 6+5-21 = -10,$

$$|\boldsymbol{B}| = \sqrt{2}$$

$$\boldsymbol{A}\cdot\boldsymbol{C} - |\boldsymbol{B}|^2 = -10+2 = \underline{\underline{-8}}$$

(e) $\frac{1}{3}\boldsymbol{A} + \frac{1}{4}\boldsymbol{C} = \left(\frac{2}{3},\frac{1}{3},-1\right) + \left(\frac{3}{4},\frac{5}{4},\frac{7}{4}\right) = (1.4167, 1.5833, 0.75)$

$$\frac{1}{2}\boldsymbol{B}\times\left(\frac{1}{3}\boldsymbol{A} + \frac{1}{4}\boldsymbol{C}\right) = \frac{1}{2}\begin{vmatrix} 0 & 1 & -1 \\ 1.4167 & 1.5833 & 0.75 \end{vmatrix} = \underline{\underline{1.1667\boldsymbol{a}_x - 0.7084\boldsymbol{a}_y - 0.7084\boldsymbol{a}_z}}$$

Prob.1.4

(a) T = $\underline{(3, -2, 1)}$ and S = $\underline{(4, 6, 2)}$

(b) $r_{TS} = r_s - r_t = (4, 6, 2) - (3, -2, 1) = \underline{\underline{a_x + 8a_y + a_z}}$

(c) distance = $|r_{TS}| = \sqrt{1+64+1} = \underline{\underline{8.124}}$ m

Prob. 1.5

Let $\boldsymbol{D} = \alpha\boldsymbol{A} + \beta\boldsymbol{B} + \boldsymbol{C}$

$$= (5\alpha - \beta + 8)\boldsymbol{a}_x + (3\alpha + 4\beta + 2)\boldsymbol{a}_y + (-2\alpha + 6\beta)\boldsymbol{a}_z$$

$$\boldsymbol{D}_x = 0 \rightarrow 5\alpha - \beta + 8 = 0 \qquad (1)$$

$$\boldsymbol{D}_z = 0 \rightarrow -2\alpha + 6\beta = 0 \rightarrow \alpha = 3\beta \qquad (2)$$

Substituting (2) into (1),

$$15\beta - \beta + 8 = 0 \rightarrow \beta = -\frac{8}{14} = -\frac{4}{7}$$

Thus

$$\underline{\underline{\alpha = -\frac{12}{7}, \beta = -\frac{4}{7}}}$$

Prob. 1.6

$$\boldsymbol{A}\cdot\boldsymbol{B}=0\rightarrow 0=3\alpha+\beta-24 \qquad (1)$$
$$\boldsymbol{A}\cdot\boldsymbol{C}=0\rightarrow 0=5\alpha-2+4\gamma \qquad (2)$$
$$\boldsymbol{B}\cdot\boldsymbol{C}=0\rightarrow 0=15-2\beta-6\gamma \qquad (3)$$

In matrix form,

$$\begin{bmatrix}24\\2\\15\end{bmatrix}=\begin{bmatrix}3&1&0\\5&0&4\\0&2&6\end{bmatrix}\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}$$

$$\Delta=\begin{vmatrix}3&1&0\\5&0&4\\0&2&6\end{vmatrix}=3(0-8)-1(30-0)+0(10-0)=-24-30=-54$$

$$\Delta_1=\begin{vmatrix}24&1&0\\2&0&4\\15&2&6\end{vmatrix}=-24\times 8-(12-60)=-144$$

$$\Delta_2=\begin{vmatrix}3&24&0\\5&2&4\\0&15&6\end{vmatrix}=3(12-60)-24\times 30=-864$$

$$\Delta_3=\begin{vmatrix}3&1&24\\5&0&2\\0&2&15\end{vmatrix}=-12-75+240=153$$

$$\alpha=\frac{\Delta_1}{\Delta}=\frac{-144}{-54}=\underline{\underline{2.667}}$$

$$\beta=\frac{\Delta_2}{\Delta}=\frac{-864}{-54}=\underline{\underline{16}}$$

$$\gamma=\frac{\Delta_3}{\Delta}=\frac{153}{-54}=\underline{\underline{-2.833}}$$

Prob. 1.7

(a) $\boldsymbol{A}\cdot\boldsymbol{B}=\boldsymbol{AB}\cos\theta_{AB}$

$\boldsymbol{A}\times\boldsymbol{B}=\boldsymbol{AB}\sin\theta_{AB}\boldsymbol{a}_n$

$(\boldsymbol{A}\cdot\boldsymbol{B})^2+|\boldsymbol{A}\times\boldsymbol{B}|^2=(\boldsymbol{AB})^2(\cos^2\theta_{AB}+\sin^2\theta_{AB})=(\boldsymbol{AB})^2$

(b) $\boldsymbol{a}_x \cdot (\boldsymbol{a}_y \times \boldsymbol{a}_z) = \boldsymbol{a}_x \cdot \boldsymbol{a}_x = 1$. Hence,

$$\frac{\boldsymbol{a}_y \times \boldsymbol{a}_z}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_x}{1} = \boldsymbol{a}_x$$

$$\frac{\boldsymbol{a}_z \times \boldsymbol{a}_x}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_y}{1} = \boldsymbol{a}_y$$

$$\frac{\boldsymbol{a}_x \times \boldsymbol{a}_y}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_z}{1} = \boldsymbol{a}_z$$

Prob. 1.8

(a) $\boldsymbol{P} + \boldsymbol{Q} = (2,2,0), \boldsymbol{P} + \boldsymbol{Q} - \boldsymbol{R} = (3,1,-2)$

$$|\boldsymbol{P} + \boldsymbol{Q} - \boldsymbol{R}| = \sqrt{9+1+4} = \sqrt{14} = \underline{\underline{3.742}}$$

(b) $$\boldsymbol{P} \cdot \boldsymbol{Q} \times \boldsymbol{R} = \begin{vmatrix} -2 & -1 & -2 \\ 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = -2(6-2) + (8+2) - 2(4+3) = -8 + 10 - 14 = \underline{\underline{-12}}$$

$$\boldsymbol{Q} \times \boldsymbol{R} = \begin{vmatrix} 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = (4,-10,7)$$

$$\boldsymbol{P} \cdot \boldsymbol{Q} \times \boldsymbol{R} = (-2,-1,-2) \cdot (4,-10,7) = -8 + 10 - 14 = \underline{\underline{-12}}$$

(c) $$\boldsymbol{Q} \times \boldsymbol{P} = \begin{vmatrix} 4 & 3 & 2 \\ -2 & -1 & -2 \end{vmatrix} = (-4,4,2)$$

$$\boldsymbol{Q} \times \boldsymbol{P} \cdot \boldsymbol{R} = (-4,4,2) \cdot (-1,1,2) = 4 + 4 + 4 = \underline{\underline{12}}$$

or $$\boldsymbol{Q} \times \boldsymbol{P} \cdot \boldsymbol{R} = \boldsymbol{R} \cdot \boldsymbol{Q} \times \boldsymbol{P} = \begin{vmatrix} -1 & 1 & 2 \\ 4 & 3 & 2 \\ -2 & -1 & -2 \end{vmatrix} = -(-6+2) - (-8+4) + 2(-4+6) = \underline{\underline{12}}$$

(d) $$(\boldsymbol{P} \times \boldsymbol{Q}) \cdot (\boldsymbol{Q} \times \boldsymbol{R}) = (4,-4,2) \cdot (4,-10,7) = 16 + 40 - 14 = \underline{\underline{42}}$$

(e) $$(\boldsymbol{P} \times \boldsymbol{Q}) \times (\boldsymbol{Q} \times \boldsymbol{R}) = \begin{vmatrix} 4 & -4 & 2 \\ 4 & -10 & 7 \end{vmatrix} = \underline{\underline{-48\boldsymbol{a}_x - 36\boldsymbol{a}_y - 24\boldsymbol{a}_z}}$$

(f) $$\cos\theta_{PR} = \frac{\boldsymbol{P} \cdot \boldsymbol{R}}{|\boldsymbol{P}||\boldsymbol{R}|} = \frac{(2-1-4)}{\sqrt{4+1+4}\sqrt{1+1+4}} = \frac{-3}{3\sqrt{6}} = \frac{-1}{\sqrt{6}}$$

$$\underline{\underline{\theta_{PR} = 114.1^\circ}}$$

(g) $\sin\theta_{PQ} = \dfrac{|\boldsymbol{P}\times\boldsymbol{Q}|}{|\boldsymbol{P}||\boldsymbol{Q}|} = \dfrac{\sqrt{16+16+4}}{3\sqrt{16+9+4}} = \dfrac{6}{3\sqrt{29}}$

$\theta_{PQ} = \underline{\underline{21.8^\circ}}$

Prob. 1.9

(a) $\boldsymbol{T}_s = \boldsymbol{T}\cdot\boldsymbol{a}_s = \dfrac{\boldsymbol{T}\cdot\boldsymbol{S}}{|\boldsymbol{S}|} = \dfrac{(2,-6,-3)\cdot(1,2,1)}{\sqrt{6}} = \dfrac{-7}{\sqrt{6}} = \underline{\underline{-2.8577}}$

(b) $\boldsymbol{S}_T = (\boldsymbol{S}\cdot\boldsymbol{a}_T)\boldsymbol{a}_T = \dfrac{(\boldsymbol{S}\cdot\boldsymbol{T})\boldsymbol{T}}{\boldsymbol{T}^2} = \dfrac{-7(2,-6,3)}{7^2}$

$= \underline{\underline{-0.2857\boldsymbol{a}_x + 0.8571\boldsymbol{a}_y - 0.4286\boldsymbol{a}_z}}$

(c) $\sin\theta_{TS} = \dfrac{|\boldsymbol{T}\times\boldsymbol{S}|}{|\boldsymbol{T}||\boldsymbol{S}|} = \begin{vmatrix} 2 & -6 & 3 \\ 1 & 2 & 1 \end{vmatrix} = \dfrac{|(-12,1,10)|}{7\sqrt{6}} = \dfrac{\sqrt{245}}{7\sqrt{6}} = 0.9129$

$\Rightarrow \theta_{TS} = \underline{\underline{65.91^\circ}}$

Prob. 1.10

(a) $\boldsymbol{A}_B = \boldsymbol{A}\cdot\boldsymbol{a}_B = \dfrac{\boldsymbol{A}\boldsymbol{B}}{|\boldsymbol{B}|} = \dfrac{-1+12+15}{\sqrt{1+4+9}} = \dfrac{26}{\sqrt{14}} = \underline{\underline{6.95}}$

(b) $\boldsymbol{B}_A = (\boldsymbol{B}\cdot\boldsymbol{a}_A)\boldsymbol{a}_A = \dfrac{(\boldsymbol{B}\cdot\boldsymbol{A})\boldsymbol{A}}{|\boldsymbol{A}|^2} = \dfrac{26(-1,6,5)}{(1+36+25)}$

$= \underline{\underline{-0.4193\boldsymbol{a}_x + 2.516\boldsymbol{a}_y + 2.097\boldsymbol{a}_z}}$

(c) $\cos\theta_{AB} = \dfrac{\boldsymbol{A}\cdot\boldsymbol{B}}{|\boldsymbol{A}||\boldsymbol{B}|} = \dfrac{26}{\sqrt{62}\sqrt{1+4+9}} = \dfrac{26}{\sqrt{62}\sqrt{14}}$

$\theta_{AB} = \underline{\underline{28.05^\circ}}$

(d) $\boldsymbol{A}\times\boldsymbol{B} = \begin{vmatrix} -1 & 6 & 5 \\ 1 & 2 & 3 \end{vmatrix} = 8\boldsymbol{a}_x + 8\boldsymbol{a}_y - 8\boldsymbol{a}_z$

A unit vector perpendicular to both $\boldsymbol{A}$ and $\boldsymbol{B}$ is

$$\boldsymbol{a}_{A\times B} = \frac{8\boldsymbol{a}_x + 8\boldsymbol{a}_y - 8\boldsymbol{a}_z}{8\sqrt{1+1+1}} = \frac{\boldsymbol{a}_x + \boldsymbol{a}_y - \boldsymbol{a}_z}{\sqrt{3}} = \underline{\underline{0.577\boldsymbol{a}_x + 0.577\boldsymbol{a}_y - 0.577\boldsymbol{a}_z}}$$

Prob. 1.11

$$\cos\theta = \frac{\boldsymbol{H}\cdot\boldsymbol{a}_x}{|\boldsymbol{H}|} = \frac{3}{\sqrt{9+25+64}} = \frac{3}{98}$$

$$\theta_x = \underline{\underline{72.36^\circ}}$$

$$\cos\theta = \frac{\boldsymbol{H}\cdot\boldsymbol{a}_y}{|\boldsymbol{H}|} = \frac{5}{\sqrt{9+25+64}} = \frac{5}{98}$$

$$\theta_y = \underline{\underline{59.66^\circ}}$$

$$\cos\theta = \frac{\boldsymbol{H}\cdot\boldsymbol{a}_z}{|\boldsymbol{H}|} = \frac{-8}{\sqrt{9+25+64}} = \frac{-8}{98}$$

$$\theta_z = \underline{\underline{143.91^\circ}}$$

Prob. 1.12

$$\boldsymbol{Q}\times\boldsymbol{R} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix} = (3,-1,-2)$$

$$\boldsymbol{P}\cdot(\boldsymbol{Q}\times\boldsymbol{R}) = (2,-1,1)\cdot(3,-1,2) = 6+1-2 = \underline{\underline{5}}$$

Prob. 1.13

(a) Using the fact that

$$(\boldsymbol{A}\times\boldsymbol{B})\times\boldsymbol{C} = (\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B} - (\boldsymbol{B}\cdot\boldsymbol{C})\boldsymbol{A},$$

we get

$$\boldsymbol{A}\times(\boldsymbol{A}\times\boldsymbol{B}) = -(\boldsymbol{A}\times\boldsymbol{B})\times\boldsymbol{A} = \underline{\underline{(\boldsymbol{B}\cdot\boldsymbol{A})\boldsymbol{A} - (\boldsymbol{A}\cdot\boldsymbol{A})\boldsymbol{B}}}$$

(b) $\boldsymbol{A}\times(\boldsymbol{A}\times(\boldsymbol{A}\times\boldsymbol{B})) = \boldsymbol{A}\times[(\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{A} - (\boldsymbol{A}\cdot\boldsymbol{A})\boldsymbol{B}]$

$$= \underline{\underline{(\boldsymbol{A}\cdot\boldsymbol{B})(\boldsymbol{A}\times\boldsymbol{A}) - (\boldsymbol{A}\cdot\boldsymbol{A})(\boldsymbol{A}\times\boldsymbol{B})}}$$

Prob. 1.14

$$\boxed{A\cdot(B\times C) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}, \quad (A\times B)\cdot C = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}}$$

Hence, $A\cdot(B\times C) = (A\times B)\cdot C$

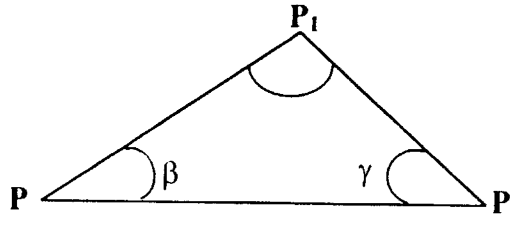
Prob. 1.15

$$P_1P_2 = r_{P_2} - r_{P_1} = (-6,0,-3)$$
$$P_1P_3 = r_{P_3} - r_{P_1} = (1,5,-6)$$
$$P_1P_2 \times P_1P_3 = \begin{vmatrix} -6 & 0 & -3 \\ 1 & 5 & -6 \end{vmatrix} = (15,39,-30)$$

$$\text{Area of the triangle} = \frac{1}{2}\left|P_1P_2 \times P_1P_3\right| = \frac{1}{2}\sqrt{15^2+39^2+30^2} = \underline{\underline{25.72}}$$

Prob. 1.16

Let $P_1 = (4,1,-3)$, $P_2 = (-2, 5, 4)$, and $P_3 = (0, 1, 6)$



$$a = r_{P_2} - r_{P_1} = (-2,5,4)-(4,1,-3) = (-6,4,7)$$
$$b = r_{P_3} - r_{P_2} = (0,1,6)-(-2,5,4) = (2,-4,2)$$
$$c = r_{P_1} - r_{P_3} = (4,1,-3)-(0,1,6) = (4,0,-9)$$

Note that $a + b + c = 0$

$$a \cdot b = ab\cos(180-\gamma) \rightarrow -\cos\gamma = \frac{a \cdot b}{|a||b|} = \frac{-12-16+14}{\sqrt{101}\sqrt{24}}$$

$$\gamma = \cos^{-1}\frac{14}{\sqrt{101}\sqrt{24}} = \underline{\underline{73.47^\circ}}$$

$$b \cdot c = bc\cos(180-\beta) \rightarrow -\cos\beta = \frac{b \cdot c}{|b||c|} = \frac{8+0-18}{\sqrt{24}\sqrt{97}}$$

$$\beta = \cos^{-1}\frac{10}{\sqrt{24}\sqrt{97}} = \underline{\underline{78.04^\circ}}$$

$$a \cdot c = ac\cos(180-\alpha) \rightarrow -\cos\alpha = \frac{a \cdot c}{|a||c|} = \frac{-24+0-63}{\sqrt{101}\sqrt{97}}$$

$$\alpha = \cos^{-1}\frac{87}{\sqrt{101}\sqrt{97}} = \underline{\underline{28.48^\circ}}$$

Prob. 1.17

(a) $\boldsymbol{r}_{PQ} = \boldsymbol{r}_Q - \boldsymbol{r}_P = (2,-1,3)-(-1,4,8) = (3,-5,-5)$

$$r_{PQ} = |\boldsymbol{r}_{PQ}| = \sqrt{9+25+25} = \underline{\underline{7.681}}$$

(b) $\boldsymbol{r}_{PR} = \boldsymbol{r}_R - \boldsymbol{r}_P = (-1,2,3)-(-1,4,8) = (0,-2,-5) = \underline{\underline{-2\boldsymbol{a}_y - 5\boldsymbol{a}_z}}$

(c) $\boldsymbol{r}_{QP} = -\boldsymbol{r}_{PQ} = -3\boldsymbol{a}_x + 5\boldsymbol{a}_y + 5\boldsymbol{a}_z$

$$\boldsymbol{r}_{QR} = \boldsymbol{r}_Q - \boldsymbol{r}_R = (2,-1,3)-(-1,2,3) = 3\boldsymbol{a}_x - 3\boldsymbol{a}_y$$

$$\cos\theta = \frac{\boldsymbol{r}_{QP}\cdot\boldsymbol{r}_{QR}}{|\boldsymbol{r}_{QP}||\boldsymbol{r}_{QR}|} = \frac{-9-15}{\sqrt{9+25+25}\sqrt{9+9}} = \frac{-24}{\sqrt{18}\sqrt{59}}$$

$$\underline{\underline{\theta = 137.43^\circ}}$$

(d) Area $= \frac{1}{2}|\boldsymbol{r}_{QP}\times\boldsymbol{r}_{QR}|$

$$\boldsymbol{r}_{QP}\times\boldsymbol{r}_{QR} = \begin{vmatrix} -3 & 5 & 5 \\ 3 & -3 & 0 \end{vmatrix} = 15\boldsymbol{a}_x + 15\boldsymbol{a}_y - 6\boldsymbol{a}_z$$

$$\text{Area} = \frac{1}{2}\sqrt{15^2+15^2+6^2} = \underline{\underline{11.02}}$$

(e) Perimeter $= QP + PR + RQ = r_{QP} + r_{PR} + r_{QR}$

$$= \sqrt{59} + \sqrt{4+25} + \sqrt{18}$$

$$= 7.681 + 5.385 + 4.243$$

$$= \underline{\underline{17.31}}$$

Prob. 1.18

(a) Let $\boldsymbol{A} = (A,B,C)$ and $\boldsymbol{r} = (x,y,z)$

$$(\boldsymbol{r}-\boldsymbol{A})\cdot\boldsymbol{A} = (x-A)A + (y-B)B + (z-C)C$$
$$= Ax + By + Cz + D$$

where $D = -A^2 - B^2 - C^2$. Hence,

$$(\boldsymbol{r}-\boldsymbol{A})\cdot\boldsymbol{A} = 0 \rightarrow Ax + By + Cz + D = 0$$

which is the equation of a plane.

(b) $(\boldsymbol{r}-\boldsymbol{A})\cdot\boldsymbol{r} = (x-A)x + (y-B)y + (z-C)z$

If $(\boldsymbol{r}-\boldsymbol{A})\cdot\boldsymbol{r} = 0$, then

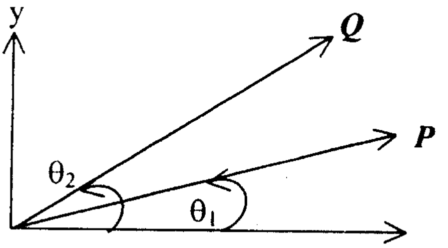
$$x^2 + y^2 + z^2 - Ax - By - Cz = 0$$

which is the equation of a sphere whose surface touches the origin.

(c) See parts (a) and (b).

Prob. 1.19

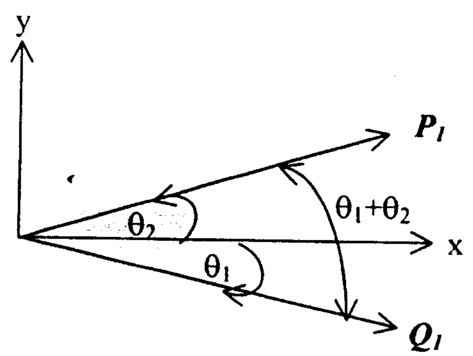
(a) Let P and Q be as shown below:



$|P| = \cos^2\theta_1 + \sin^2\theta_1 = 1, |Q| = \cos^2\theta_2 + \sin^2\theta_2 = 1,$
Hence **P** and **Q** are unit vectors.

(b) $\boldsymbol{P}\cdot\boldsymbol{Q} = (1)(1)\cos(\theta_2 - \theta_1)$
But $\boldsymbol{P}\cdot\boldsymbol{Q} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$. Thus,
$\underline{\underline{\cos(\theta_2-\theta_1) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2}}$

Let $\boldsymbol{P}_1 = \boldsymbol{P} = \cos\theta_1\boldsymbol{a}_x + \sin\theta_1\boldsymbol{a}_y$ and
$\boldsymbol{Q}_1 = \cos\theta_2\boldsymbol{a}_x - \sin\theta_2\boldsymbol{a}_y$.
$\boldsymbol{P_1}$ and $\boldsymbol{Q_1}$ are unit vectors as shown below:



$\boldsymbol{P}_1\cdot\boldsymbol{Q}_1 = (1)(1)\cos(\theta_1+\theta_2)$
But $\boldsymbol{P}_1\cdot\boldsymbol{Q}_1 = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$,
$\underline{\underline{\cos(\theta_2+\theta_1) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2}}$

Alternatively, we can obtain this formula from the previous one by replacing θ_2 by $-\theta_2$ in $\boldsymbol{Q}$.

(c)

$$\frac{1}{2}|P-Q| = \frac{1}{2}|(\cos\theta_1 - \cos\theta_2)a_x + (\sin\theta_1 - \sin\theta_2)a_y$$

$$= \frac{1}{2}\sqrt{\cos^2\theta_1 + \sin^2\theta_1 + \cos^2\theta_2 + \sin^2\theta_2 - 2\cos\theta_1\cos\theta_2 - 2\sin\theta_1\sin\theta_2}$$

$$= \frac{1}{2}\sqrt{2 - 2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)} = \frac{1}{2}\sqrt{2 - 2\cos(\theta_2 - \theta_1)}$$

Let $\theta_2 - \theta_1 = \theta$, the angle between **P** and **Q.**

$$\frac{1}{2}|P-Q| = \frac{1}{2}\sqrt{2 - 2\cos\theta}$$

But $\cos 2A = 1 - 2\sin^2 A$.

$$\frac{1}{2}|P-Q| = \frac{1}{2}\sqrt{2 - 2 + 4\sin^2\theta/2} = \sin\theta/2$$

Thus,

$$\underline{\frac{1}{2}|P-Q| = |\sin\frac{\theta_2 - \theta_1}{2}|}$$

Prob. 1.20

$$\boldsymbol{w} = \frac{w(1,-2,2)}{3} = (1,-2,2),\ \boldsymbol{r} = \boldsymbol{r}_p - \boldsymbol{r}_o = (1,3,4) - (2,-3,1) = (-1,6,3)$$

$$\boldsymbol{u} = \boldsymbol{w} \times \boldsymbol{r} = \begin{vmatrix} 1 & -2 & 2 \\ -1 & 6 & 3 \end{vmatrix} = (-18,-5,4)$$

$$\underline{\underline{\boldsymbol{u} = -18\boldsymbol{a}_x - 5\boldsymbol{a}_y + 4\boldsymbol{a}_z}}$$

Prob. 1.21

(a) At T, $\boldsymbol{A} = (-4,3,-9)$

$$|A| = \sqrt{16+9+81} = \sqrt{106} = \underline{\underline{10.3}}$$

(b) Let $\boldsymbol{r}_{TS} = \boldsymbol{B} = B\boldsymbol{a}_B$

$$B = 5.6, \boldsymbol{a}_B = \boldsymbol{a}_A = \frac{(-4,3,-9)}{10.3}$$

$$\boldsymbol{r}_{TS} = \boldsymbol{B} = \frac{5.6(-4,3,9)}{10.3}$$

$$= \underline{\underline{-2.175\boldsymbol{a}_x + 1.631\boldsymbol{a}_y - 4.893\boldsymbol{a}_z}}$$

(c) $\boldsymbol{r}_{TS} = \boldsymbol{r}_S - \boldsymbol{r}_T \rightarrow \boldsymbol{r}_S = \boldsymbol{r}_T + \boldsymbol{r}_{TS}$

$$\therefore \underline{\underline{\boldsymbol{r}_S = -0.175\boldsymbol{a}_x + 0.631\boldsymbol{a}_y - 1.893\boldsymbol{a}_Z}}$$

Prob. 1.22

(a) At (1,2,3), $\boldsymbol{E} = (2,1,6)$

$$|\boldsymbol{E}| = \sqrt{4+1+36} = \sqrt{41} = \underline{\underline{6.403}}$$

(b) At (1,2,3), $\boldsymbol{F} = (2,-4,6)$

$$\boldsymbol{E}_F = (\boldsymbol{E}\cdot\boldsymbol{a}_F)\boldsymbol{a}_F = \frac{(\boldsymbol{E}\cdot\boldsymbol{F})\boldsymbol{F}}{|\boldsymbol{F}|^2} = \frac{36}{56}(2,-4,6)$$

$$= \underline{\underline{1.286\boldsymbol{a}_x - 2.571\boldsymbol{a}_y + 3.857\boldsymbol{a}_z}}$$

(c) At (0,1,-3), $\boldsymbol{E} = (0,1,-3)$, $\boldsymbol{F} = (0,-1,0)$

$$\boldsymbol{E}\times\boldsymbol{F} = \begin{vmatrix} 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = (-3,0,0)$$

$$\boldsymbol{a}_{E\times F} = \pm\frac{\boldsymbol{E}\times\boldsymbol{F}}{|\boldsymbol{E}\times\boldsymbol{F}|} = \underline{\underline{\pm\boldsymbol{a}_x}}$$

CHAPTER 2

P. E. 2.1

(a) At P(1,3,5), x = 1, y = 3, z = 5,

$\rho = \sqrt{x^2 + y^2} = \sqrt{10}$, z = 5, $\phi = \tan^{-1} y/x = 3$

$P(\rho, \phi, z) = P(\sqrt{10}, \tan^{-1} 3, 5) = \underline{\underline{P(3.162, 71.6°, 5)}}$

Spherical system:

$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{35} = 5.916$

$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{35} = 5.916$

$\theta = \tan^{-1} \sqrt{x^2 + y^2}/z = \tan^{-1} \sqrt{10}/5 = \tan^{-1} 0.6325 = 32.31°$

$P(r, \theta, \varphi) = \underline{\underline{P(5.916, 32.31°, 71.56°)}}$

At T(0,-4,3), x = 0 y = -4, z = 3;

$\rho = \sqrt{x^2 + y^2} = 4, z = 3, \varphi = \tan^{-1} y/x = \tan^{-1} -4/0 = 270°$

$\underline{\underline{T(\rho, \varphi, z) = T(4, 270°, 3).}}$

Spherical system:

$r = \sqrt{x^2 + y^2 + z^2} = 5, \theta = \tan^{-1} \rho/z = \tan^{-1} 4/3 = 53.13°.$

$\underline{\underline{T(r, \theta, \varphi) = T(5, 53.13°, 270°).}}$

At S(-3-4-10), x = -3, y = -4, z = -10;

$\rho = \sqrt{x^2 + y^2} = 5, \ \phi = \tan^{-1} -4/-3 = 233.1°$

$\underline{\underline{S(\rho, \phi, z) = S(5, 233.1, -10).}}$

Spherical system:

$r = \sqrt{x^2 + y^2 + z^2} = 5\sqrt{5} = 11.18.$

$\theta = \tan^{-1} \rho/z = \tan^{-1} 5/-10 = 153.43°;$

$\underline{\underline{S(r, \theta, \phi) = S(11.18, 153.43°, 233.1°).}}$

(b) In Cylindrical system, $\rho = \sqrt{x^2 + y^2}$; $yz = z\rho\sin\theta$,

$Q_r = \frac{\rho}{\sqrt{\rho^2 + z^2}}$; $Q_\phi = 0$; $Q_z = \frac{z\rho\sin\phi}{\sqrt{\rho^2 + z^2}}$;

$$\begin{bmatrix} Q_\rho \\ Q_\phi \\ Q_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_x \\ 0 \\ Q_z \end{bmatrix};$$

$$Q_\rho = Q_x \cos\phi = \frac{\rho\cos\phi}{\sqrt{\rho^2 + z^2}}, \qquad Q_\phi = -Q_x \sin\phi = \frac{-\rho\sin\phi}{\sqrt{\rho^2 + z^2}}$$

Hence,

$$\bar{Q} = \frac{\rho}{\sqrt{x^2 + z^2}}(\cos\phi\, \bar{a}_\rho, -\sin\phi\, a_\phi, -z\sin\phi\, \bar{a}_z).$$

In Spherical coordinates:

$$Q_x = \frac{r\sin\phi}{r} = \sin\phi;$$

$$Q_z = -r\sin\phi\sin\theta\, r\cos\theta\,\frac{1}{r} = -r\sin\theta\cos\theta\sin\phi.$$

$$\begin{bmatrix} Q_r \\ Q_\theta \\ Q_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\phi \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} Q_x \\ 0 \\ Q_z \end{bmatrix};$$

$$Q_r = Q_x \sin\theta\cos\phi + Q_z\cos\theta = \sin^2\theta\cos\phi - r\sin\theta\cos^2\theta\sin\phi.$$

$$Q_\theta = Q_x\cos\theta\cos\phi - Q_z\sin\theta = \sin\theta\cos\theta\cos\phi + r\sin^2\theta\cos\theta\sin\phi.$$

$$Q_\phi = -Q_x\sin\phi = -\sin\theta\sin\phi.$$

$$\therefore\ \bar{Q} = \sin\theta(\sin\theta\cos\phi - r\cos^2\theta\sin\phi)\bar{a}_r + \sin\theta\cos\phi(\cos\phi + r\sin\theta\sin\phi)\bar{a}_\theta - \sin\theta\sin\phi\,\bar{a}_\phi.$$

At T :

$$\bar{Q}(x,y,z) = \frac{4}{5}\bar{a}_x + \frac{12}{5}\bar{a}_z = 0.8\bar{a}_x + 2.4\bar{a}_z;$$

$$\bar{Q}(\rho,\phi,z) = \frac{4}{5}(\cos 270^\circ\,\bar{a}_\rho - \sin 270^\circ\,\bar{a}_\phi - 3\sin 270^\circ\,\bar{a}_z$$

$$= 0.8\bar{a}_\phi + 2.4\bar{a}_z;$$

$$\bar{Q}(r,\theta,\phi) = \frac{4}{5}(0 - \frac{45}{25}(-1))\bar{a}_r + \frac{4}{5}(\frac{3}{5})(0 - \frac{20}{5}(-1))\bar{a}_\theta - \frac{4}{5}(-1)\bar{a}_\phi$$

$$= \frac{36}{25}\bar{a}_r + \frac{48}{25}\bar{a}_\theta + \frac{4}{5}\bar{a}_\phi = 1.44a_r + 1.92a_\theta + 0.8a_\phi;$$

Note, that the magnitude of vector Q = 2.53 in all 3 cases above.

P.E. 2.2 (a)

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho z \sin\phi \\ 3\rho\cos\phi \\ \rho\cos\phi\sin\phi \end{bmatrix}$$

$$\bar{A} = (\rho z \cos\phi \sin\phi - 3\rho\cos\phi\sin\phi)\bar{a}_x + (\rho z \sin^2\phi + 3\rho\cos^2\phi)\bar{a}_y + \rho\cos\phi\sin\phi\,\bar{a}_z.$$

But $\rho = \sqrt{x^2+y^2}$, $\tan\phi = \dfrac{y}{x}$, $\cos\phi = \dfrac{x}{\sqrt{x^2+y^2}}$, $\sin\phi = \dfrac{y}{\sqrt{x^2+y^2}}$;

Substituting all this yields:

$$\bar{A} = \frac{1}{\sqrt{x^2+y^2}}[(xyz - 3xy)\bar{a}_x + (zy^2 + 3x^2)\bar{a}_y + xy\bar{a}_z].$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} r^2 \\ 0 \\ \sin\theta \end{bmatrix}$$

Since $r = \sqrt{x^2+y^2+z^2}$, $\tan\theta = \dfrac{\sqrt{x^2+y^2}}{z}$, $\tan\phi = \dfrac{y}{z}$;

and $\sin\theta = \dfrac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$, $\cos\theta = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$;

and $\sin\phi = \dfrac{y}{\sqrt{x^2+y^2}}$, $\cos\phi = \dfrac{x}{\sqrt{x^2+y^2}}$;

$$B_x = r^2\sin\theta\cos\phi - \sin\theta\sin\phi = rx - \frac{y}{r} = \frac{1}{r}(r^2x - y).$$

$$B_y = r^2\sin\theta\sin\phi + \sin\theta\cos\phi = ry + \frac{y}{x} = \frac{1}{r}(r^2y + x).$$

$$B_z = r^2\cos\theta = rz = \frac{1}{r}(r^2z).$$

Hence,

$$B = \frac{1}{\sqrt{x^2+y^2+z^2}}[\{x(x^2+y^2+z^2) - y\}a_x + \{y(x^2+y^2+z^2) + x\}\bar{a}_y + z(x^2+y^2+z^2)a_z].$$

P.E.2.3 (a) At:

$(1, \pi/3, 0)$, $H = (0, 0.5, 1)$

$$a_x = \cos\phi\, \bar{a}_\rho - \sin\phi\, \bar{a}_\phi = \frac{1}{2}(\bar{a}_\rho - \sqrt{3}\,\bar{a}_\phi)$$

$$\bar{H} \bullet \bar{a}_x = -\frac{\sqrt{3}}{4} = \underline{\underline{-0.433.}}$$

(b) At:

$(1, \pi/3, 0)$, $\bar{a}_\theta = \cos\theta\, \bar{a}_\rho - \sin\theta\, \bar{a}_z = -\bar{a}_z.$

$$\bar{H} \times \bar{a}_\theta = \begin{vmatrix} \bar{a}_\rho & \bar{a}_\phi & \bar{a}_z \\ 0 & \frac{1}{2} & 1 \\ 0 & 0 & -1 \end{vmatrix} = \underline{\underline{-0.5\,\bar{a}_\rho}}$$

(c) $(H \bullet \bar{a}_\rho)\bar{a}_\rho = \underline{\underline{0\,\bar{a}_\rho}}.$

(d)

$$\bar{H} \times \bar{a}_z = \begin{vmatrix} \bar{a}_\rho & \bar{a}_\phi & \bar{a}_z \\ 0 & 1/2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0.5\,\bar{a}_\rho.$$

$$\left| H \times \bar{a}_z \right| = \underline{\underline{0.5}}$$

P.E. 2.4

(a)

$$\bar{A} \bullet \bar{B} = (3, 2, -6) \bullet (4, 0, 3) = \underline{\underline{-6.}}$$

(b)

$$\left| \bar{A} \times \bar{B} \right| = \begin{vmatrix} 3 & 2 & -6 \\ 4 & 0 & 3 \end{vmatrix} = \left| 6\,\bar{a}_r - 33\,\bar{a}_\theta - 8\,\bar{a}_\phi \right|.$$

Thus the magnitude of $A \times B = \underline{\underline{34.48.}}$

(c)

At $(1, \pi/3, 5\pi/4)$, $\theta = \pi/3$,

$$\bar{a}_z = \cos\theta\, a_r - \sin\theta\, a_\theta = \frac{1}{2} a_r - \frac{\sqrt{3}}{2} a_\theta.$$

$$(\bar{A}\bullet \bar{a}_z)\bar{a}_z = (\frac{3}{2} - \sqrt{3})(\frac{1}{2}\bar{a}_r - \frac{\sqrt{3}}{2}\bar{a}_\theta)$$

$$= \underline{\underline{-0.116\,\bar{a}_r + 0.201\,\bar{a}_\theta}}.$$

Prob. 2.1

(a)

$$x = \rho\cos\phi = 1\cos 60^\circ = 0.5;$$

$$y = \rho\sin\phi = 1\sin 120^\circ = 0.866;$$

$$z = 2;$$

$$P(x,y,z) = \underline{\underline{P\,(0.5,\ 0.866,\ 2)}}.$$

(b)

$$x = 2\cos 90^\circ = 0; \quad y = 2\sin 90^\circ = 1; \quad z = -10.$$

$$Q = \underline{\underline{Q\ (0,\ 1,\ -4)}}.$$

(c)

$$x = r\sin\theta\cos\phi = 3\sin 45^\circ\cos 210^\circ = -1.837;$$

$$y = r\sin\theta\sin\phi = 10\sin 135^\circ\sin 90^\circ = -1.061;$$

$$z = r\cos\theta = 10\cos 135^\circ = 2.121.$$

$$R(x,y,z) = \underline{\underline{R(-1.837,\ -1.061,\ 2.121)}}.$$

(d)

$$x = 4\sin 90^\circ\cos 30^\circ = 3.464.$$

$$y = 3\sin 30^\circ\sin 240^\circ = 2.$$

$$z = r\cos\theta = 4\cos 90^\circ = 0.$$

$$T(x,y,z) = \underline{\underline{T(3.464,\ 2,\ 0)}}.$$

Prob.2.2

(a) Given P(1,-4,-3), convert to cylindrical and spherical values;

$$\rho = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-4)^2} = \sqrt{17} = 4.123.$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-4}{1} = 284.04^\circ.$$

$$\therefore\ P(\rho,\phi,z) = \underline{\underline{(4.123,\ 284.04^\circ,\ -3)}}.$$

Spherical:

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{1 + 16 + 9} = 5.099.$$

$$\theta = \tan^{-1}\frac{\rho}{z} = \tan^{-1}\frac{4.123}{-3} = 126.04^\circ.$$

$$P(r,\theta,\phi) = \underline{\underline{P(5.099,\ 126.04^\circ,\ 284.04^\circ)}}.$$

Prob . 2.3

(a)

$$x = \rho\cos\phi, \qquad y = \rho\sin\phi,$$

$$V = \underline{\underline{\rho z\cos\phi - \rho^2\sin\phi\cos\phi + \rho z\sin\phi}}$$

(b)

$$U = x^2 + y^2 + z^2 + y^2 + 2z^2$$

$$= r^2 + r^2\sin^2\theta\sin^2\phi + 2r^2\cos^2\theta$$

$$= \underline{\underline{r^2[1 + \sin^2\theta\sin^2\phi + 2\cos^2\theta}}$$

Prob. 2.4

(a)

$$\begin{bmatrix} D_\rho \\ D_\phi \\ D_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ x+z \\ 0 \end{bmatrix}$$

$$D_\rho = (x+z)\sin\phi = (\rho\cos\phi + z)\sin\phi$$

$$D_\phi = (x+z)\cos\phi = (\rho\cos\phi + z)\cos\phi$$

$$\bar{D} = \underline{\underline{(\rho\cos\phi + z)[\sin\phi\,\bar{a}_\rho + \cos\phi\,\bar{a}_\phi]}}$$

Spherical:

$$\begin{bmatrix} D_r \\ D_\theta \\ D_\phi \end{bmatrix} = \begin{bmatrix} \ldots & \sin\theta\sin\phi & \ldots \\ \ldots & \cos\theta\sin\phi & \ldots \\ \ldots & \cos\phi & \ldots \end{bmatrix} \begin{bmatrix} 0 \\ x+z \\ 0 \end{bmatrix}$$

$D_r = (x+z)\sin\theta\cos\phi = r(\sin\theta\cos\phi + \cos\theta)\sin\theta\sin\phi.$

$D_\theta = (x+z)\cos\theta\sin\phi = r(\sin\theta\sin\phi + \cos\theta)\cos\theta\sin\phi.$

$D_\phi = (x+z)\cos\phi \qquad = r(\sin\theta\cos\phi + \cos\theta)\cos\phi.$

$$\bar{D} = r(\sin\theta\cos\phi + \cos\theta)[\sin\theta\sin\phi\,\bar{a}_r + \cos\theta\sin\phi\,\bar{a}_\theta + \cos\phi\,\bar{a}_\phi].$$

(b) Cylindrical:

$$\begin{bmatrix} E_\rho \\ E_\phi \\ E_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y^2 - x^2 \\ xyz \\ x^2 - z^2 \end{bmatrix}$$

$$E_\rho = (y^2 - x^2)\cos\phi + xyz\sin\phi$$

$$= \rho^2(\sin^2\phi - \cos^2\phi)\cos\phi + \rho^2 z\cos\phi\sin^2\phi$$

$$= -\rho^2\cos2\phi\cos\phi + \rho^2 z\sin^2\phi\cos\phi.$$

$$E_\phi = -(y^2 - x^2)\sin\phi + xyz\cos\phi$$

$$= \rho^2\cos2\phi\sin\phi + \rho^2\cos2\phi\sin\phi + \rho^2 z\sin\phi\cos^2\phi.$$

$$E_z = x^2 - z^2 = \rho^2\cos^2\phi - z^2.$$

$$\bar{E} = \rho^2\cos\phi(z\sin^2\phi - \cos2\phi)\bar{a}_\rho + \rho^2\sin\phi(2\cos^2\phi + \cos2\phi)\bar{a}_\phi + (\rho^2\cos\phi - z^2)\bar{a}_z.$$

In spherical:

$$\begin{bmatrix} E_r \\ E_\theta \\ E_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} y^2 - x^2 \\ xyz \\ x^2 - z^2 \end{bmatrix}$$

$$E_r = (y^2 - x^2)\sin\theta\cos\phi + xyz\sin\theta\sin\phi + (x^2 - z^2)\cos\theta;$$

but $x = r\sin\theta\cos\phi$, $\quad y = r\sin\theta\sin\phi$, $\quad z = r\cos\theta$;

$$E_r = r^2\sin^2\theta(\sin^2\phi - \cos^2\phi)\cos\phi + r^3\sin^3\theta\cos\theta\sin^2\phi\cos\phi + r^2(\sin^2\theta\cos^2\phi)\cos\theta;$$

$$E_\theta = (y^2 - x^2)\cos\theta\cos\phi + xyz\cos\theta\sin\phi - (x^2 - z^2)\sin\theta;$$

$$= -r^2\sin^2\theta\cos2\phi\cos\theta\cos\phi + r^3\sin^2\theta\cos^2\theta\sin^2\phi\cos\phi - r^2(\sin^2\theta\cos^2\phi - \cos^2\theta)\sin\theta;$$

$$E_\phi = (x^2 - y^2)\sin\phi + xyz\cos\phi$$

$$= r^2\sin^2\theta\cos2\phi\sin\phi + r^3\sin^2\theta\cos^2\phi\sin\phi\cos\theta;$$

$$\bar{E} = [-r^2\sin^3\theta\cos 2\phi + r^3\sin^3\theta\cos\theta\sin^2\phi\cos\phi + r^2(\sin^2\theta\cos^2\phi - \cos^2\theta)\cos\theta]\bar{a}_r +$$
$$[-r^2\sin^2\theta\cos 2\phi\cos\theta\cos\phi + r^3\sin^2\theta\cos^2\theta\sin^2\phi\cos\phi - r^2\sin\theta(\sin^2\theta\cos^2\phi - \cos^2\theta)]a_\theta +$$
$$+ \quad \underline{\underline{[r^2\sin^2\theta\cos 2\phi\sin\phi + r^3\sin^2\theta\cos^2\phi\sin\phi\cos\theta]\bar{a}_\phi}}$$

Prob. 2.5 (a)

$$\begin{bmatrix} F_\rho \\ F_\phi \\ F_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{x}{\sqrt{\rho^2+z^2}} \\ \dfrac{y}{\sqrt{\rho^2+z^2}} \\ \dfrac{4}{\sqrt{\rho^2+z^2}} \end{bmatrix}$$

$$F_\rho = \frac{1}{\sqrt{\rho^2+z^2}}[\rho\cos^2\phi + \rho\sin^2\phi] = \frac{\rho}{\sqrt{\rho^2+z^2}};$$

$$F_\phi = \frac{1}{\sqrt{\rho^2+z^2}}[-\rho\cos\phi\sin\phi + \rho\cos\phi\sin\phi] = 0;$$

$$F_z = \frac{4}{\sqrt{\rho^2+z^2}};$$

$$\underline{\underline{\bar{F} = \frac{1}{\sqrt{\rho^2+z^2}}(\rho\bar{a}_\rho + 4\bar{a}_z).}}$$

In Spherical:

$$\begin{bmatrix} F_r \\ F_\theta \\ F_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\theta & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \dfrac{x}{r} \\ \dfrac{y}{r} \\ \dfrac{4}{r} \end{bmatrix}$$

$$F_r = \frac{r}{r}\sin^2\theta\cos^2\theta + \frac{r}{r}\sin^2\theta\sin^2\theta + \frac{1}{r}\cos\theta \qquad = \sin^2\theta + \frac{1}{r}\cos\theta;$$

$$F_\theta = \sin\theta\cos\theta\cos^2\phi + \sin\theta\cos\theta\sin^2\phi - \frac{1}{r}\sin\theta = \sin\theta\cos\theta - \frac{1}{r}\sin\theta;$$

$$F_\phi = -\sin\theta\cos\phi\sin\phi + \sin\theta\sin\phi\cos\phi = 0;$$

$$\therefore \bar{F} = \underline{\underline{(\sin^2\theta + \frac{1}{r}\sin\theta)\bar{a}_r + \sin\theta(\cos\theta - \frac{1}{r})\bar{a}_\theta.}}$$

(b)

$$\begin{bmatrix} G_\rho \\ G_\phi \\ G_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{x\rho^2}{\sqrt{\rho^2+z^2}} \\ \dfrac{y\rho^2}{\sqrt{\rho^2+z^2}} \\ \dfrac{z\rho^2}{\sqrt{\rho^2+z^2}} \end{bmatrix}$$

$$G_\rho = \frac{\rho^2}{\sqrt{\rho^2+z^2}}[\rho\cos^2\phi + \rho\sin^2\phi] = \frac{\rho^3}{\sqrt{\rho^2+z^2}};$$

$$G_\phi = 0;$$

$$G_z = \frac{z\rho^2}{\sqrt{\rho^2+z^2}};$$

$$\bar{G} = \underline{\underline{\frac{\rho^2}{\sqrt{\rho^2+z^2}}(\rho\bar{a}_\rho + z\bar{a}_z).}}$$

Spherical :

$$\begin{bmatrix} G_r \\ G_\theta \\ G_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \dfrac{xr\sin\theta}{r} \\ y\sin\theta \\ z\sin\theta \end{bmatrix}$$

$$G_r = r\sin^2\theta\cos^2\phi + r\sin^2\theta\sin^2\phi + r\cos^2\theta\sin\theta$$
$$= r\sin^3\theta + r\cos^2\sin\theta = r\sin\theta.$$
$$G_\theta = r\sin^2\theta\cos\theta\cos^2\phi + r\sin^2\theta\cos\sin^3\phi - r\sin^3\theta\cos\theta$$
$$= r\sin^2\theta\cos\theta - r\sin^2\theta\cos\theta = 0.$$
$$G_\phi = -r\sin^2\theta\sin\phi\cos\phi + r\sin^2\theta\cos\phi\sin\phi = 0.$$

$$\therefore \bar{G} = \underline{\underline{r\sin\theta\,\bar{a}_r}}.$$

Prob. 2.6 (a)

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho(z^2+1) \\ -\rho z\cos\phi \\ 0 \end{bmatrix}$$

$$A_x = \rho(z^2 + 1)\cos\phi + \rho z\sin\phi\cos\phi$$
$$= \sqrt{x^2+y^2}\,(z^2+1)\frac{x}{\sqrt{x^2+y^2}} + \sqrt{x^2+y^2}\left(\frac{zxy}{x^2+y^2}\right)$$
$$= x(z^2+1) + \frac{xyz}{\sqrt{x^2+y^2}}.$$
$$A_y = \rho(z^2+1)\sin\phi - \rho z\cos^2\phi$$
$$= \sqrt{x^2+y^2}\,(z^2+1)\frac{y}{\sqrt{x^2+y^2}} - \frac{x^2 z}{\sqrt{x^2+y^2}}$$
$$= y(z^2+1) - \frac{x^2 z}{\sqrt{x^2+y^2}};$$
$$A_z = 0;$$

$$\therefore \bar{A} = \underline{\underline{\left[x(z^2+1) + \frac{xyz}{\sqrt{x^2+y^2}}\right]\bar{a}_x + \left[y(z^2+1) - \frac{x^2 z}{\sqrt{x^2+y^2}}\right]\bar{a}_y}}.$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 2x \\ r\cos\theta\cos\theta \\ -r\sin\phi \end{bmatrix}$$

$$B_x = 2x\sin\theta\cos\phi + r\cos^2\theta\cos^2\phi + r\sin^2\phi$$

$$= \frac{2x^2\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}} + \frac{\sqrt{x^2+y^2+z^2}}{x^2+y^2+z^2}\left(\frac{xz}{x^2+y^2}\right) + \sqrt{x^2+y^2+z^2}\left(\frac{y^2}{x^2+y^2}\right)$$

$$= \frac{2x^2}{\sqrt{x^2+y^2+z^2}} + \frac{xz}{(x^2+y^2)\sqrt{x^2+y^2+z^2}} + \frac{y^2\sqrt{x^2+y^2+z^2}}{x^2+y^2};$$

$$B_y = 2x\sin\theta\sin\phi + r\cos^2\theta\sin\phi\cos\phi - r\sin\phi\cos\phi$$

$$= \frac{2xy\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}} + \frac{\sqrt{x^2+y^2+z^2}(xyz^2)}{x^2+y^2+z^2} - \sqrt{x^2+y^2+z^2}\left(\frac{xy}{x^2+y^2}\right)$$

$$= \frac{2xy}{\sqrt{x^2+y^2+z^2}} + \frac{xyz^2}{x^2+y^2\sqrt{x^2+y^2+z^2}} - \frac{xy\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2}};$$

$$B_z = 2x\cos - r\sin\theta\cos\theta\cos\phi$$

$$= \frac{2xz}{\sqrt{x^2+y^2+z^2}} - \frac{\sqrt{x^2+y^2+z^2}\quad (xy)\sqrt{x^2+y^2}}{(x^2+y^2+z^2)\quad \sqrt{x^2+y^2}}$$

$$= \frac{2xz}{\sqrt{x^2+y^2+z^2}} - \frac{xz}{\sqrt{x^2+y^2+z^2}} = \frac{xz}{\sqrt{x^2+y^2+z^2}};$$

$$\therefore \bar{B} = \left[\frac{2x^2}{\sqrt{x^2+y^2+z^2}} + \frac{xz}{(x^2+y^2)\sqrt{x^2+y^2+z^2}} + \frac{y^2\sqrt{x^2+y^2+z^2}}{x^2+y^2}\right]\bar{a}_x +$$

$$\left[\frac{2xy}{\sqrt{x^2+y^2+z^2}} + \frac{xyz^2}{(x^2+y^2)\sqrt{x^2+y^2+z^2}} - \frac{xy\sqrt{x^2+y^2+z^2}}{x^2+y^2}\right]\bar{a}_y +$$

$$\left[\frac{xz}{\sqrt{x^2+y^2+z^2}}\right]\bar{a}_z$$

Prob 2.7 (a)

$$\begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z\sin\phi \\ -\rho\cos\phi \\ 2\rho z \end{bmatrix}$$

$$C_x = z\sin\phi\cos\phi + \rho\sin\phi\cos\phi = \frac{xyz}{x^2+y^2} + \frac{xy\sqrt{x^2+y^2}}{x^2+y^2};$$

$$C_y = z\sin^2\phi - \rho\cos^2\phi = \frac{y^2 z}{x^2+y^2} - \frac{x^2\sqrt{x^2+y^2}}{x^2+y^2};$$

$$C_z = 2\rho z = 2z\sqrt{x^2+y^2};$$

$$\therefore\ \bar{C} = \left(\frac{xyz}{x^2+y^2} + \frac{xy}{\sqrt{x^2+y^2}}\right)\bar{a}_x + \left(\frac{y^2 z}{x^2+y^2} - \frac{x^2}{\sqrt{x^2+y^2}}\right)\bar{a}_y + 2z\sqrt{x^2+y^2}\,\bar{a}_z$$

(b)

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\phi & 0 \end{bmatrix} \begin{bmatrix} \frac{\sin\theta}{r^2} \\ \frac{\cos\theta}{r^2} \\ 0 \end{bmatrix}$$

$$D_x = \frac{\sin^2\theta\cos\phi}{r^2} + \frac{\cos^2\theta\cos\phi}{r^2} = \frac{\cos\phi}{r^2} = \frac{x}{\sqrt{x^2+y^2}(x^2+y^2+z^2)};$$

$$D_y = \frac{\sin^2\theta\sin\phi}{r^2} + \frac{\cos^2\theta\sin\phi}{r^2} = \frac{\sin\phi}{r^2} = \frac{y}{\sqrt{x^2+y^2}(x^2+y^2+z^2)};$$

$$D_z = \frac{\sin\theta\cos\theta}{r^2} - \frac{\sin\theta\cos\theta}{r^2} = 0;$$

$$\therefore\ D = \frac{1}{\sqrt{x^2+y^2}(x^2+y^2+z^2)}(x\bar{a}_x + y\bar{a}_y)$$

Prob. 2.8 (a)

$$\bar{a}_x \bullet \bar{a}_\rho = (\cos\phi\,\bar{a}_\rho - \sin\phi\,\bar{a}_\phi)\bullet \bar{a}_\rho = \cos\phi$$

$$\bar{a}_x \bullet \bar{a}_\phi = (\cos\phi\,\bar{a}_\rho - \sin\phi\,\bar{a}_\phi)\bullet \bar{a}_\phi = -\sin\phi$$

$$\bar{a}_y \bullet \bar{a}_\rho = (\sin\phi\,\bar{a}_\rho + \cos\phi\,\bar{a}_\phi)\bullet \bar{a}_\rho = \sin\phi$$

$$\bar{a}_y \bullet \bar{a}_\phi = (\sin\phi\,\bar{a}_\rho + \sin\phi\,\bar{a}_\phi)\bullet \bar{a}_\phi = \cos\phi$$

(b)

Since $\bar{a}_\rho$, $\bar{a}_\phi$, and $\bar{a}_z$ are mutually orthogonal

$$\bar{a}_z \bullet \bar{a}_z = 1; \quad \bar{a}_z \bullet \bar{a}_\rho = 0; \quad \bar{a}_z \bullet \bar{a}_\phi = 0.$$

Also, $\bar{a}_x \bullet \bar{a}_z = 0; \quad \bar{a}_y \bullet \bar{a}_z = 0.$

$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \bar{a}_x \bullet \bar{a}_\rho & \bar{a}_x \bullet \bar{a}_\phi & \bar{a}_z \bullet \bar{a}_z \\ \bar{a}_y \bullet \bar{a}_\rho & \bar{a}_y \bullet \bar{a}_\phi & \bar{a}_y \bullet \bar{a}_z \\ \bar{a}_z \bullet \bar{a}_\rho & \bar{a}_z \bullet \bar{a}_\phi & \bar{a}_z \bullet \bar{a}_z \end{bmatrix}$$

(c)

In spherical system:

$$\bar{a}_x = \sin\theta\cos\phi\,\bar{a}_r + \cos\theta\cos\phi\,\bar{a}_\theta - \sin\phi\,\bar{a}_\phi.$$

$$\bar{a}_y = \sin\theta\sin\phi\,\bar{a}_r + \cos\theta\sin\phi\,\bar{a}_\theta - \cos\phi\,\bar{a}_\phi.$$

$$\bar{a}_z = \cos\theta\,\bar{a}_r - \sin\theta\,\bar{a}_\theta.$$

Hence,

$$\bar{a}_x \bullet \bar{a}_r = \sin\theta\cos\phi;$$

$$\bar{a}_x \bullet \bar{a}_\theta = \cos\theta\cos\phi;$$

$$\bar{a}_y \bullet \bar{a}_r = \sin\theta\sin\phi;$$

$$\bar{a}_y \bullet \bar{a}_\theta = \cos\theta\sin\phi;$$

$$a_z \bullet a_r = \cos\theta;$$

$$a_z \bullet a_\theta = -\sin\theta;$$

Prob 2.9 (a)

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2}.$$

$$\theta = \tan^{-1}\frac{\rho}{z}; \qquad \phi = \phi.$$

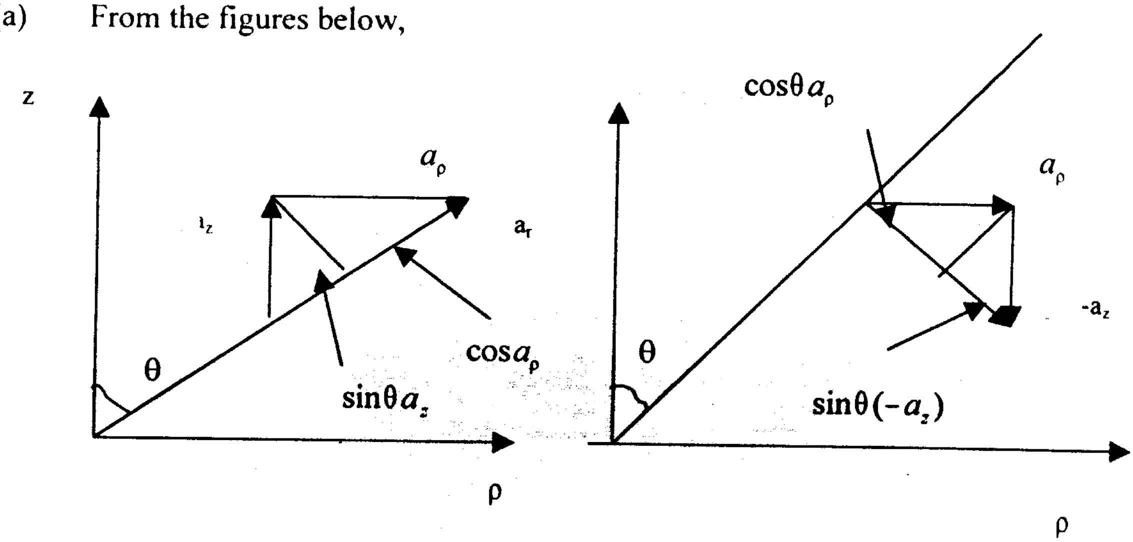
or

$$\rho = \sqrt{x^2 + y^2} = \sqrt{r^2 \sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi}.$$

$$= r\sin\theta;$$

$$z = r\cos\theta; \qquad \phi = \phi.$$

(a) From the figures below,



$\bar{a}_r = \sin\theta\,\bar{a}_z + \cos\theta\,\bar{a}_\rho$; $\quad \bar{a}_\theta = \cos\theta\,\bar{a}_\rho - \sin\theta\,\bar{a}_z$; $\quad \bar{a}_\phi = \bar{a}_\phi$

Hence,

$$\begin{bmatrix} \bar{a}_r \\ \bar{a}_\theta \\ \bar{a}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{a}_\rho \\ \bar{a}_\phi \\ \bar{a}_z \end{bmatrix}$$

From the figures below,

$\bar{a}_\rho = \cos\theta\,\bar{a}_\theta + \sin\theta\,\bar{a}_x$; $\quad \bar{a}_z = \cos\theta\,\bar{a}_x - \sin\theta\,\bar{a}_\theta$; $\quad \bar{a}_\phi = \bar{a}_\phi$.