Preface

This manual contains more or less complete solutions for every problem in the book. Should you find errors in any of the solutions, please bring them to my attention.

Over the years, I have tried to enrich my lectures by including historical information on the significant developments in thermodynamics, and biographical sketches of the people involved. The multivolume *Dictionary of Scientific Biography*, edited by Charles C. Gillispie and published by C. Scribners, New York, has been especially useful for obtaining biographical and, to some extent, historical information. [For example, the entry on Anders Celsius points out that he chose the zero of his temperature scale to be the boiling point of water, and 100 to be the freezing point. Also, the intense rivalry between the English and German scientific communities for credit for developing thermodynamics is discussed in the biographies of J.R. Mayer, J. P. Joule, R. Clausius (who introduced the word entropy) and others.] Other sources of biographical information include various encyclopedias, Asimov's Biographical Encyclopedia of Science and Technology by I. Asimov, published by Doubleday & Co., (N.Y., 1972), and, to a lesser extent, Nobel Prize Winners in Physics 1901-1951, by N.H. deV. Heathcote, published by H. Schuman, N.Y.

Historical information is usually best gotten from reading the original literature. Many of the important papers have been reproduced, with some commentary, in a series of books entitled "Benchmark Papers on Energy" distributed by Halsted Press, a division of John Wiley and Sons, N.Y. Of particular interest are:

Volume 1, *Energy: Historical Development of the Concept*, by R. Bruce Lindsay.

Volume 2, *Applications of Energy, 19th Century*, by R. Bruce Lindsay.

Volume 5, *The Second Law of Thermodynamics*, by J. Kestin and

Volume 6, *Irreversible Processes*, also by J. Kestin.

The first volume was published in 1975, the remainder in 1976.

Other useful sources of historical information are “The Early Development of the Concepts of Temperature and Heat: The Rise and Decline of the Caloric Theory” by D. Roller in Volume 1 of *Harvard Case Histories in Experimental Science* edited by J.B. Conant and published by Harvard University Press in 1957; articles in *Physics Today*, such as “A Sketch for a History of Early Thermodynamics” by E. Mendoza (February, 1961, p.32), “Carnot’s Contribution to Thermodynamics” by M.J. Klein (August, 1974, p. 23); articles in Scientific American; and various books on the history of science. Of special interest is the book *The Second Law* by P.W. Atkins published by Scientific American Books, W.H. Freeman and Company (New York, 1984) which contains a very extensive discussion of the entropy, the second law of thermodynamics, chaos and symmetry.

I also use several simple classroom demonstrations in my thermodynamics courses. For example, we have used a simple constant-volume ideal gas thermometer, and an instrumented vapor compression refrigeration cycle (heat pump or air conditioner) that can brought into the classroom. To demonstrate the pressure dependence of the melting point of ice, I do a simple regelation experiment using a cylinder of ice (produced by freezing water in a test tube), and a 0.005 inch diameter wire, both ends of which are tied to the same 500 gram weight. (The wire, when placed across the supported cylinder of ice, will cut through it in about 5 minutes, though by refreezing or regelation, the ice cylinder remains intact.—This experiment also provides an opportunity to discuss the movement of glaciers.) Scientific toys, such as “Love Meters” and drinking “Happy Birds”, available at novelty shops, have been used to illustrate how one can make practical use of the temperature dependence of the vapor pressure. I also use some professionally prepared teaching aids, such as the three-dimensional phase diagrams for carbon dioxide and water, that are available from laboratory equipment distributors.

Despite these diversions, the courses I teach are quite problem oriented. My objective has been to provide a clear exposition of the principles of thermodynamics, and then to reinforce these fundamentals by requiring the student to consider a great diversity of the applications. My approach to teaching thermodynamics is, perhaps, similar to the view of John Tyndall expressed in the quotation

> “It is thus that I should like to teach you all things; showing you the way to profitable exertion, but leaving the exertion to you—more anxious to bring out your manliness in the presence of difficulty than to make your way smooth by toning the difficulties down.”

Which appeared in The Forms of Water, published by D. Appleton (New York, 1872).

Finally, I usually conclude a course in thermodynamics with the following quotation by Albert Einstein:

> "A theory is more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended its area of applicability. Therefore, the deep impression classical thermodynamics made upon me. It is the only physical theory of universal content which, within the framework of the applicability of its basic concepts, I am convinced will never by overthrown."

1

1.1 (a) Thermostatic bath imposes its temperature T on the system.

(b) Container imposes constraint of constant volume. Thermal isolation implies that heat flow must be zero, while mechanical isolation (and constant volume) implies there is no work flow. Consequently there is no mechanism for adding or removing energy from the system. Thus, system volume and energy are constant.

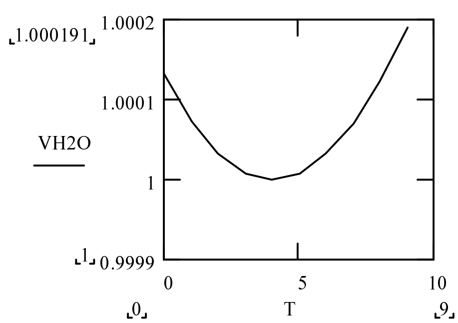
(c) Thermally isolated $\Rightarrow$ adiabatic
Frictionless piston $\Rightarrow$ pressure of system equals ambient pressure (or ambient pressure + wg/A if piston-cylinder in vertical position. Here w = weight of piston, A = its area and g is the force of gravity.)

(d) Thermostatic bath $\Rightarrow$ constant temperature T.
Frictionless piston $\Rightarrow$ constant pressure (see part c above).

(e) Since pressure difference induces a mass flow, pressure equilibrates rapidly. Temperature equilibration, which is a result of heat conduction, occurs much more slowly. Therefore, if valve between tanks is opened for only a short time and then shut, the pressure in the two tanks will be the same, but *not* the temperatures.

1.2 (a) Water is inappropriate as a thermometric fluid between 0°C and 10°C, since the volume is not a unique function of temperature in this range, i.e., two temperatures will correspond to the same specific volume,

$\hat{V}(T = 1°\text{C}) \sim V(T = 7°\text{C})$; $\hat{V}(T = 2°\text{C}) \sim V(T = 6°\text{C})$; etc.



$[T \text{ in } ^\circ\text{C and } \hat{V} \text{ in cc/g}]$

Consequently, while T uniquely determines, $\hat{V}$, $\hat{V}$ does not uniquely determine T.

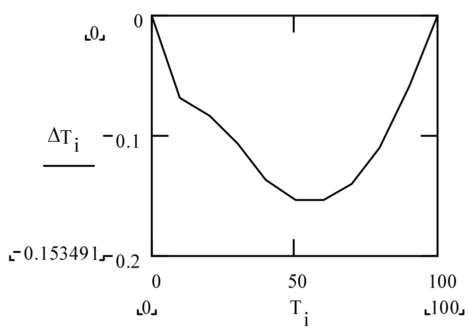
(b) Assuming that a mercury thermometer is calibrated at 0°C and 100°C, and that the specific volume of mercury varies linearly between these two temperatures yields

$$\hat{V}(T)=\hat{V}(0^{\circ}\mathrm{C})+\frac{\hat{V}(T=100^{\circ}\mathrm{C})-\hat{V}(T=0^{\circ}\mathrm{C})}{100^{\circ}\mathrm{C}-0^{\circ}\mathrm{C}}(T_{\mathrm{s}}-0^{\circ}\mathrm{C})$$
$$=0.0735560+0.000013421\ T_{\mathrm{s}} \qquad (*)$$

where T is the actual temperature, and T_s is the temperature read on the thermometer scale. At 10°C, $\hat{V}_{\exp}(T=10^{\circ}\mathrm{C})=0.0736893\ \mathrm{cc/g}$. However, the scale temperature for this specific volume is, from eqn. (*) above

$$T_{\mathrm{s}}=\frac{\hat{V}_{\exp}(T)-0.0735560}{1.3421\times10^{-5}}=\frac{0.0736893-0.0735560}{1.3421\times10^{-5}}=9.932^{\circ}\mathrm{C}$$

Thus, $T-T_s$ at 10°C $=-0.068^{\circ}\mathrm{C}$. Repeating calculation at other temperatures yields figure below.



The temperature error plotted here results from the nonlinear dependence of the volume of mercury on temperature. In a real thermometer there will also be an error associated with the imperfect bore of the capillary tube.

(c) When we use a fluid-filled thermometer to measure ΔT we really measure ΔL, where

$$\Delta L=\frac{\Delta V}{A}=\frac{M(\partial\hat{V}/\partial T)\Delta T}{A}$$

A small area A and a large mass of fluid M magnifies ΔL obtained for a given ΔT. Thus, we use a capillary tube (small A) and bulb (large M) to get an accurate thermometer, since $(\partial\hat{V}/\partial T)$ is so small.

2

2.1 (a) By an energy balance, the bicycle stops when final potential energy equals initial kinetic energy. Therefore

$$\frac{1}{2}mv_i^2 = mgh_f \quad \text{or} \quad h_f = \frac{v_i^2}{2g} = \frac{\left(20\frac{\text{km}}{\text{hr}} \times 1000\frac{\text{m}}{\text{km}} \times \frac{1\text{ hr}}{3600\text{ sec}}\right)^2}{2 \times 9.807\frac{\text{m}}{\text{sec}^2}}$$

or h=1.57 m.

(b) The energy balance now is

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgh_i \quad \text{or} \quad v_f^2 = v_i^2 + 2gh_i$$

$$v_f^2 = \left(20\ \frac{\text{km}}{\text{hr}}\right)^2 + 2 \times 9.807\frac{\text{m}}{\text{sec}^2} \times 70\text{ m} \times \left(\frac{\text{km}}{1000\text{ m}} \times \frac{3600\,\text{sec}}{\text{hr}}\right)^2$$

v_f = 134.88 km/hr. Anyone who has bicycled realizes that this number is much too high, which demonstrates the importance of air and wind resistance.

2.2 The velocity change due to the 55 m fall is

$$\left(\Delta v^2\right) = 2 \times 9.807\frac{\text{m}}{\text{sec}^2} \times 55\text{ m} \times \left(\frac{\text{km}}{1000\text{ m}} \times \frac{3600\,\text{sec}}{\text{hr}}\right)^2$$

v_f = 118.24 km/hr. Now this velocity component is in the vertical direction. The initial velocity of 8 km/hr was obviously in the horizontal direction. So the final velocity is

$$v = \sqrt{v_x^2 + v_y^2} = 118.51\frac{\text{km}}{\text{hr}}$$

2.3 (a) System: contents of the piston and cylinder
(closed isobaric = constant pressure)

M.B.: $M_2 - M_1 = \Delta M = 0 \Rightarrow M_2 = M_1 = M$

E.B.: $M_2\hat{U}_2 - M_1\hat{U}_1 = \Delta\cancel{M(\hat{H})}^{0} + Q + \cancel{W_s}^{0} - \int PdV$

$$M\left(\hat{U}_2 - \hat{U}_1\right) = Q - \int PdV = Q - P\int dV = Q - P(V_2 - V_1)$$

$$M\left(\hat{U}_2 - \hat{U}_1\right) = Q - PM\left(\hat{V}_2 - \hat{V}_1\right)$$

$$Q = M\left(\hat{U}_2 - \hat{U}_1\right) + M\left(P\hat{V}_2 - P\hat{V}_1\right) = M\left[\left(\hat{U}_2 + P\hat{V}_2\right) - \left(\hat{U}_1 + P\hat{V}_1\right)\right]$$

$$= M\left(\hat{H}_2 - \hat{H}_1\right)$$

$P = 1.013\text{ bar} \approx 0.1\text{ MPa}$

	$\hat{V}$	$\hat{U}$	$\hat{H}$	
$T = 100$	1.6958	2506.7	2676.2	
$T = 150$	1.9364	2582.8	2776.4	
Linear interpolation				
$T = 125°\text{C}$	1.8161	2544.8	2726.3	Initial state

Final state $\quad P = 0.1\text{ MPa}$, $\hat{V}_2 = 3.6322\text{ m}^3/\text{kg}$

$T = 500°\text{C}$	3.565		3488.1
$T = 600°\text{C}$	4.028		3704.7

Linear interpolation

$$\frac{3.6322-3.565}{4.028-3.565} = \frac{T_2-500}{600-500} \qquad T_2 = 514.5°\text{C}$$

$$\frac{514.5-500}{600-500} = \frac{\hat{H}_2-3488.1}{3704.7-3488.1} \qquad \hat{H}_2 = 3519.5$$

$$Q = 1\text{ kg}(3519.5-2726.3)\text{ kJ/kg} = 793.2\text{ kJ}$$

$$W = -\int PdV = -1\text{ bar}\times(V_2-V_1) = -1\text{ bar}\times(3.6322-1.8161)\ \text{m}^3/\text{kg}$$

$$= -1\text{ bar}\times 100{,}000\ \frac{\text{Pa}}{\text{bar}}\times\frac{1\text{ kg}}{\text{m}\cdot\text{s}^2\cdot\text{Pa}}\times\frac{1\text{ J}}{\text{m}^2\cdot\text{s}^2\cdot\text{kg}}\times 1.8161\ \text{m}^3/\text{kg}$$

$$= -181.6\ \text{kJ/kg}$$

(b) System is closed and constant volume

M.B.: $M_2 = M_1 = M$

E.B.: $M_2\hat{U}_2 - M_1\hat{U}_1 = \cancelto{0}{\Delta M(\hat{H})} + Q + \cancelto{0}{W_s} - \cancelto{0}{\int PdV}$

$$Q = M(\hat{U}_2 - \hat{U}_1)$$

Here final state is $P = 2\times 1.013\text{ bar} \sim 0.2\text{ MPa}$; $\hat{V}_2 = \hat{V}_1 = 1.8161\ \text{m}^3/\text{kg}$ (since piston-cylinder volume is fixed)

$P = 0.2\text{ MPa}$; $\hat{V}_2 = 1.8161$

$T(°\text{C})$	$\hat{V}$	$\hat{U}$
500	1.7814	3130.8
600	2.013	3301.4

$$\frac{1.8161-1.7814}{2.013-1.7814} = \frac{T-500}{600-500} = \frac{0.0347}{0.2316} = 0.1498 \sim 0.15$$

$$T = 515°\text{C}$$

$$\frac{\hat{U}_2-3130.8}{3301.4-3130.8} = 0.1498 \qquad \hat{U}_2 = 3156.4\ \text{kJ/kg}$$

$$Q = 1\text{ kg}\times(3156.4-2544.8)\ \text{kJ/kg} = 611.6\text{ kJ}$$

(c) Steam as an ideal gas—constant pressure

$$N = \frac{P\underline{V}}{RT} \Rightarrow \frac{P_1\underline{V}_1}{RT_1} = \frac{P_2\underline{V}_2}{RT_2}\ \text{ but }\ \underline{V}_2 = 2\underline{V}_1;\ P_1 = P_2$$

$$\frac{P_1\underline{V}_1}{T_1}=\frac{P_1 2\underline{V}_1}{T_2}\Rightarrow T_2=2\times T_1$$
$$T_1=273.15+125=398.15\text{ K}$$
$$T_2=2\times T_1=796.3\text{ K}=523.15°\text{C}$$

$$Q=N\Delta\underline{H}=\frac{1000\ \text{g/kg}}{18\ \text{g/mol}}\times 34.4\ \text{J/mol K}\times(796.3-398.15)\text{ K}\times\frac{1\text{ kJ}}{1000\text{ J}}$$
$$=760.9\text{ kJ}$$
$$W=-\int PdV=-P\Delta V=-P\left(\frac{NRT_2}{P}-\frac{NRT_1}{P_1}\right)=-NR(T_2-T_1)$$
$$=-\frac{1000}{18}\times 8.314\times 398.15=-183.9\text{ kJ}$$

(d) Ideal gas - constant volume

$$\frac{P_1\underline{V}_1}{RT_1}=\frac{P_2\underline{V}_2}{RT_2}\ \text{ here }\underline{V}_1=\underline{V}_2\text{ ; }P_2=2P_1$$

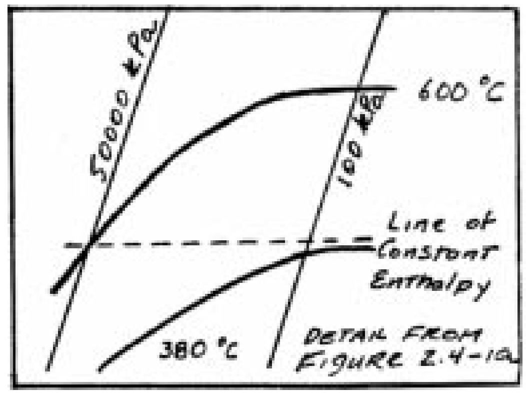
So again $\frac{P_1V_1}{T_1}=\frac{2P_1\cdot V_1}{T_2}$; $T_2=2T_1=796.3$ K .

$$Q=N\Delta\underline{U}=\frac{1000\ \text{g/kg}}{18\ \text{g/mol}}\times(34.4-8.314)\times(796.3-398.15)\times\frac{1}{1000}$$
$$C_V=C_P-R\text{ ; }Q=577.0\text{ kJ}$$

2.4

$$M_w\hat{U}_{w,\,f}-M_w\hat{U}_{w,\,i}=W_s=M_{weight}\times g\times 1\text{ m}$$
$$M_w=M_{weight}=1\text{ kg}$$
$$1\text{ kg}\times C_P(T_f-T_i)=1\text{ kg}\times 9.807\ \text{m/s}^2\times 1\text{ m}\times\frac{1\text{ J}}{\text{m}^2\text{kg/s}^2}=9.807\text{ J}$$
$$1\text{ kg}\times 4.184\ \text{J/g K}\times\frac{1000\text{ g}}{\text{kg}}\times\Delta T=9.807$$
$$\Delta T=\frac{9.807}{4.184\times 1000}\text{ K}=2.344\times 10^{-3}\text{ K}$$

2.5 From Illustration 2.3-3 we have that $\underline{H}(T_1,\ P_1)=\underline{H}(T_2,\ P_2)$ for a Joule-Thomson expansion. On the Mollier diagram for steam, Fig. 2.4-1a, the upstream and downstream conditions are connected by a horizontal line. Thus, graphically, we find that $T\sim 383$ K . (Alternatively, one could also use the Steam Tables of Appendix III.)



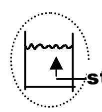
For the ideal gas, enthalpy is a function of temperature only. Thus, $\underline{H}(T_1,\ P_1) = \underline{H}(T_2,\ P_2)$ becomes $\underline{H}(T_1) = \underline{H}(T_2)$, which implies that $T_1 = T_2 = 600°\text{C}$.

2.6 System: Contents of Drum (open system)

mass balance: $M|_{t_2} - M|_{t_1} = \Delta M$

energy balance:

$M\hat{U}\Big|_{t_2} - M\hat{U}\Big|_{t_1} = \Delta M \hat{H}_{\text{in}} + Q + W_s - \int P dV$



but $Q = 0$ by problem statement, $W_s = 0$

and $\int PdV = P\Delta V$ is negligible. (Note $\hat{V}(T = 25°\text{C}) = 1.003 \times 10^{-3}\ \text{m}^3/\text{kg}$, $\hat{V}(T = 80°\text{C}) = 1.029 \times 10^{-3}\ \text{m}^3/\text{kg}$). Also from the Steam Tables

$$\hat{H}_{\text{in}} = \hat{H}(T = 300°\text{C},\ P = 3.0\ \text{bar} = 300\ \text{kPa}) = 3069.3\ \text{kJ/kg}$$

and recognizing that the internal energy of a liquid does not depend on pressure gives

$$\hat{U}\Big|_{t_1} = \hat{U}(T = 25°\text{C},\ 1.013\ \text{bar}) = \hat{U}(\text{sat.},\ T = 25°\text{C}) = 104.88\ \text{kJ/kg}$$

and

$$\hat{U}\Big|_{t_2} = \hat{U}(T = 80°\text{C},\ 1.013\ \text{bar}) = \hat{U}(\text{sat.},\ T = 80°\text{C}) = 334.86\ \text{kJ/kg}$$

Now using mass balance and energy balances with $M|_{t_1} = 100\ \text{kg}$ yields

$$M|_{t_2} \times 334.86\text{ kJ} - 100 \times 104.88\text{ kJ} = \left[M|_{t_2} - 100\right] \times 3069.3\text{ kJ}$$

Thus

$$M|_{t_2}(3069.3 - 334.86) = 100 \times (3069.3 - 104.88)$$

$M|_{t_2} = 108.41$ kg , and $\Delta M = M|_{t_2} - M|_{t_1} = 8.41$ kg of steam added.

2.7 (a) Consider a change from a given state 1 to a given state 2 in a closed system. Since initial and final states are fixed, U_1, U_2, V_1, V_2, P_1, P_2, etc. are all fixed. The energy balance for the closed system is

$$U_2 - U_1 = Q + W_s - \int PdV = Q + W$$

where $W = W_s - \int PdV =$ total work. Also, $Q = 0$ since the change of state is adiabatic. Thus, $U_2 - U_1 = W$.

Since U_1 and U_2 are fixed (that is, the end states are fixed regardless of the path), it follows that W is the same for all *adiabatic* paths. This is not in contradiction with Illustration 2.5-6, which established that the sum $Q + W$ is the same for all paths. If we consider only the subset of paths for which $Q = 0$, it follows, from that illustration that W must be path independent.

(b) Consider two different adiabatic paths between the given initial and final states, and let W^* and W^{**} be the work obtained along each of these paths, i.e.,

`Path 1:` $U_2 - U_1 = W^*$ `; Path 2:` $U_2 - U_1 = W^{**}$

Now suppose a cycle is constructed in which path 1 is followed from the initial to the final state, and path 2, in reverse, from the final state (state 2) back to state 1. The energy balance for this cycle is

$$\begin{array}{r} U_2 - U_1 = W^* \\ -(U_2 - U_1) = -W^{**} \\ \hline 0 = W^* - W^{**} \end{array}$$

Thus if the work along the two paths is different, i.e., $W^* \neq W^{**}$, we have created energy!

2.8 System = contents of tank at any time

mass balance: $M_2 - M_1 = \Delta M$

energy balance: $\left(M\hat{U}\right)_2 - \left(M\hat{U}\right)_1 = \Delta M\hat{H}_{\text{in}}$

(a) Tank is initially evacuated $\Rightarrow M_1 = 0$

Thus $M_2 = \Delta M$, and $\hat{U}_2 = \hat{H}_{\text{in}} = \hat{H}(5 \text{ bar, } 370°\text{C}) = 3209.6 \text{ kJ/kg}$ (by interpolation). Then $\hat{U}_2 = \hat{U}(P = 5 \text{ bar, } T = ?) = 3209.6 \text{ kJ/kg}$. By interpolation, using the Steam Tables (Appendix III) $T = 548°\text{C}$

$$\hat{V}(P = 5 \text{ bar, } T = 548°\text{C}) \cong 0.756 \text{ m}^3/\text{kg}$$

Therefore $M = V/\hat{V} = 1 \text{ m}^3/(0.756 \text{ m}^3/\text{kg}) = 1.3228 \text{ kg}$.

(b) Tank is initially filled with steam at 1 bar and 150°C $\Rightarrow \hat{V}_1 = \hat{V}(P = 1 \text{ bar, } T = 150°\text{C}) = 1.94 \text{ m}^3/\text{kg}$ and $\hat{U}_1 = 2583 \text{ kJ/kg}$, $M_1 = V/\hat{V} = 1/\hat{V} = 0.5155 \text{ kg}$. Thus, $M_2 = 0.5155 + \Delta M \text{ kg}$. Energy balance is $M_2\hat{U}_2 - 0.5155 \times 2583 = (M - 0.5155) \times 3209.6$. Solve by guessing value of T_2, using T_2 and $P_2 = 5 \text{ bar}$ to find $\hat{V}_2$ and $\hat{U}_2$ in the Steam Tables (Appendix III). See if energy balance and $M_2 = 1 \text{ m}^3/\hat{V}_2$ are satisfied. By trial and error: $T_2 \sim 425°\text{C}$ and $M_2 \cong 1.563 \text{ kg}$ of which 1.323 kg was present in tank intially. Thus, $\Delta M = M_2 - M_1 = 0.24 \text{ kg}$.

2.9 a) Use kinetic energy = $mv^2/2$ to find velocity.

$$1 \text{ kg} \times \frac{v^2}{2} \frac{\text{m}^2}{\text{sec}^2} = 1000 \text{ J} = 1000 \frac{\text{kg}}{\text{m}^2 \text{ sec}^2} \quad \text{so } v = 44.72 \text{ m/sec}$$

b) Heat supplied = specific heat capacity × temperature change, so

$$1000\text{g} \times \frac{1 \text{ mol}}{55.85\text{g}} \times 25.10 \frac{\text{J}}{\text{mol} \cdot \text{K}} \times \Delta T = 1000 \text{ J} \quad \text{so } \Delta T = 2.225 \text{ K.}$$

2.10 System = resistor

Energy balance: $dU/dt = \dot{W}_s + \dot{Q}$

where $\dot{W}_s = E \cdot I$, and since we are interested only in steady state $dU/dt = 0$. Thus

$$-\dot{Q} = \dot{W}_s = 1 \text{ amp} \times 10 \text{ volts} = 0.2 \times (T - 25°\text{C}) \text{ J/s}$$

and 1 watt $= 1 \text{ volt} \times 1 \text{ amp} = 1 \text{ J/s}$.

$$\Rightarrow T = \frac{10 \text{ watt} \times 1 \text{ J/s} \cdot \text{watt}}{0.2 \text{ J/s} \cdot \text{K}} + 25°\text{C} = 75.0°\text{C}$$

2.11 System = gas contained in piston and cylinder (closed)

Energy balance: $U|_{t_2} - U|_{t_1} = Q + \cancel{W_s}^{\,0} - \int PdV$

(a) $V = $ constant, $\int PdV = 0$, $Q = U|_{t_2} - U|_{t_1} = N\left(\underline{U}|_{t_2} - \underline{U}|_{t_1}\right) = NC_V(T_2 - T_1)$

From ideal gas law

$$N = \frac{PV}{RT} = \frac{114{,}367 \text{ Pa} \times 0.120 \text{ m}^3}{8.314 \text{ Pa} \cdot \text{m}^3/\text{mol} \cdot \text{K} \times 298 \text{ K}} = 5.539 \text{ mol} \text{ (see note following)}$$

Thus

$$T_2 = T_1 + \frac{Q}{NC_V} = 298\text{ K} + \frac{10{,}500\text{ J}}{5.539\text{ mol}\times 30.1\ \text{J/mol}\cdot\text{K}}$$
$$= 298 + 63.0 = 361.0\text{ K}$$

Since N and V are fixed, we have, from the ideal gas law, that

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \text{ or } P_2 = \frac{T_2}{T_1}P_1 = \frac{361.0}{298.0}\times 114.367\text{ kPa} = 1.385\times 10^5\text{ Pa}$$

(b) $P = \text{constant } = 1.14367\times 10^5\text{ Pa}$;
$C_P = C_V + R = 30.1 + 8.314 = 38.414\ \text{J/mol}\cdot\text{K}$
Energy balance $U|_{t_2} - U|_{t_1} = Q - P\Delta V$, since P = constant
$\Rightarrow NC_V(T_2 - T_1) = Q - P(V_2 - V_1) = Q - N(RT_2 - RT_1)$
$\Rightarrow Q = NC_P(T_2 - T_1)$

$$T_2 = T_1 + \frac{Q}{NC_P} = 298 + \frac{10{,}500}{5.539\times 38.414} = 347.35\text{ K}$$

and

$$\Delta V = \frac{NR\Delta T}{P} = \frac{5.539\text{ mol}\times 8.314\ \text{Pa}\cdot\text{m}^3/\text{mol}\cdot\text{K}\times 49.35\text{ K}}{114{,}367\text{ Pa}} = 0.01987\text{ m}^3$$
$$V = 0.12 + 0.0199 = 0.1399\text{ m}^3$$

Note: The initial pressure $P = P_{\text{atm}} + P_{\text{wt of piston}}$

$P_{\text{atm}} = 1.013\text{ bar} = 1.013\times 10^5\text{ kPa}$

$$P_{\text{wt piston}} = \frac{200\text{ kg}}{0.15\text{ m}^2}\times\frac{1\text{ Ns}^2}{\text{kg}\cdot\text{m}}\times 9.8\ \text{m/s}^2 = 13{,}067\ \text{N/m}^2 = 13{,}067\text{ Pa}$$
$$= 13.067\text{ kPa}$$

Thus, intial pressure $= 114.367\text{ kPa}$.

2.12 System = contents of storage tank (open system)
Mass balance: $M_2 - M_1 = \Delta M$
Energy balance: $\left(M\hat{U}\right)_2 - \left(M\hat{U}\right)_1 = (\Delta M)\hat{H}_{\text{in}}$ since $Q = W = 0$ and steam entering is of constant properties.
Initially system contains 0.02 m^3 of liquid water and $(40 - 0.02) = 39.98\text{ m}^3$ of steam.
Since vapor and liquid are in equilibrium at 50°C, from Steam Tables, $P = 12.349\text{ Pa}$. Also from Steam Tables $\hat{V}^{\text{L}} = 0.001012\ \text{m}^3/\text{kg}$, $\hat{V}^{\text{V}} = 12.03\ \text{m}^3/\text{kg}$, $\hat{H}^{\text{V}} = 2592.1\ \text{kJ/kg}$, $\hat{H}^{\text{L}} = 209.33\ \text{kJ/kg}$, $\hat{U}^{\text{L}} = 209.32\ \text{kJ/kg}$, and $\hat{U}^{\text{V}} = 2443.5\ \text{kJ/kg}$.

$$\left.\begin{aligned} M_1^{\mathrm{L}} &= \frac{0.02\ \mathrm{m}^3}{0.001012\ \ \mathrm{m}^3/\mathrm{kg}} = 19.76\ \mathrm{kg}; \\ M_1^{\mathrm{V}} &= \frac{39.98\ \mathrm{m}^3}{12.03\ \ \mathrm{m}^3/\mathrm{kg}} = 3.32\ \mathrm{kg}; \end{aligned}\right\} M_1 = M_1^{\mathrm{L}} + M_1^{\mathrm{V}} = 23.08\ \mathrm{kg}\,.$$

$$U_1 = 19.76 \times 209.32 + 3.32 \times 2443.5 = 12{,}248.6\ \mathrm{kJ}$$

Also

$$\hat{H}_{\mathrm{in}} = 0.90 \times 2676.1 + 0.10 \times 419.04 = 2450.4\ \ \mathrm{kJ/kg}$$

Possibilities for final state: 1) vapor-liquid mixture, 2) all vapor, and 3) all liquid. First possibility is most likely, so we will assume V-L mixture. Since $P = 1.013$ bar, T must be 100°C. Thus we can find properties of saturated vapor and saturated liquid in the Steam Tables: $\hat{V}^{\mathrm{L}} = 0.001044\ \ \mathrm{m}^3/\mathrm{kg}$, $\hat{V}^{\mathrm{V}} = 1.6729\ \ \mathrm{m}^3/\mathrm{kg}$, $\hat{U}^{\mathrm{L}} = 418.94\ \mathrm{kJ/kg}$, $\hat{H}^{\mathrm{V}} = 2676.1\ \ \mathrm{kJ/kg}$, and $\hat{U}^{\mathrm{V}} = 2506.5\ \mathrm{kJ/kg}$.

$\hat{V}_2 = x(1.6729) + (1-x)0.001044 = 0.001044 + 1.6719x\ \ \mathrm{m}^3/\mathrm{kg}$, where
$x =$ quality
$\hat{U}_2 = x(2506.5) + (1-x)418.94 = 418.94 + 2087.56x\ \ \mathrm{kJ/kg}$
Substituting into energy balance

$$M_2(418.94 + 2087.56x) - 12{,}248.6 = (M_2 - 23.08)\cdot 2450.4$$

where

$$M_2 = \frac{V}{\hat{V}_2} = \frac{40\ \mathrm{m}^3}{0.001044 + 1.6719x}$$

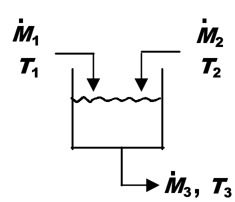
Solving by trial and error yields $x = 0.5154$ (quality), $M_2 = 46.36$ kg, and $\Delta M = 23.28$ kg. Also the final state is a vapor-liquid mixture, as assumed.

2.13 System = tank and its contents (open system)

(a) Steady state mass balance

$$\frac{dM}{dt} = 0 = \dot{M}_1 + \dot{M}_2 + \dot{M}_3$$

$$\Rightarrow \dot{M}_3 = -\left(\dot{M}_1 + \dot{M}_2\right) = -10\ \ \mathrm{kg/min}$$



Steady state energy balance

$$\frac{dU}{dt} = 0 = \dot{M}_1\hat{H}_1 + \dot{M}_2\hat{H}_2 + \dot{M}_3\hat{H}_3$$

$\hat{H}_3 = \hat{H}$ exit stream $= \hat{H}$ at temperature of tank contents

Also $T_3 = T =$ temperature of tank contents

Now $\hat{H} = \hat{H}_0 + C_P(T - T_0)$, assuming C_P is not a function of temperature

$$0 = 5\{\hat{H}_0 + C_P(T_1 - T_0)\} + 5\{\hat{H}_0 + C_P(T_2 - T_0)\} - 10\{\hat{H}_0 + C_P(T - T_0)\}$$

$$\Rightarrow T = \frac{5T_1 + 5T_2}{10} = \frac{1}{2}(T_1 + T_2) = 65°\,\text{C}$$

(b) mass balance: $\dfrac{dM}{dt} = 0 = \dot{M}_1 + \dot{M}_2 + \dot{M}_3$ (no useful information here)

energy balance: $\dfrac{dU}{dt} = \dot{M}_1\hat{H}_1 + \dot{M}_2\hat{H}_2 + \dot{M}_3\hat{H}_3$

but $\dfrac{dU}{dt} = \dfrac{d}{dt}\left(M\hat{U}\right) = M\dfrac{d\hat{U}}{dt} = MC_V\dfrac{dT}{dt} \sim MC_P\dfrac{dT}{dt}$ since $C_P \approx C_V$ for liquids. Thus $MC_P\dfrac{dT_3}{dt} = 5C_PT_1 + 5C_PT_2 - 10C_PT_3$ and $M = 50$ kg.

$$10\frac{dT_3}{dt} + 2T_3 = (80 + 50) = 130 \Rightarrow T_3 = Ae^{-t/5} + C \quad (t = \text{minutes})$$

At $t \to \infty$, $T_3 = C = 65°\,\text{C}$

At $t = 0$, $T_3 = A + C = 25°\,\text{C} \Rightarrow A = -40°\,\text{C}$

So finally $T_3 = 65°\,\text{C} - 40°\,\text{C}e^{-t/5}$ (t = minutes)

E.B: $M^F\hat{U}^F - M^i\hat{U}^i = \Delta M\hat{H}_{in}$

$$\left(M_L^F\hat{U}_L^F + M_V^F\hat{U}_V^F\right) - \left(M_L^i\hat{U}_L^i + M_V^i\hat{U}_V^i\right) = \left[M_L^F + M_V^F\right]\left[0.1\hat{H}_{L,\,in} + 0.9\hat{H}_{V,\,in}\right]$$

Also known is that $V = 60\ \text{m}^3 = M_L^F\hat{V}_L^F + M_V^F\hat{V}_V^F$. $\Rightarrow$2 equations and 2 unknowns

$$\frac{V - M_V^F\hat{V}_V^F}{\hat{V}_L^F} = M_L^F$$

$$\left(\frac{V - M_V^F\hat{V}_V^F}{\hat{V}_L^F}\hat{U}_L^F + M_V^F\hat{U}_V^F\right) - \left(M_L^i\hat{U}_L^i + M_V^i\hat{U}_V^i\right)$$
$$= \left[\frac{V - M_V^F\hat{V}_V^F}{\hat{V}_L^F} + M_V^F\right]\left[0.1\hat{H}_{L,\,in} + 0.9\hat{H}_{V,\,in}\right]$$

2.14 Thermodynamic properties of steam from the Steam Tables

Initial conditions:

Specific volume of liquid and of vapor:

$\hat{V}_L^i = 1.061 \times 10^{-3}\,\dfrac{\text{m}^3}{\text{kg}}$; $\hat{V}_V^i = 0.8857\,\dfrac{\text{m}^3}{\text{kg}}$

Specific internal energy of liquid and of vapor

$\hat{U}_L^i = 313.9\,\dfrac{\text{kJ}}{\text{kg}}$; $\hat{U}_V^i = 2475.9\,\dfrac{\text{kJ}}{\text{kg}}$

M.B: $M^{\rm f} - M^{\rm i} = \Delta M_{\rm i}$

$$M^{\rm i} = M_{\rm L}^{\rm i} + M_{\rm V}^{\rm i}\ ;\ M_{\rm L}^{\rm i} = \frac{200 \text{ liters}}{\hat{V}_{\rm L}^{\rm i}} = 194.932 \text{ kg};$$

$$M_{\rm V}^{\rm i} = \frac{60 \text{ m}^3 - 200 \text{ liters}}{\hat{V}_{\rm V}^{\rm i}} = 14.476 \text{ kg and so } M^{\rm i} = 209.408 \text{ kg}$$

E.B.

$$M^{\rm f}\hat{U}^{\rm f} - M^{\rm i}\hat{U}^{\rm i} = \Delta M \hat{H}_{\rm in}$$

$$\left(M_{\rm L}^{\rm f}\hat{U}_{\rm L}^{\rm f} + M_{\rm V}^{\rm f}\hat{U}_{\rm V}^{\rm f}\right) - \left(M_{\rm L}^{\rm i}\hat{U}_{\rm L}^{\rm i} + M_{\rm V}^{\rm i}\hat{U}_{\rm V}^{\rm i}\right) = \left[M_{\rm L}^{\rm f} + M_{\rm V}^{\rm f}\right]\left[0.1\hat{H}_{\rm L,\,in} + 0.9\hat{H}_{\rm V,\,in}\right]$$

Total internal energy of steam + water in the tank

$194.932\times313.0 + 14.476\times2475.9 = 9.686\times10^4$ kJ

Properties of steam entering, 90% quality

Specific volume = $\hat{V}_{\rm in} = 0.1\times1.061\times10^{-3} + 0.9\times0.8857 = 0.797 \text{ m}^3/\text{kg}$

Specific enthalpy = $\hat{H}_{\rm in} = 0.1\times504.70 + 0.9\times2706.7 = 2.486\times10^3$ kJ/kg

Also have that $V = 60 \text{ m}^3 = M_{\rm L}^{\rm f}\hat{V}_{\rm L}^{\rm f} + M_{\rm V}^{\rm f}\hat{V}_{\rm V}^{\rm f}$.

This gives two equations, and two unknowns, $M_{\rm L}^{\rm f}$ and $M_{\rm V}^{\rm f}$.

The solution (using MATHCAD) is $M_{\rm L}^{\rm f} = 215.306$ kg and $M_{\rm V}^{\rm f} = 67.485$ kg.

Therefore, the fraction of the tank contents that is liquid by weight is 0.761.

2.15 System = contents of both chambers (closed, adiabatic system of constant volume. Also $W_{\rm s} = 0$).

Energy balance: $U(t_2) - U(t_1) = 0$ or $U(t_2) = U(t_1)$

(a) For the ideal gas u is a function of temperature only. Thus, $U(t_2) = U(t_1) \Rightarrow T(t_2) = T(t_1) = 500$ K. From ideal gas law

$$\begin{array}{lll} P_1V_1 = N_1RT_1 & \text{but} & N_1 = N_2 \text{ since system is closed} \\ P_2V_2 = N_2RT_2 & & T_1 = T_2 \text{ see above} \\ & \text{and} & V_2 = 2V_1 \text{ see problem statement.} \end{array}$$

$$\Rightarrow P_2 = \frac{1}{2}P_1 = 5 \text{ bar} = 0.5 \text{ MPa} \Rightarrow T_2 = 500 \text{ K}, P_2 = 0.5 \text{ MPa}$$

(b) For steam the analysis above leads to $U(t_2) = U(t_1)$ or, since the system is closed $\hat{U}(t_2) = \hat{U}(t_1)$, $\hat{V}(t_2) = 2\hat{V}(t_1)$. From the Steam Tables, Appendix III,

$$\begin{aligned}\hat{U}(t_1) &= \hat{U}(T = 500 \text{ K},\ P = 1 \text{ MPa}) = \hat{U}(T = 226.85^\circ\text{C},\ P = 1 \text{ MPa}) \\ &\cong 2669.4 \text{ kJ/kg} \\ \hat{V}(t_1) &= \hat{V}(T = 226.85^\circ\text{C},\ P = 1 \text{ MPa}) \cong 0.2204 \text{ m}^3/\text{kg}\end{aligned}$$

Therefore $\hat{U}(t_2) = \hat{U}(t_1) = 2669.4$ kg/kg and $\hat{V}(t_2) = 2\hat{V}(t_1) = 0.4408 \text{ m}^3/\text{kg}$. By, essentially, trial and error, find that $T \sim 216.3^\circ\text{C}$, $P \sim 0.5$ MPa.

(c) Here $U(t_2)=U(t_1)$, as before, except that $U(t_1)=U^{\mathrm{I}}(t_1)+U^{\mathrm{II}}(t_1)$, where superscript denotes chamber.

Also, $M(t)=M^{\mathrm{I}}(t_1)+M^{\mathrm{II}}(t_1)$ {mass balance} and

$$\hat{V}(t_2)=2V_1/M(t_2)=2V_1/\left[M^{\mathrm{I}}(t_1)+M^{\mathrm{II}}(t_1)\right]$$

For the ideal gas, using mass balance, we have

$$\frac{P_2(2V_1)}{T_2}=\frac{P_1^{\mathrm{I}}V_1}{T_1^{\mathrm{I}}}+\frac{P_1^{\mathrm{II}}V_1}{T_1^{\mathrm{II}}}\Rightarrow\frac{2P_2}{T_2}=\frac{P_1^{\mathrm{I}}}{T_1^{\mathrm{I}}}+\frac{P_1^{\mathrm{II}}}{T_1^{\mathrm{II}}} \qquad (1)$$

Energy balance: $N_2\underline{U}_2=N_1^{\mathrm{I}}\underline{U}_1^{\mathrm{I}}+N_1^{\mathrm{II}}\underline{U}_1^{\mathrm{II}}$

Substitute $\underline{U}=\underline{U}_0+NC_{\mathrm{V}}(T-T_0)$, and cancel terms, use $N=PV/RT$ and get

$$2P_2=P_1^{\mathrm{I}}+P_1^{\mathrm{II}} \qquad (2)$$

Using Eqns. (1) and (2) get $P_2=7.5\times10^5\ \mathrm{Pa}=0.75\ \mathrm{MPa}$ and $T_2=529.4\ \mathrm{K}\ \ (256.25°\mathrm{C})$.

(d) For steam, solution is similar to (b). Use Steam Table to get M_1^{I} and M_1^{II} in terms of V.

Chamber 1: $\hat{U}_1^{\mathrm{I}}=2669.4\ \mathrm{kJ/kg}$; $\hat{V}_1^{\mathrm{I}}=0.2204\ \mathrm{m^3/kg}$;

$M^{\mathrm{I}}=V_1/\hat{V}_1^{\mathrm{I}}=4.537V_1$

Chamber 2: $\hat{U}_1^{\mathrm{II}}=\hat{U}(T=600\ \mathrm{K},\ P=0.5\ \mathrm{MPa})=2845.9\ \mathrm{kJ/kg}$;

$\hat{V}_1^{\mathrm{II}}=0.5483\ \mathrm{m^3/kg}$; $M^{\mathrm{II}}=1.824V_1=V_1/\hat{V}_1^{\mathrm{II}}$

Thus, $\hat{V}_2=\dfrac{2V_1}{M^{\mathrm{I}}+M^{\mathrm{II}}}=\dfrac{2V_1}{4.537V_1+1.824V_1}=0.3144\ \mathrm{m^3/kg}$;

$\hat{U}_2=\left(M^{\mathrm{I}}\hat{U}_1^{\mathrm{I}}+M^{\mathrm{II}}\hat{U}_1^{\mathrm{II}}\right)/\left(M_1^{\mathrm{I}}+M_1^{\mathrm{II}}\right)=2720.0\ \mathrm{kJ/kg}$

By trial and error: $T_2\sim302°\mathrm{C}\ (575\ \mathrm{K})$ and $P\sim0.76\ \mathrm{MPa}$.

2.16 System: contents of the turbine (open, steady state)

(a) adiabatic

mass balance: $\dfrac{dM}{dt}=0=\dot{M}_1+\dot{M}_2\Rightarrow\dot{M}_2=-\dot{M}_1$

energy balance: $\dfrac{dU}{dt}=0=\dot{M}_1\hat{H}_1+\dot{M}_2\hat{H}_2+\cancelto{0}{\dot{Q}}+\dot{W}_{\mathrm{s}}-P\cancelto{0}{\dfrac{dV}{dt}}$

$\Rightarrow\dot{W}_{\mathrm{s}}=-\dot{M}_1\left(\hat{H}_1-\hat{H}_2\right)=-\dot{M}_1(3450.9-2865.6)\ \mathrm{kJ/kg}$

$=-\dot{M}_1\left(5.853\times10^5\right)\ \mathrm{J/kg}$

But $\dot{W}_{\mathrm{s}}=-7.5\times10^5\ \mathrm{watt}=-7.5\times10^5\ \mathrm{J/s}$

$$\dot{M}_1=\frac{-7.50\times10^5\ \mathrm{J/s}}{-5.853\times10^5\ \mathrm{J/kg}}=1.281\ \mathrm{kg/s}=4.613\times10^3\ \mathrm{kg/h}$$

(b) Energy balance is

$$\frac{dU}{dt}=0=\dot{M}_1\hat{H}_1+\dot{M}_2\hat{H}_2+\dot{Q}+\dot{W}_{\mathrm{s}}-P\cancelto{0}{\frac{dV}{dt}}$$

$$\text{where } \dot{Q} = \dot{M}_1(-60 \text{ kJ/kg})$$

$$\hat{H}_2 = \hat{H}(150°\text{C}, 0.3 \text{ MPa}) = 2761.0 \text{ kJ/kg}$$

Thus

$$-\dot{W}_s = 1.281 \text{ kg/s}(3450.9 - 2761.0 - 60) \text{ kJ/kg} = 807 \text{ kJ/s}$$

$$= 8.07 \times 10^5 \text{ watt} = 807 \text{ kW}$$

2.17 System: 1 kg of water (closed system).

Work of vaporization $= \int PdV = P\int dV = P\Delta V$ since P is constant at 1.013 bar.

Also, from Steam Tables

$\hat{V}^{\text{L}} = 0.001044 \text{ m}^3/\text{kg}$; $\hat{V}^{\text{V}} = 1.6729 \text{ m}^3/\text{kg}$; $\Delta\hat{V} = 1.6719 \text{ m}^3/\text{kg}$

Energy balance for closed system (1 kg):

$$\hat{U}_2 - \hat{U}_1 = Q - \int PdV = Q - 1.013 \times 10^5 \text{ Pa} \times 1.6719 \text{ m}^3/\text{kg}$$

$$= Q - 1.6945 \times 10^5 \text{ J/kg}$$

$$\hat{U}_2 = 2506.5 \text{ kJ/kg} = 2.5065 \times 10^6 \text{ J/kg}$$

$$\hat{U}_1 = 418.94 \text{ kJ/kg} = 4.1894 \times 10^5 \text{ J/kg}$$

Thus

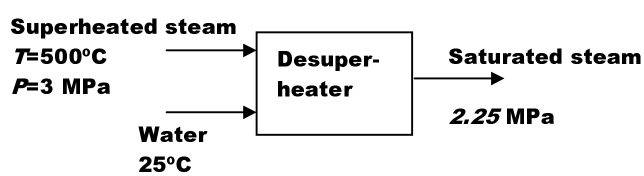
$$Q = \hat{U}_2 - \hat{U}_1 + W = 2.5065 \times 10^6 - 4.1894 \times 10^5 + 1.6945 \times 10^5$$

$$= 2.2570 \times 10^6 \text{ J/kg}$$

$$W = -\int PdV = 1.6945 \times 10^5 \text{ J/kg} .$$

So heat needed to vaporize liquid $= 2.2570 \times 10^6$ J/kg of which 0.16945×10^6 is recovered as work against the atmosphere. The remainder, 2.088×10^6 kJ/kg, goes to increase internal energy.

2.18 System = Contents of desuperheater (open, steady state)



$$\dot{M}_1 = 500 \text{ kg/hr}; \quad \hat{H}_1 = 3456.5 \text{ kJ/kg}$$

$$\dot{M}_2 = ?; \quad \hat{H}_2 = \hat{H}(\text{sat'd liq.}, T = 25°\text{C}) = 104.89 \text{ kJ/kg}$$

Mass B: $0 = \dot{M}_1 + \dot{M}_2 + \dot{M}_3$

Energy B: $0 = \dot{M}_1\hat{H}_1 + \dot{M}_2\hat{H}_2 + \dot{M}_3\hat{H}_3 + \cancel{\dot{Q}}^{0} + \cancel{\dot{W}_s}^{0} - P\cancel{\frac{dV}{dt}}^{0}$

$\dot{M}_3 = -(500 + \dot{M}_2)$ kg/hr ; $\hat{H}_3 = \hat{H}(\text{sat'd steam}, P = 2.25 \text{ MPa}) = 2801.7$ kJ/kg

Thus,

$0 = 500 \times 3456.5 + \dot{M}_2 = 104.89 - (500 + \dot{M}_2) \times 2801.7$

$\Rightarrow \dot{M}_2 = 121.4$ kg/hr

2.19 The process here is identical to that of Illustration 2.5-3, so that we can use the equation

$$T_2 = \frac{P_2}{P_1/T_1 + C_V/C_P[(P_2 - P_1)/T_{in}]}$$

developed in the illustration. Here, $P_2 = 2.0$ MPa, $T_{in} = 120°\text{C} = 393.15$ K, $C_P = 29.3$ J/mol K, $C_V = C_P - R = 20.99$ J/mol K.

Cylinder 1: $P_1 = 0$, $T_2 = \dfrac{C_P}{C_V} T_{in} = 548.8 \text{ K} = 275.65°\text{C}$

Cylinder 2: $P_1 = 0.1$ MPa, $T_1 = 20°\text{C} = 293.15$ K

$$T_2 = \frac{2.0}{0.1/293.15 + 20.99/29.3[(2.0 - 0.1)/393.15]} = 525.87 \text{ K} = 252.7°\text{C}$$

Cylinder 3: $P_1 = 1$ MPa, $T_1 = 20°\text{C} = 293.15$ K; $\Rightarrow T_2 = 382.16 \text{ K} = 109.01°\text{C}$

2.20 System: Gas contained in the cylinder (closed system)

(a) $P = 0.1013 \text{ MPa} + \dfrac{M_{\text{piston}} g}{A} = 1.0133 \times 10^5 + \dfrac{4000 \text{ kg}}{2.5 \text{ m}^2} \times \dfrac{9.8 \text{ m/s}^2}{1 \text{ kgm/Ns}^2}$

$= 1.1701 \times 10^5 \text{ Pa} = 0.117 \text{ MPa}$

$$N = \begin{matrix}\text{moles of} \\ \text{gas initially} \\ \text{in system}\end{matrix} = \frac{PV}{RT} = \frac{1.1701 \times 10^5 \text{ Pa} \times 25 \text{ m}^3}{8.314 \text{ Pa} \cdot \text{m}^3/\text{mol K} \times 293.15 \text{ K}}$$

$= 1.200 \times 10^3 \text{ mol} = 1.200 \text{ kmol}$

(b) Energy balance: $U_2 - U_1 = Q - \int P dV = Q - P\Delta V$ since P is constant.

$\Delta V = 3 \text{ m} \times 2.5 \text{ m}^2 = 7.5 \text{ m}^3$; $P\Delta V = 1.1701 \times 10^5 \text{ Pa} \times 7.5 \text{ m}^3 = 8.7758 \times 10^5$ J

Final temperature:

$$T_2 = \frac{PV_2}{NR} = \frac{1.1701 \times 10^5 \text{ Pa} \times (25 + 7.5)\text{m}^3}{1.2 \times 10^3 \text{ mol} \times 8.314 \text{ Pam}^3/\text{mol K}} = 381.2 \text{ K} = 108.05°\text{C}$$

$U_2 - U_1 = N(\underline{U}_2 - \underline{U}_1) = NC_V(T_2 - T_1)$

$= 1.2 \times 10^3 \text{ mol} \times (30 - 8.314) \text{ J/mol K} \times (381.2 - 293.15)\text{K}$

$= 2.291 \times 10^6$ J

(c)

$$Q = \underset{\substack{\Delta T\text{ of} \\ \text{gas}}}{\Delta U} + \underset{\text{work}}{P\Delta V} = 2.291\times10^{6} + \underset{\substack{\text{work}=27.7\%\text{ of energy absorbed} \\ \Delta T=72.3\%}}{8.7758\times10^{5}}$$

$$= 3.169\times10^{6}\text{ J} = 3.169\text{ MJ}$$

(d) System: Gas contained within Piston + Cylinder (open system).
[Note: Students tend to assume $dT/dt = 0$. This is true, but not obvious!]

mass balance: $\dfrac{dN}{dt} = \dot{N}$

energy balance: $\dfrac{d}{dt}(N\underline{U}) = \dot{N}\underline{H}_{\text{out}} + \cancel{\dot{Q}}^{\,0} - P\dfrac{dV}{dt}$

Here (1) P is constant, (2) Ideal Gas Law $V = NRT/P$, (3) T and P of Gas Leaving Cylinder $= T$ and P of gas in the system. Thus,

$$N\frac{d\underline{U}}{dt} + \underline{U}\frac{dN}{dt} = \underline{H}\frac{dN}{dt} - P\frac{d}{dt}\left(\frac{NRT}{P}\right)$$

$$\Rightarrow (\underline{H}-\underline{U})\frac{dN}{dt} = NC_{\text{V}}\frac{dT}{dt} + R\frac{d}{dt}(NT)$$

$$RT\frac{dN}{dt} = NC_{\text{V}}\frac{dT}{dt} + NR\frac{dT}{dt} + RT\frac{dN}{dt} \Rightarrow N(C_{\text{V}}+R)\frac{dT}{dt} = 0$$

$$\Rightarrow \frac{dT}{dt} = 0 \text{ Q.E.D.}$$

Thus $T_3 = T_2 = 381.2$ K

Now going back to

$$N\frac{d\underline{U}}{dt} + \underline{U}\frac{dN}{dt} = \underline{H}\frac{dN}{dt} - P\frac{dV}{dt} \text{ and using } \frac{dT}{dt} = 0 = \frac{d\underline{U}}{dt}$$

$$\Rightarrow (\underline{H}-\underline{U})\frac{dN}{dt} = RT\frac{dN}{dt} = P\frac{dV}{dt} \text{ or } \frac{dN}{dt} = \frac{P}{RT}\frac{dV}{dt} \qquad (**)$$

Since P and T are constants

$$\frac{N_3}{N_2} = \frac{V_3}{V_2} = \frac{25\text{ m}^3}{25+7.5\text{ m}^3} = 0.7692$$

Thus $N_3 = 0.7692\times1200\text{ mol} = 923\text{ mol}$;
$\Delta N = -277\text{ mol} = -0.277\text{ kmol}$

2.21 (a) System: Gas contained within piston-cylinder (closed system) [neglecting the potential energy change of gas]
energy balance:

$$\frac{d(N\underline{U})}{dt} = N\frac{d\underline{U}}{dt} = \dot{Q} - P\frac{dV}{dt};\; NC_{\text{V}}\frac{dT}{dt} = \dot{Q} - PA\frac{dh}{dt}$$

But $T = \dfrac{PV}{NR} \Rightarrow \dfrac{dT}{dt} = \dfrac{P}{NR}\left(\dfrac{dV}{dt}\right) = \dfrac{PA}{NR}\dfrac{dh}{dt}$.

Thus

$$Q = \frac{AC_V P}{R}\frac{dh}{dt} + AP\frac{dh}{dt} = PA\left(\frac{C_V}{R}+1\right)\frac{dh}{dt} = \frac{PAC_P}{R}\frac{dh}{dt}$$
$$= \frac{30 \text{ J/mol K}}{8.314 \text{ J/mol K}} \times 1.1701\times10^5 \text{ Pa} \times 2.5 \text{ m}^2 \times 0.2 \text{ m/s}$$
$$= 2.111\times10^5 \text{ J/s}$$

(b) System: Gas contained within piston and cylinder (open system). Start from result of Part (d), Problem **2.20** (see eqn. (**) in that illustration)

$$\frac{dN}{dt} = \frac{P}{RT}\frac{dV}{dt} = \frac{PA}{RT}\frac{dh}{dt} \text{ with } P \text{ and } T \text{ constant}$$

(See solution to Problem **2.20**)

$$\frac{dN}{dt} = \frac{1.1701\times10^5 \text{ Pa}\times 2.5 \text{ m}^2}{8.314 \text{ J/mol K}\times 381.2 \text{ K}} \times(-0.2 \text{ m/s}) = -18.46 \text{ mol/s}$$
$$= -0.01846 \text{ kmol/s}$$

[check: $-18.46 \text{ mol/sec}\times 15 \text{ sec} = -276.9 \text{ mol}$ compare with part d of Problem **2.20**]

2.22 System: gas contained in the cylinder (open system)
Important observation . . . gas leaving the system (That is, entering the exit valve of the cylinder) has same properties as gas in the cylinder.

$$\left.\begin{array}{ll}\text{mass balance} & \dfrac{dN}{dt} = \dot{N} \\ \text{energy balance} & \dfrac{d(N\underline{U})}{dt} = \dot{N}\underline{H}\end{array}\right\} \begin{array}{l}\text{Note that these are}\\ \text{Eqns. (d) and (e) of}\\ \text{Illustration 2.5-5}\end{array}$$

Proceeding as in that illustration we get Eqn. (f)

$$\left(\frac{T(t)}{T(0)}\right)^{C_P/R} = \left(\frac{P(t)}{P(0)}\right) \text{ or } \frac{T(t)}{P(t)^{R/C_P}} = \frac{320}{10^{(8.314/30)}} = 169.05 \text{ K} \qquad (1)$$

where we have used a slightly different notation. Now using the mass balance we get

$$\frac{dN}{dt} = \frac{d}{dt}\left(\frac{PV}{RT}\right) = \frac{V}{R}\frac{d(P/T)}{dt} = \dot{N}$$

or

$$\frac{d(P/T)}{dt} = \frac{\dot{N}R}{V} = \frac{-(4.5/28) \text{ mol/s}\times 8.314 \text{ Pa}\cdot\text{m}^3/\text{mol K}}{0.15 \text{ m}^3} = -8.908 \text{ Pa/K}\cdot\text{s}$$

and

$$\left.\frac{P}{T}\right|_t = \left.\frac{P}{T}\right|_{t=0} - 8.908 \times 10^{-5}\text{t} \quad \text{bar/K for } P \text{ in bar and } t \text{ in secs.} \qquad (2)$$

Using $t = 5$ minutes $= 300$ secs in Eqn. (2) and simultaneously solving Eqns. (1) and (2) yields

$$T(5\text{ min}) = 152.57\text{ K}, \; P(5\text{ min}) = 0.6907\text{ bar}$$

Computation of rates of change from mass balance

$$\frac{d}{dt}\left(\frac{P}{T}\right) = \frac{1}{T}\left(\frac{dP}{dt} - \frac{Pd\ln T}{dt}\right) = \frac{\dot{N}R}{V} \text{ or } \frac{d\ln P}{dt} - \frac{d\ln T}{dt} = \frac{\dot{N}RT}{PV} \qquad (3)$$

From energy balance (using 2 eqns. above and eqn. (f) in Illustration (2.5-5))

$$\frac{C_V}{R}\frac{d\ln T}{dt} = \frac{d\ln(P/T)}{dt} \text{ or } \frac{C_P}{R}\frac{d\ln T}{dt} = \frac{d\ln P}{dt} \qquad (4)$$

Now using Eqn. (4) in Eqn. (3). Thus,

$$\frac{C_V}{R}\frac{d\ln T}{dt} = \frac{C_V}{RT}\frac{dT}{dt} = \frac{\dot{N}RT}{PV} \text{ or}$$

$$\left.\frac{dT}{dt}\right|_{t=5\text{ min}} = \left.\frac{\dot{N}(RT)^2}{PVC_V}\right|_{t=5\text{ min}} = -1.151\text{ K/sec}$$

and

$$\left.\frac{dP}{dt}\right|_{5\text{ min}} = \left.\frac{C_P P}{RT}\frac{dT}{dt}\right|_{5\text{ min}} = -0.0188\text{ bar/s}$$

2.23 Consider a fixed mass of gas as the (closed) system for this problem. The energy balance is:

$$\frac{d(N\underline{U})}{dt} = N\frac{d\underline{U}}{dt} = NC_V\frac{dT}{dt} = -P\frac{dV}{dt}$$

From the ideal gas law we have $P = NRT/V$. Thus

$$C_V N\frac{dT}{dt} = \frac{-NRT}{V}\frac{dV}{dt} \Rightarrow \frac{C_V}{R}\frac{d\ln T}{dt} = \frac{-d\ln V}{dt}$$

or

$$\frac{C_V}{R}\ln\frac{T_2}{T_1} = -\ln\frac{V_2}{V_1} \Rightarrow \left(\frac{T_2}{T_1}\right)^{C_V/R} = \left(\frac{V_1}{V_2}\right) \qquad (*)$$

or

$$V_2 T_2^{C_V/R} = V_1 T_1^{C_V/R} = V T^{C_V/R} = \text{constant}$$

Substituting the ideal gas law gives $PV^{C_P/C_V} = PV^{\gamma} = \text{constant}$. Note that the heat capacity must be independent of temperature to do the integration in Eqn. (*) as indicated.

2.24 System: Contents of the tank (at any time)

(a) Final temperature $(T = 330 \text{ K})$ and pressure $\left(P = 1.013\times10^5 \text{ Pa}\right)$ are known. Thus, there is no need to use balance equations.

$$N_f = \frac{PV}{RT} = \frac{1.013\times10^5 \text{ Pa}\times 0.3 \text{ m}^3}{8.314 \text{ J/mol K}\cdot 330 \text{ K}} = 11.08 \text{ mol} = 0.01108 \text{ kmol}$$

(b) Assume, as usual, that enthalpy of gas leaving the cylinder is the same as gas in the cylinder . . . See Illustration 2.5-5. From Eqn. (f) of that illustration we have

$$\frac{P_f}{P_i} = \left(\frac{T_f}{T_i}\right)^{C_P/R} \text{ or } \frac{T_f}{T_i} = \left(\frac{P_f}{P_i}\right)^{R/C_P} = \left(\frac{1.0133\times10^5}{1.0\times10^6}\right)^{8.314/29} = 0.5187$$

Thus $T_f = 0.5187\times 330 \text{ K} = 171.19 \text{ K}$, $P_f = 1.013 \text{ bar}$, and $N_f = 21.36 \text{ mol} = 0.02136 \text{ kmol}$.

2.25 Except for the fact that the two cylinders have different volumes, this problem is just like Illustration 2.5-5. Following that illustration we obtain

$$\frac{2P_1^i}{T_1^i} = \frac{2P_1^f}{T_1^f} + \frac{P_2^f}{T_2^f} \qquad \text{for Eqn. (a')}$$

$$2P_1^i = 2P_1^f + P_2^f \text{ or } P^f = \frac{2}{3}P_1^i \qquad \text{for Eqn. (c')}$$

and again get Eqn. (f)

$$\left(\frac{T_1^f}{T_1^i}\right)^{C_P/R} = \left(\frac{P_1^f}{P_1^i}\right)$$

Then we obtain $P^f = 133.3 \text{ bar}$, $T_1^f = 223.4 \text{ K}$, and $T_2^f = 328.01 \text{ K}$.

2.26 From problem statement $P_1^f = P_2^f = P^f$ and $T_1^f = T_2^f = T^f$.

Mass balance on the composite system of two cylinders

$$N_1^{\mathrm{f}} + N_2^{\mathrm{f}} = N_1^{\mathrm{i}} \;\text{ or }\; \frac{2P_1^{\mathrm{f}}}{T_1^{\mathrm{f}}} + \frac{P_2^{\mathrm{f}}}{T_2^{\mathrm{f}}} = \frac{3P^{\mathrm{f}}}{T^{\mathrm{f}}} = \frac{2P^{\mathrm{i}}}{T^{\mathrm{i}}}$$

Energy balance on composite system

$$N_1^{\mathrm{i}}\underline{U}_1^{\mathrm{i}} = N_1^{\mathrm{f}}\underline{U}_1^{\mathrm{f}} + N_2^{\mathrm{f}}\underline{U}_2^{\mathrm{f}} \Rightarrow P^{\mathrm{f}} = \frac{2P^{\mathrm{i}}}{3} = \frac{2\times 200\text{ bar}}{3} = 133.3\text{ bar (as before)}$$

and $T^{\mathrm{f}} = \dfrac{3P^{\mathrm{f}}}{2P_1^{\mathrm{i}}}T_1^{\mathrm{i}} = \dfrac{3}{2}\left(\dfrac{2}{3}\right)T_1^{\mathrm{i}} = T_1^{\mathrm{i}} = 250\text{ K}$.

2.27 Even though the second cylinder is not initially evacuated, this problem still bears many similarities to Illustration 2.5-5). Proceeding as in that illustration, we obtain

$$\frac{2P_1^{\mathrm{i}}}{T_1^{\mathrm{i}}} + \frac{P_2^{\mathrm{i}}}{T_2^{\mathrm{i}}} = \frac{2P_1^{\mathrm{f}}}{T_1^{\mathrm{f}}} + \frac{P_2^{\mathrm{f}}}{T_2^{\mathrm{f}}} \text{ instead of Eqn (a')}$$
$$2P_1^{\mathrm{i}} + P_2^{\mathrm{i}} = 2P_1^{\mathrm{f}} + P_2^{\mathrm{f}} = 3P^{\mathrm{f}} \text{ instead of Eqn. (c)}$$

[Thus, $P^{\mathrm{f}} = (2\times 200 + 1\times 20)/3 = 140\text{ bar}$] and again recover Eqn. (f) for Cylinder 1

$$\left(\frac{T_1^{\mathrm{f}}}{T_1^{\mathrm{i}}}\right)^{C_{\mathrm{P}}/R} = \left(\frac{P_1^{\mathrm{f}}}{P_1^{\mathrm{i}}}\right) \qquad \text{Eqn. (f)}$$

Solution is $P_1^{\mathrm{f}} = P_2^{\mathrm{f}} = 140\text{ bar}$, $T_1^{\mathrm{f}} = 226.47\text{ K}$, $T_2^{\mathrm{f}} = 286.51\text{ K}$.

2.28 (a) System = Gas contained in room (open system)

mass balance: $\dfrac{dN}{dt} = \dot{N}$

energy balance: $\dfrac{d(N\underline{U})}{dt} = \dot{N}\underline{H} + \dot{Q} = \underline{H}\dfrac{dN}{dt} + \dot{Q}$

Thus,

$$\dot{Q} = \frac{d(N\underline{U})}{dt} - \underline{H}\frac{dN}{dt} = (\underline{U} - \underline{H})\frac{dN}{dt} + N\frac{d\underline{U}}{dt}$$

For the ideal gas, $\underline{H} - \underline{U} = P\underline{V} = RT$; $\dfrac{dN}{dt} = \dfrac{d}{dt}\left(\dfrac{PV}{RT}\right) = \dfrac{V}{R}\dfrac{d}{dt}\left(\dfrac{P}{T}\right)$

$$\dot{Q} = -RT\left(\frac{V}{R}\right)\frac{d}{dt}\left(\frac{P}{T}\right) + NC_{\mathrm{V}}\frac{dT}{dt} = -RT\cdot\frac{NT}{P}\frac{d}{dt}\left(\frac{P}{T}\right) + NC_{\mathrm{V}}\frac{dT}{dt}$$

$$\dot{Q} = \frac{-NRT}{P}\frac{dP}{dt} + NR\frac{dT}{dt} + NC_{\mathrm{V}}\frac{dT}{dt}$$

Since $P = \text{constant}$, $\frac{dP}{dt} = 0$, $\dot{Q} = \frac{NC_{\mathrm{P}}dT}{dt}$ or

$$\frac{dT}{dt} = \frac{\dot{Q}}{C_{\mathrm{P}}}\frac{RT}{PV} = \frac{1.5\times10^{3}\ \mathrm{W}\cdot 8.314\ \mathrm{J/mol\ K}\cdot 283.15\ \mathrm{K}}{29\ \mathrm{J/mol\ K}\cdot 1.0133\times10^{5}\ \mathrm{Pa}\cdot(3.5\times5\times3)\ \mathrm{m}^{3}}$$
$$= 0.0229\ \mathrm{K/s} = 1.37\ \mathrm{K/min}$$

(b) System = Gas contained in sealed room (closed system) $\dot{N} = 0$

Energy balance: $\frac{d(N\underline{U})}{dt} = N\frac{d\underline{U}}{dt} = NC_{v}\frac{dT}{dt} = \dot{Q}$

$$\left.\frac{dT}{dt}\right|_{\substack{\text{sealed}\\ \text{room}}} = \frac{\dot{Q}}{NC_{\mathrm{V}}} = \frac{C_{\mathrm{P}}}{C_{\mathrm{V}}}\left.\frac{dT}{dt}\right|_{\substack{\text{unsealed}\\ \text{room}}} = \frac{29}{29-8.314}\times1.37\ \mathrm{K/min}$$
$$= 1.925\ \mathrm{K/min}$$

2.29 In each case we must do work to get the weights on the piston, either by pushing the piston down to where it can accept the weights, or by lifting the weights to the location of the piston. We will consider both alternatives here. First, note that choosing the gas contained within piston and cylinder as the system, $\Delta U = Q + W$. But $\Delta U = 0$, since the gas is ideal and $T = \text{constant}$. Also $W = -\int PdV = -NRT\ln(V_{\mathrm{f}}/V_{\mathrm{i}})$, for the same reasons. Thus, in each case, we have that the net heat and work flows to the gas are

$$W(\text{work done on gas}) = -NRT\ln\left(\frac{V_{\mathrm{f}}}{V_{\mathrm{i}}}\right) = -2479\ln\frac{1.213\times10^{-2}}{2.334\times10^{-2}} = 1622.5\ \mathrm{J}$$
$$\text{and } Q = -W = -1622.5\ \mathrm{J}\ \text{(removed from gas)}$$

If more work is delivered to the piston, the piston will oscillate eventually dissipating the addition work as heat. Thus, more heat will be removed from the gas + piston and cylinder than if only the minimum work necessary had been used.

Note that in each case the atmosphere will provide

$$W_{\mathrm{atm}} = P\Delta V = 1.013\times10^{5}\ \mathrm{kPa}\times(2.334-1.213)\times10^{-2}\ \mathrm{m}^{3} = 1135.6\ \mathrm{J}$$

and the change in potential energy of piston

$$mg\Delta h = 5\ \mathrm{kg}\times9.8\ \mathrm{m/s^{2}}\times\frac{(2.334-1.213)\times10^{-2}\ \mathrm{m}^{3}}{1\times10^{-2}\ \mathrm{m}} = 54.9\ \mathrm{J}$$

The remainder $1622.5 - 1135.6 - 54.9 = 432.0\text{ J}$ must be supplied from other sources, as a minimum.

(a) One 100 kg weight.

An efficient way of returning the system to its original state is to slowly (i.e., at zero velocity) force the piston down by supplying 432.0 J of energy. When the piston is down to its original location, the 100 kg is slid sideways, onto the piston, with no energy expenditure.

An inefficient process would be to lift the 100 kg weight up to the present location of the piston and then put the weight on the piston. In this case we would supply

$$\begin{aligned} Mg\Delta h &= Mg\frac{\Delta V}{A} = 100\text{ kg} \times 9.8\ \frac{\text{m}}{\text{s}^2} \times \frac{\left(2.334\times10^{-2} - 1.213\times10^{-2}\right)\text{ m}^3}{1\times10^{-2}\ \text{m}^2} \\ &= 1098.6\ \text{kg m}^2/\text{s}^2 = 1098.6\text{ J} \end{aligned}$$

This energy would be transmitted to the gas as the piston moved down. Thus

$$W(\text{on gas}) = \underset{\text{(atmosphere)}}{1135.6\text{ J}} + \underset{\text{(PE of piston)}}{54.9\text{ J}} + \underset{\text{(PE of weight)}}{1098.6\text{ J}} = 2289.1$$

	$W(\text{J}) = -Q(\text{J})$	$W_{\text{cycle}} = -Q_{\text{cycle}}$
Efficient	1622.5	$1622.5 - 1190.5 = 432.0$
Inefficient	2289.1	$2289.1 - 1190.5 = 1098.6$

(b) Two 50 kg weights

In this case we also recover the potential energy of the topmost weight.

$$mg\Delta h = 50\text{ kg} \times 9.8\ \frac{\text{m}}{\text{s}^2} \times \frac{(1.597 - 1.213)\times10^{-2}\ \text{m}^3}{0.01\text{ m}^2} = 188.2\text{ J}$$

Thus in an efficient process we need supply only

$$1622.5 - 1135.6 - 54.9 - 188.2 = 243.8\text{ J}$$

An efficient process would be to move the lowest weight up to the position of the piston, by supplying

$$50\text{ kg} \times 9.8\ \frac{\text{m}}{\text{s}^2} \times \frac{(2.334 - 1.213)\times10^{-2}\ \text{m}^3}{1\times10^{-2}\ \text{m}^2} = 549.3\text{ J}$$

Slide this weight onto the piston and let go. The total work done in this case is

$$\underset{\text{(atmosphere)}}{1135.6} + \underset{\Delta\text{PE of piston}}{54.9} + \underset{\Delta\text{PE of weight}}{243.8} + \underset{\text{supplied by us}}{549.3} = 1983.6\text{ J}$$

Therefore

	$W(\text{J}) = -Q$	$W_{\text{cycle}} = -Q_{\text{cycle}}$
Efficient	1622.5	$1622.5 - 1378.7 = 243.8$ J
Inefficient	1983.6	$1983.6 - 1378.7 = 604.9$ J

(c) Four 25 kg weights.
In this case the recovered potential energy of weights is

$$25\text{ kg} \times 9.8\ \text{m/s}^2 \times \left(\frac{[(1.897 - 1.213) + (1.597 - 1.213) + (1.379 - 1.213)] \times 10^{-2}}{1 \times 10^{-2}} \right) \text{m}$$
$= 302.3$ J

Thus in an efficient process we need supply only

$$1622.5 - 1135.6 - 54.9 - 302.3 = 129.7\text{ J}$$

An inefficient process would be to raise the lowest weight up to the piston, expending

$$25\text{ kg} \times 9.8\ \text{m/s}^2 \times \frac{(2.334 - 1.213) \times 10^{-2}\ \text{m}^3}{1 \times 10^{-2}\ \text{m}} = 274.6\text{ J}$$

Thus the total work done is

$$1135.6 + 54.9 + 302.3 + 274.6 = 1767.4\text{ J}$$

and

	$W = -Q$	$W_{\text{cycle}} = -Q_{\text{cycle}}$
Efficient	1622.5	$1622.5 - 1493.0 = -129.5$
Inefficient	1767.4	$1767.4 - 1493.0 = -274.4$

(d) Grains of sand
Same analysis as above, except that since one grain of sand has essentially zero weight $W = 1622.5$ J, $Q = -1622.5$ J, $W_{\text{cycle}} = -Q_{\text{cycle}} = 0$.

2.30 System = Gas contained in the cylinder (closed system)
energy balance: $\frac{d(N\underline{U})}{dt} = N\frac{d\underline{U}}{dt} = NC_V\frac{dT}{dt} = -P\frac{dV}{dt} = \frac{-NRT}{V}\frac{dV}{dt}$ {Using the ideal gas equation of state}
Since C_V and C_P are constant

$$\frac{C_V}{R}\frac{1}{T}\frac{dT}{dt} = -\frac{1}{V}\frac{dV}{dt} \text{ or } \left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{R/C_V} = \left(\frac{L_1}{L_2}\right)^{R/C_V}$$

$$\Rightarrow T_2 = (25 + 273.15) \times \left(\frac{0.03\ \text{m}^3}{0.03 + 0.6 \times 0.05\ \text{m}^3} \right)^{8.314/(30-8.314)}$$
$$= 228.57\text{ K} = -44.58°\text{C and}$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = 20 \times \frac{1}{2} \times \frac{228.57}{298.15} = 7.666 \text{ bar}$$

From the difference (change of state) form of energy balance

$$\Delta U = \cancel{Q}^{\,0} + W = NC_V(T_2 - T_1) = -\int PdV$$

and $N = \dfrac{PV}{RT} = \dfrac{20 \text{ bar} \times 0.03 \text{ m}^3 \text{ kmol}\cdot\text{K}}{298.15 \text{ K} \times 8.314 \times 10^{-2} \text{ bar}\cdot\text{m}^3} = 0.0242 \text{ kmol}$

$\Rightarrow W = \Delta U = -0.0242 \text{ kmol} \times (30 - 8.314) \text{ kJ/kmol}\cdot\text{K} \cdot (298.15 - 228.57)\text{K}$

$= -36.52 \text{ kJ}$

Where has this work gone?

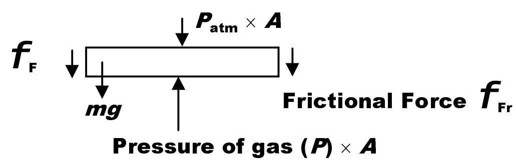
(a) To increase potential energy of piston

(b) To increase kinetic energy of piston

(c) To push back atmosphere so system can expand

(d) Work done against friction (and converted to heat).

To see this, write Newton's 2nd Law of Motion for the piston



$f = MA \Rightarrow (PA - P_{atm}A - mg - f_{fr}) = m\dfrac{dv}{dt}$; v = velocity of piston

Thus, $P = \dfrac{m}{A}\dfrac{dv}{dt} + P_{atm} + \dfrac{mg}{A} + \dfrac{f_{fr}}{A}$

$$-\Delta U = 36{,}520 \text{ J} = +\int PdV$$

$$= +\int P_{atm}dV + \frac{m}{A}\int \frac{dv}{dt}\frac{dV}{dt}dt + \frac{mg}{A}\int dV + \frac{1}{A}\int f_{fr}\frac{dv}{dt}dt \qquad (1)$$

Now $\dfrac{1}{A}\dfrac{dV}{dt} = \dfrac{dh}{dt} = v$ (h = piston height) and $v\dfrac{dv}{dt} = \dfrac{1}{2}\dfrac{d}{dt}\left(v^2\right)$

$$36{,}520 \text{ J} = \underbrace{P_{atm}\Delta V}_{\substack{3000 \text{ J} \\ \text{Work against} \\ \text{atmosphere}}} + \underbrace{\frac{mv^2}{2}}_{\substack{\text{since} \\ v_{\text{initial}}=0}} + \underbrace{mg\Delta h}_{\substack{1760 \text{ J} \\ \text{Work used to} \\ \text{increase potential} \\ \text{energy of piston}}} + \int f_{fr}vdt$$

Thus $36{,}520 \text{ J} = 3000 + \dfrac{mv^2}{2} + 1760 + \int f_{fr}vdt$.

(a) If there is no friction $f_{fr} = 0$ then

$$v^2 = \frac{(36520 - 3000 - 1760)\text{J} \times 2}{300 \text{ kg}} = 211.7 \text{ m}^2/\text{s}^2 \Rightarrow v = 14.55 \text{ m/s}$$

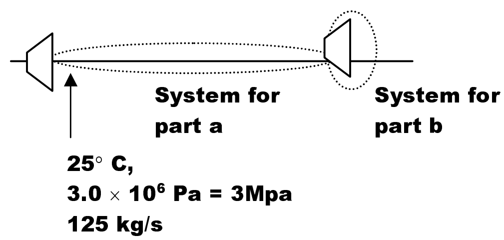
(b) If we assume only sliding friction, $f_{\text{fr}} = kv$

$$\int f_{\text{fr}} v dv = k \int v^2 dt \Rightarrow (36520 - 3000 - 1760) = \frac{m}{2} v^2 + k \int v^2 dt$$

In order to determine the velocity now we need to know the coefficient of sliding friction k, and then would have to solve the integral equation above (or integrate successively over small time steps). It is clear, however, that

$$v(\text{with friction}) < v(\text{without friction}) = 14.55 \text{ m/s}$$

2.31 25°C, 3.0×10^6 Pa = 3 MPa
125 kg/s



(a) mass balance (steady-state)

$$0 = \dot{M}_1 + \dot{M}_2$$

$$\Rightarrow \dot{M}_1 = -\dot{M}_2 = 125 \text{ kg/s}$$

Energy balance (neglecting PE terms)

$$0 = \dot{M}_1\left(\hat{H}_1 + \frac{v_1^2}{2}\right) + \dot{M}_2\left(\hat{H}_2 + \frac{v_2^2}{2}\right)$$

$\dot{M} = \rho v A = mnvA$; $\rho =$ mass density, $n =$ molar density,
$v =$ velocity, $A =$ pipe area, $m =$ molecular weight.

$$\frac{\dot{M}}{m} = \frac{P}{RT} vA$$

$$\Rightarrow \frac{125 \text{ kg/s}}{16 \text{ kg/kmol}} = \frac{3.0 \times 10^6 \text{ Pa}}{298.15 \text{ K} \times 8.314 \times 10^3 \text{ Pa} \cdot \text{m}^3} \times v(\text{m/s}) \times \pi \times 0.09 \text{ m}^2$$

$$\Rightarrow v = 22.83 \text{ m/s}$$

$$\frac{mv^2}{2} = \frac{16 \text{ kg/kmol} \times (22.83 \text{ m/s})^2}{2 \times 1 \text{ kg} \cdot \text{m/Ns}^2} = 4.170 \times 10^3 \text{ J/kmol} = 4.17 \text{ kJ/kmol}$$

Back to energy balance, now on a molar basis

$$\underline{H}_1 - \underline{H}_2 = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = C_p(T_1 - T_2)$$

As a first guess, neglect kinetic energy terms . . .

$$C_\text{p}(T_1 - T_2) = 0 \Rightarrow T_1 = T_2 = 25°\text{C}$$

Now check this assumption

$$v_2 = \frac{n_1 v_1}{n_2} = \frac{P_1 v_1}{P_2} = \frac{3.0 \times 10^6 v_1}{2.0 \times 10^6} = 34.24 \text{ m/s}$$

Recalculate including the kinetic energy terms

$$C_\text{p}(T_1 - T_2) = \frac{m}{2}\left(v_1^2 - v_2^2\right) = \frac{16}{2}\left(34.24^2 - 22.83^2\right) = 5209 \text{ J/kmol}$$

$$T_2 = T_1 - \frac{5209 \text{ J/kmol}}{36.8 \text{ J/mol} \times 1000 \text{ mol/kmol}} = T_1 - 0.14°\text{C}$$

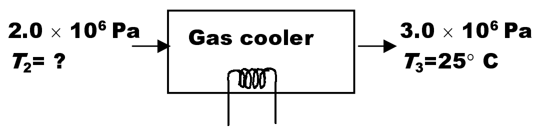
Thus the kinetic energy term makes such a small contribution, we can safely ignore it.

(b) Mass balance on compressor (steady-state) $0 = \dot{N}_1 + \dot{N}_2$

Energy balance on compressor, which is in steady-state operation

$$0 = \dot{N}_1 \underline{H}_1 + \dot{N}_2 \underline{H}_2 + \underbrace{\cancel{\dot{Q}}^{\,0}}_{\text{adiabatic compressor}} + \dot{W}_s \Rightarrow \dot{W}_s = \dot{N}_1 C_p (T_2 - \underbrace{T_1}_{=25°\text{C}})$$

Can compute $\dot{W}_s$ if T_2 is known or vice versa. However, can not compute both without further information.



Analysis as above except that $\dot{Q} \neq 0$ but $\dot{W} = 0$.

Here we get $\begin{cases} 0 = \dot{N}_2 + \dot{N}_3 \\ \dot{Q} = \dot{N}_1 C_p (\underbrace{T_3}_{=25°\text{C}} - T_2) \end{cases}$

Can not compute $\dot{Q}$ until T_2 is known.

See solution to Problem **3.10**.

2.32 a) Define the system to be the nitrogen gas. Since a Joule-Thomson expansion is isenthalpic, $\hat{H}(T_1, P_1) = \hat{H}(T_2, P_2)$. Using the pressure enthalpy diagram for nitrogen, Figure 2.4-3, we have

$\hat{H}(135 \text{ K}, 20 \text{ MPa}) = 153 \text{ kJ / kg}$ and then $T_2 = T\left(P_2 = 0.4 \text{ MPa}, \hat{H} = 153 \text{ kJ / kg}\right)$

From which we find that T = 90 K, with approximately 55% of the nitrogen as vapor, and 45% as liquid.

b) Assuming nitrogen to be an ideal gas (poor assumption), then the enthalpy depends only on temperature. Since a Joule-Thomson expansion is isenthalpic, this implies that the temperature is unchanged, so that the final state will be all vapor.

2.33 Plant produces 1.36×10^9 kwh of energy per year

$\Rightarrow$ Plant uses $1.36 \times 10^9 \times 4 = 5.44 \times 10^9$ kwh of heat

$$1 \text{ kwh} = 3.6 \times 10^6 \text{ J}$$

$$\Rightarrow \text{Plant uses } 3.6 \times 10^6 \frac{\text{J/year}}{\text{kwh}} \times 5.44 \times 10^9 \text{ kwh} = 19.584 \times 10^{15} \text{ J/year}$$

$$\begin{aligned}\Delta H \text{ of rock (total)} &= M \cdot \hat{C}_\text{p}(T_\text{f} - T_\text{i}) \\ &= 10^{12} \text{ kg} \times 1 \text{ J/g K} \times 1000 \text{ g/kg} \times (110 - 600) \text{ K} \\ &= -490 \times 10^{15} \text{ J}\end{aligned}$$

$$\Rightarrow 19.58 \times 10^{15} \text{ J/year} \times x \text{ years} = 490 \times 10^{15} \text{ J}$$

$$x = 25.02 \text{ years}$$

3

3.1 (a) System = Ball (1) + Water (2)

Energy balance: $M_1\hat{U}_1^f + M_2\hat{U}_2^f - M_1\hat{U}_1^i - M_2\hat{U}_2^i = 0$

$\Rightarrow M_1 C_{V,1}\left(T_1^f - T_1^i\right) + M_2 C_{V,2}\left(T_2^f - T_2^i\right) = 0$; also $T_1^f - T_2^f$. Thus

$$T^f = \frac{M_1 C_{V,1} T_1^i + M_2 C_{V,2} T_2^i}{M_1 C_{V,1} + M_2 C_{V,2}} = \frac{5\times10^3\times0.5\times75+12\times10^3\times4.2\times5}{5\times10^3\times0.5+12\times10^3\times4.2}$$
$$= 8.31\,°\text{C}$$

[Note: Since only ΔT's are involved, $°\text{C}$ were used instead of K)].

(b) For solids and liquids we have (eqn. 3.4-6). That $\Delta S = M\int C_P \frac{dT}{T} = MC_P \ln\frac{T_2}{T_1}$ for the case in which C_P is a constant. Thus

$$\text{Ball:}\quad \Delta S = 5\times10^3\ \text{g}\times0.5\frac{\text{J}}{\text{g}\cdot\text{K}}\times\ln\left\{\frac{8.31+273.15}{75+273.15}\right\} = -531.61\frac{\text{J}}{\text{K}}$$
$$= -531.61\ \text{s}$$

$$\text{Water:}\quad \Delta S = 12\times10^3\ \text{g}\times4.2\frac{\text{J}}{\text{g}\cdot\text{K}}\times\ln\left\{\frac{8.31+273.15}{5+273.15}\right\} = +596.22\frac{\text{J}}{\text{K}}$$

and

$$\Delta S(\text{Ball}+\text{Water}) = 596.22-531.61\frac{\text{J}}{\text{K}} = 64.61\frac{\text{J}}{\text{K}}$$

Note that the system Ball + Water is isolated. Therefore

$$\Delta S = S_{\text{gen}} = 64.61\frac{\text{J}}{\text{K}}$$

3.2 Energy balance on the combined system of casting and the oil bath

$M_c C_{V,c}\left(T^f - T_c^i\right) + M_o C_{V,o}\left(T^f - T_o^i\right) = 0$ since there is a common final temperature.

$$20\ \text{kg}\times0.5\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left(T^f-450\right)\text{K} + 150\ \text{kg}\times2.6\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left(T^f-450\right)\text{K} = 0$$

This has the solution $T^f = 60°\text{C} = 313.15$ K

Since the final temperature is known, the change in entropy of this system can be calculated from $\Delta S = 20\times0.5\times\ln\left(\frac{273.15+60}{273.15+450}\right) + 150\times2.6\times\ln\left(\frac{273.15+60}{273.15+50}\right) = 4.135\frac{\text{kJ}}{\text{K}}$

3.3 Closed system energy and entropy balances

$$\frac{dU}{dt} = \dot{Q} + \dot{W}_s - P\frac{dV}{dt};\ \frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}};$$

Thus, in general $\dot{Q} = T\frac{dS}{dt} - T\dot{S}_{\text{gen}}$ **and**

$$\dot{W}_s = \frac{dU}{dt} - \dot{Q} + P\frac{dV}{dt} = \frac{dU}{dt} - T\frac{dS}{dt} + T\dot{S}_{gen} + P\frac{dV}{dt}$$

Reversible work: $\dot{W}_s^{\text{Rev}} = \dot{W}_s^{\text{Rev}}\left(\dot{S}_{\text{gen}} = 0\right) = \frac{dU}{dt} - T\frac{dS}{dt} + P\frac{dV}{dt}$

(a) System at constant U & V $\Rightarrow \frac{dU}{dt} = 0$ and $\frac{dV}{dt} = 0$

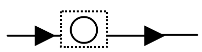
$$\dot{W}_s\left(\dot{S}_{gen} = 0\right) = \dot{W}_S^{\text{Rev}} = -T\frac{dS}{dt}$$

(b) System at constant S & P $\Rightarrow \frac{dS}{dt} = 0$ and $\frac{dP}{dt} = 0 \Rightarrow P\frac{dV}{dt} = \frac{d}{dt}(PV)$

so that

$$\dot{W}_s\left(\dot{S}_{gen} = 0\right) = \dot{W}_S^{\text{rev}} = \frac{dU}{dt} + \frac{d}{dt}(PV) = \frac{d}{dt}(U + PV) = \frac{dH}{dt}$$

3.4

700 bar, 600°C  10 bar, $T = ?$

Steady-state balance equations

$$\frac{dM}{dt} = 0 = \dot{M}_1 + \dot{M}_2$$

$$\frac{dU}{dt} = 0 = \dot{M}_1\hat{H}_1 + \dot{M}_2\hat{H}_2 + \cancel{\dot{Q}}^{\,0} + \cancel{\dot{W}_s}^{\,0} - P\cancel{\frac{dV}{dt}}^{\,0} = \dot{M}_1\hat{H}_1 + \dot{M}_2\hat{H}_2$$

or $\hat{H}_1 = \hat{H}_2$

Drawing a line of constant enthalpy on Mollier Diagram we find, at $P = 10$ bar, $T \cong 308°\text{C}$

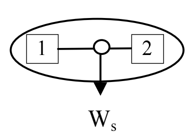
At 700 bar and 600°C	At 10 bar and 308°C
$\hat{V} = 0.003973\ \text{m}^3/\text{kg}$	$\hat{V} \approx 0.2618\ \text{m}^3/\text{kg}$
$\hat{H} = 3063\ \text{kJ/kg}$	$\hat{H} \approx 3063\ \text{kJ/kg}$
$\hat{S} = 5.522\ \text{kJ/kg K}$	$\hat{S} = 7.145\ \text{kJ/kg K}$

Also

$$\frac{dS}{dt} = 0 = \dot{M}_1\hat{S}_1 + \dot{M}_2\hat{S}_2 + \frac{\cancel{\dot{Q}}^{\,0}}{T} + \dot{S}_{\text{gen}} = 0$$

$$\Rightarrow \dot{S}_{\text{gen}} = \dot{M}_1\left(\hat{S}_2 - \hat{S}_1\right) \text{ or } \frac{\dot{S}_{\text{gen}}}{\dot{M}_1} = \hat{S}_2 - \hat{S}_1 = 7.145 - 5.522 = 1.623\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

3.5

System 

Energy balance

$$\Delta U = \left(U_2^f - U_2^i\right) + \left(U_1^f - U_1^i\right) = \cancel{\dot{Q}}^{\text{ adiabatic}} + W_S - \cancel{\int PdV}^{\text{constant volume}}$$

$$W_s = MC_p\left(T_2^f - T_2^i\right) + MC_p\left(T_1^f - T_1^i\right) = MC_p\left[\left(T_2^f - T_2^i\right) + \left(T_1^f - T_1^i\right)\right]$$

but $T_1^f = T_2^f = T^f \Rightarrow \dfrac{W_s}{MC_P} = \left[2T^f - T_1^i - T_2^i\right]$

Entropy balance

adiabatic

$$\Delta S = \left(S_2^f - S_2^i\right) + \left(S_1^f - S_1^i\right) = \cancel{\int \cancelto{0}{\frac{\dot{Q}}{T}} dt} + \cancel{S_{gen}}$$

0 for maximum wor

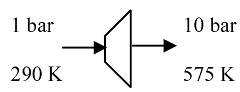
$$\left(S_2^f - S_2^i\right) + \left(S_1^f - S_1^i\right) = 0 = MC_P \ln\frac{T_2^f}{T_2^i} + MC_P \ln\frac{T_1^f}{T_1^i}$$

or $\ln\left\{\dfrac{T_2^f T_1^f}{T_2^i T_1^i}\right\} = 0$; $T_1^f T_2^f = T_1^i T_2^i$; but $T_1^f = T_2^f = T^f$

$\Rightarrow \left(T^f\right)^2 = \left(T_1^i T_2^i\right)$ or $T^f = \sqrt{T_1^i T_2^i}$ and

$$\frac{W_s}{MC_P} = \left[2T^f - T_1^i - T_2^i\right] = \left[2\sqrt{T_1^i T_2^i} - T_1^i - T_2^i\right]$$

3.6



(a) Entropy change per mole of gas

$\Delta \underline{S} = C_P \ln\dfrac{T_2}{T_1} - R\ln\dfrac{P_2}{P_1}$ **eqn. (3.4-3)**

Thus $\Delta S = 29.3\dfrac{\text{J}}{\text{mol K}}\ln\dfrac{575}{290} - 8.314\dfrac{\text{J}}{\text{mol K}}\ln\dfrac{10}{1} = 0.9118\dfrac{\text{J}}{\text{mol K}}$

(b) System = contents of turbine (steady-state system)

Mass balance $\dfrac{dN}{dt} = 0 = \dot{N}_1 + \dot{N}_2 \Rightarrow -\dot{N}_2 = \dot{N}_1 = \dot{N}$

Energy balance $\dfrac{dU}{dt} = 0 = \dot{N}_1\underline{H}_1 + \dot{N}_2\underline{H}_2 + \cancelto{0}{\dot{Q}} + \dot{W}_s - P\cancelto{0}{\dfrac{dV}{dt}}$

$\dot{W}_s = \dot{N}(\underline{H}_2 - \underline{H}_1) = \dot{N}C_P(T_2 - T_1)$

$W = \dfrac{\dot{W}_s}{\dot{N}} = C_P(T_2 - T_1) = 29.3\dfrac{\text{J}}{\text{mol K}} \times (575 - 290)\text{K}$

$= 8350.5\dfrac{\text{J}}{\text{mol}}$

(c) In Illustration 3.5-1, $W = 7834.8$ J/mol because of irreversiblitities $(\Delta S \neq 0)$, more work is done on the gas here. What happens to this additional energy input? It appears as an increase of the internal energy (temperature) of the gas.

3.7 Heat loss from metal block

$$\frac{dU}{dt} = C_P\frac{dT}{dt} = \dot{Q}$$

$$-\dot{W}=\frac{T-T_2}{T}\dot{Q}(-1)\begin{cases}\dot{Q}=\text{heat out of metal}\\ -\dot{Q}=\text{heat into heat engine}\end{cases}$$

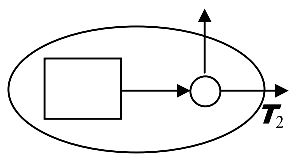
$$C_P\frac{dT}{dt}\frac{(T-T_2)}{T}=-\dot{W}\Rightarrow-\int_0^t\dot{W}dt=C_P\int_{T_1}^{T_2}\left(1-\frac{T_2}{T}\right)dT$$

$$-W=C_P(T_2-T_1)-C_P\cdot T_2\ln\frac{T_2}{T_1}=C_P\left[(T_2-T_1)-T_2\ln\frac{T_2}{T_1}\right]$$

$$-W=C_PT_2\left[\left(1-\frac{T_1}{T_2}\right)-\ln\frac{T_2}{T_1}\right]$$

$$Q=\int_{T_1}^{T_2}C_PdT=C_P(T_2-T_1)=C_PT_2\left(1-\frac{T_1}{T_2}\right)$$

Alternate way to solve the problem



System is the metal block + heat engine (closed)

E.B.: $\dfrac{dU}{dt}=C_P\dfrac{dT}{dt}=\dot{Q}+\dot{W}$

S.B.: $\dfrac{dS}{dt}=\dfrac{\dot{Q}}{T^2}+\cancel{S_{gen}}$
0 for maximum wor

$$\dot{Q}=T_2\frac{dS}{dt};\ \frac{dU}{dt}=T_2\frac{dS}{dt}+\dot{W};\ dU=C_PdT;\ dS=\frac{C_P}{T}dT$$

$$\dot{W}=\frac{dU}{dt}-T_2\frac{dS}{dt}=C_PdT-T_2\frac{C_P}{T}dT=C_P\left(1-\frac{T_2}{T}\right)dT$$

$$W=\int\dot{W}dt=\int_{T_1}^{T_2}C_P\left(1-\frac{T_2}{T}\right)dT=C_P\int_{T_1}^{T_2}\left(1-\frac{T_2}{T}\right)dT$$

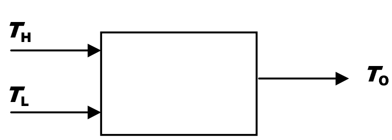
$$W=C_P(T_2-T_1)-T_2C_P\ln\frac{T_2}{T_1}=C_PT_2\left[\left(1-\frac{T_1}{T_2}\right)-\ln\frac{T_2}{T_1}\right]$$

3.8 This problem is not well posed since we do not know exactly what is happening. There are several possibilities:

(1) Water contact is very short so neither stream changes T very much. In this case we have the Carnot efficiency

$$\eta=\frac{-W}{Q}=\frac{T_{\text{high}}-T_{\text{low}}}{T_{\text{high}}}=\frac{22}{27+273}=\frac{22}{300}=0.0733=7.33\%$$

(2) Both warm surface water (27°C) and cold deep water (5°C) enter work producing device, and they leave at a common temperature.



M.B.: $\dfrac{dM}{dt} = 0 = \dot{M}_{\mathrm{H}} + \dot{M}_{\mathrm{L}} + \dot{M}_0 \Rightarrow \dot{M}_0 = -\left(\dot{M}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}\right)$

E.B.: $\dfrac{dU}{dt} = 0 = \dot{M}_{\mathrm{H}}\hat{H}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}\hat{H}_{\mathrm{L}} + \dot{M}_0\hat{H}_0 + \dot{W} = 0$

$$\begin{aligned}\dot{W} &= -\dot{M}_{\mathrm{H}}\hat{H}_{\mathrm{H}} - \dot{M}_{\mathrm{L}}\hat{H}_{\mathrm{L}} + \left(\dot{M}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}\right)\hat{H}_0 \\ &= \dot{M}_{\mathrm{H}}\left(\hat{H}_0 - \hat{H}_{\mathrm{H}}\right) + \dot{M}_{\mathrm{L}}\left(\hat{H}_0 - \hat{H}_{\mathrm{L}}\right) \\ &= \dot{M}_{\mathrm{H}}C_{\mathrm{P}}(T_0 - T_{\mathrm{H}}) + \dot{M}_{\mathrm{L}}C_{\mathrm{P}}(T_0 - T_{\mathrm{L}})\end{aligned}$$

S.B.: $\dfrac{dS}{dt} = 0 = \dot{M}_{\mathrm{H}}\hat{S}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}\hat{S}_{\mathrm{L}} + M_0\hat{S}_0 + \cancel{\dfrac{\dot{Q}}{T}}^{\,0} + \cancel{S_{\mathrm{gen}}}^{\,0}$

$$\dot{M}_{\mathrm{H}}\hat{S}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}\hat{S}_{\mathrm{L}} - \left(\dot{M}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}\right)\hat{S}_0 = 0$$

$$\dot{M}_{\mathrm{H}}\left(\hat{S}_{\mathrm{H}} - \hat{S}_0\right) + \dot{M}_{\mathrm{L}}\left(\hat{S}_{\mathrm{L}} - \hat{S}_0\right) = 0 \Rightarrow \dot{M}_{\mathrm{H}}C_{\mathrm{P}}\ln\frac{T_{\mathrm{H}}}{T_0} + \dot{M}_{\mathrm{L}}C_{\mathrm{P}}\ln\frac{T_{\mathrm{L}}}{T_0} = 0$$

$$\left(\frac{T_{\mathrm{H}}}{T_0}\right)^{\dot{M}_{\mathrm{H}}}\left(\frac{T_{\mathrm{L}}}{T_0}\right)^{\dot{M}_{\mathrm{L}}} = 1 \text{ or } T_{\mathrm{H}}^{\ \dot{M}_{\mathrm{H}}}T_{\mathrm{L}}^{\ \dot{M}_{\mathrm{L}}} = T_0^{\ \dot{M}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}}$$

$$T_0 = T_{\mathrm{H}}^{\dot{M}_{\mathrm{H}}/\left(\dot{M}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}\right)}T_{\mathrm{L}}^{\dot{M}_{\mathrm{L}}/\left(\dot{M}_{\mathrm{H}} + \dot{M}_{\mathrm{L}}\right)}$$

From this can calculate T_0. Then

$$\dot{W} = \dot{M}_{\mathrm{H}}C_{\mathrm{P}}(T_0 - T_{\mathrm{H}}) + \dot{M}_{\mathrm{L}}C_{\mathrm{P}}(T_0 - T_{\mathrm{L}})$$

This can be used for any flowrate ratio.

(3) Suppose very large amount of surface water is contacted with a small amount of deep water, i.e., $\dot{M}_{\mathrm{H}} >> \dot{M}_{\mathrm{L}}$. Then $T_0 \sim T_{\mathrm{H}}$

$$\dot{W} = \dot{M}_{\mathrm{H}}C_{\mathrm{P}}(T_{\mathrm{H}} - T_{\mathrm{H}}) + \dot{M}_{\mathrm{L}}C_{\mathrm{P}}(T_{\mathrm{H}} - T_{\mathrm{L}}) \sim \dot{M}_{\mathrm{L}}C_{\mathrm{P}}(T_{\mathrm{H}} - T_{\mathrm{L}})$$

(4) Suppose very large amount of deep water is contacted with a small amount of surface water, i.e., $\dot{M}_{\mathrm{H}} << \dot{M}_{\mathrm{L}}$, $T_0 \sim T_{\mathrm{L}}$.

$$\dot{W} = \dot{M}_{\mathrm{H}}C_{\mathrm{P}}(T_{\mathrm{L}} - T_{\mathrm{H}}) + \dot{M}_{\mathrm{L}}C_{\mathrm{P}}(T_{\mathrm{L}} - T_{\mathrm{L}}) \sim \dot{M}_{\mathrm{H}}C_{\mathrm{P}}(T_{\mathrm{L}} - T_{\mathrm{H}})$$

3.9 System = contents of the turbine. This is a steady-state, adiabatic, constant volume system.

(a) Mass balance $\dfrac{dM}{dt} = 0 = \dot{M}_1 + \dot{M}_2$ or $\dot{M}_2 = -\dot{M}_1$

Energy balance

$$\frac{dU}{dt} = 0 = \dot{M}_1\hat{H}_1 + \dot{M}_2\hat{H}_2 + \cancel{\dot{Q}}^{\text{adiabat:}} + \dot{W}_s - P\cancel{\frac{dV}{dt}}^{\text{constant volume}}$$

Entropy balance

$$\frac{dS}{dt} = 0 = \dot{M}_1\hat{S}_1 + \dot{M}_2\hat{S}_2 + \cancel{\frac{\dot{Q}}{T}} + \cancel{S_{\mathrm{gen}}}_{\text{0, by problem state}}$$

Thus

$\dot{M}_2 = -\dot{M}_1 = -4500\,\mathrm{kg/h}$	M.B.
$\dot{W}_S = -\dot{M}_1\left(\hat{H}_1 - \hat{H}_2\right)$	E.B.
$\hat{S}_2 = \hat{S}_1$	S.B.

State 1 $\quad T_1 = 500°\text{C}$, $P_1 = 60\ \text{bar}$ $\xrightarrow[\text{Tables}]{\text{Steam}}$ $\hat{H}_1 = 3422.2\ \text{kJ/kg}$, $\hat{S}_1 = 6.8803\ \text{kJ/kg}$

State 2 $\quad P_2 = 10\ \text{bar}$, $\hat{S}_2 = \hat{S}_1 = 6.8803\dfrac{\text{kJ}}{\text{kgK}}$ $\xrightarrow[\text{Tables}]{\text{Steam}}$ $T_2 \cong 240.4°\text{C}$, $\hat{H}_2 \approx 2920.5\ \text{kJ/kg}$

$$\dot{W}_s = 4500\frac{\text{kg}}{\text{h}} \times (2920.5 - 3422.2)\frac{\text{kJ}}{\text{kg}} = -2257650\frac{\text{kJ}}{\text{h}} = -627.1\ \text{kW}$$

(b) Same exit pressure $(P_2 = 10\ \text{bar})$, and still adiabatic

$\Rightarrow \dot{W}_s = -\dot{M}_1\left(\hat{H}_1 - \hat{H}_2\right)$.

Here, however,

$$\dot{W}_s = 0.8\dot{W}_s(\text{Part a}) = 0.8\left(-2.258\times 10^6\right)\frac{\text{kJ}}{\text{h}} = 4500\left(\hat{H}_2 - 3422.2\right)\frac{\text{kJ}}{\text{h}}$$

$\Rightarrow \quad \hat{H}_2 = 3020.8\ \text{kJ/kg}$, $P = 10\ \text{bar}$ $\xrightarrow[\text{Tables}]{\text{Steam}}$ $T_2 \cong 286.7\ \text{K}$, $\hat{S}_2 \approx 7.0677\ \text{kJ/kg K}$

Thus

$$\dot{S}_{\text{gen}} = -\dot{M}_1\left(\hat{S}_1 - \hat{S}_2\right) = -4500\frac{\text{kg}}{\text{h}} \times (6.8803 - 7.0677)\frac{\text{kJ}}{\text{kg K}} = 843.3\frac{\text{kJ}}{\text{K}\cdot\text{h}}$$

(c) Flow across valve is a Joule-Thompson (isenthalpic expansion) ... See Illustration 2.3-3. Thus, $\hat{H}_{\text{into valve}} = \hat{H}_{\text{out of valve}}$, and the inlet conditions to the turbine are

$$\hat{H}_1 = \hat{H}_{\text{out of valve}} = \hat{H}_{\text{into valve}} = 3422.2\ \text{kJ/kg}$$
$$P_1 = 30\ \text{bar}$$

$\xrightarrow[\text{Tables}]{\text{Steam}}$ $T_1 \approx 484.8°\text{C}$, $S_1 \approx 7.1874\ \text{kJ/kg K}$

Flow across turbine is isentropic, as in part (a)

$\hat{S}_2 = \hat{S}_1 = 7.1874\ \text{kJ/kg K}$, $P_2 = 10\ \text{bar}$ $\xrightarrow[\text{Tables}]{\text{Steam}}$ $T_2 \cong 318.1°\text{C}$, $\hat{H}_2 \approx 3090.4\ \text{kJ/kg}$

$$\dot{W}_s = 4500\frac{\text{kg}}{\text{h}} \times (3090.4 - 3422.2)\frac{\text{kJ}}{\text{kg}} = -1.493\times 10^6\frac{\text{kJ}}{\text{h}} = -414.8\ \text{kW}$$

3.10 Since compression is isentropic, and gas is ideal with constant heat capacity, we have

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{R/C_P}$$

So that $T_2 = T_1\left(\dfrac{P_2}{P_1}\right)^{R/C_P} = 298.15\left(\dfrac{3\times 10^6}{2\times 10^6}\right)^{8.314/36.8} = 326.75\ \text{K}$. Now using, from solution to Problem 2.31, that $\dot{W}_s = \dot{N}C_P(T_2 - T_1)$

$$\dot{W}_s = 125\frac{\text{kg}}{\text{s}} \times \frac{1\text{ mol}}{16\text{ g}} \times 36.8\frac{\text{J}}{\text{mol K}} \times (326.75 - 298.15)\text{K} \times \frac{1000\text{ g}}{\text{kg}}$$
$$= 8.23 \times 10^6 \text{ J/s}$$

The load on the gas cooler is, from Problem 2.31,

$$\dot{Q} = \dot{N}C_\text{p}(T_3 - T_2)$$
$$= \frac{125\text{ kg/s} \times 1000\text{ g/kg}}{16\text{ g/mol}} \times 36.8\frac{\text{J}}{\text{mol K}} \times (298.15 - 326.75)K$$
$$= -8.23 \times 10^{+6}\text{ J/s}$$

3.11 (a) This is a Joule-Thomson expansion

$$\Rightarrow \hat{H}(70\text{ bar},\ T=?) = \hat{H}(1.0133\text{ bar},\ T=400^\circ\text{C}) \approx \hat{H}(1\text{ bar},\ T=400^\circ\text{C})$$
$$= 3278.2\ \text{kJ/kg}$$

and $T = 447^\circ\text{C}$, $\hat{S} = 6.619\ \text{kJ/kg K}$

(b) If turbine is adiabatic and reversible $(\dot{S}_{\text{gen}} = 0)$, then $\hat{S}_{\text{out}} = \hat{S}_{\text{in}} = 6.619\ \text{kJ/kg K}$ and $P = 1.013$ bar. This suggests that a two-phase mixture is leaving the turbine

Let x = fraction vapor $\quad \hat{S}^{\text{V}} = 7.3594\ \text{kJ/kg K}$
$\hat{S}^{\text{L}} = 1.3026\ \text{kJ/kg K}$

Then $x(7.3594) + (1-x)(1.3026) = 6.619\ \text{kJ/kg K}$ or $x = 0.8778$. Therefore the enthalpy of fluid leaving turbine is

$$\hat{H} = 0.8788 \times \underbrace{2675.5}_{\hat{H}^{\text{V}}\ \text{(sat'd, 1 bar)}} + (1-0.8778) \times \underbrace{417.46}_{\hat{H}^{\text{L}}\ \text{(sat'd, 1 bar)}} = 2399.6\frac{\text{kJ}}{\text{kg}}$$

Energy balance

$$0 = \dot{M}_{\text{in}}\hat{H}_{\text{in}} + \dot{M}_{\text{out}}\hat{H}_{\text{out}} + \cancelto{0}{\dot{Q}} + \dot{W}_s - P\cancelto{0}{\frac{dV}{dt}}$$

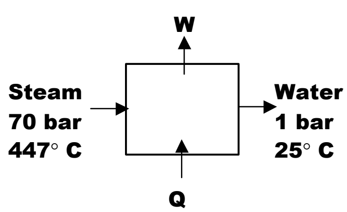
but $\dot{M}_{\text{in}} = -\dot{M}_{\text{out}}$

$$\Rightarrow -\frac{\dot{W}_s}{\dot{M}_{\text{in}}} = 3278.2 - 2399.6 = 878.6\frac{\text{kJ}}{\text{kg}}$$

(c) Saturated vapor at 1 bar

$$\hat{S} = 7.3594\ \text{kJ/kg K};\ \hat{H} = 2675.5\ \text{kJ/kg}$$
$$-\left.\frac{W_s}{\dot{M}_{\text{in}}}\right|_{\text{Actual}} = 3278.2 - 2675.5 = 602.7\ \text{kJ/kg}$$
$$\text{Efficiency }(\%) = \frac{602.7\times 100}{878.6} = 68.6\%$$
$$\frac{\dot{S}_{\text{gen}}}{\dot{M}_{\text{in}}} = 7.3594 - 6.619 = 0.740\ \text{kJ/Kh}$$

(d)



$$0 = \dot{M}_1 + \dot{M}_2 \Rightarrow \dot{M}_2 = -\dot{M}_1$$
$$0 = \dot{M}_1\left(\hat{H}_1 - \hat{H}_2\right) + \dot{W}_s + \dot{Q} - P\frac{dV}{dt}$$
$$0 = \dot{M}_1\left(\hat{S}_1 - \hat{S}_2\right) + \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}}$$

Simplifications to balance equations

$\dot{S}_{\text{gen}} = 0$ (for maximum work); $P\dfrac{dV}{dt} = 0$ (constant volume)

$\dfrac{\dot{Q}}{T} = \dfrac{\dot{Q}}{T_0}$ where $T_0 = 25^\circ\text{C}$ (all heat transfer at ambient temperature)

$$\hat{H}\text{ (sat'd liq, }25^\circ\text{C)} = 104.89\frac{\text{kJ}}{\text{kg}};\ \hat{S}\text{ (sat'd liq, }T = 25^\circ\text{C)} = 0.3674\frac{\text{kJ}}{\text{kg K}}$$

$$\frac{\dot{Q}}{\dot{M}} = T_0\left(\hat{S}_2 - \hat{S}_1\right);\ \left.\frac{-\dot{W}_s}{\dot{M}}\right|_{\max} = \hat{H}_1 - \hat{H}_2 + T_0\left(\hat{S}_2 - \hat{S}_1\right) = \left(\hat{H}_1 - T_0\hat{S}_1\right) - \left(\hat{H}_2 - T_0\hat{S}_2\right)$$

$$\left.\frac{-\dot{W}_s}{\dot{M}}\right|_{\max} = [3278.2 - 298.15\times 6.619] - [104.89 - 298.15\times 0.3674]$$

$$= 1304.75 + 4.65 = 1309.4\ \text{kJ/kg}$$

3.12 Take that portion of the methane initially in the tank that is also in the tank finally to be in the system. This system is isentropic $S_f = S_i$.

(a) The ideal gas solution

$$\underline{S}_f = \underline{S}_i \Rightarrow T_f = T_i\left(\frac{P_f}{P_i}\right)^{R/C_p} = 300\left(\frac{3.5}{70}\right)^{8.314/36} = 150.2\text{ K}$$

$$N = \frac{PV}{RT} \Rightarrow N_i = \frac{P_i V}{RT_i} = 1964.6\text{ mol};\ N_f = \frac{P_f V}{RT_f} = 196.2\text{ mol}$$

$$\Delta N = N_f - N_i = -1768.4\text{ mol}$$

(b) Using Figure 2.4-2.

70 bar ≈ 7 MPa, T = 300 K $\qquad \hat{S}_i = 5.05\ \text{kJ/kg K} = \hat{S}_f$

$$\hat{V}_i = 0.0195\frac{\text{m}^3}{\text{kg}},\ \text{so that } m_i = \frac{0.7\text{m}^3}{0.0195\frac{\text{m}^3}{\text{kg}}} = 35.90\text{ kg.}$$

$$N_i = \frac{35.90\text{ kg}\times 1000\frac{\text{g}}{\text{kg}}}{28\frac{\text{g}}{\text{mol}}} = 1282\text{ mol}$$

At 3.5 bar = 0.35 MPa and $\hat{S}_f = 5.05\ \text{kJ/kg K} \Rightarrow T \approx 138$ K. Also,

$$\hat{V}_f = 0.192\frac{\text{m}^3}{\text{kg}},\ \text{so that } m_f = \frac{0.7\text{m}^3}{0.192\frac{\text{m}^3}{\text{kg}}} = 3.646\text{ kg.}$$

$$N_f = \frac{3.646\text{ kg}\times 1000\frac{\text{g}}{\text{kg}}}{28\frac{\text{g}}{\text{mol}}} = 130.2\text{ mol}$$

$$\Delta N = N_f - N_i = 130.2 - 1282 = -1151.8\text{ mol}$$

3.13 $d\underline{S} = C\dfrac{dT}{T} + R\dfrac{d\underline{V}}{\underline{V}}$ eqn. (3.4-1)

$$\Delta S = \int\left[(a - R) + bT + cT^2 + dT^3 + \frac{e}{T^2}\right]\frac{dT}{T} + R\int\frac{d\underline{V}}{\underline{V}}$$

so that

$$\underline{S}(T_2,\ \underline{V}_2)-\underline{S}(T_1,\ \underline{V}_1)=(a-R)\ln\frac{T_2}{T_1}+b(T_2-T_1)+\frac{c}{2}\left(T_2^2-T_1^2\right)$$
$$+\frac{d}{3}\left(T_2^3-T_1^3\right)-\frac{e}{2}\left(T_2^{-2}-T_1^{-2}\right)+R\ln\frac{\underline{V}_2}{\underline{V}_1}$$

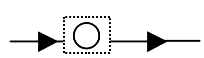
Now using

$$P\underline{V}=RT\Rightarrow\frac{\underline{V}_2}{\underline{V}_1}=\frac{T_2}{T_1}\cdot\frac{P_1}{P_2}\Rightarrow$$
$$\underline{S}(T_2,\ P_2)-S(T_1,\ P_1)=a\ln\frac{T_2}{T_1}+b(T_2-T_1)+\frac{c}{2}\left(T_2^2-T_1^2\right)$$
$$+\frac{d}{3}\left(T_2^3-T_1^3\right)-\frac{e}{2}\left(T_2^{-2}-T_1^{-2}\right)-R\ln\frac{P_2}{P_1}$$

Finally, eliminating T_2 using $T_2=T_1P_2\underline{V}_2/P_1\underline{V}_1$ yields

$$\underline{S}(P_2,\underline{V}_2)-\underline{S}(P_1,\underline{V}_1)=a\ln\left(\frac{P_2\underline{V}_2}{P_1\underline{V}_1}\right)+\frac{b}{R}(P_2\underline{V}_2-P_1\underline{V}_1)$$
$$+\frac{c}{2R^2}\left[(P_2\underline{V}_2)^2-(P_1\underline{V}_1)^2\right]$$
$$+\frac{d}{3R^3}\left[(P_2\underline{V}_2)^3-(P_1\underline{V}_1)^3\right]$$
$$-\frac{eR^2}{2}\left[\left(P_2\underline{V}_2^{-2}\right)-\left(P_1\underline{V}_1^{-2}\right)\right]-R\ln\frac{P_2}{P_1}$$

3.14 System: contents of valve (steady-state, adiabatic, constant volume system)



Mass balance $\quad 0=\dot{N}_1+\dot{N}_2$

Energy balance $\quad 0=\dot{N}_1\underline{H}_1+\dot{N}_2\underline{H}_2+\cancelto{0}{\dot{Q}}+\cancelto{0}{\dot{W}_s}-P\cancelto{0}{\frac{dV}{dt}}$

$$\Rightarrow\underline{H}_1=\underline{H}_2$$

Entropy balance $\quad 0=\dot{N}_1\underline{S}_1+\dot{N}_2\underline{S}_2+\dot{S}_{\text{gen}}+\cancelto{0}{\frac{\dot{Q}}{T}}$

$$\Rightarrow\Delta\underline{S}=\underline{S}_2-\underline{S}_1=\frac{\dot{S}_{\text{gen}}}{\dot{N}}$$

(a) Using the Mollier Diagram for steam (Fig. 2.4-1a) or the Steam Tables

$$\begin{array}{ll}T_1=600\text{ K} & P_2=7\text{ bar}\\ P_1=35\text{ bar} & \hat{H}_2=3045.3\text{ J/g}\end{array}\Rightarrow\begin{array}{l}T_2\approx 293^\circ\text{C}\\ \hat{S}_2=7.277\text{ J/g K}\end{array}$$

$\hat{H}_1=\hat{H}_2=3045.3\text{ J/g}$. Thus $\hat{S}_1=6.5598\text{ J/g K}$; $T_{\text{exit}}=293^\circ\text{C}$
$\Delta\hat{S}=\hat{S}_2-\hat{S}_1=0.717\text{ J/g K}$

(b) For the ideal gas, $\underline{H}_1=\underline{H}_2\Rightarrow T_1=T_2=600\text{ K}$