12–1.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)\ \text{m/s}^2$, where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

SOLUTION

$a = 2t - 6$

$dv = a\,dt$

$$\int_0^v dv = \int_0^t (2t - 6)\,dt$$

$v = t^2 - 6t$

$ds = v\,dt$

$$\int_0^s ds = \int_0^t (t^2 - 6t)\,dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6\ s$,

$v = 0$ **Ans.**

When $t = 11\ s$,

$s = 80.7$ m **Ans.**

Ans:
$v = 0$
$s = 80.7$ m

12–2.

The acceleration of a particle as it moves along a straight line is given by $a = (4t^3 - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 2\ \text{m}$ and $v = 5\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 5\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$\int_5^v dv = \int_0^t (4\,t^3 - 1)\,dt$$

$$v = t^4 - t + 5$$

$$\int_2^s ds = \int_0^t (t^4 - t + 5)\,dt$$

$$s = \frac{1}{5}t^5 - \frac{1}{2}t^2 + 5\,t + 2$$

When $t = 5$ s,

$$v = 625\ \text{m/s} \qquad \textbf{Ans.}$$

$$s = 639.5\ \text{m} \qquad \textbf{Ans.}$$

Since $v \neq 0$ then

$$d = 639.5 - 2 = 637.5\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$v = 625\ \text{m/s}$
$s = 639.5\ \text{m}$
$d = 637.5\ \text{m}$

12–3.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

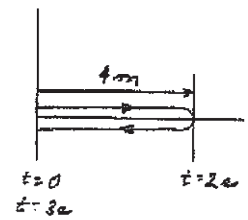
SOLUTION

$v = 6t - 3t^2$

$a = \dfrac{dv}{dt} = 6 - 6t$

At $t = 3$ s

$a = -12 \text{ m/s}^2$ **Ans.**



$ds = v\,dt$

$\int_0^s ds = \int_0^t (6t - 3t^2)dt$

$s = 3t^2 - t^3$

At $t = 3$ s

$s = 0$ **Ans.**

Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s.

when $t = 2$ s, $\quad s = 3(2)^2 - (2)^3 = 4$ m

$s_T = 4 + 4 = 8$ m **Ans.**

$(v_{sp})_{\text{avg}} = \dfrac{s_T}{t} = \dfrac{8}{3} = 2.67$ m/s **Ans.**

Ans:

$a = -12 \text{ m/s}^2$
$s = 0$
$s_T = 8$ m
$(v_{sp})_{\text{avg}} = 2.67$ m/s

***12–4.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$s = 10t^2 + 20$

(a) $s|_{1\text{ s}} = 10(1)^2 + 20 = 30 \text{ mm}$

$s|_{5\text{ s}} = 10(5)^2 + 20 = 270 \text{ mm}$

$\Delta s = 270 - 30 = 240 \text{ mm}$ **Ans.**

(b) $\Delta t = 5 - 1 = 4 \text{ s}$

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{240}{4} = 60 \text{ mm/s}$ **Ans.**

(c) $a = \dfrac{d^2s}{dt^2} = 20 \text{ mm/s}^2$ (for all t) **Ans.**

Ans:
$\Delta s = 240 \text{ mm}$
$v_{avg} = 60 \text{ mm/s}$
$a = 20 \text{ mm/s}^2$

12–5.

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

SOLUTION

$s = t^2 - 6t + 5$

$v = \dfrac{ds}{dt} = 2t - 6$

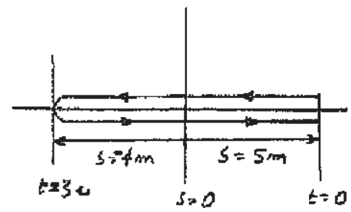
$a = \dfrac{dv}{dt} = 2$

$v = 0$ when $t = 3$

$s|_{t=0} = 5$

$s|_{t=3} = -4$

$s|_{t=6} = 5$



$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{0}{6} = 0$ **Ans.**

$(v_{sp})_{avg} = \dfrac{s_T}{\Delta t} = \dfrac{9 + 9}{6} = 3 \text{ m/s}$ **Ans.**

$a|_{t=6} = 2 \text{ m/s}^2$ **Ans.**

Ans:
$v_{avg} = 0$
$(v_{sp})_{avg} = 3 \text{ m/s}$
$a|_{t=6\text{ s}} = 2 \text{ m/s}^2$

12–6.

A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.

SOLUTION

$+\downarrow\ s = s_1 + v_1 t + \frac{1}{2}a_c t^2$

$s_A = 0 + 0 + \frac{1}{2}(9.81)(2)^2$

$s_A = 19.62\text{ m}$

$s_A = 0 + 0 + \frac{1}{2}(9.81)(1)^2$

$s_B = 4.91\text{ m}$

$\Delta s = 19.62 - 4.91 = 14.71\text{ m}$ **Ans.**

Ans:

$\Delta s = 14.71\text{ m}$

12–7.

A bus starts from rest with a constant acceleration of 1 m/s^2. Determine the time required for it to attain a speed of 25 m/s and the distance traveled.

SOLUTION

***Kinematics*:**

$v_0 = 0$, $v = 25$ m/s, $s_0 = 0$, and $a_c = 1$ m/s^2.

$(\xrightarrow{+})$ $\quad v = v_0 + a_c t$

$25 = 0 + (1)t$

$t = 25$ s **Ans.**

$(\xrightarrow{+})$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$25^2 = 0 + 2(1)(s - 0)$

$s = 312.5$ m **Ans.**

Ans:

$t = 25$ s
$s = 312.5$ m

***12–8.**

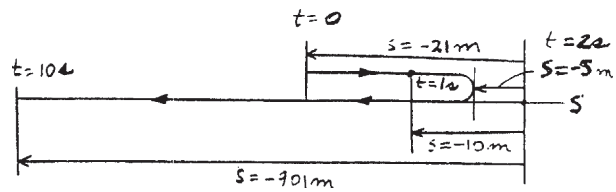
A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$$v = 12 - 3t^2 \qquad \textbf{(1)}$$

$$a = \frac{dv}{dt} = -6t\big|_{t=4} = -24 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\int_{-10}^{s} ds = \int_1^t v\,dt = \int_1^t \left(12 - 3t^2\right)dt$$



$$s + 10 = 12t - t^3 - 11$$

$$s = 12t - t^3 - 21$$

$$s\big|_{t=0} = -21$$

$$s\big|_{t=10} = -901$$

$$\Delta s = -901 - (-21) = -880 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (1):

$v = 0$ when $t = 2s$

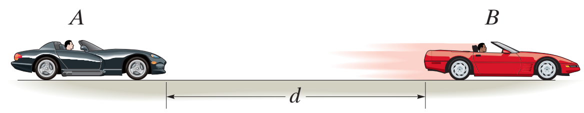
$$s\big|_{t=2} = 12(2) - (2)^3 - 21 = -5$$

$$s_T = (21 - 5) + (901 - 5) = 912 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$a = -24 \text{ m/s}^2$
$\Delta s = -880$ m
$s_T = 912$ m

12–9.

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



SOLUTION

Motion of car A:

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \qquad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B:

$$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right| \qquad \textbf{Ans.}$$

Ans:

$$S_{BA} = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right|$$

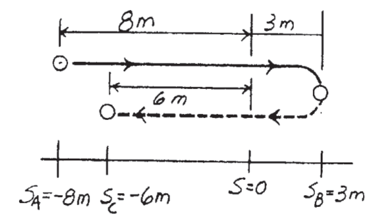
12–10.

A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

SOLUTION

Average Velocity: The displacement from A to C is $\Delta s = s_C - S_A = -6 - (-8) = 2$ m.

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{2}{4+5} = 0.222 \text{ m/s} \qquad \textbf{Ans.}$$



Average Speed: The distances traveled from A to B and B to C are $s_{A \to B} = 8 + 3 = 11.0$ m and $s_{B \to C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{Tot} = s_{A \to B} + s_{B \to C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{sp})_{avg} = \frac{s_{Tot}}{\Delta t} = \frac{20.0}{4+5} = 2.22 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{avg} = 0.222$ m/s
$(v_{sp})_{avg} = 2.22$ m/s

12–11.

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3})$ hr $= 30$ s **Ans.**

$v^2 = v_1^2 + 2\,a_c(s - s_1)$

$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792$ km $= 792$ m **Ans.**

Ans:
$t = 30$ s
$s = 792$ m

***12–12.**

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$v = 1.29$ m/s **Ans.**

Ans:
$v = 1.29$ m/s

12–13.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$a = 2t - 1$

$dv = a\,dt$

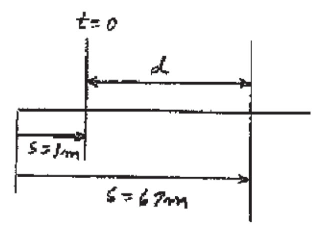
$$\int_2^v dv = \int_0^t (2t - 1)dt$$

$v = t^2 - t + 2$

$dx = v\,dt$

$$\int_t^s ds = \int_0^t (t^2 - t + 2)dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$



When $t = 6\ \text{s}$

$v = 32\ \text{m/s}$ **Ans.**

$s = 67\ \text{m}$ **Ans.**

Since $v \neq 0$ for $0 \leq t \leq 6\ \text{s}$, then

$d = 67 - 1 = 66\ \text{m}$ **Ans.**

Ans:
$v = 32\ \text{m/s}$
$s = 67\ \text{m}$
$d = 66\ \text{m}$

12–14.

A train starts from rest at station A and accelerates at $0.5\ \text{m/s}^2$ for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at $1\ \text{m/s}^2$ until it is brought to rest at station B. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60$ s, and $a_c = 0.5\ \text{m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900\ \text{m}$$

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30\ \text{m/s}$$

For stage (2) motion, $v_0 = 30\ \text{m/s}$, $s_0 = 900$ m, $a_c = 0$ and $t = 15(60) = 900$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900\ \text{m}$$

For stage (3) motion, $v_0 = 30\ \text{m/s}$, $v = 0$, $s_0 = 27\,900$ m and $a_c = -1\ \text{m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30\ \text{s}$$

$$\overset{+}{\rightarrow} \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350\ \text{m} = 28.4\ \text{km}$$ **Ans.**

Ans:
$s = 28.4$ km

12–15.

A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

SOLUTION

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2}\,ds = \int_0^t -4\,dt$$

$$-s^{-1}\big|_2^s = -4t\big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4\left(\frac{2}{8t + 1}\right)^2 = -\frac{16}{(8t + 1)^2}\ \text{m/s}$$ **Ans.**

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \frac{256}{(8t + 1)^3}\ \text{m/s}^2$$ **Ans.**

Ans:

$$v = \frac{16}{(8t + 1)^2}\ \text{m/s}$$

$$a = \frac{256}{(8t + 1)^3}\ \text{m/s}^2$$

***12–16.**

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.

SOLUTION

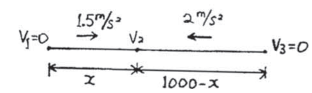
Using formulas of constant acceleration:

$v_2 = 1.5\,t_1$

$x = \frac{1}{2}(1.5)(t_1^2)$

$0 = v_2 - 2\,t_2$

$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$



Combining equations:

$t_1 = 1.33\,t_2; \quad v_2 = 2\,t_2$

$x = 1.33\,t_2^2$

$1000 - 1.33\,t_2^2 = 2\,t_2^2 - t_2^2$

$t_2 = 20.702$ s; $\qquad t_1 = 27.603$ s

$t = t_1 + t_2 = 48.3$ s **Ans.**

Ans:
$t = 48.3$ s

12–17.

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{d\nu}{dt} = -\mathrm{k}\nu^3$$

$$\int_{\nu 0}^{\nu} \nu^{-3}\, d\nu = \int_0^t -k\, dt$$

$$-\frac{1}{2}(\nu^{-2} - \nu_0^{-2}) = -kt$$

$$\nu = \left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{-\frac{1}{2}}$$ **Ans.**

$$ds = \nu\, dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \left.\frac{2\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}{2k}\right|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{\nu_0}\right]$$ Ans.

Ans:

$$v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$$

$$s = \frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0}\right]$$

12–18.

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})\ \text{m/s}^2$, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled: The distance traveled by the particle can be determined by applying Eq. 12–3.

$$ds = \frac{v\,dv}{a}$$

$$\int_0^s ds = \int_{6\text{ m/s}}^{v} \frac{v}{-1.5v^{\frac{1}{2}}}\,dv$$

$$s = \int_{6\text{ m/s}}^{v} -0.6667\,v^{\frac{1}{2}}\,dv$$

$$= \left(-0.4444v^{\frac{3}{2}} + 6.532\right)\text{m}$$

When $v = 0$, $\quad s = -0.4444\left(0^{\frac{3}{2}}\right) + 6.532 = 6.53\text{ m}$ **Ans.**

Time: The time required for the particle to stop can be determined by applying Eq. 12–2.

$$dt = \frac{dv}{a}$$

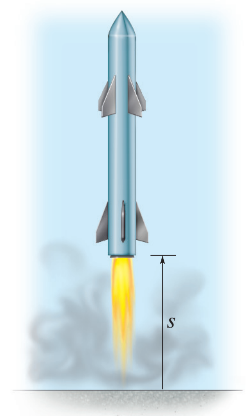
$$\int_0^t dt = -\int_{6\text{ m/s}}^{v} \frac{dv}{1.5v^{\frac{1}{2}}}$$

$$t = -1.333\left(v^{\frac{1}{2}}\right)\Big|_{6\text{ m/s}}^{v} = \left(3.266 - 1.333v^{\frac{1}{2}}\right)\text{s}$$

When $v = 0$, $\quad t = 3.266 - 1.333\left(0^{\frac{1}{2}}\right) = 3.27\text{ s}$ **Ans.**

Ans:
$s = 6.53\text{ m}$
$t = 3.27\text{ s}$

12–19. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \mathrm{m/s^2}$, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this attitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$b = 6\ \mathrm{m/s^2}$ $\quad c = 0.02\ \mathrm{s^{-2}}$ $\quad s_{p1} = 2000\ \mathrm{m}$

$$a_p = b + c\,s_p = v_p \frac{dv_p}{ds_p}$$

$$\int_0^{v_p} v_p\, dv_p = \int_0^{s_p} \left(b + c\,s_p\right) ds_p$$

$$\frac{v_p^{\,2}}{2} = b\,s_p + \frac{c}{2}s_p^{\,2}$$

$$v_p = \frac{ds_p}{dt} = \sqrt{2b\,s_p + c\,s_p^{\,2}} \qquad v_{p1} = \sqrt{2b\,s_{p1} + c\,s_{p1}^{\,2}} \qquad v_{p1} = 322.49\ \mathrm{m/s}$$ **Ans.**

$$t = \int_0^{s_p} \frac{1}{\sqrt{2b\,s_p + c\,s_p^{\,2}}}\, ds_p \qquad t_1 = \int_0^{s_{p1}} \frac{1}{\sqrt{2b\,s_p + c\,s_p^{\,2}}}\, ds_p \qquad t_1 = 19.27\ \mathrm{s}$$ **Ans.**

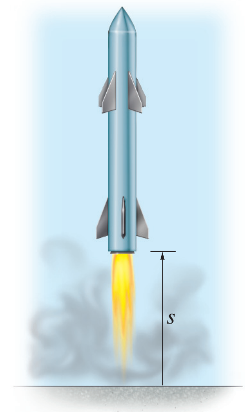
Ans:

$v_{p1} = 322.49\ \dfrac{\mathrm{m}}{\mathrm{s}}$

$t_1 = 19.27\ \mathrm{s}$

***12–20.**

The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$a\,ds = v\,dv$$

$$\int_0^s (6 + 0.02\,s)\,ds = \int_0^v v\,dv$$

$$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12\,s + 0.02\,s^2}$$

$$ds = v\,dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$$

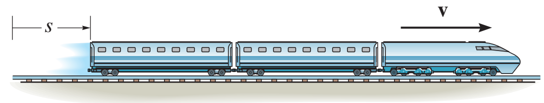
$$\frac{1}{\sqrt{0.02}}\ln\left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right]_0^{100} = t$$

$t = 5.62$ s **Ans.**

Ans:
$t = 5.62$ s

12–21.

When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60\,v^{-4})\ \text{m/s}^2$, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.



SOLUTION

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300}(v^5 - 32)$$

$$v = 3.925\ \text{m/s} = 3.93\ \text{m/s}$$ **Ans.**

$$a\,ds = v\,dv$$

$$ds = \frac{v\,dv}{a} = \frac{1}{60}v^5\,dv$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\,dv$$

$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Bigg|_2^{3.925}$$

$$= 9.98\ \text{m}$$ **Ans.**

Ans:
$v = 3.93\ \text{m/s}$
$s = 9.98\ \text{m}$

12–22.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION

$a = 2t - 9$

$$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$$

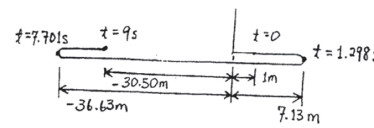
$v - 10 = t^2 - 9\,t$

$v = t^2 - 9\,t + 10$

$$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$$

$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$

$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$



Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m **Ans.**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$\quad s_{Tot} = 56.0$ m **Ans.**

(c) $\quad v = 10$ m/s **Ans.**

Ans:
(a) $s = -30.5$ m
(b) $s_{\text{Tot}} = 56.0$ m
(c) $v = 10$ m/s

12–23.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \qquad \textbf{Ans.}$$

b) If $t \rightarrow \infty$, $\dfrac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. (1)

$$v_{max} = 100 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
(a) $v = 45.5$ m/s
(b) $v_{max} = 100$ m/s

***12–24.**

A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when $t = 0$ and hits the ground when $t = 8$ s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

SOLUTION

$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h = 0 + (-6)(8) + \frac{1}{2}(9.81)(8)^2$$

$$= 265.92 \text{ m}$$

During $t = 8$ s, the balloon rises

$$h' = vt = 6(8) = 48 \text{ m}$$

Altitude $= h + h' = 265.92 + 48 = 314$ m **Ans.**

$(+\downarrow) \qquad v = v_0 + a_c t$

$v = -6 + 9.81(8) = 72.5$ m/s **Ans.**

Ans:
$h = 314$ m
$v = 72.5$ m/s

12–25.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)\ \text{m/s}^2$, where v is in meters per second. If $v = 20\ \text{m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^{v} \frac{dv}{v} = \int_{0}^{t} -2\,dt$$

$$\ln\frac{v}{20} = -2t$$

$$v = \left(20e^{-2t}\right)\ \text{m/s} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = \left(-40e^{-2t}\right)\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\int_{0}^{s} ds = v\,dt = \int_{0}^{t} (20e^{-2t})dt$$

$$s = -10e^{-2t}\big|_{0}^{t} = -10\left(e^{-2t} - 1\right)$$

$$s = 10\left(1 - e^{-2t}\right)\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$v = \left(20e^{-2t}\right)\ \text{m/s}$
$a = \left(-40e^{-2t}\right)\ \text{m/s}^2$
$s = 10\left(1 - e^{-2t}\right)\ \text{m}$

12–26.

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

SOLUTION

Velocity:

$$\left(\xrightarrow{+}\right) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_1^s \frac{1}{4}s^{1/2}ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.\frac{1}{6}s^{3/2}\right|_1^s$$

$$v = \frac{1}{\sqrt{3}}(s^{3/2} - 1)^{1/2}\text{ m/s}$$

When $s = 2$ m, $v = 0.781$ m/s. **Ans.**

Ans:
$v = 0.781$ m/s

12–27.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_f/2}{v_f - v_f/2}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

Ans:

$$t = 0.549\left(\frac{v_f}{g}\right)$$

***12–28.**

A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$$(+\downarrow) \qquad dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right) \text{ m/s} \qquad \textbf{(1)}$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \qquad t = 3.00 \text{ s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$(+\downarrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

$$s = \left(27t - t^3\right) \text{ m} \qquad \textbf{(2)}$$

At $t = 3.00$ s, from Eq. (2)

$$s = 27(3.00) - 3.00^3 = 54.0 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s = 54.0$ m

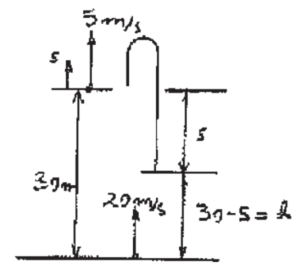
12–29.

A ball A is thrown vertically upward from the top of a 30-m-high building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

SOLUTION

Origin at roof:

Ball A:



$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = 0 + 5t - \frac{1}{2}(9.81)t^2$$

Ball B:

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = -30 + 20t - \frac{1}{2}(9.81)t^2$$

Solving,

$t = 2$ s **Ans.**

$s = 9.62$ m

Distance from ground,

$d = (30 - 9.62) = 20.4$ m **Ans.**

Also, origin at ground,

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2}(-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

Require

$$s_A = s_B$$

$$30 + 5t + \frac{1}{2}(-9.81)t^2 = 20t + \frac{1}{2}(-9.81)t^2$$

$t = 2$ s **Ans.**

$s_B = 20.4$ m **Ans.**

Ans:
$h = 20.4$ m
$t = 2$ s

12–30.

A boy throws a ball straight up from the top of a 12-m high tower. If the ball falls past him 0.75 s later, determine the velocity at which it was thrown, the velocity of the ball when it strikes the ground, and the time of flight.

SOLUTION

***Kinematics*:** When the ball passes the boy, the displacement of the ball in equal to zero.

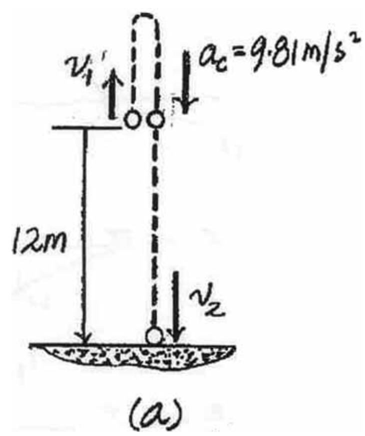
Thus, $s = 0$. Also, $s_0 = 0$, $v_0 = v_1$, $t = 0.75$ s, and $a_c = -9.81\text{ m/s}^2$.

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 0 + v_1(0.75) + \frac{1}{2}(-9.81)(0.75^2)$$

$$v_1 = 3.679\text{ m/s} = 3.68\text{ m/s}$$ **Ans.**

When the ball strikes the ground, its displacement from the roof top is $s = -12$ m. Also, $v_0 = v_1 = 3.679$ m/s, $t = t_2$, $v = v_2$, and $a_c = -9.81\text{ m/s}^2$.



$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-12 = 0 + 3.679t_2 + \frac{1}{2}(-9.81)t_2^2$$

$$4.905t_2^2 - 3.679t_2 - 12 = 0$$

$$t_2 = \frac{3.679 \pm \sqrt{(-3.679)^2 - 4(4.905)(-12)}}{2(4.905)}$$

Choosing the positive root, we have

$$t_2 = 1.983\text{ s} = 1.98\text{ s}$$ **Ans.**

Using this result,

$(+\uparrow)$ $$v = v_0 + a_c t$$

$$v_2 = 3.679 + (-9.81)(1.983)$$

$$= -15.8\text{ m/s} = 15.8\text{ m/s} \downarrow$$ **Ans.**

Ans:
$v_1 = 3.68$ m/s
$t_2 = 1.98$ s
$v_2 = 15.8\text{ m/s} \downarrow$

12–31.

The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

SOLUTION

Position:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{v_0 - ks}$$

$$t\Big|_0^t = -\frac{1}{k}\ln(v_0 - ks)\Big|_0^s$$

$$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - ks}\right)$$

$$e^{kt} = \frac{v_0}{v_0 - ks}$$

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right) \qquad \textbf{Ans.}$$

Velocity:

$$v = \frac{ds}{dt} = \frac{d}{dt}\left[\frac{v_0}{k}\left(1 - e^{-kt}\right)\right]$$

$$v = v_0 e^{-kt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(v_0 e^{-kt}\right)$$

$$a = -kv_0 e^{-kt} \qquad \textbf{Ans.}$$

Ans:

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right)$$

$$a = -kv_0 e^{-kt}$$

***12–32.**

Ball A is thrown vertically upwards with a velocity of v_0. Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_0/g$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

SOLUTION

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = v_0$, $(s_A)_0 = 0$, $s_A = h, t_A = t'$, and $(a_c)_A = -g$.

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}(a_c)_A t_A{}^2$$

$$h = 0 + v_0 t' + \frac{1}{2}(-g)(t')^2$$

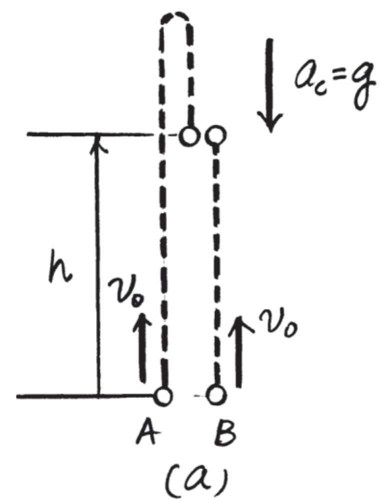
$$h = v_0 t' - \frac{g}{2}t'^2 \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad v_A = (v_A)_0 + (a_c)_A t_A$$

$$v_A = v_0 + (-g)(t')$$

$$v_A = v_0 - gt' \qquad \textbf{(2)}$$

The motion of ball B requires $(v_B)_0 = v_0$, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - t$, and $(a_c)_B = -g$.



$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}(a_c)_B t_B{}^2$$

$$h = 0 + v_0(t' - t) + \frac{1}{2}(-g)(t' - t)^2$$

$$h = v_0(t' - t) - \frac{g}{2}(t' - t)^2 \qquad \textbf{(3)}$$

$$(+\uparrow) \qquad v_B = (v_B)_0 + (a_c)_B t_B$$

$$v_B = v_0 + (-g)(t' - t)$$

$$v_B = v_0 - g(t' - t) \qquad \textbf{(4)}$$

Solving Eqs. (1) and (3),

$$v_0 t' - \frac{g}{2}t'^2 = v_0(t' - t) - \frac{g}{2}(t' - t)^2$$

$$t' = \frac{2v_0 + gt}{2g} \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (2) and (4),

$$v_A = v_0 - g\left(\frac{2v_0 + gt}{2g}\right)$$

$$= -\frac{1}{2}gt = \frac{1}{2}gt\downarrow \qquad \textbf{Ans.}$$

$$v_B = v_0 - g\left(\frac{2v_0 + gt}{2g} - t\right)$$

$$= \frac{1}{2}gt \uparrow \qquad \textbf{Ans.}$$

Ans:

$t' = \dfrac{2v_0 + gt}{2g}$

$v_A = \dfrac{1}{2}gt\downarrow$

$v_B = \dfrac{1}{2}gt\uparrow$

12–33.

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R+y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81\ \text{m/s}^2$ and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0R^2 \int_0^\infty \frac{dy}{(R+y)^2}$$

$$\left.\frac{v^2}{2}\right|_v^0 = \left.\frac{g_0R^2}{R+y}\right|_0^\infty$$

$$v = \sqrt{2g_0R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167\ \text{m/s} = 11.2\ \text{km/s}$$ **Ans.**

Ans:
$v = 11.2$ km/s

12–34.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–33), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–33.

SOLUTION

From Prob. 12–33,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R + y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0\,R^2 \int_{y_0}^{y} \frac{dy}{(R + y)^2} = \int_0^v v\,dv$$

$$g_0\,R^2 \left[\frac{1}{R + y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0\,R^2 [\frac{1}{R + y} - \frac{1}{R + y_0}] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0\,(y_0 - y)}{(R + y)(R + y_0)}} \qquad \textbf{Ans.}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356 + 500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow \qquad \textbf{Ans.}$$

Ans:

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R + y)(R + y_0)}}$$

$$v = 3.02 \text{ km/s}$$

12–35.

A train starts from station A and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station B. If the time for the whole journey is six minutes, draw the $v-t$ graph and determine the maximum speed of the train.

SOLUTION

For stage (1) motion,

$$(\overset{+}{\rightarrow}) \qquad v_1 = v_0 + (a_c)_1 t$$

$$v_{max} = 0 + (a_c)_1 t_1$$

$$v_{max} = (a_c)_1 t_1 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad v_1^2 = v_0^2 + 2(a_c)_1(s_1 - s_0)$$

$$v_{max}{}^2 = 0 + 2(a_c)_1(1000 - 0)$$

$$(a_c)_1 = \frac{v_{max}{}^2}{2000} \qquad \textbf{(2)}$$

Eliminating $(a_c)_1$ from Eqs. (1) and (2), we have

$$t_1 = \frac{2000}{v_{max}} \qquad \textbf{(3)}$$

For stage (2) motion, the train travels with the constant velocity of v_{max} for $t = (t_2 - t_1)$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s_2 = s_1 + v_1 t + \frac{1}{2}(a_c)_2 t^2$$

$$1000 + 2000 = 1000 + v_{max}(t_2 - t_1) + 0$$

$$t_2 - t_1 = \frac{2000}{v_{max}} \qquad \textbf{(4)}$$

For stage (3) motion, the train travels for $t = 360 - t_2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v_3 = v_2 + (a_c)_3 t$$

$$0 = v_{max} - (a_c)_3(360 - t_2)$$

$$v_{max} = (a_c)_3(360 - t_2) \qquad \textbf{(5)}$$

$$(\overset{+}{\rightarrow}) \qquad v_3^2 = v_2^2 + 2(a_c)_3(s_3 - s_2)$$

$$0 = v_{max}{}^2 + 2[-(a_c)_3](4000 - 3000)$$

$$(a_c)_3 = \frac{v_{max}{}^2}{2000} \qquad \textbf{(6)}$$

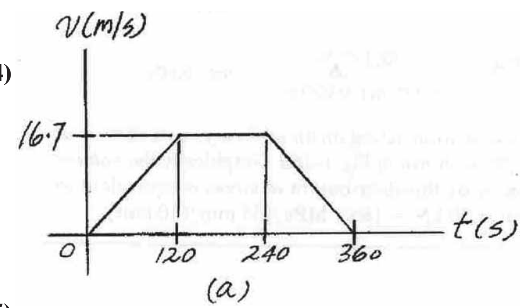
Eliminating $(a_c)_3$ from Eqs. (5) and (6) yields

$$360 - t_2 = \frac{2000}{v_{max}} \qquad \textbf{(7)}$$

Solving Eqs. (3), (4), and (7), we have

$$t_1 = 120 \text{ s} \qquad t_2 = 240 \text{ s}$$

$$v_{max} = 16.7 \text{ m/s} \qquad \textbf{Ans.}$$



Based on the above results, the $v-t$ graph is shown in Fig. a.

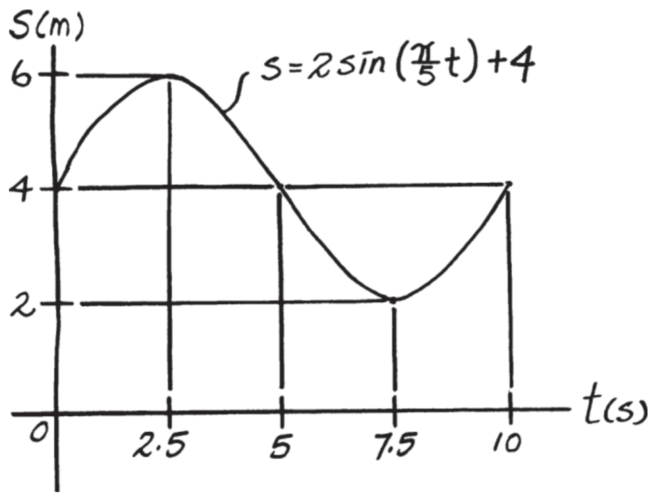
$v = v_{max}$ for 2 min < t < 4 min.

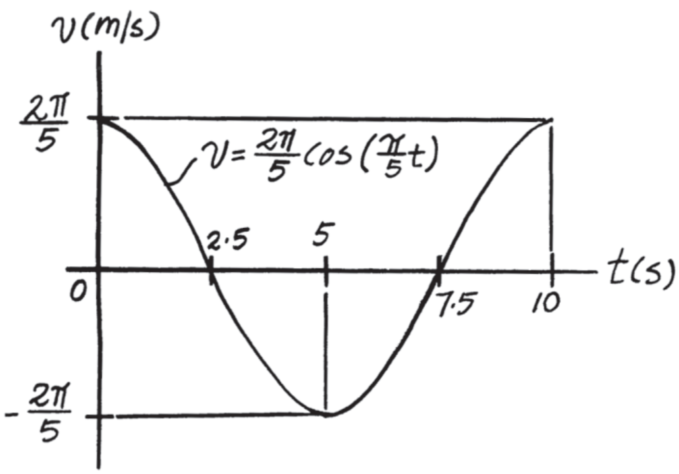
Ans:
$v_{max} = 16.7$ m/s

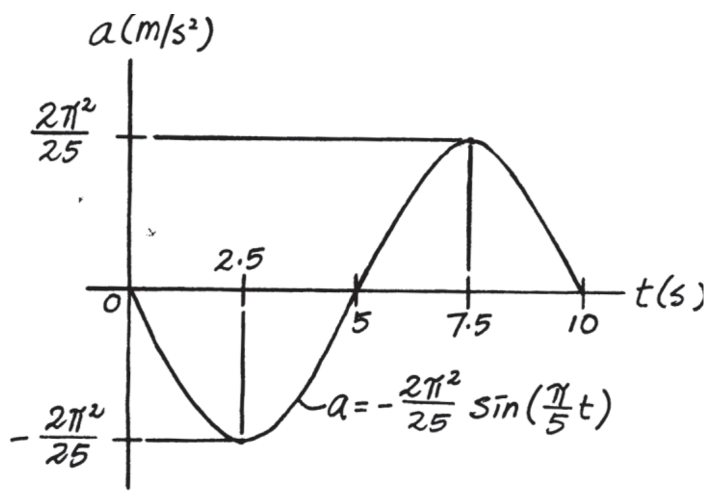
***12–36.**

If the position of a particle is defined by $s = [2\sin(\pi/5)t + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \le t \le 10$ s.

SOLUTION







Ans:

$s = 2\sin\left(\frac{\pi}{5}t\right) + 4$

$v = \frac{2\pi}{5}\cos\left(\frac{\pi}{5}t\right)$

$a = -\frac{2\pi^2}{25}\sin\left(\frac{\pi}{5}t\right)$

12–37.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \leq t \leq 4$ s.

SOLUTION

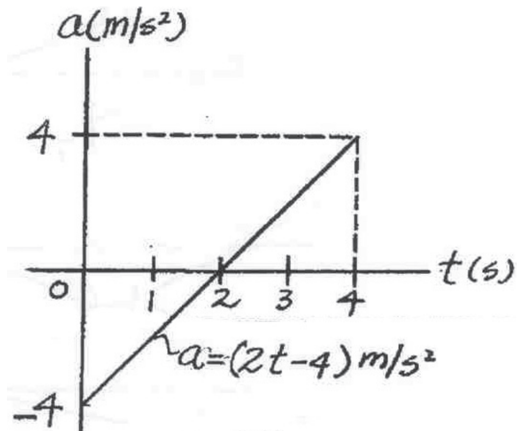
a–t Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$
$$a = (2t - 4)\text{ m/s}^2$$

Thus,

$$a|_{t=0} = 2(0) - 4 = -4\text{ m/s}^2$$
$$a|_{t=2} = 0$$
$$a|_{t=4\text{ s}} = 2(4) - 4 = 4\text{ m/s}^2$$

The $a-t$ graph is shown in Fig. a.



(a)

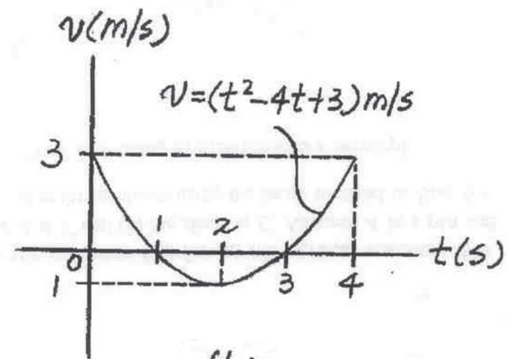
v–t Graph: The slope of the $v-t$ graph is zero when $a = \dfrac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \qquad t = 2\text{ s}$$

The velocity of the particle at $t = 0$ s, 2 s, and 4 s are

$$v|_{t=0\text{ s}} = 0^2 - 4(0) + 3 = 3\text{ m/s}$$
$$v|_{t=2\text{ s}} = 2^2 - 4(2) + 3 = -1\text{ m/s}$$
$$v|_{t=4\text{ s}} = 4^2 - 4(4) + 3 = 3\text{ m/s}$$

The $v-t$ graph is shown in Fig. b.



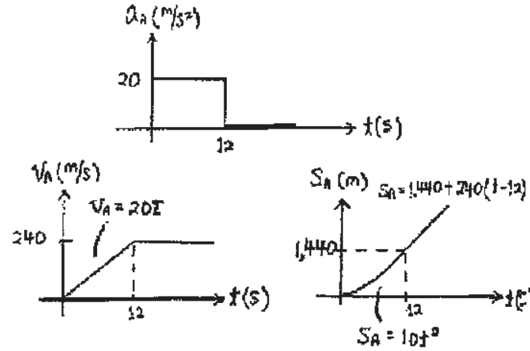
(b)

Ans:
$a|_{t=0} = -4\text{ m/s}^2$
$a|_{t=2\text{ s}} = 0$
$a|_{t=4\text{ s}} = 4\text{ m/s}^2$
$v|_{t=0} = 3\text{ m/s}$
$v|_{t=2\text{ s}} = -1\text{ m/s}$
$v|_{t=4\text{ s}} = 3\text{ m/s}$

12–38.

Two rockets start from rest at the same elevation. Rocket A accelerates vertically at 20 m/s^2 for 12 s and then maintains a constant speed. Rocket B accelerates at 15 m/s^2 until reaching a constant speed of 150 m/s. Construct the a–t, v–t, and s–t graphs for each rocket until $t = 20$ s. What is the distance between the rockets when $t = 20$ s?

SOLUTION



For rocket A

For $t < 12$ s

$$+\uparrow \quad v_A = (v_A)_0 + a_A t$$

$$v_A = 0 + 20t$$

$$v_A = 20t$$

$$+\uparrow \quad s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(20)\,t^2$$

$$s_A = 10\,t^2$$

When $t = 12$ s, $\quad v_A = 240$ m/s

$\quad s_A = 1440$ m

For $t > 12$ s

$v_A = 240$ m/s

$s_A = 1440 + 240(t - 12)$

For rocket B

For $t < 10$ s

$$+\uparrow \quad v_B = (v_B)_0 + a_B t$$

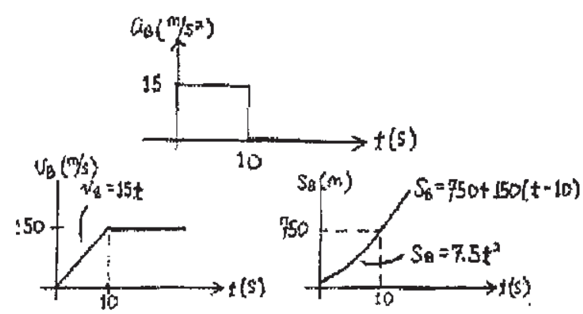
$$v_B = 0 + 15t$$

$$v_B = 15t$$

$$+\uparrow \quad s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(15)\,t^2$$

$$s_B = 7.5\,t^2$$



When $t = 10$ s, $\quad v_B = 150$ m/s

$\quad s_B = 750$ m

For $t > 10$ s

$v_B = 150$ m/s

$s_B = 750 + 150(t - 10)$

When $t = 20$ s, $\quad s_A = 3360$ m, $\quad s_B = 2250$ m

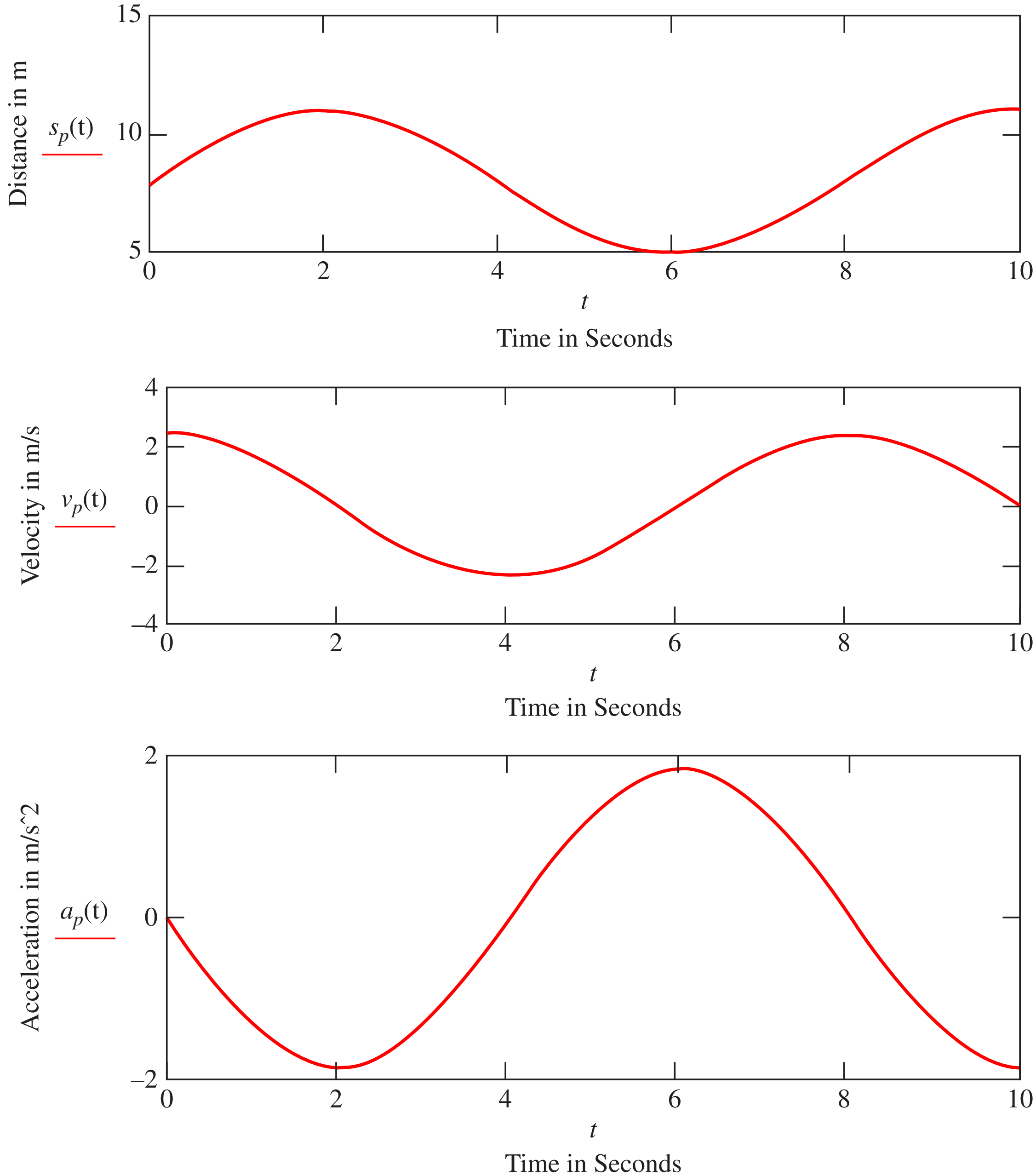
$\Delta s = 1110$ m $= 1.11$ km **Ans.**

Ans:
$\Delta s = 1.11$ km

12–39.

If the position of a particle is defined by $s = [3\sin(\pi/4)t + 8]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \leq t \leq 10$ s.

SOLUTION



Ans:

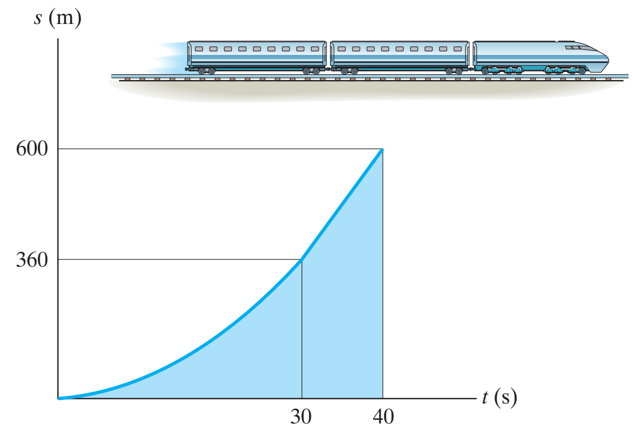
$$s = 3\sin\left(\frac{\pi}{4}t\right) + 8$$

$$v = \frac{3\pi}{4}\cos\left(\frac{\pi}{4}t\right)$$

$$a = -\frac{3\pi^2}{16}\sin\left(\frac{\pi}{4}t\right)$$

***12–40.**

The s–t graph for a train has been experimentally determined. From the data, construct the v–t and a–t graphs for the motion; $0 \leq t \leq 40$ s. For $0 \leq t \leq 30$ s, the curve is $s = (0.4t^2)$ m, and then it becomes straight for $t \geq 30$ s.



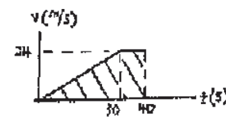
SOLUTION

$0 \leq t \leq 30$:

$$s = 0.4t^2 \qquad \textbf{Ans.}$$

$$v = \frac{ds}{dt} = 0.8t \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = 0.8 \qquad \textbf{Ans.}$$



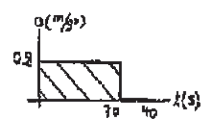
$30 \leq t \leq 40$:

$$s - 360 = \left(\frac{600 - 360}{40 - 30}\right)(t - 30)$$

$$s = 24(t - 30) + 360 \qquad \textbf{Ans.}$$

$$v = \frac{ds}{dt} = 24 \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = 0 \qquad \textbf{Ans.}$$



Ans:

$s = 0.4t^2$

$v = \frac{ds}{dt} = 0.8t$

$a = \frac{dv}{dt} = 0.8$

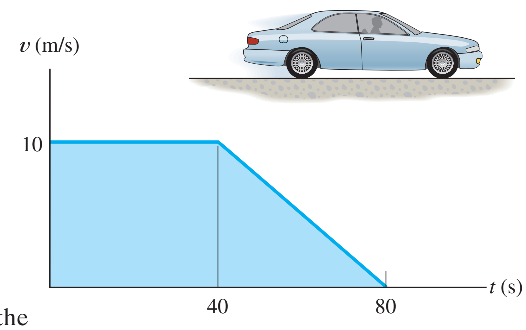
$s = 24(t - 30) + 360$

$v = \frac{ds}{dt} = 24$

$a = \frac{dv}{dt} = 0$

12–41.

The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the a–t graph.



SOLUTION

Distance Traveled: The total distance traveled can be obtained by computing the area under the $v - t$ graph.

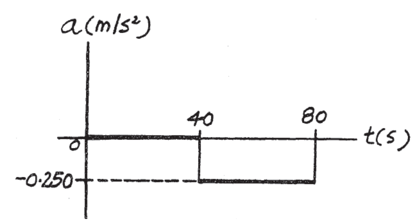
$$s = 10(40) + \frac{1}{2}(10)(80 - 40) = 600 \text{ m} \qquad \textbf{Ans.}$$

a – t Graph: The acceleration in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 40 \text{ s}$,

$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \leq 80 \text{ s}$, $\dfrac{v - 10}{t - 40} = \dfrac{0 - 10}{80 - 40}$, $v = \left(-\dfrac{1}{4}t + 20\right) \text{ m/s}$.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$

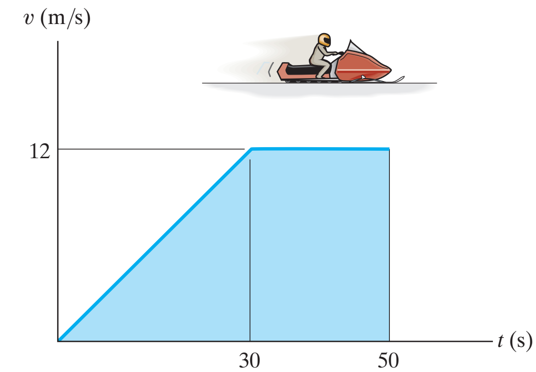


For $0 \leq t < 40$ s, $a = 0$.

For $40 \text{ s} < t \leq 80$, $a = -0.250 \text{ m/s}^2$.

Ans:
$s = 600$ m.
For $0 \leq t < 40$ s,
$a = 0$.
For $40 \text{ s} < t \leq 80$ s,
$a = -0.250 \text{ m/s}^2$

12–42. The snowmobile moves along a straight course according to the v–t graph. Construct the s–t and a–t graphs for the same 50-s time interval. When $t = 0$, $s = 0$.



SOLUTION

***s* − *t* Graph:** The position function in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $0\text{ s} \le t < 30\text{ s}$, $v = \frac{12}{30}t = \left(\frac{2}{5}t\right)$ m/s.

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{2}{5}t\,dt$$

$$s = \left(\frac{1}{5}t^2\right)\text{ m}$$

At $t = 30$ s, $\qquad s = \frac{1}{5}\left(30^2\right) = 180\text{ m}$

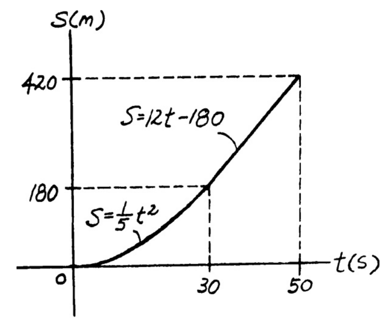
For time interval **30 s < *t* ≤ 50 s**,

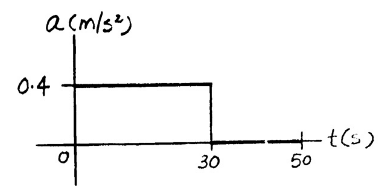
$$ds = v\,dt$$

$$\int_{180\text{ m}}^s ds = \int_{30\text{ s}}^t 12\,dt$$

$$s = (12t - 180)\text{ m}$$

At $t = 50$ s, $\qquad s = 12(50) - 180 = 420\text{ m}$

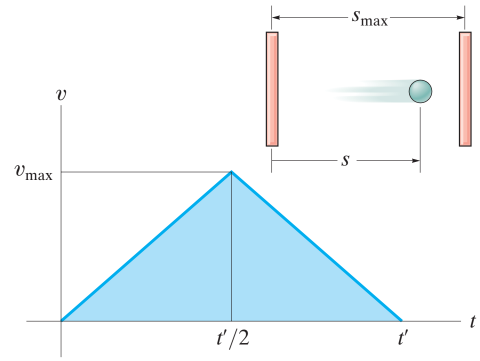




***a* − *t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval **0 s ≤ *t* < 30 s** and **30 s < *t* ≤ 50 s**, $a = \frac{dv}{dt} = \frac{2}{5} = 0.4\text{ m/s}^2$ and $a = \frac{dv}{dt} = 0$, respectively.

12–43.

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of $4\ \text{m/s}^2$. If the plates are spaced 200 mm apart, determine the maximum velocity $v_{\max}$ and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



SOLUTION

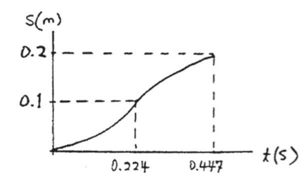
$a_c = 4\ \text{m/s}^2$

$\frac{s}{2} = 100\ \text{mm} = 0.1\ \text{m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442\ \text{m/s} \qquad = 0.894\ \text{m/s}$ **Ans.**



$v = v_0 + a_c\,t'$

$0.89442 = 0 + 4\left(\frac{t'}{2}\right)$

$t' = 0.44721\ \text{s} \qquad = 0.447\ \text{s}$ **Ans.**

$s = s_0 + v_0\,t + \frac{1}{2}\,a_c\,t^2$

$s = 0 + 0 + \frac{1}{2}(4)(t)^2$

$s = 2\,t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224$ s,

$s = 0.1$ m

$\int_{0.894}^{v} ds = -\int_{0.2235}^{t} 4\,dt$

$v = -4\,t + 1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

$s = -2\,t^2 + 1.788\,t - 0.2$

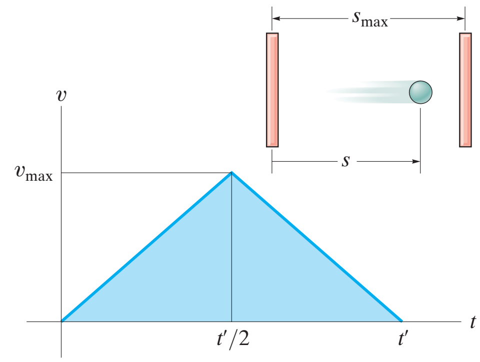
When $t = 0.447$ s,

$s = 0.2$ m **Ans.**

Ans:
$v_{max} = 0.894\ \text{m/s}$
$t' = 0.447\ \text{s}$
$s = 0.2\ \text{m}$

***12–44.**

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1\ s$,

$v = 100\,t$

$a = \dfrac{dv}{dt} = 100$

$ds = v\,dt$

$\int_0^s ds = \int_0^t 100\,t\,dt$

$s = 50\,t^2$

When $t = 0.1$ s,

$s = 0.5$ m

For $0.1\text{ s} < t < 0.2$ s,

$v = -100\,t + 20$

$a = \dfrac{dv}{dt} = -100$

$ds = v\,dt$

$\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20)\text{dt}$

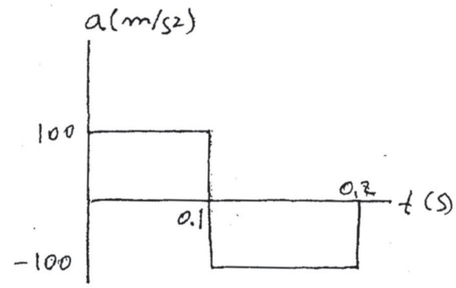
$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$

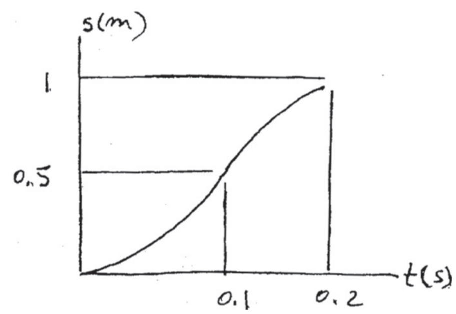
$s = -50\,t^2 + 20\,t - 1$

When $t = 0.2$ s,

$s = 1$ m

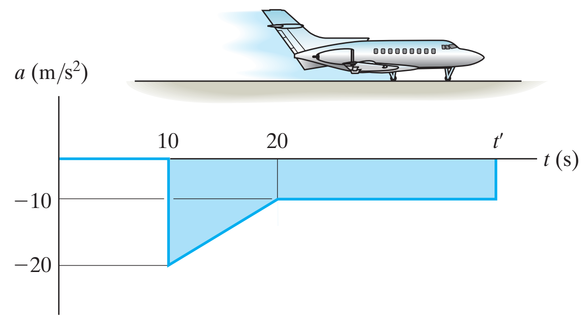
When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When t = 0.2 s, $s = 1$ m.





Ans:
When $t = 0.1$ s,
$s = 0.5$ m and a changes from
100 m/s^2 to -100 m/s^2. When $t = 0.2$ s,
$s = 1$ m.

12–45. The motion of a jet plane just after landing on a runway is described by the a–t graph. Determine the time t' when the jet plane stops. Construct the v–t and s–t graphs for the motion. Here $s = 0$, and $v = 150$ m/s when $t = 0$.



SOLUTION

***v–t* Graph**. The v–t function can be determined by integrating $dv = a\,dt$. For $0 \le t < 10$ s, a = 0. Using the initial condition $v = 150$ m/s at $t = 0$,

$$\int_{150\text{ m/s}}^{v} dv = \int_0^t 0\,dt$$

$$v - 150 = 0$$

$$v = 150\text{ m/s}$$ **Ans.**

For $10\text{ s} < t < 20\text{ s}$, $\dfrac{a - (-10)}{t - 10} = \dfrac{-5 - (-10)}{20 - 10}$, $a = \left(\dfrac{1}{2}t - 15\right)\text{m/s}^2$. Using the initial condition $v = 150$ m/s at $t = 10$ s,

$$\int_{150\text{ m/s}}^{v} dv = \int_{10\text{ s}}^{t}\left(\frac{1}{2}t - 15\right)dt$$

$$v - 150 = \left(\frac{1}{4}t^2 - 15t\right)\Big|_{10\text{ s}}^{t}$$

$$v = \left\{\frac{1}{4}t^2 - 15t + 275\right\}\text{m/s}$$ **Ans.**

At $t = 20$ s,

$$v|_{t=20\text{ s}} = \frac{1}{4}(20^2) - 15(20) + 275 = 75\text{ m/s}$$

For $20\text{ s} < t \le t'$, a = –5 m/s. Using the initial condition $v = 75$ m/s at t = 20 s,

$$\int_{75\text{ m/s}}^{v} dv = \int_{20\text{ s}}^{t} -5\,dt$$

$$v - 75 = (-5t)\Big|_{20\text{ s}}^{t}$$

$$v = \{-5t + 175\}\text{ m/s}$$ **Ans.**

It is required that at t = t′, $v = 0$. Thus

$$0 = -5t' + 175$$

$$t' = 35.0\text{ s}$$ **Ans.**

Using these results, the v–t graph shown in Fig. a can be plotted.

12–45. Continued

s-t Graph. The s–t function can be determined by integrating $ds = vdt$. For $0 \le t < 10$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 150\, dt$$

$$s = \{150\, t\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 10$ s,

$$s|_{t=10\text{ s}} = 150(10) = 1500 \text{ m}$$

For $10 \text{ s} < t < 20$ s, the initial condition is $s = 1500$ m at $t = 10$ s.

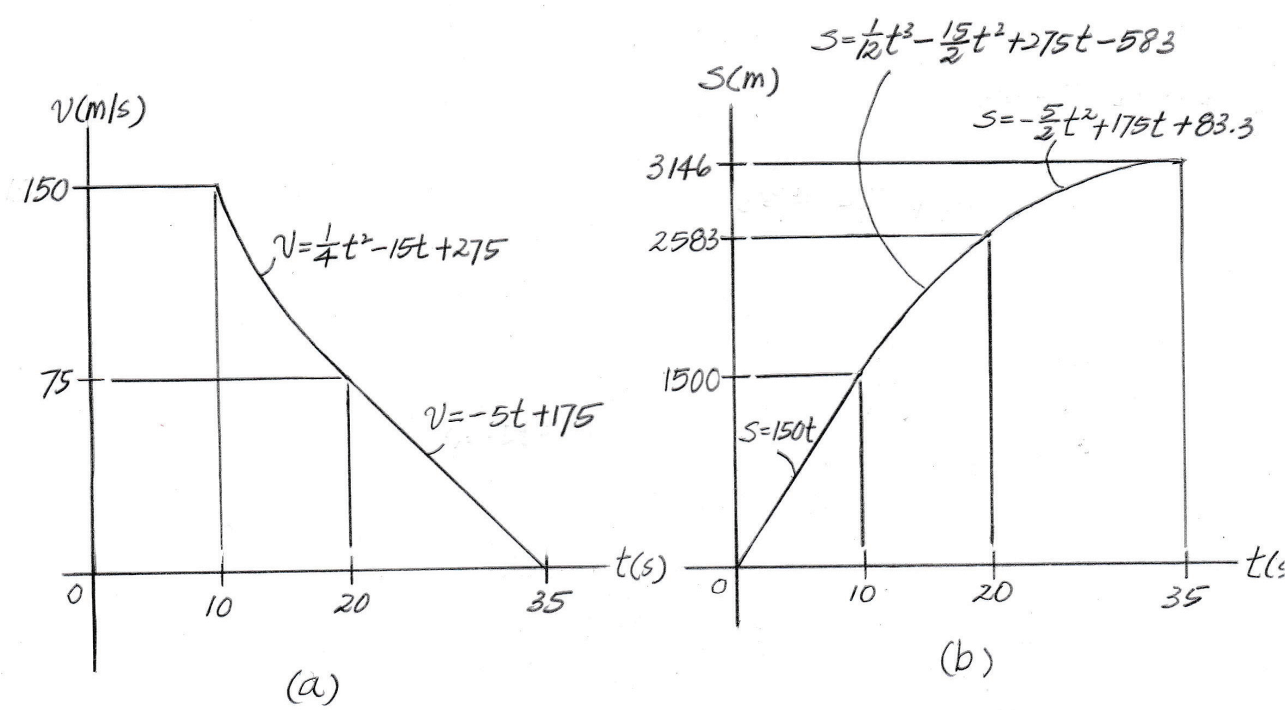
$$\int_{1500\text{ m}}^s ds = \int_{10\text{ s}}^t \left(\frac{1}{4}t^2 - 15t + 275\right)dt$$

$$s - 1500 = \left(\frac{1}{12}t^3 - \frac{15}{2}t^2 + 275t\right)\Big|_{10\text{ s}}^t$$

$$s = \left\{\frac{1}{12}t^3 - \frac{15}{2}t^2 + 275t - 583\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 20$ s,

$$s|_{t=20\text{ s}} = \frac{1}{12}(20^3) - \frac{15}{2}(20^2) + 275(20) - 583 = 2583 \text{ m}$$



12–45. Continued

For 20 s < $t \le 35$ s, the initial condition is $s = 2583$ m at $t = 20$ s.

$$\int_{2583\text{ m}}^{s} ds = \int_{20\text{ s}}^{t} (-5t + 175)\, dt$$

$$s - 2583 = \left(-\frac{5}{2}t^2 + 175t\right)\Big|_{20\text{ s}}^{t}$$

$$s = \left\{-\frac{5}{2}t^2 + 175t + 83.3\right\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 35$ s,

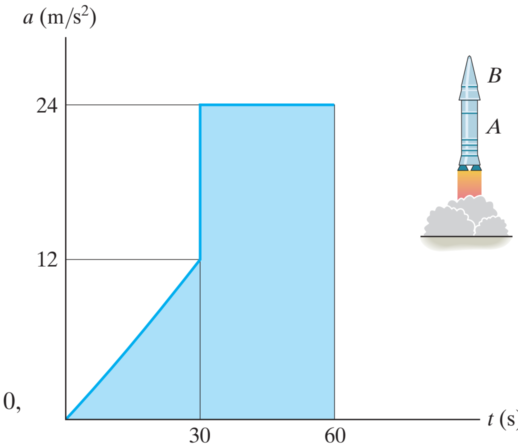
$$s|_{t=35\text{ s}} = -\frac{5}{2}(35^2) + 175(35) + 83.3 = 3146\text{ m}$$

using these results, the *s-t* graph shown in Fig. *b* can be plotted.

Ans:
$v = 150$ m/s
$v = \left\{\frac{1}{4}t^2 - 15t + 275\right\}$ m/s
$v = \{-5t + 175\}$ m/s
$t' = 35.0$ s
$s = \{150t\}$ m
$s = \left\{\frac{1}{12}t^3 - \frac{15}{2}t^2 + 275t - 583\right\}$ m
$s = \left\{\frac{-5}{2}t^2 + 175t + 83.3\right\}$ m

12–46.

A two-stage rocket is fired vertically from rest at $s = 0$ with the acceleration as shown. After 30 s the first stage, A, burns out and the second stage, B, ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \le t \le 60$ s.



SOLUTION

***v–t* Graph.** The $v-t$ function can be determined by integrating $dv = a\,dt$.

For $0 \le t < 30$ s, $a = \dfrac{12}{30}t = \left(\dfrac{2}{5}t\right)$ m/s². Using the initial condition $v = 0$ at $t = 0$,

$$\int_0^v dv = \int_0^t \frac{2}{5}t\,dt$$

$$v = \left\{\frac{1}{5}t^2\right\}\text{ m/s} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$v\Big|_{t=30\text{ s}} = \frac{1}{5}(30^2) = 180\text{ m/s}$$

For $30 < t \le 60$ s, $a = 24$ m/s². Using the initial condition $v = 180$ m/s at $t = 30$ s,

$$\int_{180\text{ m/s}}^v dv = \int_{30\text{ s}}^t 24\,dt$$

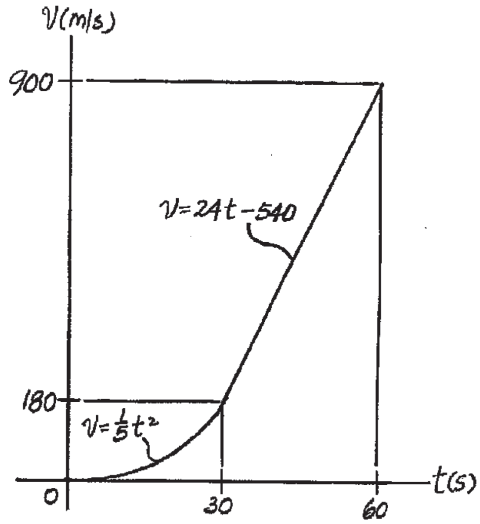
$$v - 180 = 24\,t\Big|_{30\text{ s}}^t$$

$$v = \{24t - 540\}\text{ m/s} \qquad \textbf{Ans.}$$

At $t = 60$ s,

$$v\Big|_{t=60\text{ s}} = 24(60) - 540 = 900\text{ m/s}$$

Using these results, $v-t$ graph shown in Fig. a can be plotted.



(a)

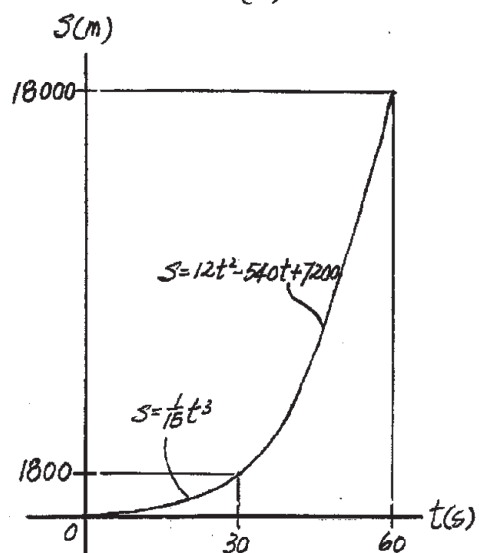
***s–t* Graph.** The $s-t$ function can be determined by integrating $ds = v\,dt$. For $0 \le t < 30$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{1}{5}t^2 dt$$

$$s = \left\{\frac{1}{15}t^3\right\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$s\Big|_{t=30\text{ s}} = \frac{1}{15}(30^3) = 1800\text{ m}$$



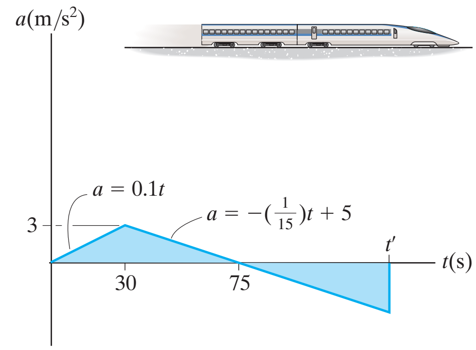
(b)

Ans:
For $0 \le t < 30$ s, $v = \left\{\dfrac{1}{5}t^2\right\}$ m/s, $s = \left\{\dfrac{1}{15}t^3\right\}$ m

For $30 \le t \le 60$ s, $v = \{24t - 540\}$ m/s,

12–47.

The *a–t* graph of the bullet train is shown. If the train starts from rest, determine the elapsed time t' before it again comes to rest. What is the total distance traveled during this time interval? Construct the *v–t* and *s–t* graphs.



SOLUTION

$v - t$ Graph: For the time interval $0 \le t < 30$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.1t\,dt$$

$$v = \left(0.05t^2\right)\ \text{m/s}$$

When $t = 30$ s,

$$v\big|_{t=30\ \text{s}} = 0.05\left(30^2\right) = 45\ \text{m/s}$$

or the time interval $30\ \text{s} < t \le t'$, the initial condition is $v = 45$ m/s at $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_{45\ \text{m/s}}^{v} dv = \int_{30\ \text{s}}^{t}\left(-\frac{1}{15}t + 5\right)dt$$

$$v = \left(-\frac{1}{30}t^2 + 5t - 75\right)\ \text{m/s}$$

Thus, when $v = 0$,

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

Choosing the root $t' > 75$ s,

$$t' = 133.09\ \text{s} = 133\ \text{s} \qquad \textbf{Ans.}$$

Also, the change in velocity is equal to the area under the *a–t* graph. Thus,

$$\Delta v = \int a\,dt$$

$$0 = \frac{1}{2}(3)(75) + \frac{1}{2}\left[\left(-\frac{1}{15}t' + 5\right)(t' - 75)\right]$$

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

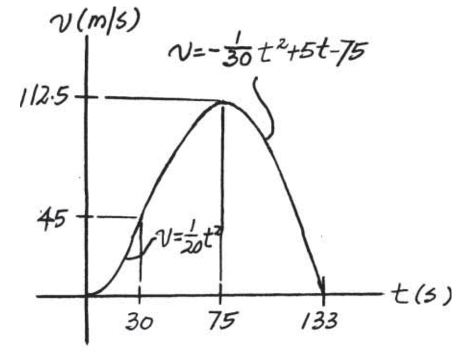
This equation is the same as the one obtained previously.

The slope of the v–$t$graph is zero when $t = 75$ s, which is the instant $a = \dfrac{dv}{dt} = 0$. Thus,

$$v\big|_{t=75\ \text{s}} = -\frac{1}{30}\left(75^2\right) + 5(75) - 75 = 112.5\ \text{m/s}$$

12–47. continued

The v–t graph is shown in Fig. *a*.



(a)

$s-t$ Graph: Using the result of v, the equation of the s–t graph can be obtained by integrating the kinematic equation $ds = v dt$. For the time interval $0 \leq t < 30$ s, the initial condition $s = 0$ at $t = 0$ s will be used as the integration limit. Thus,

$$(\xrightarrow{+}) \qquad ds = v dt$$

$$\int_0^s ds = \int_0^t 0.05t^2 \, dt$$

$$s = \left(\frac{1}{60}t^3\right) \text{m}$$

When $t = 30$ s,

$$s\big|_{t=30\text{ s}} = \frac{1}{60}\left(30^3\right) = 450 \text{ m}$$

For the time interval $30 \text{ s} < t \leq t' = 133.09$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\xrightarrow{+}) \qquad ds = v dt$$

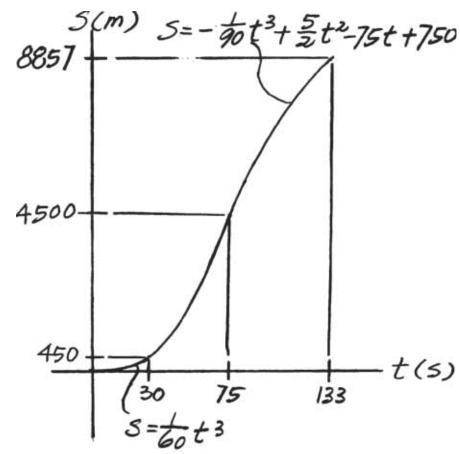
$$\int_{450\text{ m}}^s ds = \int_{30\text{ s}}^t \left(-\frac{1}{30}t^2 + 5t - 75\right)dt$$

$$s = \left(-\frac{1}{90}t^3 + \frac{5}{2}t^2 - 75t + 750\right) \text{m}$$

When $t = 75$ s and $t' = 133.09$ s,

$$s\big|_{t=75\text{ s}} = -\frac{1}{90}\left(75^3\right) + \frac{5}{2}\left(75^2\right) - 75(75) + 750 = 4500 \text{ m}$$

$$s\big|_{t=133.09\text{ s}} = -\frac{1}{90}\left(133.09^3\right) + \frac{5}{2}\left(133.09^2\right) - 75(133.09) + 750 = 8857 \text{ m} \qquad \textbf{Ans.}$$



(b)

The s–t graph is shown in Fig. *b*.

When $t = 30$ s,

$$v = 45 \text{ m/s} \text{ and } s = 450 \text{ m}.$$

When $t = 75$ s,

$$v = v_{\text{max}} = 112.5 \text{ m/s} \text{ and } s = 4500 \text{ m}.$$

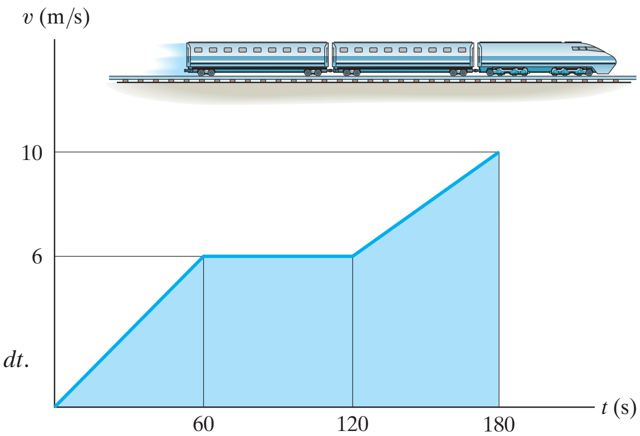
When $t = 133$ s,

$$v = 0 \text{ and } s = 8857 \text{ m}.$$

Ans:
$t' = 133$ s
$s\big|_{t=133.09\text{ s}} = 8857$ m

***12–48.**

The v–t graph for a train has been experimentally determined. From the data, construct the s–t and a–t graphs for the motion for $0 \leq t \leq 180$ s. When $t = 0$, $s = 0$.



SOLUTION

***s–t* Graph.** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \leq t < 60$ s, $v = \dfrac{6}{60}t = \left(\dfrac{1}{10}t\right)$ m/s. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t \left(\frac{1}{10}t\right)dt$$

$$s = \left\{\frac{1}{20}t^2\right\}\text{ m} \qquad \textbf{Ans.}$$

When $t = 60$ s,

$$s\big|_{t-60\text{ s}} = \frac{1}{20}\left(60^2\right) = 180\text{ m}$$

For $60\text{ s} < t < 120$ s, $v = 6$ m/s. Using the initial condition $s = 180$ m at $t = 60$ s,

$$\int_{180\text{ m}}^s ds = \int_{60\text{ s}}^t 6\,dt$$

$$s - 180 = 6t\Big|_{60\text{ s}}^t$$

$$s = \{6t - 180\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 120$ s,

$$s\big|_{t-120\text{ s}} = 6(120) - 180 = 540\text{ m}$$

For $120\text{ s} < t \leq 180$ s, $\dfrac{v - 6}{t - 120} = \dfrac{10 - 6}{180 - 120}$; $v = \left\{\dfrac{1}{15}t - 2\right\}$ m/s. Using the initial condition $s = 540$ m at $t = 120$ s,

$$\int_{540\text{ m}}^s ds = \int_{120\text{ s}}^t \left(\frac{1}{15}\text{t} - 2\right)dt$$

$$s - 540 = \left(\frac{1}{30}t^2 - 2t\right)\Big|_{120\text{ s}}^t$$

$$s = \left\{\frac{1}{30}t^2 - 2t + 300\right\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 180$ s,

$$s\big|_{t=180\text{ s}} = \frac{1}{30}\left(180^2\right) - 2(180) + 300 = 1020\text{ m}$$

Using these results, s–t graph shown in Fig. a can be plotted.

***12–48. Continued**

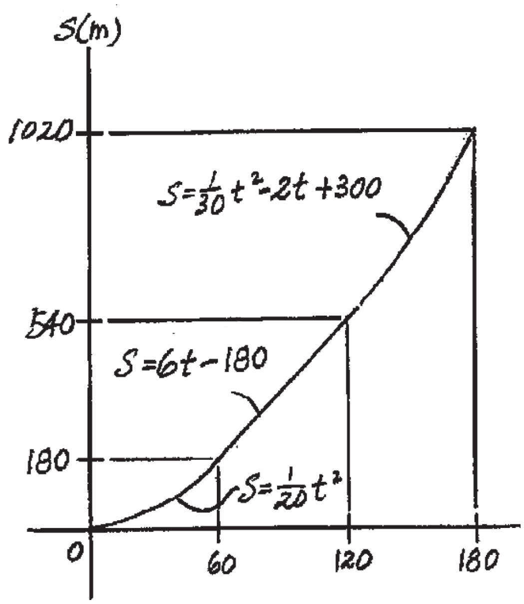
***a–t* Graph.** The a–t function can be determined using $a = \dfrac{dv}{dt}$.

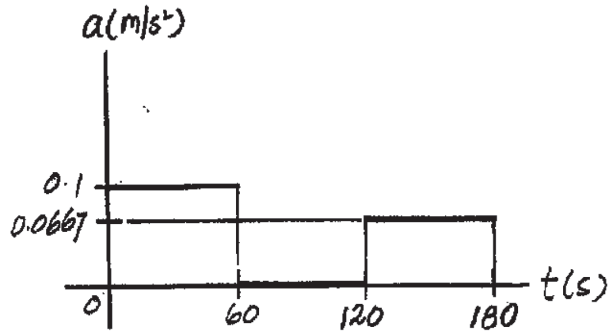
For $0 \le t < 60$ s, $\quad a = \dfrac{d\left(\frac{1}{10}t\right)}{dt} = 0.1\ \text{m/s}^2$ **Ans.**

For $60\ \text{s} < t < 120$ s, $\quad a = \dfrac{d(6)}{dt} = 0$ **Ans.**

For $120\ \text{s} < t \le 180$ s, $\quad a = \dfrac{d\left(\frac{1}{15}t - 2\right)}{dt} = 0.0667\ \text{m/s}^2$ **Ans.**

Using these results, a–t graph shown in Fig. b can be plotted.

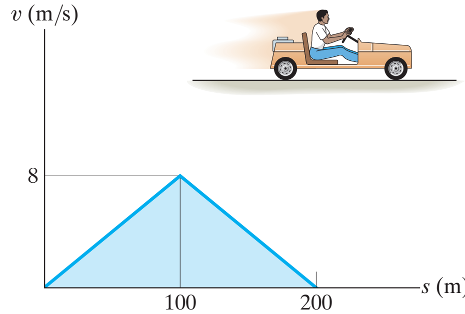




Ans:
For $0 \le t < 60$ s,
$s = \left\{\dfrac{1}{20}t^2\right\}$ m,
$a = 0.1\ \text{m/s}^2$.
For $60\ \text{s} < t < 120$ s,
$s = \{6t - 180\}$ m,
$a = 0$. For $120\ \text{s} < t \le 180$ s,
$s = \left\{\dfrac{1}{30}t^2 - 2t + 300\right\}$ m,
$a = 0.0667\ \text{m/s}^2$.

12–49.

The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at $s = 50$ m and $s = 150$ m. Draw the a–s graph.



SOLUTION

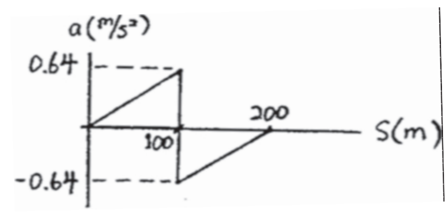
For $0 \leq s < 100$

$v = 0.08\,s, \qquad dv = 0.08\,ds$

$a\,ds = (0.08\,s)(0.08\,ds)$

$a = 6.4(10^{-3})\,s$

At $s = 50$ m, $\quad a = 0.32\text{ m/s}^2$ **Ans.**



For $100 < s < 200$

$v = -0.08\,s + 16,$

$dv = -0.08\,ds$

$a\,ds = (-0.08\,s + 16)(-0.08\,ds)$

$a = 0.08(0.08\,s - 16)$

At $s = 150$ m, $\quad a = -0.32\text{ m/s}^2$ **Ans.**

Also,

$v\,dv = a\,ds$

$a = v(\frac{dv}{ds})$

At $s = 50$ m,

$a = 4(\frac{8}{100}) = 0.32\text{ m/s}^2$ **Ans.**

At $s = 150$ m,

$a = 4(\frac{-8}{100}) = -0.32\text{ m/s}^2$ **Ans.**

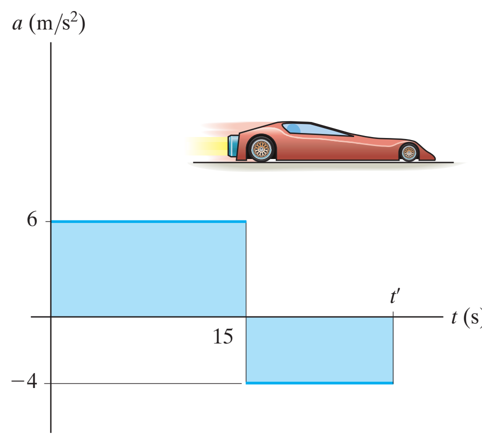
At $s = 100$ m, a changes from $a_{max} = 0.64\text{ m/s}^2$

to $a_{min} = -0.64\text{ m/s}^2$.

Ans:
$a = 0.32\text{ m/s}^2,$
$a = -0.32\text{ m/s}^2$

12–50.

The jet car is originally traveling at a velocity of 10 m/s when it is subjected to the acceleration shown. Determine the car's maximum velocity and the time t' when it stops. When $t = 0, s = 0$.



SOLUTION

***v–t* Function.** The $v-t$ function can be determined by integrating $dv = a\,dt$. For $0 \le t < 15$ s, $a = 6$ m/s². Using the initial condition $v = 10$ m/s at $t = 0$,

$$\int_{10\text{ m/s}}^{v} dv = \int_0^t 6dt$$

$$v - 10 = 6t$$

$$v = \{6t + 10\}\text{ m/s}$$

The maximum velocity occurs when $t = 15$ s. Then

$$v_{\max} = 6(15) + 10 = 100\text{ m/s}$$ **Ans.**

For $15\text{ s} < t \le t'$, $a = -4$ m/s, Using the initial condition $v = 100$ m/s at $t = 15$ s,

$$\int_{100\text{ m/s}}^{v} dv = \int_{15\text{ s}}^{t} -4dt$$

$$v - 100 = (-4t)\Big|_{15\text{ s}}^{t}$$

$$v = \{-4t + 160\}\text{ m/s}$$

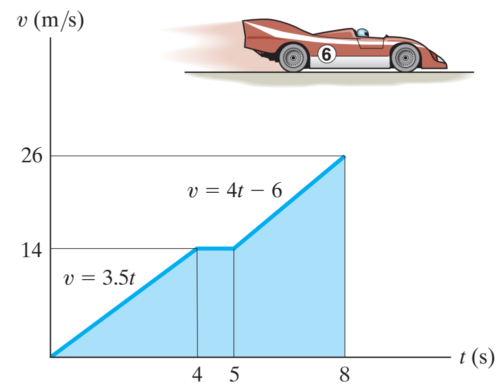
It is required that $v = 0$ at $t = t'$. Then

$$0 = -4t' + 160 \qquad t' = 40\text{ s}$$ **Ans.**

Ans:
$v_{\max} = 100$ m/s
$t' = 40$ s

12–51.

The race car starts from rest and travels along a straight road until it reaches a speed of 26 m/s in 8 s as shown on the v–t graph. The flat part of the graph is caused by shifting gears. Draw the a–t graph and determine the maximum acceleration of the car.



SOLUTION

For $0 \leq t < 4$ s

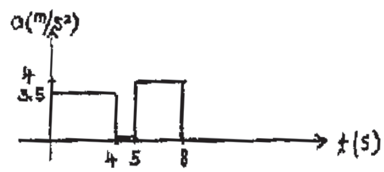
$$a = \frac{\Delta v}{\Delta t} = \frac{14}{4} = 3.5 \text{ m/s}^2$$

For $4 \text{ s} \leq t < 5$ s

$$a = \frac{\Delta v}{\Delta t} = 0$$

For $5 \text{ s} \leq t < 8$ s

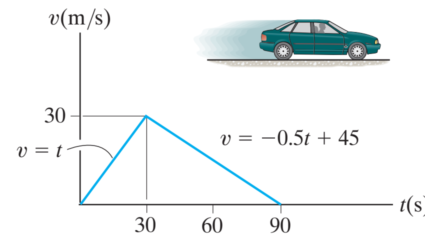
$$a = \frac{\Delta v}{\Delta t} = \frac{26 - 14}{8 - 5} = 4 \text{ m/s}^2$$



$a_{\text{max}} = 4.00 \text{ m/s}^2$ **Ans.**

Ans:
$a_{\text{max}} = 4.00 \text{ m/s}^2$

***12–52.** A car starts from rest and travels along a straight road with a velocity described by the graph. Determine the total distance traveled until the car stops. Construct the s–t and a–t graphs.



SOLUTION

$s-t$ Graph: For the time interval $0 \le t < 30$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$(\xrightarrow{+}) \qquad ds = vdt$$

$$\int_0^s ds = \int_0^t tdt$$

$$s = \left(\frac{t^2}{2}\right) \text{m}$$

When $t = 30$ s,

$$s = \frac{30^2}{2} = 450 \text{ m}$$

For the time interval $30 \text{ s} < t \le 90$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\xrightarrow{+}) \qquad ds = vdt$$

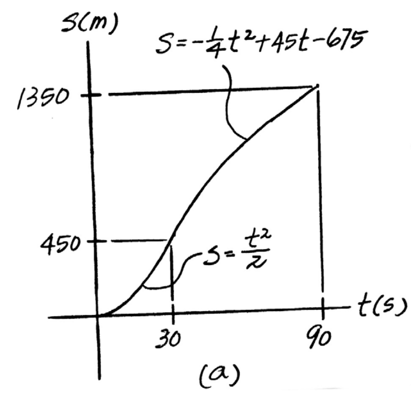
$$\int_{450 \text{ m}}^s ds = \int_{30\text{s}}^t (-0.5t + 45)dt$$

$$s = \left(-\frac{1}{4}t^2 + 45t - 675\right) \text{m}$$

When $t = 90$ s,

$$s\big|_{t=90 \text{ s}} = -\frac{1}{4}\left(90^2\right) + 45(90) - 675 = 1350 \text{ m} \qquad \textbf{Ans.}$$

The $s-t$ graph shown is in Fig. *a*.



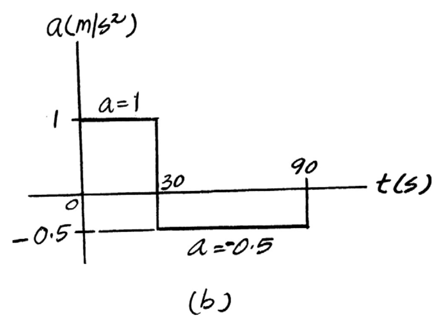
$a-t$ Graph: For the time interval $0 < t < 30$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(t) = 1 \text{ m/s}^2$$

For the time interval $30 \text{ s} < t \le 90$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(-0.5t + 45) = -0.5 \text{ m/s}^2$$

The $a-t$ graph is shown in Fig. *b*.



Note: Since the change in position of the car is equal to the area under the $v-t$ graph, the total distance traveled by the car is

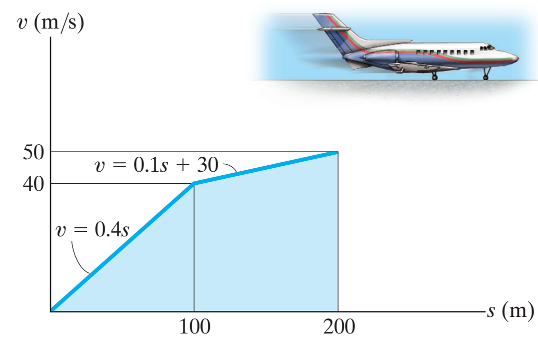
$$\Delta s = \int vdt$$

$$s\big|_{t=90 \text{ s}} - 0 = \frac{1}{2}(90)(30)$$

$$s\big|_{t=90 \text{ s}} = 1350 \text{ m}$$

Ans:
$s\big|_{t=90 \text{ s}} = 1350$ m

12–53. The v–s graph for an airplane traveling on a straight runway is shown. Determine the acceleration of the plane at $s = 100$ m and $s = 150$ m. Draw the a–s graph.



SOLUTION

$$a = v\frac{dv}{ds}$$

$s_1 = 100$ m $\quad s_4 = 150$ m

$s_2 = 200$ m $\quad v_1 = 40$ m/s

$s_3 = 50$ m $\quad v_2 = 50$ m/s

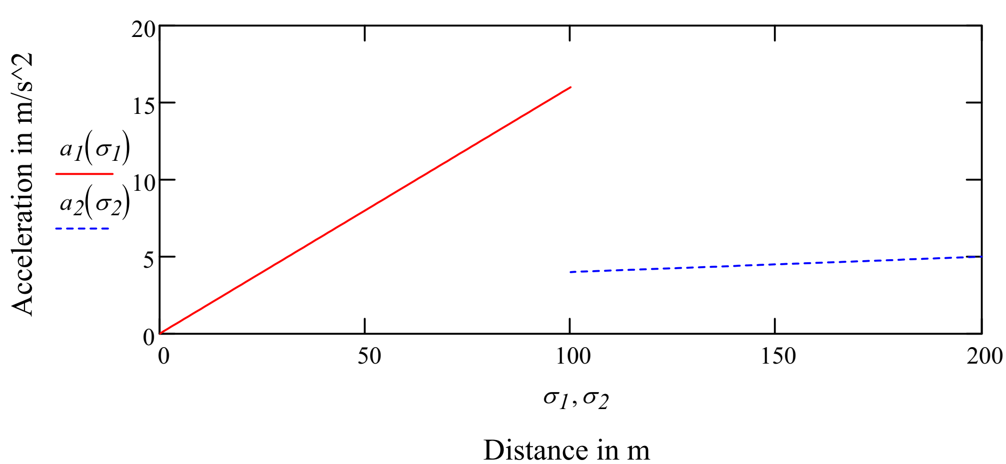
$0 < s_3 < s_1 \qquad a_3 = \left(\frac{s_3}{s_1}\right)v_1\left(\frac{v_1}{s_1}\right) \qquad a_3 = 8.00\ \text{m/s}^2$ **Ans.**

$s_1 < s_4 < s_2 \qquad a_4 = \left[v_1 + \frac{s_4 - s_1}{s_2 - s_1}(v_2 - v_1)\right]\frac{v_2 - v_1}{s_2 - s_1} \qquad a_4 = 4.50\ \text{m/s}^2$ **Ans.**

The graph

$\sigma_1 = 0, 0.01s_1 .. s_1 \qquad a_1(\sigma_1) = \frac{\sigma_1}{s_1}\frac{v_1^2}{s_1}\ \text{s}^2/\text{m}$

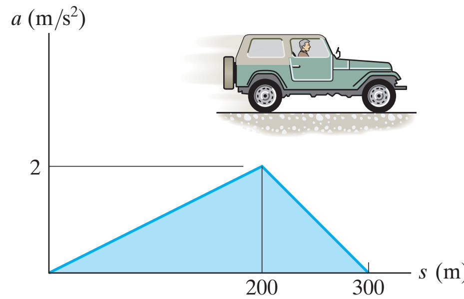
$\sigma_2 = s_1, 1.01s_1 .. s_2 \qquad a_2(\sigma_2) = \left[v_1 + \frac{\sigma_2 - s_1}{s_2 - s_1}(v_2 - v_1)\right]\frac{v_2 - v_1}{s_2 - s_1}\ \text{s}^2/\text{m}$



Ans:
$a_3 = 8.00\ \text{m/s}^2$
$a_4 = 4.50\ \text{m/s}^2$

12–54.

The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v–s graph. At $s = 0, v = 0$.



SOLUTION

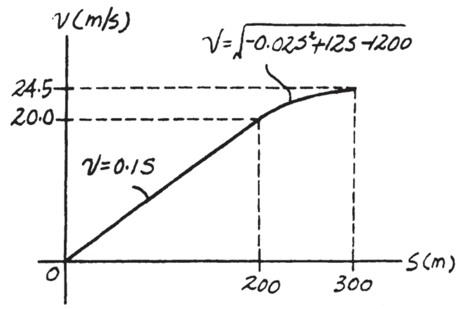
***a–s* Graph**: The function of acceleration a in terms of s for the interval $0 \text{ m} \le s < 200 \text{ m}$ is

$$\frac{a-0}{s-0} = \frac{2-0}{200-0} \qquad a = (0.01s) \text{ m/s}^2$$

For the interval $200 \text{ m} < s \le 300 \text{ m}$,

$$\frac{a-2}{s-200} = \frac{0-2}{300-200} \qquad a = (-0.02s + 6) \text{ m/s}^2$$

***v–s* Graph**: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $\mathbf{0\ m \le s < 200\ m}$,



$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 0.01s\,ds$$

$$v = (0.1s) \text{ m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0 \text{ m/s}$

For the interval $\mathbf{200\ m < s \le 300\ m}$,

$$vdv = ads$$

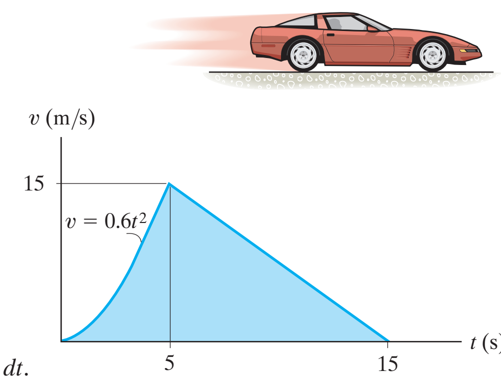
$$\int_{20.0 \text{ m/s}}^v vdv = \int_{200 \text{ m}}^s (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right) \text{ m/s}$$

At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5 \text{ m/s}$

12–55.

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t and a–t graphs. Also determine the average speed and the distance traveled for the 15-s time interval. When $t = 0$, $s = 0$.



SOLUTION

***s*–*t* Graph.** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \le t < 5$ s, $v = 0.6t^2$. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t 0.6t^2 dt$$

$$s = \{0.2t^3\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 5$ s,

$$s|_{t=5\text{ s}} = 0.2(5^3) = 25\text{ m}$$

For $5\text{ s} < t \le 15$ s, $\dfrac{v - 15}{t - 5} = \dfrac{0 - 15}{15 - 5}$; $v = \dfrac{1}{2}(45 - 3t)$. Using the initial condition $s = 25$ m at $t = 5$ s,

$$\int_{25\text{ m}}^s ds = \int_{5\text{ s}}^t \frac{1}{2}(45 - 3t)dt$$

$$s - 25 = \frac{45}{2}t - \frac{3}{4}t^2 - 93.75$$

$$s = \left\{\frac{1}{4}(90t - 3t^2 - 275)\right\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 15$ s,

$$s = \frac{1}{4}\left[90(15) - 3(15^2) - 275\right] = 100\text{ m} \qquad \textbf{Ans.}$$

Thus the average speed is

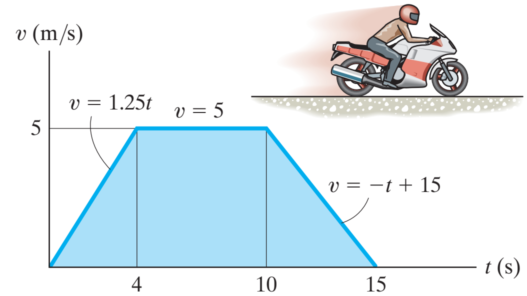
$$v_{avg} = \frac{s_T}{t} = \frac{100\text{ m}}{15\text{ s}} = 6.67\text{ m/s} \qquad \textbf{Ans.}$$

using these results, the s–t graph shown in Fig. *a* can be plotted.

Ans:
$s = \{0.2t^3\}$ m
$s = \left\{\frac{1}{4}(90t - 3t^2 - 275)\right\}$ m
$s = 100$ m
$v_{avg} = 6.67$ m/s

***12–56.**

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the total distance the motorcycle travels until it stops when $t = 15$ s. Also plot the a–t and s–t graphs.



SOLUTION

For $t < 4$ s

$$a = \frac{dv}{dt} = 1.25$$

$$\int_0^s ds = \int_0^t 1.25\, t\, dt$$

$$s = 0.625\, t^2$$

When $t = 4$ s, $\quad s = 10$ m

For 4 s $< t <$ 10 s

$$a = \frac{dv}{dt} = 0$$

$$\int_{10}^s ds = \int_4^t 5\, dt$$

$$s = 5\, t - 10$$

When $t = 10$ s, $\quad s = 40$ m

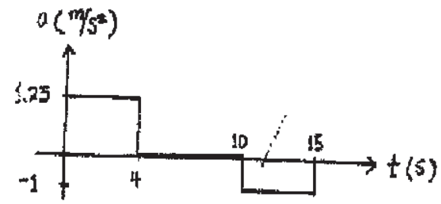
For 10 s $< t <$ 15 s

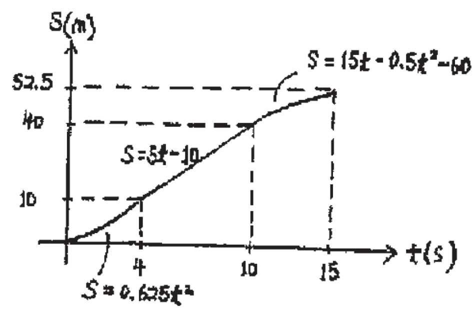
$$a = \frac{dv}{dt} = -1$$

$$\int_{40}^s ds = \int_{10}^t (15 - t)\, dt$$

$$s = 15\, t - 0.5\, t^2 - 60$$

When $t = 15$ s, $\quad s = 52.5$ m **Ans.**

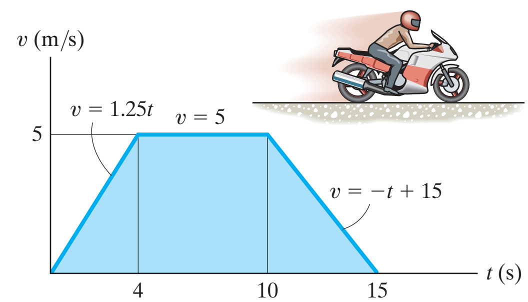




Ans:
When $t = 15$ s, $\quad s = 52.5$ m

12–57.

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



SOLUTION

At $t = 8$ s

$$a = \frac{dv}{dt} = 0$$ **Ans.**

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8 - 4)(5) = 30$$

$s = 30$ m **Ans.**

At $t = 12$ s

$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2$$ **Ans.**

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (10 - 4)(5) + \frac{1}{2}(15 - 10)(5) - \frac{1}{2}\left(\frac{3}{5}\right)(5)\left(\frac{3}{5}\right)(5)$$

$s = 48$ m **Ans.**

Ans:
At $t = 8$ s,
$a = 0$ and $s = 30$ m.
At $t = 12$ s,
$a = -1 \text{ m/s}^2$
and $s = 48$ m.

12–58.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at $4\ \text{m/s}^2$ for 10 s and then maintains a constant speed. Car B accelerates at $5\ \text{m/s}^2$ until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

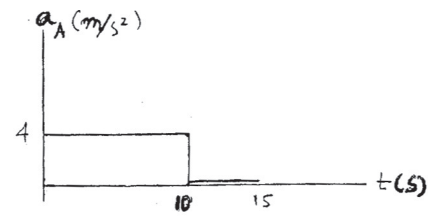
SOLUTION

Car A:

$v = v_0 + a_c t$

$v_A = 0 + 4t$

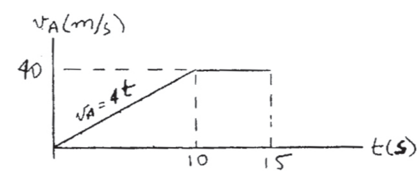
At $t = 10$ s, $\quad v_A = 40$ m/s



$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$

At $t = 10$ s, $\qquad s_A = 200$ m

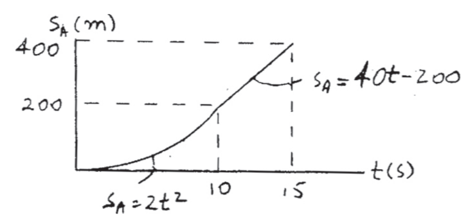


$t > 10$ s, $\qquad ds = v\,dt$

$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$$s_A = 40t - 200$$

At $t = 15$ s, $\qquad s_A = 400$ m



Car B:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $\qquad t = \frac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10$ s, $v_A = (v_A)_{\max} = 40$ m/s and $s_A = 200$ m.

When $t = 5$ s, $s_B = 62.5$ m.

When $t = 15$ s, $s_A = 400$ m and $s_B = 312.5$ m.

12–58. Continued

At $t = 5$ s, $\quad s_B = 62.5$ m

$t > 5$ s, $\quad ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_5^t 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

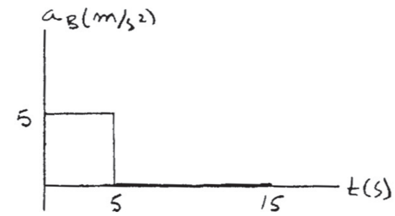
$$s_B = 25t - 62.5$$

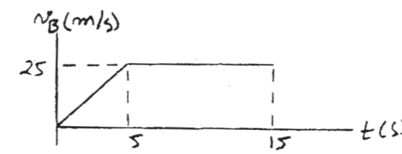
When $t = 15$ s, $\quad s_B = 312.5$

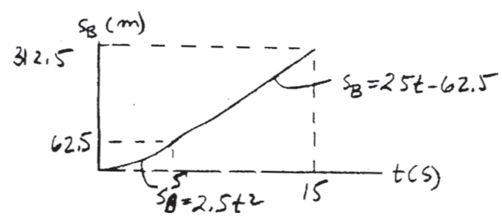
Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans.**

Car A is ahead of car B.







Ans:
When $t = 5$ s,
$s_B = 62.5$ m.
When $t = 10$ s,
$v_A = (v_A)_{\max} = 40$ m/s and
$s_A = 200$ m.
When $t = 15$ s,
$s_A = 400$ m and $s_B = 312.5$ m.
$\Delta s = s_A - s_B = 87.5$ m

12–59. A particle travels along a curve defined by the equation $s = (t^3 - 3t^2 + 2t)$ m. where t is in seconds. Draw the $s - t$, $v - t$, and $a - t$ graphs for the particle for $0 \leq t \leq 3$ s.

SOLUTION

$s = t^3 - 3t^2 + 2t$

$v = \dfrac{ds}{dt} = 3t^2 - 6t + 2$

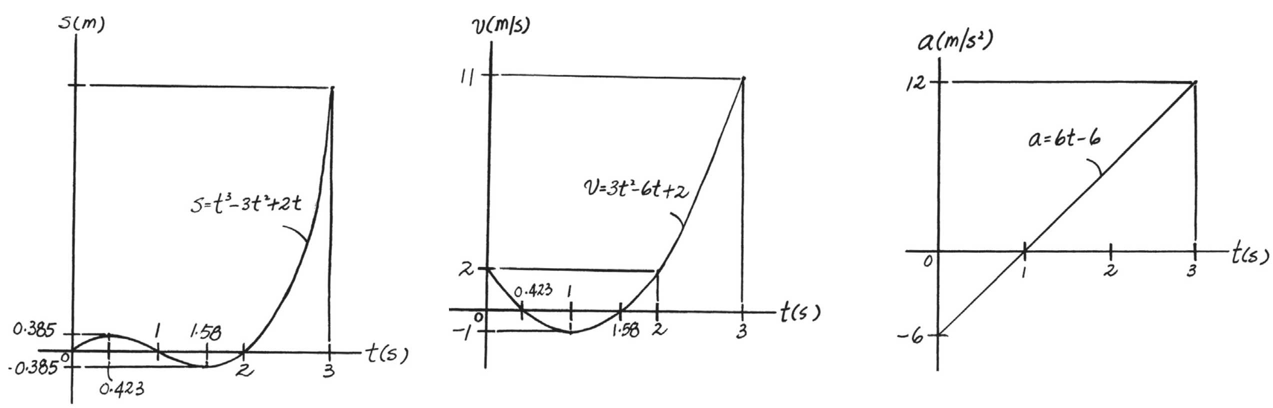
$a = \dfrac{dv}{dt} = 6t - 6$

$v = 0$ at $0 = 3t^2 - 6t + 2$

$t = 1.577$ s, and $t = 0.4226$ s,

$s|_{t=1.577} = -0.386$ m

$s|_{t=0.4226} = 0.385$ m

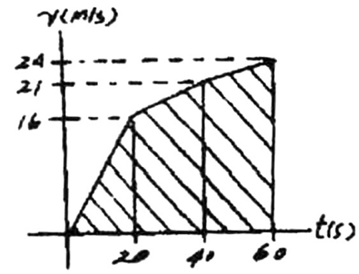


***12–60.**

The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.



SOLUTION

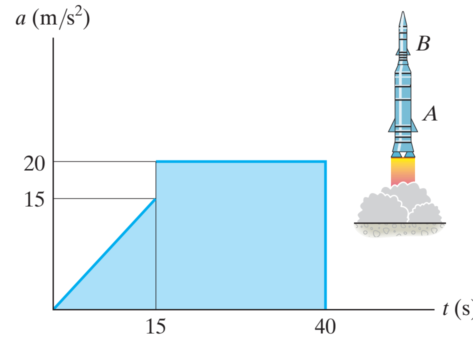
The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_T = 980$ m

12–61.

A two-stage rocket is fired vertically from rest with the acceleration shown. After 15 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \le t \le 40$ s.



SOLUTION

For $0 \le t < 15$

$a = t$

$$\int_0^v dv = \int_0^t t\,dt$$

$$v = \frac{1}{2}t^2$$

$v = 112.5$ when $t = 15$ s

$$\int_0^s ds = \int_0^t \frac{1}{2}t^2\,dt$$

$$s = \frac{1}{6}t^3$$

$s = 562.5$ when $t = 15$ s

For $15 < t < 40$

$a = 20$

$$\int_{112.5}^v dv = \int_{1.5}^t 20\,dt$$

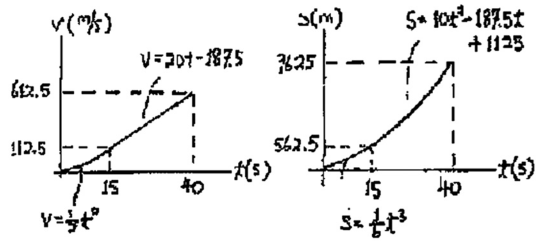
$v = 20t - 187.5$

$v = 612.5$ when $t = 40$ s

$$\int_{562.5}^s ds = \int_{15}^t (20\,t - 187.5)\,dt$$

$s = 10\,t^2 - 187.5\,t + 1125$

$s = 9625$ when $t = 40$ s



Ans:

For $0 \le t < 15$ s,

$$v = \left\{\frac{1}{2}t^2\right\} \text{m/s}$$

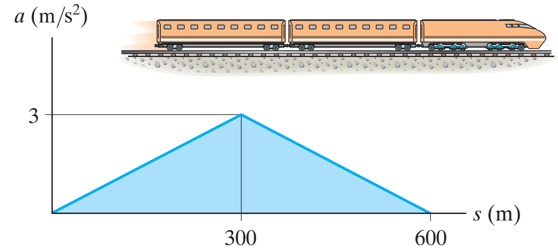
$$s = \left\{\frac{1}{6}t^3\right\} \text{m}$$

For $15 \text{ s} < t \le 40$ s,

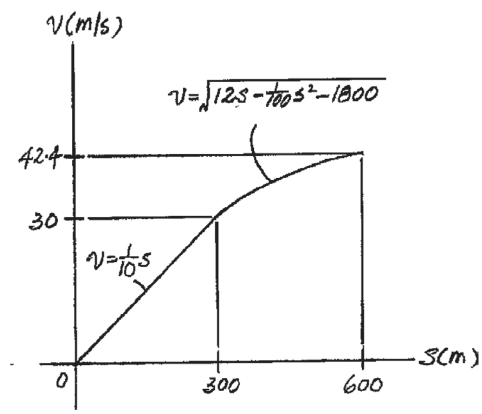
$v = \{20t - 187.5\}$ m/s

$s = \{10t^2 - 187.5t + 1125\}$ m

12–62. The motion of a train is described by the a–s graph shown. Draw the v–s graph if $v = 0$ at $s = 0$.



SOLUTION



***v–s* Graph.** The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 300$ m, $a = \left(\frac{3}{300}\right)s = \left(\frac{1}{100}s\right)$ m/s^2. Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(\frac{1}{100}s\right)ds$$

$$\frac{v^2}{2} = \frac{1}{200}s^2$$

$$v = \left\{\frac{1}{10}s\right\}\text{ m/s} \qquad \textbf{Ans.}$$

At $s = 300$ m,

$$v|_{s=300\text{ m}} = \frac{1}{10}(300) = 30\text{ m/s}$$

For $300\text{ m} < s \le 600$ m, $\frac{a-3}{s-300} = \frac{0-3}{600-300}$; $a = \left\{-\frac{1}{100}s + 6\right\}$ m/s^2, using the initial condition $v = 30$ m/s at $s = 300$ m,

$$\int_{30\text{ m/s}}^v v\,dv = \int_{300\text{ m}}^s \left(-\frac{1}{100}s + 6\right)ds$$

$$\left.\frac{v^2}{2}\right|_{30\text{ m/s}}^{v} = \left.\left(-\frac{1}{200}s^2 + 6s\right)\right|_{300\text{ m}}^{s}$$

$$\frac{v^2}{2} - 450 = 6s - \frac{1}{200}s^2 - 1350$$

$$v = \left\{\sqrt{12s - \frac{1}{100}s^2 - 1800}\right\}\text{ m/s} \qquad \textbf{Ans.}$$

At $s = 600$ m,

$$v = \sqrt{12\,(600) - \frac{1}{100}(600^2) - 1800} = 42.43\text{ m/s}$$

Using these results, the $v-s$ graph shown in Fig. a can be plotted.

Ans:
$v = \left\{\frac{1}{10}s\right\}$ m/s

12–63. If the position of a particle is defined as $s = (5t - 3t^2)$ m, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \le t \le 2.5$ s.

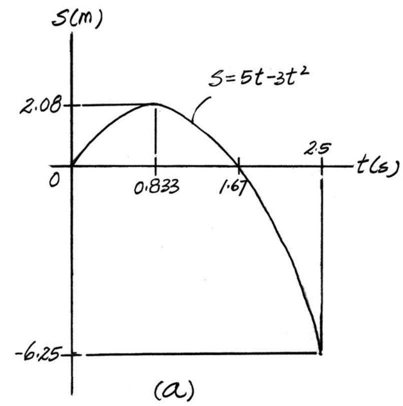
SOLUTION

***s–t* Graph.** When s = 0,

$$0 = 5t - 3t^2 \quad 0 = t(5 - 3t) \quad t = 0 \quad \text{and} \quad t = 1.675$$

At $t = 10$ s,

$$s|_{t=2\text{ s}} = 5(2.5) - 3(2.5^2) = -6.25 \text{ m}$$



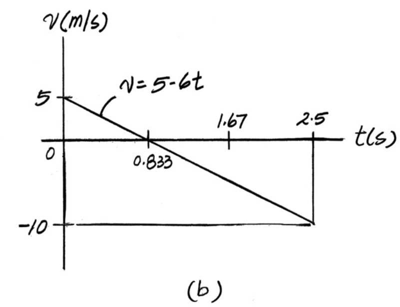
***v–t* Graph.** The velocity function can be determined using $v = \dfrac{ds}{dt}$, Thus $v = \dfrac{ds}{dt} = \{5 - 6t\}$ m/s

At $t = 0$, $v|_{t=0} = 5 - 6(0) = 5$ m/s

At $t = 2.5$ s, $v|_{t=2.5\text{ s}} = 5 - 6(2.5) = -10$ m/s

when $v = 0$,

$$0 = 5 - 6t \quad t = 0.833 \text{ s}$$

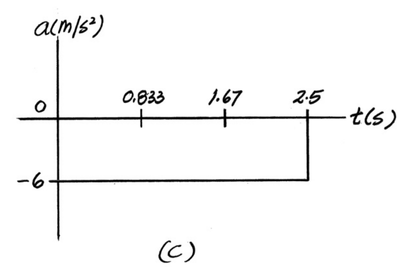


Then, at $t = 0.833$ s,

$$s|_{t=0.833\text{ s}} = 5(0.833) - 3(0.833^2) = 2.08 \text{ m}$$

***a–t* Graph.** The acceleration function can be determined using $a = \dfrac{dv}{dt}$. Thus

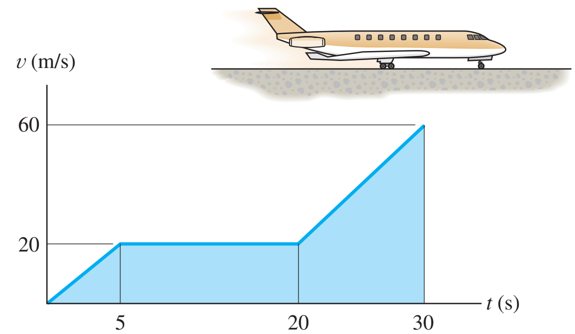
$$a = \frac{dv}{dt} = -6 \text{ m/s}^2 \qquad \textbf{Ans.}$$



s–t, v–t and a–t graph shown in Fig. a, b, and c can be plotted using these results.

Ans:
$v = \{5 - 6t\}$ m/s
$a = -6$ m/s^2

***12–64.** From experimental data, the motion of a jet plane while traveling along a runway is defined by the v–t graph. Construct the s–t and a–t graphs for the motion. When $t = 0, s = 0$.



SOLUTION

s − t Graph: The position in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$. For time interval $\mathbf{0\ s \le t < 5\ s}$, $v = \dfrac{20}{5}t = (4t)$ m/s.

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 4t\,dt$$

$$s = \left(2t^2\right) \text{ m}$$

When $t = 5$ s, $\quad s = 2\left(5^2\right) = 50$ m,

For time interval $\mathbf{5\ s < t < 20\ s}$,

$$ds = v\,dt$$

$$\int_{50\text{ m}}^s ds = \int_{5\text{ a}}^t 20\,dt$$

$$s = (20t - 50) \text{ m}$$

When $t = 20$ s, $\quad s = 20(20) - 50 = 350$ m

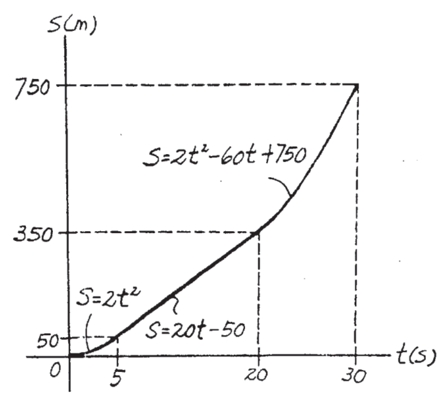
For time interval $\mathbf{20\ s < t \le 30\ s}$, $\dfrac{v - 20}{t - 20} = \dfrac{60 - 20}{30 - 20}$, $v = (4t - 60)$ m/s.

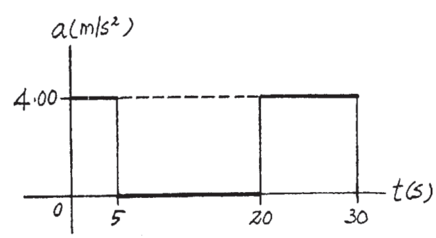
$$ds = v\,dt$$

$$\int_{350\text{ m}}^s ds = \int_{20\text{ a}}^t (4t - 60)\,dt$$

$$s = \left(2t^2 - 60t + 750\right) \text{ m}$$

When $t = 30$ s, $\quad s = 2\left(30^2\right) - 60(30) + 750 = 750$ m

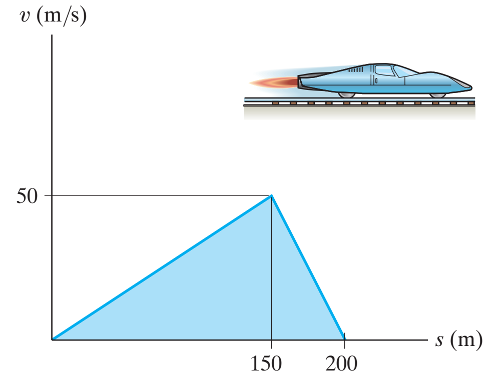




a − t Graph: The acceleration function in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $\mathbf{0\ s \le t < 5\ s}$, $\mathbf{5\ s < t < 20\ s}$ and $\mathbf{20\ s < t \le 30\ s}$, $a = \dfrac{dv}{dt} = 4.00 \text{ m/s}^2$, $a = \dfrac{dv}{dt} = 0$ and $a = \dfrac{dv}{dt} = 4.00 \text{ m/s}^2$, respectively.

Ans:
For $0 \le t < 5$ s,
$s = \left\{2t^2\right\}$ m and $a = 4 \text{ m/s}^2$.
For $5 \text{ s} < t < 20$ s,
$s = \{20t - 50\}$ m and $a = 0$.
For $20 \text{ s} < t \le 30$ s,
$s = \left\{2t^2 - 60t + 750\right\}$ m
and $a = 4 \text{ m/s}^2$.

12–65. The v–s graph for a test vehicle is shown. Determine its acceleration when $s = 100$ m and when $s = 175$ m.



SOLUTION

$0 \leq s \leq 150$ m: $\qquad v = \frac{1}{3}s$

$$dv = \frac{1}{3}ds$$

$$v\,dv = a\,ds$$

$$\frac{1}{3}s\left(\frac{1}{3}ds\right) = a\,ds$$

$$a = \frac{1}{9}s$$

At $s = 100$ m, $\qquad a = \frac{1}{9}(100) = 11.1 \text{ m/s}^2$ **Ans.**

$150\text{ m} \leq s \leq 200$ m; $\qquad v = 200 - s,$

$$dv = -ds$$

$$v\,dv = a\,ds$$

$$(200 - s)(-ds) = a\,ds$$

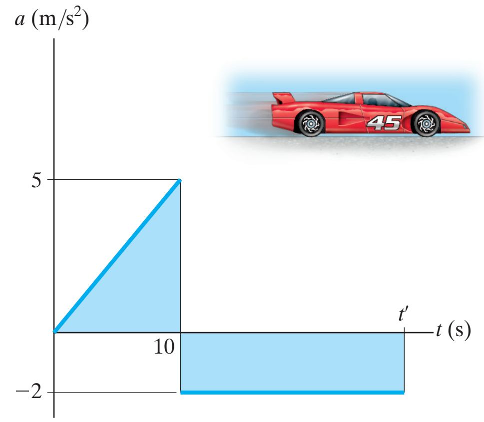
$$a = s - 200$$

At $s = 175$ m, $\qquad a = 175 - 200 = -25 \text{ m/s}^2$ **Ans.**

Ans:
At $s = 100$ m, $a = 11.1 \text{ m/s}^2$
At $s = 175$ m, $a = -25 \text{ m/s}^2$

12–66.

The a–t graph for a car is shown. Construct the v–t and s–t graphs if the car starts from rest at $t = 0$. At what time t' does the car stop?



SOLUTION

$a_1 = 5\ \text{m/s}^2$

$a_2 = -2\ \text{m/s}^2$

$t_1 = 10\ \text{s}$

$k = \dfrac{a_1}{t_1}$

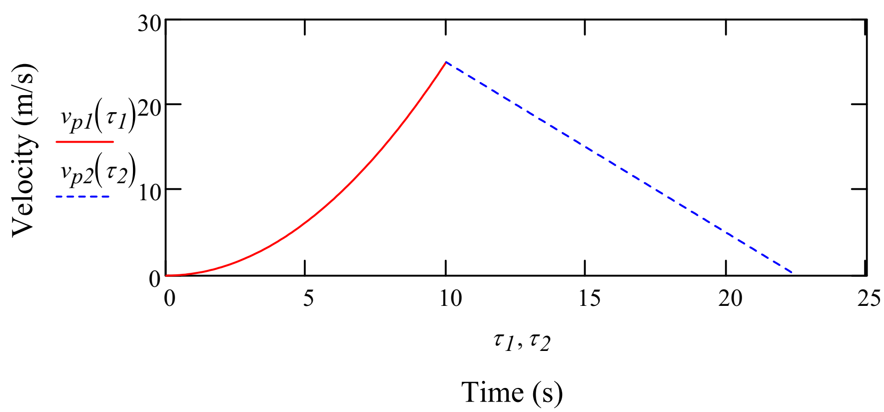
$a_{p1}(t) = kt \qquad v_{p1}(t) = k\dfrac{t^2}{2} \qquad s_{p1}(t) = k\dfrac{t^3}{6}$

$a_{p2}(t) = a_2 \qquad v_{p2}(t) = v_{p1}(t_1) + a_2(t - t_1)$

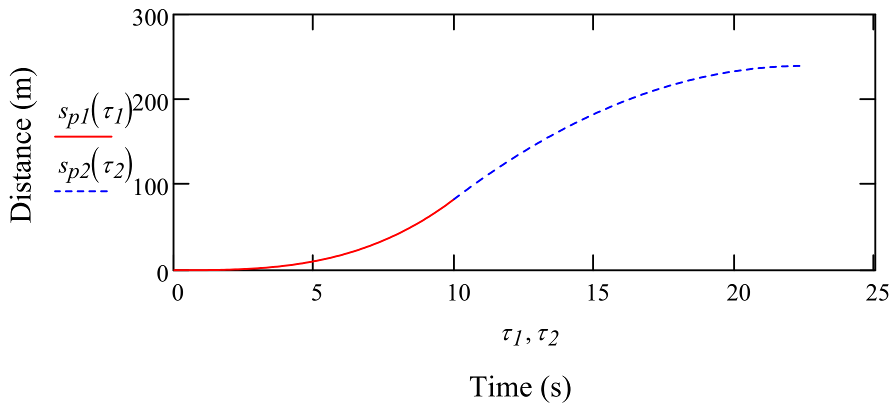
$$s_{p2}(t) = s_{p1}(t_1) + v_{p1}(t_1)(t - t_1) + \frac{1}{2}a_2(t - t_1)^2$$

Guess $t' = 12\ \text{s}$ Given $v_{p2}(t') = 0$ $t' = \text{Find}(t')$ $t' = 22.5\ \text{s}$ **Ans.**

$\tau_1 = 0, 0.01t_1 .. t_1 \qquad \tau_2 = t_1, 1.01t_1 .. t'$



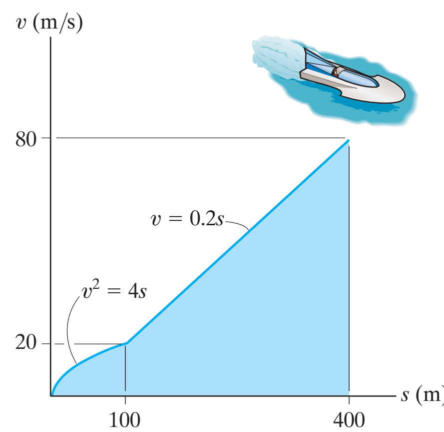
12–66. Continued



Ans:
$t' = 22.5$ s

12–67.

The boat travels along a straight line with the speed described by the graph. Construct the s–t and a–s graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



SOLUTION

***s–t* Graph:** For $0 \leq s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{2s^{1/2}}$$

$$t = s^{1/2}$$

$$s = \left(t^2\right) \text{ m}$$

When $s = 100$ m,

$$100 = t^2 \qquad t = 10 \text{ s} \qquad \textbf{Ans.}$$

For $100 \text{ m} < s \leq 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_{10\text{ s}}^t dt = \int_{100\text{ m}}^s \frac{ds}{0.2s}$$

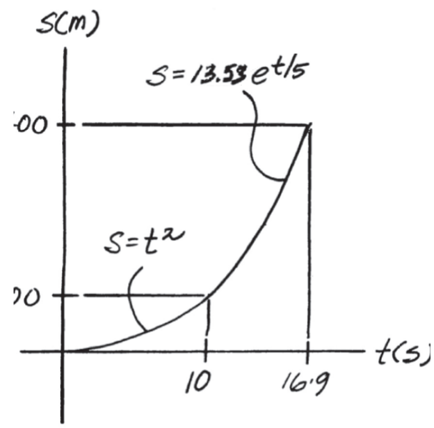
$$t - 10 = 5\ln\frac{s}{100}$$

$$\frac{t}{5} - 2 = \ln\frac{s}{100}$$

$$e^{t/5-2} = \frac{s}{100}$$

$$\frac{e^{t/5}}{e^2} = \frac{s}{100}$$

$$s = \left(13.53e^{t/5}\right) \text{ m}$$



(a)

When $s = 400$ m,

$$400 = 13.53e^{t/5}$$

$$t = 16.93 \text{ s} = 16.9 \text{ s} \qquad \textbf{Ans.}$$

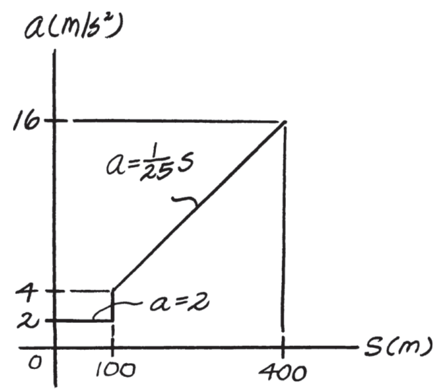
The s–t graph is shown in Fig. *a*.

***a–s* Graph:** For $0 \text{ m} \leq s < 100$ m,

$$a = v\frac{dv}{ds} = \left(2s^{1/2}\right)\left(s^{-1/2}\right) = 2 \text{ m/s}^2$$

For $100 \text{ m} < s \leq 400$ m,

$$a = v\frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$



(b)

When $s = 100$ m and 400 m,

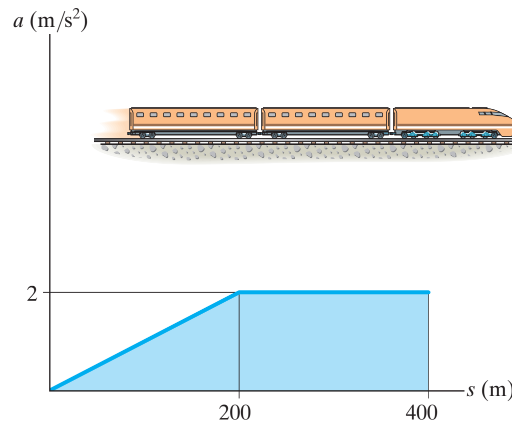
$$a\big|_{s=100\text{ m}} = 0.04(100) = 4 \text{ m/s}^2$$

$$a\big|_{s=400\text{ m}} = 0.04(400) = 16 \text{ m/s}^2$$

The a–s graph is shown in Fig. *b*.

Ans:
When $s = 100$ m,
$t = 10$ s.
When $s = 400$ m,
$t = 16.9$ s.
$a\big|_{s=100\text{ m}} = 4 \text{ m/s}^2$
$a\big|_{s=400\text{ m}} = 16 \text{ m/s}^2$

***12–68.** The *a–s* graph for a train traveling along a straight track is given for the first 400 m of its motion. Plot the *v–s* graph. $v = 0$ at $s = 0$.



SOLUTION

$s_1 = 200\ \text{m}$

$s_2 = 400\ \text{m}$

$a_1 = 2\ \dfrac{\text{m}}{\text{s}^2}$

$\sigma_1 = 0, 0.01s_1 .. s_1$

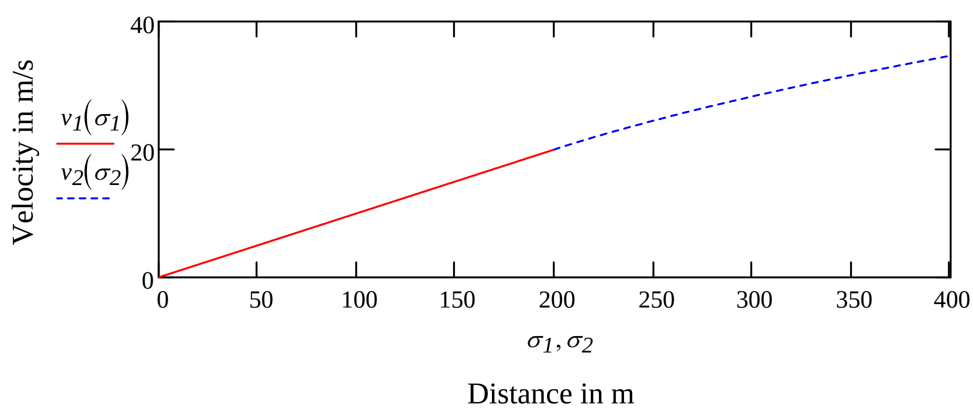
$\sigma_2 = s_1, 1.01s_1 .. s_2$

For $0 < s < s_1$ $\quad k = \dfrac{a_1}{s_1} \quad a_{c1} = ks$

$$a = ks = v\frac{dv}{ds} \qquad \int_0^v v\,dv = \int_0^s ks\,ds \qquad \frac{v^2}{2} = \frac{k}{2}s^2 \qquad v_1(\sigma_1) = \sqrt{k}\,\sigma_1\frac{\text{s}}{\text{m}}$$

For $s_1 < s < s_2$ $\quad a_{c2} = a_1 \quad a = ks = v\dfrac{dv}{ds} \quad \displaystyle\int_{v_1}^{v} v\,dv = \int_{s_1}^{s} a_1\,ds$

$$\frac{v^2}{2} - \frac{v_1^2}{2} = a_1(s - s_1) \qquad v_2(\sigma_2) = \sqrt{2a_1(\sigma_2 - s_1) + ks_1^2}\,\frac{\text{s}}{\text{m}}$$



12–69.

The velocity of a particle is given by $\boldsymbol{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\boldsymbol{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12(2^2)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}$ m/s^2. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \boldsymbol{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

Thus, the coordinate of the particle is

$$(42.7,\ 16.0,\ 14.0)\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$a = 80.2$ m/s^2
(42.7, 16.0, 14.0) m

12–70. The position of a particle is defined by $r = \{5(\cos 2t)\mathbf{i} + 4(\sin 2t)\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10 \sin 2t\mathbf{i} + 8 \cos 2t\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $v = -10 \sin 2(1)\mathbf{i} + 8 \cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$\mathbf{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The acceleration expressed in Cartesian vector from can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20 \cos 2t\mathbf{i} - 16 \sin 2t\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20 \cos 2(1)\mathbf{i} - 16 \sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Traveling Path: Here, $x = 5 \cos 2t$ and $y = 4 \sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad \textbf{(1)}$$

$$\frac{y^2}{16} = \sin^2 2t \qquad \textbf{(2)}$$

Adding Eqs (1) and (2) yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \qquad \textbf{(Equation of an Ellipse) (Q.E.D.)}$$

Ans:
$v = 9.68$ m/s
$a = 169.8$ m/s^2

12–71.

The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

Position: The position $\mathbf{r}$ of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \boldsymbol{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$d\mathbf{r} = \boldsymbol{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left[3\mathbf{i} + (6 - 2t)\mathbf{j}\right]dt$$

$$\mathbf{r} = \left[3t\mathbf{i} + \left(6t - t^2\right)\mathbf{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$r\big|_{t=1\text{ s}} = 3(1)\mathbf{i} + \left[6(1) - 1^2\right]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}]\text{ m/s}$$

$$r\big|_{t=3\text{ s}} = 3(3)\mathbf{i} + \left[6(3) - 3^2\right]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}]\text{ m/s}$$

Thus, the displacement of the particle is

$$\Delta\mathbf{r} = \mathbf{r}\big|_{t=3\text{ s}} - \mathbf{r}\big|_{t=1\text{ s}}$$

$$= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= \{6\mathbf{i} + 4\mathbf{j}\}\text{ m}$$ **Ans.**

Ans:
$\Delta\mathbf{r} = \{6\mathbf{i} + 4\mathbf{j}\}$ m

***12–72.**

If the velocity of a particle is defined as $\mathbf{v}(t) = \{0.8t^2\mathbf{i} + 12t^{1/2}\mathbf{j} + 5\mathbf{k}\}$ m/s, determine the magnitude and coordinate direction angles α, β, γ of the particle's acceleration when $t = 2$ s.

SOLUTION

$\mathbf{v}(t) = 0.8t^2\,\mathbf{i} + 12t^{1/2}\,\mathbf{j} + 5\mathbf{k}$

$\mathbf{a} = \dfrac{dv}{dt} = 1.6t\,\mathbf{i} + 6t^{1/2}\mathbf{j}$

When $t = 2$ s, $\quad \mathbf{a} = 3.2\mathbf{i} + 4.243\mathbf{j}$

$a = \sqrt{(3.2)^2 + (4.243)^2} = 5.31\text{ m/s}^2$ **Ans.**

$u_o = \dfrac{\mathbf{a}}{a} = 0.6022\mathbf{i} + 0.7984\mathbf{j}$

$\alpha = \cos^{-1}(0.6022) = 53.0°$ **Ans.**

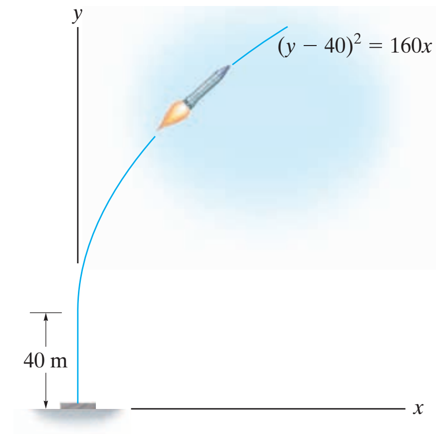
$\beta = \cos^{-1}(0.7984) = 37.0°$ **Ans.**

$\gamma = \cos^{-1}(0) = 90.0°$ **Ans.**

Ans:
$a = 5.31\text{ m/s}^2$
$\alpha = 53.0°$
$\beta = 37.0°$
$\gamma = 90.0°$

12–73.

When a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y - 40)^2 = 160x$, where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at $v_y = 180$ m/s, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



SOLUTION

$v_y = 180$ m/s

$(y - 40)^2 = 160\,x$

$2(y - 40)v_y = 160v_x$ **(1)**

$2(80 - 40)(180) = 160v_x$

$v_x = 90$ m/s

$v = \sqrt{90^2 + 180^2} = 201$ m/s **Ans.**

$$a_y = \frac{d\,v_y}{dt} = 0$$

From Eq. 1,

$2\,v_y^2 + 2(y - 40)a_y = 160\,a_x$

$2(180)^2 + 0 = 160\,a_x$

$a_x = 405$ m/s^2

$a = 405$ m/s^2 **Ans.**

Ans:
$v = 201$ m/s
$a = 405$ m/s^2

12–74. A particle is traveling with a velocity of $\mathbf{v} = \{3\sqrt{t}e^{-0.2t}\mathbf{i} + 4e^{-0.8t^2}\mathbf{j}\}$ m/s, where t is in seconds. Determine the magnitude of the particle's displacement from $t = 0$ to $t = 3$ s. Use Simpson's rule with $n = 100$ to evaluate the integrals. What is the magnitude of the particle's acceleration when $t = 2$ s?

SOLUTION

Given: $a = 3\ \text{m/s}^{3/2}$ $\quad b = -0.2\ \text{s}^{-1}$ $\quad c = 4\ \text{m/s}$ $\quad d = -0.8\ \text{s}^{-2}$ $\quad t_1 = 3\ \text{s}$ $\quad t_2 = 2\ \text{s}$

$n = 100$

Displacement

$$x_1 = \int_0^{t_1} a\sqrt{t}e^{bt}\,dt \qquad x_1 = 7.34\ \text{m} \qquad y_1 = \int_0^{t_1} ce^{dt^2}\,dt \qquad y_1 = 3.96\ \text{m}$$

$$d_1 = \sqrt{x_1^2 + y_1^2} \qquad d_1 = 8.34\ \text{m}$$ **Ans.**

Acceleration

$$a_x = \frac{d}{dt}\left(a\sqrt{t}e^{bt}\right) = \frac{a}{2\sqrt{t}}e^{bt} + ab\sqrt{t}e^{bt} \qquad a_{x2} = \frac{a}{\sqrt{t_2}}e^{bt_2}\left(\frac{1}{2} + bt_2\right)$$

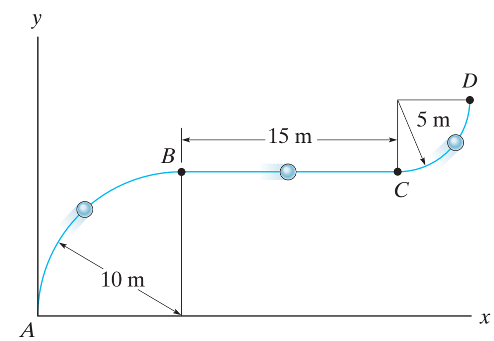
$$a_y = \frac{d}{dt}\left(ce^{dt^2}\right) = 2cdte^{dt^2} \qquad a_{y2} = 2cdt_2e^{dt_2^2}$$

$$a_{x2} = 0.14\ \text{m/s}^2 \qquad a_{y2} = -0.52\ \text{m/s}^2 \qquad a_2 = \sqrt{a_{x2}^2 + a_{y2}^2} \qquad a_2 = 0.541\ \text{m/s}^2$$ **Ans.**

Ans:
$d_1 = 8.34$ m,
$a_2 = 0.541\ \text{m/s}^2$

12–75.

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



SOLUTION

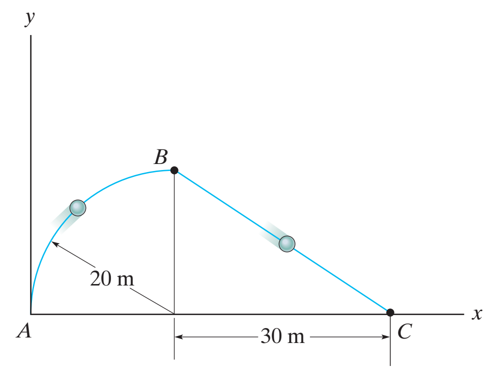
$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$ **Ans.**

Ans:
$(v_{\text{sp}})_{\text{avg}} = 4.28 \text{ m/s}$

***12–76.**

A particle travels along the curve from A to B in 5 s. It takes 8 s for it to go from B to C and then 10 s to go from C to A. Determine its average speed when it goes around the closed path.



SOLUTION

The total distance traveled is

$$S_{\text{Tot}} = S_{AB} + S_{BC} + S_{CA}$$

$$= 20\left(\frac{\pi}{2}\right) + \sqrt{20^2 + 30^2} + (30 + 20)$$

$$= 117.47 \text{ m}$$

The total time taken is

$$t_{\text{Tot}} = t_{AB} + t_{BC} + t_{CA}$$

$$= 5 + 8 + 10$$

$$= 23 \text{ s}$$

Thus, the average speed is

$$(v_{sp})_{avg} = \frac{S_{\text{Tot}}}{t_{\text{Tot}}} = \frac{117.47 \text{ m}}{23 \text{ s}} = 5.107 \text{ m/s} = 5.11 \text{ m/s}$$ **Ans.**

Ans:
$(v_{sp})_{avg} = 5.11 \text{ m/s}$

12–77.

The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: By taking the time derivative of x, y, and z, we obtain the x, y, and z components of the crate's velocity.

$$v_x = \dot{x} = \frac{d}{dt}\left(0.25t^3\right) = \left(0.75t^2\right) \text{ m/s}$$

$$v_y = \dot{y} = \frac{d}{dt}\left(1.5t^2\right) = \left(3t\right) \text{ m/s}$$

$$v_z = \dot{z} = \frac{d}{dt}\left(6 - 0.75t^{5/2}\right) = \left(-1.875t^{3/2}\right) \text{ m/s}$$

When $t = 2$ s,

$$v_x = 0.75\left(2^2\right) = 3 \text{ m/s} \qquad v_y = 3(2) = 6 \text{ m/s} \qquad v_z = -1.875\left(2\right)^{3/2} = -5.303 \text{ m/s}$$

Thus, the magnitude of the crate's velocity is

$$v = \sqrt{v_x^{\,2} + v_y^{\,2} + v_z^{\,2}} = \sqrt{3^2 + 6^2 + (-5.303)^2} = 8.551 \text{ m/s} = 8.55 \text{ m} \qquad \textbf{Ans.}$$

Acceleration: The x, y, and z components of the crate's acceleration can be obtained by taking the time derivative of the results of v_x, v_y, and v_z, respectively.

$$a_x = \dot{v}_x = \frac{d}{dt}\left(0.75t^2\right) = (1.5t) \text{ m/s}^2$$

$$a_y = \dot{v}_y = \frac{d}{dt}(3t) = 3 \text{ m/s}^2$$

$$a_z = \dot{v}_z = \frac{d}{dt}\left(-1.875t^{3/2}\right) = \left(-2.815t^{1/2}\right) \text{ m/s}^2$$

When $t = 2$ s,

$$a_x = 1.5(2) = 3 \text{ m/s}^2 \qquad a_y = 3 \text{ m/s}^2 \qquad a_z = -2.8125\left(2^{1/2}\right) = -3.977 \text{ m/s}^2$$

Thus, the magnitude of the crate's acceleration is

$$a = \sqrt{a_x^{\,2} + a_y^{\,2} + a_z^{\,2}} = \sqrt{3^2 + 3^2 + (-3.977)^2} = 5.815 \text{ m/s}^2 = 5.82 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:
$v = 8.55$ m/s
$a = 5.82$ m/s^2

12–78.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s^2, where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t.

$$\int dv_x = \int a_x dt$$

$$\int_0^{v_x} dv_x = \int_0^t \frac{1}{4}t^2 dt$$

$$v_x = \left(\frac{1}{12}t^3\right) \text{ m/s}$$

$$\int dx = \int v_x dt$$

$$\int_0^x dx = \int_0^t \frac{1}{12}t^3 dt$$

$$x = \left(\frac{1}{48}t^4\right) \text{ m}$$

Substituting the result of x into the equation of the path,

$$y^2 = 120\left(10^3\right)\left(\frac{1}{48}t^4\right)$$

$$y = \left(50t^2\right) \text{ m}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}\left(50t^2\right) = \left(100t\right) \text{ m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}\left(10^3\right) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

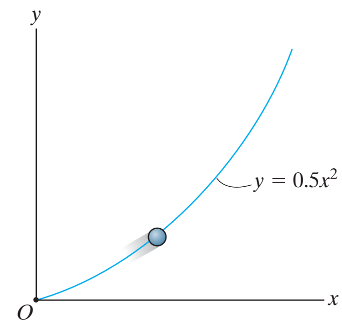
$$a_x = \frac{1}{4}\left(10^2\right) = 25 \text{ m/s}^2$$

Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 1003$ m/s
$a = 103$ m/s^2

12–79. The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ m/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \dfrac{dx}{dt}$.

$$dx = v_x dt$$

$$\int_0^x dx = \int_0^t 5t\,dt$$

$$x = \left(2.50t^2\right)\ \text{m}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ m. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ m and $y = 3.125\left(1^4\right) = 3.125$ m. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00\ \text{m} \qquad \textbf{Ans.}$$

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad \textbf{(1)}$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + xa_x \qquad \textbf{(2)}$$

When $t = 1$ s, $v_x = 5(1) = 5$ m/s $a_x = \dfrac{dv_x}{dt} = 5$ m/s^2, and $x = 2.50$ m. Then, from Eq. (2)

$$a_y = 5^2 + 2.50(5) = 37.5\ \text{m/s}^2$$

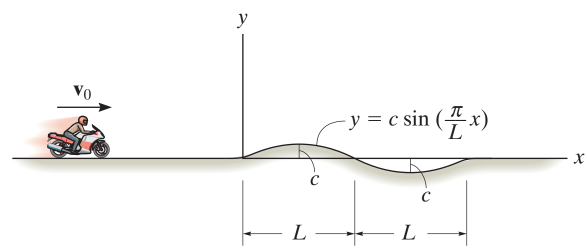
Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$d = 4.00$ m
$a = 37.8$ m/s^2

***12–80.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c \sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\, v_x\left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

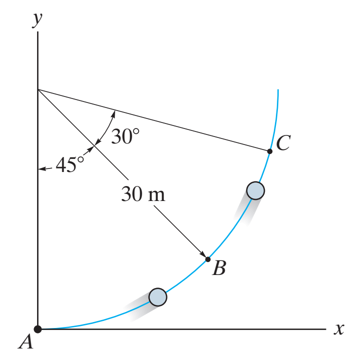
Ans:

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

12–81.

A particle travels along the circular path from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



SOLUTION

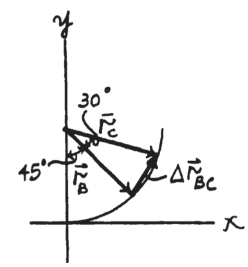
Position: The coordinates for points B and C are [30 sin 45°, 30 − 30 cos 45°] and [30 sin 75°, 30 − 30 cos 75°]. Thus,

$$\mathbf{r}_B = (30\sin 45° - 0)\mathbf{i} + [(30 - 30\cos 45°) - 30]\mathbf{j}$$

$$= \{21.21\mathbf{i} - 21.21\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_C = (30\sin 75° - 0)\mathbf{i} + [(30 - 30\cos 75°) - 30]\mathbf{j}$$

$$= \{28.98\mathbf{i} - 7.765\mathbf{j}\}\text{ m}$$



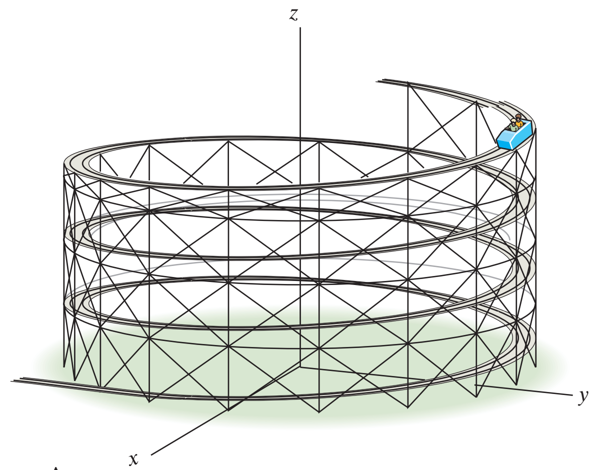
Average Velocity: The displacement from point B to C is $\Delta\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = (28.98\mathbf{i} - 7.765\mathbf{j}) - (21.21\mathbf{i} - 21.21\mathbf{j}) = \{7.765\mathbf{i} + 13.45\mathbf{j}\}$ m.

$$(\mathbf{v}_{BC})_{avg} = \frac{\Delta\mathbf{r}_{BC}}{\Delta t} = \frac{7.765\mathbf{i} + 13.45\mathbf{j}}{3 - 1} = \{3.88\mathbf{i} + 6.72\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{BC})_{avg} = \{3.88\mathbf{i} + 6.72\mathbf{j}\}$ m/s

12–82.

The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c, h, and b are constants. Determine the magnitudes of its velocity and acceleration.



SOLUTION

$x = c \sin kt \qquad \dot{x} = ck \cos kt \qquad \ddot{x} = -ck^2 \sin kt$

$y = c \cos kt \qquad \dot{y} = -ck \sin kt \qquad \ddot{y} = -ck^2 \cos kt$

$z = h - bt \qquad \dot{z} = -b \qquad \ddot{z} = 0$

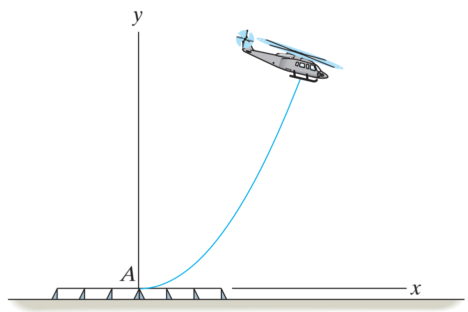
$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2} = \sqrt{c^2k^2 + b^2}$ **Ans.**

$a = \sqrt{(-ck^2 \sin kt)^2 + (-ck^2 \cos kt)^2 + 0} = ck^2$ **Ans.**

Ans:
$v = \sqrt{c^2 k^2 + b^2}$
$a = ck^2$

12–83.

The flight path of the helicopter as it takes off from A is defined by the parametric equations $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in seconds. Determine the distance the helicopter is from point A and the magnitudes of its velocity and acceleration when $t = 10$ s.



SOLUTION

$x = 2t^2 \qquad y = 0.04t^3$

At $t = 10$ s, $\qquad x = 200$ m $\qquad y = 40$ m

$d = \sqrt{(200)^2 + (40)^2} = 204$ m **Ans.**

$$v_x = \frac{dx}{dt} = 4t$$

$$a_x = \frac{dv_x}{dt} = 4$$

$$v_y = \frac{dy}{dt} = 0.12t^2$$

$$a_y = \frac{dv_y}{dt} = 0.24t$$

At $t = 10$ s,

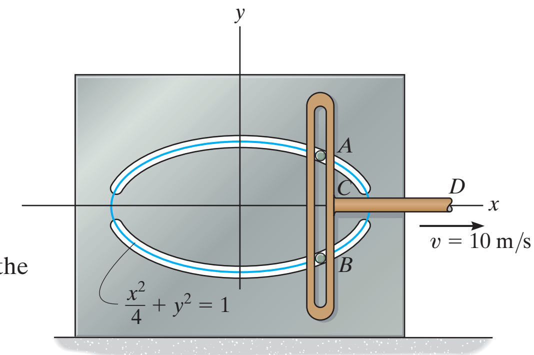
$v = \sqrt{(40)^2 + (12)^2} = 41.8$ m/s **Ans.**

$a = \sqrt{(4)^2 + (2.4)^2} = 4.66$ m/s^2 **Ans.**

Ans:
$d = 204$ m
$v = 41.8$ m/s
$a = 4.66$ m/s^2

***12–84.**

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1\text{ m}$.



SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1\text{ m}$,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad\qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10\text{ m/s}$ and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad\qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s}\downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left(v_x^{\,2} + xa_x\right) + 2\left(v_y^{\,2} + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1\text{ m}$, $y = \frac{\sqrt{3}}{2}\text{ m}$, $v_x = 10\text{ m/s}$, and $v_y = -2.887\text{ m/s}$. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2\downarrow$$

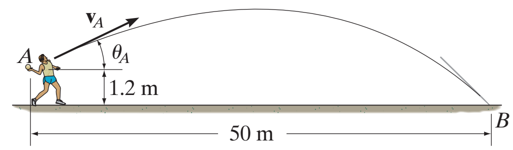
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_x^{\,2} + a_y^{\,2}} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 10.4\text{ m/s}$
$a = 38.5\text{ m/s}^2$

12–85.

It is observed that the time for the ball to strike the ground at B is 2.5 s. Determine the speed v_A and angle θ_A at which the ball was thrown.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

x-Motion: Here, $(v_A)_x = v_A \cos\theta_A$, $x_A = 0$, $x_B = 50$ m, and $t = 2.5$ s. Thus,

$$(\xrightarrow{+}) \qquad x_B = x_A + (v_A)_x t$$

$$50 = 0 + v_A \cos\theta_A (2.5)$$

$$v_A \cos\theta_A = 20 \qquad \textbf{(1)}$$

y-Motion: Here, $(v_A)_y = v_A \sin\theta_A$, $y_A = 0$, $y_B = -1.2$ m, and $a_y = -g = -9.81\ \text{m/s}^2$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.2 = 0 + v_A \sin\theta_A (2.5) + \frac{1}{2}(-9.81)\left(2.5^2\right)$$

$$v_A \sin\theta_A = 11.7825 \qquad \textbf{(2)}$$

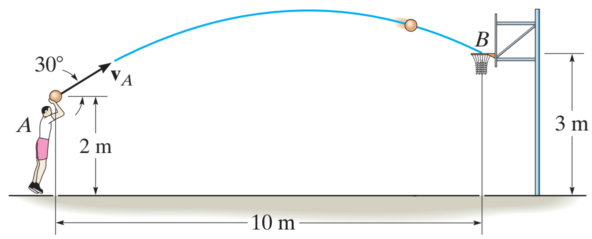
Solving Eqs. (1) and (2) yields

$$\theta_A = 30.5^\circ \qquad v_A = 23.2\ \text{m/s} \qquad \textbf{Ans.}$$

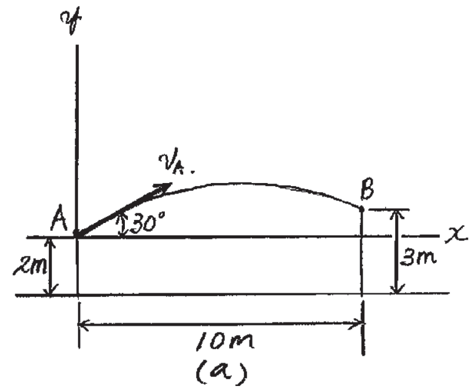
Ans:
$v_A = 23.2$ m/s

12–86.

Neglecting the size of the ball, determine the magnitude v_A of the basketball's initial velocity and its velocity when it passes through the basket.



SOLUTION



Coordinate System. The origin of the *x-y* coordinate system will be set to coinside with point *A* as shown in Fig. *a*

Horizontal Motion. Here $(v_A)_x = v_A \cos 30° \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 10 \text{ m} \rightarrow$.

$$(\overset{+}{\rightarrow}) \quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$10 = 0 + v_A \cos 30° \, t$$

$$t = \frac{10}{v_A \cos 30°} \qquad \textbf{(1)}$$

Also,

$$(\overset{+}{\rightarrow}) \quad (v_B)_x = (v_A)_x = v_A \cos 30° \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = v_A \sin 30° \uparrow$, $(s_A)_y = 0$, $(s_B)_y = 3 - 2 = 1 \text{ m} \uparrow$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$(+\uparrow) \quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$1 = 0 + v_A \sin 30° \, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 0.5 \, v_A \, t + 1 = 0 \qquad \textbf{(3)}$$

Also

$$(+\uparrow) \quad (v_B)_y = (v_A)_y + a_y t$$

$$(v_B)_y = v_A \sin 30° + (-9.81)t$$

$$(v_B)_y = 0.5 \, v_A - 9.81t \qquad \textbf{(4)}$$

Solving Eq. (1) and (3)

$$v_A = 11.705 \text{ m/s} = 11.7 \text{ m/s} \qquad \textbf{Ans.}$$

$$t = 0.9865 \text{ s}$$

Substitute these results into Eq. (2) and (4)

$$(v_B)_x = 11.705 \cos 30° = 10.14 \text{ m/s} \rightarrow$$

$$(v_B)_y = 0.5(11.705) - 9.81(0.9865) = -3.825 \text{ m/s} = 3.825 \text{ m/s} \downarrow$$

Thus, the magnitude of $\mathbf{v}_B$ is

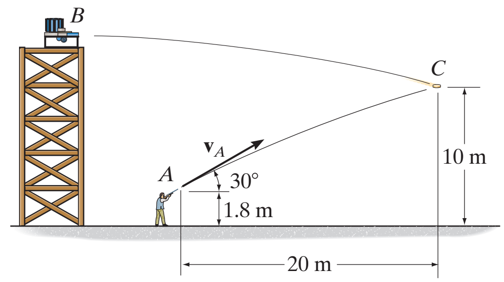
$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{10.14^2 + 3.825^2} = 10.83 \text{ m/s} = 10.8 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{3.825}{10.14}\right) = 20.67° = 20.7° \qquad \textbf{Ans.}$$

Ans:
$v_A = 11.7 \text{ m/s}$
$v_B = 10.8 \text{ m/s}$
$\theta = 20.7°$ ⦩

12–87. A projectile is fired from the platform at B. The shooter fires his gun from point A at an angle of 30°. Determine the muzzle speed of the bullet if it hits the projectile at C.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $x_A = 0$ and $x_C = 20$ m. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad x_C = x_A + (v_A)_x t$$

$$20 = 0 + v_A \cos 30^\circ\, t \qquad \textbf{(1)}$$

***y*-Motion:** Here, $y_A = 1.8$, $(v_A)_y = v_A \sin 30^\circ$, and $a_y = -g = -9.81 \text{ m/s}^2$. Thus,

$$(+\uparrow) \qquad y_C = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$10 = 1.8 + v_A \sin 30^\circ (t) + \frac{1}{2}(-9.81)(t)^2$$

Thus,

$$10 - 1.8 = \left(\frac{20 \sin 30^\circ}{\cos 30^\circ (t)}\right)(t) - 4.905(t)^2$$

$$t = 0.8261 \text{ s}$$

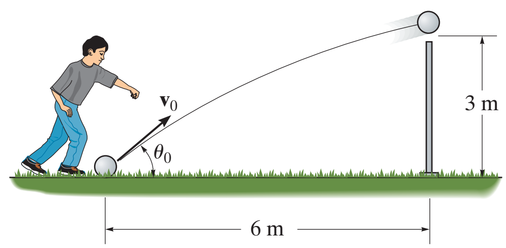
So that

$$v_A = \frac{20}{\cos 30^\circ (0.8261)} = 28.0 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 28.0$ m/s

***12–88.**

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set so that its origin coincides with the ball's initial position.

x-Motion: Here, $(v_0)_x = v_0 \cos\theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$\left(\xrightarrow{+}\right) \qquad x = x_0 + (v_0)x^t$$

$$6 = 0 + (v_0 \cos\theta)t$$

$$t = \frac{6}{v_0 \cos\theta} \qquad \textbf{(1)}$$

y-Motion: Here, $(v_0)_x = v_0 \sin\theta$, $a_y = -g = -9.81 \text{ m/s}^2$, and $y_0 = 0$. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$3 = 0 + v_0 (\sin\theta) t + \frac{1}{2}(-9.81)t^2$$

$$3 = v_0 (\sin\theta) t - 4.905t^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2\theta}} \qquad \textbf{(3)}$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2\theta$ is maximum. This requires $\frac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57°$$

$$\theta = 58.28° = 58.3° \qquad \textbf{Ans.}$$

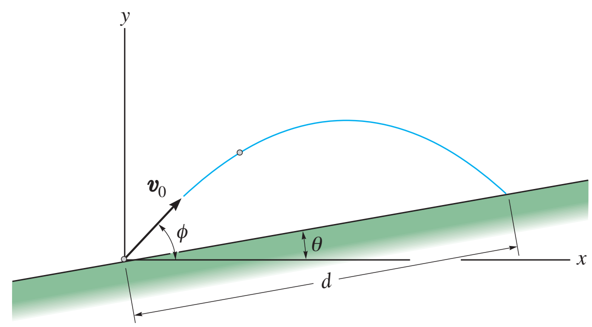
Substituting the result of θ into Eq. (2), we have

$$(v_0)_{min} = \sqrt{\frac{58.86}{\sin 116.57° - \cos^2 58.28°}} = 9.76 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\theta = 58.3°$
$(v_0)_{\text{min}} = 9.76 \text{ m/s}$

12–89.

A projectile is given a velocity $\mathbf{v}_0$ at an angle ϕ above the horizontal. Determine the distance d to where it strikes the sloped ground. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

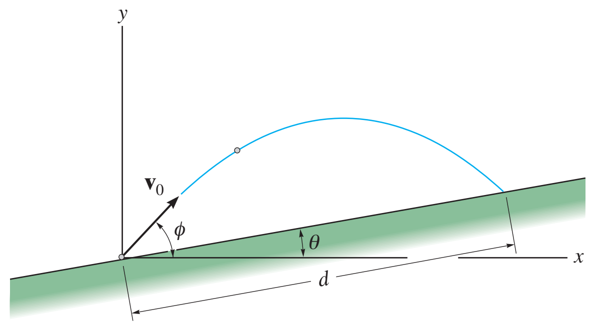
$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right) \qquad \textbf{Ans.}$$

Ans:

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$

12-90.

A projectile is given a velocity $\mathbf{v}_0$. Determine the angle ϕ at which it should be launched so that d is a maximum. The acceleration due to gravity is g.



SOLUTION

$$\left(\xrightarrow{+}\right) \qquad s_x = s_0 + v_0 t$$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$$(+\uparrow) \qquad s_y = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$

Require:

$$\frac{d(d)}{d\phi} = \frac{v_0^2}{g\cos\theta}\left[\cos 2\phi(2) - 2\tan\theta(2\cos\phi)(-\sin\phi)\right] = 0$$

$$\cos 2\phi + \tan\theta\sin 2\phi = 0$$

$$\frac{\sin 2\phi}{\cos 2\phi}\tan\theta + 1 = 0$$

$$\tan 2\phi = -\text{ctn}\,\theta$$

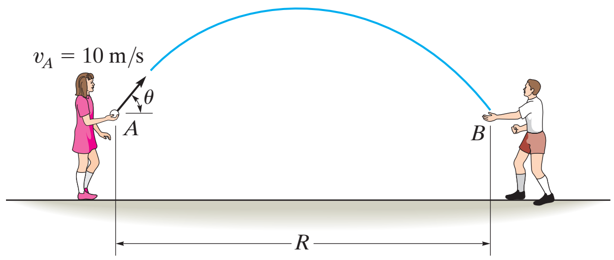
$$\phi = \frac{1}{2}\tan^{-1}(-\text{ctn}\,\theta) \qquad \textbf{Ans.}$$

Ans:

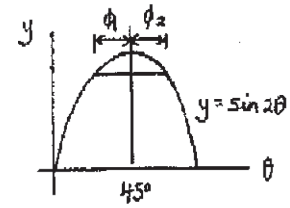
$$\phi = \frac{1}{2}\tan^{-1}(-\text{ctn}\,\theta)$$

12–91.

The girl at A can throw a ball at $v_A = 10$ m/s. Calculate the maximum possible range $R = R_{max}$ and the associated angle θ at which it should be thrown. Assume the ball is caught at B at the same elevation from which it is thrown.



SOLUTION



$(\xrightarrow{+})\quad s = s_0 + v_0 t$

$$R = 0 + (10 \cos \theta)t$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$-10 \sin \theta = 10 \sin \theta - 9.81t$$

$$t = \frac{20}{9.81} \sin \theta$$

Thus, $\quad R = \dfrac{200}{9.81} \sin \theta \cos \theta$

$$R = \frac{100}{9.81} \sin 2\theta \qquad \textbf{(1)}$$

Require,

$$\frac{dR}{d\theta} = 0$$

$$\frac{100}{9.81} \cos 2\theta(2) = 0$$

$$\cos 2\theta = 0$$

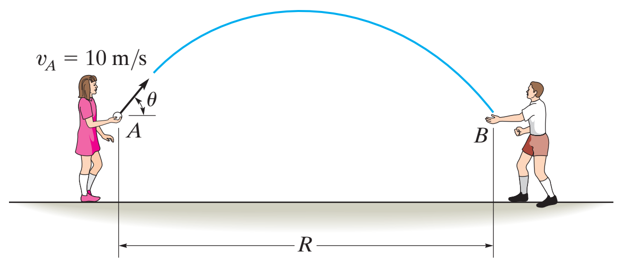
$$\theta = 45^\circ \qquad \textbf{Ans.}$$

$$R = \frac{100}{9.81} (\sin 90^\circ) = 10.2 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$R_{max} = 10.2$ m
$\theta = 45^\circ$

***12–92.**

Show that the girl at A can throw the ball to the boy at B by launching it at equal angles measured up or down from a 45° inclination. If $v_A = 10$ m/s, determine the range R if this value is 15°, i.e., $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$. Assume the ball is caught at the same elevation from which it is thrown.



SOLUTION

$(\xrightarrow{+})$ $s = s_0 + v_0 t$

$R = 0 + (10 \cos\theta)t$

$(+\uparrow)$ $v = v_0 + a_c t$

$-10 \sin\theta = 10 \sin\theta - 9.81t$

$$t = \frac{20}{9.81}\sin\theta$$

Thus, $R = \dfrac{200}{9.81}\sin\theta\cos\theta$

$$R = \frac{100}{9.81}\sin 2\theta \qquad \textbf{(1)}$$

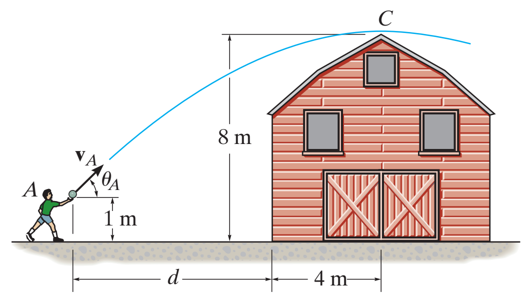
Since the function $y = \sin 2\theta$ is symmetric with respect to $\theta = 45°$ as indicated, Eq. (1) will be satisfied if $|\phi_1| = |\phi_2|$

Choosing $\phi = 15°$ or $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$, and substituting into Eq. (1) yields

$R = 8.83$ m **Ans.**

Ans:
$R = 8.83$ m

12–93. The boy at A attempts to throw a ball over the roof of a barn with an initial speed of $v_A = 15$ m/s. Determine the angle θ_A at which the ball must be thrown so that it reaches its maximum height at C. Also, find the distance d where the boy should stand to make the throw.



SOLUTION

Vertical Motion: The vertical component, of initial and final velocity are $(v_0)_y = (15 \sin \theta_A)$ m/s and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1$ m.

$$(+\uparrow) \qquad v_y = (v_0) + a_c t$$

$$0 = 15 \sin \theta_A + (-9.81)t \qquad \textbf{[1]}$$

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$8 = 1 + 15 \sin \theta_A t + \frac{1}{2}(-9.81)t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$\theta_A = 51.38° = 51.4° \qquad \textbf{Ans.}$$

$$t = 1.195 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos \theta_A = 15 \cos 51.38° = 9.363$ m/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4)$ m, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

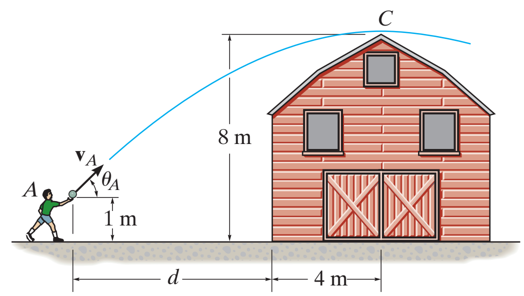
$$d + 4 = 0 + 9.363(1.195)$$

$$d = 7.18 \text{ m} \qquad \textbf{Ans.}$$

Ans:

$\theta_A = 51.4°$
$d = 7.18$ m

12–94. The boy at A attempts to throw a ball over the roof of a barn such that it is launched at an angle $\theta_A = 40°$. Determine the minimum speed v_A at which he must throw the ball so that it reaches its maximum height at C. Also, find the distance d where the boy must stand so that he can make the throw.



SOLUTION

Vertical Motion: The vertical components of initial and final velocity are $(v_0)_y = (v_A \sin 40°)$ m/s and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1$ m.

$(+\uparrow)$ $$v_y = (v_0) + a_c t$$

$$0 = v_A \sin 40° + (-9.81)\,t \qquad \textbf{[1]}$$

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$8 = 1 + v_A \sin 40° t + \frac{1}{2}(-9.81)\,t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$v_A = 18.23 \text{ m/s} = 18.2 \text{ m/s}$$

$$t = 1.195 \text{ s} \qquad \textbf{Ans.}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos\theta_A = 18.23 \cos 40° = 13.97$ m/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4)$ m, respectively.

$(\overset{+}{\rightarrow})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$d + 4 = 0 + 13.97(1.195)$$

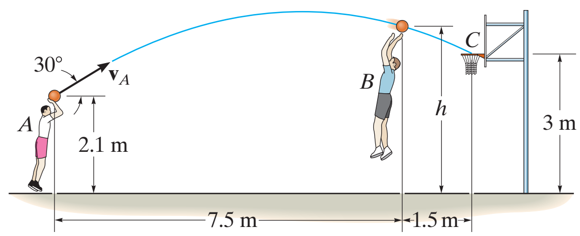
$$d = 12.7 \text{ m} \qquad \textbf{Ans.}$$

Ans:

$t = 1.195$ s
$d = 12.7$ m

12-95.

Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



SOLUTION

$a = 2.1\ \text{m}$

$b = 7.5\ \text{m}$

$c = 1.5\ \text{m}$

$d = 3\ \text{m}$

$\theta = 30°$

$g = 9.81\ \text{m/s}^2$

Guesses $\quad v_A = 3\ \text{m/s} \quad t_B = 1\,\text{s} \quad t_C = 1\,\text{s} \quad h = 3.5\,\text{m}$

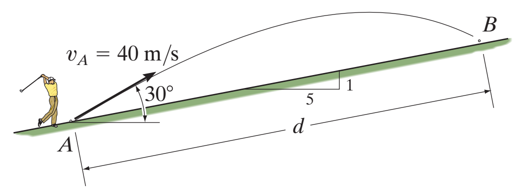
Given $\quad b + c = v_A\cos(\theta)t_C \qquad b = v_A\cos(\theta)t_B$

$$d = \frac{-g}{2}t_C^{\,2} + v_A\sin(\theta)t_C + a \qquad h = \frac{-g}{2}t_B^{\,2} + v_A\sin(\theta)t_B + a$$

$$\begin{pmatrix} v_A \\ t_B \\ t_C \\ h \end{pmatrix} = \text{Find}(v_A, t_B, t_C, h) \qquad \begin{pmatrix} t_B \\ t_C \end{pmatrix} = \begin{pmatrix} 0.78 \\ 0.94 \end{pmatrix}\text{s} \qquad v_A = 11.1\,\text{m/s} \qquad h = 3.45\,\text{m} \quad \textbf{Ans.}$$

Ans:
$v_A = 11.1\ \text{m/s}$
$h = 3.45\ \text{m}$

***12–96.** The golf ball is hit at A with a speed of $v_A = 40\text{ m/s}$ and directed at an angle of 30° with the horizontal as shown. Determine the distance d where the ball strikes the slope at B.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = 40\cos 30° = 34.64\text{ m/s}$, $x_A = 0$, and $x_B = d\left(\dfrac{5}{\sqrt{5^2+1}}\right) = 0.9806d$. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad x_B = x_A + (v_A)_x t$$

$$0.9806d = 0 + 34.64t$$

$$t = 0.02831d \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 40\sin 30° = 20\text{ m/s}$, $y_A = 0$, $y_B = d\left(\sqrt{\dfrac{1}{\sqrt{5^2+1}}}\right) = 0.1961d$, and $a_y = -g = -9.81\text{ m/s}^2$.

Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$0.1961d = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 20t + 0.1961d = 0 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$4.905(0.02831d)^2 - 20(0.02831d) + 0.1961d = 0$$

$$3.9303\left(10^{-3}\right)d^2 - 0.37002d = 0$$

$$d\left[3.9303\left(10^{-3}\right)d - 0.37002\right] = 0$$

Since $d \neq 0$, then

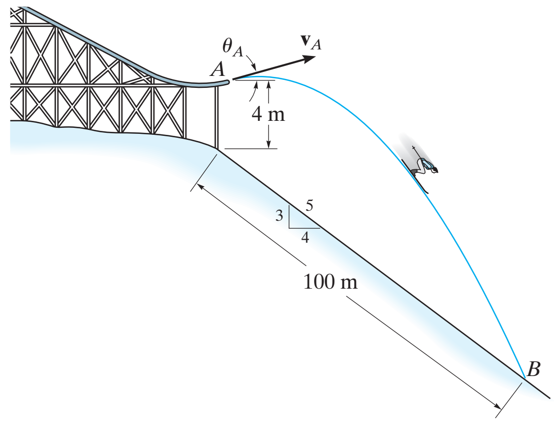
$$3.9303\left(10^{-3}\right)d = 0.37002 = 0$$

$$d = 94.1\text{ m} \qquad \textbf{Ans.}$$

Ans:
$d = 94.1$ m

12–97.

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.



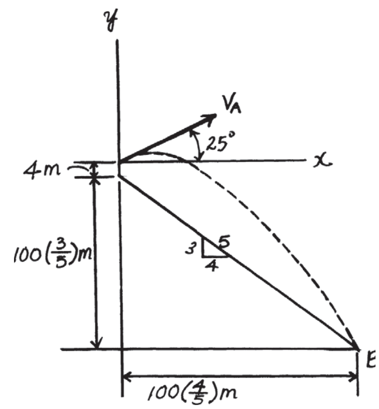
SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81) t_{AB}^2$$



Solving,

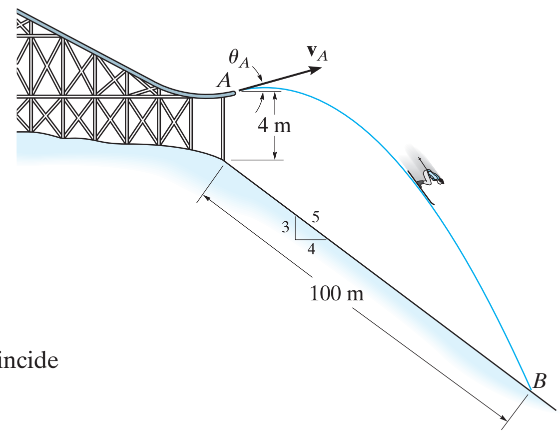
$$v_A = 19.4 \text{ m/s} \qquad \textbf{Ans.}$$

$$t_{AB} = 4.54 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 19.4$ m/s
$t_{AB} = 4.54$ s

12–98.

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.



SOLUTION

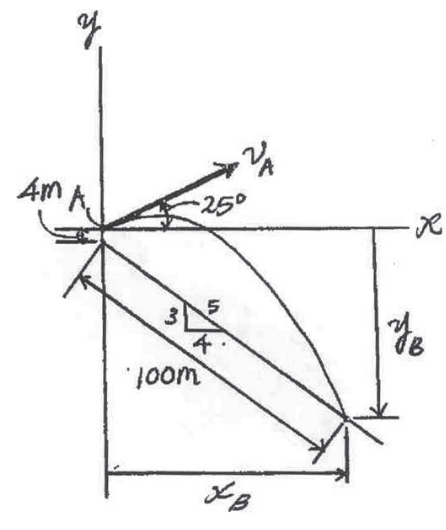
Coordinate System: $x-y$ coordinate system will be set with its origin to coincide with point A as shown in Fig. a.

x-motion: Here, $x_A = 0,\ x_B = 100\left(\frac{4}{5}\right) = 80\text{ m}$ and $(v_A)_x = v_A \cos 25°$.

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$80 = 0 + (v_A \cos 25°)t$$

$$t = \frac{80}{v_A \cos 25°} \tag{1}$$



(a)

y-motion: Here, $y_A = 0, y_B = -[4 + 100\left(\frac{3}{5}\right)] = -64\text{ m}$ and $(v_A)_y = v_A \sin 25°$ and $a_y = -g = -9.81\text{ m/s}^2$.

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-64 = 0 + v_A \sin 25°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 25°\, t = 64 \tag{2}$$

Substitute Eq. (1) into (2) yields

$$4.905\left(\frac{80}{v_A \cos 25°}\right)^2 = v_A \sin 25°\left(\frac{80}{v_A \cos 25°}\right) = 64$$

$$\left(\frac{80}{v_A \cos 25°}\right)^2 = 20.65$$

$$\frac{80}{v_A \cos 25°} = 4.545$$

$$v_A = 19.42\text{ m/s} = 19.4\text{ m/s}$$ **Ans.**

Substitute this result into Eq. (1),

$$t = \frac{80}{19.42 \cos 25°} = 4.54465\text{ s}$$

12–98. Continued

Using this result,

$(+\uparrow)$ $$(v_B)_y = (v_A)_y + a_y t$$

$$= 19.42 \sin 25^\circ + (-9.81)(4.5446)$$

$$= -36.37 \text{ m/s} = 36.37 \text{ m/s} \downarrow$$

And

$(\overset{+}{\rightarrow})$ $$(v_B)_x = (v_A)_x = v_A \cos 25^\circ = 19.42 \cos 25^\circ = 17.60 \text{ m/s} \rightarrow$$

Thus,

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2}$$

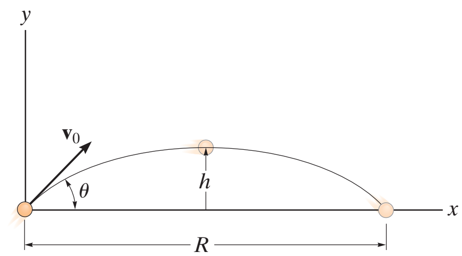
$$= \sqrt{36.37^2 + 17.60^2}$$

$$= 40.4 \text{ m/s}$$ **Ans.**

Ans:
$v_A = 19.4 \text{ m/s}$
$v_B = 40.4 \text{ m/s}$

12–99.

The projectile is launched with a velocity $\mathbf{v}_0$. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$$R = 0 + (v_0 \cos\theta)t$$

$(+\uparrow)\qquad s = s_0 + v_0\, t + \frac{1}{2}a_c t^2$

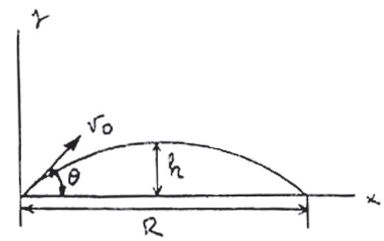
$$0 = 0 + (v_0 \sin\theta)\, t + \frac{1}{2}(-g)t^2$$

$$0 = v_0 \sin\theta - \frac{1}{2}(g)\left(\frac{R}{v_0 \cos\theta}\right)$$

$$R = \frac{v_0^2}{g}\sin 2\theta \qquad \textbf{Ans.}$$

$$t = \frac{R}{v_0 \cos\theta} = \frac{v_0^2\,(2\sin\theta\cos\theta)}{v_0\, g\cos\theta}$$

$$= \frac{2v_0}{g}\sin\theta \qquad \textbf{Ans.}$$



$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_0 \sin\theta)^2 + 2(-g)(h - 0)$$

$$h = \frac{v_0^2}{2g}\sin^2\theta \qquad \textbf{Ans.}$$

Ans:

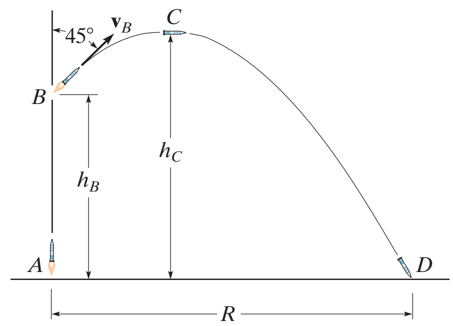
$R = \dfrac{v_0}{g}\sin 2\theta$

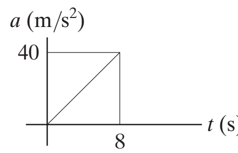
$t = \dfrac{2v_0}{g}\sin\theta$

$h = \dfrac{v_0^2}{2g}\sin^2\theta$

***12–100.**

The missile at A takes off from rest and rises vertically to B, where its fuel runs out in 8 s. If the acceleration varies with time as shown, determine the missile's height h_B and speed v_B. If by internal controls the missile is then suddenly pointed 45° as shown, and allowed to travel in free flight, determine the maximum height attained, h_C, and the range R to where it crashes at D.





SOLUTION

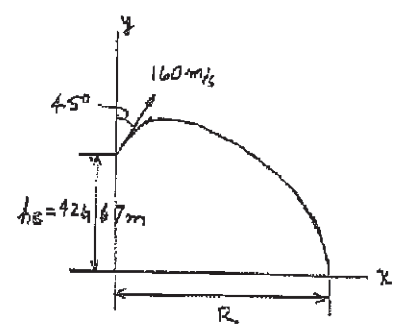
$$a = \frac{40}{8}t = 5t$$

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 5t\,dt$$

$$v = 2.5t^2$$

When $t = 8$ s, $\quad v_B = 2.5(8)^2 = 160$ m/s **Ans.**



$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 2.5t^2\,dt$$

$$x = \frac{2.5}{3}t^3$$

$$h_B = \frac{2.5}{3}(8)^3 = 426.67 = 427 \text{ m}$$ **Ans.**

$$(v_B)_x = 160 \sin 45° = 113.14 \text{ m/s}$$

$$(v_B)_y = 160 \cos 45° = 113.14 \text{ m/s}$$

$$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = (113.14)^2 + 2(-9.81)(s_c - 426.67)$$

$$h_c = 1079.1 \text{ m} = 1.08 \text{ km}$$ **Ans.**

$$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$$

$$R = 0 + 113.14t$$

$$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 426.67 + 113.14t + \frac{1}{2}(-9.81)t^2$$

Solving for the positive root, $t = 26.36$ s

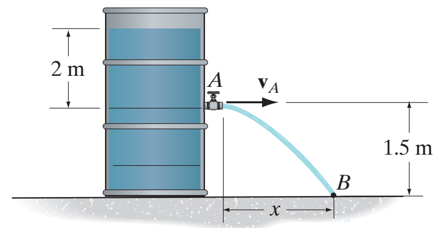
Then,

$$R = 113.14\,(26.36) = 2983.0 = 2.98 \text{ km}$$ **Ans.**

Ans:
$v_B = 160$ m/s
$h_B = 427$ m
$h_C = 1.08$ km
$R = 2.98$ km

12–101.

The velocity of the water jet discharging from the orifice can be obtained from $v = \sqrt{2gh}$, where $h = 2\text{ m}$ is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x where it hits the surface.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A. The speed of the water that the jet discharges from A is

$$v_A = \sqrt{2(9.81)(2)} = 6.264 \text{ m/s}$$

***x*-Motion:** Here, $(v_A)_x = v_A = 6.264$ m/s, $x_A = 0$, $x_B = x$, and $t = t_A$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$x = 0 + 6.264 t_A \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 0$, $a_y = -g = -9.81 \text{ m/s}^2$, $y_A = 0$ m, $y_B = -1.5$ m, and $t = t_A$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.5 = 0 + 0 + \frac{1}{2}(-9.81) t_A{}^2$$

$$t_A = 0.553 \text{ s} \qquad \textbf{Ans.}$$

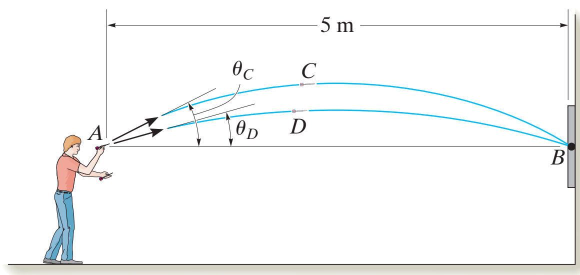
Thus,

$$x = 0 + 6.264(0.553) = 3.46 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$t_A = 0.553$ s
$x = 3.46$ m

12–102.

The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at θ_C $(>\theta_D)$, then the second dart is thrown at θ_D.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$5 = 0 + (10 \cos \theta)\, t$ **(1)**

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$-10 \sin \theta = 10 \sin \theta - 9.81\, t$

$$t = \frac{2(10 \sin \theta)}{9.81} = 2.039 \sin \theta$$

From Eq. (1),

$5 = 20.39 \sin \theta \cos \theta$

Since $\quad \sin 2\theta = 2 \sin \theta \cos \theta$

$\sin 2\theta = 0.4905$

The two roots are $\theta_D = 14.7°$ **Ans.**

$\theta_C = 75.3°$ **Ans.**

From Eq. (1): $t_D = 0.517$ s

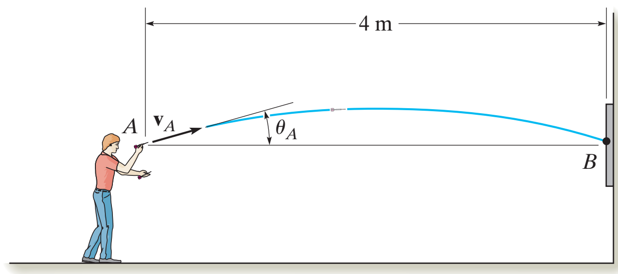
$t_C = 1.97$ s

So that $\Delta t = t_C - t_D = 1.45$ s **Ans.**

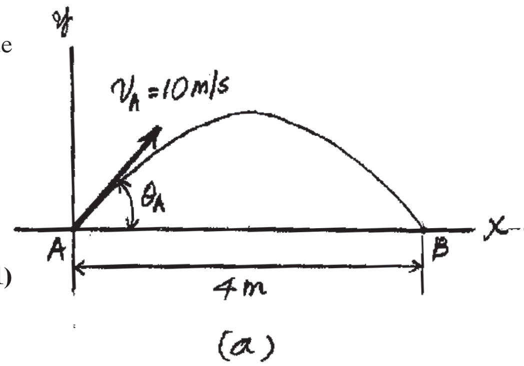
Ans:
$\theta_D = 14.7°$
$\theta_C = 75.3°$
$\Delta t = 1.45$ s

12–103.

If the dart is thrown with a speed of 10 m/s, determine the shortest possible time before it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



SOLUTION



Coordinate System. The origin of the x-y coordinate system will be set to coincide with point A as shown in Fig. a.

Horizontal Motion. Here, $(v_A)_x = 10 \cos \theta_A \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 4 \text{ m} \rightarrow$.

$$(\overset{+}{\rightarrow}) \quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$4 = 0 + 10 \cos \theta_A \, t$$

$$t = \frac{4}{10 \cos \theta_A} \qquad \textbf{(1)}$$

Also,

$$(\overset{+}{\rightarrow}) \quad (v_B)_x = (v_A)_x = 10 \cos \theta_A \rightarrow \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = 10 \sin \theta_A \uparrow$, $(s_A)_y = (s_B)_y = 0$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$(+\uparrow) \quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$0 = 0 + (10 \sin \theta_A)\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - (10 \sin \theta_A)\, t = 0$$

$$t\,(4.905t - 10 \sin \theta_A) = 0$$

Since $t \neq 0$, then

$$4.905t - 10 \sin \theta_A = 0 \qquad \textbf{(3)}$$

Also

$$(+\uparrow) \quad (v_B)_y^2 = (v_A)_y^2 + 2\, a_y \left[(s_B)_y - (s_A)_y\right]$$

$$(v_B)_y^2 = (10 \sin \theta_A)^2 + 2\,(-9.81)\,(0 - 0)$$

$$(v_B)_y = -10 \sin \theta_A = 10 \sin \theta_A \downarrow \qquad \textbf{(4)}$$

Substitute Eq. (1) into (3)

$$4.905\left(\frac{4}{10 \cos \theta_A}\right) - 10 \sin \theta_A = 0$$

$$1.962 - 10 \sin \theta_A \cos \theta_A = 0$$

Using the trigonometry identity $\sin 2\theta_A = 2 \sin \theta_A \cos \theta_A$, this equation becomes

$$1.962 - 5 \sin 2\theta_A = 0$$

$$\sin 2\theta_A = 0.3924$$

$$2\theta_A = 23.10° \text{ and } 2\theta_A = 156.90°$$

$$\theta_A = 11.55° \text{ and } \theta_A = 78.45°$$

12–103. Continued

Since the shorter time is required, Eq. (1) indicates that smaller θ_A must be choosen. Thus

$$\theta_A = 11.55^\circ = 11.6^\circ \qquad \textbf{Ans.}$$

and

$$t = \frac{4}{10 \cos 11.55^\circ} = 0.4083 \text{ s} = 0.408 \text{ s} \qquad \textbf{Ans.}$$

Substitute the result of θ_A into Eq. (2) and (4)

$$(v_B)_x = 10 \cos 11.55^\circ = 9.7974 \text{ m/s} \rightarrow$$

$$(v_B)_y = 10 \sin 11.55^\circ = 2.0026 \text{ m/s} \downarrow$$

Thus, the magnitude of v_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{9.7974^2 + 2.0026^2} = 10 \text{ m/s} \qquad \textbf{Ans.}$$

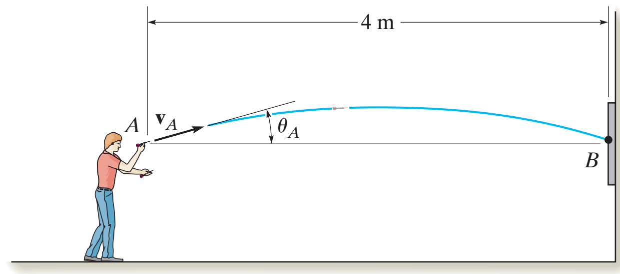
And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{2.0026}{9.7974}\right) = 11.55^\circ = 11.6^\circ \qquad \textbf{Ans.}$$

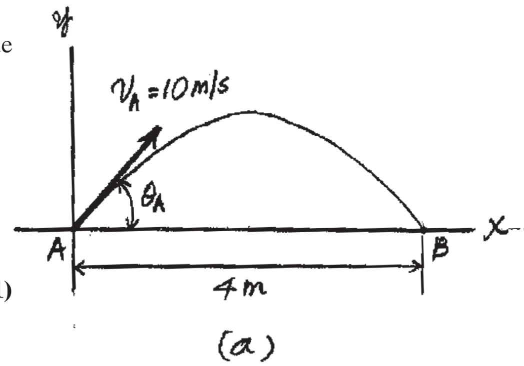
Ans:
$\theta_A = 11.6^\circ$
$t = 0\ 408$ s
$v_B = 10$ m/s
$\theta_B = 11.6^\circ$

***12–104.**

If the dart is thrown with a speed of 10 m/s, determine the longest possible time when it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



SOLUTION



Coordinate System. The origin of the x-y coordinate system will be set to coincide with point A as shown in Fig. a.

Horizontal Motion. Here, $(v_A)_x = 10 \cos \theta_A \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 4\text{ m} \rightarrow$.

$$(\overset{+}{\rightarrow}) \quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$4 = 0 + 10 \cos \theta_A\, t$$

$$t = \frac{4}{10 \cos \theta_A} \qquad \textbf{(1)}$$

Also,

$$(\overset{+}{\rightarrow}) \quad (v_B)_x = (v_A)_x = 10 \cos \theta_A \rightarrow \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = 10 \sin \theta_A \uparrow$, $(s_A)_y = (s_B)_y = 0$ and $a_y = -9.81\text{ m/s}^2 \downarrow$.

$$(+\uparrow) \quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$0 = 0 + (10 \sin \theta_A)\, t + \frac{1}{2}(-9.81)\, t^2$$

$$4.905t^2 - (10 \sin \theta_A)\, t = 0$$

$$t\,(4.905t - 10 \sin \theta_A) = 0$$

Since $t \neq 0$, then

$$4.905\, t - 10 \sin \theta_A = 0 \qquad \textbf{(3)}$$

Also,

$$(v_B)_y^2 = (v_A)_y^2 + 2\, a_y\, [(s_B)_y - (s_A)_y]$$

$$(v_B)_y^2 = (10 \sin \theta_A)^2 + 2\,(-9.81)\,(0 - 0)$$

$$(v_B)_y = -10 \sin \theta_A = 10 \sin \theta_A \downarrow \qquad \textbf{(4)}$$

Substitute Eq. (1) into (3)

$$4.905\left(\frac{4}{10 \cos \theta_A}\right) - 10 \sin \theta_A = 0$$

$$1.962 - 10 \sin \theta_A \cos \theta_A = 0$$

Using the trigonometry identity $\sin 2\theta_A = 2 \sin \theta_A \cos \theta_A$, this equation becomes

$$1.962 - 5 \sin 2\theta_A = 0$$

$$\sin 2\theta_A = 0.3924$$

$$2\theta_A = 23.10° \text{ and } 2\theta_A = 156.90°$$

$$\theta_A = 11.55° \text{ and } \theta_A = 78.44°$$

12–104. Continued

Since the longer time is required, Eq. (1) indicates that larger θ_A must be choosen. Thus,

$$\theta_A = 78.44° = 78.4° \qquad \textbf{Ans.}$$

and

$$t = \frac{4}{10 \cos 78.44°} = 1.9974 \text{ s} = 2.00 \text{ s} \qquad \textbf{Ans.}$$

Substitute the result of θ_A into Eq. (2) and (4)

$$(v_B)_x = 10 \cos 78.44° = 2.0026 \text{ m/s} \rightarrow$$

$$(v_B)_y = 10 \sin 78.44° = 9.7974 \text{ m/s} \downarrow$$

Thus, the magnitude of v_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{2.0026^2 + 9.7974^2} = 10 \text{ m/s} \qquad \textbf{Ans.}$$

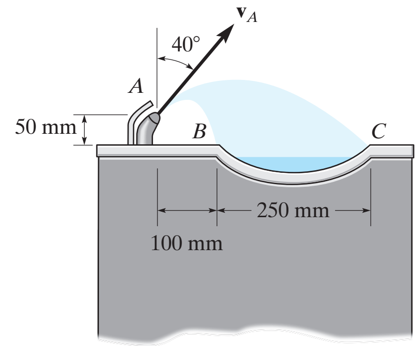
And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{9.7974}{2.0026}\right) = 78.44° = 78.4° \qquad \textbf{Ans.}$$

Ans:
$\theta_A = 78.4°$
$t = 2.00$ s
$\theta_B = 78.4°$

12–105.

The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.



SOLUTION

Horizontal Motion:

$$(\xrightarrow{+}) \qquad s = v_0 t$$

$$R = v_A \sin 40^\circ\, t \quad t = \frac{R}{v_A \sin 40^\circ} \qquad \textbf{(1)}$$

Vertical Motion:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-\,0.05 = 0 + v_A \cos 40^\circ t + \frac{1}{2}(-9.81)t^2 \qquad \textbf{(2)}$$

Substituting Eq.(1) into (2) yields:

$$-0.05 = v_A \cos 40^\circ \left(\frac{R}{v_A \sin 40^\circ}\right) + \frac{1}{2}(-9.81)\left(\frac{R}{v_A \sin 40^\circ}\right)^2$$

$$v_A = \sqrt{\frac{4.905 R^2}{\sin 40^\circ\,(R \cos 40^\circ + 0.05 \sin 40^\circ)}}$$

At point B, $\quad R = 0.1$ m.

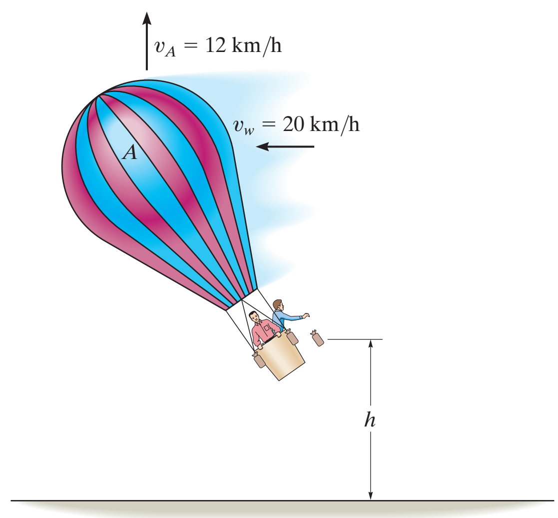
$$v_{\min} = v_A = \sqrt{\frac{4.905\,(0.1)^2}{\sin 40^\circ\,(0.1 \cos 40^\circ + 0.05 \sin 40^\circ)}} = 0.838 \text{ m/s} \qquad \textbf{Ans.}$$

At point C, $\quad R = 0.35$ m.

$$v_{\max} = v_A = \sqrt{\frac{4.905\,(0.35)^2}{\sin 40^\circ\,(0.35 \cos 40^\circ + 0.05 \sin 40^\circ)}} = 1.76 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{\min} = 0.838$ m/s
$v_{\max} = 1.76$ m/s

12–106. The balloon A is ascending at the rate $v_A = 12$ km/h and is being carried horizontally by the wind at $v_w = 20$ km/h. If a ballast bag is dropped from the balloon at the instant $h = 50$ m, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground?



SOLUTION

$v_A = 12$ km/h

$v_w = 20$ km/h

$h = 50$ m

$g = 9.81$ m/s^2

$a_x = 0 \qquad a_y = -g$

$v_x = v_w \qquad v_y = -g t + v_A$

$s_x = v_w t \qquad s_y = \frac{-1}{2} g t^2 + v_A t + h$

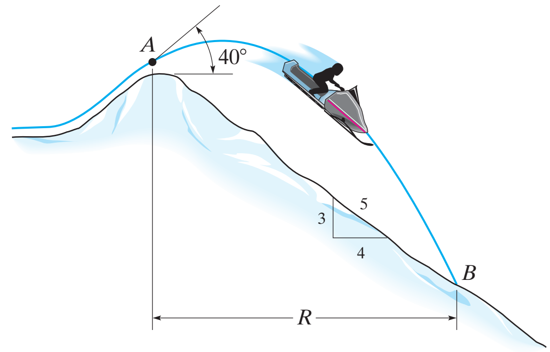
Thus $\quad 0 = \frac{-1}{2} g t^2 + v_A t + h \qquad t = \dfrac{v_A + \sqrt{v_A^2 + 2 g h}}{g} \qquad t = 3.55$ s **Ans.**

$v_x = v_w \qquad v_y = -g t + v_A \qquad v = \sqrt{v_x^2 + v_y^2} \qquad v = 32.0$ m/s **Ans.**

Ans:
$t = 3.55$ s
$v = 32.0$ m/s

12–107.

The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.



SOLUTION

$(\xrightarrow{+})$ $\quad s_B = s_A + v_A t$

$$R = 0 + 10 \cos 40^\circ\, t$$

$(+\uparrow)$ $\quad s_B = s_A + v_A t + \frac{1}{2} a_c t^2$

$$-R\left(\frac{3}{4}\right) = 0 + 10 \sin 40^\circ t - \frac{1}{2}(9.81)\, t^2$$

Solving:

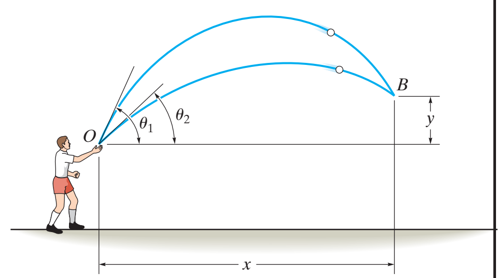
$R = 19.0$ m **Ans.**

$t = 2.48$ s **Ans.**

Ans:
$R = 19.0$ m
$t = 2.48$ s

***12–108.**

A boy throws a ball at O in the air with a speed v_0 at an angle θ_1. If he then throws another ball with the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in midair at B.



SOLUTION

Vertical Motion: For the first ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_1$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_1 t_1 + \frac{1}{2}(-g)t_1^2 \qquad \textbf{(1)}$$

For the second ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_2$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_2 t_2 + \frac{1}{2}(-g)t_2^2 \qquad \textbf{(2)}$$

Horizontal Motion: For the first ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_1$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_1\, t_1 \qquad \textbf{(3)}$$

For the second ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_2\, t_2 \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4), we have

$$t_2 = \frac{\cos\theta_1}{\cos\theta_2} t_1 \qquad \textbf{(5)}$$

Equating Eqs. (1) and (2), we have

$$v_0 t_1 \sin\theta_1 - v_0 t_2 \sin\theta_2 = \frac{1}{2} g\left(t_1^2 - t_2^2\right) \qquad \textbf{(6)}$$

Solving Eq. [5] into [6] yields

$$t_1 = \frac{2v_0 \cos\theta_2 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$t_2 = \frac{2v_0 \cos\theta_1 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

Thus, the time between the throws is

$$\Delta t = t_1 - t_2 = \frac{2v_0 \sin(\theta_1 - \theta_2)(\cos\theta_2 - \cos\theta_1)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

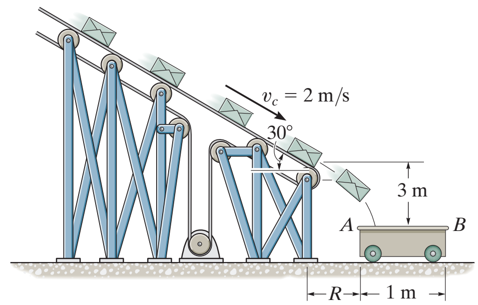
$$= \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos\theta_2 + \cos\theta_1)} \qquad \textbf{Ans.}$$

Ans:

$$\Delta t = \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos\theta_2 + \cos\theta_1)}$$

12–109.

Small packages traveling on the conveyor belt fall off into a l-m-long loading car. If the conveyor is running at a constant speed of $v_C = 2\text{ m/s}$, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 2\sin 30° = 1.00\text{ m/s}$. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 3\text{ m}$, respectively.

$$(+\downarrow) \qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$3 = 0 + 1.00(t) + \frac{1}{2}(9.81)\left(t^2\right)$$

Choose the positive root $t = 0.6867\text{ s}$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 2\cos 30° = 1.732\text{ m/s}$ and the initial horizontal position is $(s_0)_x = 0$. If $s_x = R$, then

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$R = 0 + 1.732(0.6867) = 1.19\text{ m} \qquad \textbf{Ans.}$$

If $s_x = R + 1$, then

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$R + 1 = 0 + 1.732(0.6867)$$

$$R = 0.189\text{ m} \qquad \textbf{Ans.}$$

Thus, $R_{\min} = 0.189\text{ m}$, $R_{\max} = 1.19\text{ m}$ **Ans.**

Ans:

$R_{\min} = 0.189\text{ m}$
$R_{\max} = 1.19\text{ m}$

12–110.

The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

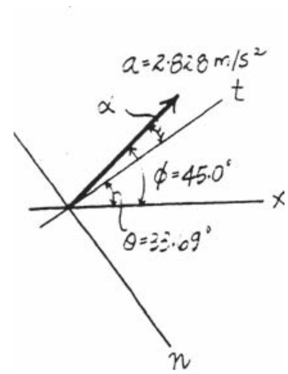
Velocity: Here, $\mathbf{r} = \left\{\left(2t + t^2\right)\mathbf{i} + t^2\mathbf{j}\right\}$ m. To determine the velocity $\mathbf{v}$, apply Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\,\mathbf{i} + 2t\mathbf{j}\}\text{ m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2}$ $= 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \quad \text{and} \quad v_t = 7.21\text{ m/s} \qquad \textbf{Ans.}$$

The velocity $\mathbf{v}$ makes an angle $\theta = \tan^{-1}\dfrac{4}{6} = 33.69°$ with the x axis.



Acceleration: To determine the acceleration $\mathbf{a}$, apply Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\}\text{ m/s}^2$$

Then

$$a = \sqrt{2^2 + 2^2} = 2.828\text{ m/s}^2$$

The acceleration $\mathbf{a}$ makes an angle $\phi = \tan^{-1}\dfrac{2}{2} = 45.0°$ with the x axis. From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a\sin\alpha = 2.828\sin 11.31° = 0.555\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_t = a\cos\alpha = 2.828\cos 11.31° = 2.77\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v_n = 0$ and $v_t = 7.21$ m/s
$a_n = 0.555$ m/s^2
$a_t = 2.77$ m/s^2

12–111.

Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

SOLUTION

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s}$$ **Ans.**

Ans:
$v = 38.7 \text{ m/s}$

***12–112.**

A particle moves along the curve $y = b\sin(cx)$ with a constant speed v. Determine the normal and tangential components of its velocity and acceleration at any instant.

SOLUTION

$$v = 2\ \frac{\text{m}}{\text{s}} \qquad b = 1\ \text{m} \qquad c = \frac{1}{\text{m}}$$

$$y = b\sin(cx) \qquad y' = bc\cos(cx) \qquad y'' = -bc^2\sin(cx)$$

$$\rho = \frac{\sqrt{\left(1 + y'^2\right)^3}}{y''} = \frac{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}}{-bc^2\sin(cx)}$$

$$a_n = \frac{v^2 bc\sin(cx)}{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}} \qquad a_t = 0 \qquad v_t = 0 \qquad v_n = 0 \qquad \textbf{Ans.}$$

Ans:

$$a_n = \frac{v^2 bc\sin(cx)}{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}}$$

$a_t = 0$

$v_t = 0$

$v_n = 0$

12–113.

The position of a particle is defined by $\mathbf{r} = \{4(t - \sin t)\mathbf{i} + (2t^2 - 3)\mathbf{j}\}$ m, where t is in seconds and the argument for the sine is in radians. Determine the speed of the particle and its normal and tangential components of acceleration when $t = 1$ s.

SOLUTION

$\mathbf{r} = 4(t - \sin t)\,\mathbf{i} + (2\,t^2 - 3)\mathbf{j}$

$\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 4(1 - \cos t)\mathbf{i} + (4\,t)\mathbf{j}$

$\mathbf{v}|_{t=1} = 1.83879\mathbf{i} + 4\mathbf{j}$

$v = \sqrt{(1.83879)^2 + (4)^2} = 4.40 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{4}{1.83879}\right) = 65.312° \measuredangle\theta$

$\mathbf{a} = 4 \sin \mathbf{r}\mathbf{i} + 4\mathbf{j}$

$\mathbf{a}|_{t=1} = 3.3659\mathbf{i} + 4\mathbf{j}$

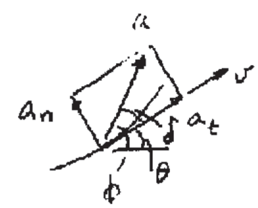
$a = \sqrt{(3.3659)^2 + (4)^2} = 5.22773 \text{ m/s}^2$

$\phi = \tan^{-1}\left(\dfrac{4}{3.3659}\right) = 49.920° \measuredangle\theta$

$\delta = \theta - \phi = 15.392°$

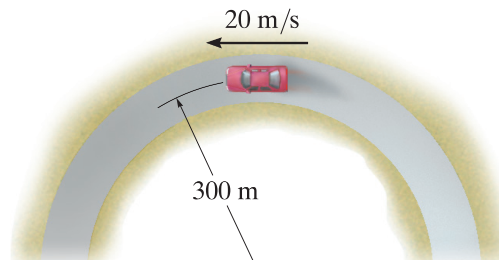
$a_2 = 5.22773 \cos 15.392° = 5.04 \text{ m/s}^2$ **Ans.**

$a_n = 5.22773 \sin 15.392° = 1.39 \text{ m/s}^2$ **Ans.**



Ans:
$v = 4.40$ m/s
$a_t = 5.04$ m/s^2
$a_n = 1.39$ m/s^2

12–114. The car travels along the curve having a radius of 300 m. If its speed is uniformly increased from 15 m/s to 27 m/s in 3 s, determine the magnitude of its acceleration at the instant its speed is 20 m/s.



SOLUTION

$v_1 = 15 \text{ m/s} \qquad t = 3 \text{ s}$

$R = 300 \text{ m}$

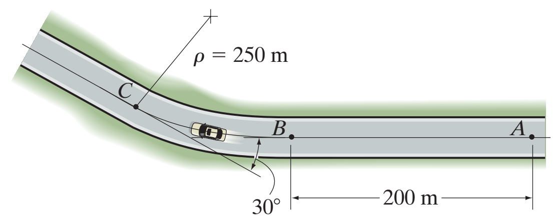
$v_2 = 27 \text{ m/s}$

$v_3 = 20 \text{ m/s}$

$$a_t = \frac{v_2 - v_1}{t} \qquad a_n = \frac{v_3^2}{R} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 4.22 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:
$a = 4.22 \text{ m/s}^2$

12–115. When the car reaches point A it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (-\frac{1}{4}t^{1/2})$ m/s^2. Determine the magnitude of acceleration of the car just before it reaches point C.



SOLUTION

Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$dv = a_t\,dt$$

$$\int_{25\text{ m/s}}^{v} dv = \int_0^t -\frac{1}{4}t^{1/2}\,dt$$

$$v = \left(25 - \frac{1}{6}t^{3/2}\right)\text{ m/s} \qquad \textbf{(1)}$$

Position: Using the initial conditions $s = 0$ when $t = 0$ s,

$$\int ds = \int v\,dt$$

$$\int_0^s ds = \int_0^t \left(25 - \frac{1}{6}t^{3/2}\right)dt$$

$$s = \left(25t - \frac{1}{15}t^{5/2}\right)\text{ m}$$

Acceleration: When the car reaches C, $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90$ m. Thus,

$$330.90 = 25t - \frac{1}{15}t^{5/2}$$

Solving by trial and error,

$$t = 15.942\text{ s}$$

Thus, using Eq. (1).

$$v_C = 25 - \frac{1}{6}(15.942)^{3/2} = 14.391\text{ m/s}$$

$$(a_t)_C = \dot{v} = -\frac{1}{4}\left(15.942^{1/2}\right) = -0.9982\text{ m/s}^2$$

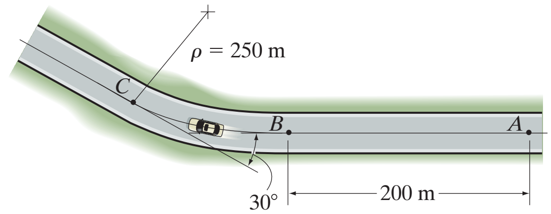
$$\left(a_n\right)_C = \frac{v_C^{\,2}}{\rho} = \frac{14.391^2}{250} = 0.8284\text{ m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{\left(a_t\right)_C^{\,2} + \left(a_n\right)_C^{\,2}} = \sqrt{(-0.9982)^2 + 0.8284^2} = 1.30\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 1.30\text{ m/s}^2$

***12–116.** When the car reaches point A, it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (0.001s - 1)\ \text{m/s}^2$. Determine the magnitude of acceleration of the car just before it reaches point C.



SOLUTION

Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$dv = a\,ds$$

$$\int_{25\ \text{m/s}}^{v} v\,dv = \int_0^s (0.001s - 1)\,ds$$

$$v = \sqrt{0.001s^2 - 2s + 625}$$

Acceleration: When the car is at point C, $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90$ m. Thus, the speed of the car at C is

$$v_C = \sqrt{0.001\left(330.90^2\right) - 2(330.90) + 625} = 8.526\ \text{m/s}^2$$

$$(a_t)_C = \dot{v} = [0.001(330.90) - 1] = -0.6691\ \text{m/s}^2$$

$$(a_n)_C = \frac{v_C{}^2}{\rho} = \frac{8.526^2}{250} = 0.2908\ \text{m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{(a_t)_C{}^2 + (a_n)_C{}^2} = \sqrt{(-0.6691)^2 + 0.2908^2} = 0.730\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.730\ \text{m/s}^2$

12–117. At a given instant, a car travels along a circular curved road with a speed of 20 m/s while decreasing its speed at the rate of 3 m/s^2. If the magnitude of the car's acceleration is 5 m/s^2, determine the radius of curvature of the road.

SOLUTION

Acceleration: Here, the car's tangential component of acceleration of $a_t = -3\ m/s^2$. Thus,

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}}$$

$$5 = \sqrt{(-3)^2 + a_n^{\,2}}$$

$$a_n = 4\ m/s^2$$

$$a_n = \frac{v^2}{\rho}$$

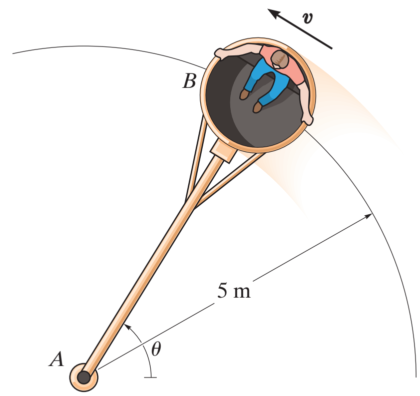
$$4 = \frac{20^2}{\rho}$$

$$\rho = 100\ m$$ **Ans.**

Ans:
$\rho = 100$ m

12–118.

Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



SOLUTION

Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5\left(e^t - 1\right) \qquad \textbf{(1)}$$

When $\theta = 30°$, the car has traveled a distance of $s = r\theta = 5\left(\dfrac{30°}{180°}\pi\right) = 2.618\ \text{m}$. The time required for the car to travel this distance can be obtained by applying $v = \dfrac{ds}{dt}$.

$$ds = v\,dt$$

$$\int_0^{2.618\ \text{m}} ds = \int_0^t 0.5\left(e^t - 1\right) dt$$

$$2.618 = 0.5\left(e^t - t - 1\right)$$

Solving by trial and error $\quad t = 2.1234\ \text{s}$

Substituting $t = 2.1234\ \text{s}$ into Eq. (1) yields

$$v = 0.5\left(e^{2.1234} - 1\right) = 3.680\ \text{m/s} = 3.68\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration for the car at $t = 2.1234\ \text{s}$ is $a_t = 0.5e^{2.1234} = 4.180\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.680^2}{5} = 2.708\ \text{m/s}^2$$

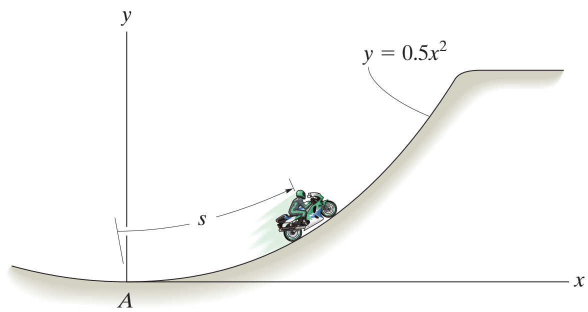
The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.180^2 + 2.708^2} = 4.98\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 3.68\ \text{m/s}$
$a = 4.98\ \text{m/s}^2$

12–119.

The motorcycle is traveling at 1 m/s when it is at A. If the speed is then increased at $\dot{v} = 0.1\ \text{m/s}^2$, determine its speed and acceleration at the instant $t = 5$ s.



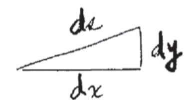
SOLUTION

$a_t = \nu = 0.1\,\text{m/s}^2$

$$s = s_0 + \nu_0\, t + \frac{1}{2} a_c t^2$$

$$s = 0 + 1(5) + \frac{1}{2}(0.1)(5)^2 = 6.25\ \text{m}$$

$$\int_0^{6.25} ds = \int_0^{x} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$



$y = 0.5x^2$

$$\frac{dy}{dx} = x$$

$$\frac{d^2y}{dx^2} = 1$$

$$6.25 = \int_0^x \sqrt{1 + x^2}\, dx$$

$$6.25 = \frac{1}{2}\left[x\sqrt{1 + x^2} + 1\text{n}\left(x + \sqrt{1 + x^2}\right)\right]_0^x$$

$$x\sqrt{1 + x^2} + 1\text{n}\left(x + \sqrt{1 + x^2}\right) = 12.5$$

Solving,

$x = 3.184$ m

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{[1 + x^2]^{\frac{3}{2}}}{|1|}\right|_{x=3.184} = 37.17\ \text{m}$$

$\nu = \nu_0 + a_c t$

$\quad = 1 + 0.1(5) = 1.5\ \text{m/s}$ **Ans.**

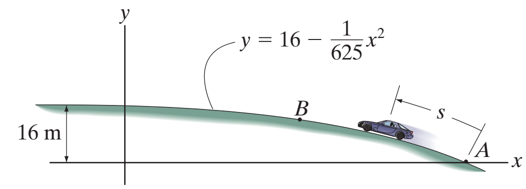
$$a_n = \frac{\nu^2}{\rho} = \frac{(1.5)^2}{37.17} = 0.0605\ \text{m/s}^2$$

$a = \sqrt{(0.1)^2 + (0.0605)^2} = 0.117\ \text{m/s}^2$ **Ans.**

Ans:
$v = 1.5\ \text{m/s}$
$a = 0.117\ \text{m/s}^2$

***12–120.**

The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m and $x = 50$ m.



SOLUTION

Velocity: The speed of the car at B is

$$v_B = \left[25 - 0.15(51.5)\right] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=50\text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{v_B{}^2}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v\frac{dv}{ds} = (25 - 0.15s)(-0.15) = (0.225s - 3.75) \text{ m/s}^2$$

When the car is at B $(s = 51.5 \text{ m})$

$$a_t = \left[0.225(51.5) - 3.75\right] = -2.591 \text{ m/s}^2$$

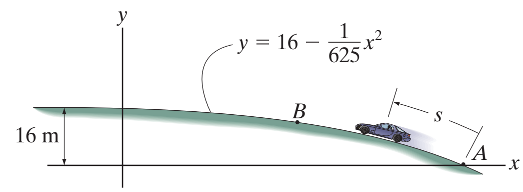
Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2$$ **Ans.**

Ans:
$a = 2.75 \text{ m/s}^2$

12–121.

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5\ \text{m/s}^2$, determine the magnitude of the car's acceleration when $s = 101.68$ m and $x = 0$.



SOLUTION

Velocity: The speed of the car at C is

$$v_C^{\,2} = v_A^{\,2} + 2a_t\,(s_C - s_A)$$

$$v_C^{\,2} = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361\ \text{m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=0} = 312.5\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}$$

$$a_n = \frac{v_C^{\,2}}{\rho} = \frac{22.361^2}{312.5} = 1.60\ \text{m/s}^2$$

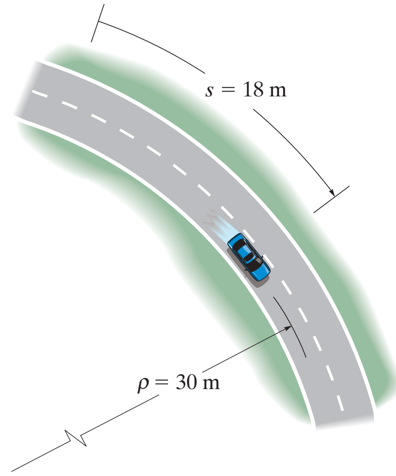
The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 1.68\ \text{m/s}^2$

12–122.

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18\ \text{m}$ starting from rest. Neglect the size of the car.



SOLUTION

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064\ \text{s}$$

$$v = 0.5(e^{3.7064} - 1) = 19.85\ \text{m/s} = 19.9\ \text{m/s} \qquad \textbf{Ans.}$$

$$a_t = \dot{v} = 0.5e^t\big|_{t=3.7064\ \text{s}} = 20.35\ \text{m/s}^2$$

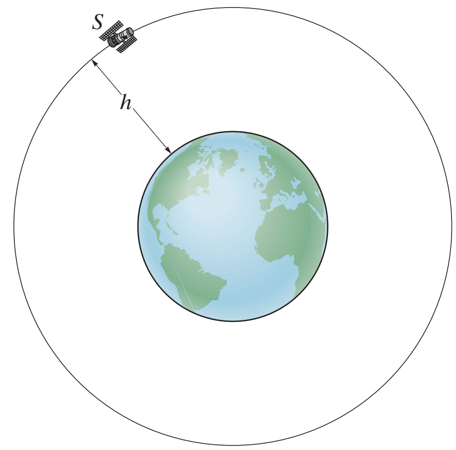
$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 19.9\ \text{m/s}$
$a = 24.2\ \text{m/s}^2$

12–123.

The satellite S travels around the earth in a circular path with a constant speed of 20 Mm/h. If the acceleration is 2.5 m/s^2, determine the altitude h. Assume the earth's diameter to be 12 713 km.



SOLUTION

$$v = 20\text{ Mm/h} = \frac{20(10^6)}{3600} = 5.56(10^3)\text{ m/s}$$

Since $a_t = \dfrac{dv}{dt} = 0$, then,

$$a = a_n = 2.5 = \frac{v^2}{\rho}$$

$$\rho = \frac{(5.56(10^3))^2}{2.5} = 12.35(10^6)\text{ m}$$

The radius of the earth is

$$\frac{12\,713(10^3)}{2} = 6.36(10^6)\text{ m}$$

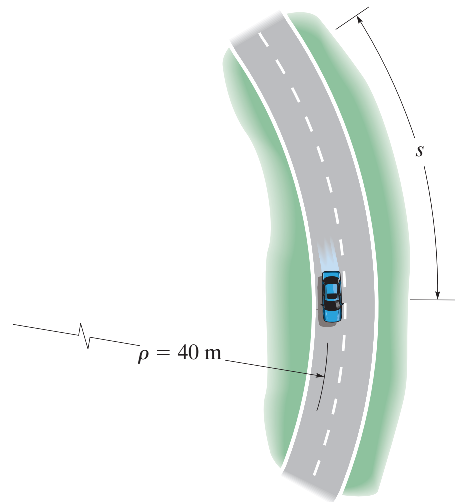
Hence,

$$h = 12.35(10^6) - 6.36(10^6) = 5.99(10^6)\text{ m} = 5.99\text{ Mm}$$ **Ans.**

Ans:
$h = 5.99$ Mm

***12–124.**

The car has an initial speed $v_0 = 20$ m/s. If it increases its speed along the circular track at $s = 0$, $a_t = (0.8s)$ m/s^2, where s is in meters, determine the time needed for the car to travel $s = 25$ m.



SOLUTION

The distance traveled by the car along the circular track can be determined by integrating $v\,dv = a_t\,ds$. Using the initial condition $v = 20$ m/s at $s = 0$,

$$\int_{20\text{ m/s}}^{v} v\,dv = \int_0^s 0.8\,s\,ds$$

$$\left.\frac{v^2}{2}\right|_{20\text{ m/s}}^{v} = 0.4\,s^2$$

$$v = \left\{\sqrt{0.8\,(s^2 + 500)}\right\}\text{ m/s}$$

The time can be determined by integrating $dt = \dfrac{ds}{v}$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^t dt = \int_0^{25\text{ m}} \frac{ds}{\sqrt{0.8(s^2 + 500)}}$$

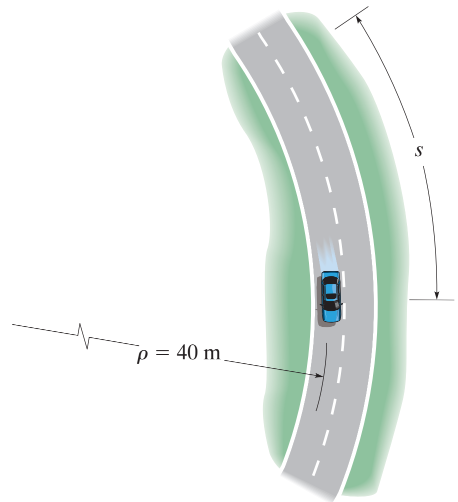
$$t = \frac{1}{\sqrt{0.8}}\left[\ln\left(s + \sqrt{s^2 + 500}\right)\right]\Big|_0^{25\text{ m}}$$

$$= 1.076\text{ s} = 1.08\text{ s}$$ **Ans.**

Ans:
$t = 1.08$ s

12–125.

The car starts from rest at $s = 0$ and increases its speed at $a_t = 4\ \text{m/s}^2$. Determine the time when the magnitude of acceleration becomes $20\ \text{m/s}^2$. At what position s does this occur?



SOLUTION

Acceleration. The normal component of the acceleration can be determined from

$$\theta_r = \frac{v^2}{\rho}; \qquad a_r = \frac{v^2}{40}$$

From the magnitude of the acceleration

$$a = \sqrt{a_t^2 + a_n^2}; \quad 20 = \sqrt{4^2 + \left(\frac{v^2}{40}\right)^2} \quad v = 28.00\ \text{m/s}$$

Velocity. Since the car has a constant tangential accelaration of $a_t = 4\ \text{m/s}^2$,

$$v = v_0 + a_t t; \quad 28.00 = 0 + 4t$$

$t = 6.999\ \text{s} = 7.00\ \text{s}$ **Ans.**

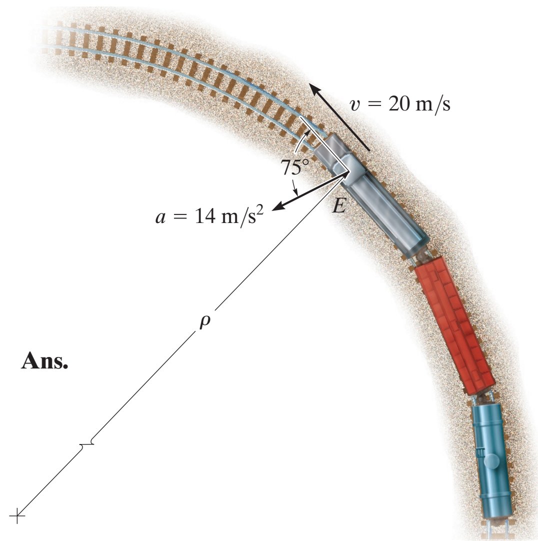
$$v^2 = v_0^2 + 2a_t s; \ \ 28.00^2 = 0^2 + 2(4)\ s$$

$s = 97.98\ \text{m} = 98.0\ \text{m}$ **Ans.**

Ans:
$t = 7.00\ \text{s}$
$s = 98.0\ \text{m}$

12–126.

At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



SOLUTION

$a_t = 14 \cos 75° = 3.62 \text{ m/s}^2$ **Ans.**

$a_n = 14 \sin 75°$

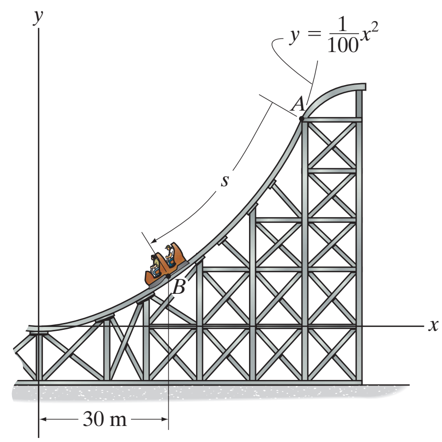
$$a_n = \frac{(20)^2}{\rho}$$

$\rho = 29.6 \text{ m}$ **Ans.**

Ans:
$a_t = 3.62 \text{ m/s}^2$
$\rho = 29.6 \text{ m}$

12–127.

When the roller coaster is at B, it has a speed of 25 m/s, which is increasing at $a_t = 3\text{ m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x axis.



SOLUTION

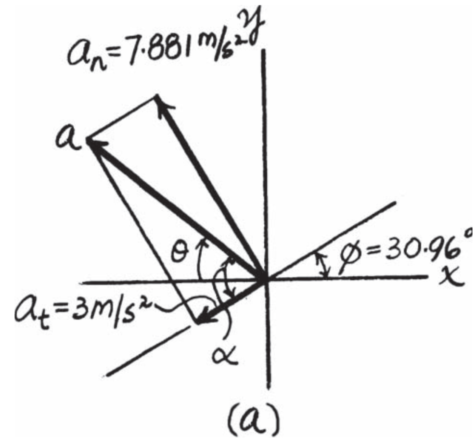
Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\text{ m}$$



Acceleration:

$$a_t = \dot{v} = 3\text{ m/s}^2$$

$$a_n = \frac{v_B{}^2}{\rho} = \frac{25^2}{79.30} = 7.881\text{ m/s}^2$$

The magnitude of the roller coaster's acceleration is

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{3^2 + 7.881^2} = 8.43\text{ m/s}^2 \qquad \textbf{Ans.}$$

The angle that the tangent at B makes with the x axis is $\phi = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=30\text{ m}}\right) = \tan^{-1}\left[\frac{1}{50}(30)\right] = 30.96°$.

As shown in Fig. a, $\mathbf{a}_n$ is always directed towards the center of curvature of the path. Here, $\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{7.881}{3}\right) = 69.16°$. Thus, the angle θ that the roller coaster's acceleration makes with the x axis is

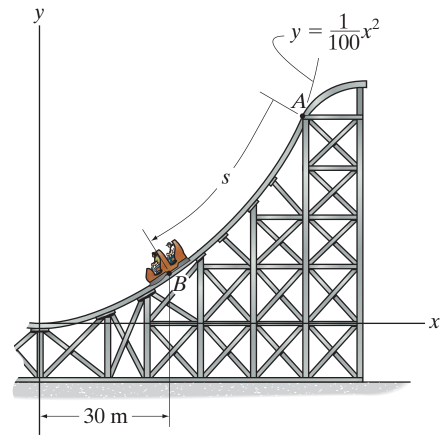
$$\theta = \alpha - \phi = 38.2° \qquad \textbf{Ans.}$$

Ans:

$\theta = 38.2°$

***12–128.**

If the roller coaster starts from rest at A and its speed increases at $a_t = (6 - 0.06s)\ \text{m/s}^2$, determine the magnitude of its acceleration when it reaches B where $s_B = 40$ m.



SOLUTION

Velocity: Using the initial condition $v = 0$ at $s = 0$,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^s \left(6 - 0.06s\right)ds$$

$$v = \left(\sqrt{12s - 0.06s^2}\right)\ \text{m/s} \qquad \textbf{(1)}$$

Thus,

$$v_B = \sqrt{12\left(40\right) - 0.06\left(40\right)^2} = 19.60\ \text{m/s}$$

Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 6 - 0.06(40) = 3.600\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.60^2}{79.30} = 4.842\ \text{m/s}^2$$

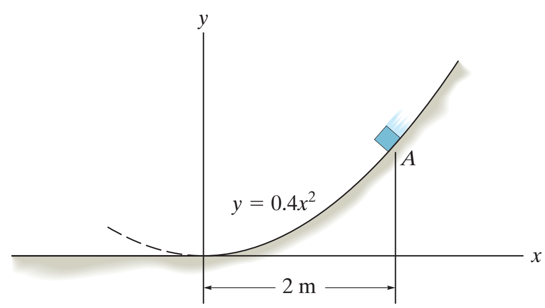
The magnitude of the roller coaster's acceleration at B is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{3.600^2 + 4.842^2} = 6.03\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 6.03\ \text{m/s}^2$

12–129.

The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v = 8$ m/s and the increase in speed is $dv/dt = 4$ m/s^2. Determine the magnitude of the acceleration of the box at this instant.



SOLUTION

$$y = 0.4\,x^2$$

$$\left.\frac{dy}{dx}\right|_{x=2\text{ m}} = \left.0.8x\right|_{x=2\text{ m}} = 1.6$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2\text{ m}} = 0.8$$

$$\rho = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=2\text{ m}} = \frac{\left[1 + (1.6)^2\right]^{3/2}}{|0.8|} = 8.396\text{ m}$$

$$a_n = \frac{v_B{}^2}{\rho} = \frac{8^2}{8.396} = 7.622\text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61\text{ m/s}^2$$ **Ans.**

Ans:
$a = 8.61$ m/s^2

12–130.

The position of a particle traveling along a curved path is $s = (3t^3 - 4t^2 + 4)$ m, where t is in seconds. When $t = 2$ s, the particle is at a position on the path where the radius of curvature is 25 m. Determine the magnitude of the particle's acceleration at this instant.

SOLUTION

Velocity:

$$v = \frac{d}{dt}\left(3t^3 - 4t^2 + 4\right) = \left(9t^2 - 8t\right)\text{m/s}$$

When $t = 2$ s,

$$v|_{t=2\text{ s}} = 9\left(2^2\right) - 8(2) = 20\text{ m/s}$$

Acceleration:

$$a_t = \frac{dv}{ds} = \frac{d}{dt}\left(9t^2 - 8t\right) = \left(18t - 8\right)\text{m/s}^2$$

$$a_t|_{t=2\text{ s}} = 18(2) - 8 = 28\text{ m/s}^2$$

$$a_n = \frac{\left(v|_{t=2\text{s}}\right)^2}{\rho} = \frac{20^2}{25} = 16\text{ m/s}^2$$

Thus,

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{28^2 + 16^2} = 32.2\text{ m/s}^2$$ **Ans.**

Ans:
$a = 32.2\text{ m/s}^2$

12–131.

A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05\,v)$ m/s^2, determine the magnitude of the particle's acceleraton four seconds later.

SOLUTION

Velocity: Using the initial condition $v = 10$ m/s at $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{10\text{ m/s}}^{v} \frac{dv}{0.05v}$$

$$t = 20\ln\frac{v}{10}$$

$$v = (10e^{t/20})\text{ m/s}$$

When $t = 4$ s,

$$v = 10e^{4/20} = 12.214\text{ m/s}$$

Acceleration: When $v = 12.214$ m/s ($t = 4$ s),

$$a_t = 0.05(12.214) = 0.6107\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(12.214)^2}{50} = 2.984\text{ m/s}^2$$

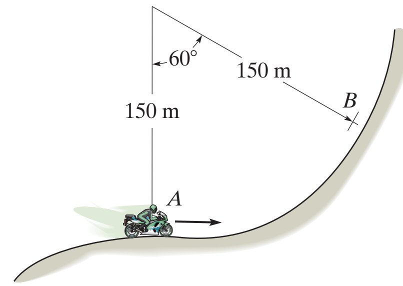
Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6107^2 + 2.984^2} = 3.05\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 3.05$ m/s^2

***12–132.**

The motorcycle is traveling at 40 m/s when it is at A. If the speed is then decreased at $\dot{v} = -(0.05\,s)$ m/s^2, where s is in meters measured from A, determine its speed and acceleration when it reaches B.



SOLUTION

Velocity. The velocity of the motorcycle along the circular track can be determined by integrating $vdv = a_d s$ with the initial condition $v = 40$ m/s at $s = 0$. Here, $a_t = -0.05s$.

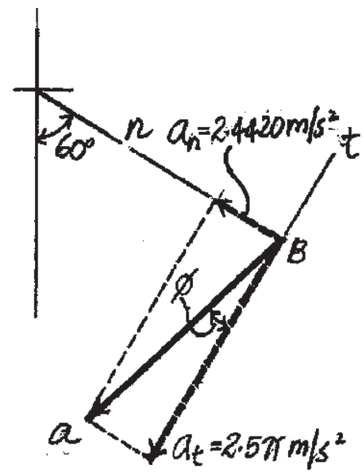
$$\int_{40\text{ m/s}}^{v} vdv = \int_0^s -0.05\,sds$$

$$\left.\frac{v^2}{2}\right|_{40\text{ m/s}}^{v} = -0.025\,s^2\Big|_0^s$$

$$v = \left\{\sqrt{1600 - 0.05\,s^2}\right\}\text{ m/s}$$

At B, $s = r\theta = 150\left(\dfrac{\pi}{3}\right) = 50\pi$ m. Thus

$$v_B = v|_{s=50\pi\text{m}} = \sqrt{1600 - 0.05(50\pi)^2} = 19.14\text{ m/s} = 19.1\text{ m/s}$$ **Ans.**



(a)

Acceleration. At B, the tangential and normal components are

$$a_t = 0.05(50\pi) = 2.5\pi\text{ m/s}^2$$

$$a_n = \frac{v_B^2}{\rho} = \frac{19.14^2}{150} = 2.4420\text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(2.5\pi)^2 + 2.4420^2} = 8.2249\text{ m/s}^2 = 8.22\text{ m/s}^2$$ **Ans.**

And its direction is defined by angle ϕ measured from the negative t-axis, Fig. a.

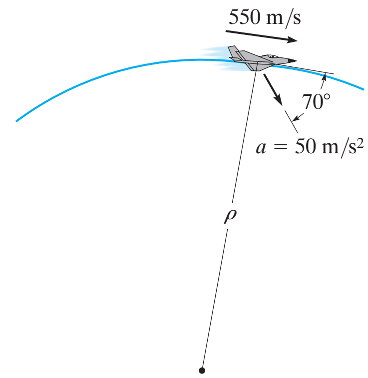
$$\phi = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{2.4420}{2.5\pi}\right)$$

$$= 17.27° = 17.3°$$ **Ans.**

Ans:
$v_B = 19.1$ m/s
$a = 8.22$ m/s^2
$\phi = 17.3°$
up from negative$-t$ axis

12–133.

At a given instant the jet plane has a speed of 550 m/s and an acceleration of 50 m/s^2 acting in the direction shown. Determine the rate of increase in the plane's speed, and also the radius of curvature ρ of the path.



SOLUTION

Acceleration. With respect to the *n–t* coordinate established as shown in Fig. *a*, the tangential and normal components of the acceleration are

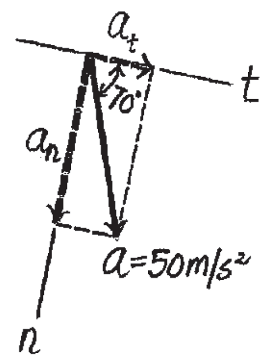
$$a_t = 50 \cos 70° = 17.10 \text{ m/s}^2 = 17.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = 50 \sin 70° = 46.98 \text{ m/s}^2$$

However,

$$a_n = \frac{v^2}{\rho}; \qquad 46.98 = \frac{550^2}{\rho}$$

$$\rho = 6438.28 \text{ m} = 6.44 \text{ km} \qquad \textbf{Ans.}$$



(a)

Ans:
$a_t = 17.1 \text{ m/s}^2$
$a_n = 46.98 \text{ m/s}^2$
$\rho = 6.44 \text{ km}$

12–134.

A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5\text{ m/s}$ and the rate of increase in the speed is $\dot{v} = 2\text{ m/s}^2$.

SOLUTION

$a_t = 2\text{ m/s}^2$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25\text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36\text{ m/s}^2$$ **Ans.**

Ans:
$a = 2.36\text{ m/s}^2$

12–135.

Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

SOLUTION

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$t = 4.147$ s

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s}$$ **Ans.**

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\Big|_{t=4.147\text{ s}} = 0.8465 \text{ m/s}^2$$

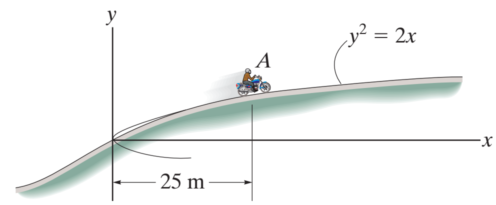
$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 1.96$ m/s
$a = 0.930$ m/s^2

***12–136.**

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|}\right|_{x=25\text{ m}} = 364.21\text{ m}$$

Acceleration: The speed of the motorcycle at a is

$$v = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67\text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627\text{ m/s}^2$$

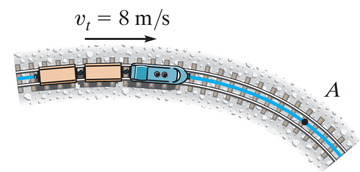
Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at A is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{0^2 + 0.7627^2} = 0.763\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.763\text{ m/s}^2$

12–137.

When $t = 0$, the train has a speed of 8 m/s, which is increasing at 0.5 m/s^2. Determine the magnitude of the acceleration of the engine when it reaches point A, at $t = 20$ s. Here the radius of curvature of the tracks is $\rho_A = 400$ m.



SOLUTION

Velocity. The velocity of the train along the track can be determined by integrating $dv = a_t\, dt$ with initial condition $v = 8$ m/s at $t = 0$.

$$\int_{8\text{ m/s}}^{v} dv = \int_0^t 0.5\, dt$$

$$v - 8 = 0.5\,t$$

$$v = \{0.5\,t + 8\}\text{ m/s}$$

At $t = 20$ s,

$$v|_{t = 20\,s} = 0.5(20) + 8 = 18\text{ m/s}$$

Acceleration. Here, the tangential component is $a_t = 0.5\text{ m/s}^2$. The normal component can be determined from

$$a_n = \frac{v^2}{\rho} = \frac{18^2}{400} = 0.81\text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2}$$

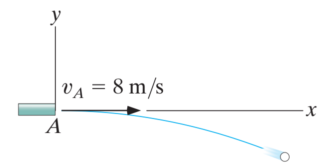
$$= \sqrt{0.5^2 + 0.81^2}$$

$$= 0.9519\text{ m/s}^2 = 0.952\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.952\text{ m/s}^2$

12–138.

The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.

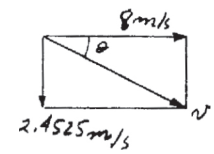


SOLUTION

$v_x = 8$ m/s

$(\overset{+}{\rightarrow})\qquad s = v_0 t$

$x = 8t$

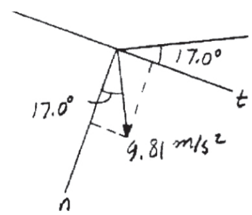


$$(+\uparrow)\qquad s = s_0 + v_0\,t + \frac{1}{2}a_c\,t^2$$

$$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$$

$$y = -4.905t^2$$

$$y = -4.905\left(\frac{x}{8}\right)^2$$



$$y = -0.0766x^2 \qquad \text{(Parabola)}$$ **Ans.**

$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = \; -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans.**

$$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04^\circ$$

$a_x = 0 \quad a_y = 9.81\text{ m/s}^2$

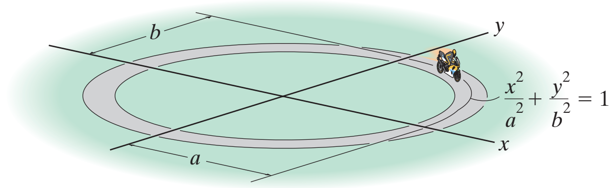
$a_n = 9.81 \cos 17.04^\circ = 9.38\text{ m/s}^2$ **Ans.**

$a_t = 9.81 \sin 17.04^\circ = 2.88\text{ m/s}^2$ **Ans.**

Ans:
$y = -0.0766x^2$
$v = 8.37$ m/s
$a_n = 9.38\text{ m/s}^2$
$a_t = 2.88\text{ m/s}^2$

12–139.

The motorcycle travels along the elliptical track at a constant speed v. Determine its greatest acceleration if $a > b$.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{b^2x}{a^2y}\right)^2\right]^{3/2}}{\frac{-b^4}{a^2y^3}}$$

At $x = a$, $y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

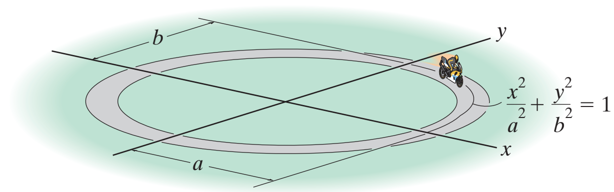
$$a_{\max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{a}} = \frac{v^2a}{b^2}$$ **Ans.**

Ans:

$$a_{\max} = \frac{v^2a}{b^2}$$

***12–140.**

The motorcycle travels along the elliptical track at a constant speed v. Determine its smallest acceleration if $a > b$.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{b^2x}{a^2y}\right)^2\right]^{3/2}}{\frac{-b^4}{a^2y^3}}$$

At $x = 0$, $y = b$,

$$|\rho| = \frac{a^2}{b}$$

Thus

$$a_t = 0$$

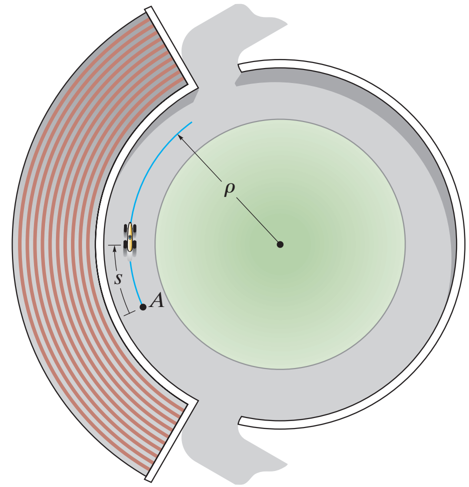
$$a_{\min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^3}{b}} = \frac{v^2b}{a^2} \qquad \textbf{Ans.}$$

Ans:

$$a_{\min} = \frac{v^2b}{a^2}$$

12–141.

The race car has an initial speed $v_A = 15\text{ m/s}$ at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)\text{ m/s}^2$, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150\text{ m}$.



SOLUTION

$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^{v} v\,dv$$

$$\frac{0.4s^2}{2}\Big|_0^s = \frac{v^2}{2}\Big|_{15}^{v}$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$1\text{n}\left(s + \sqrt{s^2 + 562.5}\right)\Big|_0^s = 0.632\,456t$$

$$1\text{n}\left(s + \sqrt{s^2 + 562.5}\right) - 3.166\,196 = 0.632\,456t$$

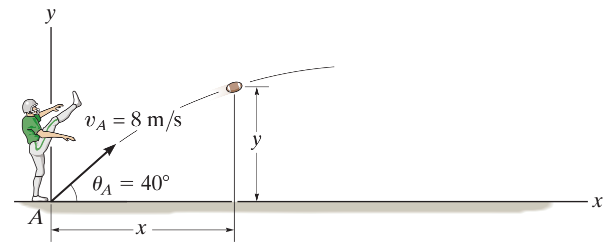
At $s = 20\text{ m}$,

$t = 1.21\text{ s}$ **Ans.**

Ans:
$t = 1.21\text{ s}$

12–142.

The ball is kicked with an initial speed $v_A = 8$ m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the normal and tangential components of its acceleration when $t = 0.25$ s.

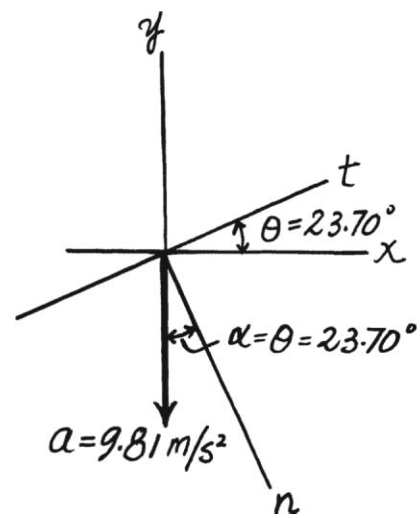


SOLUTION

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 8 \cos 40° = 6.128$ m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 6.128t \qquad \textbf{(1)}$$



Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 8 \sin 40° = 5.143$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 5.143t + \frac{1}{2}(-9.81)\left(t^2\right) \qquad \textbf{(2)}$$

Eliminate t from Eqs (1) and (2),we have

$$y = \{0.8391x - 0.1306x^2\} \text{ m} = \{0.839x - 0.131x^2\} \text{ m} \qquad \textbf{Ans.}$$

Acceleration: When $t = 0.25$ s, from Eq. (1), $x = 0 + 6.128(0.25) = 1.532$ m. Here, $\frac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\frac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1} 0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81$ m/s^2 and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

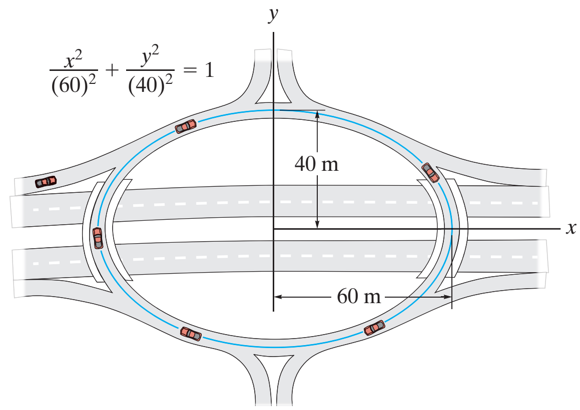
$$a_t = -a \sin \alpha = -9.81 \sin 23.70° = -3.94 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = a \cos \alpha = 9.81 \cos 23.70° = 8.98 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$y = \left\{0.839x - 0.131x^2\right\}$ m
$a_t = -3.94$ m/s^2
$a_n = 8.98$ m/s^2

12–143.

Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the minimum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{b^4}{a^2y^2}\right)\left(\frac{x^2}{a^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2}\left(1 - \frac{y^2}{b^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2} + \frac{b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^2}{a^2y^3}\right|}$$

At $x = 0$, $y = h$,

$$\rho = \frac{a^2}{b}$$

12–143. Continued

Thus

$a_t = 0$

$$a_{\min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^2}{b}} = \frac{v^2 b}{a^2}$$

Set $a = 60$ m, $b = 40$ m,

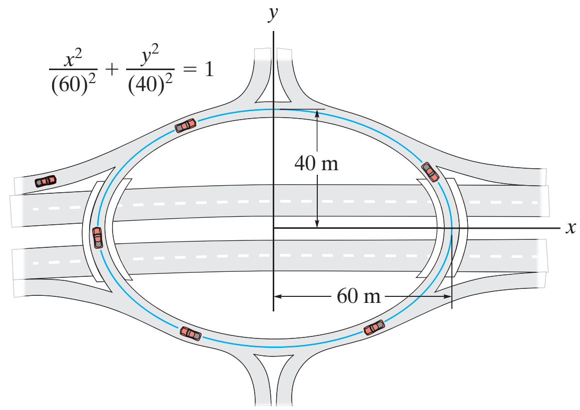
$$v = \frac{60(10)^3}{3600} = 16.67 \text{ m/s}$$

$$a_{\min} = \frac{(16.67)^2(40)}{(60)^2} = 3.09 \text{ m/s}^2$$ **Ans.**

Ans:
$a_{\min} = 3.09 \text{ m/s}^2$

***12–144.**

Cars move around the “traffic circle” which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the maximum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^4}{a^2y^3}\right|}$$

At $x = a$, $y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

$$a_{\max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{n}} = \frac{v^2a}{b^3}$$

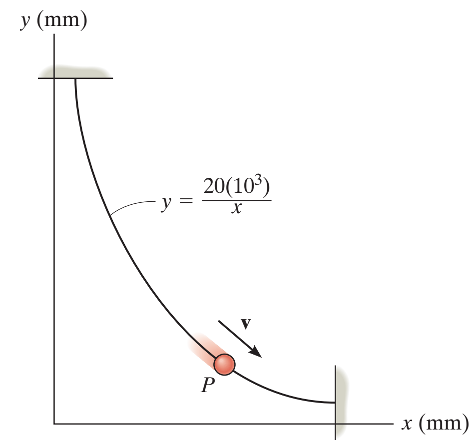
Set $a = 60$ m, $b = 40$ m, $v = \frac{60(10^3)}{3600} = 16.67$ m/s

$$a_{\max} = \frac{(16.67)^2(60)}{(40)^2} = 10.4\ \text{m/s}^2$$ **Ans.**

Ans:
$a_{\max} = 10.4\ \text{m/s}^2$

12–145.

The particle travels with a constant speed of 300 mm/s along the curve. Determine the particle's acceleration when it is located at point (200 mm, 100 mm) and sketch this vector on the curve.



SOLUTION

$v = 300$ mm/s

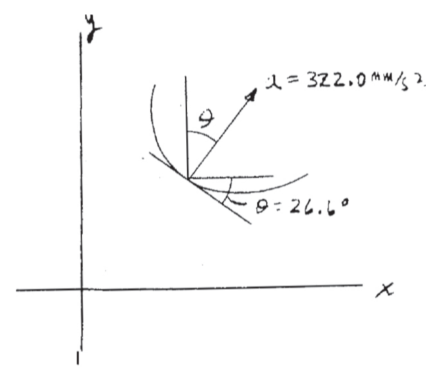
$$a_t = \frac{dv}{dt} = 0$$

$$y = \frac{20(10^3)}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=200} = -\frac{20(10^3)}{x^2} = -0.5$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=200} = \frac{40(10^3)}{x^3} = 5(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-0.5)^2]^{\frac{3}{2}}}{\left|5(10^{-3})\right|} = 279.5 \text{ mm}$$



$$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322 \text{ mm/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{(0)^2 + (322)^2} = 322 \text{ mm/s}^2$$ **Ans.**

Since $\frac{dy}{dx} = -0.5$,

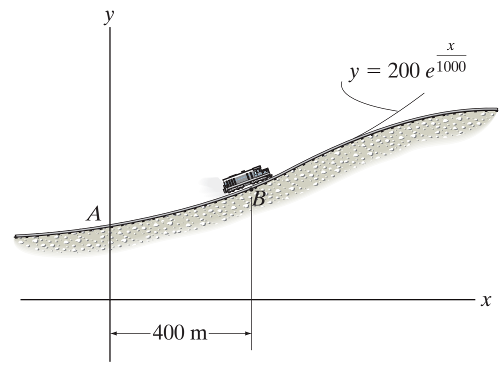
$$\theta = \tan^{-1}(-0.5) = 26.6° \measuredangle$$ **Ans.**

Ans:

$a = 322$ mm/s^2
$\theta = 26.6°$

12–146.

The train passes point B with a speed of 20 m/s which is decreasing at $a_t = -0.5\ \text{m/s}^2$. Determine the magnitude of acceleration of the train at this point.



SOLUTION

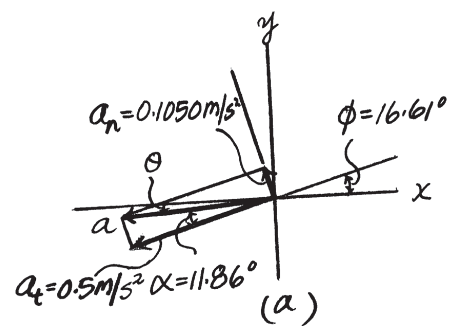
Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 200\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}.$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2\,e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96\text{ m}$$



Acceleration:

$$a_t = \dot{v} = -0.5\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{20^2}{3808.96} = 0.1050\ \text{m/s}^2$$

The magnitude of the train's acceleration at B is

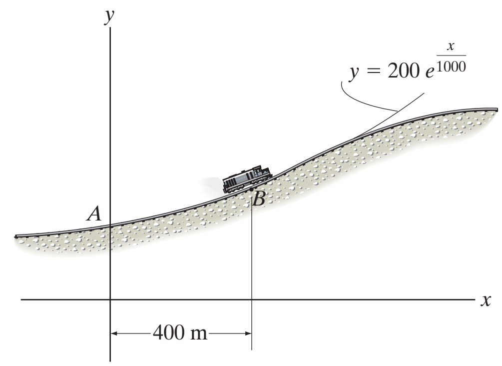
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.5)^2 + 0.1050^2} = 0.511\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$a = 0.511\ \text{m/s}^2$

12–147.

The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of $a_t = -0.25 \text{ m/s}^2$. Determine the magnitude of the acceleration of the train when it reaches point B, where $s_{AB} = 412$ m.



SOLUTION

Velocity: The speed of the train at B can be determined from

$$v_B^{\,2} = v_A^{\,2} + 2a_t\,(s_B - s_A)$$

$$v_B^{\,2} = 30^2 + 2(-0.25)(412 - 0)$$

$$v_B = 26.34 \text{ m/s}$$

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}$$

$$\rho = \frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = -0.25 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{26.34^2}{3808.96} = 0.1822 \text{ m/s}^2$$

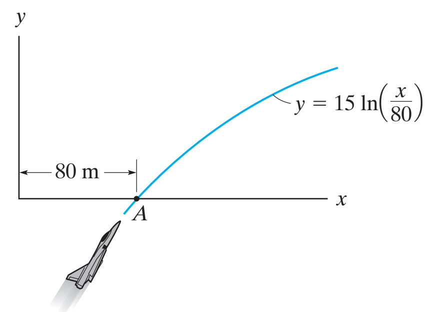
The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\left(-0.5\right)^2 + 0.1822^2} = 0.309 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.309 \text{ m/s}^2$

***12–148.**

The jet plane is traveling with a speed of 120 m/s which is decreasing at 40 m/s^2 when it reaches point A. Determine the magnitude of its acceleration when it is at this point. Also, specify the direction of flight, measured from the x axis.



SOLUTION

$$y = 15\ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x=80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x=80\text{ m}} = -0.002344$$

$$\rho\bigg|_{x=80\text{ m}} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=80\text{ m}}$$

$$= \frac{[1+(0.1875)^2]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(120)^2}{449.4} = 32.04\text{ m/s}^2$$

$$a_n = -40\text{ m/s}^2$$

$$a = \sqrt{(-40)^2 + (32.04)^2} = 51.3\text{ m/s}^2$$ **Ans.**

Since

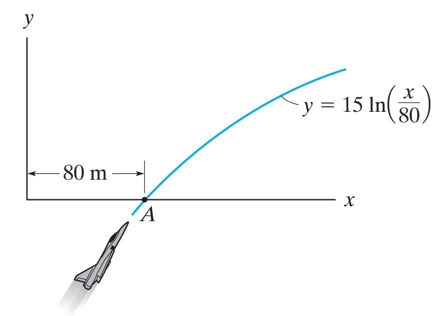
$$\frac{dy}{dx} = \tan\theta = 0.1875$$

$$\theta = 10.6^\circ$$ **Ans.**

Ans:
$a = 51.3\text{ m/s}^2$
$\theta = 10.6^\circ$

12–149.

The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point $A(y = 0)$.



SOLUTION

$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x = 80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x = 80\text{ m}} = -0.002344$$

$$\rho\bigg|_{x = 80\text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x = 80\text{ m}}$$

$$= \frac{\left[1 + (0.1875)^2\right]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(110)^2}{449.4} = 26.9\text{ m/s}^2$$

Since the plane travels with a constant speed, $a_t = 0$. Hence

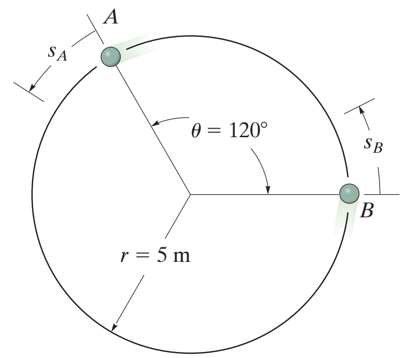
$a = a_n = 26.9\text{ m/s}^2$ **Ans.**

Ans:

$a = 26.9\text{ m/s}^2$

12–150.

Particles A and B are traveling counterclockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A begins to increase by $(a_t)_A = (0.4s_A)\ \text{m/s}^2$, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?



SOLUTION

Distance Traveled: Initially the distance between the particles is

$$d_0 = \rho d\theta = 5\left(\frac{120°}{180°}\right)\pi = 10.47 \text{ m}$$

When $t = 1$ s, B travels a distance of

$$d_B = 8(1) = 8 \text{ m}$$

The distance traveled by particle A is determined as follows:

$$vdv = ads$$

$$\int_{8\text{ m/s}}^{v} vdv = \int_0^s 0.4\, sds$$

$$v = 0.6325\sqrt{s^2 + 160} \qquad \textbf{(1)}$$

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.6325\sqrt{s^2 + 160}}$$

$$1 = \frac{1}{0.6325}\left(\text{In}\left[\frac{\sqrt{s^2 + 160} + s}{\sqrt{160}}\right]\right)$$

$$s = 8.544 \text{ m}$$

Thus the distance between the two cyclists after $t = 1$ s is

$$d = 10.47 + 8.544 - 8 = 11.0 \text{ m} \qquad \textbf{Ans.}$$

Acceleration:

For A, when $t = 1$ s,

$$(a_t)_A = \dot{v}_A = 0.4(8.544) = 3.4176 \text{ m/s}^2$$

$$v_A = 0.6325\sqrt{8.544^2 + 160} = 9.655 \text{ m/s}$$

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{9.655^2}{5} = 18.64 \text{ m/s}^2$$

The magnitude of the A's acceleration is

$$a_A = \sqrt{3.4176^2 + 18.64^2} = 19.0 \text{ m/s}^2 \qquad \textbf{Ans.}$$

For B, when $t = 1$ s,

$$(a_t)_B = \dot{v}_A = 0$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80 \text{ m/s}^2$$

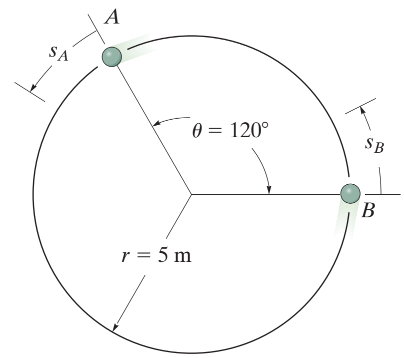
The magnitude of the B's acceleration is

$$a_B = \sqrt{0^2 + 12.80^2} = 12.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$d = 11.0$ m
$a_A = 19.0\ \text{m/s}^2$
$a_B = 12.8\ \text{m/s}^2$

12–151.

Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increasing by $(a_t)_B = 4\text{ m/s}^2$, and at the same instant A has an increase in speed of $(a_t)_A = 0.8t\text{ m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



SOLUTION

Distance Traveled: Initially the distance between the two particles is $d_0 = \rho\theta = 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)\,t^2 = \left(8t + 2t^2\right)\text{ m} \qquad [1]$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A\,dt$$

$$\int_{8\text{ m/s}}^{v_A} dv_A = \int_0^t 0.8\,t\,dt$$

$$v_A = \left(0.4t^2 + 8\right)\text{ m/s} \qquad [2]$$

$$ds_A = v_A\,dt$$

$$\int_0^{s_A} ds_A = \int_0^t \left(0.4t^2 + 8\right)dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $\quad t = 2.5074\text{ s} = 2.51\text{ s}$ **Ans.**

Note: If particle A strikes B then, $s_A = 5\left(\frac{240°}{180°}\pi\right) + s_B$. This equation will result in $t = 14.6\text{ s} > 2.51\text{ s}$.

Acceleration: The tangential acceleration for particle A and B when $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8\,(2.5074) = 2.006\text{ m/s}^2$ and $(a_t)_B = 4\text{ m/s}^2$, respectively. When $t = 2.5074$ s, from Eq. [1], $v_A = 0.4\left(2.5074^2\right) + 8 = 10.51$ m/s and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03$ m/s. To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11\text{ m/s}^2$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01\text{ m/s}^2$$

The magnitude of the acceleration for particles A and B just before collision are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$t = 2.51$ s
$a_A = 22.2\text{ m/s}^2$
$a_B = 65.1\text{ m/s}^2$

***12–152.**

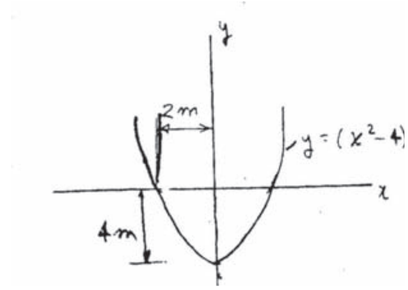
A particle P moves along the curve $y = (x^2 - 4)$ m with a constant speed of 5 m/s. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

SOLUTION

$y = (x^2 - 4)$

$a_t = \dfrac{dv}{dt} = 0,$

To obtain maximum $a = a_n$, ρ must be a minimum.



This occurs at:

$x = 0, \quad y = -4 \text{ m}$ **Ans.**

Hence,

$$\left.\frac{dy}{dx}\right|_{x=0} = 2x = 0; \quad \frac{d^2y}{dx^2} = 2$$

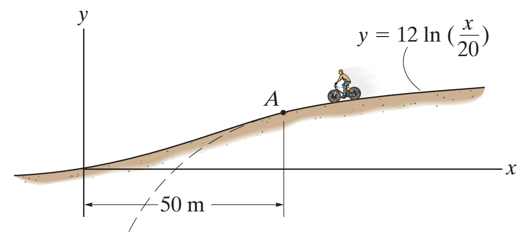
$$\rho_{min} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+0]^{\frac{3}{2}}}{|2|} = \frac{1}{2}$$

$$(a)_{max} = (a_n)_{max} = \frac{v^2}{\rho_{min}} = \frac{5^2}{\frac{1}{2}} = 50 \text{ m/s}^2$$ **Ans.**

Ans:
$x = 0, \quad y = -4$ m
$(a)_{max} = 50$ m/s^2

12–153.

When the bicycle passes point A, it has a speed of 6 m/s, which is increasing at the rate of $\dot{v} = (0.5)\ \text{m/s}^2$. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = 12\ \ln\left(\frac{x}{20}\right)$$

$$\frac{dy}{dx} = 12\left(\frac{1}{x/20}\right)\left(\frac{1}{20}\right) = \frac{12}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{12}{x^2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\frac{12}{x}\right)^2\right]^{3/2}}{\left|-\frac{12}{x^2}\right|}\Bigg|_{x=50\text{ m}} = 226.59\text{ m}$$

Acceleration:

$$a_t = v = 0.5\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{6^2}{226.59} = 0.1589\ \text{m/s}^2$$

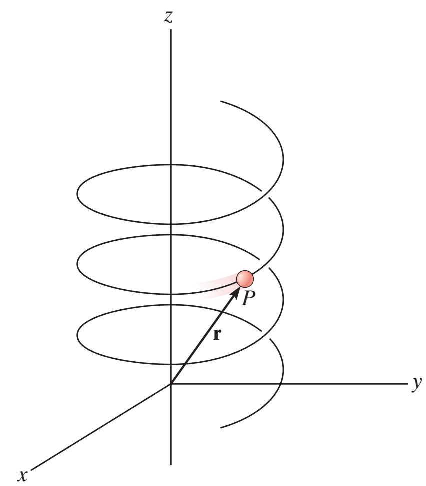
The magnitude of the bicycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 0.1589^2} = 0.525\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.525\ \text{m/s}^2$

12–154.

A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



SOLUTION

$$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$$

$$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$$

When $t = 8$ s,

$$\mathbf{v}_P = -0.2\sin(0.8\text{ rad})\mathbf{i} + 0.15\cos(0.8\text{ rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = -0.02\cos(0.8\text{ rad})\mathbf{i} - 0.015\sin(0.8\text{ rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$$

Since the binormal vector is perpendicular to the plane containing the n–t axis, and $\mathbf{a}_p$ and $\mathbf{v}_p$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.14\,347 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$$

$$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans.**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans.**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans.**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_p \times \mathbf{v}_p$.

For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans.**

Ans:

$\alpha = 52.5°$
$\beta = 142°$
$\gamma = 85.1°$
$\alpha = 128°, \beta = 37.9°, \gamma = 94.9°$

12–155.

If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

When $t = 2$ s,

$r = 4(1+\sin t) = 7.637$

$\dot{r} = 4 \cos t = -1.66459$

$\ddot{r} = -4 \sin t = -3.6372$

$\theta = 2\, e^{-t}$

$\dot{\theta} = -2\, e^{-t} = -0.27067$

$\ddot{\theta} = 2\, e^{-t} = 0.270665$

$v_r = \dot{r} = -1.66$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97$ m/s^2 **Ans.**

Ans:
$v_r = -1.66$ m/s
$v_\theta = -2.07$ m/s
$a_r = -4.20$ m/s
$a_\theta = 2.97$ m/s^2

***12–156.**

A particle travels around a limaçon, defined by the equation $r = b - a \cos \theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

SOLUTION

$$r = b - a\cos\theta$$

$$\dot{r} = a\sin\theta\dot{\theta}$$

$$\dot{r} = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta}$$

$$v_r = \dot{r} = a\sin\theta\dot{\theta} \qquad \textbf{Ans.}$$

$$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta} \qquad \textbf{Ans.}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta} - (b - a\cos\theta)\dot{\theta}^2$$

$$= (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\,\ddot{\theta} \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (b - a\cos\theta)\ddot{\theta} + 2\left(a\sin\theta\dot{\theta}\right)\dot{\theta}$$

$$= (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta \qquad \textbf{Ans.}$$

Ans:
$v_r = \dot{r} = a\sin\theta\dot{\theta}$
$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta}$
$a_r = (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\,\ddot{\theta}$
$a_\theta = (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta$

12–157. A particle moves along a circular path of radius 300 mm. If its angular velocity is $\dot{\theta} = (2t^2)$ rad/s, where t is in seconds, determine the magnitude of the particle's acceleration when $t = 2$ s.

SOLUTION

Time Derivatives:

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = 2t^2\big|_{t=2\text{ s}} = 8 \text{ rad/s} \qquad \ddot{\theta} = 4t\big|_{t=2\text{ s}} = 8 \text{ rad/s}^2$$

Velocity: The radial and transverse components of the particle's velocity are

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 0.3(8) = 2.4 \text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2} = \sqrt{0^2 + 2.4^2} = 2.4 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.3\left(8^2\right) = -19.2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3(8) + 0 = 2.4 \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2} = \sqrt{(-19.2)^2 + 2.4^2} = 19.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

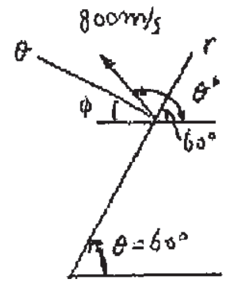
Ans:

$v = 2.4$ m/s
$a = 19.3$ m/s^2

12–158.

For a short time a rocket travels up and to the right at a constant speed of 800 m/s along the parabolic path $y = 600 - 35x^2$. Determine the radial and transverse components of velocity of the rocket at the instant $\theta = 60°$, where θ is measured counterclockwise from the x axis.

SOLUTION



$y = 600 - 35x^2$

$\dot{y} = -70x\dot{x}$

$\dfrac{dy}{dx} = -70x$

$\tan 60° = \dfrac{y}{x}$

$y = 1.732051x$

$1.732051x = 600 - 35x^2$

$x^2 + 0.049487x - 17.142857 = 0$

Solving for the positive root,

$x = 4.1157$ m

$\tan \theta' = \dfrac{dy}{dx} = -288.1$

$\theta' = 89.8011°$

$\phi = 180° - 89.8011° - 60° = 30.1989°$

$v_r = 800 \cos 30.1989° = 691$ m/s **Ans.**

$v_\theta = 800 \sin 30.1989° = 402$ m/s **Ans.**

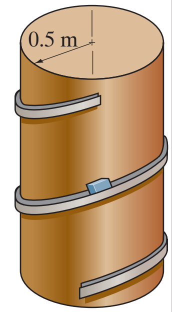
Ans:

$v_r = 691$ m/s

$v_\theta = 402$ m/s

12–159.

The box slides down the helical ramp with a constant speed of $v = 2$ m/s. Determine the magnitude of its acceleration. The ramp descends a vertical distance of 1 m for every full revolution. The mean radius of the ramp is $r = 0.5$ m.



SOLUTION

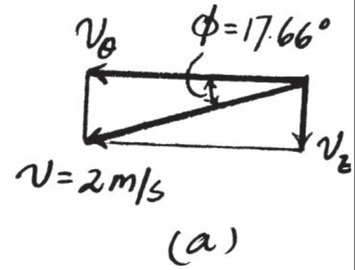
Velocity: The inclination angle of the ramp is $\phi = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\left[\dfrac{1}{2\pi(0.5)}\right] = 17.66°$.

Thus, from Fig. a, $v_\theta = 2\cos 17.66° = 1.906$ m/s and $v_z = 2\sin 17.66° = 0.6066$ m/s. Thus,

$$v_\theta = r\dot{\theta}$$

$$1.906 = 0.5\dot{\theta}$$

$$\dot{\theta} = 3.812 \text{ rad/s}$$



Acceleration: Since $r = 0.5$ m is constant, $\dot{r} = \ddot{r} = 0$. Also, $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$. Using the above results,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(3.812)^2 = -7.264 \text{ m/s}^2$$

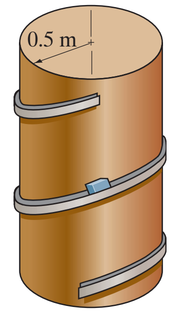
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(0) + 2(0)(3.812) = 0$$

Since $\mathbf{v}_z$ is constant $a_z = 0$. Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2 + a_z{}^2} = \sqrt{(-7.264)^2 + 0^2 + 0^2} = 7.26 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 7.26 \text{ m/s}^2$

***12–160.** The box slides down the helical ramp such that $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



SOLUTION

Time Derivatives:

$$r = 0.5 \text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = \left(1.5t^2\right) \text{ rad/s} \qquad \ddot{\theta} = (3t) \text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t) \text{ m/s} \qquad \ddot{z} = -0.4 \text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \qquad t = 2.325 \text{ s}$$

Thus,

$$\dot{\theta}\Big|_{t=2.325 \text{ s}} = 1.5(2.325)^2 = 8.108 \text{ rad/s}$$

$$\ddot{\theta}\Big|_{t=2.325 \text{ s}} = 3(2.325) = 6.975 \text{ rad/s}^2$$

$$\dot{z}\Big|_{t=2.325 \text{ s}} = -0.4(2.325) = -0.92996 \text{ m/s}$$

$$\ddot{z}\Big|_{t=2.325 \text{ s}} = -0.4 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385 \text{ m/s}$$

$$v_z = \dot{z} = -0.92996 \text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{v_r{}^2 + v_\theta{}^2 + v_z{}^2} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487 \text{ m/s}^2$$

$$a_z = \ddot{z} = -0.4 \text{ m/s}^2$$

Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2 + a_z{}^2} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 4.16$ m/s
$a = 33.1$ m/s^2

12–161.

If a particle's position is described by the polar coordinates $r = (2 \sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and transverse components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s,

$\theta = 4\,t = 4$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 2 \sin 2\theta = 1.9787$

$\dot{r} = 4 \cos 2\theta\,\dot{\theta} = -2.3280$

$\ddot{r} = -8 \sin 2\theta(\dot{\theta})^2 + 8 \cos 2\theta\,\ddot{\theta} = -126.638$

$v_r = \dot{r} = -2.33 \text{ m/s}$ **Ans.**

$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = -2.33 \text{ m/s}$
$v_\theta = 7.91 \text{ m/s}$
$a_r = -158 \text{ m/s}^2$
$a_\theta = -18.6 \text{ m/s}^2$

12–162.

If a particle moves along a path such that $r = (e^{at})$ m and $\theta = t$, where t is in seconds, plot the path $r = f(\theta)$, and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$r = e^{at} \qquad \dot{r} = ae^{at} \qquad \ddot{r} = a^2e^{et}$

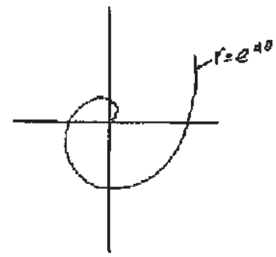
$\theta = t \qquad \dot{\theta} = 1 \qquad \ddot{\theta} = 0$

$v_r = \dot{r} = ae^{at}$ **Ans.**

$v_a = r\dot{\theta} = e^{at}(1) = e^{at}$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = a^2e^{at} - e^{at}(1)^2 = e^{at}(a^2 - 1)$ **Ans.**

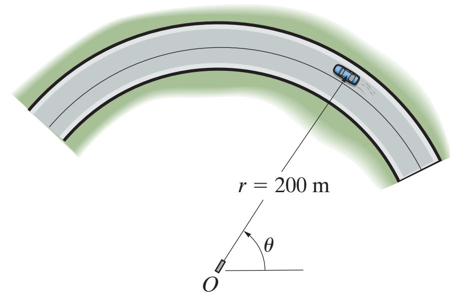
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{at}(0) + 2(ae^{at})(1) = 2ae^{at}$ **Ans.**



Ans:
$v_r = ae^{at}$
$v_\theta = e^{at}$
$a_r = e^{at}(a^2-1)$
$a_\theta = 2ae^{at}$

12–163.

A radar gun at O rotates with the angular velocity of $\dot{\theta} = 0.1$ rad/s and angular acceleration of $\ddot{\theta} = 0.025$ rad/s^2, at the instant $\theta = 45°$, as it follows the motion of the car traveling along the circular road having a radius of $r = 200$ m. Determine the magnitudes of velocity and acceleration of the car at this instant.



SOLUTION

Time Derivatives: Since r is constant,

$$\dot{r} = \ddot{r} = 0$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 200(0.1) = 20 \text{ m/s}$$

Thus, the magnitude of the car's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 20^2} = 20 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \dot{r} - r\dot{\theta}^2 = 0 - 200(0.1^2) = -2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(0.025) + 0 = 5 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration is

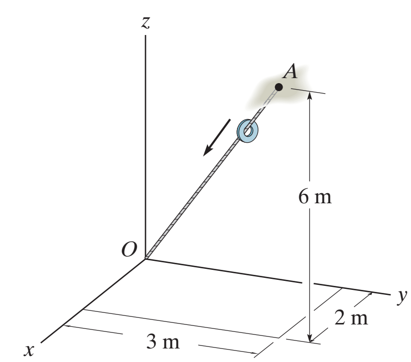
$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-2)^2 + 5^2} = 5.39 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v = 20$ m/s
$a = 5.39$ m/s^2

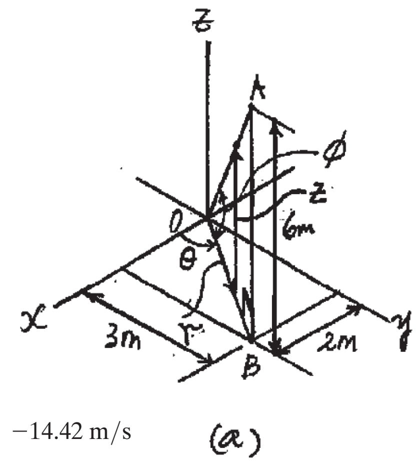
***12–164.**

The small washer is sliding down the cord OA. When it is at the midpoint, its speed is 28 m/s and its acceleration is 7 m/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



SOLUTION

The position of the washer can be defined using the cylindrical coordinate system (r, θ and z) as shown in Fig. a. Since θ is constant, there will be no transverse component for **v** and **a**. The velocity and acceleration expressed as Cartesian vectors are



$$\mathbf{v} = v\left(-\frac{\mathbf{r}_{AO}}{r_{AO}}\right) = 28\left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}}\right] = \{-8\mathbf{i} - 12\mathbf{j} - 24\mathbf{k}\}\text{ m/s}$$

$$\mathbf{a} = a\left(-\frac{\mathbf{r}_{AO}}{r_{AO}}\right) = 7\left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}}\right] = \{-2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}\}\text{ m}^2/\text{s}$$

$$\mathbf{u}_r = \frac{\mathbf{r}_{OB}}{r_{OB}} = \frac{2\mathbf{i} + 3\mathbf{j}}{\sqrt{2^2 + 3^2}} = \frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}$$

$$\mathbf{u}_z = \mathbf{k}$$

Using vector dot product

$$v_r = \mathbf{v}\cdot\mathbf{u}_r = (-8\,\mathbf{i} - 12\,\mathbf{j} - 24\,\mathbf{k})\cdot\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = -8\left(\frac{2}{\sqrt{13}}\right) + \left[-12\left(\frac{3}{\sqrt{13}}\right)\right] = -14.42\text{ m/s}$$

$$v_z = \mathbf{v}\cdot\mathbf{u}_z = (-8\,\mathbf{i} - 12\,\mathbf{j} - 24\,\mathbf{k})\cdot(\mathbf{k}) = -24.0\text{ m/s}$$

$$a_r = \mathbf{a}\cdot\mathbf{u}_r = (-2\,\mathbf{i} - 3\,\mathbf{j} - 6\,\mathbf{k})\cdot\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = -2\left(\frac{2}{\sqrt{13}}\right) + \left[-3\left(\frac{3}{\sqrt{13}}\right)\right] = -3.606\text{ m/s}^2$$

$$a_z = \mathbf{a}\cdot\mathbf{u}_z = (-2\,\mathbf{i} - 3\,\mathbf{j} - 6\,\mathbf{k})\cdot\mathbf{k} = -6.00\text{ m/s}^2$$

Thus, in vector form

$\mathbf{v} = \{-14.2\,\mathbf{u}_r - 24.0\,\mathbf{u}_z\}$ m/s **Ans.**

$\mathbf{a} = \{-3.61\,\mathbf{u}_r - 6.00\,\mathbf{u}_z\}$ m/s^2 **Ans.**

These components can also be determined using trigonometry by first obtain angle ϕ shown in Fig. a.

$$OA = \sqrt{2^2 + 3^2 + 6^2} = 7\text{ m} \qquad OB = \sqrt{2^2 + 3^2} = \sqrt{13}$$

Thus,

$\sin\phi = \dfrac{6}{7}$ and $\cos\phi = \dfrac{\sqrt{13}}{7}$. Then

$$v_r = -v\cos\phi = -28\left(\frac{\sqrt{13}}{7}\right) = -14.42\text{ m/s}$$

$$v_z = -v\sin\phi = -28\left(\frac{6}{7}\right) = -24.0\text{ m/s}$$

$$a_r = -a\cos\phi = -7\left(\frac{\sqrt{13}}{7}\right) = -3.606\text{ m/s}^2$$

$$a_z = -a\sin\phi = -7\left(\frac{6}{7}\right) = -6.00\text{ m/s}^2$$

Ans:

$\mathbf{v} = \{-14.2\mathbf{u}_r - 24.0\mathbf{u}_z\}$ m/s
$\mathbf{a} = \{-3.61\mathbf{u}_r - 6.00\mathbf{u}_z\}$ m/s^2

12–165.

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

SOLUTION

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\,\mathbf{u}_z$$

$$\dot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\,\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\mathbf{u}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z$$ **Ans.**

Ans:
$\dot{\mathbf{a}} = (\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta})\mathbf{u}_r$
$+ (3\dot{r}\ddot{\theta} + \dot{r}\ddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3)\mathbf{u}_\theta + (\dddot{z})\mathbf{u}_z$

12–166. A particle is moving along a circular path having a 400-mm radius. Its position as a function of time is given by $\theta = (2t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

SOLUTION

$t = \sqrt{\frac{\theta_1}{b}}$ $\qquad t = 0.51\text{ s}$

$r = 400\text{ mm} \qquad b = 2\text{ rad}/\text{s}^2 \qquad \theta_1 = 30°$

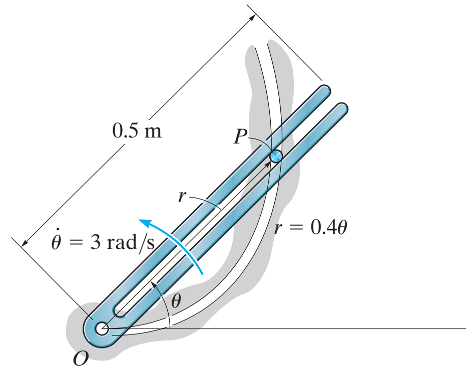
$\theta = bt^2 \qquad \theta' = 2bt \qquad \theta'' = 2b$

$a = \sqrt{\left(-r\theta'^2\right)^2 + \left(r\theta''\right)^2} \qquad a = 2.32\text{ m/s}^2$ **Ans.**

Ans:
$a = 2.32\text{ m/s}^2$

12–167.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$$\dot{r} = 0.4\,\dot{\theta}$$

$$\ddot{r} = 0.4\,\ddot{\theta}$$

At $\theta = \dfrac{\pi}{3}, \quad r = 0.4189$

$$\dot{r} = 0.4(3) = 1.20$$

$$\ddot{r} = 0.4(0) = 0$$

$v = \dot{r} = 1.20$ m/s **Ans.**

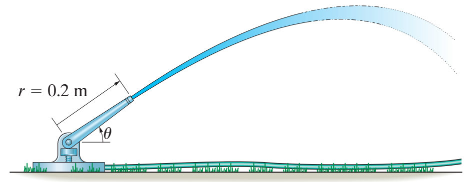
$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20$ m/s^2 **Ans.**

Ans:
$v_r = 1.20$ m/s
$v_\theta = 1.26$ m/s
$a_r = -3.77$ m/s^2
$a_\theta = 7.20$ m/s^2

***12–168.** At the instant shown, the watersprinkler is rotating with an angular speed $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 3$ rad/s^2. If the nozzle lies in the vertical plane and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 0.2$ m.



SOLUTION

$\theta' = 2$ rad/s

$\theta'' = 3$ rad/s^2

$r' = 3$ m/s

$r = 0.2$ m

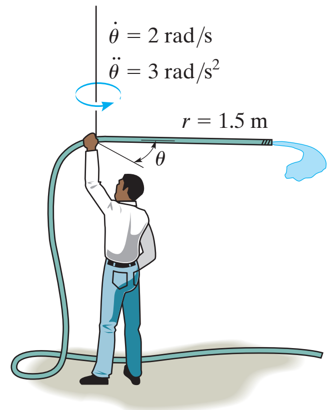
$v = \sqrt{r'^2 + (r\theta')^2}$ $\qquad v = 3.03$ m/s **Ans.**

$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2}$ $\qquad a = 12.63$ m/s^2 **Ans.**

Ans:
$v = 3.03$ m/s
$a = 12.63$ m/s^2

12–169.

At the instant shown, the man is twirling a hose over his head with an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 3$ rad/s^2. If it is assumed that the hose lies in a horizontal plane, and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 1.5$ m.



SOLUTION

$r = 1.5$

$\dot{r} = 3$

$\ddot{r} = 0$

$\dot{\theta} = 2$

$\ddot{\theta} = 3$

$v_r = \dot{r} = 3$

$v_\theta = r\dot{\theta} = 1.5(2) = 3$

$v = \sqrt{(3)^2 + (3)^2} = 4.24 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(2)^2 = 6$

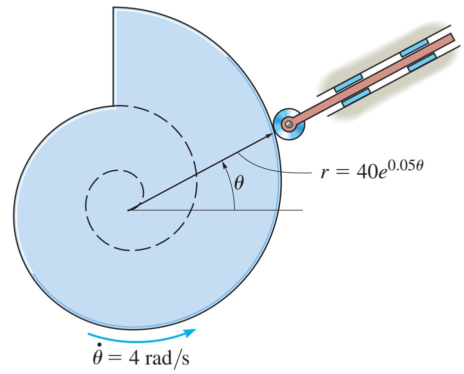
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.5(3) + 2(3)(2) = 16.5$

$a = \sqrt{(6)^2 + (16.5)^2} = 17.6 \text{ m/s}^2$ **Ans.**

Ans:
$v = 4.24$ m/s
$a = 17.6$ m/s^2

12–170.

The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam is rotating at a constant angular rate of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod at the instant $\theta = 30°$.



SOLUTION

$r = 40e^{0.05\,\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.05\,\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\,\theta}\ddot{\theta}$

$\theta = \dfrac{\pi}{6}$

$\dot{\theta} = -4$

$\ddot{\theta} = 0$

$r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\left(\frac{\pi}{6}\right)}(-4) = -8.2122$

$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 0 = 1.64244$

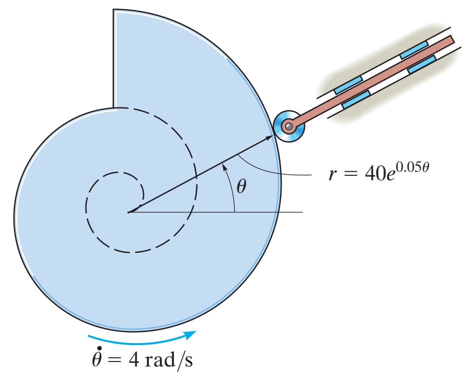
$v = \dot{r} = -8.2122 = 8.21$ mm/s **Ans.**

$a = \ddot{r} - r\dot{\theta}^2 = 1.642\,44 - 41.0610(-4)^2 = -665.33 = -665$ mm/s^2 **Ans.**

Ans:
$v_r = 8.21$ mm/s
$a = -665$ mm/s^2

12–171.

Solve Prob. 12–170, if the cam has an angular acceleration of $\ddot{\theta} = 2 \text{ rad/s}^2$ when its angular velocity is $\dot{\theta} = 4 \text{ rad/s}$ at $\theta = 30°$.



SOLUTION

$r = 40e^{0.05\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.05\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\theta}\ddot{\theta}$

$\theta = \dfrac{\pi}{6}$

$\dot{\theta} = -4$

$\ddot{\theta} = -2$

$r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\left(\left(\frac{\pi}{6}\right)\right.}(-4) = -8.2122$

$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 2e^{0.05\left(\frac{\pi}{6}\right)}(-2) = -2.4637$

$v = \dot{r} = 8.2122 = 8.21 \text{ mm/s}$ **Ans.**

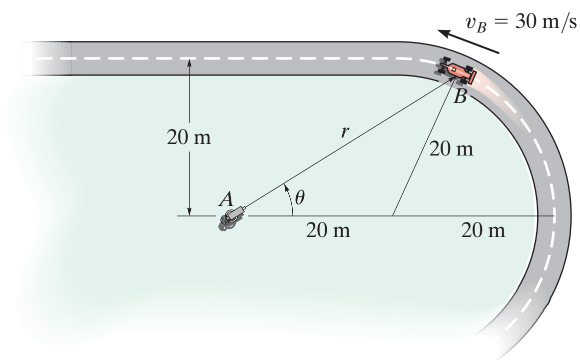
$a = \ddot{r} - r\dot{\theta}^2 = -2.4637 - 41.0610(-4)^2 = -659 \text{ mm/s}^2$ **Ans.**

Ans:

$v = 8.21 \text{ mm/s}$
$a = -659 \text{ mm/s}^2$

***12–172.**

A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at a constant speed of 30 m/s. Determine the angular rate $\dot{\theta}$ at which the man must turn in order to keep the camera directed on the car at the instant $\theta = 30°$.



SOLUTION

$r = 2(20)\cos\theta = 40\cos\theta$

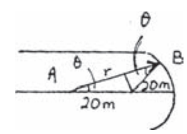
$\dot{r} = -(40\sin\theta)\,\dot{\theta}$

$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\,\dot{\theta}\,\mathbf{u}_\theta$

$v = \sqrt{(\dot{r})^2 + (r\,\dot{\theta})^2}$

$(30)^2 = (-40\sin\theta)^2(\dot{\theta})^2 + (40\cos\theta)^2(\dot{\theta})^2$

$(30)^2 = (40)^2[\sin^2\theta + \cos^2\theta](\dot{\theta})^2$

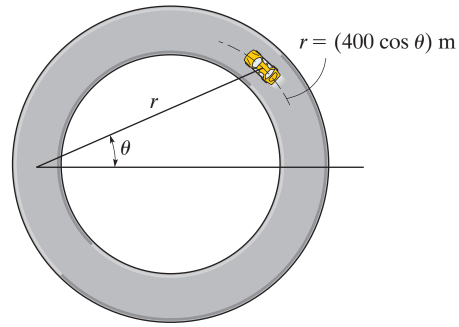


$\dot{\theta} = \dfrac{30}{40} = 0.75$ rad/s **Ans.**

Ans:
$\dot{\theta} = 0.75$ rad/s

12–173.

The car travels around the circular track with a constant speed of 20 m/s. Determine the car's radial and transverse components of velocity and acceleration at the instant $\theta = \pi/4$ rad.



SOLUTION

$v = 20 \text{ m/s}$

$\theta = \dfrac{\pi}{4} = 45°$

$r = 400 \cos \theta$

$\dot{r} = -400 \sin \theta\, \dot{\theta}$

$\ddot{r} = -400(\cos \theta (\dot{\theta})^2 + \sin \theta\, \ddot{\theta})$

$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$0 = \dot{r}\ddot{r} + r\dot{\theta}(\dot{r}\dot{\theta} + r\ddot{\theta})$

Thus

$r = 282.84$

$(20)^2 = [-400 \sin 45°\, \dot{\theta}]^2 + [282.84\, \dot{\theta}]^2$

$\dot{\theta} = 0.05$

$\dot{r} = -14.14$

$0 = -14.14[-400(\cos 45°)(0.05)^2 + \sin 45°\, \ddot{\theta}] + 282.84(0.05)[-14.14(0.05) + 282.84\ddot{\theta}]$

$\ddot{\theta} = 0$

$\ddot{r} = -0.707$

$v_r = \dot{r} = -14.1 \text{ m/s}$ **Ans.**

$v_\theta = r\dot{\theta} = 282.84(0.05) = 14.1 \text{ m/s}$ **Ans.**

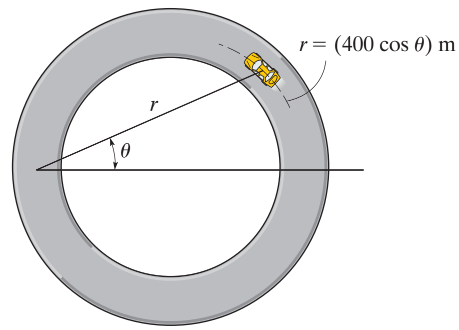
$a_r = \ddot{r} - r(\dot{\theta})^2 = -0.707 - 282.84(0.05)^2 = -1.41 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \dot{\theta} + 2(-14.14)(0.05) = -1.41 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = -14.1 \text{ m/s}$
$v_\theta = 14.1 \text{ m/s}$
$a_r = -1.41 \text{ m/s}^2$
$a_\theta = -1.41 \text{ m/s}^2$

12–174.

The car travels around the circular track such that its transverse component is $\theta = (0.006t^2)$ rad, where t is in seconds. Determine the car's radial and transverse components of velocity and acceleration at the instant $t = 4$ s.



SOLUTION

$\theta = 0.006\, t^2\big|_{t=4} = 0.096 \text{ rad} = 5.50°$

$\dot{\theta} = 0.012\, t\big|_{t=4} = 0.048 \text{ rad/s}$

$\ddot{\theta} = 0.012 \text{ rad/s}^2$

$r = 400 \cos \theta$

$\dot{r} = -400 \sin \theta\, \dot{\theta}$

$\ddot{r} = -400(\cos \theta\, (\dot{\theta})^2 + \sin\theta\, \ddot{\theta})$

At $\theta = 0.096$ rad

$r = 398.158$ m

$\dot{r} = -1.84037$ m/s

$\ddot{r} = -1.377449 \text{ m/s}^2$

$v_r = \dot{r} = -1.84$ m/s **Ans.**

$v_\theta = r\,\dot{\theta} = 398.158(0.048) = 19.1$ m/s **Ans.**

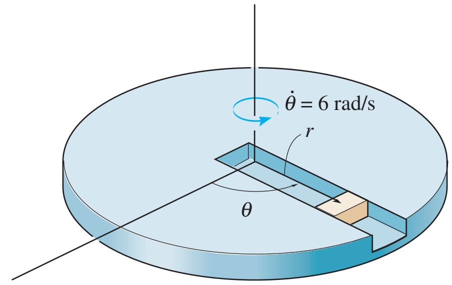
$a_r = \ddot{r} - r\,(\dot{\theta})^2 = -1.377449 - 398.158(0.048)^2 = -2.29 \text{ m/s}^2$ **Ans.**

$a_\theta = r\,\ddot{\theta} = 2\dot{r}\,\dot{\theta} = 398.158\,(0.012) + 2(-1.84037)(0.048) = 4.60 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = -1.84$ m/s
$v_\theta = 19.1$ m/s
$a_r = -2.29 \text{ m/s}^2$
$a_\theta = 4.60 \text{ m/s}^2$

12–175.

A block moves outward along the slot in the platform with a speed of $\dot{r} = (4t)$ m/s, where t is in seconds. The platform rotates at a constant rate of 6 rad/s. If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



SOLUTION

$\dot{r} = 4t|_{t=1} = 4 \qquad \ddot{r} = 4$

$\dot{\theta} = 6 \qquad \ddot{\theta} = 0$

$$\int_0^1 dr = \int_0^1 4t\,dt$$

$r = 2t^2\big]_0^1 = 2$ m

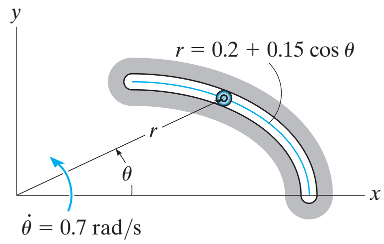
$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = \sqrt{(4)^2 + [2(6)]^2} = 12.6$ m/s **Ans.**

$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\,\dot{r}\dot{\theta})^2} = \sqrt{[4 - 2(6)^2]^2 + [0 + 2(4)(6)]^2}$ **Ans.**

$= 83.2$ m/s^2

Ans:
$v = 12.6$ m/s
$a = 83.2$ m/s^2

***12–176.** The pin follows the path described by the equation $r = (0.2 + 0.15\cos\theta)$ m. At the instant $\theta = 30°$, $\dot{\theta} = 0.7$ rad/s and $\ddot{\theta} = 0.5$ rad/s^2. Determine the magnitudes of the pin's velocity and acceleration at this instant. Neglect the size of the pin.



SOLUTION

$a = 0.2\ \text{m}$

$b = 0.15\ \text{m}$

$\theta_1 = 30°$

$\theta' = 0.7\ \text{rad/s}$

$\theta'' = 0.5\ \text{rad/s}^2$

$\theta = \theta_1$

$r = a + b\cos(\theta) \qquad r' = -b\sin(\theta)\theta' \qquad r'' = -b\cos(\theta)\theta'^2 - b\sin(\theta)\theta''$

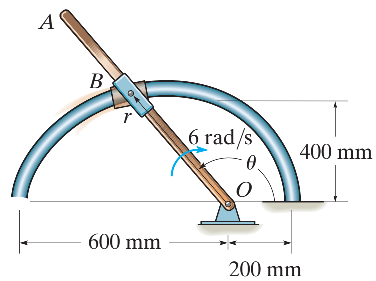
$v = \sqrt{r'^2 + (r\theta')^2} \qquad v = 0.237\ \text{m/s}$ **Ans.**

$a = \sqrt{(r'' - r\theta'^2)^2 + (r\theta'' + 2r'\theta')^2} \qquad a = 0.278\ \text{m/s}^2$ **Ans.**

Ans:
$v = 0.237\ \text{m/s}$
$a = 0.278\ \text{m/s}^2$

12–177.

The rod OA rotates clockwise with a constant angular velocity of 6 rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 200(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 150°$.



SOLUTION

Velocity. Using the chain rule, the first and second time derivatives of r can be determined.

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\,\dot{\theta} = \{200(\sin\theta)\,\dot{\theta}\}\ \text{mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\ \text{mm/s}^2$$

The radial and transverse components of the velocity are

$$v_r = \dot{r} = \{200(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$v_\theta = r\dot{\theta} = \{200(2 - \cos\theta)\dot{\theta}\}\ \text{mm/s}$$

Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$,

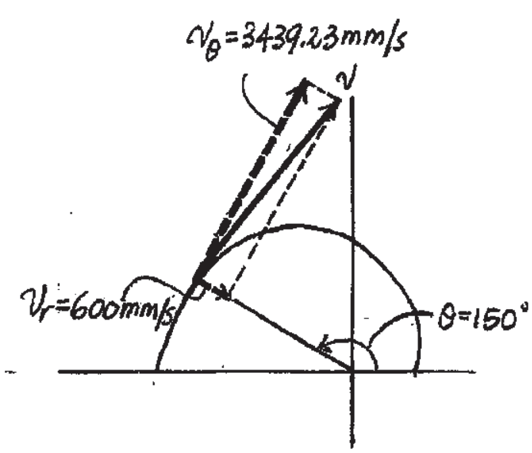
$$v_r = 200(\sin 150°)(-6) = -600\ \text{mm/s}$$

$$v_\theta = 200(2 - \cos 150°)(-6) = -3439.23\ \text{mm/s}$$

Thus, the magnitude of the velocity is

$$v = \sqrt{v_r^2 + v_\theta^2}\quad \sqrt{(-600)^2 + (-3439.23)^2} = 3491\ \text{mm/s} = 3.49\ \text{m/s} \qquad \textbf{Ans.}$$

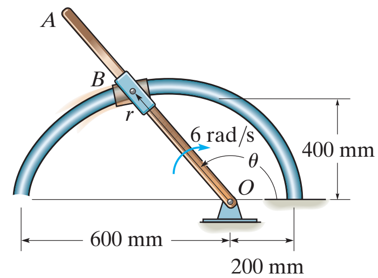
These components are shown in Fig. *a*



Ans:
$v = 3.49$ m/s

12–178.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–177 when $\theta = 150°$.



SOLUTION

Acceleration. Using the chain rule, the first and second time derivatives of r can be determined

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200\,(\sin\theta)\dot{\theta} = \{200\,(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\ \text{mm/s}^2$$

Here, since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$

$$r = 200(2 - \cos 150°) = 573.21\ \text{mm}$$

$$\dot{r} = 200(\sin 150°)(-6) = -600\ \text{mm/s}$$

$$\ddot{r} = 200\left[(\cos 150°)(-6)^2 + \sin 150°(0)\right] = -6235.38\ \text{mm/s}^2$$

The radial and transverse components of the acceleration are

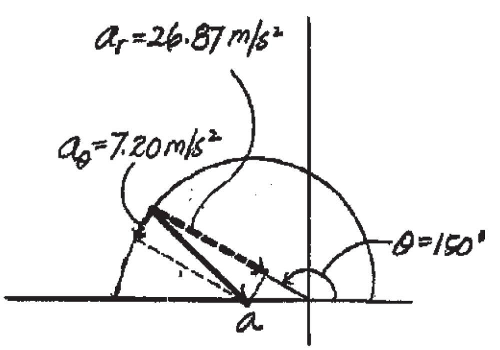
$$a_r = \ddot{r} - r\dot{\theta}^2 = -6235.38 - 573.21\,(-6)^2 = -26870.77\ \text{mm/s}^2 = -26.87\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 573.21(0) + 2(-600)(-6) = 7200\ \text{mm/s}^2 = 7.20\ \text{m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-26.87)^2 + 7.20^2} = 27.82\ \text{m/s}^2 = 27.8\ \text{m/s}^2 \qquad \textbf{Ans.}$$

These components are shown in Fig. *a*.

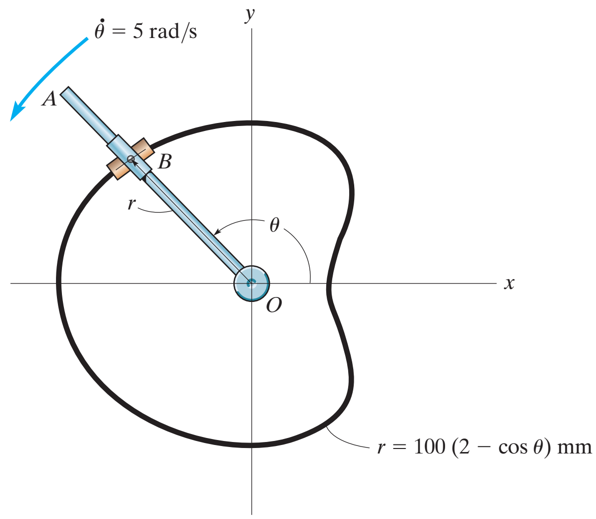


(a)

Ans:
$a = 27.8\ \text{m/s}^2$

12–179.

The rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 100(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 120°$.



SOLUTION

$\dot{\theta} = 5$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100\sin\theta\dot{\theta} = 500\sin\theta$

$\ddot{r} = 500\cos\theta\dot{\theta} = 2500\cos\theta$

At $\theta = 120°$,

$v_r = \dot{r} = 500\sin 120° = 433.013$

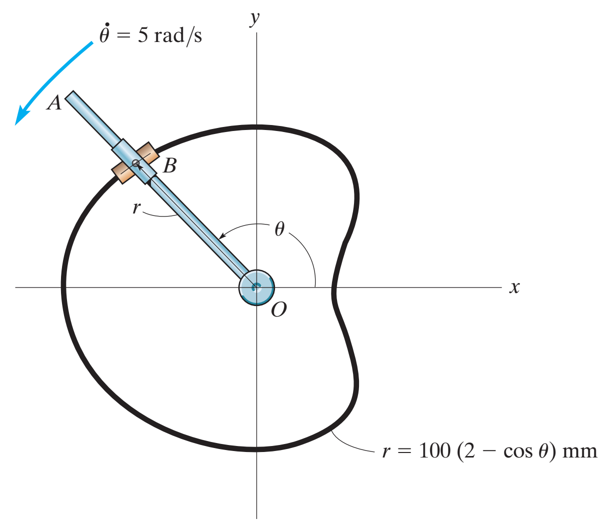
$v_\theta = r\dot{\theta} = 100\,(2 - \cos 120°)(5) = 1250$

$v = \sqrt{(433.013)^2 + (1250)^2} = 1322.9\text{ mm/s} = 1.32\text{ m/s}$ **Ans.**

Ans:
$v = 1.32$ m/s

***12–180.**

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–179 when $\theta = 120°$.



SOLUTION

$\dot{\theta} = 5$

$\dot{\theta} = 0$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100 \sin\theta\dot{\theta} = 500 \sin\theta$

$\ddot{r} = 500 \cos\theta\dot{\theta} = 2500 \cos\theta$

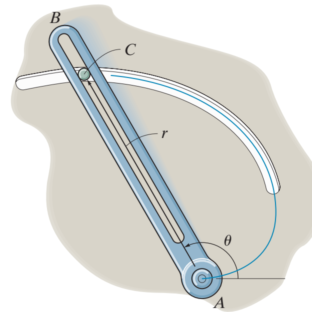
$a_r = \ddot{r} - r\dot{\theta}^2 = 2500 \cos\theta - 100(2 - \cos\theta)(5)^2 = 5000(\cos 120° - 1) = -7500 \text{ mm/s}^2$

$a_s = r\dot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(500 \sin\theta)(5) = 5000 \sin 120° = 4330.1 \text{ mm/s}^2$

$a = \sqrt{(-7500)^2 + (4330.1)^2} = 8660.3 \text{ mm/s}^2 = 8.66 \text{ m/s}^2$ **Ans.**

Ans:
$a = 8.66 \text{ m/s}^2$

12–181. The slotted arm AB drives pin C through the spiral groove described by the equation $r = a\theta$. If the angular velocity is constant at $\dot{\theta}$, determine the radial and transverse components of velocity and acceleration of the pin.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$.

$$r = a\theta \qquad \dot{r} = a\dot{\theta} \qquad \ddot{r} = a\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = a\dot{\theta} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = a\theta\dot{\theta} \qquad \textbf{Ans.}$$

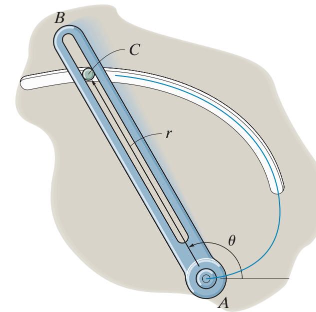
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 = -a\theta\dot{\theta}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(a\dot{\theta})(\dot{\theta}) = 2a\dot{\theta}^2 \qquad \textbf{Ans.}$$

Ans:
$v_r = a\,\dot{\theta}$
$v_\theta = a\,\theta\dot{\theta}$
$a_r = -a\theta\dot{\theta}^2$
$a_\theta = 2a\dot{\theta}^2$

12–182. The slotted arm AB drives pin C through the spiral groove described by the equation $r = (1.5\,\theta)$ m, where θ is in radians. If the arm starts from rest when $\theta = 60°$ and is driven at an angular velocity of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin C when $t = 1$ s.



SOLUTION

Time Derivatives: Here, $\dot{\theta} = 4t$ and $\ddot{\theta} = 4$ rad/s^2.

$$r = 1.5\theta \qquad \dot{r} = 1.5\dot{\theta} = 1.5(4t) = 6t \qquad \ddot{r} = 1.5\ddot{\theta} = 1.5(4) = 6 \text{ m/s}^2$$

Velocity: Integrate the angular rate, $\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t\,dt$, we have $\theta = \frac{1}{3}(6t^2 + \pi)$ rad.

Then, $r = \left\{\frac{1}{2}(6t^2 + \pi)\right\}$ m. At $t = 1$ s, $r = \frac{1}{2}\left[6(1^2) + \pi\right] = 4.571$ m, $\dot{r} = 6(1) = 6.00$ m/s and $\dot{\theta} = 4(1) = 4$ rad/s. Applying Eq. 12–25, we have

$$v_r = \dot{r} = 6.00 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4.571\,(4) = 18.3 \text{ m/s} \qquad \textbf{Ans.}$$

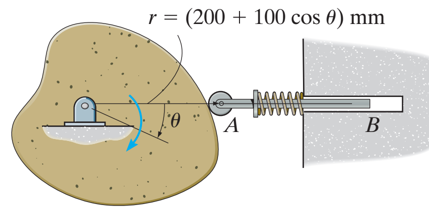
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.571\left(4^2\right) = -67.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.571(4) + 2(6)\,(4) = 66.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_r = 6.00$ m/s
$v_\theta = 18.3$ m/s
$a_r = -67.1$ m/s^2
$a_\theta = 66.3$ m/s^2

12–183. If the cam rotates clockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB at the instant $\theta = 30°$. The surface of the cam has a shape of limaçon defined by $r = (200 + 100 \cos \theta)$ mm.



SOLUTION

Time Derivatives:

$$r = (200 + 100 \cos \theta) \text{ mm}$$

$$\dot{r} = (-100 \sin \theta \dot{\theta}) \text{ mm/s} \qquad \dot{\theta} = 5 \text{ rad/s}$$

$$\ddot{r} = -100\left[\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2\right] \text{ mm/s}^2 \qquad \ddot{\theta} = 0$$

When $\theta = 30°$,

$$r\big|_{\theta=30°} = 200 + 100 \cos 30° = 286.60 \text{ mm}$$

$$\dot{r}\big|_{\theta=30°} = -100 \sin 30°(5) = -250 \text{ mm/s}$$

$$\dot{r}\big|_{\theta=30°} = -100\left[0 + \cos 30°\left(5^2\right)\right] = -2165.06 \text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = -250 \text{ mm/s} \qquad \textbf{Ans.}$$

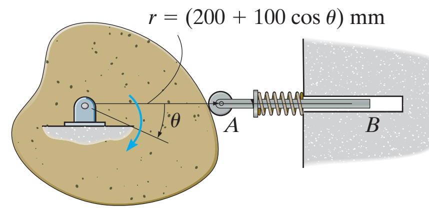
Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2156.06 - 286.60(5^2) = -9330 \text{ mm/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v_r = -250$ mm/s
$a_r = -9330$ mm/s^2

***12–184.** At the instant $\theta = 30^\circ$, the cam rotates with a clockwise angular velocity of $\dot{\theta} = 5$ rad/s and, angular acceleration of $\ddot{\theta} = 6$ rad/s^2. Determine the magnitudes of the velocity and acceleration of the follower rod AB at this instant. The surface of the cam has a shape of a limaçon defined by $r = (200 + 100\cos\theta)$ mm.



SOLUTION

Time Derivatives:

$$r = (200 - 100\cos\theta)\text{ mm}$$

$$\dot{r} = (-100\sin\theta\dot{\theta})\text{ mm/s}$$

$$\dot{r} = -100\left[\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right]\text{ mm/s}^2$$

When $\theta = 30^\circ$,

$$r\big|_{\theta=30^\circ} = 200 + 100\cos 30^\circ = 286.60\text{ mm}$$

$$\dot{r}\big|_{\theta=30^\circ} = -100\sin 30^\circ(5) = -250\text{ mm/s}$$

$$\ddot{r}\big|_{\theta=30^\circ} = -100\left[\sin 30^\circ(6) + \cos 30^\circ\left(5^2\right)\right] = -2465.06\text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = -250\text{ mm/s}$$ **Ans.**

Acceleration: The radial component gives the rod's acceleration.

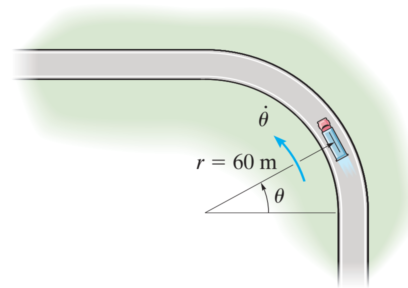
$$a_r = \ddot{r} - r\dot{\theta}^2 = -2465.06 - 286.60\left(5^2\right) = -9630\text{ mm/s}^2$$ **Ans.**

Ans:

$v_r = -250$ mm/s
$a_r = -9630$ mm/s^2

12–185.

A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the truck's acceleration.



SOLUTION

$r = 60$

$\dot{r} = 0$

$\ddot{r} = 0$

$v = 20$

$v_r = \dot{r} = 0$

$v_\theta = r\,\dot{\theta} = 60\,\dot{\theta}$

$v = \sqrt{(v_r)^2 + (v_\theta)^2}$

$20 = 60\,\dot{\theta}$

$\dot{\theta} = 0.333$ rad/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2$

$= 0 - 60(0.333)^2$

$= -6.67$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 60\ddot{\theta}$

Since

$v = r\dot{\theta}$

$\dot{v} = \dot{r}\dot{\theta} + r\ddot{\theta}$

$0 = 0 + 60\,\ddot{\theta}$

$\ddot{\theta} = 0$

Thus,

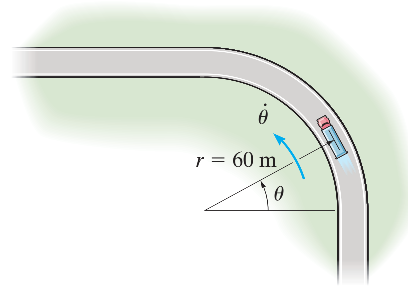
$a_\theta = 0$

$a = \left|a_r\right| = 6.67$ m/s^2 **Ans.**

Ans:
$\dot{\theta} = 0.333$ rad/s
$a = 6.67$ m/s^2

12–186.

A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at 3 m/s^2. Determine the truck's radial and transverse components of acceleration.



SOLUTION

$r = 60$

$a_t = 3 \text{ m/s}^2$

$$a_n = \frac{v^2}{r} = \frac{(20)^2}{60} = 6.67 \text{ m/s}^2$$

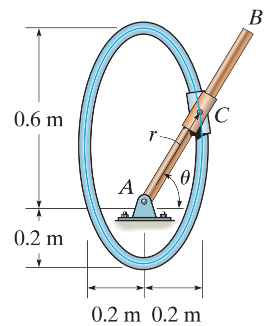
$a_r = -a_n = -6.67 \text{ m/s}^2$ **Ans.**

$a_\theta = a_t = 3 \text{ m/s}^2$ **Ans.**

Ans:
$a_r = -6.67 \text{ m/s}^2$
$a_\theta = 3 \text{ m/s}^2$

12–187.

The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB. If the angular velocity of AB is given as $\dot{\theta} = (e^{0.5\,t^2})$ rad/s, where t is in seconds, and the path defined by the fixed rod is $r = |(0.4 \sin\theta + 0.2)|$ m, determine the radial and transverse components of the collar's velocity and acceleration when $t = 1$ s. When $t = 0$, $\theta = 0$. Use Simpson's rule with $n = 50$ to determine θ at $t = 1$ s.



SOLUTION

$$\dot{\theta} = e^{0.5\,t^2}\big|_{t=1} = 1.649 \text{ rad/s}$$

$$\ddot{\theta} = e^{0.5\,t^2}\,t\big|_{t=1} = 1.649 \text{ rad/s}^2$$

$$\theta = \int_0^1 e^{0.5\,t^2}dt = 1.195 \text{ rad} = 68.47^\circ$$

$$r = 0.4 \sin\theta + 0.2$$

$$\dot{r} = 0.4 \cos\theta\,\dot{\theta}$$

$$\ddot{r} = -0.4 \sin\theta\,\dot{\theta}^2 + 0.4\cos\theta\,\ddot{\theta}$$

At $t = 1$ s,

$$r = 0.5721$$

$$\dot{r} = 0.2421$$

$$\ddot{r} = -0.7697$$

$$v_r = \dot{r} = 0.242 \text{ m/s}$$ **Ans.**

$$v_\theta = r\,\dot{\theta} = 0.5721(1.649) = 0.943 \text{ m/s}$$ **Ans.**

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.7697 - 0.5721(1.649)^2$$

$$a_r = -2.33 \text{ m/s}^2$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

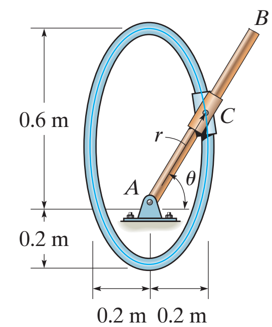
$$= 0.5721(1.649) + 2(0.2421)(1.649)$$

$$a_\theta = 1.74 \text{ m/s}^2$$ **Ans.**

Ans:
$v_r = 0.242$ m/s
$v_\theta = 0.943$ m/s
$a_r = -2.33$ m/s^2
$a_\theta = 1.74$ m/s^2

***12–188.**

The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB. If the mechanism is to be designed so that the largest speed given to the collar is 6 m/s, determine the required constant angular velocity $\dot{\theta}$ of rod AB. The path defined by the fixed rod is $r = (0.4 \sin\theta + 0.2)$ m.



SOLUTION

$r = 0.4 \sin\theta + 0.2$

$\dot{r} = 0.4 \cos\theta\,\dot{\theta}$

$v_r = \dot{r} = 0.4 \cos\theta\,\dot{\theta}$

$v_\theta = r\,\dot{\theta} = (0.4 \sin\theta + 0.2)\,\dot{\theta}$

$v^2 = v_r^2 + v_\theta^3$

$(6)^2 = [(0.4 \cos\theta)^2 + (0.4 \sin\theta + 0.2)^2](\dot{\theta})^2$

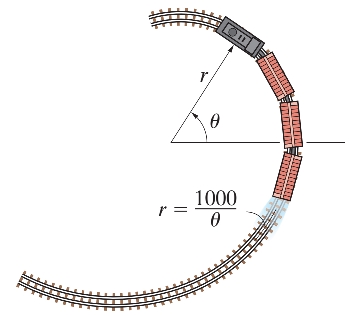
$36 = [0.2 + 0.16 \sin\theta](\dot{\theta})^2$

The greatest speed occurs when $\theta = 90°$.

$\dot{\theta} = 10.0$ rad/s **Ans.**

Ans:
$\dot{\theta} = 10.0$ rad/s

12–189. For a *short distance* the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If it maintains a constant speed $v = 20$ m/s, determine the radial and transverse components of its velocity when $\theta = (9\pi/4)$ rad.



SOLUTION

$$r = \frac{1000}{\theta}$$

$$\dot{r} = -\frac{1000}{\theta^2}\dot{\theta}$$

Since

$$v^2 = (\dot{r})^2 + (r\,\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(\dot{\theta})^2 + \frac{(1000)^2}{\theta^2}(\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(1 + \theta^2)(\dot{\theta})^2$$

Thus,

$$\dot{\theta} = \frac{0.02\theta^2}{\sqrt{1 + \theta^2}}$$

At $\theta = \frac{9\pi}{4}$

$$\dot{\theta} = 0.140$$

$$\dot{r} = \frac{-1000}{(9\pi/4)^2}(0.140) = -2.80$$

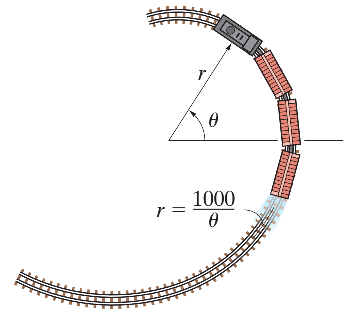
$v_r = \dot{r} = -2.80$ m/s **Ans.**

$v_\theta = r\dot{\theta} = \frac{1000}{(9\pi/4)}(0.140) = 19.8$ m/s **Ans.**

Ans:
$v_r = -2.80$ m/s
$v_\theta = 19.8$ m/s

12–190.

For a *short distance* the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = (9\pi/4)$ rad.



SOLUTION

$\dot{\theta} = 0.2$

$\ddot{\theta} = 0$

$$r = \frac{1000}{\theta}$$

$\dot{r} = -1000(\theta^{-2})\dot{\theta}$

$\ddot{r} = 2000(\theta^{-3})(\dot{\theta})^2 - 1000(\theta^{-2})\ddot{\theta}$

When $\theta = \dfrac{9\pi}{4}$

$r = 141.477$

$\dot{r} = -4.002812$

$\ddot{r} = 0.226513$

$v_r = \dot{r} = -4.00$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 141.477(0.2) = 28.3$ m/s **Ans.**

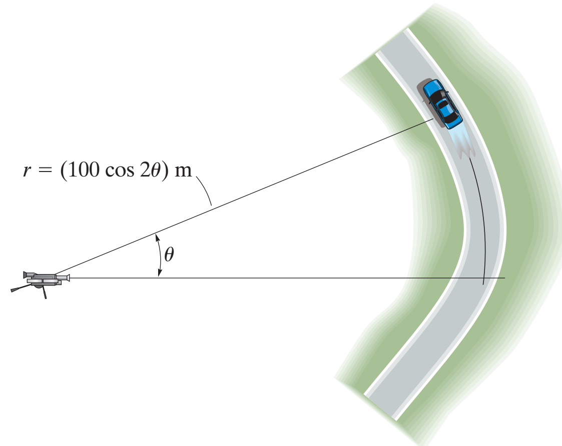
$a_r = \ddot{r} - r(\dot{\theta})^2 = 0.226513 - 141.477(0.2)^2 = -5.43$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-4.002812)(0.2) = -1.60$ m/s^2 **Ans.**

Ans:
$v_r = -4.00$ m/s
$v_\theta = 28.3$ m/s
$a_r = -5.43$ m/s^2
$a_\theta = -1.60$ m/s

12–191.

The driver of the car maintains a constant speed of 40 m/s. Determine the angular velocity of the camera tracking the car when $\theta = 15°$.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\dot{\theta}) \text{ m/s}$$

At $\theta = 15°$,

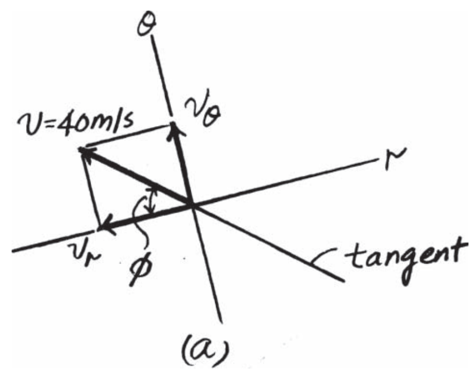
$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

Velocity: Referring to Fig. *a*, $v_r = -40 \cos \phi$ and $v_\theta = 40 \sin \phi$.

$$v_r = \dot{r}$$

$$-40 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$



and

$$v_\theta = r\dot{\theta}$$

$$40 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

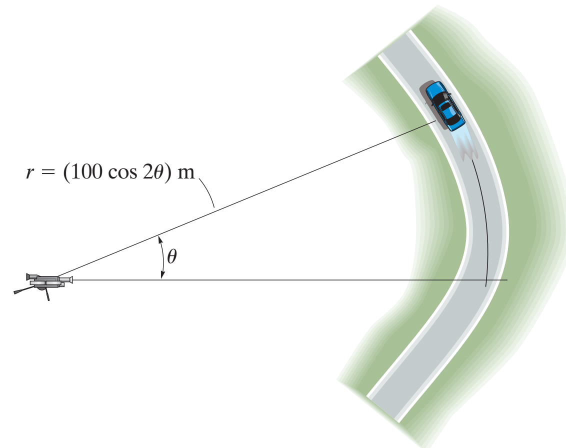
$$\phi = 40.89°$$

$$\dot{\theta} = 0.3024 \text{ rad/s} = 0.302 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\dot{\theta} = 0.302$ rad/s

***12–192.**

When $\theta = 15°$, the car has a speed of 50 m/s which is increasing at 6 m/s^2. Determine the angular velocity of the camera tracking the car at this instant.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\,\dot{\theta}) \text{ m/s}$$

$$\ddot{r} = -200\left[(\sin\ 2\theta)\,\ddot{\theta} + 2\,(\cos 2\theta)\,\dot{\theta}^2\right] \text{ m/s}^2$$

At $\theta = 15°$,

$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

$$\ddot{r}\big|_{\theta=15°} = -200\left[\sin 30°\,\ddot{\theta} + 2 \cos 30°\,\dot{\theta}^2\right] = \left(-100\ddot{\theta} - 346.41\dot{\theta}^2\right) \text{ m/s}^2$$

Velocity: Referring to Fig. *a*, $v_r = -50 \cos \phi$ and $v_\theta = 50 \sin \phi$. Thus,

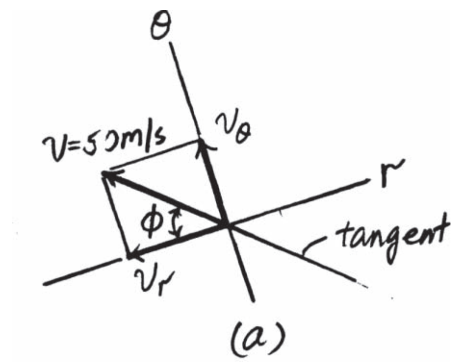
$$v_r = \dot{r}$$

$$-50 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$50 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

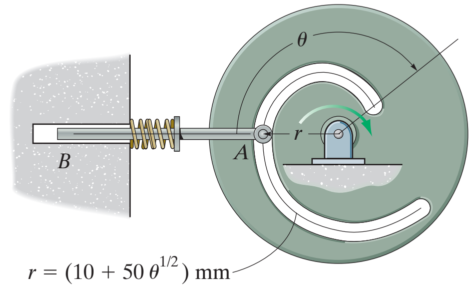
$$\phi = 40.89°$$

$$\dot{\theta} = 0.378 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\dot{\theta} = 0.378$ rad/s

12–193.

If the circular plate rotates clockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB when $\theta = 2/3\pi$ rad.



SOLUTION

Time Derivaties:

$$r = \left(10 + 50\theta^{1/2}\right) \text{mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta} \text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right] \text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36 \text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91 \text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[0 - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = -9.279 \text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = 25.9 \text{ mm/s} \qquad \textbf{Ans.}$$

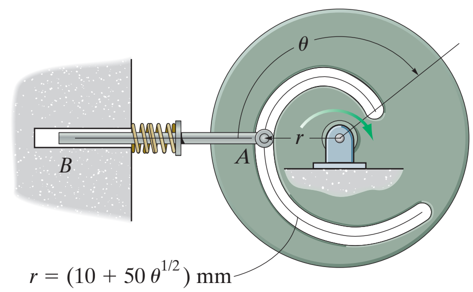
Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -9.279 - 82.36\left(1.5^2\right) = -195 \text{ mm/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_r = 25.9$ mm/s
$a_r = -195$ mm/s^2

12–194.

When $\theta = 2/3\pi$ rad, the angular velocity and angular acceleration of the circular plate are $\dot{\theta} = 1.5\,\text{rad/s}$ and $\ddot{\theta} = 3\,\text{rad/s}^2$, respectively. Determine the magnitudes of the velocity and acceleration of the rod AB at this instant.



SOLUTION

Time Derivatives:

$$r = (10 + 50\theta^{1/2})\text{ mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[\left(\frac{2\pi}{3}\right)^{-1/2}(3) - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = 42.55\text{ mm/s}^2$$

For the rod,

$$v = \dot{r} = 25.9\text{ mm/s}$$ **Ans.**

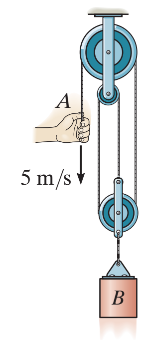
$$a = \ddot{r} = 42.5\text{ mm/s}^2$$ **Ans.**

Ans:

$v = 25.9$ mm/s
$a = 42.5$ mm/s^2

12–195.

If the end of the cable at A is pulled down with a speed of 5 m/s, determine the speed at which block B rises.



SOLUTION

Position Coordinate. The positions of pulley B and point A are specified by position coordinates s_B and s_A, respectively, as shown in Fig. *a*. This is a single-cord pulley system. Thus,

$$s_B + 2(s_B - a) + s_A = l$$

$$3s_B + s_A = l + 2a \qquad \textbf{(1)}$$

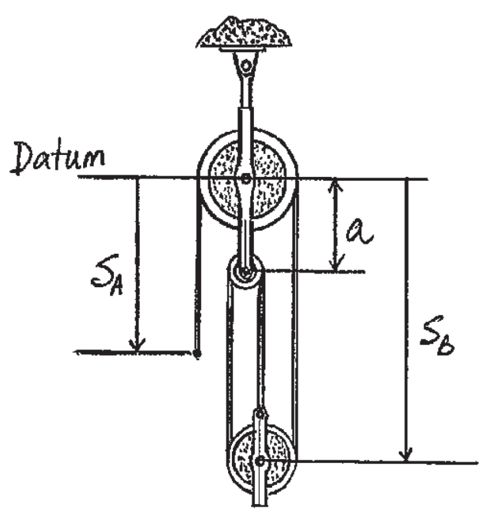
Time Derivative. Taking the time derivative of Eq. (1),

$$3v_B + v_A = 0 \qquad \textbf{(2)}$$

Here $v_A = +5$ m/s, since it is directed toward the positive sense of s_A. Thus,

$$3v_B + 5 = 0 \qquad v_B = -1.667 \text{ m/s} = 1.67 \text{ m/s}\uparrow \qquad \textbf{Ans.}$$

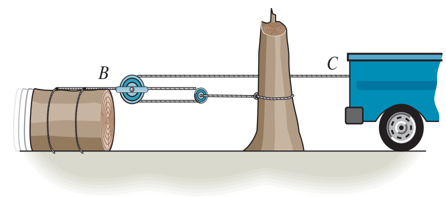
The negative sign indicates that v_B is directed toward the negative sense of s_B.



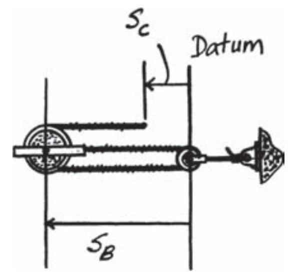
(a)

Ans:
$v_B = 1.67$ m/s

***12–196.** Determine the displacement of the log if the truck at C pulls the cable 1.2 m to the right.



SOLUTION



$2s_B + (s_B - s_C) = l$

$3s_B - s_C = l$

$3\Delta s_B - \Delta s_C = 0$

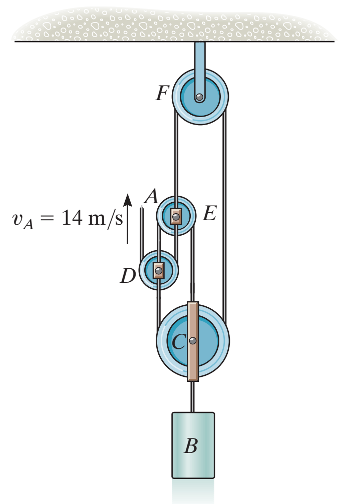
Since $\Delta s_C = -1.2$, then

$3\Delta s_B = -1.2$

$\Delta s_B = -0.4 \text{ m} = 0.4 \text{ m} \rightarrow$ **Ans.**

Ans:
$\Delta s_B = 0.4 \text{ m} \rightarrow$

12–197. If the end A of the cable is moving upwards at $v_A = 14$ m/s, determine the speed of block B.



SOLUTION

$v_A = -14$ m/s

$L_1 = (s_D - s_A) + (s_D - s_E)$

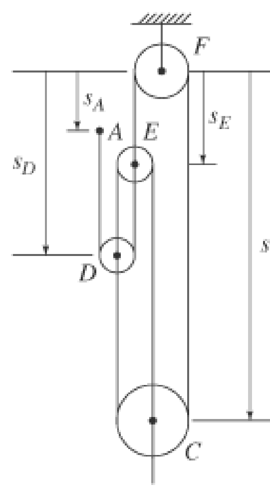
$0 = 2v_D - v_A - v_E$

$L_2 = (s_D - s_E) + (s_C - s_E)$

$0 = v_D + v_C - 2v_E$

$L_3 = (s_C - s_D) + s_C + s_E$

$0 = 2v_C - v_D + v_E$



Guesses $\quad v_C = 1$ m/s $\quad v_D = 1$ m/s $\quad v_E = 1$ m/s

Given $\quad 0 = 2v_D - v_A - v_E$

$0 = v_D + v_C - 2v_E$

$0 = 2v_C - v_D + v_E$

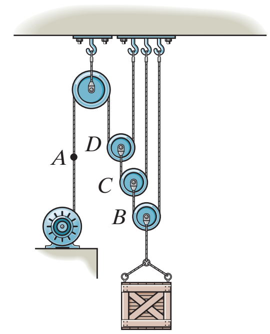
$$\begin{pmatrix} v_C \\ v_D \\ v_E \end{pmatrix} = \text{Find}(v_C, v_D, v_E) \qquad \begin{pmatrix} v_C \\ v_D \\ v_E \end{pmatrix} = \begin{pmatrix} -2 \\ -10 \\ -6 \end{pmatrix} \text{m/s}$$

$v_B = v_C \qquad v_B = -2$ m/s $\quad$ Positive means down, Negative means up $\quad$ **Ans.**

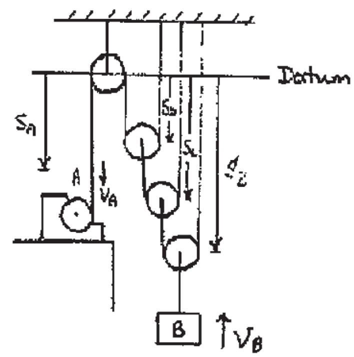
Ans:
$v_B = -2$ m/s Positive means down, Negative means up.

12–198.

Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load 6 m in 1.5 s.



SOLUTION



$$v_B = \frac{6}{1.5} = 4 \text{ m/s} \uparrow$$

$$s_B + (s_B - s_C) = l_1$$

$$s_C + (s_C - s_D) = l_2$$

$$s_A + 2\,s_D = l_3$$

Thus,

$$2s_B - s_C = l_1$$

$$2s_C - s_D = l_2$$

$$s_A + 2s_D = l_3$$

$$2v_A = v_C$$

$$2v_C = v_D$$

$$v_A = -2v_D$$

$$2(2v_B) = v_D$$

$$v_A = -2(4v_B)$$

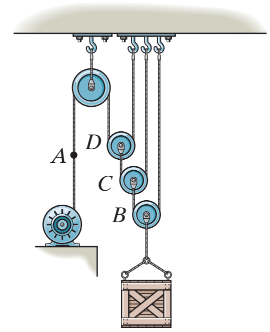
$$v_A = -8v_B$$

$$v_A = -8(-4) = 32 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

Ans:
$v_A = 32 \text{ m/s}\downarrow$

12–199.

Starting from rest, the cable can be wound onto the drum of the motor at a rate of $v_A = (3t^2)$ m/s, where t is in seconds. Determine the time needed to lift the load 7 m.



SOLUTION

$$v_B = \frac{6}{1.5} = 4\text{ m/s} \uparrow$$

$$s_B + (s_B - s_C) = l_1$$

$$s_C + (s_C - s_D) = l_2$$

$$s_A + 2s_D) = l_3$$

Thus,

$$2s_B - s_C = l_1$$

$$2s_C - s_D = l_2$$

$$s_A + 2s_D = l_3$$

$$2v_B = v_C$$

$$2v_C = v_D$$

$$v_A = -2v_D$$

$$v_A = -8v_B$$

$$3\,t^2 = -8v_B$$

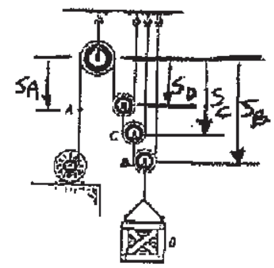
$$v_B = \frac{-3}{8}t^2$$

$$s_B = \int_0^t \frac{-3}{8}t^2\,dt$$

$$s_B = \frac{-1}{8}t^3$$

$$-7 = \frac{-1}{8}t^3$$

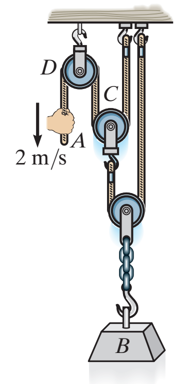
$t = 3.83$ s **Ans.**



Ans:
$t = 3.83$ s

***12–200.**

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



SOLUTION

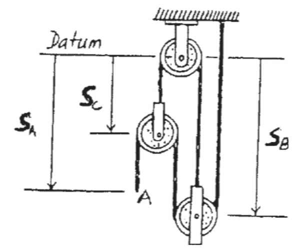
Position-Coordinate Equation: Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B, and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad \textbf{(1)}$$

$$s_B + s_C = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2) yields

$$s_A + 4s_B = l_1 = 2l_2$$



Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad \textbf{(3)}$$

Since $v_A = 2$ m/s, from Eq. (3)

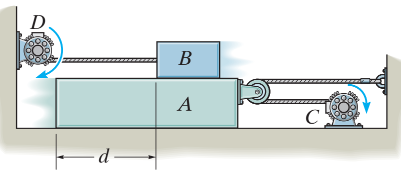
$(+\downarrow)$ $$2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Ans:
$v_B = 0.5$ m/s

12–201.

The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



SOLUTION

For A:

$s_A + (s_A - s_C) = l$

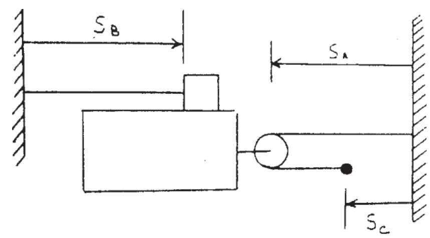
$2v_A = v_C$

$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \quad \rightarrow$

$v_A = 0.5t^3 \quad \rightarrow$

$s_A = 0.125\,t^4 \quad \rightarrow$



For B:

$a_B = 5\ \text{m/s}^2 \quad \leftarrow$

$v_B = 5t \quad \leftarrow$

$s_B = 2.5t^2 \quad \leftarrow$

Require $s_A + s_B = d$

$0.125t^4 + 2.5t^2 = 3$

Set $u = t^2 \qquad 0.125u^2 + 2.5u = 3$

The positive root is $u = 1.1355$. Thus,

$t = 1.0656 = 1.07$ s **Ans.**

$v_A = 0.5(1.0656)^3 = 0.6050$

$v_B = 5(1.0656) = 5.3281$ m/s

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$0.6050\mathbf{i} = -5.3281\mathbf{i} + v_{A/B}\,\mathbf{i}$

$v_{A/B} = 5.93$ m/s $\quad \rightarrow$ **Ans.**

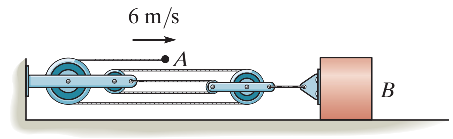
Ans:

$t = 1.07$ s

$v_{A/B} = 5.93$ ms/s $\rightarrow$

12–202.

Determine the speed of the block at B.



SOLUTION

Position Coordinate. The positions of pulley B and point A are specified by position coordinates s_B and s_A respectively as shown in Fig. *a*. This is a single cord pulley system. Thus,

$$s_B + 2(s_B - a - b) + (s_B - a) + s_A = l$$

$$4s_B + s_A = l + 3a + 2b \qquad \textbf{(1)}$$

Time Derivative. Taking the time derivative of Eq. (1),

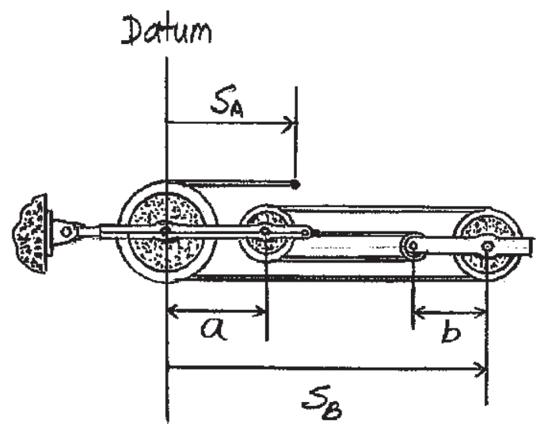
$$4v_B + v_A = 0 \qquad \textbf{(2)}$$

Here, $v_A = +6$ m/s since it is directed toward the positive sense of s_A. Thus,

$$4v_B + 6 = 0$$

$$v_B = -1.50 \text{ m/s} = 1.50 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

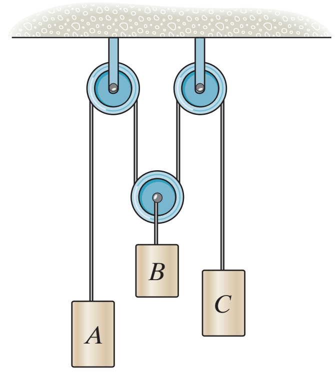
The negative sign indicates that $\mathbf{v}_B$ is directed towards negative sense of s_B.



(a)

Ans:
$v_B = 1.50$ m/s

12-203. If block A is moving downward with a speed of 2 m/s while C is moving up at 1 m/s, determine the speed of block B.



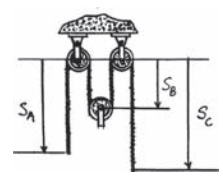
SOLUTION

$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

$2 + 2v_B - 1 = 0$

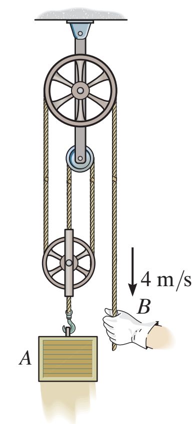
$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow$ **Ans.**



Ans:
$2 + 2v_B - 1 = 0$

***12–204.**

Determine the speed of block A if the end of the rope is pulled down with a speed of 4 m/s.



SOLUTION

Position Coordinates: By referring to Fig. *a*, the length of the cord written in terms of the position coordinates s_A and s_B is

$$s_B + s_A + 2(s_A - a) = l$$

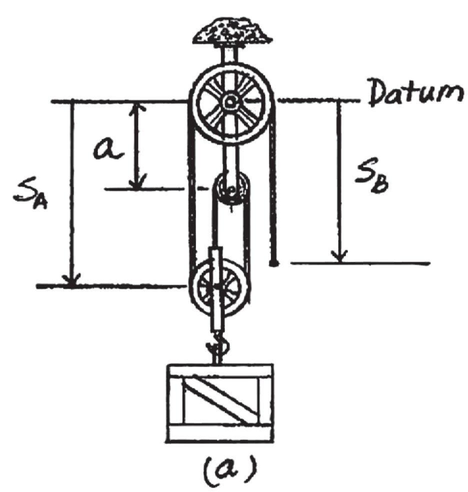
$$s_B + 3s_A = l + 2a$$

Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad v_B + 3v_A = 0$$

Here, $v_B = 4$ m/s. Thus,

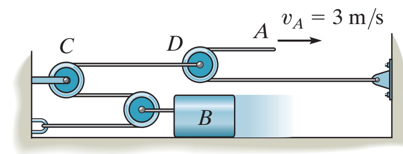
$$4 + 3v_A = 0 \qquad v_A = -133 \text{ m/s} = 1.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



Ans:
$v_A = 1.33$ m/s

12–205.

If the end A of the cable is moving at $v_A = 3$ m/s, determine the speed of block B.



SOLUTION

Position Coordinates. The positions of pulley B, D and point A are specified by position coordinates s_B, s_D and s_A respectively as shown in Fig. a. The pulley system consists of two cords which give

$$2\,s_B + s_D = l_1 \quad \textbf{(1)}$$

and

$$(s_A - s_D) + (b - s_D) = l_2$$

$$s_A - 2\,s_D = l_2 - b \quad \textbf{(2)}$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$2v_B + v_D = 0 \quad \textbf{(3)}$$

$$v_A - 2v_D = 0 \quad \textbf{(4)}$$

Eliminate v_0 from Eqs. (3) and (4),

$$v_A + 4v_B = 0 \quad \textbf{(5)}$$

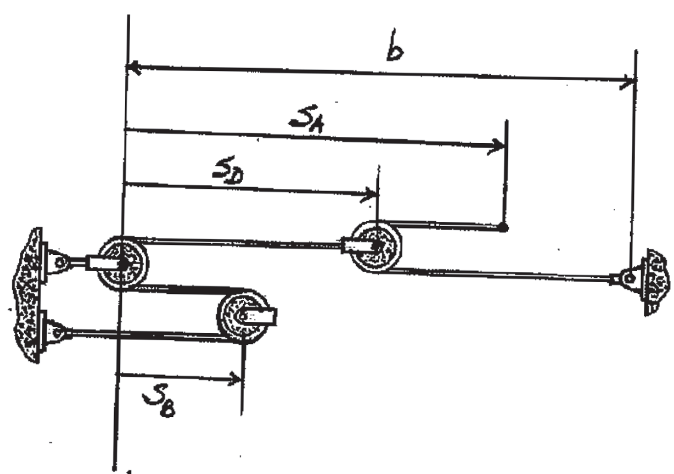
Here $v_A = +3$ m/s since it is directed toward the positive sense of s_A.

Thus

$$3 + 4v_B = 0$$

$$v_B = -0.75 \text{ m/s} = 0.75 \text{ m/s} \leftarrow \quad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_D$ is directed toward the negative sense of $\mathbf{s}_B$.

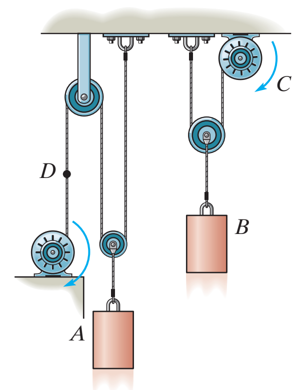


(a)

Ans:
$v_B = 0.75$ m/s

12–206.

The motor draws in the cable at C with a constant velocity of $v_C = 4$ m/s. The motor draws in the cable at D with a constant acceleration of $a_D = 8\ \text{m/s}^2$. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise 3 m, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION

(a) $a_D = 8\ \text{m/s}^2$

$v_D = 8\,t$

$s_D = 4\,t^2$

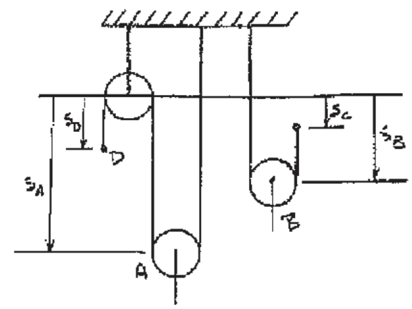
$s_D + 2s_A = l$

$\Delta s_D = -2\Delta s_A$ **(1)**

$\Delta s_A = -2\,t^2$

$-3 = -2\,t^2$

$t = 1.2247 = 1.22\ \text{s}$ **Ans.**



(b) $v_A = s_A = -4\,t = -4(1.2247) = -4.90\ \text{m/s} = 4.90\ \text{m/s}\uparrow$

$s_B + (s_B - s_C) = l$

$2v_B = v_C = -4$

$v_B = -2\ \text{m/s} = 2\ \text{m/s}\uparrow$

$(+\downarrow)$ $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

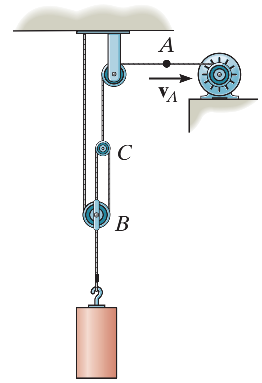
$-4.90 = -2 + v_{A/B}$

$v_{A/B} = -2.90\ \text{m/s} = 2.90\ \text{m/s}\uparrow$ **Ans.**

Ans:
$v_{A/B} = 2.90\ \text{m/s}\uparrow$

12–207.

Determine the time needed for the load at B to attain a speed of 10 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 3 m/s^2.



SOLUTION

Position Coordinates. The position of pulleys B, C and point A are specified by position coordinates s_B, s_C and s_A respectively as shown in Fig. *a*. The pulley system consists of two cords which gives

$$s_B + 2(s_B - s_C) = l_1$$

$$3s_B - 2s_C = l_1 \qquad \textbf{(1)}$$

And

$$s_C + s_A = l_2 \qquad \textbf{(2)}$$

Time Derivative. Taking the time derivative twice of Eqs. (1) and (2),

$$3a_B - 2a_C = 0 \qquad \textbf{(3)}$$

And

$$a_C + a_A = 0 \qquad \textbf{(4)}$$

Eliminate a_C from Eqs. (3) and (4)

$$3a_B + 2a_A = 0$$

Here, $a_A = +3\text{ m/s}^2$ since it is directed toward the positive sense of s_A. Thus,

$$3a_B + 2(3) = 0 \qquad a_B = -2\text{ m/s}^2 = 2\text{ m/s}^2 \uparrow$$

The negative sign indicates that $\mathbf{a}_B$ is directed toward the negative sense of s_B. Applying kinematic equation of constant acceleration,

$$+\uparrow \qquad v_B = (v_B)_0 + a_B t$$

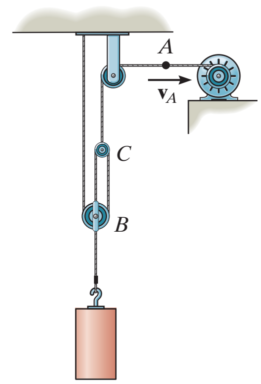
$$10 = 0 + 2\,t$$

$$t = 5.00\text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 5.00\text{ s}$

***12–208.**

The cable at A is being drawn toward the motor at $v_A = 8$ m/s. Determine the velocity of the block.



SOLUTION

Position Coordinates. The position of pulleys B, C and point A are specified by position coordinates s_B, s_C and s_A respectively as shown in Fig. a. The pulley system consists of two cords which give

$$s_B + 2(s_B - s_C) = l_1$$

$$3s_B - 2s_C = l_1 \tag{1}$$

And

$$s_C + s_A = l_2 \tag{2}$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$3v_B - 2v_C = 0 \tag{3}$$

And

$$v_C + v_A = 0 \tag{4}$$

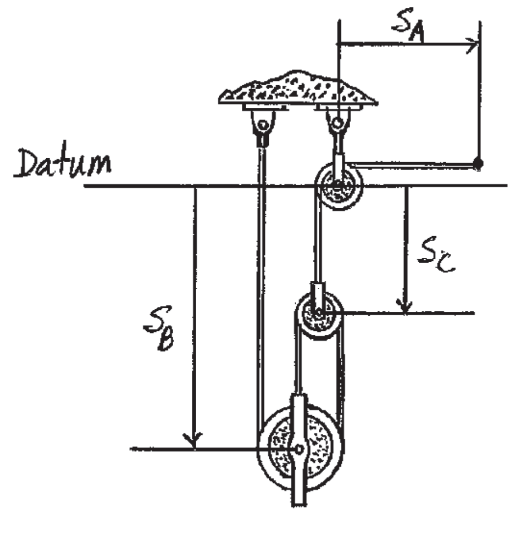
Eliminate v_C from Eqs. (3) and (4),

$$3v_B + 2v_A = 0$$

Here $v_A = +8$ m/s since it is directed toward the positive sense of s_A. Thus,

$$3v_B + 2(8) = 0 \qquad v_B = -5.33 \text{ m/s} = 5.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

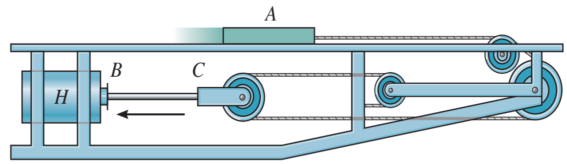
The negative sign indicates that $\mathbf{v}_B$ is directed toward the negative sense of s_B.



(a)

Ans:
$v_B = 5.33$ m/s$\uparrow$

12-209. If the hydraulic cylinder H draws in rod BC at 1 m/s, determine the speed of slider A.



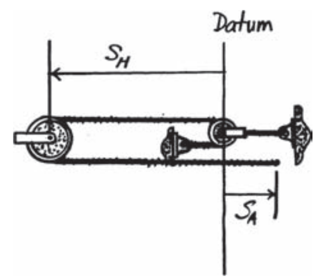
SOLUTION

$2s_H + s_A = l$

$2v_H = -v_A$

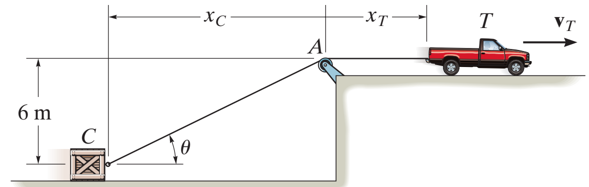
$2(1) = -v_A$

$v_A = -2 \text{ m/s} = 2 \text{ m/s} \leftarrow$ **Ans.**



Ans:
$v_A = 2 \text{ m/s} \leftarrow$

12–210. If the truck travels at a constant speed of $v_T = 1.8\text{ m/s}$, determine the speed of the crate for any angle θ of the rope. The rope has a length of 30 m and passes over a pulley of negligible size at A. *Hint:* Relate the coordinates x_T and x_C to the length of the rope and take the time derivative. Then substitute the trigonometric relation between x_C and θ.



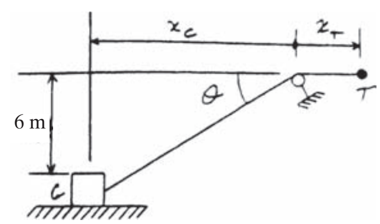
SOLUTION

$$\sqrt{(6)^2 + x_C^2} + x_T = l = 30$$

$$\frac{1}{2}\left((6)^2 + (x_C)^2\right)^{-\frac{1}{2}}\left(2x_C\dot{x}_C\right) + \dot{x}_T = 0$$

Since $\dot{x}_T = v_T = 1.8\text{ m/s}$, $v_c = \dot{x}_c$, and

$$x_C = 6\text{ ctn}\,\theta$$



Then,

$$\frac{(6\text{ ctn}\,\theta)v_C}{(36 + 36\text{ ctn}^2\,\theta)^{\frac{1}{2}}} = -1.8$$

Since $1 + \text{ctn}^2\,\theta = \csc^2\theta$,

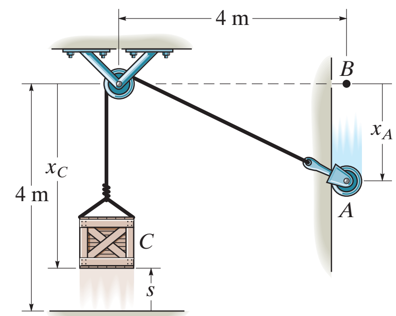
$$\left(\frac{\text{ctn}\,\theta}{\csc\theta}\right)v_C = \cos\theta v_C = -1.8$$

$$v_C = -1.8\sec\theta = (1.8\sec\theta)\text{ m/s} \rightarrow$$ **Ans.**

Ans:
$v_C = (1.8\sec\theta)\text{ m/s} \rightarrow$

12–211.

The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2\text{ m/s}$ along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1\text{ m}$. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.

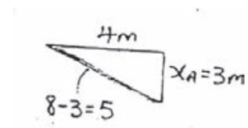


SOLUTION

$$x_C + \sqrt{x_A^2 + (4)^2} = l$$

$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$



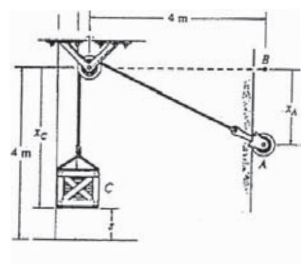
$l = 8\text{ m}$, and when $s = 1\text{ m}$,

$x_C = 3\text{ m}$

$x_A = 3\text{ m}$

$v_A = \dot{x}_A = 2\text{ m/s}$

$a_A = \ddot{x}_A = 0$



Thus,

$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$

$v_C = -1.2\text{ m/s} = 1.2\text{ m/s}\uparrow$ **Ans.**

$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$

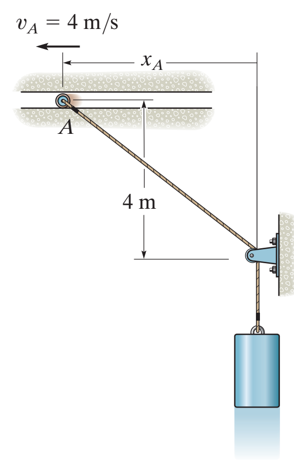
$a_C = -0.512\text{ m/s}^2 = 0.512\text{ m/s}^2\uparrow$ **Ans.**

Ans:

$v_C = 1.2\text{ m/s}\uparrow$
$a_C = 0.512\text{ m/s}^2\uparrow$

***12–212.**

The roller at A is moving with a velocity of $v_A = 4\text{ m/s}$ and has an acceleration of $a_A = 2\text{ m/s}^2$ when $x_A = 3\text{ m}$. Determine the velocity and acceleration of block B at this instant.



SOLUTION

Position Coordinates. The position of roller A and block B are specified by position coordinates x_A and y_B respectively as shown in Fig. a. We can relate these two position coordinates by considering the length of the cable, which is constant

$$\sqrt{x_A^2 + 4^2} + y_B = l$$

$$y_B = l - \sqrt{x_A^2 + 16} \qquad \textbf{(1)}$$

Velocity. Taking the time derivative of Eq. (1) using the chain rule,

$$\frac{dy_B}{dt} = 0 - \frac{1}{2}\left(x_A^2 + 16\right)^{-\frac{1}{2}}(2x_A)\frac{dx_A}{dt}$$

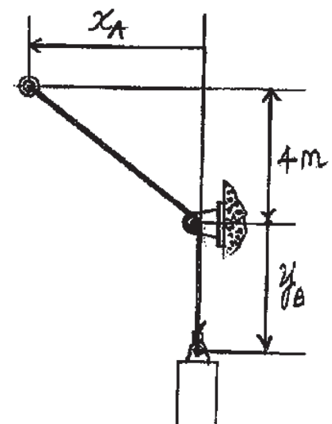
$$\frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 16}}\frac{dx_A}{dt}$$

However, $\dfrac{dy_B}{dt} = v_B$ and $\dfrac{dx_A}{dt} = v_A$. Then

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 16}}v_A \qquad \textbf{(2)}$$

At $x_A = 3\text{ m}$, $v_A = +4\text{ m/s}$ since $\mathbf{v}_A$ is directed toward the positive sense of x_A. Then Eq. (2) give

$$v_B = -\frac{3}{\sqrt{3^2 + 16}}(4) = -2.40\text{ m/s} = 2.40\text{ m/s}\uparrow \qquad \textbf{Ans.}$$



(a)

The negative sign indicates that $\mathbf{v}_B$ is directed toward the negative sense of y_B.

Acceleration. Taking the time derivative of Eq. (2),

$$\frac{dv_B}{dt} = -\left[x_A\left(-\frac{1}{2}\right)(x_A^2 + 16)^{-3/2}(2x_A)\frac{dx_A}{dt} + (x_A^2 + 16)^{-1/2}\frac{dx_A}{dt}\right]$$

$$v_A - x_A(x_A^2 + 16)^{-1/2}\frac{dv_A}{dt}$$

However, $\dfrac{dv_B}{dt} = a_B$, $\dfrac{dv_A}{dt} = a_A$ and $\dfrac{dx_A}{dt} = v_A$. Then

$$a_B = \frac{x_A^2 v_A^2}{\left(x_A^2 + 16\right)^{3/2}} - \frac{v_A^2}{\left(x_A^2 + 16\right)^{1/2}} - \frac{x_A a_A}{\left(x_A^2 + 16\right)^{1/2}}$$

$$a_B = -\frac{16\,v_A^2 + a_A x_A\left(x_A^2 + 16\right)}{\left(x_A^2 + 16\right)^{3/2}}$$

At $x_A = 3\text{ m}$, $v_A = +4\text{ m/s}$, $a_A = +2\text{ m/s}^2$ since $\mathbf{v}_A$ and a_A are directed toward the positive sense of x_A.

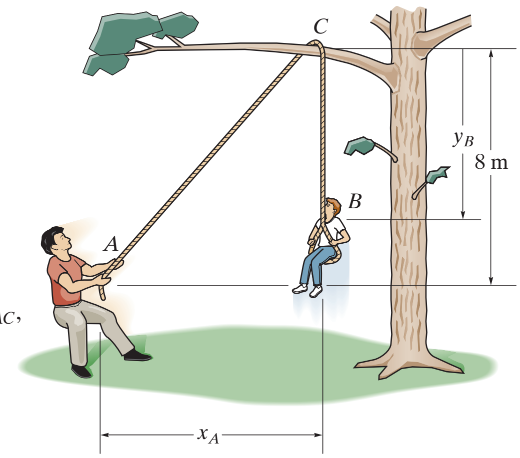
$$a_B = -\frac{16\left(4^2\right) + 2(3)\left(3^2 + 16\right)}{\left(3^2 + 16\right)^{3/2}} = -3.248\text{ m/s}^2 = 3.25\text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{a}_B$ is directed toward the negative sense of y_B.

Ans:
$v_B = 2.40\text{ m/s}\uparrow$
$a_B = 3.25\text{ m/s}^2\uparrow$

12–213.

The man pulls the boy up to the tree limb C by walking backward at a constant speed of 1.5 m/s. Determine the speed at which the boy is being lifted at the instant $x_A = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) and realizing that $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

At the instant $x_A = 4$ m, from Eq. [2]

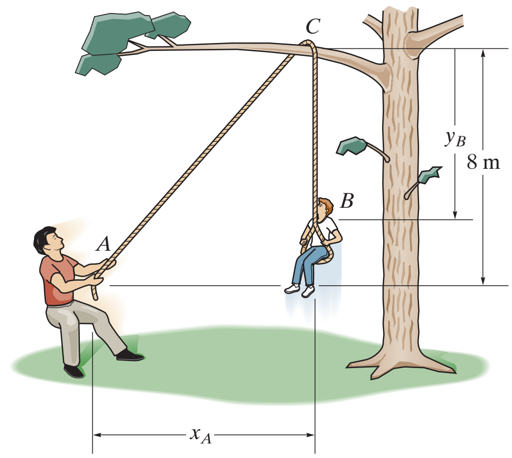
$$v_B = -\frac{4}{\sqrt{4^2 + 64}}(1.5) = -0.671 \text{ m/s} = 0.671 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

Ans:
$v_B = 0.671 \text{ m/s} \uparrow$

12–214.

The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) Where $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

At the instant $y_B = 4$ m, from Eq. (1), $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

$$v_A^2 = (v_0)_A^2 + 2(a_c)_A\left[s_A - (s_0)_A\right]$$

$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$

$$v_A = 1.891\ \text{m/s}$$

Substitute the above results into Eq. (2) yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41\ \text{m/s} = 1.41\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$

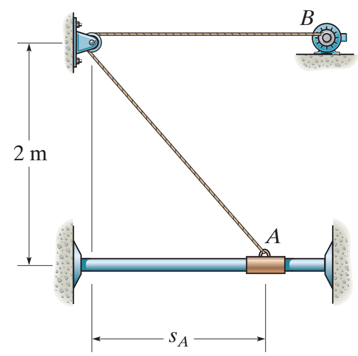
Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

Ans:

$v_B = 1.41\ \text{m/s}\uparrow$

12–215.

The motor draws in the cord at B with an acceleration of $a_B = 2\text{ m/s}^2$. When $s_A = 1.5$ m, $v_B = 6$ m/s. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

Position Coordinates. The position of collar A and point B are specified by s_A and s_C respectively as shown in Fig. *a*. We can relate these two position coordinates by considering the length of the cable, which is constant.

$$s_B + \sqrt{s_A^2 + 2^2} = l$$

$$s_B = l - \sqrt{s_A^2 + 4} \qquad \textbf{(1)}$$

Velocity. Taking the time derivative of Eq. (1),

$$\frac{ds_B}{dt} = 0 - \frac{1}{2}\left(s_A^2 + 4\right)^{-1/2}\left(2s_A\frac{ds_A}{dt}\right)$$

$$\frac{ds_B}{dt} = -\frac{s_A}{\sqrt{s_A^2 + 4}}\frac{ds_A}{dt}$$

However, $\dfrac{ds_B}{dt} = v_B$ and $\dfrac{ds_A}{dt} = v_A$. Then this equation becomes

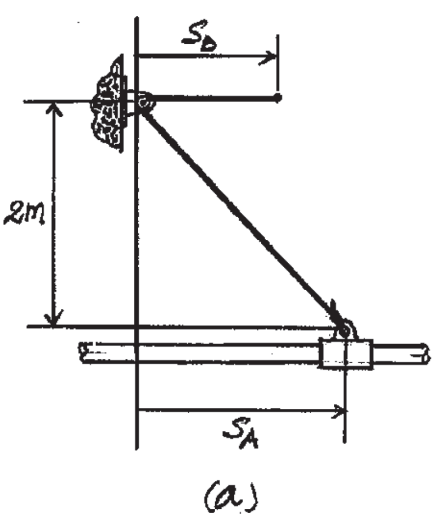
$$v_B = -\frac{s_A}{\sqrt{s_A^2 + 4}}v_A \qquad \textbf{(2)}$$

At the instant $s_A = 1.5$ m, $v_B = +6$ m/s. v_B is positive since it is directed toward the positive sense of s_B.

$$6 = -\frac{1.5}{\sqrt{1.5^2 + 4}}v_A$$

$$v_A = -10.0\text{ m/s} = 10.0\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that v_A is directed toward the negative sense of s_A.



Acceleration. Taking the time derivative of Eq. (2),

$$\frac{dv_B}{dt} = -\left[s_A\left(-\frac{1}{2}\right)\left(s_A^2 + 4\right)^{-3/2}\left(2s_A\frac{ds_A}{dt}\right) + \left(s_A^2 + 4\right)^{-1/2}\frac{ds_A}{dt}\right]$$

$$v_A - s_A\left(s_A^2 + 4\right)^{-1/2}\frac{dv_A}{dt}$$

However, $\dfrac{dv_B}{dt} = a_B$, $\dfrac{dv_A}{dt} = a_A$ and $\dfrac{ds_A}{dt} = v_A$. Then

$$a_B = \frac{s_A^2 v_A^2}{\left(s_A^2 + 4\right)^{3/2}} - \frac{v_A^2}{\left(s_A^2 + 4\right)^{1/2}} - \frac{a_A s_A}{\left(s_A^2 + 4\right)^{1/2}}$$

$$a_B = -\frac{4v_A^2 + a_A s_A\left(s_A^2 + 4\right)}{\left(s_A^2 + 4\right)^{3/2}}$$

At the instant $s_A = 1.5$ m, $a_B = +2\text{ m/s}^2$. $\mathbf{a}_B$ is positive since it is directed toward the positive sense of s_B. Also, $v_A = -10.0$ m/s. Then

$$2 = -\left[\frac{4(-10.0)^2 + a_A(1.5)(1.5^2 + 4)}{(1.5^2 + 4)^{3/2}}\right]$$

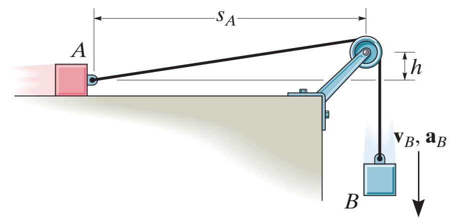
$$a_A = -46.0\text{ m/s}^2 = 46.0\text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{a}_A$ is directed toward the negative sense of s_A.

Ans:
$v_A = 10.0\text{ m/s} \leftarrow$
$a_A = 46.0\text{ m/s}^2 \leftarrow$

***12–216.**

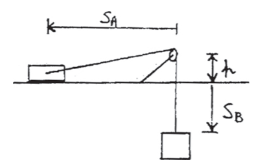
If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



SOLUTION

$$l = s_B + \sqrt{s_B^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}\, 2s_A\, \dot{s}_A$$



$$v_A = \dot{s}_A = \frac{-\,\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B\,(1 + \left(\frac{h}{s_A}\right)^2)^{1/2} \qquad \textbf{Ans.}$$

$$a_A = \dot{v}_A = \; -\,\dot{v}_B(1 + \left(\frac{h}{s_A}\right)^2)^{1/2} - v_B\left(\frac{1}{2}\right)(1 + \left(\frac{h}{s_A}\right)^2)^{-1/2}(h^2)(\,-\,2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -\,a_B(1+\left(\frac{h}{s_B}\right)^2)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\,(1+\left(\frac{h}{s_A}\right)^2)^{-1/2} \qquad \textbf{Ans.}$$

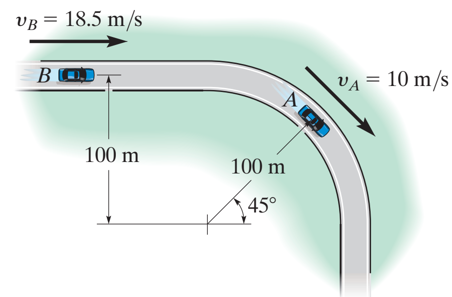
Ans:

$$v_A = -v_B\,(1+\left(\frac{h}{s_A}\right)^2)^{1/2}$$

$$a_A = -a_B\,(1+\left(\frac{h}{s_B}\right)^2)^{1/2} + \frac{v_A v_B h^2}{s_A^3}(1+\left(\frac{h}{S_A}\right)^2)^{-1/2}$$

12–217.

At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s^2. The car at B is traveling at 18.5 m/s along the straightaway and increasing its speed at 2 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$v_A = 10 \cos 45°\mathbf{i} - 10 \sin 45°\mathbf{j} = \{7.071\mathbf{i} - 7.071\mathbf{j}\}$ m/s

$v_B = \{18.5\mathbf{i}\}$ m/s

$v_{A/B} = v_A - v_B$

$= (7.071\mathbf{i} - 7.071\mathbf{j}) - 18.5\mathbf{i} = \{-11.429\mathbf{i} - 7.071\mathbf{j}\}$ m/s

$v_{A/B} = \sqrt{(-11.429)^2 + (-7.071)^2} = 13.4$ m/s **Ans.**

$\theta = \tan^{-1}\dfrac{7.071}{11.429} = 31.7°$ ⦣ **Ans.**

$(a_A)_n = \dfrac{v_A^2}{\rho} = \dfrac{10^2}{100} = 1$ m/s^2 $\qquad (a_A)_t = 5$ m/s^2

$\mathbf{a}_A = (5 \cos 45° - 1 \cos 45°)\mathbf{i} + (-1 \sin 45° - 5 \sin 45°)\mathbf{j}$

$= \{2.828\mathbf{i} - 4.243\mathbf{j}\}$ m/s^2

$\mathbf{a}_B = \{2\mathbf{i}\}$ m/s^2

$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{u}_B$

$= (2.828\mathbf{i} - 4.243\mathbf{j}) - 2\mathbf{i} = \{0.828\mathbf{i} - 4.24\mathbf{j}\}$ m/s^2

$a_{A/B} = \sqrt{0.828^2 + (-4.243)^2} = 4.32$ m/s^2 **Ans.**

$\theta = \tan^{-1}\dfrac{4.243}{0.828} = 79.0°$ ⦣ **Ans.**

Ans:

$v_{A/B} = 13.4$ m/s

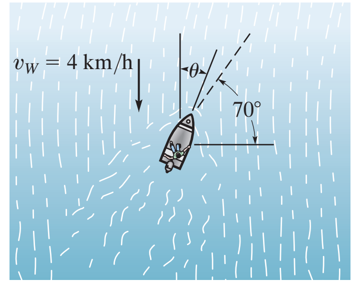
$\theta_v = 31.7°$ ⦣

$a_{A/B} = 4.32$ m/s^2

$\theta_a = 79.0°$ ⦣

12–218.

The boat can travel with a speed of 16 km/h in still water. The point of destination is located along the dashed line. If the water is moving at 4 km/h, determine the bearing angle θ at which the boat must travel to stay on course.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_W + \mathbf{v}_{B/W}$

$v_B \cos 70°\mathbf{i} + v_B \sin 70°\mathbf{j} = -4\mathbf{j} + 16 \sin \theta\mathbf{i} + 16 \cos \theta\mathbf{j}$

$(\xrightarrow{+})\qquad v_B \cos 70° = 0 + 16 \sin \theta$

$(+\uparrow)\qquad v_B \sin 70° = -4 + 16 \cos \theta$

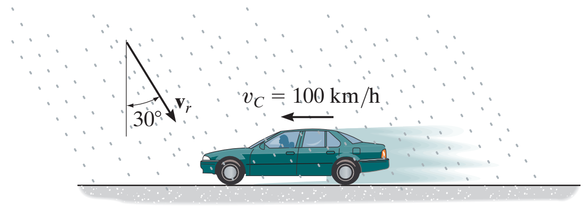
$2.748 \sin \theta - \cos \theta + 0.25 = 0$

Solving,

$\theta = 15.1°$ **Ans.**

Ans:
$\theta = 15.1°$

12-219. The car is traveling at a constant speed of 100 km/h. If the rain is falling at 6 m/s in the direction shown, determine the velocity of the rain as seen by the driver.



SOLUTION

Solution I

Vector Analysis: The speed of the car is $v_c = \left(100\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 27.78$ m/s. The velocity of the car and the rain expressed in Cartesian vector form are $\mathbf{v}_c = [-27.78\mathbf{i}]$ m/s and $\mathbf{v}_r = [6\sin 30°\mathbf{i} - 6\cos 30°\mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}]$ m/s. Applying the relative velocity equation, we have

$$\mathbf{v}_r = \mathbf{v}_c + \mathbf{v}_{r/c}$$

$$3\mathbf{i} - 5.196\mathbf{j} = -27.78\mathbf{i} + \mathbf{v}_{r/c}$$

$$\mathbf{v}_{r/c} = [30.78\mathbf{i} - 5.196\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{r/c}$ is given by

$$v_{r/c} = \sqrt{30.78^2 + (-5.196)^2} = 31.2\text{ m/s} \qquad \textbf{Ans.}$$

and the angle $x_{r/c}$ makes with the x axis is

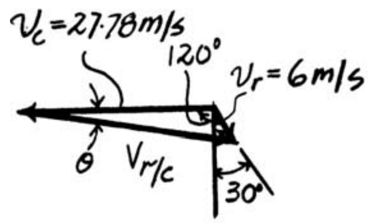
$$\theta = \tan^{-1}\left(\frac{5.196}{30.78}\right) = 9.58° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

$$v_{r/c} = \sqrt{27.78^2 + 6^2 - 2(27.78)(6)\cos 120°}$$

$$= 19.91\text{ m/s} = 19.9\text{ m/s} \qquad \textbf{Ans.}$$



(a)

Using the result of $v_{r/c}$ and applying the law of sines,

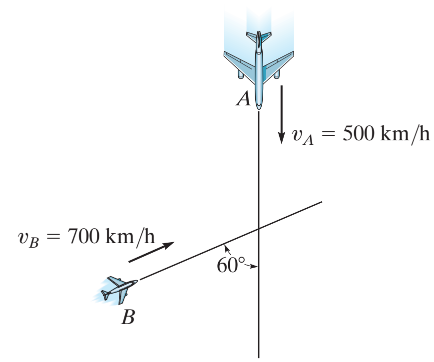
$$\frac{\sin\theta}{6} = \frac{\sin 120°}{31.21}$$

$$\theta = 9.58° \qquad \textbf{Ans.}$$

Ans:
$v_{r/c} = 31.2$ m/s
$\theta = 9.58°$
$v_{r/c} = 19.9$ m/s
$\theta = 9.58°$

***12–220.**

Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 500\text{ km/h}$ and $v_B = 700\text{ km/h}$ such that the angle between their straight-line courses is $\theta = 60°$, determine the velocity of plane B with respect to plane A.



SOLUTION

Relative Velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form,

$$\mathbf{v}_A = \{-500\,\mathbf{j}\}\text{ km/h}$$

$$\mathbf{v}_B = \{700\sin 60°\mathbf{i} + 700\cos 60°\mathbf{j}\}\text{ km/h} = \left\{350\sqrt{3}\mathbf{i} + 350\mathbf{j}\right\}\text{ km/h}$$

Applying the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$350\sqrt{3}\mathbf{i} + 350\mathbf{j} = -500\mathbf{j} + \mathbf{v}_{B/A}$$

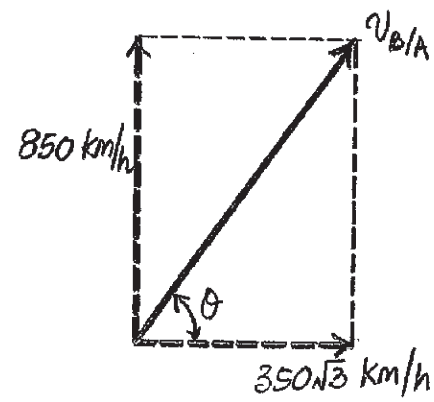
$$\mathbf{v}_{B/A} = \left\{350\sqrt{3}\mathbf{i} + 850\mathbf{j}\right\}\text{km/h}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(350\sqrt{3})^2 + 850^2} = 1044.03\text{ km/h} = 1044\text{ km/h} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. a.

$$\theta = \tan^{-1}\left(\frac{850}{350\sqrt{3}}\right) = 54.50° = 54.5° \measuredangle \qquad \textbf{Ans.}$$



(a)

Ans:
$v_{B/A} = 1044\text{ km/h}$
$\theta = 54.5° \measuredangle$

12–221.

A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is coming from the east. If the car's speed is 80 km/h, the instrument indicates that the wind is coming from the northeast. Determine the speed and direction of the wind.

SOLUTION

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_1\,\mathbf{i}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 50\mathbf{j} + (v_{w/c})_1\,\mathbf{i}$$

$$\mathbf{v}_w = (v_{w/c})_1\mathbf{i} + 50\mathbf{j} \qquad \textbf{(1)}$$

For the second case, $v_C = [80\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_2 \cos 45°\mathbf{i} + (v_{W/C})_2 \sin 45°\,\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 80\mathbf{j} + (v_{w/c})_2 \cos 45°\mathbf{i} + (v_{w/c})_2 \sin 45°\,\mathbf{j}$$

$$\mathbf{v}_w = (v_{w/c})_2 \cos 45°\,\mathbf{i} + \left[80 + (v_{w/c})_2 \sin 45°\right]\mathbf{j} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) and then the **i** and **j** components,

$$(v_{w/c})_1 = (v_{w/c})_2 \cos 45° \qquad \textbf{(3)}$$

$$50 = 80 + (v_{w/c})_2 \sin 45° \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43 \text{ km/h} \qquad (v_{w/c})_1 = -30 \text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}] \text{ km/h}$$

Thus, the magnitude of $\mathbf{v}_W$ is

$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ km/h} \qquad \textbf{Ans.}$$

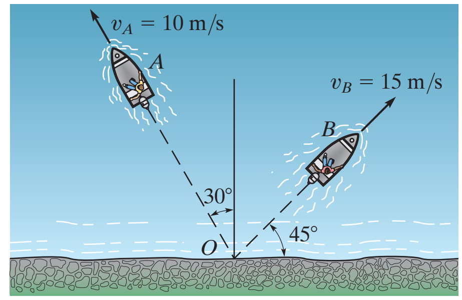
and the directional angle θ that $\mathbf{v}_W$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{50}{30}\right) = 59.0° \measuredangle \qquad \textbf{Ans.}$$

Ans:
$v_w = 58.3$ km/h
$\theta = 59.0° \measuredangle$

12–222.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 10$ m/s and $v_B = 15$ m/s, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 600 m apart?



SOLUTION

Relative Velocity. The velocity triangle shown in Fig. *a* is drawn based on the relative velocity equation $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$. Using the cosine law,

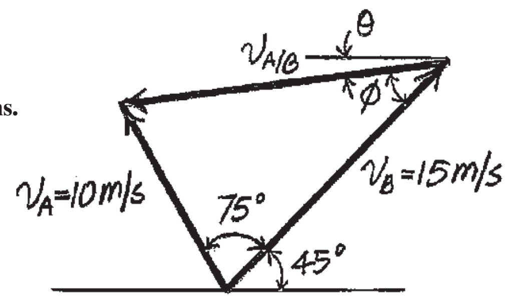
$$v_{A/B} = \sqrt{10^2 + 15^2 - 2(10)(15)\cos 75^\circ} = 15.73 \text{ m/s} = 15.7 \text{ m/s} \qquad \textbf{Ans.}$$

Then, the sine law gives

$$\frac{\sin\phi}{10} = \frac{\sin 75^\circ}{15.73} \qquad \phi = 37.89^\circ$$

The direction of $\mathbf{v}_{A/B}$ is defined by

$$\theta = 45^\circ - \phi = 45^\circ - 37.89^\circ = 7.11^\circ \ \measuredangle \qquad \textbf{Ans.}$$



(a)

Alternatively, we can express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form

$$\mathbf{v}_A = \{-10\sin 30^\circ\mathbf{i} + 10\cos 30^\circ\mathbf{j}\} \text{ m/s} = \{-5.00\mathbf{i} + 5\sqrt{3}\mathbf{j}\} \text{ m/s}$$

$$\mathbf{v}_B = \{15\cos 45^\circ\mathbf{i} + 15\sin 45^\circ\mathbf{j}\} \text{ m/s} = \{7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j}\} \text{ m/s}.$$

Applying the relative velocity equation

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-500\mathbf{i} + 5\sqrt{3}\mathbf{j} = 7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-15.61\mathbf{i} - 1.946\mathbf{j}\} \text{ m/s}$$

Thus the magnitude of $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-15.61)^2 + (-1.946)^2} = 15.73 \text{ m/s} = 15.7 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. *b*,

$$\theta = \tan^{-1}\left(\frac{1.946}{15.61}\right) = 7.1088^\circ = 7.11^\circ \ \measuredangle \qquad \textbf{Ans.}$$

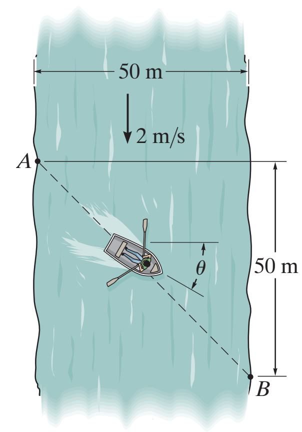
(b)

Here $s_{A/B} = 600$ m. Thus

$$t = \frac{s_{A/B}}{v_{A/B}} = \frac{600}{15.73} = 38.15 \text{ s} = 38.1 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_{A/B} = 15.7$ m/s
$\theta = 7.11^\circ \ \measuredangle$
$t = 38.1$ s

12-223. A man can row a boat at 5 m/s in still water. He wishes to cross a 50-m-wide river to point B, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the boat and the time needed to make the crossing.



SOLUTION

Relative Velocity:

$$\mathbf{v}_b = \mathbf{v}_r + \mathbf{v}_{b/r}$$

$$v_b \sin 45°\mathbf{i} - v_b \cos 45°\mathbf{j} = -2\mathbf{j} + 5\cos\theta\mathbf{i} - 5\sin\theta\mathbf{j}$$

Equating **i** and **j** component, we have

$$v_b \sin 45° = 5\cos\theta \qquad \textbf{[1]}$$

$$-v_b \cos 45° = -2 - 5\sin\theta \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$\theta = 28.57°$$

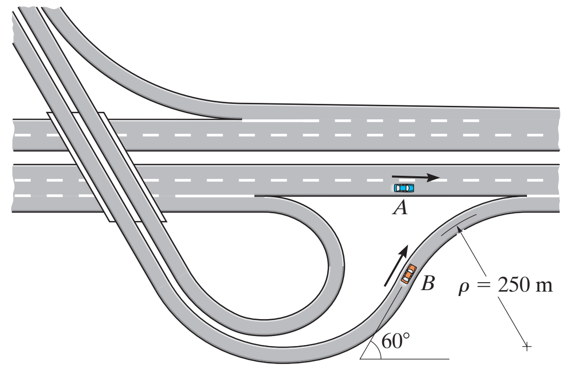
$$v_b = 6.210\text{ m/s} = 6.21\text{ m/s} \qquad \textbf{Ans.}$$

Thus, the time t required by the boat to travel from point A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{50^2 + 50^2}}{6.210} = 11.4\text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_b = 6.21$ m/s
$t = 11.4$ s

***12–224.** At the instant shown car A is traveling with a velocity of 30 m/s and has an acceleration of 2 m/s^2 along the highway. At the same instant B is traveling on the trumpet interchange curve with a speed of 15 m/s, which is decreasing at 0.8 m/s^2. Determine the relative velocity and relative acceleration of B with respect to A at this instant.



SOLUTION

$$\mathbf{v_{Av}} = \begin{pmatrix} v_A \\ 0 \end{pmatrix} \qquad \mathbf{a_{Av}} = \begin{pmatrix} a_A \\ 0 \end{pmatrix}$$

$$\mathbf{v_{Bv}} = v_B\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \qquad \mathbf{a_{Bv}} = v'_B\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} + \frac{v_B^{\,2}}{\rho}\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \end{pmatrix}$$

$v_A = 30 \text{ m/s}$

$v_B = 15 \text{ m/s}$

$a_A = 2 \text{ m/s}^2$

$v'_B = -0.8 \text{ m/s}^2$

$\rho = 250 \text{ m}$

$\theta = 60°$

$$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}} \qquad \mathbf{v_{BA}} = \begin{pmatrix} -22.5 \\ 12.99 \end{pmatrix} \text{ m/s} \qquad \left|\mathbf{v_{BA}}\right| = 26.0 \qquad \textbf{Ans.}$$

$$\mathbf{a_{BA}} = \mathbf{a_{Bv}} - \mathbf{a_{Av}} \qquad \mathbf{a_{BA}} = \begin{pmatrix} -1.621 \\ -1.143 \end{pmatrix} \text{m/s}^2 \qquad \left|\mathbf{a_{BA}}\right| = 1.983 \text{ m/s}^2 \qquad \textbf{Ans.}$$

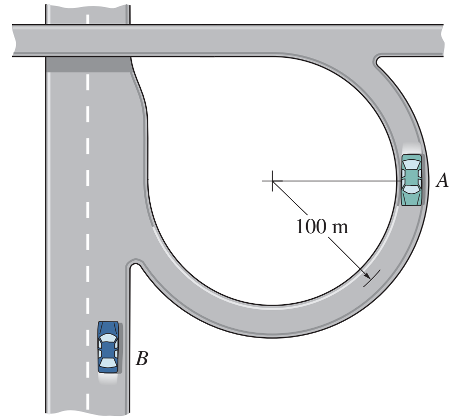
Ans:

$\left|\mathbf{v_{BA}}\right| = 26.0$

$\left|\mathbf{a_{BA}}\right| = 1.983 \text{ m/s}^2$

12–225.

At the instant shown, car A has a speed of 20 km/h, which is being increased at the rate of 300 km/h^2 as the car enters an expressway. At the same instant, car B is decelerating at 250 km/h^2 while traveling forward at 100 km/h. Determine the velocity and acceleration of A with respect to B.



SOLUTION

$\mathbf{v}_A = \{-20\mathbf{j}\}$ km/h $\qquad \mathbf{v}_B = \{100\mathbf{j}\}$ km/h

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (-20\mathbf{j} - 100\mathbf{j}) = \{-120\mathbf{j}\} \text{ km/h}$$

$$v_{A/B} = 120 \text{ km/h} \downarrow$$ **Ans.**

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{20^2}{0.1} = 4000 \text{ km/h}^2 \qquad (a_A)_t = 300 \text{ km/h}^2$$

$$\mathbf{a}_A = -4000\mathbf{i} + (-300\mathbf{j})$$

$$= \{-4000\mathbf{i} - 300\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_B = \{-250\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$$

$$= (-4000\mathbf{i} - 300\mathbf{j}) - (-250\mathbf{j}) = \{-4000\mathbf{i} - 50\mathbf{j}\} \text{ km/h}^2$$

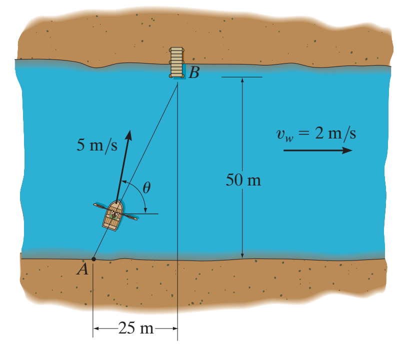
$$a_{A/B} = \sqrt{(-4000)^2 + (-50)^2} = 4000 \text{ km/h}^2$$ **Ans.**

$$\theta = \tan^{-1}\frac{50}{4000} = 0.716^\circ \measuredangle$$ **Ans.**

Ans:
$v_{A/B} = 120$ km/h $\downarrow$
$a_{A/B} = 4000$ km/h^2
$\theta = 0.716^\circ \measuredangle$

12–226.

The man can row the boat in still water with a speed of 5 m/s. If the river is flowing at 2 m/s, determine the speed of the boat and the angle θ he must direct the boat so that it travels from A to B.



SOLUTION

Solution I

Vector Analysis: Here, the velocity $\mathbf{v}_b$ of the boat is directed from A to B. Thus, $\phi = \tan^{-1}\left(\frac{50}{25}\right) = 63.43°$. The magnitude of the boat's velocity relative to the flowing river is $v_{b/w} = 5$ m/s. Expressing $\mathbf{v}_b$, $\mathbf{v}_w$, and $\mathbf{v}_{b/w}$ in Cartesian vector form, we have $\mathbf{v}_b = v_b \cos 63.43\mathbf{i} + v_b \sin 63.43\mathbf{j} = 0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j}$, $\mathbf{v}_w = [2\mathbf{i}]$ m/s, and $\mathbf{v}_{b/w} = 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_b = \mathbf{v}_w + \mathbf{v}_{b/w}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = 2\mathbf{i} + 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = (2 + 5\cos\theta)\mathbf{i} + 5\sin\theta\mathbf{j}$$

Equating the **i** and **j** components, we have

$$0.4472v_b = 2 + 5\cos\theta \qquad \textbf{(1)}$$

$$0.8944v_b = 5\sin\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_b = 5.56 \text{ m/s} \qquad \theta = 84.4° \qquad \textbf{Ans.}$$

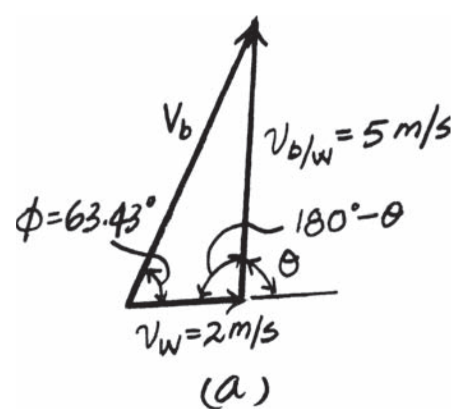
Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

$$5^2 = 2^2 + v_b{}^2 - 2(2)(v_b)\cos 63.43°$$

$$v_b{}^2 - 1.789v_b - 21 = 0$$

$$v_b = \frac{-(-1.789) \pm \sqrt{(-1.789)^2 - 4(1)(-21)}}{2(1)}$$



Choosing the positive root,

$$v_b = 5.563 \text{ m/s} = 5.56 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of v_b and applying the law of sines,

$$\frac{\sin(180° - \theta)}{5.563} = \frac{\sin 63.43°}{5}$$

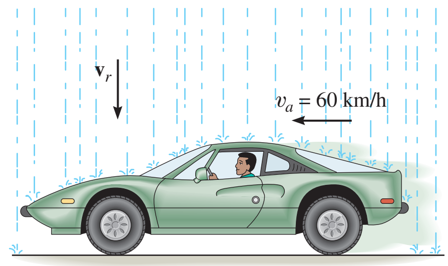
$$\theta = 84.4° \qquad \textbf{Ans.}$$

Ans:

$v_b = 5.56$ m/s
$\theta = 84.4°$

12–227.

A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant) velocity $\mathbf{v}_r$ of the rain if it is assumed to fall vertically.



SOLUTION

$v_r = v_a + v_{r/a}$

$-v_r\mathbf{j} = -60\mathbf{i} + v_{r/a}\cos 30°\mathbf{i} - v_{r/a}\sin 30°\mathbf{j}$

$(\overset{+}{\rightarrow})$ $\quad 0 = -60 + v_{r/a}\cos 30°$

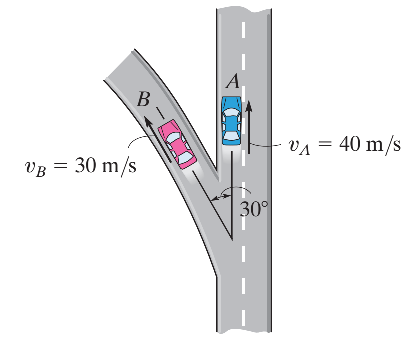
$(+\uparrow)$ $\quad -v_r = 0 - v_{r/a}\sin 30°$

$v_{r/a} = 69.3$ km/h

$v_r = 34.6$ km/h **Ans.**

Ans:
$v_r = 34.6\ \text{km/h}\downarrow$

***12–228.** At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If B is increasing its velocity by 2 m/s^2, while A maintains a constant velocity, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



SOLUTION

Relative velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ as Cartesian vectors.

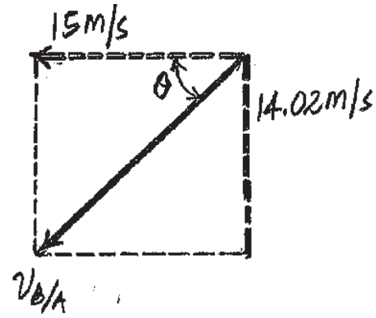
$\mathbf{v}_A = \{40\mathbf{j}\}$ m/s $\quad \mathbf{v}_B = \{-30 \sin 30°\mathbf{i} + 30 \cos 30°\mathbf{j}\}$ m/s $= \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\}$ m/s

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{-15\mathbf{i} - 14.02\mathbf{j}\} \text{ m/s}$$



(a)

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53 \text{ m/s} = 20.5 \text{ m/s} \qquad \textbf{Ans.}$$

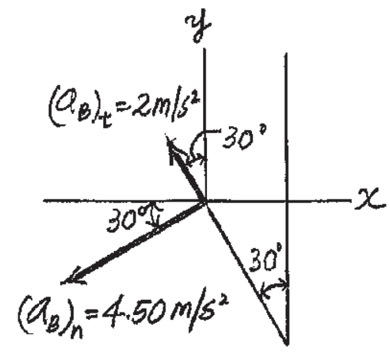
And its direction is defined by angle θ, Fig. a

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06° = 43.1° \qquad \textbf{Ans.}$$

Relative Acceleration. Here, $(a_B)_t = 2$ m/s^2 and $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{30^2}{200} = 4.50$ m/s^2 and their directions are shown in Fig. b. Then, express $\mathbf{a}_B$ as a Cartesian vector,

$$\mathbf{a}_B = (-2 \sin 30° - 4.50 \cos 30°)\mathbf{i} + (2 \cos 30° - 4.50 \sin 30°)\mathbf{j}$$

$$= \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\} \text{ m/s}^2$$



(b)

Applying the relative acceleration equation with $\mathbf{a}_A = \mathbf{0}$,

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-4.8971\mathbf{i} - 0.5179\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

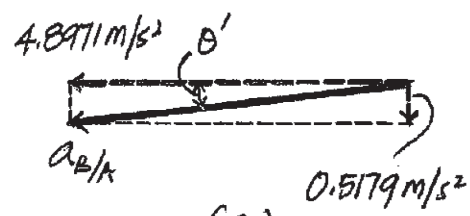
$$\mathbf{a}_{B/A} = \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\} \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$\mathbf{a}_{B/A} = \sqrt{(-4.8971)^2 + (-0.5179)^2} = 4.9244 \text{ m/s}^2 = 4.92 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by angle θ', Fig. c,

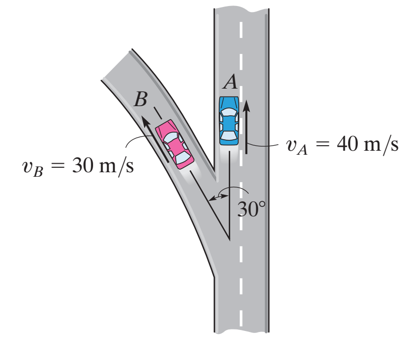
$$\theta' = \tan^{-1}\left(\frac{0.5179}{4.8971}\right) = 6.038° = 6.04° \qquad \textbf{Ans.}$$



(c)

Ans:
$v_{B/A} = 20.5$ m/s
$\theta_v = 43.1°$
$a_{B/A} = 4.92$ m/s^2
$\theta_a = 6.04°$

12–229. At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If A is increasing its speed at 4 m/s^2, whereas the speed of B is decreasing at 3 m/s^2, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



SOLUTION

Relative velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ as Cartesian vectors.

$$\mathbf{v}_A = \{40\mathbf{j}\}\ \text{m/s} \quad \mathbf{v}_B = \{-30 \sin 30°\mathbf{i} + 30 \cos 30°\mathbf{j}\}\ \text{m/s} = \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\}\ \text{m/s}$$

Applying the relative velocity equation,

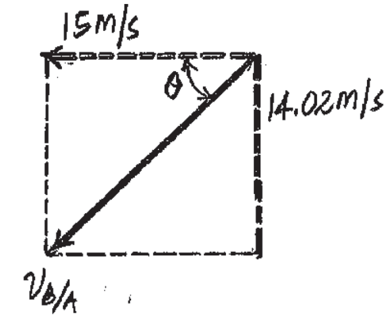
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{-15\mathbf{i} - 14.02\mathbf{j}\}\ \text{m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53\ \text{m/s} = 20.5\ \text{m/s} \qquad \textbf{Ans.}$$

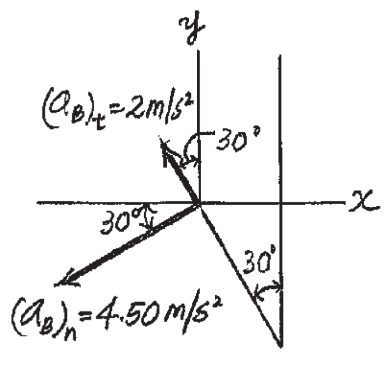


(a)

And its direction is defined by angle θ, Fig. *a*

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06° = 43.1° \measuredangle \qquad \textbf{Ans.}$$

Relative Acceleration. Here, $(a_B)_t = 2\ \text{m/s}^2$ and $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{30^2}{200} = 4.50\ \text{m/s}^2$ and their directions are shown in Fig. *b*. Then, express $\mathbf{a}_B$ as a Cartesian vector,



(b)

$$\mathbf{a}_B = (-2 \sin 30° - 4.50 \cos 30°)\mathbf{i} + (2 \cos 30° - 4.50 \sin 30°)\mathbf{j}$$

$$= \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\}\ \text{m/s}^2$$

Applying the relative acceleration equation with $\mathbf{a}_A = \mathbf{0}$,

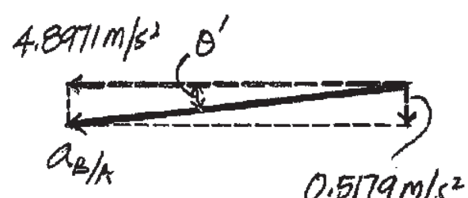
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-4.8971\mathbf{i} - 0.5179\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\}\ \text{m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$\mathbf{a}_{B/A} = \sqrt{(-4.8971)^2 + (-0.5179)^2} = 4.9244\ \text{m/s}^2 = 4.92\ \text{m/s}^2 \qquad \textbf{Ans.}$$



(c)

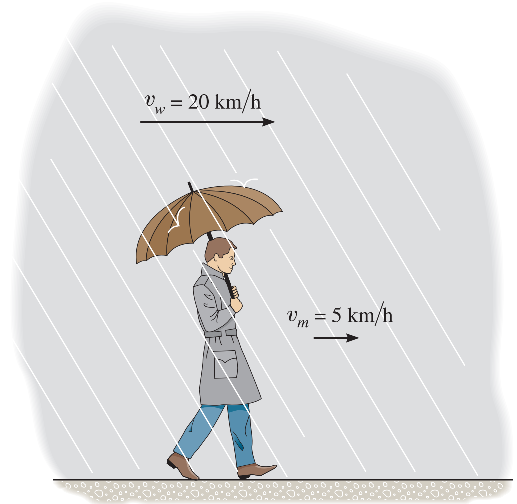
And its direction is defined by angle θ', Fig. *c*,

$$\theta' = \tan^{-1}\left(\frac{0.5179}{4.8971}\right) = 6.038° = 6.04° \measuredangle \qquad \textbf{Ans.}$$

Ans:
$v_{B/A} = 20.5$ m/s
$\theta_v = 43.1° \measuredangle$
$a_{B/A} = 4.92$ m/s^2
$\theta_a = 6.04° \measuredangle$

12–230.

A man walks at 5 km/h in the direction of a 20-km/h wind. If raindrops fall vertically at 7 km/h in *still air*, determine the direction in which the drops appear to fall with respect to the man.



SOLUTION

Relative Velocity: The velocity of the rain must be determined first. Applying Eq. 12–34 gives

$$\mathbf{v}_r = \mathbf{v}_w + \mathbf{v}_{r/w} = 20\,\mathbf{i} + (-7\,\mathbf{j}) = \{20\,\mathbf{i} - 7\,\mathbf{j}\}\ \text{km/h}$$

Thus, the relatives velocity of the rain with respect to the man is

$$\mathbf{v}_r = \mathbf{v}_m + \mathbf{v}_{r/m}$$

$$20\,\mathbf{i} - 7\,\mathbf{j} = 5\,\mathbf{i} + \mathbf{v}_{r/m}$$

$$\mathbf{v}_{r/m} = \{15\,\mathbf{i} - 7\,\mathbf{j}\}\ \text{km/h}$$

The magnitude of the relative velocity $\mathbf{v}_{r/m}$ is given by

$$v_{r/m} = \sqrt{15^2 + (-7)^2} = 16.6\ \text{km/h} \qquad \textbf{Ans.}$$

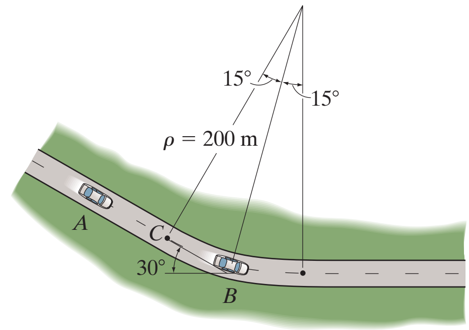
And its direction is given by

$$\theta = \tan^{-1}\frac{7}{15} = 25.0° \qquad \textbf{Ans.}$$

Ans:
$v_{r/m} = 16.6$ km/h
$\theta = 25.0°$

12–231.

At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.

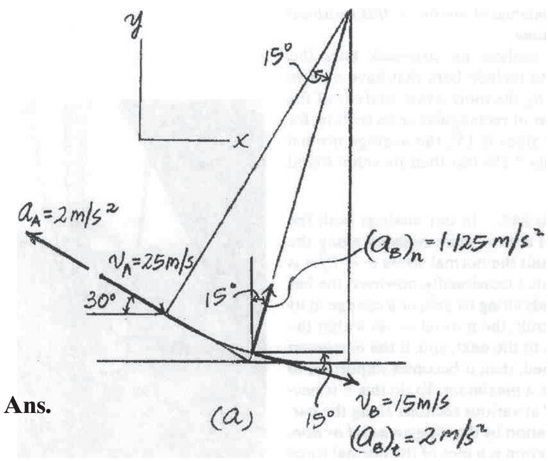


SOLUTION

Velocity: Referring to Fig. a, the velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [25\cos 30°\,\mathbf{i} - 25\sin 30°\,\mathbf{j}]\text{ m/s} = [21.65\mathbf{i} - 12.5\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_B = [15\cos 15°\,\mathbf{i} - 15\sin 15°\,\mathbf{j}]\text{ m/s} = [14.49\mathbf{i} - 3.882\mathbf{j}]\text{ m/s}$$



Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49\mathbf{i} - 3.882\mathbf{j} = 21.65\mathbf{i} - 12.5\mathbf{j} + \mathbf{v}_{B/A}$$

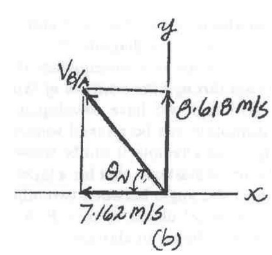
$$\mathbf{v}_{B/A} = [-7.162\mathbf{i} + 8.618\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2\text{ m/s} \qquad \textbf{Ans.}$$

The direction angle θ_v of $\mathbf{v}_{B/A}$ measured down from the negative x axis, Fig. b is

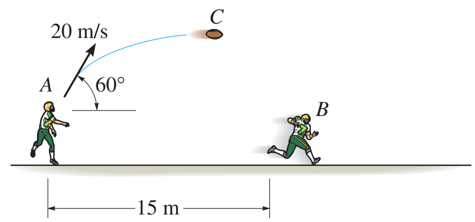
$$\theta_v = \tan^{-1}\left(\frac{8.618}{7.162}\right) = 50.3° \qquad \textbf{Ans.}$$



Ans:
$v_{B/A} = 11.2$ m/s
$\theta = 50.3°$

***12–232.**

At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



SOLUTION

Ball:

$(\xrightarrow{+})\, s = s_0 + v_0 t$

$s_C = 0 + 20 \cos 60° \, t$

$(+\uparrow) \quad v = v_0 + a_c t$

$-20 \sin 60° = 20 \sin 60° - 9.81\, t$

$t = 3.53\ s$

$s_C = 35.31$ m

Player B:

$(\xrightarrow{+})\ s_B = s_0 + \nu_B t$

Require,

$35.31 = 15 + v_B(3.53)$

$v_B = 5.75$ m/s **Ans.**

At the time of the catch

$(v_C)_x = 20 \cos 60° = 10 \text{ m/s} \rightarrow$

$(v_C)_y = 20 \sin 60° = 17.32 \text{ m/s} \downarrow$

$v_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$10\mathbf{i} - 17.32\mathbf{j} = 5.751\mathbf{i} + (v_{C/B})_x \mathbf{i} + (v_{C/B})_y \mathbf{j}$

$(\xrightarrow{+}) \quad 10 = 5.75 + (v_{C/B})_x$

$(+\uparrow) \quad -17.32 = (v_{C/B})_y$

$(v_{C/B})_x = 4.25 \text{ m/s} \rightarrow$

$(v_{C/B})_y = 17.32 \text{ m/s} \downarrow$

$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{17.32}{4.25}\right) = 76.2°$ **Ans.**

$a_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

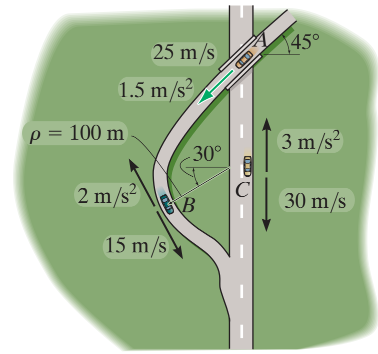
$-9.81\,\mathbf{j} = 0 + \mathbf{a}_{C/B}$

$a_{C/B} = 9.81 \text{ m/s}^2 \downarrow$ **Ans.**

Ans:
$v_B = 5.75$ m/s
$v_{C/B} = 17.8$ m/s
$\theta = 76.2°$
$a_{C/B} = 9.81 \text{ m/s}^2 \downarrow$

12–233.

Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car A relative to car C.



SOLUTION

Velocity: The velocity of cars A and C expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25\cos 45°\mathbf{i} - 25\sin 45°\mathbf{j}] \text{ m/s} = [-17.68\mathbf{i} - 17.68\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68\mathbf{i} - 17.68\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68\mathbf{i} + 12.32\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9° \qquad \textbf{Ans.}$$

Acceleration: The acceleration of cars A and C expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5\cos 45°\mathbf{i} - 1.5\sin 45°\mathbf{j}] \text{ m/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}] \text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}] \text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061\mathbf{i} - 1.061\mathbf{j} = 3\mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061\mathbf{i} - 4.061\mathbf{j}] \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20 \text{ m/s}^2 \qquad \textbf{Ans.}$$

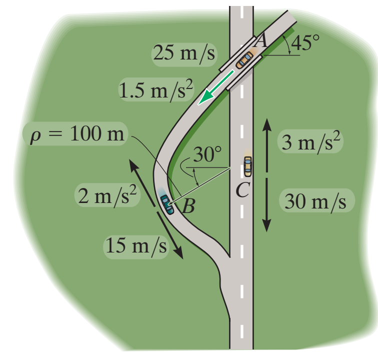
and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4° \qquad \textbf{Ans.}$$

Ans:
$v_{A/C} = 21.5$ m/s
$\theta_v = 34.9°$
$a_{A/C} = 4.20$ m/s^2
$\theta_a = 75.4°$

12–234.

Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car B relative to car C.



SOLUTION

Velocity: The velocity of cars B and C expressed in Cartesian vector form are

$$\mathbf{v}_B = [15 \cos 60° \,\mathbf{i} - 15 \sin 60° \,\mathbf{j}] \text{ m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{B/C} = \sqrt{7.5^2 + 17.01^2} = 18.6 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2° \qquad \textbf{Ans.}$$

Acceleration: The normal component of car B's acceleration is $(a_B)_n = \dfrac{{v_B}^2}{\rho} = \dfrac{15^2}{100} = 2.25 \text{ m/s}^2$. Thus, the tangential and normal components of car B's acceleration and the acceleration of car C expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2 \cos 60° \,\mathbf{i} + 2 \sin 60° \mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}] \text{ m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25 \cos 30° \,\mathbf{i} + 2.25 \sin 30° \,\mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}] \text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}] \text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}] \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959 \text{ m/s}^2 \qquad \textbf{Ans.}$$

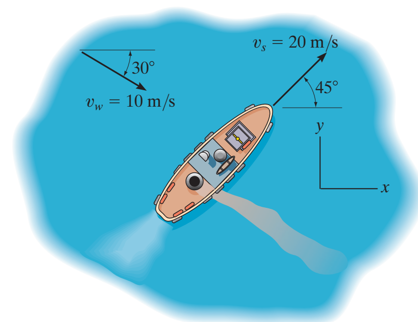
and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57° \qquad \textbf{Ans.}$$

Ans:
$v_{B/C} = 18.6$ m/s
$\theta_v = 66.2°$
$a_{B/C} = 0.959$ m/s^2
$\theta_a = 8.57°$

12–235.

The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



SOLUTION

Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20\cos 45°\,\mathbf{i} + 20\sin 45°\,\mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10\cos 30°\,\mathbf{i} - 10\sin 30°\,\mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9 \text{ m/s}$$ **Ans.**

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

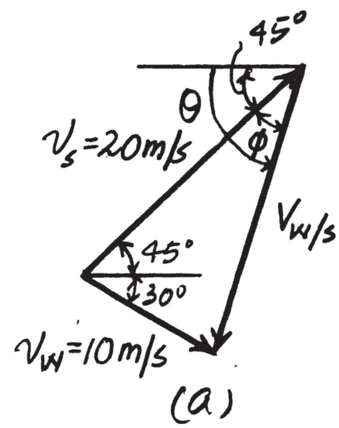
$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0° \;\measuredangle$$ **Ans.**

Solution II

Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. *a*,

$$v_{w/s} = \sqrt{20^2 + 10^2 - 2(20)(10)\cos 75°}$$

$$= 19.91 \text{ m/s} = 19.9 \text{ m/s}$$ **Ans.**



Using the result of $v_{w/s}$ and applying the law of sines,

$$\frac{\sin\phi}{10} = \frac{\sin 75°}{19.91} \qquad \phi = 29.02°$$

Thus,

$$\theta = 45° + \phi = 74.0° \;\measuredangle$$ **Ans.**

Ans:
$v_{w/s} = 19.9$ m/s
$\theta = 74.0° \;\measuredangle$