

Chapter 1

1.1

$$(a) \ m = \frac{30 \text{ lb}}{5.32 \text{ ft/s}^2} = 5.639 \text{ slugs} \blacklozenge (133.5 \text{ N})/(1.62 \text{ m/s}^2) = 82.4 \text{ kg}$$

$$(b) \ W = mg = (5.639)(32.2) = 181.6 \text{ lb} \blacklozenge (82.4 \text{ kg})(9.81 \text{ m/s}^2) = 808 \text{ N}$$

1.2

$$W = \rho g V = (7850)(9.81) [\pi(0.04^2)(0.110)] = 42.58 \text{ N} \blacktriangleleft$$

1.3

$$(a) \ 1 \text{ kN} \cdot \text{mm} = 1 \text{ kN} \cdot \text{mm} \times \frac{1000 \text{ N}}{1.0 \text{ kN}} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 1 \text{ N} \cdot \text{m} \blacktriangleleft$$

1.4

$$\begin{aligned} 30 \text{ m/mL} &= \frac{30 \text{ m}}{\text{mL}} \times \frac{1 \text{ km}}{1000 \text{ m}} \frac{1000 \text{ mL}}{1 \text{ L}} \\ &= 30 \text{ km/L} \blacktriangleleft \end{aligned}$$

1.5

$$\begin{aligned} E &= \frac{1}{2}(1000 \text{ kg}) \left(6 \frac{\text{m}}{\text{s}}\right)^2 = 18\,000 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = 18\,000 \left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right) (\text{m}) \\ &= 18\,000 \text{ N} \cdot \text{m} = 18 \text{ kN} \cdot \text{m} \blacktriangleleft \end{aligned}$$

1.6

$$\text{The dimensions of } \frac{gkx}{W} \text{ are: } [g][k][x] \left[\frac{1}{W}\right] = \left[\frac{L}{T^2}\right] \left[\frac{F}{L}\right] [L] \left[\frac{1}{F}\right] = \left[\frac{L}{T^2}\right] = [a] \quad \text{Q.E.D.}$$

1.7

$$\text{The dimensions of } k = \frac{F}{x} \text{ are: } [k] = \left[\frac{F}{x}\right] = \left[\frac{ML}{T^2}\right] \left[\frac{1}{L}\right] = \left[\frac{M}{T^2}\right] \blacklozenge$$

1.8

$$8 \text{ mm}/\mu\text{s} = \frac{8 \text{ mm}}{\mu\text{s}} \times \frac{1.0 \text{ m}}{1000 \text{ mm}} \times \frac{1.0 \mu\text{s}}{10^{-6}\text{s}} = 8000 \text{ m/s} \blacktriangleleft$$

1.9

$$y = kx^2 \text{ (where } k = 1.0\text{)}$$

$$\text{The dimensions of } k = \frac{y}{x^2} \text{ are: } \therefore [k] = \left[\frac{y}{x^2}\right] = \left[\frac{L}{L^2}\right] = \left[\frac{1}{L}\right]$$

$y = x^2$ can be dimensionally correct if the units of the constant 1.0 (not shown explicitly) are understood to be m^{-1} .

1.10

$$\left[\frac{L}{T^2} \right] = [A] [L^2] + [B] [L] [T]$$

$$\therefore [A] = \left[\frac{1}{LT^2} \right] \blacktriangleleft \quad [B] = \left[\frac{1}{T^3} \right] \blacktriangleleft$$

1.11

(a) The dimensions of $x = At^2 - Bvt$ are

$$[L] = [A][T^2] - [B][LT^{-1}][T]$$

$$\therefore [A] = [LT^{-2}] \blacktriangleleft \quad [B] = [1](\text{dimensionless}) \blacktriangleleft$$

(b) The dimensions of $x = Avte^{-Bt}$ are

$$[L] = [A][LT^{-1}][T]e^{[e][T]}$$

$$[B][T] = [1] \quad \therefore [B] = [T^{-1}] \blacktriangleleft$$

$$[L] = [A][LT^{-1}][T] \quad \therefore [A] = [1] \blacktriangleleft$$

1.12

$$\left[\frac{d^4 y}{dx^4} \right] = \left[\frac{L}{L^4} \right] = [L^{-3}]$$

$$\left[\frac{\omega^2 \gamma}{D} y \right] = \frac{[T^{-2}][ML^{-1}]}{[FL^2]} [L] = \left[\frac{M}{T^2 FL^2} \right]$$

Substituting $[F] = [MLT^{-2}]$ —see Eq. (1.2b)— we get

$$\left[\frac{\omega^2 \gamma}{D} y \right] = \left[\frac{M}{T^2 L^2} \right] \left[\frac{T^2}{ML} \right] = [L^{-3}] \text{ Q.E.D.}$$

Substituting $[F] = [MLT^{-2}]$ —see Eq. (1.2b)— we get

$$\left[\frac{\omega^2 \gamma}{D} y \right] = \left[\frac{M}{T^2 L^2} \right] \left[\frac{T^2}{ML} \right] = [L^{-3}] \text{ Q.E.D.}$$

1.13

The argument of the sine function must be dimensionless:

$$\left[\frac{Bx}{k} \right] = [1] \quad [B][L] \left[\frac{L}{F} \right] = [1] \quad [B] = [FL^{-2}] \blacktriangleleft$$

$$[F] = [Akx^2] = [A][FL^{-1}][L^2] \quad [A] = [L^{-1}] \blacktriangleleft$$

1.14

$$110 \text{ kJ/s} = 110 \text{ kJ/s} \times \frac{1 \text{ kW}}{1 \text{ kJ/s}} = 110 \text{ kW} \blacktriangleleft$$

1.15

$$F = G \frac{m_A m_B}{R^2} = (6.67 \times 10^{-11}) \frac{(12)(12)}{0.4^2} = 6.003 \times 10^{-8} \text{ N}$$

$$W = mg = (12)(9.81) = 117.7 \text{ N}$$

$$\% \text{ of weight} = \frac{F}{W} \times 100\% = \frac{6.003 \times 10^{-8}}{117.7} \times 100\% = 5.10 \times 10^{-8} \% \blacktriangleleft$$

1.16

$$F = G \frac{m_A m_B}{R^2} = (6.67 \times 10^{-11}) \frac{(8.9 \text{ N}/9.81)(8.9 \text{ N}/9.81)}{(0.406)^2} = 3.38 \times 10^{-10} \text{ N} \blacktriangleright$$

1.17

On earth: $W_e = \frac{GM_e m}{R_e^2}$ At elevation h : $W = \frac{GM_e m}{(R_e + h)^2}$

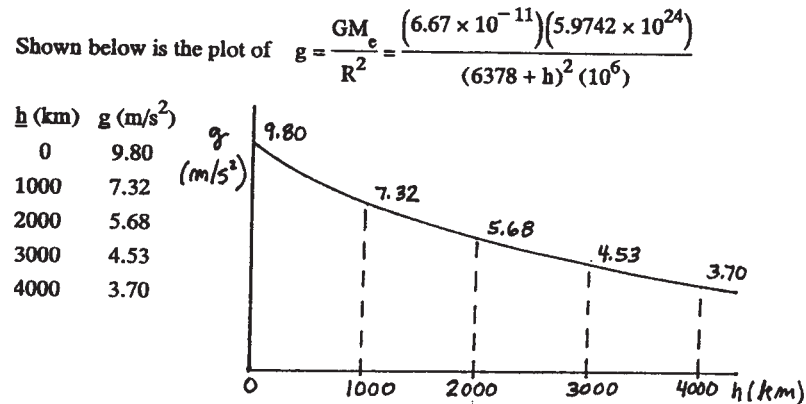
$$W = W_e \frac{R_e^2}{(R_e + h)^2} = 756 \frac{6378^2}{(6378 + 8.5)^2} = 755 \text{ N} \blacktriangleleft$$

1.18

$$g_m = \frac{GM_m}{R_m^2} \quad g_e = \frac{GM_e}{R_e^2}$$

$$\frac{g_m}{g_e} = \frac{M_m R_e^2}{M_e R_m^2} = \frac{0.07348(6378)^2}{5.974(1737)^2} = 0.1658 \approx \frac{1}{6} \text{ Q.E.D.}$$

1.19



1.20

On earth: $W_e = \frac{GM_e m}{R_e^2}$ At elevation h : $W = \frac{GM_e m}{(R_e + h)^2}$

$$W = \frac{W_e}{10} \quad \frac{GM_e m}{(R_e + h)^2} = \frac{GM_e m}{10R_e^2} \quad (R_e + h)^2 = 10R_e^2$$

$$(6378 + h)^2 = 10(6378)^2 \quad h = 13\,790 \text{ km} \blacktriangleleft$$

1.21

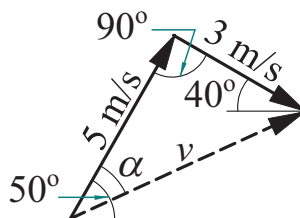
$$R = R_e + R_m + d = 6378 + 1737 + 387 \times 10^3$$

$$= 392.1 \times 10^3 \text{ km} = 392.1 \times 10^6 \text{ m}$$

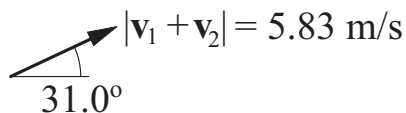
$$F = G \frac{M_e M_m}{R^2} = (6.67 \times 10^{-11}) \frac{(5.974 \times 10^{24})(0.07348 \times 10^{24})}{(392.1 \times 10^6)^2}$$

$$= 1.904 \times 10^{20} \text{ N} \blacktriangleleft$$

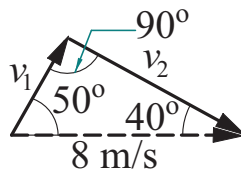
1.22



$$v = \sqrt{5^2 + 3^2} = 5.83 \text{ m/s} \quad \alpha = \tan^{-1} \frac{3}{5} = 31.0^\circ$$

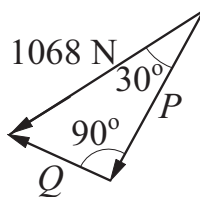


1.23



$$v_1 = 8 \sin 40^\circ = 5.14 \text{ m/s} \blacktriangleleft \quad v_2 = 8 \sin 50^\circ = 6.13 \text{ m/s} \blacktriangleleft$$

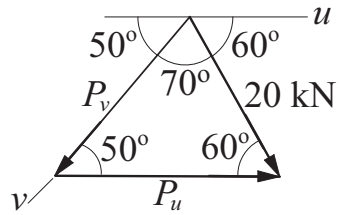
1.24



$$\text{Component parallel to } AB : P = 1068 \cos 30^\circ = 924.9 \text{ N} \blacktriangleleft$$

$$\text{Component Pependicular to } AB : Q = 1068 \sin 30^\circ = 534.0 \text{ N} \blacktriangleleft$$

1.25

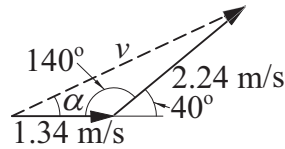


$$\frac{P_v}{\sin 60^\circ} = \frac{P_u}{\sin 70^\circ} = \frac{20}{\sin 50^\circ}$$

$$P_v = 20 \frac{\sin 60^\circ}{\sin 50^\circ} = 22.6 \text{ kN} \quad \blacktriangleleft$$

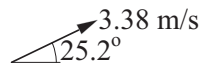
$$P_u = 20 \frac{\sin 70^\circ}{\sin 50^\circ} = 24.5 \text{ kN} \quad \blacktriangleleft$$

1.26

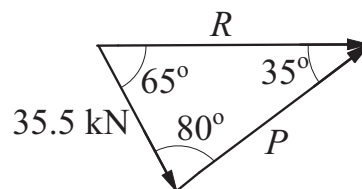


Law of cosines: $v = \sqrt{1.34^2 + 2.24^2 - 2(1.34)(2.24)\cos 140^\circ} = 3.378 \text{ m/s}$

Law of sines: $\frac{2.24}{\sin \alpha} = \frac{3.378}{\sin 140^\circ} \quad \sin \alpha = 0.4262 \quad \alpha = 25.2^\circ$

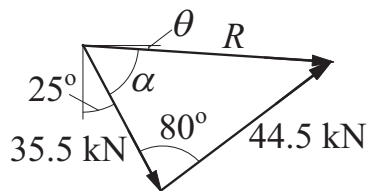


1.27



$$\frac{P}{\sin 65^\circ} = \frac{35.5}{\sin 35^\circ} \quad P = 35.5 \frac{\sin 65^\circ}{\sin 35^\circ} = 56.1 \text{ kN} \quad \blacktriangleleft$$

1.28



Law of cosines:

$$R = \sqrt{35.5 \text{ kN}^2 + 44.5 \text{ kN}^2 - 2(35.5 \text{ kN})(44.5 \text{ kN}) \cos 80^\circ}$$

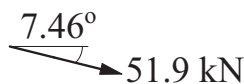
$$= 51.9 \text{ kN} \quad \blacktriangleleft$$

Law of sines:

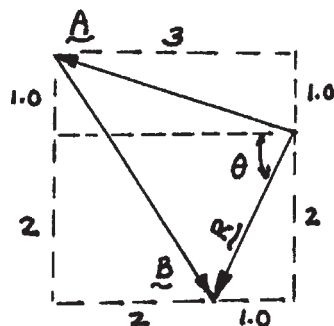
$$\frac{44.5}{\sin \alpha} = \frac{51.9}{\sin 80^\circ} \quad \sin \alpha = \frac{44.5}{51.9} \sin 80^\circ = 0.8444$$

$$\alpha = \sin^{-1}(0.8438) = 57.61^\circ$$

$$\theta = 90^\circ - 25^\circ - 57.61^\circ = 7.39^\circ \quad \blacktriangleleft$$



1.29

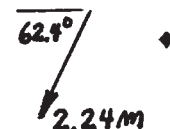


$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

By inspection of the triangle (dimensions in meters)

$$R = \sqrt{2^2 + 1.0^2} = 2.24 \text{ m} \quad \text{and} \quad \theta = \tan^{-1} 2 = 62.4^\circ$$

Therefore, the resultant of A and B is:



1.30

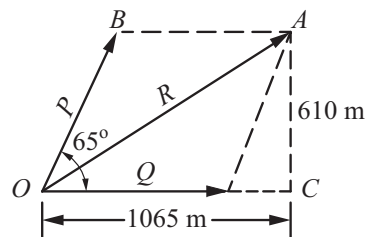
$$R = P + Q$$

$$P = \frac{610}{\sin 65^\circ} = 673 \text{ m}$$

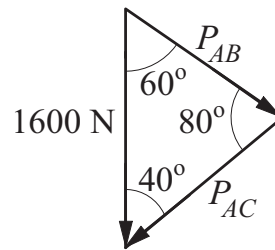
$$Q = 1065 - P \cos 65^\circ$$

$$= 1065 - 673 \cos 65^\circ = 781 \text{ m}$$

Therefore, the components are: 673 m along OB and 781 m along OC $\blacklozenge$



1.31



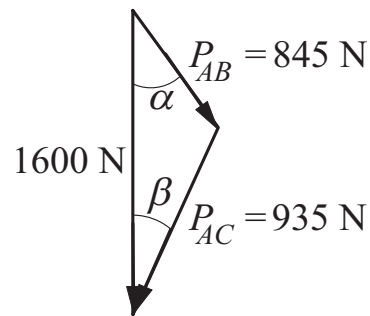
Law of sines:

$$\frac{1600}{\sin 80^\circ} = \frac{P_{AB}}{\sin 40^\circ} = \frac{P_{AC}}{\sin 60^\circ}$$

$$P_{AB} = \frac{1600 \sin 40^\circ}{\sin 80^\circ} = 1044 \text{ N} \quad \blacktriangleleft$$

$$P_{AC} = \frac{1600 \sin 60^\circ}{\sin 80^\circ} = 1407 \text{ N} \quad \blacktriangleleft$$

1.32



Law of cosines:

$$935^2 = 1600^2 + 845^2 - 2(1600)(845) \cos \alpha$$

$$\alpha = 27.4^\circ \quad \blacktriangleleft$$

$$845^2 = 1600^2 + 935^2 - 2(1600)(935) \cos \beta$$

$$\beta = 24.6^\circ \quad \blacktriangleleft$$

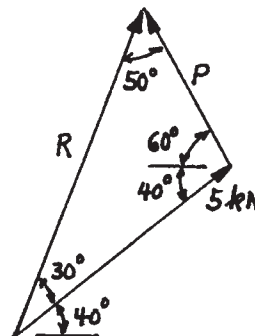
1.33

Law of sines:

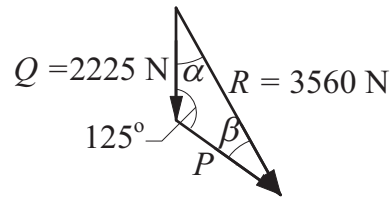
$$\frac{P}{\sin 30^\circ} = \frac{5}{\sin 50^\circ}$$

which gives

$$P = \frac{5 \sin 30^\circ}{\sin 50^\circ} = 3.26 \text{ kN} \quad \blacklozenge$$



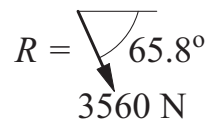
1.34



Law of sines:

$$\frac{2225}{\sin \beta} = \frac{3560}{\sin 125^\circ} \quad \beta = 30.8^\circ$$

$$\alpha = 180^\circ - (125^\circ + 30.8^\circ) = 24.2^\circ$$

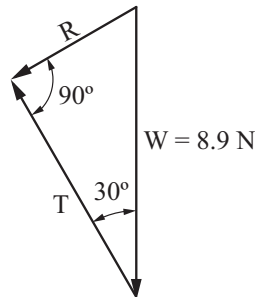


$$\frac{3560}{\sin 125^\circ} = \frac{P}{\sin 24.2^\circ} \quad P = 1780 \text{ N} \quad \blacktriangleleft$$

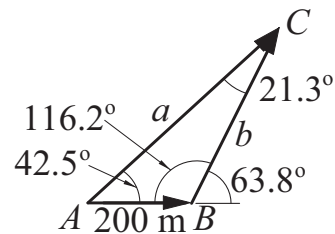
1.35

$$R = W + T$$

$$T = 8.9 \cos 30^\circ = 7.708 \text{ N} \quad \blacklozenge$$



1.36

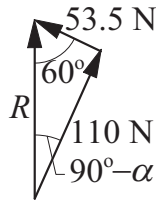


Law of sines: $\frac{200}{\sin 21.3^\circ} = \frac{a}{\sin 116.2^\circ} = \frac{b}{\sin 42.5^\circ}$

$$\therefore a = \frac{200 \sin 116.2^\circ}{\sin 21.3^\circ} = 494 \text{ m} \blacktriangleleft$$

$$b = \frac{200 \sin 42.5^\circ}{\sin 21.3^\circ} = 372 \text{ m} \blacktriangleleft$$

1.37

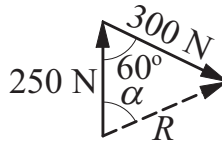


$$\frac{53.5}{\sin(90^\circ - \alpha)} = \frac{110}{\sin 60^\circ}$$

$$\sin(90^\circ - \alpha) = \frac{53.5 \sin 60^\circ}{110} = 0.4212$$

$$90^\circ - \alpha = 24.91^\circ \quad \alpha = 65.1^\circ \blacktriangleleft$$

***1.38**



First compute the resultant **R** of the two known forces. The smallest required **F** has the same direction as **R** and its magnitude is $500 \text{ N} - R$.

Law of cosines: $R = \sqrt{250^2 + 300^2 - 2(250)(300) \cos 60^\circ}$

$$= 278.4 \text{ N}$$

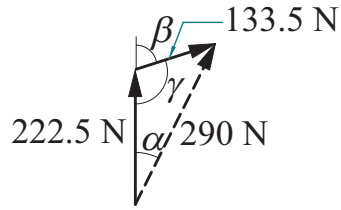
$$\therefore F = 500 - 278.4 = 222 \text{ N}$$

Law of sines: $\frac{300}{\sin \alpha} = \frac{278.4}{\sin 60^\circ}$

$$\alpha = \sin^{-1} \frac{300 \sin 60^\circ}{278.4} = 68.9^\circ$$



1.39



Law of cosines: $290^2 = 222.5^2 + 133.5^2 - 2(222.5)(133.5) \cos \gamma$

$$\gamma = \cos^{-1} \frac{290^2 + 222.5^2 + 133.5^2}{2(222.5)(133.5)} = 106.38^\circ$$

$$\beta = 180^\circ - \gamma = 180^\circ - 106.38^\circ = 73.6^\circ \quad \blacktriangleleft$$

Law of sines: $\frac{133.5}{\sin \alpha} = \frac{290}{\sin 106.38^\circ}$

$$\alpha = \sin^{-1} \frac{133.5 \sin 106.38^\circ}{290} = 26.2^\circ \quad \blacktriangleleft$$

1.40

$$\mathbf{P} = -135 \cos 50^\circ \sin 30^\circ \mathbf{i} + 135 \cos 50^\circ \cos 30^\circ \mathbf{j} + 135 \sin 50^\circ \mathbf{k} \text{ N}$$

$$\therefore \mathbf{P} = -43.4 \mathbf{i} + 75.2 \mathbf{j} + 130.4 \mathbf{k} \text{ N} \quad \blacklozenge$$

1.41

(a) $\mathbf{r} = 240 \sin 40^\circ \cos 50^\circ \mathbf{i} + 240 \sin 40^\circ \sin 50^\circ \mathbf{j} + 240 \cos 40^\circ \mathbf{k} \text{ mm}$

$$\therefore \mathbf{r} = 99.16 \mathbf{i} + 118.2 \mathbf{j} + 183.8 \mathbf{k} \text{ mm} \quad \blacklozenge$$

(b) $\lambda_x = \frac{r_x}{r} = \frac{99.16}{240} = 0.413 \quad \lambda_y = \frac{r_y}{r} = \frac{118.2}{240} = 0.493 \quad \lambda_z = \frac{r_z}{r} = \frac{183.8}{240} = 0.766$

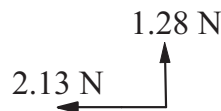
$$\therefore \vec{\lambda} = 0.413 \mathbf{i} + 0.493 \mathbf{j} + 0.766 \mathbf{k} \quad \blacklozenge$$

1.42

$$\vec{AB} = -1.52 \mathbf{i} + 0.915 \mathbf{j} \text{ m} \quad \left| \vec{AB} \right| = \sqrt{1.52^2 + 0.915^2} = 1.774 \text{ m}$$

$$\lambda = \frac{\vec{AB}}{\left| \vec{AB} \right|} = \frac{-1.52 \mathbf{i} + 0.915 \mathbf{j}}{1.774} = -0.8568 \mathbf{i} + 0.5159 \mathbf{j}$$

$$\mathbf{F} = F \lambda = 2.49(-0.8568 \mathbf{i} + 0.5159 \mathbf{j}) = -2.13 \mathbf{i} + 1.28 \mathbf{j} \text{ N} \quad \blacktriangleleft$$



1.43

$$(a) \quad \vec{AB} = 7\mathbf{i} + \mathbf{j} + 5\mathbf{k} \text{ m} \quad \therefore |\vec{AB}| = \sqrt{7^2 + 1^2 + 5^2} = 8.66 \text{ m} \blacklozenge$$

$$(b) \quad \vec{\lambda}_{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{7\mathbf{i} + \mathbf{j} + 5\mathbf{k}}{8.66} = 0.808\mathbf{i} + 0.115\mathbf{j} + 0.577\mathbf{k} \text{ m} \blacklozenge$$

1.44

$$(a) \quad \lambda_{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{-2.2\mathbf{i} + 7.5\mathbf{j} + 3\mathbf{k}}{8.372} = -0.2628\mathbf{i} + 0.8958\mathbf{j} + 0.3583\mathbf{k} \blacktriangleleft$$

$$(b) \quad \mathbf{v} = 8\lambda_{AB} = 8(-0.2628\mathbf{i} + 0.8958\mathbf{j} + 0.3583\mathbf{k}) \\ = -2.10\mathbf{i} + 7.17\mathbf{j} + 2.87\mathbf{k} \text{ m/s} \blacktriangleleft$$

1.45

$$\lambda_{OA} = \frac{\vec{OA}}{|\vec{OA}|} = \frac{-3\mathbf{i} + 4\mathbf{j} + 2.5\mathbf{k}}{5.590} = -0.5367\mathbf{i} + 0.7156\mathbf{j} + 0.4472\mathbf{k}$$

$$\mathbf{F} = F\lambda_{OA} = 320(-0.5367\mathbf{i} + 0.7156\mathbf{j} + 0.4472\mathbf{k}) \\ = -172\mathbf{i} + 229\mathbf{j} + 143\mathbf{k} \text{ N} \blacktriangleleft$$

1.46

$$\lambda_{BA} = \frac{\vec{BA}}{|\vec{BA}|} = \frac{4.27\mathbf{i} - 3.05\mathbf{j} - 5.5\mathbf{k}}{7.602} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

$$\mathbf{F} = F\lambda_{AB} = 712(0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}) \\ = 399.9\mathbf{i} - 285.7\mathbf{j} - 515.1\mathbf{k} \text{ N} \blacktriangleleft$$

1.47

$$\vec{AB} = 48.8\mathbf{i} + 67\mathbf{j} - 21.3\mathbf{k} \text{ m}$$

$$\lambda = \frac{\vec{AB}}{|\vec{AB}|} = \frac{48.8\mathbf{i} + 67\mathbf{j} - 21.3\mathbf{k}}{\sqrt{48.8^2 + 67^2 + 21.3^2}} = 0.5696\mathbf{i} + 0.7832\mathbf{j} - 0.2492\mathbf{k}$$

$$\mathbf{v} = v\lambda = 427(0.5696\mathbf{i} + 0.7832\mathbf{j} - 0.2492\mathbf{k}) \\ = 106.3\mathbf{i} + 334.3\mathbf{j} - 106.3\mathbf{k} \text{ m/s} \blacktriangleleft$$

1.48

(a)

$$\vec{BA} = -6.1\mathbf{i} + 18.3\mathbf{j} - 27.4\mathbf{k} \text{ ft} \quad |\vec{BA}| = \sqrt{20^2 + 60^2 + 90^2} = 33.5 \text{ m}$$

$$\lambda = \frac{\vec{BA}}{|\vec{BA}|} = \frac{-6.1\mathbf{i} + 18.3\mathbf{j} - 27.4\mathbf{k}}{33.5} = -0.1818\mathbf{i} + 0.5455\mathbf{j} - 0.8182\mathbf{k}$$

$$F_x = F\lambda_x = 2.67(-0.1818) = -486 \text{ N} \blacktriangleleft$$

$$F_y = F\lambda_y = 2.67(0.5455) = 1458 \text{ N} \blacktriangleleft$$

$$F_z = F\lambda_z = 2.67(-0.8182) = -2184 \text{ N} \blacktriangleleft$$

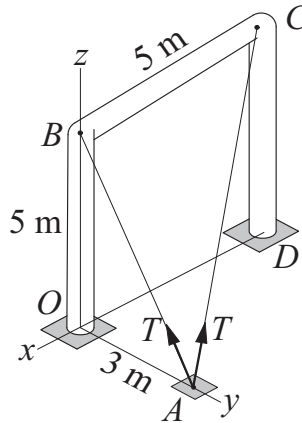
(b)

$$\theta_x = \cos^{-1} \lambda_x = \cos^{-1}(-0.1818) = 100.5^\circ \blacktriangleleft$$

$$\theta_y = \cos^{-1} \lambda_y = \cos^{-1}(0.5455) = 56.9^\circ \blacktriangleleft$$

$$\theta_z = \cos^{-1} \lambda_z = \cos^{-1}(-0.8182) = 144.9^\circ \blacktriangleleft$$

1.49



$$\overrightarrow{AB} = -3\mathbf{j} + 5\mathbf{k} \text{ m} \quad \left| \overrightarrow{AB} \right| = \sqrt{3^2 + 5^2} = 5.831 \text{ m}$$

$$\mathbf{T}_{AB} = T \frac{\overrightarrow{AB}}{\left| \overrightarrow{AB} \right|} = 35 \frac{-3\mathbf{j} + 5\mathbf{k}}{5.831} = -18.01\mathbf{j} + 30.01\mathbf{k} \text{ kN}$$

$$\overrightarrow{AC} = -5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} \text{ m} \quad \left| \overrightarrow{AC} \right| = \sqrt{5^2 + 3^2 + 5^2} = 7.681 \text{ m}$$

$$\mathbf{T}_{AC} = T \frac{\overrightarrow{AC}}{\left| \overrightarrow{AC} \right|} = 35 \frac{-5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}}{7.681} = -22.78\mathbf{i} - 13.67\mathbf{j} + 22.78\mathbf{k} \text{ kN}$$

$$\begin{aligned} \mathbf{R} &= \mathbf{T}_{AB} + \mathbf{T}_{AC} \\ &= -22.78\mathbf{i} + (-18.01 - 13.67)\mathbf{j} + (30.01 + 22.78)\mathbf{k} \\ &= -22.8\mathbf{i} - 31.7\mathbf{j} + 52.8\mathbf{k} \text{ kN} \blacktriangleleft \end{aligned}$$

1.50

$$\vec{AB} = 2.44\mathbf{i} - 2.44\mathbf{j} + 1.22\mathbf{k} \text{ m} \quad \left| \vec{AB} \right| = \sqrt{2.44^2 + 2.44^2 + 1.22^2} = 3.66 \text{ m}$$

$$\mathbf{F}_{AB} = F \frac{\vec{AB}}{\left| \vec{AB} \right|} = F \frac{2.44\mathbf{i} - 2.44\mathbf{j} + 1.22\mathbf{k}}{3.66}$$

$$\vec{AC} = -1.22\mathbf{i} - 2.44\mathbf{j} + 1.22\mathbf{k} \text{ ft} \quad \left| \vec{AC} \right| = \sqrt{1.22^2 + 2.44^2 + 1.22^2} = 2.99 \text{ m}$$

$$\mathbf{F}_{AC} = 890 \frac{\vec{AC}}{\left| \vec{AC} \right|} = 890 \frac{-1.22\mathbf{i} - 2.44\mathbf{j} + 1.22\mathbf{k}}{2.99}$$

The resultant $\mathbf{R}$ lies in the yz -plane if

$$R_x = (F_{AB})_x + (F_{AC})_x = 0 \quad F \frac{2.44}{3.66} - 890 \frac{1.22}{2.99} = 0$$

$$F = 545 \text{ N} \quad \blacktriangleleft$$

1.51

(a)

$$\begin{aligned} \mathbf{R} &= (F_1 + F_2 \sin 35^\circ)\mathbf{i} + (F_2 \cos 35^\circ + F_3 \cos 65^\circ)\mathbf{j} + (F_3 \sin 65^\circ)\mathbf{k} \\ &= (1.6 + 1.2 \sin 35^\circ)\mathbf{i} + (1.2 \cos 35^\circ + 1.0 \cos 65^\circ)\mathbf{j} + (1.0 \sin 65^\circ)\mathbf{k} \\ &= 2.288\mathbf{i} + 1.4056\mathbf{j} + 0.9063\mathbf{k} \text{ kN} \quad \blacktriangleleft \end{aligned}$$

(b)

$$R = \sqrt{2.288^2 + 1.4056^2 + 0.9063^2} = 2.834 \text{ kN}$$

$$\begin{aligned} \lambda &= \frac{\mathbf{R}}{R} = \frac{2.288\mathbf{i} + 1.4056\mathbf{j} + 0.9063\mathbf{k}}{2.834} \\ &= 0.807\mathbf{i} + 0.496\mathbf{j} + 0.320\mathbf{k} \\ \therefore R &= 2.83(0.807\mathbf{i} + 0.496\mathbf{j} + 0.320\mathbf{k}) \text{ kN} \quad \blacktriangleleft \end{aligned}$$

1.52

$$\begin{aligned} \mathbf{P} &= 534 \left(\frac{4\mathbf{i} + 3\mathbf{j}}{5} \right) = 427\mathbf{i} + 320\mathbf{j} \text{ N} \quad \mathbf{Q} = 578 \left(\frac{5\mathbf{i} - 12\mathbf{j}}{13} \right) = 222\mathbf{i} - 534\mathbf{j} \text{ N} \\ \therefore \mathbf{P} + \mathbf{Q} &= (427\mathbf{i} + 320\mathbf{j}) + (222\mathbf{i} - 534\mathbf{j}) = 649\mathbf{i} - 214\mathbf{j} \text{ N} \quad \blacklozenge \end{aligned}$$

1.53

$$\mathbf{P} = 400 \frac{4\mathbf{i} + 3\mathbf{j}}{5} = 320\mathbf{i} + 240\mathbf{j} \text{ N} \quad \mathbf{NQ} = Q \frac{5\mathbf{i} - 12\mathbf{j}}{13}$$

Because $\mathbf{R} = \mathbf{P} + \mathbf{Q}$ lies in x -direction, we have

$$R_y = 0 \quad 240 - \frac{12}{13}Q = 0 \quad Q = 260 \text{ N} \quad \blacktriangleleft$$

$$R = R_x = 320 + 260 \frac{5}{13} = 420 \text{ N} \quad \blacktriangleleft$$

1.54

$$\begin{aligned} P_x + Q_x &= R_x & P \cos 30^\circ - Q \sin 30^\circ &= 1600 \cos 25^\circ \\ P_y + Q_y &= R_y & P \sin 30^\circ - Q \cos 30^\circ &= -1600 \sin 25^\circ \end{aligned}$$

Solution is : $P = 3190 \text{ N}$ ◀ $Q = 2620 \text{ N}$ ◀

1.55

$$\begin{aligned} P_x + Q_x &= R_x & 3 \cos \theta &= 2 \sin 55^\circ \\ \theta &= \cos^{-1} \left(\frac{2}{3} \sin 55^\circ \right) & &= 56.90^\circ \text{ ◀} \\ P_y + Q_y &= R_y & 3 \sin \theta - Q &= -2 \sin 55^\circ \\ Q &= 2 \sin 55^\circ + 3 \sin 56.90^\circ & &= 4.15 \text{ kN} \text{ ◀} \end{aligned}$$

1.56

$$\begin{aligned} \lambda_P &= \frac{1.83\mathbf{i} + 2.44\mathbf{j} - 3.66\mathbf{k}}{\sqrt{1.83^2 + 2.44^2 + (-3.66)^2}} = 0.3841\mathbf{i} + 0.5121\mathbf{j} - 0.7682\mathbf{k} \\ \lambda_Q &= \frac{-1.83\mathbf{i} + 1.83\mathbf{j} - 3.66\mathbf{k}}{\sqrt{(-1.83)^2 + 1.83^2 + (-3.66)^2}} = -0.4082\mathbf{i} + 0.4082\mathbf{j} - 0.8165\mathbf{k} \\ \lambda_F &= \frac{-2.44\mathbf{j} - 3.66\mathbf{k}}{\sqrt{(-2.44)^2 + (-3.66)^2}} = -0.5547\mathbf{j} - 0.8321\mathbf{k} \end{aligned}$$

$$\begin{aligned} P_x + Q_x + F_x &= 0 & 0.3841P - 0.4082Q + 0 &= 0 \\ P_y + Q_y + F_y &= 0 & 0.5121P + 0.4082Q - 0.5547(534\text{N}) &= 0 \end{aligned}$$

Solution is: $P = 330 \text{ N}$ ◀ $Q = 310 \text{ N}$ ◀

1.57

$$\begin{aligned} \text{(a)} \quad \mathbf{A} \cdot \mathbf{B} &= 12(-2) + 8(3) = 0 \text{ ◀} \\ \text{(b)} \quad \mathbf{A} \cdot \mathbf{B} &= 5(7) = 35 \text{ N} \cdot \text{m} \text{ ◀} \\ \text{(c)} \quad \mathbf{A} \cdot \mathbf{B} &= 3(-6) + 2(2) + (-1)(-8) = -6 \text{ m}^2 \text{ ◀} \end{aligned}$$

1.58

$$\begin{aligned} \text{(a)} \quad \mathbf{C} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 12 & 8 \\ 4 & -2 & 3 \end{vmatrix} = 52\mathbf{i} + 32\mathbf{j} - 48\mathbf{k} \text{ m}^2 \text{ ◀} \\ \text{(b)} \quad \mathbf{C} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 0 \\ 7 & 0 & -12 \end{vmatrix} = -36\mathbf{i} + 60\mathbf{j} - 21\mathbf{k} \text{ N} \cdot \text{m} \text{ ◀} \\ \text{(c)} \quad \mathbf{C} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ -6 & 2 & -8 \end{vmatrix} = -14\mathbf{i} + 30\mathbf{j} + 18\mathbf{k} \text{ m}^2 \text{ ◀} \end{aligned}$$

1.59

$$\mathbf{r} \times \mathbf{F} \cdot \boldsymbol{\lambda} = \begin{vmatrix} 4 & -6 & 2 \\ 20 & 40 & -30 \\ 0 & 0.8 & 0.6 \end{vmatrix} = 296 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

$$\boldsymbol{\lambda} \times \mathbf{r} \cdot \mathbf{F} = \begin{vmatrix} 0 & 0.8 & 0.6 \\ 4 & -6 & 2 \\ 20 & 40 & -30 \end{vmatrix} = 296 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

1.60

$$\mathbf{A} = 2\mathbf{i} + 1.2\mathbf{j} \text{ m} \quad \mathbf{B} = 2\mathbf{i} + 1.2\mathbf{j} + 1.5\mathbf{k} \text{ m} \quad \mathbf{C} = -1.5\mathbf{k} \text{ m}$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1.2 & 0 \\ 2 & 1.2 & 1.5 \end{vmatrix} = 1.8\mathbf{i} - 3\mathbf{j} \text{ m}^2 \quad \blacktriangleleft$$

$$\mathbf{C} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1.5 \\ 2 & 1.2 & 1.5 \end{vmatrix} = 1.8\mathbf{i} - 3\mathbf{j} \text{ m}^2 \quad \blacktriangleleft$$

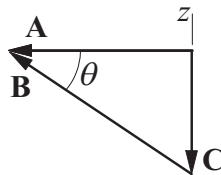
1.61

$$\mathbf{A} = 2\mathbf{i} + 1.2\mathbf{j} \text{ m} \quad \mathbf{B} = 2\mathbf{i} + 1.2\mathbf{j} + 1.5\mathbf{k} \text{ m}$$

$$A = \sqrt{2^2 + 1.2^2} = 2.332 \text{ m} \quad B = \sqrt{2^2 + 1.2^2 + 1.5^2} = 2.773 \text{ m}$$

$$\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{2(2) + 1.2(1.2)}{(2.332)(2.773)} = 0.8412$$

$$\therefore \theta = 32.7^\circ \quad \blacktriangleleft$$



Because the three vectors form a right triangle, we have in this case

$$\theta = \cos^{-1} \frac{A}{B} = \cos^{-1} \frac{2.332}{2.773} = 32.8^\circ$$

The difference in the results is due to round-off error.

1.62

$$B_z = A_z = \sqrt{4.27^2 + 2.74^2} \csc 50^\circ = 6.62 \text{ m}$$

$$\mathbf{A} = 2.74\mathbf{i} + 4.27\mathbf{j} + 6.62\mathbf{k} \text{ m} \quad \mathbf{B} = 1.83\mathbf{i} + 6.62\mathbf{k} \text{ m}$$

$$A = \sqrt{2.74^2 + 4.27^2 + 6.62^2} = 8.34 \text{ m}$$

$$B = \sqrt{1.83^2 + 6.62^2} = 6.87 \text{ m}$$

$$\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{2.74(1.83) + 6.62(6.62)}{8.34(6.87)} = 0.8524$$

$$\theta = 31.5^\circ \blacktriangleleft$$

1.63

Statement (ii) is true. ♦

Proof $\mathbf{A} \times \mathbf{B}$ is perpendicular to $\mathbf{A}$ and $\mathbf{B}$, that is, normal to plane S . Therefore, $\mathbf{C} = \mathbf{A} \times (\mathbf{A} \times \mathbf{B})$ is "normal to the normal" of S . Then $\mathbf{C}$ lies in plane S .
(Note: $\mathbf{C}$ is also normal to $\mathbf{A}$.)

1.64

$$\mathbf{P} = 76.2\mathbf{i} + 101.6\mathbf{k} \text{ mm} \quad \mathbf{Q} = -50.8\mathbf{j} + 101.6\mathbf{k} \text{ mm}$$

$$P = \sqrt{76.2^2 + 101.6^2} = 127 \text{ mm} \quad Q = \sqrt{50.8^2 + 101.6^2} = 113.6 \text{ mm}$$

(a)

$$\cos \theta = \frac{\mathbf{P} \cdot \mathbf{Q}}{PQ} = \frac{76.2(-50.8) + 101.6(101.6)}{127(113.6)} = 0.4472$$

$$\theta = 63.4^\circ \blacktriangleleft$$

(b)

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 76.2 & 0 & 101.6 \\ 0 & -50.8 & 101.6 \end{vmatrix} = 8\mathbf{i} - 12\mathbf{j} - 6\mathbf{k} \text{ mm}$$

$$\lambda = \frac{\mathbf{P} \times \mathbf{Q}}{|\mathbf{P} \times \mathbf{Q}|} = \frac{8\mathbf{i} - 12\mathbf{j} - 6\mathbf{k}}{\sqrt{8^2 + 12^2 + 6^2}}$$

$$= 0.512\mathbf{i} - 0.768\mathbf{j} - 0.384\mathbf{k} \blacktriangleleft$$

1.65

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -3 & -2 \\ -2 & -4 & 3 \end{vmatrix} = -17\mathbf{i} - 8\mathbf{j} - 22\mathbf{k} \text{ m}^2$$

$$\lambda = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|} = \pm \frac{-17\mathbf{i} - 8\mathbf{j} - 22\mathbf{k}}{\sqrt{17^2 + 8^2 + 22^2}}$$

$$= \pm (-0.588\mathbf{i} - 0.277\mathbf{j} - 0.760\mathbf{k}) \blacktriangleleft$$

1.66

$$\overrightarrow{CA} = (0 - 3)\mathbf{i} + (-2 - 0)\mathbf{j} + (2 - 0)\mathbf{k} = -3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \text{ mm}$$

$$\overrightarrow{CB} = (-1 - 3)\mathbf{i} + (4 - 0)\mathbf{j} + (1 - 0)\mathbf{k} = -4\mathbf{i} + 4\mathbf{j} + \mathbf{k} \text{ mm}$$

$$|\overrightarrow{CA}| = \sqrt{(-3)^2 + (-2)^2 + 2^2} = 4.123 \text{ mm}$$

$$|\overrightarrow{CB}| = \sqrt{(-4)^2 + 4^2 + 1^2} = 5.745 \text{ mm}$$

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & 2 \\ -4 & 4 & 1 \end{vmatrix} = -10\mathbf{i} - 5\mathbf{j} - 20\mathbf{k} \text{ mm}^2$$

$$\lambda = \pm \frac{\overrightarrow{CA} \times \overrightarrow{CB}}{|\overrightarrow{CA} \times \overrightarrow{CB}|} = \pm \frac{-10\mathbf{i} - 5\mathbf{j} - 20\mathbf{k}}{4.123(5.745)} = \pm (-0.422\mathbf{i} - 0.211\mathbf{j} - 0.844\mathbf{k}) \blacktriangleleft$$

1.67

$$\mathbf{P} = 3\mathbf{i} + 4\mathbf{k} \text{ m} \quad \mathbf{Q} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k} \text{ m}$$

$$\lambda = \frac{\overrightarrow{OA}}{|\overrightarrow{OA}|} = \frac{3\mathbf{i} + 4\mathbf{j}}{\sqrt{3^2 + 4^2}} = 0.6\mathbf{i} + 0.8\mathbf{j}$$

The component of $\mathbf{P} \times \mathbf{Q}$ in direction of λ is

$$\mathbf{P} \times \mathbf{Q} \cdot \lambda = \begin{vmatrix} 3 & 0 & 4 \\ 3 & 4 & 5 \\ 0.6 & 0.8 & 0 \end{vmatrix} = -12.0 \text{ m} \blacktriangleleft$$

1.68

$$\vec{\lambda}_A = \frac{\mathbf{A}}{A} = \frac{0.61\mathbf{i} + 0.914\mathbf{j} - 1.52\mathbf{k}}{\sqrt{1.876}} = 0.3244\mathbf{i} - 0.4867\mathbf{j} + 0.8111\mathbf{k}$$

$$F_A = \mathbf{F} \cdot \vec{\lambda}_A = 26.7(0.3244) + 89(-0.4867) + (-53.4)(0.8111) = -77.92 \text{ N} \blacklozenge$$

1.69

$$\mathbf{A} \cdot \mathbf{B} = 0$$

$$3(4) - a(1) - 2(1) = 0 \quad a = 10.0 \quad \blacktriangleleft$$

*1.70

$$\vec{\lambda}_B = \frac{\mathbf{B}}{B} = \frac{152.4\mathbf{i} + 50.8\mathbf{k}}{\sqrt{152.4^2 + 50.8^2}} = 0.950\mathbf{i} + 0.313\mathbf{k}$$

The component of A parallel to B is : $A_B = \mathbf{A} \cdot \vec{\lambda}_B = 3(76.2)(0.950) - 102(0.313) = 40.46 \text{ mm}$

$$\therefore A_B = 40.46(0.950\mathbf{i} - 0.313\mathbf{k}) \text{ mm} \quad \blacklozenge$$

Letting A_C be the component of A that is perpendicular to B, we have

$$A_C = A - A_B$$

$$A_C = (76.2\mathbf{i} + 127\mathbf{j} - 102\mathbf{k}) - 40.46(0.950\mathbf{i} - 0.313\mathbf{k}) = 37.8\mathbf{i} + 127\mathbf{j} - 114.7\mathbf{k} \text{ mm}$$

$$\therefore A_C = \sqrt{37.8^2 + 127^2 - 114.7^2} = 175.3 \text{ mm}$$

The Unit vector in the direction of A_C is

$$\vec{\lambda}_C = \frac{A_C}{A_C} = \frac{37.8\mathbf{i} + 127\mathbf{j} - 114.7\mathbf{k}}{175.3} = 0.216\mathbf{i} + 0.724\mathbf{j} - 0.654\mathbf{k}$$

$$\therefore A_C = 175.3(0.216\mathbf{i} + 0.724\mathbf{j} - 0.654\mathbf{k}) \text{ mm} \quad \blacklozenge$$

1.71 By inspection, a unit vector perpendicular to the door is

$$\boldsymbol{\lambda} = \sin 20^\circ \mathbf{i} + \cos 20^\circ \mathbf{j} = 0.3420\mathbf{i} + 0.9397\mathbf{j}$$

The component of $\mathbf{F}$ perpendicular to the plane of the door is

$$F_\perp = \mathbf{F} \cdot \boldsymbol{\lambda} = -22.2(0.3420) + 53.4(0.9397) = 42.6 \text{ N} \quad \blacktriangleleft$$

1.72

For the three vectors to lie in the same plane, $\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = 0$.

$$\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \begin{vmatrix} 2 & -1 & 2 \\ 6 & 3 & a \\ 16 & 46 & 7 \end{vmatrix} = 2(21 - 46a) + 1(42 - 16a) + 2(276 - 48) = 0$$

$$\text{which gives: } 540 - 108a = 0 \quad \therefore a = 5 \text{ m} \quad \blacklozenge$$

***1.73**

We first compute a unit vector $\vec{\lambda}$ that is perpendicular to plane ABC:

$$\begin{aligned}\vec{AB} &= -50.8\mathbf{i} + 127\mathbf{k} \text{ mm} & \vec{AC} &= -50.8\mathbf{i} + 152.5\mathbf{j} \text{ mm} \\ \vec{AC} \times \vec{AB} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50.8 & 152.5 & 0 \\ -50.8 & 0 & 127 \end{vmatrix} = 19,368\mathbf{i} + 6452\mathbf{j} + 7747\mathbf{k} \text{ mm}^2 \\ \therefore \vec{\lambda} &= \frac{\vec{AC} \times \vec{AB}}{|\vec{AC} \times \vec{AB}|} = \frac{19,368\mathbf{i} + 6452\mathbf{j} + 7747\mathbf{k}}{\sqrt{19,368^2 + 6452^2 + 7747^2}} = 0.887\mathbf{i} + 0.296\mathbf{j} + 0.355\mathbf{k}\end{aligned}$$

The normal component of $F = 89\mathbf{i} + 133.4\mathbf{j} + 222\mathbf{k}$ N is:

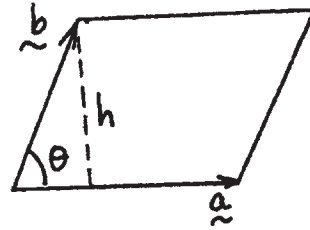
$$\begin{aligned}F_n &= F \bullet \vec{\lambda} = 89(0.887) + 133.4(0.296) + 222(0.355) = 197.2 \text{ N} \\ F_n &= F_n \vec{\lambda} = 197.2(0.887\mathbf{i} + 0.296\mathbf{j} + 0.355\mathbf{k}) = 174.9\mathbf{i} + 58.4\mathbf{j} + 70\mathbf{k} \text{ N}\end{aligned}$$

Therefore, the component of F that lies in plane ABC is:

$$F_t = F - F_n = -85.9\mathbf{i} + 75\mathbf{j} + 152\mathbf{k} \text{ N} \blacklozenge$$

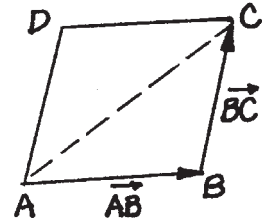
1.74

Since $\mathbf{a} \times \mathbf{b}$ is perpendicular to the area,
it has the correct direction.
Check of magnitude:
 $|\mathbf{a} \times \mathbf{b}| = ab \sin \theta = ah = A$ It checks!



1.75

$$\begin{aligned}|\vec{AB} \times \vec{BC}| &= \text{area of parallelogram ABCD} \\ &= 2(\text{area of triangle ABC}) \\ \vec{AB} &= -5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} \text{ mm} & \vec{BC} &= 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} \text{ mm} \\ \vec{AB} \times \vec{BC} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 3 & 3 \\ 2 & -2 & 1 \end{vmatrix} = 9\mathbf{i} + 11\mathbf{j} + 4\mathbf{k} \text{ mm}^2\end{aligned}$$



$$\begin{aligned}\text{Therefore, area} &= \frac{1}{2} |\vec{AB} \times \vec{BC}| = \\ &= \frac{1}{2} \sqrt{9^2 + 11^2 + 4^2} = 14.76 \text{ mm}^2 \blacklozenge\end{aligned}$$

1.76

$$|\mathbf{a} \times \mathbf{b} \cdot \mathbf{c}| = |\mathbf{a} \times \mathbf{b}| |\mathbf{c}| |\cos \theta|$$

From Prob. 1.74:

$$|\mathbf{a} \times \mathbf{b}| = \text{area of base}$$

(shown shaded in figure)

Note that $|\mathbf{c}| |\cos \theta| = h$

$$\therefore |\mathbf{a} \times \mathbf{b} \cdot \mathbf{c}| = (\text{area of base}) \times h$$

$$= \text{vol. of parallelepiped} \quad \text{Q.E.D.}$$

