Chapter 1

Equations and Inequalities

Section 1.1: Linear Equations

Classroom Example 1 (page 85)

$$\begin{aligned}-4(3x-5) &= 3-(8x+7)\\ -12x+20 &= 3-8x-7\\ -12x+20 &= -4-8x\\ 24 &= 4x\\ 6 &= x\end{aligned}$$

Solution set: $\{6\}$

Classroom Example 2 (page 85)

$$\begin{aligned}\frac{3s+6}{10}-\frac{1}{2}s &= \frac{2}{5}s+\frac{33}{5}\\ 10\left(\frac{3s+6}{10}-\frac{1}{2}s\right) &= 10\left(\frac{2}{5}s+\frac{33}{5}\right)\\ 3s+6-5s &= 4s+66\\ -2s+6 &= 4s+66\\ -6s &= 60\\ s &= -10\end{aligned}$$

Solution set: $\{-10\}$

Classroom Example 3 (page 86)

(a) $$\begin{aligned}6x-9 &= 4x+13\\ 2x &= 22 \Rightarrow x=11\end{aligned}$$

conditional equations; $\{11\}$

(b) $$\begin{aligned}10+14x &= 7(2x-5)\\ 10+14x &= 14x-35\\ 10 &= -35\end{aligned}$$

contradiction; $\varnothing$

(c) $$\begin{aligned}-3(2x-1)+5x &= 3-x\\ -6x+3+5x &= 3-x\\ -x+3 &= 3-x\end{aligned}$$

identity; {all real numbers}

Classroom Example 4 (page 87)

(a) $d=rt \Rightarrow \dfrac{d}{r}=t$

(b) $$\begin{aligned}S &= kr^2+kr\ell\\ S &= k\left(r^2+r\ell\right)\\ \frac{S}{r^2+r\ell} &= k\end{aligned}$$

(c) $$\begin{aligned}11y+8 &= 2(4y+5w)-6z\\ 11y+8 &= 8y+10w-6z\\ 3y &= 10w-6z-8\\ y &= \frac{10w-6z-8}{3}\end{aligned}$$

Classroom Example 5 (page 88)

$P=\$2580, t=\frac{9}{12}=\frac{3}{4}$ yr, and $r=.06$

$I=Prt=2580(.06)\left(\frac{3}{4}\right)=\$116.10$

Section 1.2: Applications and Modeling with Linear Equations

Classroom Example 1 (page 91)

Let x = the length of the original rectangle. Then,
$x-2$ = the width of the original rectangle.
$x+3$ = the length of the new rectangle, so
$(x-2)+3=x+1$ = the width of the new rectangle.
The perimeter of the new rectangle is
$2(x+3)+2(x+1)=2x+6+2x+2=4x+8$. The perimeter of the new rectangle is 4 in. less than 8 times the width of the original rectangle, so we have $4x+8=8(x-2)-4$. Solving, we have

$4x+8=8(x-2)-4 \Rightarrow 4x+8=8x-16-4 \Rightarrow$
$4x+8=8x-20 \Rightarrow 28=4x \Rightarrow 7=x$

Thus, the length of the original rectangle is 7 in. and the width of the original rectangle is 5 in.

Classroom Example 2 (page 92)

Let x = the distance to her grandmother's house.

	r	t	d
Going	40	$\frac{x}{40}$	x
Returning	48	$\frac{x}{48}$	x

The driving time returning was 1 hour less than the driving time going, so we have

$\frac{x}{40}-1=\frac{x}{48}$. Solving, we have

$$\frac{x}{40}-1=\frac{x}{48}$$
$$40\cdot 48\left(\frac{x}{40}-1\right)=40\cdot 48\left(\frac{x}{48}\right)$$
$$48x-1920=40x$$
$$-1920=-8x \Rightarrow 240=x$$

The distance from Krissa's home to her grandmother's home is 240 mi.

Classroom Example 3 (page 93)

Let x = the amount of 25% antifreeze solution (in liters).

Strength	Gallons of Solution	Gallons of Pure Antifreeze
25%	x	$.25x$
10%	5	$.10\cdot 5=.5$
15%	$x+5$	$.15(x+5)$

The number of gallons of pure antifreeze in the 25% solution plus the number of gallons of pure antifreeze in the 10% solution must equal the number of gallons of pure antifreeze in the 15% solution.

$$.25x+.5=.15(x+5)$$
$$.25x+.5=.15x+.75$$
$$.1x=.25 \Rightarrow x=2.5\text{ L}$$

2.5 liters of the 25% solution should be added.

Classroom Example 4 (page 94)

Let x = amount invested at 4.8%.
Then 28,000 − x = amount invested at 5.5%.

Amount in Account	Interest Rate	Interest
x	4.8%	$.048x$
28,000 − x	5.5%	$.055(28{,}000-x)$
		1456

The amount of interest from the 4.8% account plus the amount of interest from the 5.5% account must equal the total amount of interest.

$$.048x+.055(28{,}000-x)=1456$$
$$.048x+1540-.055x=1456$$
$$-.007x=-84$$
$$x=12{,}000$$

Owen invested \$12,000 at 4.8% and \$28,000 − \$12,000 = \$16,000 at 5.5%.

Classroom Example 5 (page 95)

$P=1.06F+7.18$. Since P = 70, we have

$$70=1.06F+7.18$$
$$62.82=1.06F \Rightarrow 59.26\approx F$$

The flow rate must be approximately 59.26 L per second to remove 70% of the contaminants.

Classroom Example 6 (page 96)

(a) The year 2007 is 7 years after the year 2000. Let x = 7 and find the value of y.

$$y=390x+4710$$
$$y=390(7)+4710=7440$$

The per capita health care expenditures in the year 2007 were \$7440.

(b) Let y = 8220 and find the value of x

$$y=390x+4710$$
$$8220=390x+4710$$
$$3510=390x$$
$$9=x$$

Per capita health care expenditures are projected to reach \$8220 9 years after 2000, or in 2009.

Section 1.3: Complex Numbers

Classroom Example 1 (page 104)

(a) $\sqrt{-81}=i\sqrt{81}=9i$

(b) $\sqrt{-55}=i\sqrt{55}$

(c) $\sqrt{-98}=i\sqrt{98}=i\sqrt{49\cdot 2}=7i\sqrt{2}$

Classroom Example 2 (page 105)

(a) $\sqrt{-21}\cdot\sqrt{-21}=i\sqrt{21}\cdot i\sqrt{21}=i^2\left(\sqrt{21}\right)^2=-21$

(b) $\sqrt{-5}\cdot\sqrt{-30}=i\sqrt{5}\cdot i\sqrt{30}=i^2\sqrt{150}$
$=i^2\sqrt{25\cdot 6}=-5\sqrt{6}$

(c) $\dfrac{\sqrt{-42}}{\sqrt{-3}}=\dfrac{i\sqrt{42}}{i\sqrt{3}}=\sqrt{\dfrac{42}{3}}=\sqrt{14}$

(d) $\dfrac{\sqrt{-63}}{\sqrt{21}}=\dfrac{i\sqrt{63}}{\sqrt{21}}=i\sqrt{\dfrac{63}{21}}=i\sqrt{3}$

Classroom Example 3 (page 105)

$$\frac{15-\sqrt{-75}}{5}=\frac{15-i\sqrt{75}}{5}=\frac{15-i\sqrt{25\cdot 3}}{5}$$
$$=\frac{15-5i\sqrt{3}}{5}=\frac{5\left(3-i\sqrt{3}\right)}{5}=3-i\sqrt{3}$$

Classroom Example 4 (page 106)

(a) $(4-5i)+(-5+8i)=-1+3i$

(b) $(-6+3i)+(12-9i)=6-6i$

(c) $(-10+7i)-(5-3i)=-15+10i$

(d) $(15-8i)-(-10+4i)=25-12i$

Classroom Example 5 (page 107)

(a)
$$\begin{aligned}&(5+3i)(2-7i)\\&=10+5(-7i)+(3i)(2)+(3i)(-7i)\\&=10-35i+6i-21i^2\\&=10-29i+21=31-29i\end{aligned}$$

(b)
$$\begin{aligned}(4-5i)^2&=4^2-2(4)(5i)+(5i)^2\\&=16-40i+25i^2\\&=16-40i-25=-9-40i\end{aligned}$$

(c)
$$\begin{aligned}(9-8i)(9+8i)&=9^2-(8i)^2 \quad \text{difference of two squares}\\&=81-64i^2\\&=81-64(-1)\\&=81+64=145,\ \text{or } 145+0i\end{aligned}$$

Classroom Example 6 (page 107)

(a) $i^{33}=i^{32}\cdot i=\left(i^4\right)^8\cdot i=1^8\cdot i=i$

(b)
$$\begin{aligned}i^{-14}&=i^{-16}\cdot i^2=\left(i^{16}\right)^{-1}\cdot i^2=\left(\left(i^4\right)^4\right)^{-1}\cdot i^2\\&=1^{-1}\cdot i^2=i^2=-1\end{aligned}$$

Classroom Example 7 (page 108)

(a)
$$\begin{aligned}\frac{5-5i}{3+i}&=\frac{5-5i}{3+i}\cdot\frac{3-i}{3-i}=\frac{15-5i-15i+5i^2}{9-i^2}\\&=\frac{15-20i-5}{9-(-1)}=\frac{10-20i}{10}=1-2i\end{aligned}$$

(b) $\frac{15}{-i}=\frac{15}{-i}\cdot\frac{i}{i}=\frac{15i}{-i^2}=\frac{15i}{1}=15i,\ \text{or } 0+15i$

Section 1.4: Quadratic Equations

Classroom Example 1 (page 111)

$$\begin{aligned}10x^2+x-2&=0\\(2x+1)(5x-2)&=0\end{aligned}$$
$$\begin{array}{lcl}2x+1=0 & \text{or} & 5x-2=0\\ 2x=-1 & \text{or} & 5x=2\\ x=-\frac{1}{2} & \text{or} & x=\frac{2}{5}\end{array}$$

Solution set: $\left\{-\frac{1}{2},\frac{2}{5}\right\}$

Classroom Example 2 (page 112)

(a) $x^2=29\Rightarrow x=\pm\sqrt{29}$

(b) $x^2=-144\Rightarrow x=\pm 12i$

(c) $(x-8)^2=24\Rightarrow x-8=\pm\sqrt{24}\Rightarrow$
$x-8=\pm 2\sqrt{6}\Rightarrow x=8\pm 2\sqrt{6}$

Classroom Example 3 (page 113)

$$\begin{aligned}x^2+10x-20&=0\\x^2+10x&=20\\x^2+10x+\left(\tfrac{1}{2}\cdot 10\right)^2&=20+\left(\tfrac{1}{2}\cdot 10\right)^2\\x^2+10x+25&=20+25\\(x+5)^2&=45\\x+5&=\pm\sqrt{45}=\pm 3\sqrt{5}\\x&=-5\pm 3\sqrt{5}\end{aligned}$$

Classroom Example 4 (page 113)

$$\begin{aligned}4x^2+6x+5&=0\\x^2+\tfrac{6}{4}x+\tfrac{5}{4}&=0\\x^2+\tfrac{6}{4}x&=-\tfrac{5}{4}\\x^2+\tfrac{6}{4}x+\left(\tfrac{1}{2}\cdot\tfrac{6}{4}\right)^2&=-\tfrac{5}{4}+\left(\tfrac{1}{2}\cdot\tfrac{6}{4}\right)^2\\x^2+\tfrac{3}{2}x+\tfrac{9}{16}&=-\tfrac{5}{4}+\tfrac{9}{16}\\\left(x+\tfrac{3}{4}\right)^2&=-\tfrac{11}{16}\\x+\tfrac{3}{4}&=\pm i\sqrt{\tfrac{11}{16}}=\pm\tfrac{\sqrt{11}}{4}i\\x&=-\tfrac{3}{4}\pm\tfrac{\sqrt{11}}{4}i\end{aligned}$$

Classroom Example 5 (page 115)

$$x^2+6x=3 \Rightarrow x^2+6x-3=0$$
$$a=1, b=6, c=-3$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-6\pm\sqrt{6^2-4(1)(-3)}}{2(1)}$$
$$=\frac{-6\pm\sqrt{36+12}}{2}=\frac{-6\pm\sqrt{48}}{2}$$
$$=\frac{-6\pm4\sqrt{3}}{2}=-3\pm2\sqrt{3}$$

Classroom Example 6 (page 115)

$$4x^2=3x-5 \Rightarrow 4x^2-3x+5=0$$
$$a=4, b=-3, c=5$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-3)\pm\sqrt{(-3)^2-4(4)(5)}}{2(4)}$$
$$=\frac{3\pm\sqrt{9-80}}{8}=\frac{3\pm\sqrt{-71}}{8}=\frac{3\pm i\sqrt{71}}{8}$$
$$=\frac{3}{8}\pm\frac{\sqrt{71}}{8}i$$

Classroom Example 7 (page 116)

$$x^3-125=(x-5)\left(x^2+5x+25\right)$$ difference of cubes (section R.4)

$$x-5=0 \quad \text{or} \quad x^2+5x+25=0$$
$$x=5 \quad \text{or} \quad x=\frac{-5\pm\sqrt{5^2-4(1)(25)}}{2(1)}$$
$$x=\frac{-5\pm\sqrt{25-100}}{2}$$
$$x=\frac{-5\pm\sqrt{-75}}{2}$$
$$x=\frac{-5\pm5\sqrt{3}i}{2}$$
$$x=-\frac{5}{2}\pm\frac{5\sqrt{3}}{2}i$$

Classroom Example 8 (page 116)

(a) $V=\frac{1}{3}\pi r^2h \Rightarrow 3V=\pi r^2h \Rightarrow \frac{3V}{\pi h}=r^2 \Rightarrow$
$r=\pm\sqrt{\frac{3V}{\pi h}} \Rightarrow r=\pm\sqrt{\frac{3V}{\pi h}}\cdot\frac{\sqrt{\pi h}}{\sqrt{\pi h}} \Rightarrow$
$r=\frac{\pm\sqrt{3V\pi h}}{\pi h}$

(b) $2my^2-ny=3p \Rightarrow 2my^2-ny-3p=0$
Use the quadratic formula with $a = 2m$, $b = -n$, and $c = -3p$ to solve for y:
$$y=\frac{-(-n)\pm\sqrt{(-n)^2-4(2m)(-3p)}}{2(2m)}$$
$$=\frac{n\pm\sqrt{n^2+24mp}}{4m}$$

Classroom Example 9 (page 118)

(a) $4x^2-12x+9=0$
$a=4, b=-12, c=9$
$b^2-4ac=(-12)^2-4(4)(9)=0$
Therefore, there is one distinct rational solution.

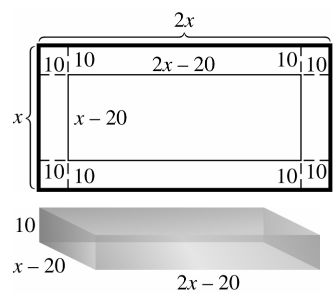
(b) $3x^2+x=-5 \Rightarrow 3x^2+x+5=0$
$a=3, b=1, c=5$
$b^2-4ac=1^2-4(3)(5)=-59$
Therefore, there are two distinct nonreal complex solutions.

(c) $2x^2=6x+7 \Rightarrow 2x^2-6x-7=0$
$a=2, b=-6, c=-7$
$b^2-4ac=(-6)^2-4(2)(-7)=92$
Therefore, there are two distinct irrational solutions.

Section 1.5: Applications and Modeling with Quadratic Equations

Classroom Example 1 (page 122)

Let x = the width of the rectangle. Then $2x$ = the length of the rectangle. The box is formed by cutting 10 cm + 10 cm from both the length and the width. Thus the width of the bottom of the box is $x - 20$, the length of the bottom of the box is $2x - 20$, and the height of the box is 10 cm.



$$V = lwh$$
$$7500 = (2x-20)(x-20)(10)$$
$$7500 = 20x^2 - 600x + 4000$$
$$0 = 20x^2 - 600x - 3500$$
$$0 = x^2 - 30x - 175$$
$$0 = (x+5)(x-35)$$
$$x+5=0 \quad \text{or} \quad x-35=0$$
$$x=-5 \quad \text{or} \quad x=35$$

Reject the negative solution because length cannot be negative. The dimensions of the box are 35 cm by 70 cm.

Classroom Example 2 (page 123)

Let s = the length of the shorter leg.
Then $2s - 10$ = the length of the longer leg, and $(2s - 10) + 20 = 2s + 10$ = the length of the hypotenuse. Using the Pythagorean theorem, we have

$$s^2 + (2s-10)^2 = (2s+10)^2$$
$$s^2 + 4s^2 - 40s + 100 = 4s^2 + 40s + 100$$
$$5s^2 - 40s + 100 = 4s^2 + 40s + 100$$
$$s^2 - 80s = 0$$
$$s(s-80) = 0$$

$s = 0$ or $s - 80 = 0 \Rightarrow s = 80$. Since length cannot be 0, we reject that solution. The shorter leg is 80 m, the longer leg is $2(80) - 10 = 150$ m, and the hypotenuse is $2(80) + 10 = 170$ m.

Classroom Example 3 (page 124)

(a) $s = -4.9t^2 + 73.5t$
$100 = -4.9t^2 + 73.5t \Rightarrow$
$4.9t^2 - 73.5t + 100 = 0$
Solve using the quadratic formula, with $a = 4.9$, $b = -73.5$, and $c = 100$.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-73.5) \pm \sqrt{(-73.5)^2 - 4(4.9)(100)}}{2(4.9)} = \frac{73.5 \pm \sqrt{5402.25 - 1960}}{9.8} = \frac{73.5 \pm \sqrt{3442.25}}{9.8} \approx 1.51 \text{ or } 13.49$$

The projectile will be 100 m above the ground after 1.51 sec and after 13.49 sec.

(b) When the projectile returns to the ground, the height s will be 0 ft, so let $s = 0$ and solve for t:
$0 = -4.9t^2 + 73.5t \Rightarrow$
$t(-4.9t + 73.5) = 0 \Rightarrow t = 0$ or
$-4.9t + 73.5 = 0 \Rightarrow t = 15$
The first solution, 0, represents the time at which the projectile was on the ground before being launched. The projectile will return to the ground 15 sec after being launched.

Classroom Example 4 (page 126)

(a) $S = .016x^2 + .124x + .787$
For 2000, $x = 10$.
$S = .016(10)^2 + .124(10) + .787 \approx 3.6$ million
For 2001, $x = 11$
$S = .016(11)^2 + .124(11) + .787 \approx 4.1$ million
For 2000, the prediction is equal to the actual figure of 3.6 million. For 2001, the prediction is greater than the actual figure of 3.7 million.

(b) Let $S = 3$, then solve for x:
$3 = .016x^2 + .124x + .787 \Rightarrow$
$0 = .016x^2 + .124x - 2.213$

$$x = \frac{-.124 \pm \sqrt{.124^2 - 4(.016)(-2.213)}}{2(.016)}$$

$x \approx 8.5$ or $x \approx -16.3$
Reject the negative solution. According to the model, sales reached 3 million about 8 years after 1990, or 1998.

Section 1.6: Other Types of Equations and Applications

Classroom Example 1 (page 133)

(a) $\dfrac{2x-3}{2} + \dfrac{5x}{x+1} = x$
The least common denominator is $2(x + 1)$, which equals 0 when $x = -1$. Therefore, -1 cannot be a solution of the equation.

$$2(x+1)\left(\frac{2x-3}{2} + \frac{5x}{x+1}\right) = 2(x+1)x$$
$$(x+1)(2x-3) + 2(5x) = 2x^2 + 2x$$
$$2x^2 - 3x + 2x - 3 + 10x = 2x^2 + 2x$$
$$2x^2 + 9x - 3 = 2x^2 + 2x$$
$$7x = 3$$
$$x = \tfrac{3}{7}$$

The restriction $x \neq -1$ does not affect the result. Therefore, the solution set is $\left\{\tfrac{3}{7}\right\}$.

(b) $\dfrac{x}{x-5}+5=\dfrac{5}{x-5}$

The least common denominator is $x-5$, which equals 0 when $x = 5$. Therefore, 5 cannot be a solution of the equation.

$$(x-5)\left(\frac{x}{x-5}+5\right)=(x-5)\left(\frac{5}{x-5}\right)$$
$$x+5(x-5)=5$$
$$6x-25=5 \Rightarrow 6x=30 \Rightarrow x=5$$

The only possible solution is 5. However, the variable is restricted to real numbers except 5. Therefore, the solution set is $\varnothing$.

Classroom Example 2 (page 134)

(a) $\dfrac{x-5}{x-3}+\dfrac{1}{x}=\dfrac{-7}{x^2-3x}$

The least common denominator is $x(x-3)=x^2-3x$, which is equal to 0 if $x = 3$ or $x = 0$. Therefore, 0 and 3 cannot possibly be solutions of this equation.

$$x(x-3)\left(\frac{x-5}{x-3}+\frac{1}{x}\right)=x(x-3)\left(\frac{-7}{x^2-3x}\right)$$
$$x(x-5)+(x-3)=-7$$
$$x^2-5x+x-3=-7$$
$$x^2-4x+4=0$$
$$(x-2)(x-2)=0 \Rightarrow x=2$$

The restrictions $x \neq 0$ or $x \neq 3$ do not affect the result. Therefore, the solution set is $\{2\}$.

(b) $\dfrac{x}{x+5}+\dfrac{5}{x-5}=\dfrac{50}{x^2-25}$

The least common denominator is $(x+5)(x-5)=x^2-25$, which is equal to 0 if $x=\pm 5$.

$$(x+5)(x-5)\left(\frac{x}{x+5}+\frac{5}{x-5}\right)$$
$$=(x+5)(x-5)\left(\frac{50}{x^2-25}\right)$$
$$x(x-5)+5(x+5)=50$$
$$x^2-5x+5x+25=50$$
$$x^2=25 \Rightarrow x=\pm 5$$

Because of the restriction $x \neq \pm 5$ the solution set is $\varnothing$.

Classroom Example 3 (page 135)

Let x = the amount of time (in hours) it takes Lisa and Keith to rake the leaves.

	r	t	Part of the Job Accomplished
Lisa	$\frac{1}{5}$	x	$\frac{1}{5}x$
Keith	$\frac{1}{4}$	x	$\frac{1}{4}x$

Since Lisa and Keith must accomplish 1 job (raking the leaves), we must solve the following equation.

$$\tfrac{1}{5}x+\tfrac{1}{4}x=1$$
$$20\left[\tfrac{1}{5}x+\tfrac{1}{4}x\right]=20\cdot 1$$
$$4x+5x=20 \Rightarrow 9x=20 \Rightarrow x=\frac{20}{9}=2\tfrac{2}{9}$$

It takes Lisa and Keith $2\frac{2}{9}$ hr working together to rake the leaves.

Classroom Example 4 (page 138)

$$x-\sqrt{4x+12}=0$$
$$x=\sqrt{4x+12}$$
$$x^2=4x+12$$
$$x^2-4x-12=0$$
$$(x-6)(x+2)=0 \Rightarrow x=6 \text{ or } x=-2$$

Check $x = -2$:

$$-2-\sqrt{4(-2)+12}\stackrel{?}{=}0$$
$$-2-\sqrt{-8+12}=0$$
$$-2-\sqrt{4}=0$$
$$-2-2=0$$
$$-4\neq 0$$

Thus, -2 is not a solution.

Check $x = 6$:

$$6-\sqrt{4(6)+12}\stackrel{?}{=}0$$
$$6-\sqrt{24+12}=0$$
$$6-\sqrt{36}=0$$
$$6-6=0$$
$$0=0$$

Thus, 6 is a solution.

Solution set: $\{6\}$

Classroom Example 5 (page 138)

$$\sqrt{3x+1}-\sqrt{x+4}=1$$
$$\sqrt{3x+1}=\sqrt{x+4}+1$$
$$\left(\sqrt{3x+1}\right)^2=\left(\sqrt{x+4}+1\right)^2$$
$$3x+1=\left(\sqrt{x+4}\right)^2+2\sqrt{x+4}+1$$
$$3x+1=x+4+2\sqrt{x+4}+1$$
$$2x-4=2\sqrt{x+4}$$
$$(2x-4)^2=\left(2\sqrt{x+4}\right)^2$$
$$4x^2-16x+16=4x+16$$
$$4x^2-20x=0$$
$$x^2-5x=0\Rightarrow x(x-5)=0\Rightarrow x=0 \text{ or } x=5$$

Check $x = 0$:

$$\sqrt{3(0)+1}-\sqrt{(0)+4}\stackrel{?}{=}1$$
$$1-2=1$$
$$-1\neq 1$$

Thus, $x = 0$ is not a solution.
Check $x = 5$:

$$\sqrt{3(5)+1}-\sqrt{(5)+4}\stackrel{?}{=}1$$
$$4-3=1\Rightarrow 1=1$$

Thus, $x = 5$ is a solution. The solution set is $\{5\}$.

Classroom Example 6 (page 139)

$$\sqrt[3]{5x^2-12x+6}-\sqrt[3]{x}=0$$
$$\sqrt[3]{5x^2-12x+6}=\sqrt[3]{x}$$
$$\left(\sqrt[3]{5x^2-12x+6}\right)^3=\left(\sqrt[3]{x}\right)^3$$
$$5x^2-12x+6=x$$
$$5x^2-13x+6=0$$
$$(5x-3)(x-2)=0\Rightarrow x=\tfrac{3}{5} \text{ or } x=2$$

Check $x=\frac{3}{5}$:

$$\sqrt[3]{5\left(\tfrac{3}{5}\right)^2-12\left(\tfrac{3}{5}\right)+6}-\sqrt[3]{\tfrac{3}{5}}\stackrel{?}{=}0$$
$$\sqrt[3]{\tfrac{9}{5}-\tfrac{36}{5}+6}-\sqrt[3]{\tfrac{3}{5}}=0$$
$$\sqrt[3]{\tfrac{3}{5}}-\sqrt[3]{\tfrac{3}{5}}=0\Rightarrow 0=0$$

Thus, $x=\frac{3}{5}$ is a solution.

Check $x = 2$:

$$\sqrt[3]{5(2)^2-12(2)+6}-\sqrt[3]{2}\stackrel{?}{=}0$$
$$\sqrt[3]{20-24+6}-\sqrt[3]{2}=0$$
$$\sqrt[3]{2}-\sqrt[3]{2}=0\Rightarrow 0=0$$

Thus, $x = 2$ is a solution.

Solution set: $\left\{\frac{3}{5},2\right\}$

Classroom Example 7 (page 140)

(a) $(x-3)^{1/2}-6(x-3)^{1/4}+8=0$

Let $u=(x-3)^{1/4}$, so

$$u^2=\left[(x-3)^{1/4}\right]^2=(x-3)^{1/2}.$$

Substituting, we have

$$u^2-6u+8=0$$
$$(u-4)(u-2)=0\Rightarrow u=4 \text{ or } u=2$$

Now solve for *x*, by replacing *u* with $(x-3)^{1/4}$:

$$4=(x-3)^{1/4}\Rightarrow 4^4=x-3\Rightarrow 259=x$$
$$2=(x-3)^{1/4}\Rightarrow 2^4=x-3\Rightarrow 19=x$$

Check $x = 259$:

$$(259-3)^{1/2}-6(259-3)^{1/4}+8\stackrel{?}{=}0$$
$$\sqrt{256}-6\sqrt[4]{256}+8=0$$
$$16-6(4)+8=0$$
$$0=0$$

Thus, $x = 259$ is a solution.
Check $x = 19$:

$$(19-3)^{1/2}-6(19-3)^{1/4}+8\stackrel{?}{=}0$$
$$\sqrt{16}-6\sqrt[4]{16}+8=0$$
$$4-6(2)+8=0$$
$$0=0$$

Thus, $x = 19$ is a solution.
Solution set: $\{19, 259\}$.

(b) $15x^{-2}-4x^{-1}=3\Rightarrow 15x^{-2}-4x^{-1}-3=0$

Let $u=x^{-1}$. Substituting, we have

$$15u^2-4u-3=0$$
$$(5u-3)(3u+1)=0\Rightarrow u=\frac{3}{5} \text{ or } u=-\frac{1}{3}$$

Now solve for *x*, by replacing *u* with x^{-1}:

$$x^{-1}=\frac{3}{5}\Rightarrow x=\frac{5}{3};\ x^{-1}=-\frac{1}{3}\Rightarrow x=-3$$

Check $x=\frac{5}{3}$:

$$15\left(\frac{5}{3}\right)^{-2}-4\left(\frac{5}{3}\right)^{-1}\stackrel{?}{=}3$$
$$15\left(\frac{3}{5}\right)^2-4\left(\frac{3}{5}\right)=3\Rightarrow 15\left(\frac{9}{25}\right)-\frac{12}{5}=3$$
$$\frac{27}{5}-\frac{12}{5}=3\Rightarrow 3=3$$

Thus, $x=\frac{5}{3}$ is a solution.

(*continued on next page*)

(*continued from page 15*)

Check $x=-3$:

$15(-3)^{-2}-4(-3)^{-1}\stackrel{?}{=}3$

$15\left(-\frac{1}{3}\right)^2-4\left(-\frac{1}{3}\right)=3 \Rightarrow 15\left(\frac{1}{9}\right)+\frac{4}{3}=3$

$\frac{5}{3}+\frac{4}{3}=3 \Rightarrow 3=3$

Thus, $x=-3$ is a solution.

Solution set: $\left\{-3,\frac{5}{3}\right\}$

Classroom Example 8 (page 141)

$18x^4-29x^2+3=0$

Let $u=x^2$; then $u^2=x^4$. With this substitution, the equation becomes $18u^2-29u+3=0$. Solving, we have $18u^2-29u+3=0$

$(2u-3)(9u-1)=0 \Rightarrow u=\frac{3}{2}$ or $u=\frac{1}{9}$

Now solve for x: $x^2=\frac{3}{2} \Rightarrow x=\pm\sqrt{\frac{3}{2}}=\pm\frac{\sqrt{6}}{2}$ or

$x^2=\frac{1}{9} \Rightarrow x=\pm\sqrt{\frac{1}{9}}=\pm\frac{1}{3}$. Checking each proposed solution, we find that all are solutions.

Solution set: $\left\{\pm\frac{\sqrt{6}}{2},\pm\frac{1}{3}\right\}$

Classroom Example 9 (page 142)

$x=\left(10x^2-24\right)^{1/4}$

$x^4=10x^2-24$

$x^4-10x^2+24=0$

Let $u=x^2$, then substitute:

$u^2-10u+24=0$

$(u-6)(u-4)=0 \Rightarrow u=6$ or $u=4$

Now solve for x:

$x^2=6 \Rightarrow x=\pm\sqrt{6}$

$x^2=4 \Rightarrow x=\pm 2$

Check $x=-\sqrt{6}$:

$-\sqrt{6}\stackrel{?}{=}\left(10\left(-\sqrt{6}\right)^2-24\right)^{1/4}$

$-\sqrt{6}=(36)^{1/4}$

$-\sqrt{6}\neq\sqrt{6}$

Thus, $x=-\sqrt{6}$ is not a solution.

Check $x=\sqrt{6}$:

$\sqrt{6}\stackrel{?}{=}\left(10\left(\sqrt{6}\right)^2-24\right)^{1/4}$

$\sqrt{6}=(36)^{1/4}$

$\sqrt{6}=\sqrt{6}$

Thus, $x=\sqrt{6}$ is not a solution.

Check $x=-2$:

$-2\stackrel{?}{=}\left(10(-2)^2-24\right)^{1/4}$

$-2=(16)^{1/4}$

$-2\neq 2$

Thus, $x=-2$ is not a solution.

Check $x=2$:

$2\stackrel{?}{=}\left(10(2)^2-24\right)^{1/4}$

$2=(16)^{1/4}$

$2=2$

Thus $x=2$.

Solution set: $\left\{\sqrt{6},2\right\}$

Section 1.7: Inequalities

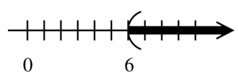
Classroom Example 1 (page 146)

$-2x+7<-5$

$-2x<-12$

$x>6$

Solution set: $\{x\,|\,x>6\}$



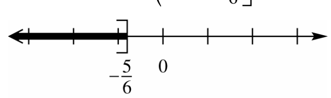
Classroom Example 2 (page 147)

$3-4x\geq 2x+8$

$-5\geq 6x$

$-\frac{5}{6}\geq x \Rightarrow x\leq-\frac{5}{6}$

Solution set: $\left(-\infty,-\frac{5}{6}\right]$



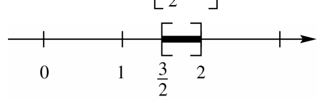
Classroom Example 3 (page 148)

$1\leq 6x-8\leq 4$

$9\leq 6x\leq 12$

$\frac{3}{2}\leq x\leq 2$

Solution set: $\left[\frac{3}{2},2\right]$

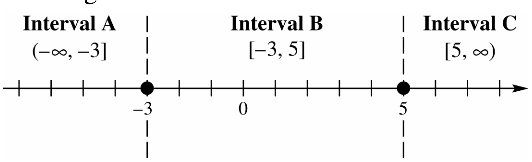


Classroom Example 4 (page 148)

$R = 45x$ and $C = 30x + 5250$
Set $R \geq C$, and solve for x:
$45x \geq 30x + 5250$
$15x \geq 5250$
$x \geq 350$
The break-even point is at $x = 350$. This product will at least break even only if the number of units produced and sold is in the interval $[350, \infty)$.

Classroom Example 5 (page 149)

$x^2 - 2x - 15 \leq 0$
Step 1: Find the values of x that satisfy
$x^2 - 2x - 15 = 0$.
$x^2 - 2x - 15 = 0 \Rightarrow (x-5)(x+3) = 0 \Rightarrow$
$x = 5$ or $x = -3$
Step 2: The two numbers divide a number line into three regions.



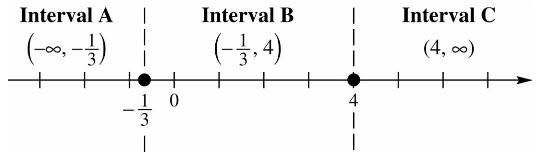
Step 3: Choose a test value to see if it satisfies the inequality $x^2 - 2x - 15 \leq 0$.

Interval	Test Value	Is $x^2 - 2x - 15 \leq 0$ True or False?
A: $(-\infty, -3]$	−5	$(-5)^2 - 2(-5) - 15 \overset{?}{\leq} 0$ $20 \leq 0$ False
B: $[-3, 5]$	0	$0^2 - 2(0) - 15 \overset{?}{\leq} 0$ $-15 \leq 0$ True
C: $[5, \infty)$	6	$6^2 - 2(6) - 15 \overset{?}{\leq} 0$ $9 \leq 0$ False

Solution set: $[-3, 5]$

Classroom Example 6 (page 150)

$3x^2 - 11x - 4 > 0$
Step 1: Find the values of x that satisfy
$3x^2 - 11x - 4 = 0$.
$3x^2 - 11x - 4 = 0 \Rightarrow (3x+1)(x-4) = 0 \Rightarrow$
$x = -\frac{1}{3}$ or $x = 4$
Step 2: The two numbers divide a number line into three regions.



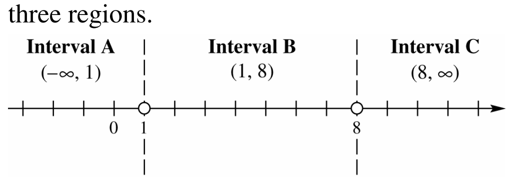
Step 3: Choose a test value to see if it satisfies the inequality $3x^2 - 11x - 4 > 0$.

Interval	Test Value	Is $3x^2 - 11x - 4 > 0$ True or False?
A: $\left(-\infty, -\frac{1}{3}\right)$	−1	$3(-1)^2 - 11(-1) - 4 \overset{?}{>} 0$ $10 > 0$ True
B: $\left(-\frac{1}{3}, 4\right)$	0	$3(0)^2 - 11(0) - 4 \overset{?}{>} 0$ $-4 > 0$ False
C: $(4, \infty)$	5	$3(5)^2 - 11(5) - 4 \overset{?}{>} 0$ $16 > 0$ True

Solution set: $\left(-\infty, -\frac{1}{3}\right) \cup (4, \infty)$

Classroom Example 7 (page 151)

$-16t^2 + 144t > 128$
Step 1: Find the values of x that satisfy
$-16t^2 + 144t = 128$.
$-16t^2 + 144t = 128 \Rightarrow 0 = 16t^2 - 144t + 128 \Rightarrow$
$16(t-8)(t-1) = 0 \Rightarrow t = 8$ or $t = 1$
Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality $-16t^2 + 144t > 128$.

Interval	Test Value	Is $-16t^2 + 144t > 128$ True or False?
A: $(-\infty, 1)$	0	$-16(0)^2 + 144(0) \overset{?}{>} 128$ $0 > 128$ False
B: $(1, 8)$	2	$-16(2)^2 + 144(2) \overset{?}{>} 128$ $224 > 128$ True
C: $(8, \infty)$	10	$-16(10)^2 + 144(10) \overset{?}{>} 128$ $-160 > 128$ False

(*continued on next page*)

(*continued from page 17*)

Solution set: $(1,8)$

The object will be greater than 128 ft above ground level between 1 and 8 seconds after it is launched.

Classroom Example 8 (page 152)

$$\frac{6}{x-3} \ge 4$$

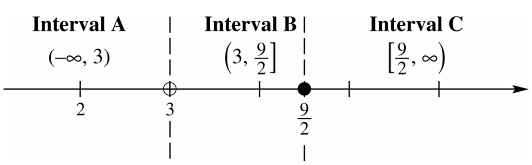
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{6}{x-3} \ge 4 \Rightarrow \frac{6}{x-3} - 4 \ge 0 \Rightarrow$$
$$\frac{6}{x-3} - \frac{4(x-3)}{x-3} \ge 0 \Rightarrow$$
$$\frac{6-4x+12}{x-3} \ge 0 \Rightarrow \frac{18-4x}{x-3} \ge 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$18-4x=0 \Rightarrow x=\frac{9}{2}$ or $x-3=0 \Rightarrow x=3$

The values $\frac{9}{2}$ and 3 divide the number line into three regions. Use an open circle on 3 because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{6}{x-3} \ge 4$

Interval	Test Value	Is $\frac{6}{x-3} \ge 4$ True or False?
A: $(-\infty,3)$	0	$\frac{6}{0-3} \overset{?}{\ge} 4$ $-2 > 4$ False
B: $\left(3,\frac{9}{2}\right]$	4	$\frac{6}{4-3} \overset{?}{\ge} 4$ $6 > 4$ True
C: $\left[\frac{9}{2},\infty\right)$	5	$\frac{6}{5-3} \overset{?}{\ge} 4$ $3 > 4$ False

Solution set: $\left(3,\frac{9}{2}\right]$

Classroom Example 9 (page 153)

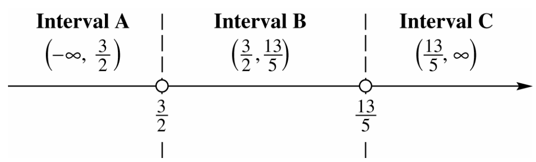
$$\frac{3x+1}{2x-3} < 4$$

Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{3x+1}{2x-3} < 4 \Rightarrow \frac{3x+1}{2x-3} - 4 < 0 \Rightarrow$$
$$\frac{3x+1}{2x-3} - \frac{4(2x-3)}{2x-3} < 0 \Rightarrow$$
$$\frac{3x+1-8x+12}{2x-3} < 0 \Rightarrow$$
$$\frac{-5x+13}{2x-3} < 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-5x+13=0 \Rightarrow x=\frac{13}{5}$ or $2x-3=0 \Rightarrow x=\frac{3}{2}$. The values $\frac{3}{2}$ and $\frac{13}{5}$ divide the number line into three regions. Use an open circle on $\frac{3}{2}$ because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{3x+1}{2x-3} < 4$

Interval	Test Value	Is $\frac{3x+1}{2x-3} < 4$ True or False?
A: $\left(-\infty,\frac{3}{2}\right)$	0	$\frac{3(0)+1}{2(0)-3} \overset{?}{<} 4$ $-\frac{1}{3} < 4$ True
B: $\left(\frac{3}{2},\frac{13}{5}\right)$	2	$\frac{3(2)+1}{2(2)-3} \overset{?}{<} 4$ $7 < 4$ False
C: $\left(\frac{13}{5},\infty\right)$	5	$\frac{3(5)+1}{2(5)-3} \overset{?}{<} 4$ $\frac{16}{7} < 4$ True

Solution set: $\left(-\infty,\frac{3}{2}\right) \cup \left(\frac{13}{5},\infty\right)$

Section 1.8: Absolute Value Equations and Inequalities

Classroom Example 1 (page 159)

(a) $|9-4x|=7$

$9-4x=7 \Rightarrow -4x=-2 \Rightarrow x=\frac{1}{2}$ or
$9-4x=-7 \Rightarrow -4x=-16 \Rightarrow x=4$
Solution set: $\left\{\frac{1}{2},4\right\}$

(b) $|3x+2| = |x-5|$

$3x+2 = x-5 \Rightarrow 2x = -7 \Rightarrow x = -\frac{7}{2}$ or
$3x+2 = -(x-5) \Rightarrow 3x+2 = -x+5 \Rightarrow$
$4x = 3 \Rightarrow x = \frac{3}{4}$

Solution set: $\left\{-\frac{7}{2}, \frac{3}{4}\right\}$

Classroom Example 2 (page 160)

(a) $|4x-6| < 10$

$-10 < 4x-6 < 10 \Rightarrow -4 < 4x < 16 \Rightarrow$
$-1 < x < 4$

Solution set: $(-1, 4)$

(b) $|4x-6| > 10$

$$\begin{aligned} 4x-6 &< -10 \quad \text{or} \quad 4x-6 > 10 \\ 4x &< -4 \quad \text{or} \quad 4x > 16 \\ x &< -1 \quad \text{or} \quad x > 4 \end{aligned}$$

Solution set: $(-\infty, -1) \cup (4, \infty)$

Classroom Example 3 (page 161)

$$\begin{aligned} |5-8x| + 6 &\geq 14 \\ |5-8x| &\geq 8 \end{aligned}$$

$$\begin{aligned} 5-8x &\leq -8 \quad \text{or} \quad 5-8x \geq 8 \\ -8x &\leq -13 \quad \text{or} \quad -8x \geq 3 \\ x &\geq \tfrac{13}{8} \quad \text{or} \quad x \leq -\tfrac{3}{8} \end{aligned}$$

Solution set: $\left(-\infty, -\frac{3}{8}\right] \cup \left[\frac{13}{8}, \infty\right)$

Classroom Example 4 (page 161)

(a) $|7x+28| = 0$

The absolute value of a number will be 0 if that number is 0. Therefore $|7x+28| = 0$ is equivalent to $7x+28 = 0,$ which has solution set $\{-4\}$.

(b) $|6x-9| > -2$

Since the absolute value of a number is always nonnegative, the inequality $|6x-9| > -2$ is always true. The solution set is $(-\infty, \infty)$.

(c) $|2-5x| \leq -5$

There is no number whose absolute value is less than any negative number. The solution set of $|2-5x| \leq -5$ is $\varnothing$.

Classroom Example 5 (page 162)

(a) "m is no more than 9 units from 3" means that m is 9 units or less from 3. Thus the distance between m and 3 is less than or equal to 9, or $|m-3| \leq 9$.

(b) "t is within .02 unit of 5.8" means that t is less than .02 unit from 5.8. Thus the distance between t and 5.8 is less than .02, or $|t-5.8| < .02$.

Classroom Example 6 (page 162)

Since we want y to be within .001 unit of 6, we have $|y-6| < .001$ or $|(5x-2)-6| < .001$.

$$\begin{aligned} |5x-8| &< .001 \\ -.001 < 5x-8 &< .001 \\ 7.999 < 5x &< 8.001 \\ 1.5998 < x &< 1.6002 \end{aligned}$$

Values of x in the interval $(1.5998, 1.6002)$ will satisfy the condition.