# Chapter 2
## Investment Decisions: The Certainty Case

1. (a) Cash flows adjusted for the depreciation tax shelter

| | |
|---|---|
| Sales = cash inflows | $140,000 |
| Operating costs = cash outflows | 100,000 |
| Earnings before depreciation, interest and taxes | 40,000 |
| Depreciation (Dep) | 10,000 |
| EBIT | 30,000 |
| Taxes @ 40% | 12,000 |
| Net income | $18,000 |

Using equation 2-13:

$$\begin{aligned} CF &= (\Delta Rev - \Delta VC)(1-\tau_c) + \tau_c \Delta dep \\ &= (140,000 - 100,000)(1-.4) + .4(10,000) = 28,000 \end{aligned}$$

Alternatively, equation 2-13a can be used:

$$\begin{aligned} CF &= \Delta NI + \Delta dep + (1-\tau_c)\Delta\ k_d D \\ &= 18,000 + 10,000 + (1-.4)(0) = 28,000 \end{aligned}$$

(b) Net present value using straight-line depreciation

$$\begin{aligned} NPV &= \sum_{t=1}^{N} \frac{(Rev_t - VC_t)(1-\tau_c) + \tau_c(dep_t)}{(1+\ WACC)^t} - I_0 \\ &= \text{(annual cash inflow) (present value annuity factor @12\%, 10 years)} - I_0 \\ &= (5.650)(28,000) - 100,000 \\ &= 158,200 - 100,000 \\ &= 58,200 \end{aligned}$$

2. (a)

| | |
|---|---|
| Earnings before depreciation, interest and taxes | $22,000 |
| Depreciation (straight-line) | 10,000 |
| EBIT | 12,000 |
| Taxes @ 40% | 4,800 |
| Net income | $7,200 |

Net present value using straight-line depreciation

$$\begin{aligned} CF &= (\Delta Rev - \Delta VC)(1-\tau_c) + \tau_c \Delta dep \\ &= (22{,}000)(1-.4) + .4(10{,}000) = 17{,}200 \end{aligned}$$

$$\begin{aligned} NPV &= \sum_{t=1}^{N} \frac{CF_t}{(1+WACC)^t} - I_0 \\ &= \text{(annual cash flow) (present value annuity factor @ 12\%, 10 years)} - I_0 \\ &= 17{,}200(5.650) - 100{,}000 \\ &= 97{,}180 - 100{,}000 = -2{,}820 \end{aligned}$$

(b) NPV using sum-of-years digits accelerated depreciation

In each year the depreciation allowance is:

$$Dep_t = \frac{T+1-t}{\sum_{i=1}^{T} i} = \frac{T+1-t}{55}, \text{ where } T = 10$$

In each year the cash flows are as given in the table below:

| **(1) Year** | **(2) $Rev_t - VC_t$** | **(3) $Dep_t$** | **(4) $(Rev_t - VC_t)(1-\tau_c) + \tau_c dep$** | **(5) PV Factor** | **(6) PV** |
|---|---|---|---|---|---|
| 1 | 22,000 | (10/55)100,000 | 13,200 + 7,272.72 | .893 | 18,282.14 |
| 2 | 22,000 | (9/55)100,000 | 13,200 + 6,545.45 | .797 | 15,737.12 |
| 3 | 22,000 | (8/55)100,000 | 13,200 + 5,818.18 | .712 | 13,540.94 |
| 4 | 22,000 | (7/55)100,000 | 13,200 + 5,090.91 | .636 | 11,633.02 |
| 5 | 22,000 | (6/55)100,000 | 13,200 + 4,363.64 | .567 | 9,958.58 |
| 6 | 22,000 | (5/55)100,000 | 13,200 + 3,636.36 | .507 | 8,536.03 |
| 7 | 22,000 | (4/55)100,000 | 13,200 + 2,909.09 | .452 | 7,281.31 |
| 8 | 22,000 | (3/55)100,000 | 13,200 + 2,181.82 | .404 | 6,214.26 |
| 9 | 22,000 | (2/55)100,000 | 13,200 + 1,454.54 | .361 | 5,290.29 |
| 10 | 22,000 | (1/55)100,000 | 13,200 + 727.27 | .322 | 4,484.58 |
| | | | | | 100,958.27 |

$$NPV = PV \text{ of inflows} - I_0$$

$$NPV = 100{,}958.27 - 100{,}000 = 958.27$$

Notice that using accelerated depreciation increases the depreciation tax shield enough to make the project acceptable.

3. Replacement

| | **Amount before Tax** | **Amount after Tax** | **Year** | **PVIF @ 12%** | **Present Value** |
|---|---|---|---|---|---|
| Outflows at t = 0 | | | | | |
| Cost of new equipment | $100,000 | $100,000 | 0 | 1.0 | $100,000 |
| Inflows, years 1–8 | | | | | |
| Savings from new investment | 31,000 | 18,600 | 1–8 | 4.968 | 92,405 |
| Tax savings on depreciation | 12,500 | 5,000 | 1–8 | 4.968 | 24,840 |
| | | | | Present value of inflows = | $117,245 |
| Net present value = $117,245 – 100,000 = $17,245 | | | | | |

If the criterion of a positive NPV is used, buy the new machine.

4. Replacement with salvage value

| | **Amount before Tax** | **Amount after Tax** | **Year** | **PVIF @ 12%** | **Present Value** |
|---|---|---|---|---|---|
| Outflows at t = 0 | | | | | |
| Investment in new machine | $100,000 | $100,000 | 0 | 1.00 | $100,000 |
| Salvage value of old | –15,000 | –15,000 | 0 | 1.00 | –15,000 |
| Tax loss on sale | –25,000 | –10,000 | 0 | 1.00 | –10,000 |
| | | | | Net cash outlay = | $75,000 |
| Inflows, years 1–8 | | | | | |
| Savings from new machine | $31,000 | $18,600 | 1–8 | 4.968 | $92,405 |
| Depreciation saving on new | 11,000 | 4,400 | 1–8 | 4.968 | 21,859 |
| Depreciation lost on old | –5,000 | –2,000 | 1–8 | 4.968 | –9,936 |
| Salvage value of new | 12,000 | 12,000 | 8 | .404 | 4,848 |
| | | | | Net cash inflows = | $109,176 |
| Net present value = $109,176 – 75,000 = $34,176 | | | | | |

Using the NPV rule the machine should be replaced.

5. The correct definition of cash flows for capital budgeting purposes (equation 2-13) is:

$$CF = (\Delta Rev - \Delta VC)(1 - \tau_c) + \tau_c \Delta dep$$

In this problem

Rev = revenues. There is no change in revenues.

VC = cash savings from operations = –3,000

$\tau_c$ = the tax rate = .4

dep = depreciation = 2,000

Therefore, the annual net cash flows for years one through five are

$$CF = 3{,}000(1 - .4) + .4(2{,}000) = 2{,}600$$

The net present value of the project is

$$NPV = -10{,}000 + 2{,}600(2.991) = -2{,}223.40$$

Therefore, the project should be rejected.

6. The NPV at different positive rates of return is[1]

| Discounted Cash Flows | | | | |
|---|---|---|---|---|
| **@ 0%** | **@ 10%** | **@ 15%** | **@ 16%** | **@ 20%** |
| 400 | 363.64 | 347.83 | 344.83 | 333.33 |
| 400 | 330.58 | 302.46 | 297.27 | 277.78 |
| –1,000 | –751.32 | –657.52 | –640.66 | –578.70 |
| –200 | –57.10 | –7.23 | 1.44 | 32.41 |

Figure S2.1 graphs NPV versus the discount rate. The IRR on this project is approximately 15.8 percent.

At an opportunity cost of capital of 10 percent, the project has a negative NPV; therefore, it should be rejected (even though the IRR is greater than the cost of capital).

This is an interesting example which demonstrates another difficulty with the IRR technique; namely, that it does not consider the order of cash flows.

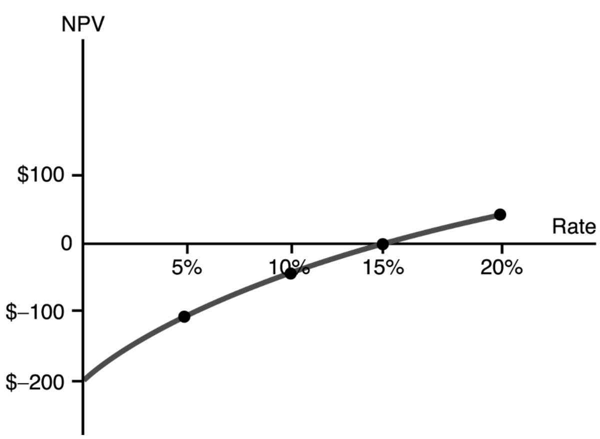


*Figure S2.1* The internal rate of return ignores the order of cash flows

[1]There is a second IRR at –315.75%, but it has no economic meaning. Note also that the function is undefined at IRR = –1.

7. These are the cash flows for project A which was used as an example in section E of the chapter. We are told that the IRR for these cash flows is –200%. But how is this determined? One way is to graph the NPV for a wide range of interest rates and observe which rates give NPV = 0. These rates are the

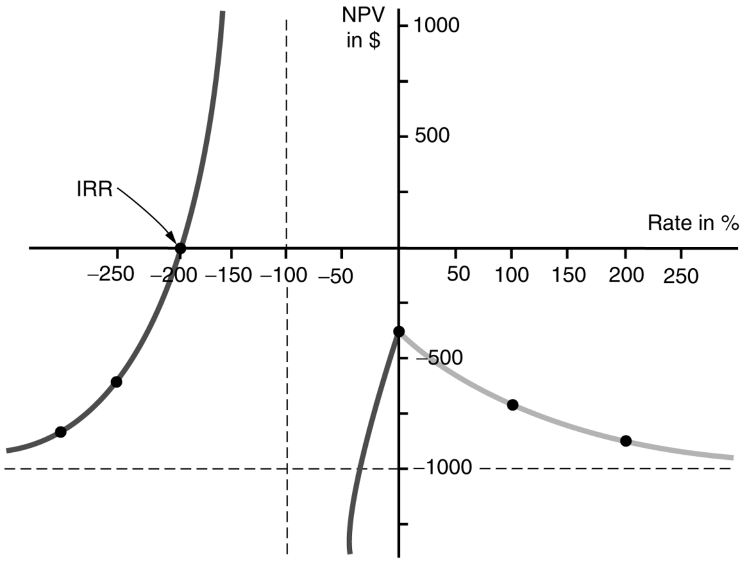


*Figure S2.2* An IRR calculation

internal rates of return for the project. Figure S2.2 plots NPV against various discount rates for this particular set of cash flows. By inspection, we see that the IRR is –200%.

8. All of the information about the financing of the project is irrelevant for computation of the correct cash flows for capital budgeting. Sources of financing, as well as their costs, are included in the computation of the cost of capital. Therefore, it would be "double counting" to include financing costs (or the tax changes which they create) in cash flows for capital budgeting purposes.
   The cash flows are:

$$(\Delta\text{Rev} - \Delta\text{VC} - \Delta\text{FCC} - \Delta\text{dep})(1-\tau_c) + \tau_c\Delta\text{dep} = (200-(-360)-0-0)(1-.4)+.4(400)$$
$$= 336 + 160$$
$$= 496$$

$$\text{NPV} = 496\ (\text{PVIF}_a\text{: 10\%, 3 years})^* - 1{,}200$$
$$= 496\ (2.487) - 1{,}200 = 33.55$$

The project should be accepted.

9. First calculate cash flows for capital budgeting purposes:

$$\text{CF}_t = (\Delta\text{Rev}_t - \Delta\text{VC}_t)(1-\tau_c) + \tau_c\Delta\text{dep}$$
$$= (0-(-290))(1-.5)+.5(180)$$
$$= 145 + 90 = 235$$

* Note: $\text{PVIF}_a$: 10%, 3 years, the discount factor for a three year annuity paid in arrears (at 10%).

Next, calculate the NPV:

$$\begin{aligned} \text{NPV} &= \sum_{t=1}^{5} \frac{\text{CF}_t}{(1+\text{WACC})^t} - I_0 \\ &= (\text{CF}_t)\text{ (present value annuity factor @ 10\%, 5 years) } - I_0 \\ &= 235(3.791) - 900.00 \\ &= 890.89 - 900.00 = -9.12 \end{aligned}$$

The project should be rejected because it has negative net present value.

10. The net present values are calculated below:

| Year | PVIF | A | PV (A) | B | PV (B) | C | PV (C) | A + C | B + C |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1.000 | –1 | –1.00 | –1 | –1.00 | –1 | –1.00 | –2 | –2 |
| 1 | .909 | 0 | 0 | 1 | .91 | 0 | 0 | 0 | 1 |
| 2 | .826 | 2 | 1.65 | 0 | 0 | 0 | 0 | 2 | 0 |
| 3 | .751 | –1 | –.75 | 1 | .75 | 3 | 2.25 | 2 | 4 |
| | | | –.10 | | .66 | | 1.25 | | |

$$\text{NPV}(A + C) = 1.15$$

$$\text{NPV}(B + C) = 1.91$$

Project A has a two-year payback.

Project B has a one-year payback.

Project C has a three-year payback.

Therefore, if projects A and B are mutually exclusive, project B would be preferable according to both capital budgeting techniques.

Project (A + C) has a two-year payback, NPV = $1.15.

Project (B + C) has a three-year payback, NPV = $1.91.

Once Project C is combined with A or B, the results change if we use the payback criterion. Now A + C is preferred. Previously, B was preferred. Because C is an independent choice, it should be irrelevant when considering a choice between A and B. However, with payback, this is not true. Payback violates value additivity. On the other hand, NPV does not. B + C is preferred. Its NPV is simply the sum of the NPV's of B and C separately. Therefore, NPV does obey the value additivity principle.

11. Using the method discussed in section F.3 of this chapter, in the first year the firm invests $5,000 and expects to earn IRR. Therefore, at the end of the first time period, we have

$$5{,}000(1 + \text{IRR})$$

During the second period the firm borrows from the project at the opportunity cost of capital, k.
The amount borrowed is

$$(10{,}000 - 5{,}000(1 + \text{IRR}))$$

By the end of the second time period this is worth

$$(10{,}000 - 5{,}000(1 + \text{IRR}))\,(1 + k)$$

The firm then lends 3,000 at the end of the second time period:

$$3{,}000 = (10{,}000 - 5{,}000(1 + \text{IRR}))\,(1.10)$$

Solving for IRR, we have

$$\frac{\frac{3{,}000}{1.10} - 10{,}000}{-5{,}000} - 1 = \text{IRR} = 45.45\%$$