

2.1

a.
$$\begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{bmatrix}$$

3 x 3, square, symmetric

b.
$$\begin{Bmatrix} x \\ x^2 \\ x^3 \\ x^4 \end{Bmatrix}$$

4 x 1 column

c.
$$\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$$

2 x 2, square, diagonal

d.
$$[1 \quad y \quad y^2 \quad y^3]$$

1 x 4, row

e.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3 x 3, square, diagonal, identity

f.
$$\begin{bmatrix} 3 & -1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 0 & 0 \\ 0 & 4 & 1 & 4 & 0 \\ 0 & 0 & 5 & 4 & 2 \\ 0 & 0 & 0 & 7 & 8 \end{bmatrix}$$

5 x 5, square, banded

g.
$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 x 4, square, upper triangular

h.
$$\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix}$$

4 x 4, square, diagonal

$$\text{a. } [A] + [B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 12 & 3 & -4 \\ 5 & 0 & -4 \end{bmatrix}$$

$$\text{b. } [A] - [B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -3 & -10 \\ -3 & -10 & 10 \end{bmatrix}$$

$$\text{c. } 3[A] = 3 \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 6 & 3 \\ 21 & 0 & -21 \\ 3 & -15 & 9 \end{bmatrix}$$

$$\text{d. } [A][B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 18 & 19 & -5 \\ -21 & -21 & 42 \\ -12 & 2 & -37 \end{bmatrix}$$

$$\text{e. } [A]\{C\} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ -2 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 4 \\ -21 \\ 23 \end{Bmatrix}$$

$$\text{f. } [A]^2 = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 31 & 3 & -7 \\ 21 & 49 & -14 \\ -28 & -13 & 45 \end{bmatrix}$$

$$\text{g. } [I][A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}$$

$$[A][I] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}$$

2.3

$$[A_{11}] = \begin{bmatrix} 5 & 7 & 2 \\ 3 & 8 & -3 \end{bmatrix}$$

$$[A_{12}] = \begin{bmatrix} 0 & 3 & 5 \\ -5 & 0 & 8 \end{bmatrix}$$

$$[A_{21}] = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 10 & 5 \\ 2 & -5 & 9 \end{bmatrix}$$

$$[A_{22}] = \begin{bmatrix} 7 & 15 & 9 \\ 12 & 3 & -1 \\ 2 & 18 & -10 \end{bmatrix}$$

$$\{B_{11}\} = \begin{Bmatrix} 2 \\ 8 \\ -5 \end{Bmatrix}$$

$$[B_{12}] = \begin{bmatrix} 10 & 0 \\ 7 & 5 \\ 2 & -4 \end{bmatrix}$$

$$\{B_{21}\} = \begin{Bmatrix} 4 \\ 3 \\ 1 \end{Bmatrix}$$

$$[B_{22}] = \begin{bmatrix} 8 & 13 \\ 12 & 0 \\ 5 & 7 \end{bmatrix}$$

$$[A_{11}]\{B_{11}\} + [A_{12}]\{B_{21}\} = \begin{Bmatrix} 70 \\ 73 \end{Bmatrix}$$

$$[A_{21}]\{B_{11}\} + [A_{22}]\{B_{21}\} = \begin{Bmatrix} 116 \\ 111 \\ -29 \end{Bmatrix}$$

$$[A_{11}][B_{12}] + [A_{12}][B_{22}] = \begin{bmatrix} 164 & 62 \\ 80 & 43 \end{bmatrix}$$

$$[A_{21}][B_{12}] + [A_{22}][B_{22}] = \begin{bmatrix} 319 & 174 \\ 207 & 179 \\ 185 & -105 \end{bmatrix}$$

$$[A][B] = \begin{bmatrix} 70 & 164 & 62 \\ 73 & 80 & 43 \\ 116 & 319 & 174 \\ 111 & 207 & 179 \\ -29 & 185 & -105 \end{bmatrix}$$

2.4

a.

$$[A] = \begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix}$$

$$[A]^T = \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix}$$

$$[B]^T = \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix}$$

b.

$$\begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix}^T = \begin{bmatrix} 1 & 9 & 1 \\ 5 & 4 & 13 \\ 9 & 5 & -6 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 & 9 \\ 9 & 4 & 5 \\ 1 & 13 & -6 \end{bmatrix}$$

$$[A]^T + [B]^T = \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix} + \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 9 \\ 9 & 4 & 5 \\ 1 & 13 & -6 \end{bmatrix}$$

c.

$$([A][B])^T = \left(\begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} -8 & 17 & 19 \\ 3 & 67 & -11 \\ -7 & 28 & 8 \end{bmatrix}^T = \begin{bmatrix} -8 & 3 & -7 \\ 17 & 67 & 28 \\ 19 & -11 & 8 \end{bmatrix}$$

$$[B]^T[A]^T = \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix} \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix} = \begin{bmatrix} -8 & 3 & -7 \\ 17 & 67 & 28 \\ 19 & -11 & 8 \end{bmatrix}$$

a.

$$\begin{vmatrix} 2 & 10 & 0 \\ 16 & 6 & 14 \\ 12 & -4 & 18 \end{vmatrix} = (2)(6)(18) + (10)(14)(12) + (0)(16)(-4) \\ - (10)(16)(18) - (2)(14)(-4) - (6)(6)(12)$$

$$\underline{\underline{\det(A) = -872}}$$



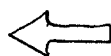
$$\begin{vmatrix} 2 & 10 & 0 \\ 16 & 6 & 14 \\ 12 & -4 & 18 \end{vmatrix} = 2 \begin{vmatrix} 6 & 14 \\ -4 & 18 \end{vmatrix} - 10 \begin{vmatrix} 16 & 14 \\ 12 & 18 \end{vmatrix} + 0 \begin{vmatrix} 16 & 6 \\ 12 & -4 \end{vmatrix} \\ = 2[(6)(18) - (14)(-4)] - 10[(16)(18) - (14)(12)] + 0$$

$$\underline{\underline{\det(A) = -872}}$$



matrix [B] is singular because elements of second row and first row are linearly dependent.

$$\underline{\underline{\det(B) = 0}}$$



This result can be shown by direct expansion as well.

b.

$$\underline{\underline{\det([A]^T) = \det(A) = -872}}$$



2.5
Cont.

C.

$$\det(5[A]) = \begin{vmatrix} 10 & 50 & 0 \\ 80 & 30 & 70 \\ 60 & -20 & 90 \end{vmatrix} = (10)(30)(90) + (50)(70)(60) + 0 \\ - (50)(80)(90) - (10)(70)(-20) - 0$$

$$\underline{\underline{\det(5[A]) = -109000}}$$

Since matrix $[A]$ is 3×3 , alternatively,

$$\det(5[A]) = 5^3 \det(A) = (125)(-872) = \underline{\underline{-109000}}$$

2.6

$$\det(A) = \begin{vmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{vmatrix} = (0)(3)(9) + (5)(7)(9) + (0)(8)(-2) \\ - (5)(8)(9) - (0)(7)(-2) - (0)(3)(9)$$

$$\underline{\underline{\det(A) = -45}} \quad \leftarrow$$

$$\underline{\underline{\det([A]^T) = \det(A) = -45}} \quad \leftarrow$$

2.7

$$\begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

Following the steps discussed in Section 2.7, we get

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

2.8

$$\begin{bmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{bmatrix}$$

Because of the zero elements in Row 1, the lower triangular matrix will not have a triangular form, instead it becomes

$$[L] = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0.8889 & 0.9556 & 1.0000 \\ 1.0000 & 0 & 0 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 9 & -2 & 9 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Check:

$$[L][U] = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0.8889 & 0.9556 & 1.0000 \\ 1.0000 & 0 & 0 \end{bmatrix} \begin{bmatrix} 9 & -2 & 9 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{bmatrix}$$

2.9

$$[A] = \begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix}$$

$$\{b\} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$[L] = \begin{bmatrix} 1.0000 & 0 & 0 \\ -0.1667 & 1.0000 & 0 \\ 0 & -0.7500 & 1.0000 \end{bmatrix}$$

$$[u] = 10^7 \begin{bmatrix} 1.0875 & -0.1812 & 0 \\ 0 & 0.6042 & -0.4531 \\ 0 & 0 & 0.1133 \end{bmatrix}$$

$$\{z\} = [L]^{-1} \{b\} = \begin{bmatrix} 1.0000 & 0 & 0 \\ -0.1667 & 1.0000 & 0 \\ 0 & -0.7500 & 1.0000 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$\{U\} = [u]^{-1} \{z\} = 10^7 \begin{bmatrix} 1.0875 & -0.1812 & 0 \\ 0 & 0.6042 & -0.4531 \\ 0 & 0 & 0.1133 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$\{U\} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

Note the difference between **u** denoting upper triangular matrix and **U** denoting the displacement results

2.10

$$[A] = \begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix}$$

$$\{b\} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$[A]^{-1} = 10^{-6} \begin{bmatrix} 0.1103 & 0.1103 & 0.1103 \\ 0.1103 & 0.6621 & 0.6621 \\ 0.1103 & 0.6621 & 0.8828 \end{bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = [A]^{-1} \{b\} = 10^{-6} \begin{bmatrix} 0.1103 & 0.1103 & 0.1103 \\ 0.1103 & 0.6621 & 0.6621 \\ 0.1103 & 0.6621 & 0.8828 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

2.11

(a) Using Gaussian method

$$x_1 + x_2 + x_3 = 6$$

$$2x_1 + 5x_2 + x_3 = 15$$

$$-3x_1 + x_2 + 5x_3 = 14$$

$$\begin{cases} 2x_1 + 5x_2 + x_3 = 15 \\ -2x_1 - 2x_2 - 2x_3 = -12 \end{cases}$$

$$3x_2 - x_3 = 3$$

$$\begin{cases} -3x_1 + x_2 + 5x_3 = 14 \\ 3x_1 + 3x_2 + 3x_3 = 18 \end{cases}$$

$$4x_2 + 8x_3 = 32$$

$$\begin{cases} 3x_2 - x_3 = 3 \\ 4x_2 + 8x_3 = 32 \end{cases} \rightarrow \begin{cases} x_2 - \frac{1}{3}x_3 = 1 \\ 4x_2 + 8x_3 = 32 \end{cases}$$

$$\begin{cases} 4x_2 + 8x_3 = 32 \\ -4x_2 + \frac{4}{3}x_3 = -4 \end{cases}$$

$$\frac{28}{3}x_3 = 28$$

$$\rightarrow \underline{\underline{x_3 = 3}}$$



$$x_2 = 1 + \frac{1}{3}x_3 = 1 + \frac{1}{3}(3) = 2$$

$$\underline{\underline{x_2 = 2}}$$



$$x_1 = 6 - x_2 - x_3 = 6 - 2 - 3 = 1$$

$$\underline{\underline{x_1 = 1}}$$



2.11
Cont.

(b) Using the LU decomposition method

$$[A] = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} \quad \{b\} = \begin{bmatrix} 6 \\ 15 \\ 14 \end{bmatrix}$$

$$u_{11} = a_{11} = 1 \quad u_{12} = a_{12} = 1 \quad u_{13} = a_{13} = 1$$

$$l_{21} = \frac{a_{21}}{u_{11}} = \frac{2}{1} = 2 \quad l_{31} = \frac{a_{31}}{u_{11}} = \frac{-3}{1} = -3$$

$$u_{22} = a_{22} - l_{21}u_{12} = 5 - (2)(1) = 3$$

$$u_{23} = a_{23} - l_{21}u_{13} = 1 - (2)(1) = -1$$

$$l_{32} = \frac{a_{32} - l_{31}u_{12}}{u_{22}} = \frac{1 - (-3)(1)}{3} = \frac{4}{3}$$

$$u_{33} = a_{33} - (l_{31}u_{13} + l_{32}u_{23}) = 5 - [(-3)(1) + (\frac{4}{3})(-1)] = \frac{28}{3}$$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{4}{3} & 1 \end{bmatrix}}_{[L]} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{28}{3} \end{bmatrix}}_{[U]}$$

$$[L]\{z\} = \{b\} \quad \rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{4}{3} & 1 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\{z\} = \begin{Bmatrix} 6 \\ 3 \\ 28 \end{Bmatrix}$$

$$[U]\{x\} = \{z\} \quad \rightarrow \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{28}{3} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 3 \\ 28 \end{Bmatrix}$$

$$\{x\} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \quad \leftarrow$$

2.11
Cont.

(c) by finding the inverse of the coefficient matrix

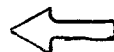
$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}^{-1} \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 0.8571 & -0.1429 & -0.1429 \\ -0.4643 & 0.2857 & 0.0357 \\ 0.6071 & -0.1429 & 0.1071 \end{bmatrix} \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \quad \leftarrow$$

$$[A]^{-1} = \begin{bmatrix} \frac{1}{5} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$



$$[B]^{-1} = \begin{bmatrix} 0.8571 & -0.1429 & -0.1429 \\ -0.4643 & 0.2857 & 0.0357 \\ 0.6071 & -0.1429 & 0.1071 \end{bmatrix}$$



$$[C] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} x_{11} \\ x_{21} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \rightarrow x_{11} = \frac{K_{22}}{K_{11}K_{22} - K_{12}K_{21}}$$

$$x_{21} = \frac{-K_{21}}{K_{11}K_{22} - K_{12}K_{21}}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} x_{12} \\ x_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \rightarrow x_{12} = \frac{-K_{12}}{K_{11}K_{22} - K_{12}K_{21}}$$

$$x_{22} = \frac{K_{11}}{K_{11}K_{22} - K_{12}K_{21}}$$



For a 2×2 matrix:

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}) \stackrel{?}{=} \alpha^2 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\alpha \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{bmatrix}$$

$$\det \begin{bmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{bmatrix} = (\alpha a_{11})(\alpha a_{22}) - (\alpha a_{12})(\alpha a_{21})$$

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}) = \alpha^2 (a_{11}a_{22} - a_{12}a_{21}) = \alpha^2 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

For a 3×3 matrix:

Q.E.D.

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}) \stackrel{?}{=} \alpha^3 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\det \begin{bmatrix} \alpha a_{11} & \alpha a_{12} & \alpha a_{13} \\ \alpha a_{21} & \alpha a_{22} & \alpha a_{23} \\ \alpha a_{31} & \alpha a_{32} & \alpha a_{33} \end{bmatrix} = (\alpha a_{11})(\alpha a_{22})(\alpha a_{33}) +$$

$$(\alpha a_{12})(\alpha a_{23})(\alpha a_{31}) + \dots$$

$$\dots - (\alpha a_{13})(\alpha a_{22})(\alpha a_{31})$$

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}) = \alpha^3 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Q.E.D.

In general for a $n \times n$ matrix, we have

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}) = \alpha^n \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

2.14

Using Equation (2.83), we have

$$\begin{bmatrix} -\omega^2 + \frac{2K}{m_1} & -\frac{K}{m_1} \\ -\frac{K}{m_2} & -\omega^2 + \frac{2K}{m_2} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0$$

$$\begin{vmatrix} -\omega^2 + \frac{2(100)}{0.1} & -\frac{100}{0.1} \\ -\frac{100}{0.2} & -\omega^2 + \frac{2(100)}{0.2} \end{vmatrix} = 0$$

$$\omega^4 - 3000\omega^2 + 1.5 \times 10^6 = 0$$

$$\omega_1^2 = 2366 \text{ (rad/s)}^2$$

$$\omega_2^2 = 634 \text{ (rad/s)}^2$$

$$\omega_1 = 48.6 \text{ rad/s}$$

$$\omega_2 = 25.2 \text{ rad/s}$$



$$(-2366 + 2000)X_1 - 1000X_2 = 0 \rightarrow \frac{X_2}{X_1} = -0.366$$

$$(-634 + 2000)X_1 - 1000X_2 = 0 \rightarrow \frac{X_2}{X_1} = 1.366$$

2.15

```
>> a=[4 2 1;7 0 -7;1 -5 3]
```

```
a =
```

```
4 2 1
7 0 -7
1 -5 3
```

```
>> b=[1 2 -1;5 3 3;4 5 -7]
```

```
b =
```

```
1 2 -1
5 3 3
4 5 -7
```

```
>> c=[1;-2;4]
```

```
c =
```

```
1
-2
4
```

```
>> a+b
```

```
ans =
```

```
5 4 0
12 3 -4
5 0 -4
```

```
>> a-b
```

```
ans =
```

```
3 0 2
2 -3 -10
-3 -10 10
```

2.15
Cont.

```
>> 3*a
```

```
ans =
```

```
12  6  3
21  0 -21
 3 -15  9
```

```
>> a*b
```

```
ans =
```

```
18 19 -5
-21 -21 42
-12  2 -37
```

```
>> a*c
```

```
ans =
```

```
 4
-21
23
```

```
>> a*a
```

```
ans =
```

```
31  3 -7
21 49 -14
-28 -13 45
```

```
>> i=[1 0 0;0 1 0;0 0 1]
```

```
i =
```

```
1  0  0
0  1  0
0  0  1
```

2.15
Cont.

>> i*a

ans =

4	2	1
7	0	-7
1	-5	3

>> a*i

ans =

4	2	1
7	0	-7
1	-5	3

>>

2.16

```
>> A=[1 4 2;8 3 6;7 1 -2]
```

```
A =
```

```
1 4 2
8 3 6
7 1 -2
```

```
>> B=[0 5 -1;-3 1 7;2 4 -4]
```

```
B =
```

```
0 5 -1
-3 1 7
2 4 -4
```

```
>> A'
```

```
ans =
```

```
1 8 7
4 3 1
2 6 -2
```

```
>> B'
```

```
ans =
```

```
0 -3 2
5 1 4
-1 7 -4
```

```
>> (A+B)'
```

```
ans =
```

```
1 5 9
9 4 5
1 13 -6
```

2.16
Cont.

>> A'+B'

ans =

1	5	9
9	4	5
1	13	-6

>> (A*B)'

ans =

-8	3	-7
17	67	28
19	-11	8

>> B'*A'

ans =

-8	3	-7
17	67	28
19	-11	8

>>

2.17

```
>> A=[2 10 0;16 6 14;12 -4 18]
```

```
A =
```

```
    2    10     0
   16     6    14
   12    -4    18
```

```
>> B=[2 10 0;4 20 0;12 -4 18]
```

```
B =
```

```
    2    10     0
    4    20     0
   12    -4    18
```

```
>> det(A)
```

```
ans =
```

```
-872
```

```
>> det(B)
```

```
ans =
```

```
0
```

```
>> det((A)')
```

```
ans =
```

```
-872
```

```
>> det(5*(A))
```

```
ans =
```

```
-109000
```

```
>>
```

2.18

```
>> A=[0 5 0;8 3 7;9 -2 9]
```

```
A =
```

```
    0    5    0  
    8    3    7  
    9   -2    9
```

```
>> det(A)
```

```
ans =
```

```
-45
```

```
>> det((A)')
```

```
ans =
```

```
-45
```

```
>>
```

2.19

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]
```

```
A =
```

10875000	-1812500	0
-1812500	6343750	-4531250
0	-4531250	4531250

```
>> b=[0;0;800]
```

```
b =
```

0
0
800

```
>> x=A\b
```

```
x =
```

1.0e-003 *

0.0883
0.5297
0.7062

```
>>
```

2.20

```
>> A=[0 5 0;8 3 7;9 -2 9]
```

```
A =
```

```
0 5 0
8 3 7
9 -2 9
```

```
>> [l,u]=lu(A)
```

```
l =
```

```
0 1.0000 0
0.8889 0.9556 1.0000
1.0000 0 0
```

```
u =
```

```
9 -2 9
0 5 0
0 0 -1
```

```
>> l*u
```

```
ans =
```

```
0 5 0
8 3 7
9 -2 9
```

2.21

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]
```

A =

```
10875000 -1812500 0
-1812500 6343750 -4531250
0 -4531250 4531250
```

```
>> b=[0;0;800]
```

b =

```
0
0
800
```

```
>> [l,u]=lu(A)
```

l =

```
1.0000 0 0
-0.1667 1.0000 0
0 -0.7500 1.0000
```

u =

1.0e+007 *

```
1.0875 -0.1812 0
0 0.6042 -0.4531
0 0 0.1133
```

```
>> z=inv(l)*b
```

z =

```
0
0
800
```

```
>> U=inv(u)*z
```

U =

1.0e-003 *

```
0.0883
0.5297
0.7062
```

Note the difference between **u** denoting upper triangular matrix and **U** denoting the displacement results

2.22

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]
```

```
A =
```

```
10875000 -1812500      0
-1812500  6343750 -4531250
      0    -4531250  4531250
```

```
>> b=[0;0;800]
```

```
b =
```

```
0
0
800
```

```
>> Ainverse=inv(A)
```

```
Ainverse =
```

```
1.0e-006 *
```

```
0.1103  0.1103  0.1103
0.1103  0.6621  0.6621
0.1103  0.6621  0.8828
```

```
>> u=Ainverse*b
```

```
u =
```

```
1.0e-003 *
```

```
0.0883
0.5297
0.7062
```

```
>>
```

2.23

```
>> A=[1 1 1;2 5 1;-3 1 5]
```

A =

1	1	1
2	5	1
-3	1	5

```
>> b=[6 15 14]
```

b =

6	15	14
---	----	----

```
>> b=[6;15;14]
```

b =

6
15
14

(a) using the Gaussian method

```
>> x=A\b
```

x =

1.0000
2.0000
3.0000

(b) using the LU decomposition method

```
>> [l,u]=lu(A)
```

l =

-0.3333	0.2353	1.0000
-0.6667	1.0000	0
1.0000	0	0

u =

-3.0000	1.0000	5.0000
0	5.6667	4.3333
0	0	1.6471

2.23
Cont.

```
>> z=inv(l)*b
```

```
z =
```

```
14.0000  
24.3333  
4.9412
```

```
>> x=inv(u)*z
```

```
x =
```

```
1.0000  
2.0000  
3.0000
```

(c) by finding the inverse of the coefficient matrix

```
>> x=inv(A)*b
```

```
x =
```

```
1.0000  
2.0000  
3.0000
```

```
>>
```

2.24

For example, consider the following 4 x 4 matrix, and $\alpha = 2$ and $\alpha = 3$.

```
>> A=[1 2 1 3;2 1 4 1;5 3 0 1;4 1 5 7]
```

A =

1	2	1	3
2	1	4	1
5	3	0	1
4	1	5	7

```
>> det(A)
```

ans =

219

```
>> det(2*A)
```

ans =

3504

Since matrix A is 4 x 4 then let us examine to see if $\det(2*A) = 2^4 * \det(A)$?

```
>> 2^4*det(A)
```

ans =

3504

```
>> det(3*A)
```

ans =

17739

Or is $\det(3*A) = 3^4 * \det(A)$?

```
>> 3^4*det(A)
```

ans =

17739

2.24
Cont.

Let us now consider the following 3 x 3 matrix, and $\alpha = 2$ and $\alpha = 3$.

```
>> B=[1 2 1;2 1 4;5 3 0]
```

B =

```
1 2 1
2 1 4
5 3 0
```

```
>> det(B)
```

ans =

29

```
>> det(2*B)
```

ans =

232

Is $\det(2*B) = 2^3 * \det(B)$?

```
>> 2^3*det(B)
```

ans =

232

```
>> det(3*B)
```

ans =

783

Or is $\det(3*B) = 3^3 * \det(B)$?

```
>> 3^3*det(B)
```

ans =

783

```
>>
```

2.25

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]
```

A =

7.1100	-1.2300	0	0	0
-1.2300	1.9900	-0.7600	0	0
0	-0.7600	0.8510	-0.0910	0
0	0	-0.0910	2.3110	-2.2200
0	0	0	-2.2200	3.6900

```
>> b=[5.88*20; 0; 0; 0; 1.47*70]
```

b =

117.6000
0
0
0
102.9000

```
>> T=A\b
```

T =

20.5898
23.4091
27.9719
66.0789
67.6410

```
>>
```

2.26

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]
```

A =

7.1100	-1.2300	0	0	0
-1.2300	1.9900	-0.7600	0	0
0	-0.7600	0.8510	-0.0910	0
0	0	-0.0910	2.3110	-2.2200
0	0	0	-2.2200	3.6900

```
>> b=[5.88*20; 0; 0; 0; 1.47*70]
```

b =

117.6000
0
0
0
102.9000

```
>> Ainverse=inv(A)
```

Ainverse =

0.1681	0.1585	0.1430	0.0133	0.0080
0.1585	0.9160	0.8263	0.0771	0.0464
0.1430	0.8263	1.9323	0.1803	0.1085
0.0133	0.0771	0.1803	1.0421	0.6269
0.0080	0.0464	0.1085	0.6269	0.6482

```
>> T=Ainverse*b
```

T =

20.5898
23.4091
27.9719
66.0789
67.6410

2.27

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;-0.76 0.851 -0.091 0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]
```

A =

```
7.1100 -1.2300 0 0 0
-1.2300 1.9900 -0.7600 0 0
0 -0.7600 0.8510 -0.0910 0
0 0 -0.0910 2.3110 -2.2200
0 0 0 -2.2200 3.6900
```

```
>> b=[5.88*20; 0; 0; 0; 1.47*70]
```

b =

```
117.6000
0
0
0
102.9000
```

```
>> [l,u]=lu(A)
```

l =

```
1.0000 0 0 0 0
-0.1730 1.0000 0 0 0
0 -0.4276 1.0000 0 0
0 0 -0.1730 1.0000 0
0 0 0 -0.9672 1.0000
```

u =

```
7.1100 -1.2300 0 0 0
0 1.7772 -0.7600 0 0
0 0 0.5260 -0.0910 0
0 0 0 2.2953 -2.2200
0 0 0 0 1.5428
```

```
>> z=inv(l)*b
```

z =

```
117.6000
20.3443
8.6999
1.5051
104.3558
```

```
>> T=inv(u)*z
```

T =

```
20.5898
23.4091
27.9719
66.0789
67.6410
```

```
>>
```

```
>> A=[1 0 0 0 0;-0.0408 0.0888 -0.0408 0 0;0 -0.0408 0.0888 -0.0408 0;0 0 -0.0408 0.0888 -0.0408;
0 0 0 -0.0408 0.04455]
```

```
A =
```

```
1.0000    0    0    0    0
-0.0408  0.0888 -0.0408    0    0
    0 -0.0408  0.0888 -0.0408    0
    0    0 -0.0408  0.0888 -0.0408
    0    0    0 -0.0408  0.0445
```

```
>> b=[100;0.144;0.144;0.144;0.075]
```

```
b =
```

```
100.0000
  0.1440
  0.1440
  0.1440
  0.0750
```

```
>> T=A\b
```

```
T =
```

```
100.0000
 75.0387
 59.7901
 51.5633
 48.9064
```

```
>>
```

2.29

```
>> A=10^5*[7.2 0 0 0 -1.49 -1.49;0 7.2 0 -4.22 -1.49 -1.49;0 0 8.44 0 -4.22 0;-4.22 0 4.22 0 0;-1.49 -1.49  
-4.22 0 5.71 1.49;-1.49 -1.49 0 0 1.49 1.49]
```

A =

720000	0	0	0	-149000	-149000
0	720000	0	-422000	-149000	-149000
0	0	844000	0	-422000	0
0	-422000	0	422000	0	0
-149000	-149000	-422000	0	571000	149000
-149000	-149000	0	0	149000	149000

```
>> b=[0;0;0;-500;0;-500]
```

b =

0
0
0
-500
0
-500

```
>> U=A\b
```

U =

-0.0036
-0.0103
0.0012
-0.0115
0.0024
-0.0195

```
>>
```

2.30

```
>> A=[2000 -1000;-500 1000]
```

```
A =
```

```
    2000    -1000  
    -500     1000
```

The eigenvalues are:

```
>> eig(A)
```

```
ans =
```

```
1.0e+003 *  
  
    2.3660  
    0.6340
```

Note the natural frequencies of the system are equal to the square root of the eigenvalues.

```
>> sqrt(eig(A))
```

```
ans =
```

```
    48.6418  
    25.1789
```

The eigenvector and eigenvalues are given by:

```
>> [v,e]=eig(A)
```

```
v =
```

```
    0.9391    0.5907  
   -0.3437    0.8069
```

```
e =
```

```
1.0e+003 *  
  
    2.3660     0  
     0    0.6340
```

2.30
Cont.

Normalizing the eigenvector with respect to X_1 , we get:

$\gg -0.3437/0.9391$

ans =

-0.3660

Therefore, the first mode is given by $X_2/X_1 = -0.3660$.

$\gg .8069/0.5907$

ans =

1.3660

The second mode is then given by $X_2/X_1 = 1.3660$.

Problem 2-31

$$[K] =$$

7.11	-1.23	0	0	0
-1.23	1.99	-0.76	0	0
0	-0.76	0.851	-0.091	0
0	0	-0.091	2.31	-2.22
0	0	0	-2.22	3.69

$$[K]^{-1} =$$

0.1681	0.1585	0.143	0.0134	0.008
0.1585	0.9161	0.8264	0.0772	0.0464
0.143	0.8264	1.9324	0.1805	0.1086
0.0134	0.0772	0.1805	1.0431	0.6276
0.008	0.0464	0.1086	0.6276	0.6486

$$\{T\} =$$

T ₁
T ₂
T ₃
T ₄
T ₅

$$\{T\} =$$

T ₁
T ₂
T ₃
T ₄
T ₅

$$[F] =$$

117.6
0
0
0
102.9

$$= [K]^{-1} [F] =$$

20.59
23.41
27.98
66.15
67.68

Problem 2-32

$$[K] =$$

1	0	0	0	0
-0.041	0.0888	-0.0408	0	0
0	-0.0408	0.0888	-0.041	0
0	0	-0.0408	0.0888	-0.0408
0	0	0	-0.041	0.04455

$$[K]^{-1} =$$

1	0	0	0	0
0.688	16.8623	12.191	9.6701	8.85613
0.4974	12.1906	26.532	21.047	19.2751
0.3945	9.67011	21.047	36.137	33.0956
0.3613	8.85613	19.275	33.096	52.7564

$$\{T\} =$$

T ₁
T ₂
T ₃
T ₄
T ₅

$$[F] =$$

100
0.144
0.144
0.144
0.075

$$\{T\} = [K]^{-1} [F]$$

T ₁	100.00
T ₂	75.04
T ₃	59.79
T ₄	51.56
T ₅	48.91

Problem 2-33

$$[K] =$$

7.2	0	0	0	-1.49	-1.49
0	7.2	0	-4.22	-1.49	-1.49
0	0	8.44	0	-4.22	0
0	-4.22	0	4.22	0	0
-1.49	-1.49	-4.22	0	5.71	1.49
-1.49	-1.49	0	0	1.49	1.49

$$[K]^{-1} =$$

0.23697	0.23697	0	0.237	0	0.473934
0.23697	0.90811	0	0.9081	0	1.145075
0	-1E-17	0.237	-1E-17	0.23697	-0.23697
0.23697	0.90811	0	1.1451	0	1.145075
0	-2E-17	0.237	-2E-17	0.47393	-0.47393
0.47393	1.14507	-0.237	1.1451	-0.4739	2.764083

$$\{T\} =$$

U2x
U2y
U4x
U4y
U5x
U5y

$$[F] =$$

0
0
0
-0.005
0
-0.005

$$\{U\} =$$

U2x
U2y
U4x
U4y
U5x
U5y

$$= [K]^{-1} [F] =$$

-0.0036
-0.0103
0.0012
-0.0115
0.0024
-0.0195