CHAPTER 1
Preparation for Calculus

CHAPTER 1
Preparation for Calculus

Section 1.1 Graphs and Models

1. To find the x-intercepts of the graph of an equation, let y be zero and solve the equation for x. To find the y-intercepts of the graph of an equation, let x be zero and solve the equation for y.

2. Symmetry helps in sketching a graph because you need only half as many points to plot. Answers will vary.

3. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

4. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

5. $y = 3 - x^2$

x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y-intercept: $(0, 3)$

Matches graph (a).

6. $y = x^3 - x$

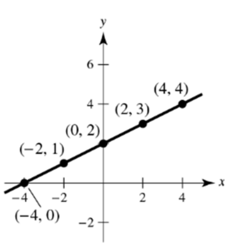
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

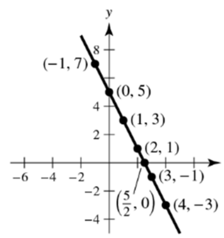
7. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



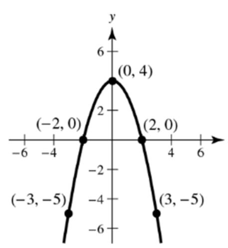
8. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



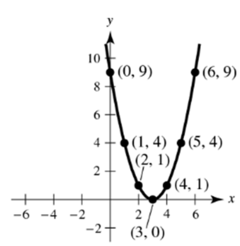
9. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



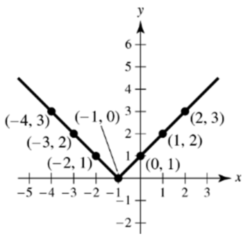
10. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



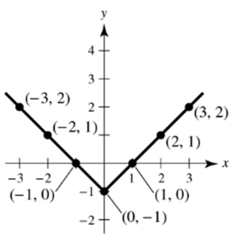
11. $y = |x + 1|$

x	-4	-3	-2	-1	0	1	2
y	3	2	1	0	1	2	3



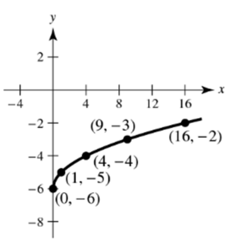
12. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



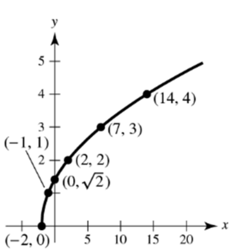
13. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



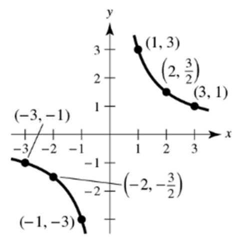
14. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



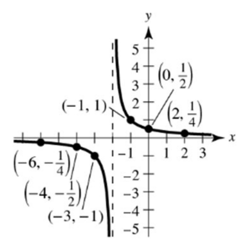
15. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

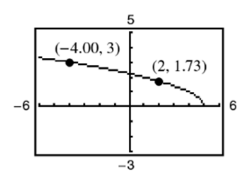


16. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



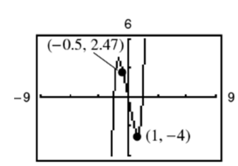
17. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

18. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

19. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5;\ (0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2};\ \left(\frac{5}{2}, 0\right)$

20. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3;\ (0, 3)$

x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None. y cannot equal 0.

21. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

22. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2;\ (0, 0), (\pm 2, 0)$

23. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0;\ (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4;\ (0, 0), (4, 0), (-4, 0)$

24. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1;\ (1, 0)$

25. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4;\ (4, 0)$

26. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{\left[3(0) + 1\right]^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3;\ (0, 0), (-3, 0)$

27. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

28. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3};\ \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

29. Symmetric with respect to the y-axis because
$y = (-x)^2 - 6 = x^2 - 6.$

30. $y = 9x - x^2$

No symmetry with respect to either axis or the origin.

31. Symmetric with respect to the x-axis because
$(-y)^2 = y^2 = x^3 - 8x.$

32. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

33. Symmetric with respect to the origin because
$(-x)(-y) = xy = 4.$

34. Symmetric with respect to the x-axis because
$x(-y)^2 = xy^2 = -10.$

35. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

36. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

37. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

38. Symmetric with respect to the origin because

$$-y = \frac{(-x)^5}{4 - (-x)^2}$$
$$-y = \frac{-x^5}{4 - x^2}$$
$$y = \frac{x^5}{4 - x^2}.$$

39. $y = |x^3 + x|$ is symmetric with respect to the y-axis because $y = |(-x)^3 + (-x)| = |-(x^3 + x)| = |x^3 + x|.$

40. $|y| - x = 3$ is symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

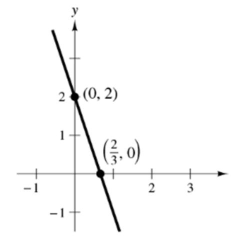
41. $y = 2 - 3x$

$y = 2 - 3(0) = 2$, y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$, x-intercept

Intercepts: $(0, 2)$, $\left(\frac{2}{3}, 0\right)$

Symmetry: none



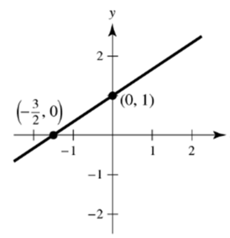
42. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1$, y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2}$, x-intercept

Intercepts: $(0, 1)$, $\left(-\frac{3}{2}, 0\right)$

Symmetry: none



43. $y = 9 - x^2$

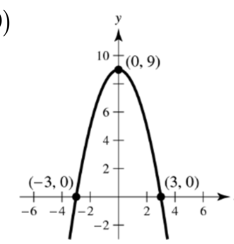
$y = 9 - (0)^2 = 9$, y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$, x-intercepts

Intercepts: $(0, 9)$, $(3, 0)$, $(-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



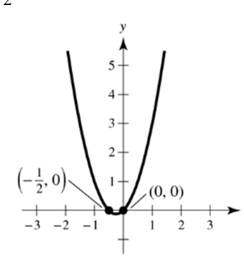
44. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0$, y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2}$, x-intercepts

Intercepts: $(0, 0)$, $\left(-\frac{1}{2}, 0\right)$

Symmetry: none



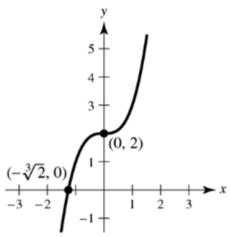
45. $y = x^3 + 2$

$y = 0^3 + 2 = 2$, y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2}$, x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



46. $y = x^3 - 4x$

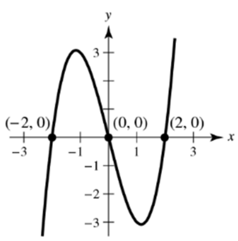
$y = 0^3 - 4(0) = 0$, y-intercept

$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x(x + 2)(x - 2) = 0$$
$$x = 0, \pm 2, \; x\text{-intercepts}$$

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



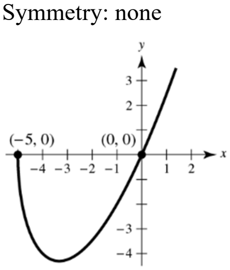
47. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



48. $y = \sqrt{25 - x^2}$

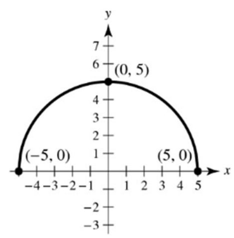
$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$$\sqrt{25 - x^2} = 0$$
$$25 - x^2 = 0$$
$$(5 + x)(5 - x) = 0$$
$$x = \pm 5, \; x\text{-intercept}$$

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



49. $x = y^3$

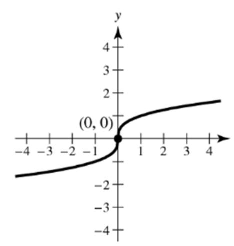
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: $(0, 0)$

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



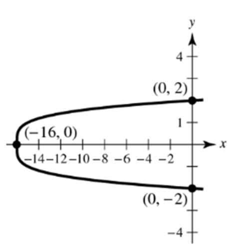
50. $x = y^4 - 16$

$$y^4 - 16 = 0$$
$$(y^2 - 4)(y^2 + 4) = 0$$
$$(y - 2)(y + 2)(y^2 + 4) = 0$$
$$y = \pm 2, \; y\text{-intercepts}$$

$x = 0^4 - 16 = -16$, x-intercept

Intercepts: $(0, 2), (0, -2), (-16, 0)$

Symmetry: x-axis because $x = (-y)^4 - 16 = y^4 - 16$



51. $y = \frac{8}{x}$

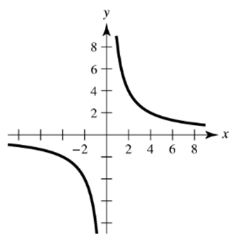
$y = \frac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\frac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \frac{8}{-x} \Rightarrow y = \frac{8}{x}$

Symmetry: origin



52. $y = \frac{10}{x^2 + 1}$

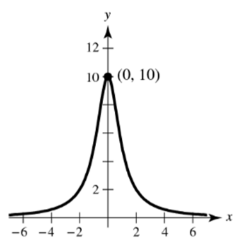
$y = \frac{10}{0^2 + 1} = 10$, y-intercept

$\frac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: (0, 10)

$y = \frac{10}{(-x)^2 + 1} = \frac{10}{x^2 + 1}$

Symmetry: y-axis



53. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

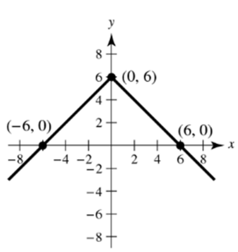
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6), (-6, 0), (6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



54. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

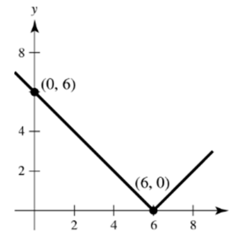
$|6 - x| = 0$

$6 - x = 0$

$6 = x$, x-intercept

Intercepts: (0, 6), (6, 0)

Symmetry: none



55. $x^2 + y^2 = 9$

$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$

$y = \pm\sqrt{9 - 0} = \pm 3$, y-intercepts

$\pm\sqrt{9 - x^2} = 0$

$9 - x^2 = 0$

$9 = x^2$

$\pm 3 = x$, x-intercepts

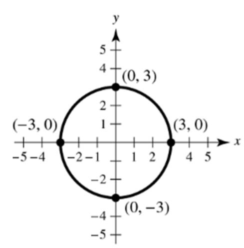
Intercepts: $(\pm 3, 0), (0, \pm 3)$

$(-x)^2 + y^2 = 9 \Rightarrow x^2 + y^2 = 9$

$x^2 + (-y)^2 = 9 \Rightarrow x^2 + y^2 = 9$

$(-x)^2 + (-y)^2 = 9 \Rightarrow x^2 + y^2 = 9$

Symmetry: x-axis, y-axis, origin



56. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

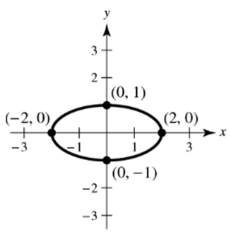
$$y = \pm\frac{\sqrt{4 - 0^2}}{2} = \pm\frac{\sqrt{4}}{2} = \pm 1, \text{ } y\text{-intercepts}$$

$$x^2 + 4(0)^2 = 4$$
$$x^2 = 4$$
$$x = \pm 2, \text{ } x\text{-intercepts}$$

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



57. $3y^2 - x = 9$

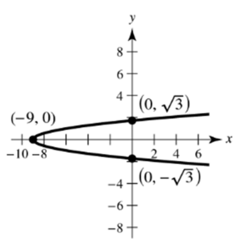
$$3y^2 = x + 9$$
$$y^2 = \tfrac{1}{3}x + 3$$
$$y = \pm\sqrt{\tfrac{1}{3}x + 3}$$
$$y = \pm\sqrt{0 + 3} = \pm\sqrt{3}, \text{ } y\text{-intercepts}$$

$$\pm\sqrt{\tfrac{1}{3}x + 3} = 0$$
$$\tfrac{1}{3}x + 3 = 0$$
$$x = -9, \text{ } x\text{-intercept}$$

Intercepts: $(0, \sqrt{3}), (0, -\sqrt{3}), (-9, 0)$

$3(-y)^2 - x = 3y^2 - x = 9$

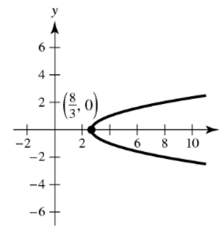
Symmetry: x-axis



58. $3x - 4y^2 = 8$

$$-4y^2 = -3x + 8$$
$$y^2 = \tfrac{3}{4}x - 2$$
$$y = \pm\sqrt{\tfrac{3}{4}x - 2}$$
$$y = \pm\sqrt{0 - 2}$$
$\Rightarrow$ No real solution $\Rightarrow$ No y-intercepts

$$\pm\sqrt{\tfrac{3}{4}x - 2} = 0$$
$$\tfrac{3}{4}x - 2 = 0$$
$$x = \tfrac{8}{3}, \text{ } x\text{-intercept}$$



Intercept: $\left(\tfrac{8}{3}, 0\right)$

Symmetry: x-axis

59. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$$8 - x = 4x - 7$$
$$15 = 5x$$
$$3 = x$$

The corresponding y-value is $y = 5$.

Point of intersection: (3, 5)

60. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$$\frac{3x + 4}{2} = \frac{-4x - 10}{2}$$
$$3x + 4 = -4x - 10$$
$$7x = -14$$
$$x = -2$$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

61. $x^2 + y = 15 \Rightarrow y = -x^2 + 15$

$-3x + y = 11 \Rightarrow y = 3x + 11$

$$-x^2 + 15 = 3x + 11$$
$$0 = x^2 + 3x - 4$$
$$0 = (x + 4)(x - 1)$$
$$x = -4, 1$$

The corresponding y-values are $y = -1$ (for $x = -4$) and $y = 14$ (for $x = 1$).

Points of intersection: $(-4, -1), (1, 14)$

62. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$
$y = x - 1$
$3 - x = (x-1)^2$
$3 - x = x^2 - 2x + 1$
$0 = x^2 - x - 2 = (x+1)(x-2)$
$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

63. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$
$x - y = 1 \Rightarrow y = x - 1$
$5 - x^2 = (x-1)^2$
$5 - x^2 = x^2 - 2x + 1$
$0 = 2x^2 - 2x - 4 = 2(x+1)(x-2)$
$x = -1$ or $x = 2$

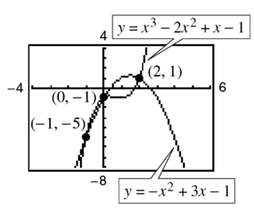
The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

64. $x^2 + y^2 = 16$
$x + 2y = 4 \quad \Rightarrow \quad x = 4 - 2y$
$(4 - 2y)^2 + y^2 = 16$
$5y^2 - 16y + 16 = 16$
$y(5y - 16) = 0 \quad \Rightarrow \quad y = 0, \frac{16}{5}$
$x = 4 - 2(0) \Rightarrow x = 4$
$x = 4 - 2\left(\frac{16}{5}\right) \Rightarrow x = -\frac{12}{5}$

Points of intersection: $(4, 0), \left(-\frac{12}{5}, \frac{16}{5}\right)$

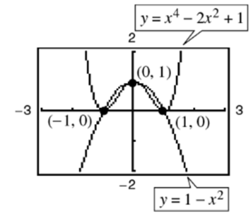
65. $y = x^3 - 2x^2 + x - 1$
$y = -x^2 + 3x - 1$



Points of intersection:
$(-1, -5), (0, -1), (2, 1)$

Analytically,
$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$
$x^3 - x^2 - 2x = 0$
$x(x-2)(x+1) = 0$
$x = -1, 0, 2.$

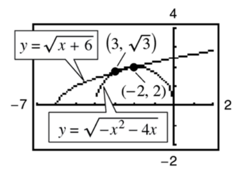
66. $y = x^4 - 2x^2 + 1$
$y = 1 - x^2$



Points of intersection:
$(-1, 0), (0, 1), (1, 0)$

Analytically,
$1 - x^2 = x^4 - 2x^2 + 1$
$0 = x^4 - x^2$
$0 = x^2(x+1)(x-1)$
$x = -1, 0, 1.$

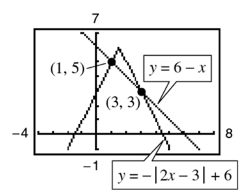
67. $y = \sqrt{x + 6}$
$y = \sqrt{-x^2 - 4x}$



Points of intersection: $(-2, 2), \left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically, $\sqrt{x+6} = \sqrt{-x^2 - 4x}$
$x + 6 = -x^2 - 4x$
$x^2 + 5x + 6 = 0$
$(x+3)(x+2) = 0$
$x = -3, -2.$

68. $y = -|2x - 3| + 6$
$y = 6 - x$

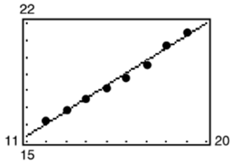


Points of intersection: (3, 3), (1, 5)
Analytically, $-|2x - 3| + 6 = 6 - x$
$|2x - 3| = x$
$2x - 3 = x$ or $2x - 3 = -x$
$x = 3$ or $x = 1.$

69. (a) Using a graphing utility, you obtain

$y = 0.74t + 7.2.$

(b)



The model is a good fit for the data.

(c) For 2029, $t = 29$:

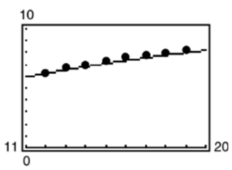
$y = 0.74(29) + 7.2 \approx 28.7$

The GDP in 2029 will be about \$28.7 trillion.

70. (a) Using a graphing utility, you obtain

$y = -0.007t^2 + 0.47t + 1.6.$

(b)



The model is a good fit for the data.

(c) For 2029, $t = 29$:

$y = -0.007(29)^2 + 0.47(29) + 1.6 \approx 9.3$

The number of cellphone subscriptions worldwide in 2029 will be about 9.3 billion.

71.

$$\begin{aligned} C &= R \\ 2.04x + 5600 &= 3.29x \\ 5600 &= 3.29x - 2.04x \\ 5600 &= 1.25x \\ x &= \frac{5600}{1.25} = 4480 \end{aligned}$$

To break even, 4480 units must be sold.

72. $y^2 = 4kx$

(a) $(1, 1)$: $1^2 = 4k(1)$

$1 = 4k$

$k = \frac{1}{4}$

(b) $(2, 4)$: $(4)^2 = 4k(2)$

$16 = 8k$

$k = 2$

(c) $(0, 0)$: $0^2 = 4k(0)$

k can be any real number.

(d) $(3, 3)$: $(3)^2 = 4k(3)$

$9 = 12k$

$k = \frac{9}{12} = \frac{3}{4}$

73. Answers may vary. *Sample answer*:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at

$x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

74. Yes. If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

75. Yes. Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

76. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0)\ (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because

$-y = (-x)^3 - (-x) = -x^3 + x.$

Symmetry with respect to the y-axis for $y = x^2 + 2$ because

$y = (-x)^2 + 2 = x^2 + 2.$

(c)
$$x^3 - x = x^2 + 2$$
$$x^3 - x^2 - x - 2 = 0$$
$$(x - 2)(x^2 + x + 1) = 0$$
$$x = 2 \Rightarrow y = 6$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

77 False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

78. True. $f(4) = f(-4)$.

79. True. The x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

80. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

Section 1.2 Linear Models and Rates of Change

1. In the form $y = mx + b$, m is the slope and b is the y-intercept.

2. No. Perpendicular lines have slopes that are negative reciprocals of each other. So, one line has a positive slope and the other line has a negative slope.

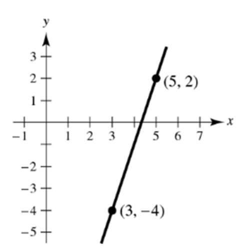
3. $m = 2$

4. $m = 0$

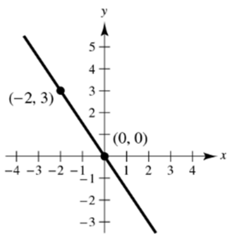
5. $m = -1$

6. $m = -12$

7. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

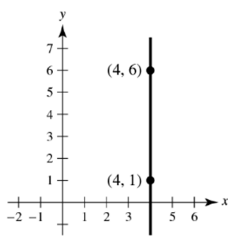


8. $m = \dfrac{3 - 0}{-2 - 0} = -\dfrac{3}{2}$



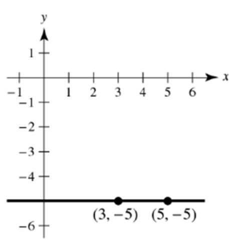
9. $m = \dfrac{1 - 6}{4 - 4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

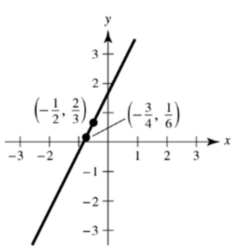


10. $m = \dfrac{-5 - (-5)}{5 - 3} = \dfrac{0}{2} = 0$

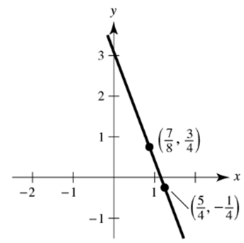
The line is horizontal.



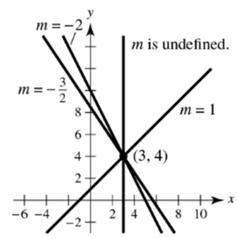
11. $m = \dfrac{\dfrac{2}{3} - \dfrac{1}{6}}{-\dfrac{1}{2} - \left(-\dfrac{3}{4}\right)} = \dfrac{\dfrac{1}{2}}{\dfrac{1}{4}} = 2$



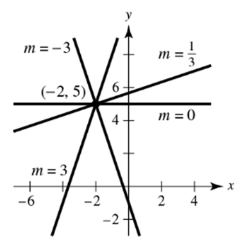
12. $m = \dfrac{\left(\dfrac{3}{4}\right) - \left(-\dfrac{1}{4}\right)}{\left(\dfrac{7}{8}\right) - \left(\dfrac{5}{4}\right)} = \dfrac{1}{-\dfrac{3}{8}} = -\dfrac{8}{3}$



13.



14.



15. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2)$, $(1, 2)$, $(5, 2)$.

16. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0)$, $(-4, 1)$, $(-4, 2)$.

17. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

18. The equation of this line is

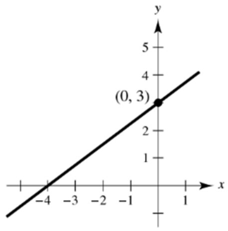
$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

19. $y = \frac{3}{4}x + 3$

$4y = 3x + 12$

$0 = 3x - 4y + 12$



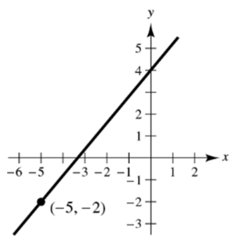
20. $y - (-2) = \frac{6}{5}[x - (-5)]$

$y + 2 = \frac{6}{5}(x + 5)$

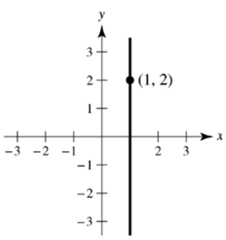
$y + 2 = \frac{6}{5}x + 6$

$y = \frac{6}{5}x + 4$

$0 = 6x - 5y + 20$

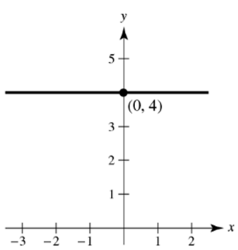


21. Because the slope is undefined, the line is vertical and its equation is $x = 1$.



22. $y = 4$

$y - 4 = 0$

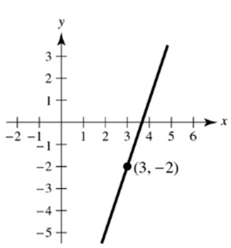


23. $y + 2 = 3(x - 3)$

$y + 2 = 3x - 9$

$y = 3x - 11$

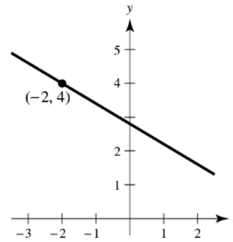
$0 = 3x - y - 11$



24. $y - 4 = -\frac{3}{5}(x + 2)$

$5y - 20 = -3x - 6$

$3x + 5y - 14 = 0$



25. $\dfrac{6}{100} = \dfrac{x}{200}$

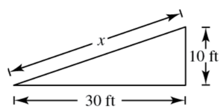
$100x = 1200$

$x = 12$

Since the grade of the road is $\dfrac{6}{100}$, if you drive 200 feet, the vertical rise in the road will be 12 feet.

26. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

(b)

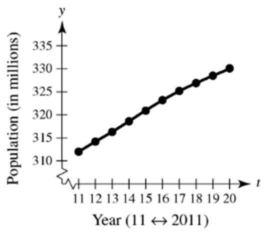


By the Pythagorean Theorem,

$x^2 = 30^2 + 10^2 = 1000$

$x = 10\sqrt{10} \approx 31.623$ feet.

27. (a)



Slopes: $314.2 - 312.0 = 2.2$

$316.3 - 314.2 = 2.1$

$318.6 - 316.3 = 2.3$

$320.9 - 318.6 = 2.3$

$323.2 - 320.9 = 2.3$

$325.2 - 323.2 = 2.0$

$326.9 - 325.2 = 1.7$

$328.5 - 326.9 = 1.6$

$330.1 - 328.5 = 1.6$

The population increased least rapidly from 2018 to 2019 and from 2019 to 2020.

(b) Average rate of change:

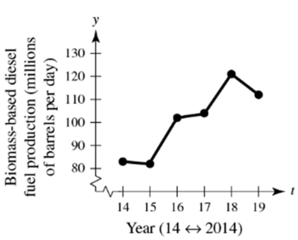
$$\frac{330.1 - 312.0}{20 - 11} = \frac{18.1}{9}$$

≈ 2.01 million people per year

(c) For 2030, $t = 30$: $330.1 + 10(2.01) = 350.2$

The population of the United States in 2030 will be about 350.2 million people.

28. (a)



Slopes: $82 - 83 = -1$

$102 - 82 = 20$

$104 - 102 = 2$

$121 - 104 = 17$

$112 - 121 = -9$

The biomass-based diesel fuel production increased most rapidly from 2015 to 2016.

(b) Average rate of change:

$$\frac{112 - 83}{19 - 14} = \frac{29}{5} = 5.8 \text{ million barrels per day}$$

(c) No. The production seems to randomly increase and decrease year to year.

29. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

30. $-x + y = 1$

$y = x + 1$

The slope is $m = 1$ and the y-intercept is $(0, 1)$.

31. $5x + y = 20$

$y = -5x + 20$

The slope is $m = -5$ and the y-intercept is $(0, 20)$.

32. $6x - 5y = 15$

$y = \frac{6}{5}x - 3$

The slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

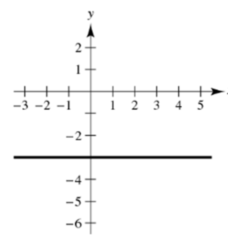
33. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

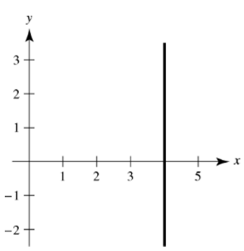
34. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

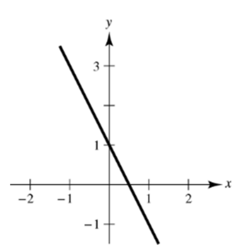
35. $y = -3$



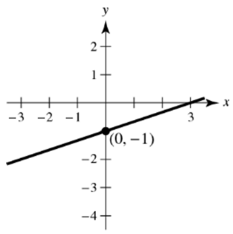
36. $x = 4$



37. $y = -2x + 1$

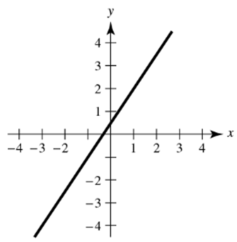


38. $y = \frac{1}{3}x - 1$



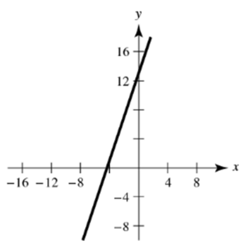
39. $y - 2 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{1}{2}$



40. $y - 1 = 3(x + 4)$

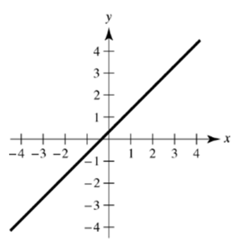
$y = 3x + 13$



41. $3x - 3y + 1 = 0$

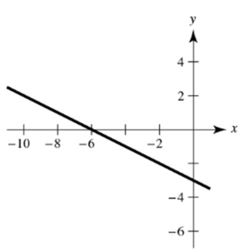
$3y = 3x + 1$

$y = x + \frac{1}{3}$



42. $x + 2y + 6 = 0$

$y = -\frac{1}{2}x - 3$

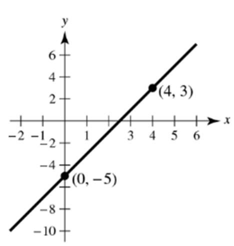


43. $m = \dfrac{-5 - 3}{0 - 4} = \dfrac{-8}{-4} = 2$

$y - (-5) = 2(x - 0)$

$y + 5 = 2x$

$0 = 2x - y - 5$



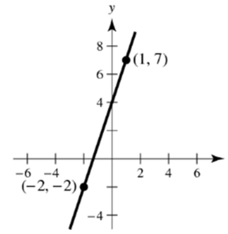
44. $m = \dfrac{7 - (-2)}{1 - (-2)} = \dfrac{9}{3} = 3$

$y - (-2) = 3(x - (-2))$

$y + 2 = 3(x + 2)$

$y = 3x + 4$

$0 = 3x - y + 4$

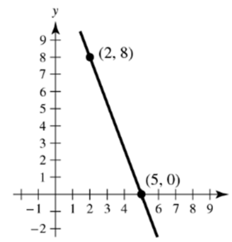


45. $m = \dfrac{8 - 0}{2 - 5} = -\dfrac{8}{3}$

$y - 0 = -\dfrac{8}{3}(x - 5)$

$y = -\dfrac{8}{3}x + \dfrac{40}{3}$

$8x + 3y - 40 = 0$

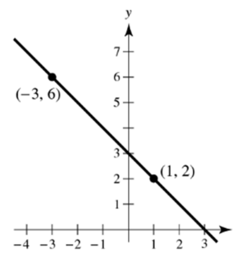


46. $m = \dfrac{6 - 2}{-3 - 1} = \dfrac{4}{-4} = -1$

$y - 2 = -1(x - 1)$

$y - 2 = -x + 1$

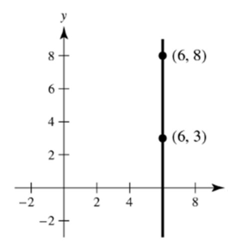
$x + y - 3 = 0$



47. $m = \dfrac{8 - 3}{6 - 6} = \dfrac{5}{0}$, undefined

The line is vertical.

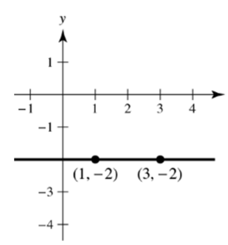
$x = 6$ or $x - 6 = 0$



48. $m = \dfrac{-2 - (-2)}{3 - 1} = \dfrac{0}{2} = 0$

$y = -2$

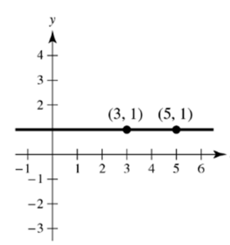
$y + 2 = 0$



49. $m = \dfrac{1 - 1}{5 - 3} = 0$

The line is horizontal.

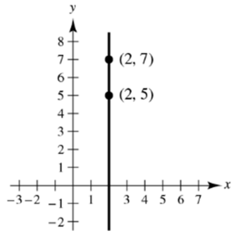
$y = 1$ or $y - 1 = 0$



50. $m = \dfrac{7-5}{2-2} = \dfrac{2}{0}$, undefined

The line is vertical.

$x = 2$ or $x - 2 = 0$



51. The slope is $\dfrac{1-b}{3-0} = \dfrac{1-b}{3}$.

The y-intercept is $(0, b)$. Hence,

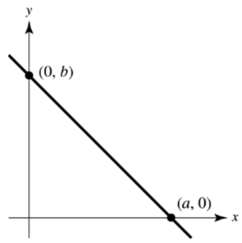
$$y = mx + b = \left(\frac{1-b}{3}\right)x + b.$$

52. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$

$$\frac{b}{a}x + y = b$$

$$\frac{x}{a} + \frac{y}{b} = 1$$



53. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$$3x + 2y - 6 = 0$$

54. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$$\frac{-3x}{2} - \frac{y}{2} = 1$$

$$3x + y = -2$$

$$3x + y + 2 = 0$$

55. $\dfrac{x}{2a} + \dfrac{y}{a} = 1$

$$\frac{9}{2a} + \frac{-2}{a} = 1$$

$$\frac{9-4}{2a} = 1$$

$$5 = 2a$$

$$a = \frac{5}{2}$$

$$\frac{x}{2\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$$

$$\frac{x}{5} + \frac{2y}{5} = 1$$

$$x + 2y = 5$$

$$x + 2y - 5 = 0$$

56. $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$$\frac{\left(-\frac{2}{3}\right)}{a} + \frac{(-2)}{-a} = 1$$

$$-\frac{2}{3} + 2 = a$$

$$a = \frac{4}{3}$$

$$\frac{x}{\left(\frac{4}{3}\right)} + \frac{y}{\left(-\frac{4}{3}\right)} = 1$$

$$x - y = \frac{4}{3}$$

$$3x - 3y - 4 = 0$$

57. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

58. The given line is horizontal.

(a) $y = 0$

(b) $x = -1$, or $x + 1 = 0$

59. $x + y = 7$

$$y = -x + 7$$

$$m = -1$$

(a) $y - 2 = -1(x + 3)$

$$y - 2 = -x - 3$$

$$x + y + 1 = 0$$

(b) $y - 2 = 1(x + 3)$

$$y - 2 = x + 3$$

$$0 = x - y + 5$$

60. $x - y = -2$

$$y = x + 2$$

$$m = 1$$

(a) $y - 5 = 1(x - 2)$

$$y - 5 = x - 2$$

$$x - y + 3 = 0$$

(b) $y - 5 = -1(x - 2)$

$$y - 5 = -x + 2$$

$$x + y - 7 = 0$$

61. $5x - 3y = 0$

$y = \frac{5}{3}x$

$m = \frac{5}{3}$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$

$24y - 21 = 40x - 30$

$0 = 40x - 24y - 9$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$

$40y - 35 = -24x + 18$

$24x + 40y - 53 = 0$

62. $7x + 4y = 8$

$4y = -7x + 8$

$y = \frac{-7}{4}x + 2$

$m = -\frac{7}{4}$

(a) $y + \frac{1}{2} = \frac{-7}{4}\left(x - \frac{5}{6}\right)$

$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$

$24y + 12 = -42x + 35$

$42x + 24y - 23 = 0$

(b) $y + \frac{1}{2} = \frac{4}{7}\left(x - \frac{5}{6}\right)$

$42y + 21 = 24x - 20$

$24x - 42y - 41 = 0$

63. The slope is 250.

$V = 1850$ when $t = 1$.

$V = 250(t - 1) + 1850$

$= 250t + 1600$

64. The slope is -1600.

$V = 17{,}200$ when $t = 1$.

$V = -1600(t - 1) + 17{,}200$

$= -1600t + 18{,}800$

65. $m_1 = \frac{1 - 0}{-2 - (-1)} = -1$

$m_2 = \frac{-2 - 0}{2 - (-1)} = -\frac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

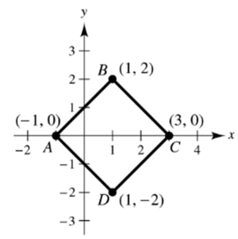
66. $m_1 = \frac{-6 - 4}{7 - 0} = -\frac{10}{7}$

$m_2 = \frac{11 - 4}{-5 - 0} = -\frac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

67.



The four sides are of equal length: $\sqrt{8} = 2\sqrt{2}$.

For example, the length of segment AB is

$$\sqrt{(1 - (-1))^2 + (2 - 0)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2} \text{ units.}$$

Furthermore, the adjacent sides are perpendicular because the slope of $\overline{AB}$ is $\frac{2 - 0}{1 - (-1)} = \frac{2}{2} = 1$, whereas the slope of $\overline{BC}$ is $\frac{2 - 0}{1 - 3} = -1$.

68. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

(b) The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(c) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$-5x + 8y = 4$

$y = \frac{1}{8}(5x + 4) = \frac{5}{8}x + \frac{1}{2}$

(d) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$5x + 2y = 4$

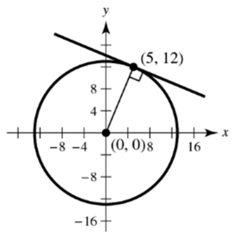
$y = \frac{1}{2}(-5x + 4) = -\frac{5}{2}x + 2$

(e) $a = \frac{5}{2}$ and $b = 3$.

$\frac{5}{2}x + 3y = 4$

$5x + 6y = 8$

69. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).

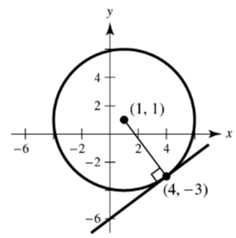


Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$
$$y = \frac{-5}{12}x + \frac{169}{12}$$
$$5x + 12y - 169 = 0.$$

70. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is

$\frac{1 + 3}{1 - 4} = \frac{-4}{3}$.

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$
$$y = \frac{3}{4}x - 6$$
$$0 = 3x - 4y - 24$$

71. (a) The slope of the segment joining (b, c) and $(a, 0)$ is $\frac{c}{(b - a)}$. The slope of the perpendicular bisector of this segment is $\frac{(a - b)}{c}$. The midpoint of this segment is $\left(\frac{a + b}{2}, \frac{c}{2}\right)$.

So, the equation of the perpendicular bisector to this segment is

$$y - \frac{c}{2} = \frac{a - b}{c}\left(x - \frac{a + b}{2}\right).$$

Similarly, the equation of the perpendicular bisector of the segment joining $(-a, 0)$ and $(a, 0)$ is

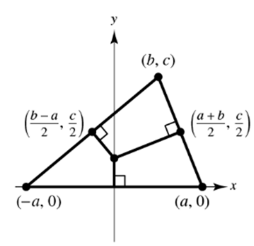
$$y - \frac{c}{2} = \frac{a - b}{-c}\left(x - \frac{b - a}{2}\right).$$

Equating the right-hand sides of each equation, you obtain $x = 0$.

Letting $x = 0$ in either equation yields the point of intersection:

$$y = \frac{c}{2} + \frac{a - b}{c}\left(0 - \frac{a + b}{2}\right) = \frac{c^2}{2c} + \frac{b^2 - a^2}{2c} = \frac{c^2 + b^2 - a^2}{2c}.$$

The point of intersection is $\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right)$.

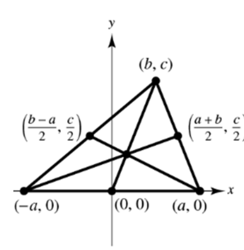


(b) The equations of the medians are:

$$y = \frac{c}{b}x$$

$$y = \frac{c/2}{\left(\frac{b - a}{2}\right) - a}(x - a) = \frac{c}{b - 3a}(x - a)$$

$$y = \frac{c/2}{\left(\frac{a + b}{2} + a\right)}(x + a) = \frac{c}{3a + b}(x + a).$$



Solving these equation simultaneously for (x, y), you obtain the point of intersection $\left(\frac{b}{3}, \frac{c}{3}\right)$.

72. (a) Lines *c*, *d*, *e* and *f* have positive slopes.

(b) Lines *a* and *b* have negative slopes.

(c) Lines *c* and *e* appear parallel.
Lines *d* and *f* appear parallel.

(d) Lines *b* and *f* appear perpendicular.
Lines *b* and *d* appear perpendicular.

73. Find the equation of the line through the points (0, 32) and (100, 212).

$$m = \frac{180}{100} = \frac{9}{5}$$

$$F - 32 = \frac{9}{5}(C - 0)$$

$$F = \frac{9}{5}C + 32$$

or

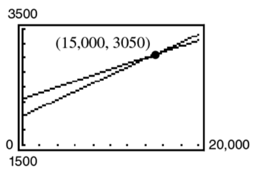
$$C = \frac{1}{9}(5F - 160)$$

$$5F - 9C - 160 = 0$$

For $F = 72°$, $C \approx 22.2°$.

74. (a) Current job: $W_1 = 0.07s + 2000$
New job offer: $W_2 = 0.05s + 2300$

(b)



Using a graphing utility, the point of intersection is (15,000, 3050).

Analytically, $W_1 = W_2$

$$0.07s + 2000 = 0.05s + 2300$$

$$0.02s = 300$$

$$s = 15{,}000$$

So, $W_1 = W_2 = 0.07(15{,}000) + 2000 = 3050.$

When sales exceed \$15,000, the current job pays more.

(c) No, if you can sell \$20,000 worth of goods, then $W_1 > W_2$.
(**Note:** $W_1 = 3400$ and $W_2 = 3300$ when $s = 20{,}000$.)

75. (a) Two points are (50, 780) and (47, 825).
The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$

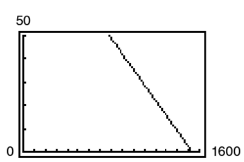
$$p - 780 = -15(x - 50)$$

$$p = -15x + 750 + 780 = -15x + 1530$$

or

$$x = \frac{1}{15}(1530 - p)$$

(b)

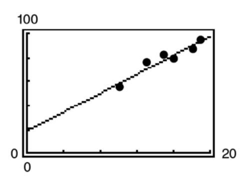


If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units

76. (a) $y = 18.91 + 3.97x$
$(x = \text{quiz score},\ y = \text{test score})$

(b)



(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a *y*-intercept 4 greater than before: $y = 22.91 + 3.97x$.

77. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$ then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$
$$Ay - Ay_1 = Bx - Bx_1$$
$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad \Rightarrow \quad A^2x + ABy = -AC \qquad (1)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \qquad (2)$$
$$(A^2 + B^2)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$
$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad \Rightarrow \quad ABx + B^2y = -BC \qquad (3)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} \qquad (4)$$
$$(A^2 + B^2)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$
$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

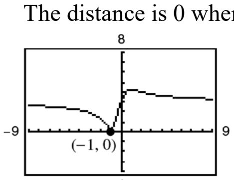
The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$d = \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2}$$
$$= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2}$$
$$= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

78. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).



79. $x - y - 2 = 0 \Rightarrow d = \dfrac{\left|1(-2) + (-1)(1) - 2\right|}{\sqrt{1^2 + 1^2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

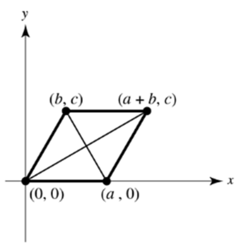
80. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{\left|4(2) + 3(3) - 10\right|}{\sqrt{4^2 + 3^2}} = \dfrac{7}{5}$

81. For simplicity, let the vertices of the rhombus be (0, 0), $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure.

The slopes of the diagonals are then $m_1 = \dfrac{c}{a + b}$ and $m_2 = \dfrac{c}{b - a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

$$m_1 m_2 = \frac{c}{a + b} \cdot \frac{c}{b - a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



82. For simplicity, let the vertices of the quadrilateral be (0, 0), (a, 0), (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

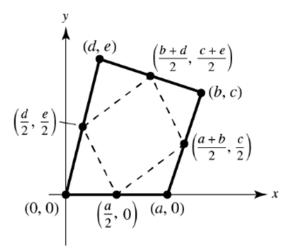
$$\left(\frac{a}{2}, 0\right), \left(\frac{a + b}{2}, \frac{c}{2}\right), \left(\frac{b + d}{2}, \frac{c + e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a + b}{2} - \frac{a}{2}} = \frac{\frac{c + e}{2} - \frac{e}{2}}{\frac{b + d}{2} - \frac{d}{2}} = \frac{c}{b}$$

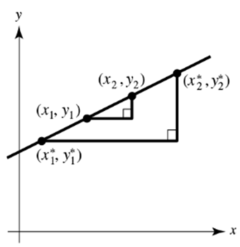
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c + e}{2}}{\frac{a + b}{2} - \frac{b + d}{2}} = -\frac{e}{a - d}$$

Therefore, the figure is a parallelogram.



83. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



84. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. So, $m_2 = m_3 \Rightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

85. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

86. True. The slope must be positive.

Section 1.3 Functions and Their Graphs

1. All functions are relations, but not all relations are functions. For example, $\{(1, 1), (1, 2)\}$ is a relation, but not a function because two different outputs correspond to the same input. Functions have exactly one output for every input.

2. For a function f from X to Y, the domain is the set X. If y is the image of x, then the range is a subset of Y consisting of all images of numbers in X.

3. The three basic types are vertical shifts, horizontal shifts, and reflections.

4. Consider the nonconstant polynomial

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0.$

If n is even and $a_n > 0$, then the graph of f moves up to the left and up to the right.

If n is even and $a_n < 0$, then the graph of f moves down to the left and down to the right.

If n is odd and $a_n > 0$, then the graph of f moves down to the left and up to the right.

If n is odd and $a_n < 0$, then the graph of f moves up to the left and down to the right.

5. $f(x) = 3x - 2$

(a) $f(0) = 3(0) - 2 = -2$

(b) $f(5) = 3(5) - 2 = 13$

(c) $f(b) = 3(b) - 2 = 3b - 2$

(d) $f(x - 1) = 3(x - 1) - 2 = 3x - 5$

6. (a) $f(0) = 7(0) - 4 = -4$

(b) $f(-3) = 7(-3) - 4 = -25$

(c) $f(b) = 7(b) - 4 = 7b - 4$

(d) $f(x - 1) = 7(x - 1) - 4 = 7x - 11$

7. (a) $f(-2) = \sqrt{(-2)^2 + 4} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

(b) $f(3) = \sqrt{3^2 + 4} = \sqrt{9 + 4} = \sqrt{13}$

(c) $f(2) = \sqrt{2^2 + 4} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

(d) $f(x + bx) = \sqrt{(x + bx)^2 + 4}$

$= \sqrt{x^2 + 2bx^2 + b^2x^2 + 4}$

8. (a) $f(-4) = \sqrt{-4 + 5} = \sqrt{1} = 1$

(b) $f(11) = \sqrt{11 + 5} = \sqrt{16} = 4$

(c) $f(4) = \sqrt{4 + 5} = \sqrt{9} = 3$

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 5}$

9. (a) $g(0) = 5 - 0^2 = 5$

(b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = 5 - 5 = 0$

(c) $g(-2) = 5 - (-2)^2 = 5 - 4 = 1$

(d) $g(t - 1) = 5 - (t - 1)^2 = 5 - (t^2 - 2t + 1)$

$= 4 + 2t - t^2$

10. (a) $g(4) = 4^2(4 - 4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $g(t + 4) = (t + 4)^2(t + 4 - 4)$

$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

11. $$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2,\ \Delta x \neq 0$$

12. $$\frac{f(x) - f(1)}{x - 1} = \frac{3x - 1 - (3 - 1)}{x - 1} = \frac{3(x - 1)}{x - 1} = 3,\ x \neq 1$$

13. $f(x) = 4x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

14. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

15. $h(x) = 4 - x^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

16. $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

17. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$

$x \geq 0 \Rightarrow [0, \infty)$

Range: $[0, \infty)$

18. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \geq 0 \Rightarrow [-3, \infty)$

Range: $(-\infty, 0]$

19. $f(x) = \sqrt{16 - x^2}$

$16 - x^2 \geq 0 \Rightarrow x^2 \leq 16$

Domain: $[-4, 4]$

Range: $[0, 4]$

Note: $y = \sqrt{16 - x^2}$ is a semicircle of radius 4.

20. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

21. $f(x) = \dfrac{3}{x}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

22. $f(x) = \dfrac{x - 2}{x + 4}$

Domain: all $x \neq -4$

Range: all $y \neq 1$

[**Note:** You can see that the range is all $y \neq 1$ by graphing *f*.]

23. $f(x) = \sqrt{x} + \sqrt{1 - x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

24. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \geq 0$

$(x - 2)(x - 1) \geq 0$

Domain: $x \geq 2$ or $x \leq 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

25. $f(x) = \dfrac{1}{|x + 3|}$

$|x + 3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

26. $g(x) = \dfrac{1}{|x^2 - 4|}$

$|x^2 - 4| \neq 0$

$(x - 2)(x + 2) \neq 0$

Domain: all $x \neq \pm 2$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

27. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all *t*.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

28. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 + 2 = 2s^4 + 8s^2 + 10$

(**Note:** $s^2 + 2 > 1$ for all *s*.)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

29. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

30. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

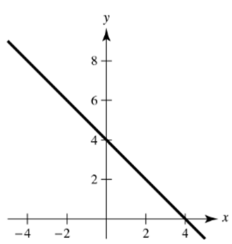
Domain: $[-4, \infty)$

Range: $[0, \infty)$

31. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

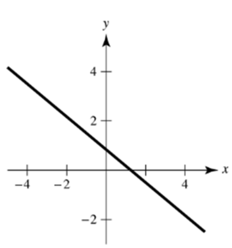
Range: $(-\infty, \infty)$



32. $f(x) = \frac{5}{6} - \frac{2}{3}x$

Domain: $(-\infty, \infty)$

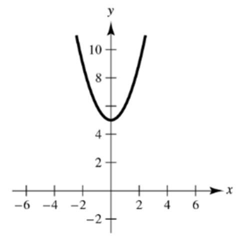
Range: $(-\infty, \infty)$



33. $f(x) = x^2 + 5$

Domain: $(-\infty, \infty)$

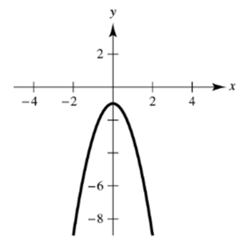
Range: $[5, \infty)$



34. $f(x) = -2x^2 - 1$

Domain: $(-\infty, \infty)$

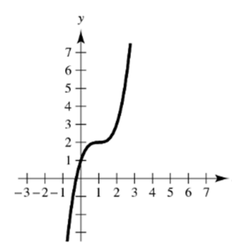
Range: $(-\infty, \infty)$



35. $f(x) = (x - 1)^3 + 2$

Domain: $(-\infty, \infty)$

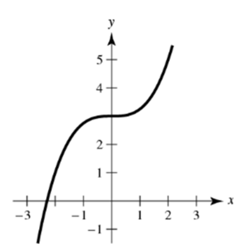
Range: $(-\infty, \infty)$



36. $f(x) = \frac{1}{4}x^3 + 3$

Domain: $(-\infty, \infty)$

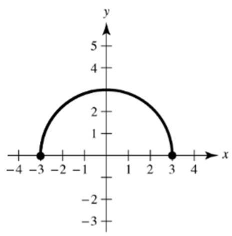
Range: $(-\infty, \infty)$



37. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



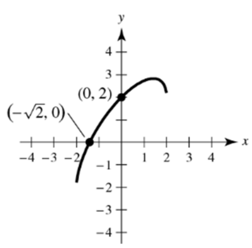
38. $f(x) = x + \sqrt{4 - x^2}$

Domain: $[-2, 2]$

Range: $\left[-2, 2\sqrt{2}\right] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

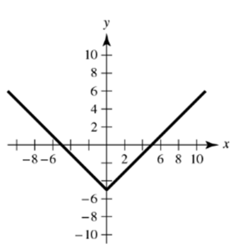
x-intercept: $\left(-\sqrt{2}, 0\right)$



39. $g(x) = |x| - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$



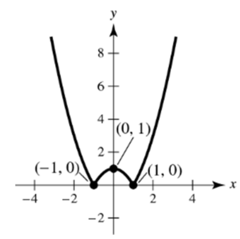
40. $f(x) = \left|x^2 - 1\right| = \left|(x - 1)(x + 1)\right|$

Intercepts: $(1, 0), (-1, 0), (0, 1)$

Symmetry: y-axis

Domain: $(-\infty, \infty)$

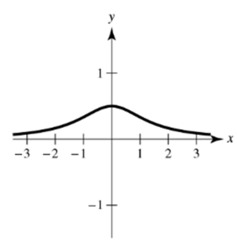
Range: $[0, \infty)$



41. $g(x) = \dfrac{1}{x^2 + 2}$

Domain: $(-\infty, \infty)$

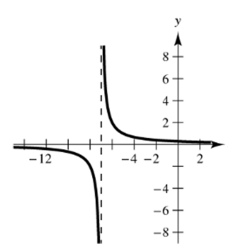
Range: $\left(0, \frac{1}{2}\right]$



42. $f(t) = \dfrac{2}{7 + t}$

Domain: $(-\infty, -7) \cup (-7, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$



43. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

44. $\sqrt{x^2 - 4} - y = 0 \Rightarrow y = \sqrt{x^2 - 4}$

y is a function of x. Vertical lines intersect the graph at most once.

45. y is a function of x. Vertical lines intersect the graph at most once.

46. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4 - x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

47. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16 - x^2}$

y is not a function of x because there are two values of y for some x.

48. $x^2 + y = 16 \Rightarrow y = 16 - x^2$

y is a function of x because there is one value of y for each x.

49. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x because there are two values of y for some x.

50. $x^2y - x^2 + 4y = 0 \Rightarrow y = \dfrac{x^2}{x^2 + 4}$

y is a function of x because there is one value of y for each x.

51. The transformation is a horizontal shift two units to the right of the function $f(x) = \sqrt{x}$.

Shifted function: $y = \sqrt{x - 2}$

52. The transformation is a vertical shift 2 units upward of the function $f(x) = |x|$.

Shifted function: $y = |x| + 2$

53. The transformation is a horizontal shift 2 units to the right and a vertical shift 1 unit downward of the function $f(x) = x^2$.

Shifted function: $y = (x - 2)^2 - 1$

54. The transformation is a horizontal shift 1 unit to the left and a vertical shift 2 units upward of the function $f(x) = x^3$.

Shifted function: $y = (x + 1)^3 + 2$

55. $y = f(x + 5)$ is a horizontal shift 5 units to the left.

Matches d.

56. $y = f(x) - 5$ is a vertical shift 5 units downward.

Matches b.

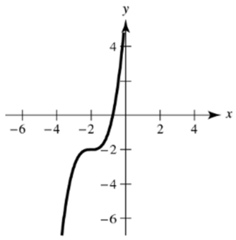
57. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

58. $y = -f(x - 4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

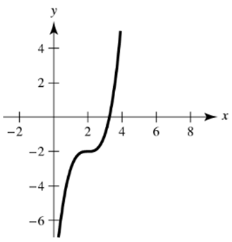
59. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

60. $y = f(x - 1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

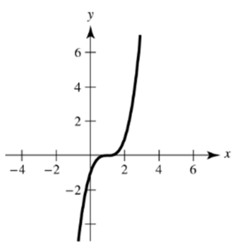
61. (a) The graph is shifted 3 units to the left.



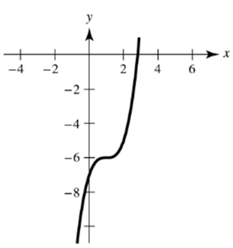
(b) The graph is shifted 1 unit to the right.



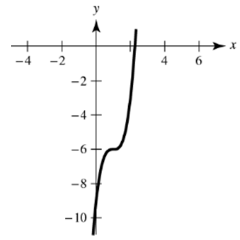
(c) The graph is shifted 2 units upward.



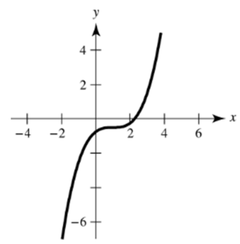
(d) The graph is shifted 4 units downward.



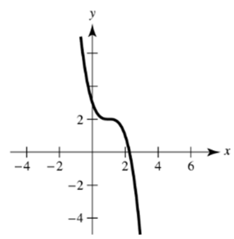
(e) The graph is stretched vertically by a factor of 3.



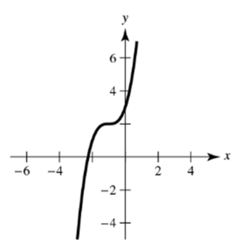
(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.



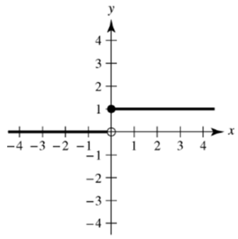
(g) The graph is a reflection in the *x*-axis.



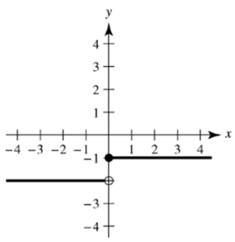
(h) The graph is a reflection about the origin.



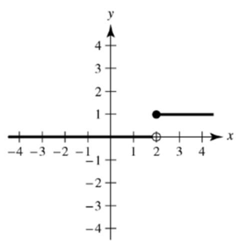
62. $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$



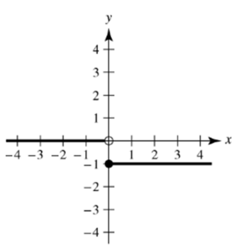
(a) $H(x) - 2 = \begin{cases} -1, & x \geq 0 \\ -2, & x < 0 \end{cases}$



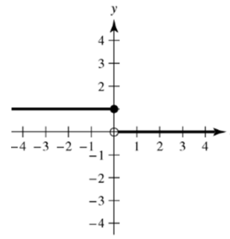
(b) $H(x - 2) = \begin{cases} 1, & x \geq 2 \\ 0, & x < 2 \end{cases}$



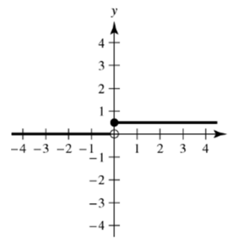
(c) $-H(x) = \begin{cases} -1, & x \geq 0 \\ 0, & x < 0 \end{cases}$



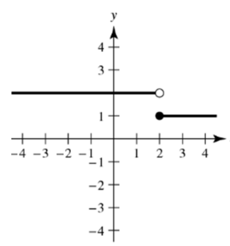
(d) $H(-x) = \begin{cases} 1, & x \leq 0 \\ 0, & x > 0 \end{cases}$



(e) $\frac{1}{2}H(x) = \begin{cases} \frac{1}{2}, & x \geq 0 \\ 0, & x < 0 \end{cases}$



(f) $-H(x - 2) + 2 = \begin{cases} 1, & x \geq 2 \\ 2, & x < 2 \end{cases}$



63. $f(x) = 2x - 5, g(x) = 4 - 3x$

(a) $f(x) + g(x) = (2x - 5) + (4 - 3x) = -x - 1$

(b) $f(x) - g(x) = (2x - 5) - (4 - 3x) = 5x - 9$

(c) $f(x) \cdot g(x) = (2x - 5)(4 - 3x) = -6x^2 + 8x + 15x - 20 = -6x^2 + 23x - 20$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{2x - 5}{4 - 3x}$

64. $f(x) = x^2 + 5x + 4, \quad g(x) = x + 1$

(a) $f(x) + g(x) = (x^2 + 5x + 4) + (x + 1) = x^2 + 6x + 5$

(b) $f(x) - g(x) = (x^2 + 5x + 4) - (x + 1) = x^2 + 4x + 3$

(c) $f(x) \cdot g(x) = (x^2 + 5x + 4)(x + 1) = x^3 + 5x^2 + 4x + x^2 + 5x + 4 = x^3 + 6x^2 + 9x + 4$

(d) $f(x)/g(x) = \dfrac{x^2 + 5x + 4}{x + 1} = \dfrac{(x + 4)(x + 1)}{x + 1} = x + 4, x \neq -1$

65. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \ (x \geq 0)$

66. $f(x) = 2x^3, g(x) = 4x + 3$

(a) $f(g(0)) = f(3) = 2(3)^3 = 54$

(b) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(4\left(\frac{1}{2}\right) + 3\right) = f(5) = 2(5^3) = 250$

(c) $g(f(0)) = g(0) = 3$

(d) $g\left(f\left(-\frac{1}{4}\right)\right) = g\left(2\left(-\frac{1}{4}\right)^3\right) = g\left(-\frac{1}{32}\right) = 4\left(-\frac{1}{32}\right) + 3 = \frac{23}{8}$

(e) $f(g(x)) = f(4x + 3) = 2(4x + 3)^3 = 2(64x^3 + 144x^2 + 108x + 27) = 128x^3 + 288x^2 + 216x + 54$

(f) $g(f(x)) = g(2x^3) = 4(2x^3) + 3 = 8x^3 + 3$

67. $f(x) = x, g(x) = x^2$

$(f \circ g)(x) = f(g(x)) = f(x^2) = x^2$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x) = x^2$

Domain: $(-\infty, \infty)$

The composite functions are equal.

68. $f(x) = \sqrt[3]{x - 5},\ g(x) = x^3 + 5$

$(f \circ g)(x) = f(g(x)) = f(x^3 + 5) = \sqrt[3]{(x^3 + 5) - 5} = \sqrt[3]{x^3} = x$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x - 5}) = (\sqrt[3]{x - 5})^3 + 5 = x - 5 + 5 = x$

Domain: $(-\infty, \infty)$

The composite functions are equal.

69. $f(x) = x^2,\ g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x,\ x \ge 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \ge 0$.

70. $f(x) = x^2 - 1, g(x) = -x$

$(f \circ g)(x) = f(g(x)) = f(-x) = (-x)^2 - 1 = x^2 - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = -(x^2 - 1) = 1 - x^2$

Domain: $(-\infty, \infty)$

$f \circ g \ne g \circ f$

71. $f(x) = \dfrac{3}{x},\ g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \dfrac{3}{x^2 - 1}$

Domain: all $x \ne \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{3}{x}\right) = \left(\dfrac{3}{x}\right)^2 - 1 = \dfrac{9}{x^2} - 1 = \dfrac{9 - x^2}{x^2}$

Domain: all $x \ne 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

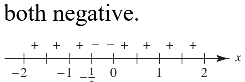
$f \circ g \ne g \circ f$

72. $(f \circ g)(x) = f(\sqrt{x + 2}) = \dfrac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 2} = \sqrt{\dfrac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

73. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined

74. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

75. $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x$, $g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

(Other answers possible.)

76. $F(x) = \dfrac{1}{4x^6}$

Let $f(x) = \dfrac{1}{x}$, $g(x) = 4x$, and $h(x) = x^6$.

Then $(f \circ g \circ h)(x) = f(g(x^6)) = f(4x^6) = \dfrac{1}{4x^6}$.

(Other answers possible.)

77. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

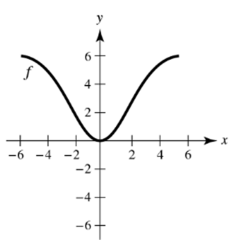
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

78. (a) If f is even, then $(-4, 9)$ is on the graph.

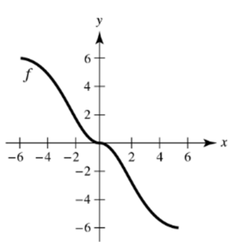
(b) If f is odd, then $(-4, -9)$ is on the graph.

79. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

80. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.



81. $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2\left(4 - (-x)^2\right) = x^2(4 - x^2) = f(x)$

f is even.

$f(x) = x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

Zeros: $x = 0, -2, 2$

82. $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{(-x)} = -\sqrt[3]{x} = -f(x)$

f is odd.

$f(x) = \sqrt[3]{x} = 0 \Rightarrow x = 0$ is the zero.

83. $f(x) = 2\sqrt[6]{x}$

The domain of f is $x \ge 0$ and the range is $y \ge 0$. Hence, the function is neither even nor odd. The only zero is $x = 0$.

84. $f(x) = 4x^4 - 3x^2$

$f(-x) = 4(-x)^4 - 3(-x)^2 = 4x^4 - 3x^2 = f(x)$

f is even.

$4x^4 - 3x^2 = x^2(4x^2 - 3) = 0$

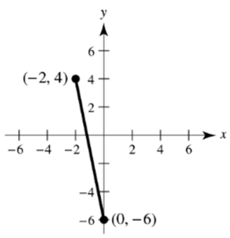
Zeros: $x = 0, \pm\dfrac{\sqrt{3}}{2}$

85. Slope $= \dfrac{4 - (-6)}{-2 - 0} = \dfrac{10}{-2} = -5$

$$y - 4 = -5(x - (-2))$$
$$y - 4 = -5x - 10$$
$$y = -5x - 6$$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, -2 \le x \le 0$

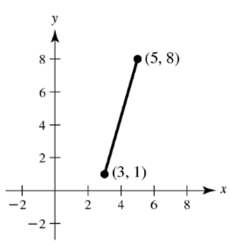


86. Slope $= \dfrac{8 - 1}{5 - 3} = \dfrac{7}{2}$

$$y - 1 = \frac{7}{2}(x - 3)$$
$$y - 1 = \frac{7}{2}x - \frac{21}{2}$$
$$y = \frac{7}{2}x - \frac{19}{2}$$

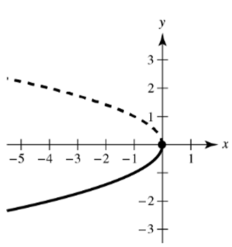
For the line segment, you must restrict the domain.

$f(x) = \dfrac{7}{2}x - \dfrac{19}{2}, 3 \le x \le 5$



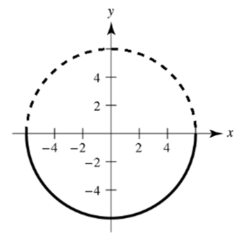
87. $x + y^2 = 0$

$$y^2 = -x$$
$$y = -\sqrt{-x}$$
$$f(x) = -\sqrt{-x}, \ x \le 0$$

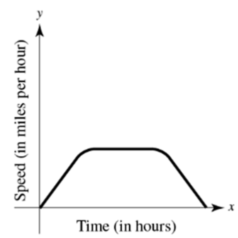


88. $x^2 + y^2 = 36$

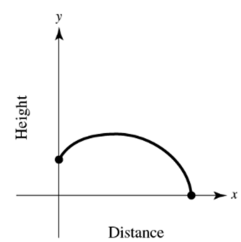
$$y^2 = 36 - x^2$$
$$y = -\sqrt{36 - x^2}, \ -6 \le x \le 6$$



89. Answers will vary. *Sample answer*: Speed begins and ends at 0. The speed might be constant in the middle:

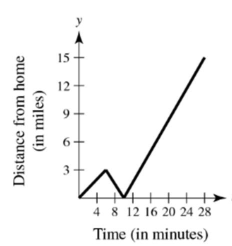


90. Answers will vary. *Sample answer*: Height begins a few feet above 0, and ends at 0.



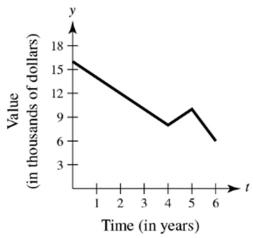
91. Answers will vary. *Sample answer:*

Distance begins at 0, then the graph has a sharp turn after a few minutes and goes back to 0. Then the graph goes back upward steeply.



92. Answers will vary. *Sample answer:*

The graph begins at time 0, then decreases until year 4. The graph then increases slightly for a few years and then decreases again.



93. $y = \sqrt{c - x^2}$

$y^2 = c - x^2$

$x^2 + y^2 = c$, a circle.

For the domain to be $[-5, 5]$, $c = 25$.

94. For the domain to be the set of all real numbers, you must require that $x^2 + 3cx + 6 \neq 0$. So, the discriminant must be less than zero:

$$(3c)^2 - 4(6) < 0$$
$$9c^2 < 24$$
$$c^2 < \tfrac{8}{3}$$
$$-\sqrt{\tfrac{8}{3}} < c < \sqrt{\tfrac{8}{3}}$$
$$-\tfrac{2}{3}\sqrt{6} < c < \tfrac{2}{3}\sqrt{6}$$

95. No. If a horizontal line intersects the graph more than once, then there is more than one *x*-value corresponding to the same *y*-value.

96. Answers will vary. *Sample answer:*

$f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x^2}$

$$(f \circ g)(x) = f\left(\frac{1}{x^2}\right) = x^2$$

$$(g \circ f)(x) = g\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)^2} = x^2$$

97. No. For example, $y = x^3 + x + 2$ is not odd since $f(-x) \neq -f(x)$.

98. $f(x) = 0$ is even and odd.

$$f(-x) = 0 = f(x) = -f(x)$$

99. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

(d) The points at the endpoints of the individual functions that form each "piece" appear to be $(0, 60)$, $(6, 60)$, $(7, 72)$, $(20, 72)$, $(21, 60)$, and $(24, 60)$. Note that the value $t = 24$ is chosen for the last ordered pair because that is when the day ends and the cycle starts over.

From $t = 0$ to $t = 6$: This is the constant function $T(t) = 60$.

From $t = 6$ to $t = 7$: Use the points (6, 60) and (7, 72).

$$m = \frac{72 - 60}{7 - 6} = 12$$

$y - 60 = 12(x - 6) \Rightarrow y = 12x - 12$, or $T(t) = 12t - 12$.

From $t = 7$ to $t = 20$: This is the constant function $T(t) = 72$.

From $t = 20$ to $t = 21$: Use the points (20, 72) and (21, 60).

$$m = \frac{72 - 60}{20 - 21} = -12$$

$y - 60 = -12(x - 21) \Rightarrow y = -12x + 312$, or $T(t) = -12 + 312$

From $t = 21$ to $t = 24$: This is the constant function $T(t) = 60$.

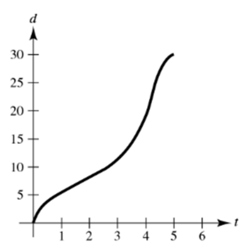
A piecewise-defined function is $T(t) = \begin{cases} 60, & x \le 0 \le 6 \\ 12t - 12, & 6 < T < 7 \\ 72, & 7 \le T \le 20 \\ -12t + 312, & 20 < t < 21 \\ 60, & 21 \le t \le 24 \end{cases}$

100. (a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

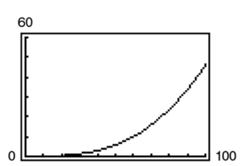
Range: $0 \le d \le 30$

(c)



(d) $d(4) \approx 18$. At time 4 seconds, the depth is approximately 18 cm.

101. (a)



(b) $H\left(\frac{x}{1.6}\right) = 0.00004636\left(\frac{x}{1.6}\right)^3 \approx 0.00001132x^3$

102. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, you have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Because the graph has no breaks, the graph must cross the x-axis at least one time.

103.
$$\begin{aligned} f(-x) &= a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x) \\ &= -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right] \\ &= -f(x) \end{aligned}$$

Odd

104. $f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$

$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$= f(x)$

Even

105. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.

So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x)$.

So, $F(x)$ is even.

106. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then

$F(-x) = f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x) = -F(x)$.

So, $F(x)$ is odd.

107. By equating slopes, $\dfrac{y-2}{0-3} = \dfrac{0-2}{x-3}$

$$y - 2 = \frac{6}{x-3}$$

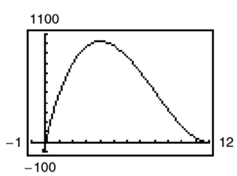
$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

108. (a) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(b)



Maximum volume occurs at $x = 4$. So, the dimensions of the box would be $4 \times 16 \times 16$ cm.

109. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

110. True

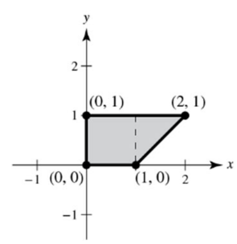
111. True. The function is even.

112. False. If $f(x) = x^2$ then, $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So, $3f(x) \neq f(3x)$.

113. False. The constant function $f(x) = 0$ has symmetry with respect to the x-axis.

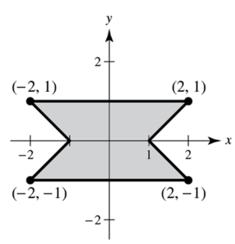
114. True. If the domain is $\{a\}$, then the range is $\{f(a)\}$.

115. First consider the portion of R in the first quadrant: $x \geq 0,\ 0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

116. Let $g(x) = c$ be constant polynomial.

Then $f(g(x)) = f(c)$ and $g(f(x)) = c$.

So, $f(c) = c$. Because this is true for all real numbers c, f is the identity function: $f(x) = x$.

Section 1.4 Review of Trigonometric Functions

1. In general, if θ is any angle in radians, then the angle $\theta + n(2\pi)$, where n is a nonzero integer, is coterminal with θ.

2. If the degree measure of angle θ is x, then the radian measure of θ is $x\left(\frac{\pi}{180°}\right)$.

3. $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{7}{25}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{24}{25}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{7}{24}$

4. The amplitude of the graph of $y = a\sin bx$ or $y = a\cos bx$ is $|a|$. This value is the maximum value of the function, and $-|a|$ is the minimum value.

A function is periodic if there exists a positive real number p such that $f(x + p) = f(x)$ for all x in the domain of f. The period of f is the least positive value of p.

5. (a) $\theta + 360° = 36° + 360° = 396°$

$\theta - 360° = 36° - 360° = -324°$

(b) $\theta + 360° = -120° + 360° = 240°$

$\theta - 360° = -120° - 360° = -480°$

6. (a) $\theta + 360° = 300° + 360° = 660°$

$\theta - 360° = 300° - 360° = -60°$

(b) $\theta + 2(360°) = -420° + 720° = 300°$

$\theta + 360° = -420° + 360° = -60°$

7. (a) $\theta + 2\pi = \frac{\pi}{9} + 2\pi = \frac{19\pi}{9}$

$\theta - 2\pi = \frac{\pi}{9} - 2\pi = -\frac{17\pi}{9}$

(b) $\theta + 2\pi = \frac{4\pi}{3} + 2\pi = \frac{10\pi}{3}$

$\theta - 2\pi = \frac{4\pi}{3} - 2\pi = -\frac{2\pi}{3}$

8. (a) $\theta + 2\pi = -\frac{9\pi}{4} + 2\pi = -\frac{\pi}{4}$

$\theta + 4\pi = -\frac{9\pi}{4} + 4\pi = \frac{7\pi}{4}$

(b) $\theta + 2\pi = \frac{8\pi}{9} + 2\pi = \frac{26\pi}{9}$

$\theta - 2\pi = \frac{8\pi}{9} - 2\pi = -\frac{10\pi}{9}$

9. (a) $30°\left(\frac{\pi}{180°}\right) = \frac{\pi}{6} \approx 0.524$

(b) $150°\left(\frac{\pi}{180°}\right) = \frac{5\pi}{6} \approx 2.618$

(c) $315°\left(\frac{\pi}{180°}\right) = \frac{7\pi}{4} \approx 5.498$

(d) $120°\left(\frac{\pi}{180°}\right) = \frac{2\pi}{3} \approx 2.094$

10. (a) $-20°\left(\frac{\pi}{180°}\right) = -\frac{\pi}{9} \approx -0.349$

(b) $-240°\left(\frac{\pi}{180°}\right) = -\frac{4\pi}{3} \approx -4.189$

(c) $-330°\left(\frac{\pi}{180°}\right) = -\frac{11}{6}\pi \approx -5.760$

(d) $144°\left(\frac{\pi}{180°}\right) = -\frac{4\pi}{5} \approx 2.513$

11. (a) $\frac{3\pi}{2}\left(\frac{180°}{\pi}\right) = 270°$

(b) $\frac{7\pi}{6}\left(\frac{180°}{\pi}\right) = 210°$

(c) $-\frac{7\pi}{12}\left(\frac{180°}{\pi}\right) = -105°$

(d) $-2.367\left(\frac{180°}{\pi}\right) \approx -135.62°$

12. (a) $\frac{7\pi}{3}\left(\frac{180°}{\pi}\right) = 420°$

(b) $-\frac{11\pi}{30}\left(\frac{180°}{\pi}\right) = -66°$

(c) $\frac{11\pi}{6}\left(\frac{180°}{\pi}\right) = 330°$

(d) $0.438\left(\frac{180°}{\pi}\right) \approx 25.1°$

13.

r	8 ft	15 in.	85 cm	24 in.	$\frac{12{,}963}{\pi}$ mi
s	12 ft	24 in.	63.75π cm	96 in.	8642 mi
θ	1.5	1.6	$\frac{3\pi}{4}$	4	$\frac{2\pi}{3}$

14. (a) $50 \text{ mi/h} = \dfrac{50(5280)}{60} = 4400 \text{ ft/min}$

Circumference of tire: $C = 2.5\pi$ feet

Revolutions per minute: $\dfrac{4400}{2.5\pi} \approx 560.2$

(b) $\theta = \dfrac{4400}{2.5\pi}(2\pi) = 3520$ radians

Angular speed:
$\dfrac{\theta}{t} = \dfrac{3520 \text{ radians}}{1 \text{ minute}} = 3520 \text{ rad/min}$

15. (a) $x = 3, y = 4, r = 5$

$\sin\theta = \frac{4}{5}$ $\qquad \csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\qquad \sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\qquad \cot\theta = \frac{3}{4}$

(b) $x = -12, y = -5, r = 13$

$\sin\theta = -\frac{5}{13}$ $\qquad \csc\theta = -\frac{13}{5}$

$\cos\theta = -\frac{12}{13}$ $\qquad \sec\theta = -\frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\qquad \cot\theta = \frac{12}{5}$

16. (a) $x = 8, y = -15, r = 17$

$\sin\theta = -\dfrac{15}{17}$ $\qquad \csc\theta = -\dfrac{17}{15}$

$\cos\theta = \dfrac{8}{17}$ $\qquad \sec\theta = \dfrac{17}{8}$

$\tan\theta = -\dfrac{15}{8}$ $\qquad \cot\theta = -\dfrac{8}{15}$

(b) $x = 1, y = -1, r = \sqrt{2}$

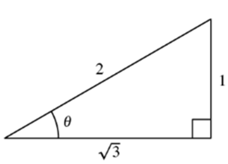
$\sin\theta = -\dfrac{\sqrt{2}}{2}$ $\qquad \csc\theta = -\sqrt{2}$

$\cos\theta = \dfrac{\sqrt{2}}{2}$ $\qquad \sec\theta = \sqrt{2}$

$\tan\theta = -1$ $\qquad \cot\theta = -1$

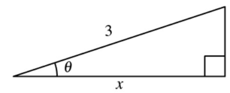
17. $x^2 + 1^2 = 2^2 \Rightarrow x = \sqrt{3}$

$\cos\theta = \dfrac{x}{2} = \dfrac{\sqrt{3}}{2}$



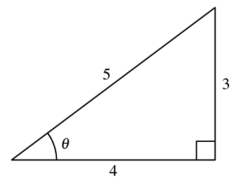
18. $x^2 + 1^2 = 3^2 \Rightarrow x = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \dfrac{1}{x} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$



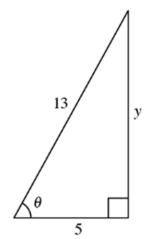
19. $4^2 + y^2 = 5^2 \Rightarrow y = 3$

$\cot\theta = \dfrac{4}{y} = \dfrac{4}{3}$



20. $5^2 + y^2 = 13^2 \Rightarrow y = 12$

$\csc\theta = \dfrac{13}{y} = \dfrac{13}{12}$



21. (a) $\sin 225° = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos 225° = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

$\tan 225° = \tan 45° = 1$

(b) $\sin(-225°) = \sin 45° = \dfrac{\sqrt{2}}{2}$

$\cos(-225°) = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

$\tan(-225°) = -\tan 45° = -1$

(c) $\sin\dfrac{5\pi}{3} = -\sin\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos\dfrac{5\pi}{3} = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

$\tan\dfrac{5\pi}{3} = -\tan\dfrac{\pi}{3} = -\sqrt{3}$

(d) $\sin\dfrac{11\pi}{6} = -\sin\dfrac{\pi}{6} = -\dfrac{1}{2}$

$\cos\dfrac{11\pi}{6} = \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

$\tan\dfrac{11\pi}{6} = -\tan\dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{3}$

22. (a) $\sin 60° = \dfrac{\sqrt{3}}{2}$

$\cos 60° = \dfrac{1}{2}$

$\tan 60° = \sqrt{3}$

(b) $\sin 120° = \sin 60° = \dfrac{\sqrt{3}}{2}$

$\cos 120° = -\cos 60° = -\dfrac{1}{2}$

$\tan 120° = -\tan 60° = -\sqrt{3}$

(c) $\sin \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\cos \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\tan \dfrac{\pi}{4} = 1$

(d) $\sin \dfrac{5\pi}{4} - \sin \dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$

$\cos \dfrac{5\pi}{4} = \cos \dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$

$\tan \dfrac{5\pi}{4} = \tan \dfrac{\pi}{4} = 1$

23. (a) $\cos(-60°) = \cos(60°) = \frac{1}{2}$

(b) $\cot \dfrac{\pi}{6} = \dfrac{1}{\tan(\pi/6)} = \dfrac{1}{(\sqrt{3}/3)} = \sqrt{3}$

24. (a) $\csc \dfrac{\pi}{3} = \dfrac{1}{\sin(\pi/3)} = \dfrac{1}{(\sqrt{3}/2)} = \dfrac{2\sqrt{3}}{3}$

(b) $\tan(-30°) = -\tan(30°) = -\dfrac{\sqrt{3}}{3}$

25. (a) $\sin 10° \approx 0.1736$

(b) $\csc 10° \approx 5.759$

26. (a) $\sec 225° \approx -1.414$

(b) $\sec 135° \approx -1.414$

27. (a) $\tan \dfrac{\pi}{9} \approx 0.3640$

(b) $\tan \dfrac{10\pi}{9} \approx 0.3640$

28. (a) $\cot 1.35 \approx 0.2245$

(b) $\tan 1.35 \approx 4.455$

29. (a) $\sin\theta < 0 \Rightarrow \theta$ is in Quadrant III or IV.

$\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II or III.

$\sin\theta < 0$ **and** $\cos\theta < 0 \Rightarrow \theta$ is in Quadrant III.

(b) $\sec\theta > 0 \Rightarrow \theta$ is in Quadrant I or IV.

$\cot\theta < 0 \Rightarrow \theta$ is in Quadrant II or IV.

$\sec\theta > 0$ **and** $\cot\theta < 0 \Rightarrow \theta$ is in Quadrant IV.

30. (a) $\sin\theta > 0 \Rightarrow \theta$ is in Quadrant I or II.

$\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II or III.

$\sin\theta > 0$ **and** $\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II.

(b) $\csc\theta < 0 \Rightarrow \theta$ is in Quadrant III or IV.

$\tan\theta > 0 \Rightarrow \theta$ is in Quadrant I or III.

$\csc\theta < 0$ **and** $\tan\theta > 0 \Rightarrow \theta$ is in Quadrant III.

31. (a) $\cos\theta = \dfrac{\sqrt{2}}{2}$

$\theta = \dfrac{\pi}{4}, \dfrac{7\pi}{4}$

(b) $\cos\theta = -\dfrac{\sqrt{2}}{2}$

$\theta = \dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

32. (a) $\sec\theta = 2$

$\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

(b) $\sec\theta = -2$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

33. (a) $\tan\theta = 1$

$\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

(b) $\cot\theta = -\sqrt{3}$

$\theta = \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

34. (a) $\sin\theta = \dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

(b) $\sin\theta = -\dfrac{1}{2} \Rightarrow \theta = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

35. $2\sin^2\theta = 1$

$$\sin\theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

36. $\tan^2\theta = 3$

$$\tan\theta = \pm\sqrt{3}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

37. $\tan^2\theta - \tan\theta = 0$

$$\tan\theta(\tan\theta - 1) = 0$$

$\tan\theta = 0$ $\qquad$ $\tan\theta = 1$

$\theta = 0, \pi, 2\pi$ $\qquad$ $\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

38. $2\cos^2\theta - \cos\theta - 1 = 0$

$$(2\cos\theta + 1)(\cos\theta - 1) = 0$$

$\cos\theta = -\dfrac{1}{2}$ $\qquad$ $\cos\theta = 1$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ $\qquad$ $\theta = 0, 2\pi$

39. $2\cos^2\theta + \cos\theta - 1 = 0$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$2\cos\theta - 1 = 0$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$$

or $\cos\theta + 1 = 0$

$$\cos\theta = -1$$

$$\theta = \pi + 2n\pi$$

40. $\csc^2\theta + \csc\theta = 2$

$$\csc^2\theta + \csc\theta - 2 = 0$$

$$(\csc\theta + 2)(\csc\theta - 1) = 0$$

$$\csc\theta + 2 = 0$$

$$\csc\theta = -2$$

$$\theta = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$$

or $\csc\theta - 1 = 0$

$$\csc\theta = 1$$

$$\theta = \frac{\pi}{2} + 2n\pi$$

41. $\sec\theta\csc\theta - 2\csc\theta = 0$

$$\csc\theta(\sec\theta - 2) = 0$$

($\csc\theta \neq 0$ for any value of θ)

$$\sec\theta = 2$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

42. $\sin\theta = \cos\theta$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

43. $\cos^2\theta + \sin\theta = 1$

$$1 - \sin^2\theta + \sin\theta = 1$$

$$\sin^2\theta - \sin\theta = 0$$

$$\sin\theta(\sin\theta - 1) = 0$$

$\sin\theta = 0$ $\qquad$ $\sin\theta = 1$

$\theta = 0, \pi, 2\pi$ $\qquad$ $\theta = \dfrac{\pi}{2}$

44. $\tan^2\theta = \sec\theta - 1$

$$\sec^2\theta - 1 = \sec\theta - 1$$

$$\sec^2\theta - \sec\theta = 0$$

$$\sec^2\theta(\sec\theta - 1) = 0$$

$$\sec\theta = 0 \text{ or } \theta - 1 = 0$$

No Solutions $\qquad$ $\sec\theta = 1$

$$\theta = 0$$

45. $\cos\left(\dfrac{\theta}{2}\right) - \cos\theta = 1$

$$\cos\left(\frac{\theta}{2}\right) = \cos\theta + 1$$

$$\sqrt{\left(\frac{1}{2}\right)(1 + \cos\theta)} = \cos\theta + 1$$

$$\left(\frac{1}{2}\right)(1 + \cos\theta) = \cos^2\theta + 2\cos\theta + 1$$

$$0 = \cos^2\theta + \left(\frac{3}{2}\right)\cos\theta + \left(\frac{1}{2}\right)$$

$$0 = \left(\frac{1}{2}\right)(2\cos^2\theta + 3\cos\theta + 1)$$

$$0 = \left(\frac{1}{2}\right)(2\cos\theta + 1)(\cos\theta + 1)$$

$\cos\theta = -\dfrac{1}{2}$ $\qquad$ $\cos\theta = -1$

$\theta = \dfrac{2\pi}{3}$ $\qquad$ $\theta = \pi$

($0 = 4\pi/3$ is extraneous)

46.

$$\cos\frac{\theta}{2} - \sin\theta = 0$$
$$\pm\sqrt{\frac{1+\cos\theta}{2}} = \sin\theta$$
$$\frac{1+\cos\theta}{2} = \sin^2\theta$$
$$1+\cos\theta = 2\sin^2\theta$$
$$1+\cos\theta = 2 - 2\cos^2\theta$$
$$2\cos^2\theta + \cos\theta - 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$2\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$

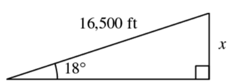
$\cos\theta = \frac{1}{2}$ $\qquad$ $\cos\theta = -1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $\theta = \pi$

$$\theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

47. $(275 \text{ ft/sec})(60 \text{ sec}) = 16{,}500$ feet

$$\sin 18^\circ = \frac{a}{16{,}500}$$
$$a = 16{,}500 \sin 18^\circ \approx 5099 \text{ feet}$$



48.

$$\tan 3.5^\circ = \frac{h}{13+x} \text{ and } \tan 9^\circ = \frac{h}{x}$$

$(13+x)\tan 3.5^\circ = h$ $\qquad$ $x\tan 9^\circ = h$

$$13\tan 3.5^\circ + x\tan 3.5^\circ = x\tan 9^\circ$$
$$13\tan 3.5^\circ = x(\tan 9^\circ - \tan 3.5^\circ)$$
$$\frac{13\tan 3.5^\circ}{\tan 9^\circ - \tan 3.5^\circ} = x$$
$$h = x\tan 9^\circ = \frac{13\tan 3.5^\circ \tan 9^\circ}{\tan 9^\circ - \tan 3.5^\circ}$$
$$\approx 1.295 \text{ mi or } 6839.307 \text{ ft}$$

49. $y = 2\sin 2x$

Period $= \frac{2\pi}{2} = \pi$

Amplitude $= |2| = 2$

50. $y = \frac{3}{2}\cos\frac{x}{2}$

Period $= \frac{2\pi}{(1/2)} = 4\pi$

Amplitude $= \left|\frac{3}{2}\right| = \frac{3}{2}$

51. $y = -3\sin 4\pi x$

Period $= \frac{2\pi}{4\pi} = \frac{1}{2}$

Amplitude $= |-3| = 3$

52. $y = \frac{2}{3}\cos\frac{\pi x}{10}$

Period $= \frac{2\pi}{(\pi/10)} = 20$

Amplitude $= \left|\frac{2}{3}\right| = \frac{2}{3}$

53. $y = 5\tan 2x$

Period $= \frac{\pi}{2}$

54. $y = 7\tan 2\pi x$

Period $= \frac{\pi}{2\pi} = \frac{1}{2}$

55. $y = \sec 5x$

Period $= \frac{2\pi}{5}$

56. $y = \csc 4x$

Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

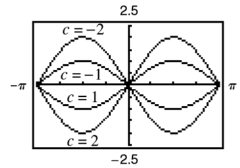
57. (a) $f(x) = c \sin x$; changing c changes the amplitude.

When $c = -2$: $f(x) = -2 \sin x$.

When $c = -1$: $f(x) = -\sin x$.

When $c = 1$: $f(x) = \sin x$. $f(x) = \cos(cx)$;

When $c = 2$: $f(x) = 2 \sin x$.



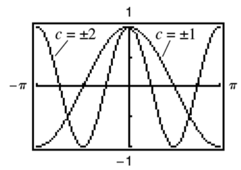
(b) changing c changes the period.

When $c = -2$: $f(x) = \cos(-2x) = \cos 2x$.

When $c = -1$: $f(x) = \cos(-x) = \cos x$.

When $c = 1$: $f(x) = \cos x$.

When $c = 2$: $f(x) = \cos 2x$.



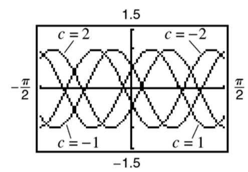
(c) $f(x) = \cos(\pi x - c)$; changing c causes a horizontal shift.

When $c = -2$: $f(x) = \cos(\pi x + 2)$.

When $c = -1$: $f(x) = \cos(\pi x + 1)$.

When $c = 1$: $f(x) = \cos(\pi x - 1)$.

When $c = 2$: $f(x) = \cos(\pi x - 2)$.



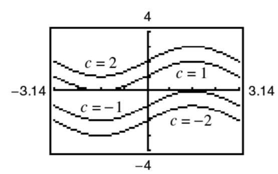
58. (a) $f(x) = \sin x + c$; changing c causes a vertical shift.

When $c = -2$: $f(x) = \sin x - 2$.

When $c = -1$: $f(x) = \sin x - 1$.

When $c = 1$: $f(x) = \sin x + 1$.

When $c = 2$: $f(x) = \sin x + 2$.



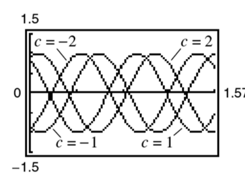
(b) $f(x) = -\sin(2\pi x - c)$; changing c causes a horizontal shift.

When $c = -2$: $f(x) = -\sin(2\pi x + 2)$.

When $c = -1$: $f(x) = -\sin(2\pi x + 1)$.

When $c = 1$: $f(x) = -\sin(2\pi x - 1)$.

When $c = 2$: $f(x) = -\sin(2\pi x - 2)$.



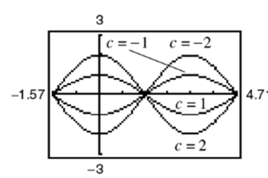
(c) $f(x) = c \cos x$; changing c changes the amplitude.

When $c = -2$: $f(x) = -2 \cos x$.

When $c = -1$: $f(x) = -\cos x$.

When $c = 1$: $f(x) = \cos x$.

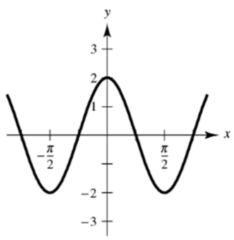
When $c = 2$: $f(x) = 2 \cos x$.



59. $y = 2\cos 2x$

Period: π

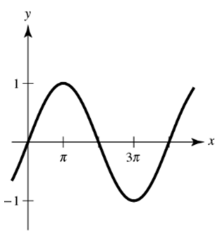
Amplitude: 2



60. $y = \sin\dfrac{x}{2}$

Period: 4π

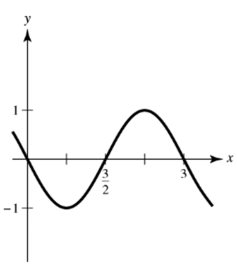
Amplitude: 1



61. $y = -\sin\dfrac{2\pi x}{3}$

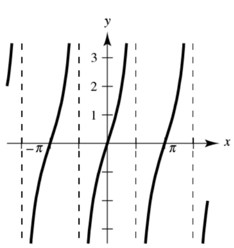
Period: 3

Amplitude: 1



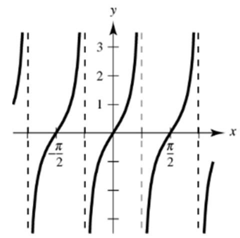
62. $y = 2\tan x$

Period: π



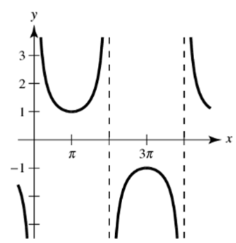
63. $y = \tan 2x$

Period: $\dfrac{\pi}{2}$



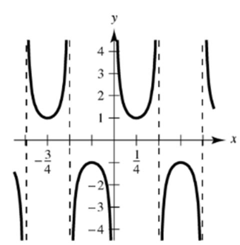
64. $y = \csc\dfrac{x}{2}$

Period: 4π



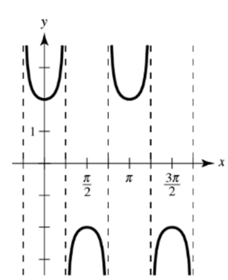
65. $y = \csc 2\pi x$

Period: 1



66. $y = 2\sec 2x$

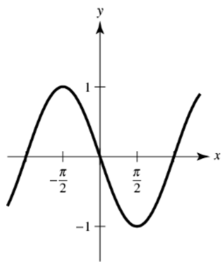
Period: π



67. $y = \sin(x + \pi)$

Period: 2π

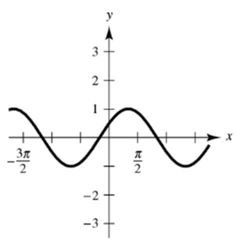
Amplitude: 1



68. $y = \cos\left(x - \dfrac{\pi}{3}\right)$

Period: 2π

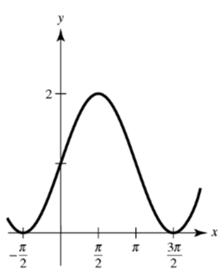
Amplitude: 1



69. $y = 1 + \cos\left(x - \dfrac{\pi}{2}\right)$

Period: 2π

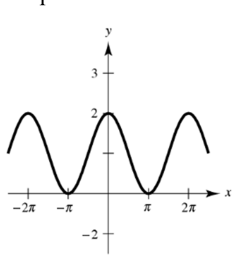
Amplitude: 1



70. $y = 1 + \sin\left(x + \dfrac{\pi}{2}\right)$

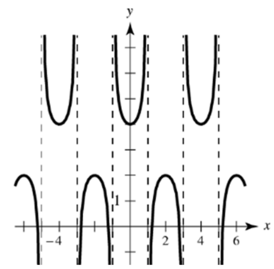
Period: 2π

Amplitude: 1



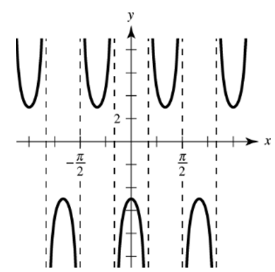
71. $y = 3 + \sec\dfrac{\pi x}{2}$

Period: $\dfrac{2\pi}{\frac{\pi}{2}} = 4$



72. $y = -1 - 4\csc\left(3x - \dfrac{3\pi}{2}\right)$

Period: $\dfrac{2\pi}{3}$



73. $y = a\cos(bx - c)$

From the graph, we see that the amplitude is 3, the period is 4π, and the horizontal shift is π. Thus,

$$a = 3$$

$$\frac{2\pi}{b} = 4\pi \Rightarrow b = \frac{1}{2}$$

$$\frac{c}{b} = \pi \Rightarrow c = \frac{\pi}{2}.$$

Therefore, $y = 3\cos\left[(1/2)x - (\pi/2)\right]$.

74. $y = a\sin(bx - c)$

From the graph, we see that the amplitude is $\frac{1}{2}$, the period is π, and the horizontal shift is 0. Also, the graph is reflected about the x-axis. Thus,

$$a = -\frac{1}{2}$$

$$\frac{2\pi}{b} = \pi \Rightarrow b = 2$$

$$\frac{c}{b} = 0 \Rightarrow c = 0.$$

Therefore, $y = -\frac{1}{2}\sin 2x$.

75. Yes. Use the right-triangle definitions of the trigonometric functions.

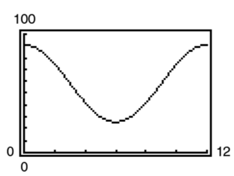
76. The sine function is one-to-one on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Other intervals are possible.

77. The range of the cosine function is $[-1, 1]$. The range of the secant function is $(-\infty, -1] \cup [1, \infty)$.

78. As θ increases from $0°$ to $90°$ with $r = 12$ centimeters,
x decreases from 12 to 0 centimeters,
y increases from 0 to 12 centimeters,
$\sin\theta$ increases from 0 to 1,
$\cos\theta$ decreases from 1 to 0, and
$\tan\theta$ increases from 0 to (positive) infinity.

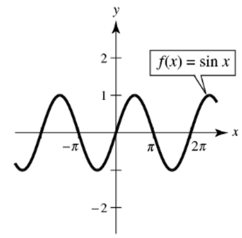
79. $h = 50 + 50 \sin 8\pi t$ satisfies $h = 50$ when $t = 0$. Now let $h = 51 + 50 \sin\left(8\pi t - \frac{\pi}{2}\right)$. Then $h = 1$ when $t = 0$.

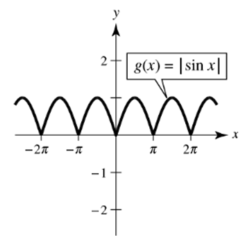
80. $S = 58.3 + 32.5 \cos\frac{\pi t}{6}$

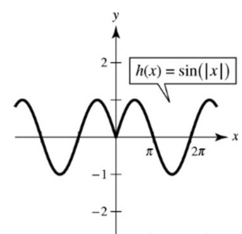


Sales exceed 75,000 during the months of January, November, and December.

81. $f(x) = \sin x$
$g(x) = |\sin x|$
$h(x) = \sin|x|$





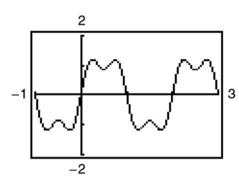


The graph of $|f(x)|$ will reflect any parts of the graph of $f(x)$ below the x-axis about the y-axis.

The graph of $f(|x|)$ will reflect the part of the graph of $f(x)$ to the right of the y-axis about the y-axis.

82. $f(x) = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x\right)$

$g(x) = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x\right)$



Pattern: $f(x) = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x + \cdots + \frac{1}{2n-1}\sin(2n-1)\pi x\right)$, $n = 1, 2, 3\ldots$

83. False. 4π radians (not 4 radians) corresponds to two complete revolutions from the initial side to the terminal side of an angle.

84. True

85. False. The amplitude of the function $y = \frac{1}{2}\sin 2x$ is one-half the amplitude of the function $y = \sin x$.

86. True

Section 1.5 Inverse Functions

1. The graphs of f and f^{-1} are mirror images with respect to the line $y = x$.

2. No. The domain of f^{-1} is the range of f.

3. arccos x is the angle θ whose cosine is x, where $0 \le \theta \le \pi$.

4. The inverse trigonometric functions are not one-to-one. Therefore, their domains must be restricted to intervals upon which they are one-to-one.

5. $f(x) = 6x$ and $g(x) = \dfrac{x}{6}$

$$f(g(x)) = f\left(\frac{x}{6}\right) = 6\left(\frac{x}{6}\right) = x$$

$$g(f(x)) = g(6x) = \frac{6x}{6} = x$$

So, f and g are inverse functions of each other.

6. $f(x) = \dfrac{x}{4}$ and $g(x) = 4x$

$$f(g(x)) = f(4x) = \frac{4x}{4} = x$$

$$g(f(x)) = g\left(\frac{x}{4}\right) = 4\left(\frac{x}{4}\right) = x$$

So, f and g are inverse functions of each other.

7. $f(x) = 5x + 1$ and $g(x) = \dfrac{x-1}{5}$

$$f(g(x)) = f\left(\frac{x-1}{5}\right) = 5\left(\frac{x-1}{5}\right) + 1$$
$$= (x - 1) + 1 = x$$

$$g(f(x)) = g(5x + 1) = \frac{[(5x + 1) - 1]}{5}$$
$$= \frac{5x}{5} = x$$

So, f and g are inverse functions of each other.

8. $f(x) = \dfrac{x-2}{3}$ and $g(x) = 3x + 2$

$$f(g(x)) = f(3x + 2) = \frac{[(3x + 2) - 2]}{3}$$
$$= \frac{3x}{3} = x$$

$$g(f(x)) = g\left(\frac{x-2}{3}\right) = 3\left(\frac{x-2}{3}\right) + 2$$
$$= (x - 2) + 2 = x$$

So, f and g are inverse functions of each other.

9. Matches (c)

10. Matches (b)

11. Matches (a)

12. Matches (d)

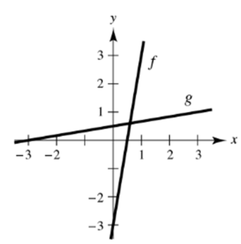
13. $f(x) = 6x - 3$

$$g(x) = \frac{x + 3}{6}$$

(a) $$f(g(x)) = f\left(\frac{x+3}{6}\right) = 6\left(\frac{x+3}{6}\right) - 3$$
$$= (x + 3) - 3 = x$$

$$g(f(x)) = g(6x - 3) = \frac{(6x - 3) + 3}{6}$$
$$= \frac{6x}{6} = x$$

(b)



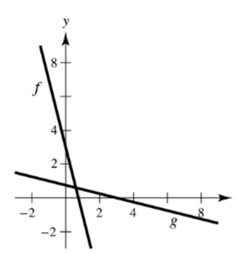
14. (a) $$f(x) = 3 - 4x$$

$$g(x) = \frac{3 - x}{4}$$

$$f(g(x)) = f\left(\frac{3-x}{4}\right) = 3 - 4\left(\frac{3-x}{4}\right) = x$$

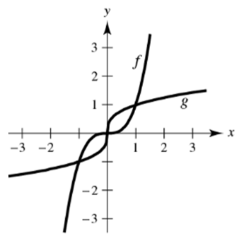
$$g(f(x)) = g(3 - 4x) = \frac{3 - (3 - 4x)}{4} = x$$

(b)



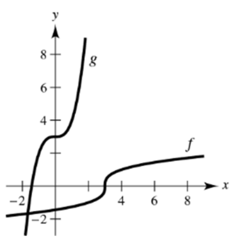
15. (a)
$$f(x) = x^3$$
$$g(x) = \sqrt[3]{x}$$
$$f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$$
$$g(f(x)) = g\left(x^3\right) = \sqrt[3]{x^3} = x$$

(b)



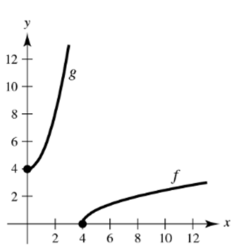
16. (a)
$$f(x) = \sqrt[3]{x-3}$$
$$g(x) = 3 + x^3$$
$$f(g(x)) = f\left(3 + x^3\right) = \sqrt[3]{\left(3 + x^3\right) - 3} = \sqrt[3]{x^3} = x$$
$$g(f(x)) = g\left(\sqrt[3]{x-3}\right) = 3 + \left(\sqrt[3]{x-3}\right)^3 = 3 + (x-3) = x$$

(b)



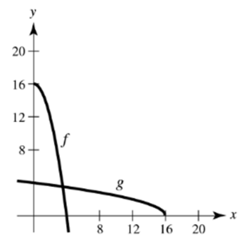
17. (a)
$$f(x) = \sqrt{x-4}$$
$$g(x) = x^2 + 4, \quad x \ge 0$$
$$f(g(x)) = f\left(x^2 + 4\right) = \sqrt{\left(x^2 + 4\right) - 4} = \sqrt{x^2} = x$$
$$g(f(x)) = g\left(\sqrt{x-4}\right) = \left(\sqrt{x-4}\right)^2 + 4 = x - 4 + 4 = x$$

(b)



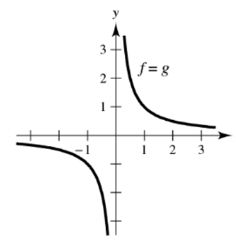
18. (a)
$$f(x) = 16 - x^2, \quad x \ge 0$$
$$g(x) = \sqrt{16 - x}$$
$$f(g(x)) = f\left(\sqrt{16-x}\right) = 16 - \left(\sqrt{16-x}\right)^2 = 16 - (16 - x) = x$$
$$g(f(x)) = g\left(16 - x^2\right) = \sqrt{16 - \left(16 - x^2\right)} = \sqrt{x^2} = x$$

(b)



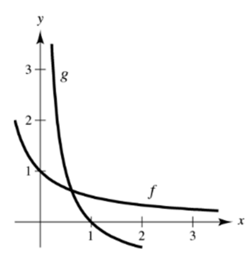
19. (a)
$$f(x) = \frac{1}{x}$$
$$g(x) = \frac{1}{x}$$
$$f(g(x)) = \frac{1}{1/x} = x$$
$$g(f(x)) = \frac{1}{1/x} = x$$

(b)



20. (a)
$$f(x) = \frac{1}{1+x}, \qquad x \ge 0$$
$$g(x) = \frac{1-x}{x}, \qquad 0 < x \le 1$$
$$f(g(x)) = f\left(\frac{1-x}{x}\right) = \frac{1}{1 + \frac{1-x}{x}} = \frac{1}{\frac{1}{x}} = x$$
$$g(f(x)) = g\left(\frac{1}{1+x}\right) = \frac{1 - \frac{1}{1+x}}{\frac{1}{1+x}} = \frac{x}{1+x} \cdot \frac{1+x}{1} = x$$

(b)



21. $f(\theta) = \sin\theta$

Not one-to-one; does not have an inverse

22. $f(x) = 5x - 3$

One-to-one; has an inverse

23. $f(x) = 2 - x - x^3$

One-to-one; has an inverse

24. $f(x) = \dfrac{x^4}{4} - 2x^2$

Not one-to-one; does not have an inverse

25. $f(x) = \dfrac{1}{3x + 1}$

One-to-one; has an inverse

26. $f(x) = \sqrt[3]{x + 1}$

One-to-one; has an inverse

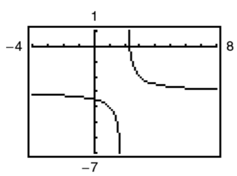
27. $f(x) = \tan 2\pi x$

Not one-to-one; does not have an inverse

28. $f(x) = \sin\dfrac{3x}{2}$

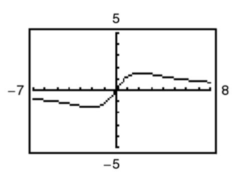
Not one-to-one; does not have an inverse

29. $h(s) = \dfrac{1}{s - 2} - 3$



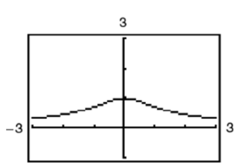
One-to-one; has an inverse

30. $f(x) = \dfrac{6x}{x^2 + 4}$



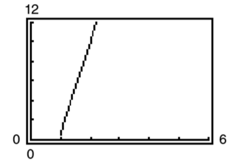
Not one-to-one; does not have an inverse

31. $g(t) = \dfrac{1}{\sqrt{t^2 + 1}}$



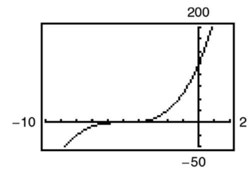
Not one-to-one; does not have an inverse

32. $f(x) = 5x\sqrt{x - 1}$



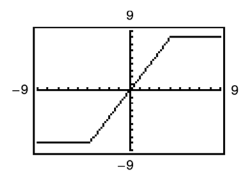
One-to-one; has an inverse

33. $g(x) = (x + 5)^3$



One-to-one; has an inverse

34. $h(x) = |x + 4| - |x - 4|$



Not one-to-one; does not have an inverse

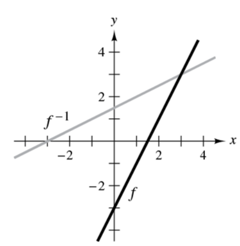
35. (a) $f(x) = 2x - 3 = y$

$$x = \frac{y + 3}{2}$$

$$y = \frac{x + 3}{2}$$

$$f^{-1}(x) = \frac{x + 3}{2}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

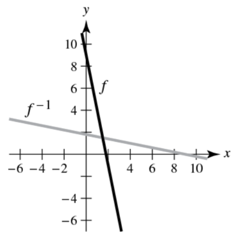
(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

36. (a) $f(x) = 9 - 5x = y$ (b)

$$x = \frac{9 - y}{5}$$

$$y = \frac{9 - x}{5}$$

$$f^{-1}(x) = \frac{1}{5}(9 - x)$$



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: all real numbers

Range of f: all real numbers

Domain of f^{-1}: all real numbers

Range of f^{-1}: all real numbers

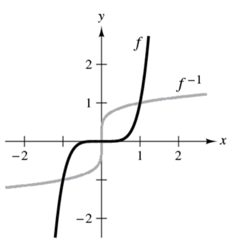
37. (a) $f(x) = x^5 = y$

$$x = \sqrt[5]{y}$$

$$y = \sqrt[5]{x}$$

$$f^{-1}(x) = \sqrt[5]{x} = x^{1/5}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers

Range of f: all real numbers

Domain of f^{-1}: all real numbers

Range of f^{-1}: all real numbers

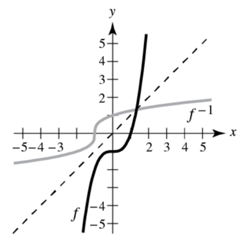
38. (a) $f(x) = x^3 - 1 = y$

$$x = \sqrt[3]{y + 1}$$

$$y = \sqrt[3]{x + 1}$$

$$f^{-1}(x) = \sqrt[3]{x + 1} = (x + 1)^{1/3}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers

Range of f: all real numbers

Domain of f^{-1}: all real numbers

Range of f^{-1}: all real numbers

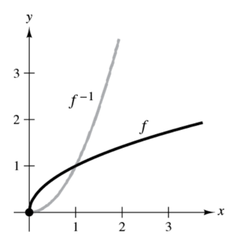
39. (a) $f(x) = \sqrt{x} = y$

$$x = y^2$$

$$y = x^2$$

$$f^{-1}(x) = x^2, \quad x \ge 0$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \ge 0$

Range of f: $y \ge 0$

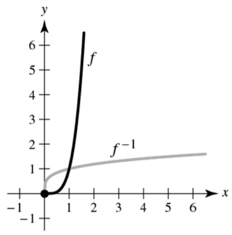
Domain of f^{-1}: $x \ge 0$

Range of f^{-1}: $y \ge 0$

40. (a) $f(x) = x^4, x \geq 0$

$$y = x^4$$
$$x = y^{1/4}$$
$$f^{-1}(x) = x^{1/4} = \sqrt[4]{x}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: $x \geq 0$

Range of f: $y \geq 0$

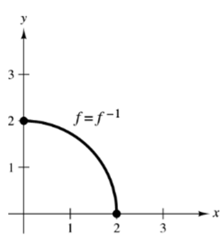
Domain of f^{-1}: $x \geq 0$

Range of f^{-1}: $y \geq 0$

41. (a) $f(x) = \sqrt{4 - x^2} = y, \quad 0 \leq x \leq 2$

$$4 - x^2 = y^2$$
$$x^2 = 4 - y^2$$
$$x = \sqrt{4 - y^2}$$
$$y = \sqrt{4 - x^2}$$
$$f^{-1}(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$. In fact, the graphs are identical.

(d) Domain of f: $0 \leq x \leq 2$

Range of f: $0 \leq y \leq 2$

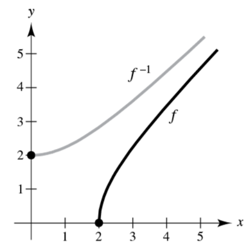
Domain of f^{-1}: $0 \leq x \leq 2$

Range of f^{-1}: $0 \leq y \leq 2$

42. (a) $f(x) = \sqrt{x^2 - 4} = y, \quad x \geq 2$

$$x^2 = y^2 + 4$$
$$x = \sqrt{y^2 + 4}$$
$$y = \sqrt{x^2 + 4}$$
$$f^{-1}(x) = \sqrt{x^2 + 4}, \quad x \geq 0$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \geq 2$

Range of f: $y \geq 0$

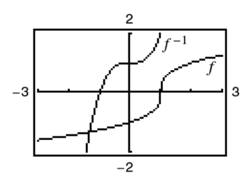
Domain of f^{-1}: $x \geq 0$

Range of f^{-1}: $y \geq 2$

43. (a) $f(x) = \sqrt[3]{x - 1} = y$

$$x - 1 = y^3$$
$$x = y^3 + 1$$
$$y = x^3 + 1$$
$$f^{-1}(x) = x^3 + 1$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers

Range of f: all real numbers

Domain of f^{-1}: all real numbers

Range of f^{-1}: all real numbers

44. (a) $f(x) = 3\sqrt[5]{2x-1} = y$

$$2x - 1 = \left(\frac{y}{3}\right)^5 = \frac{y^5}{243}$$

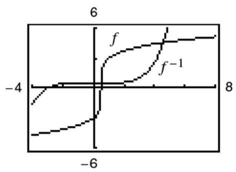
$$2x = \frac{y^5 + 243}{243}$$

$$x = \frac{y^5 + 243}{486}$$

$$y = \frac{x^5 + 243}{486}$$

$$f^{-1}(x) = \frac{x^5 + 243}{486}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

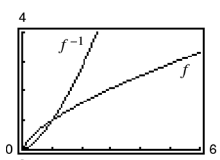
45. (a) $f(x) = x^{2/3} = y, \quad x \ge 0$

$$x = y^{3/2}$$

$$y = x^{3/2}$$

$$f^{-1}(x) = x^{3/2}, \ x \ge 0$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \ge 0$
Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$
Range of f^{-1}: $y \ge 0$

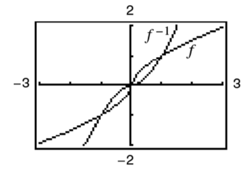
46. (a) $f(x) = x^{3/5} = y$

$$x = y^{5/3}$$

$$y = x^{5/3}$$

$$f^{-1}(x) = x^{5/3}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

47. (a) $f(x) = \dfrac{x}{\sqrt{x^2 + 7}} = y$

$$x = y\sqrt{x^2 + 7}$$

$$x^2 = y^2(x^2 + 7) = y^2x^2 + 7y^2$$

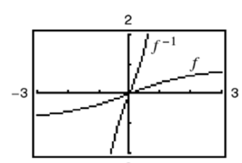
$$x^2(1 - y^2) = 7y^2$$

$$x = \frac{\sqrt{7}y}{\sqrt{1 - y^2}}$$

$$y = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}$$

$$f^{-1}(x) = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

(b)



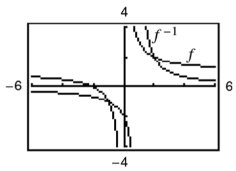
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: $-1 < y < 1$
Domain of f^{-1}: $-1 < x < 1$
Range of f^{-1}: all real numbers

48. (a) $f(x) = \dfrac{x+2}{x} = y, \quad x \neq 0$

$$x + 2 = yx$$
$$x(1 - y) = -2$$
$$x = \frac{2}{y-1}$$
$$y = \frac{2}{x-1}$$
$$f^{-1}(x) = \frac{2}{x-1}, \quad x \neq 1$$

(b)



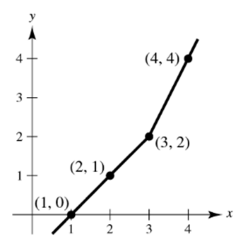
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all $x \neq 0$
Range of f: all $y \neq 1$
Domain of f^{-1}: all $x \neq 1$
Range of f^{-1}: all $y \neq 0$

49.

x	0	1	2	4
$f(x)$	1	2	3	4

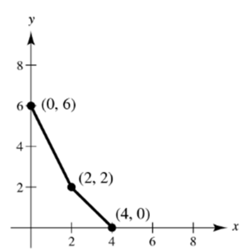
x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



50.

x	0	2	6
$f(x)$	4	2	0

x	0	2	4
$f^{-1}(x)$	6	2	0



51. (a) Let x be the number of pounds of the commodity costing \$1.25 per pound. Because there are 50 pounds total, the amount of the second commodity is 50-x. The total cost is

$$f(x) = y = 1.25x + 2.75(50 - x)$$
$$= -1.5x + 137.5, 0 \leq x \leq 50$$

(b) $$y = -1.5x + 137.5$$
$$1.5x = 137.5 - y$$
$$x = \frac{(137.5 - y)}{1.5}$$
$$y = f^{-1}(x) = \frac{2}{3}(137.5 - x)$$

x represents the total cost and y represents the number of pounds of the less expensive commodity.

(c) The range of f is $[62.5, 137.5]$, so the domain of f^{-1} is the same. $50(1.25) = 62.5$ gives the total cost when purchasing 50 pounds of the less expensive commodity, and $50(2.75) = 137.5$ gives the total cost when purchasing 50 pounds of the more expensive commodity.

(d) If $x = 73$, then $f^{-1}(73) = 43$ pounds.

52. $C = \frac{5}{9}(F - 32), \quad F \geq -459.6$

(a) $$\frac{9}{5}C = F - 32$$
$$F = 32 + \frac{9}{5}C$$

(b) The inverse function gives the Fahrenheit temperature F corresponding to the Celsius temperature C.

(c) For $F \geq -459.6$, $C = \frac{5}{9}(F - 32) \geq -273.1\overline{1}$.

So, the domain is $C \geq -273.\overline{1} = -273\frac{1}{9}$.

(d) If $C = 22°$, then $F = 32 + \frac{9}{5}(22) = 71.6°$F.

53. $f(x) = \sqrt{x - 2}, \ x \geq 2$

f is one-to-one; has an inverse.

$$y = \sqrt{x-2}, \ x \geq 2, \ y \geq 0$$
$$y^2 = x - 2$$
$$x = y^2 + 2$$
$$f^{-1}(x) = x^2 + 2, \ x \geq 0$$

54. $f(x) = \sqrt{9 - x^2}$ is not one-to-one.

For example, $f(3) = f(-3) = 0$.

55. $f(x) = -3$

Not one-to-one; does not have an inverse.

56. $f(x) = |x - 2|, \; x \le 2$

$= -(x - 2)$

$= 2 - x$

f is one-to-one; has an inverse.

$2 - x = y$

$2 - y = x$

$f^{-1}(x) = 2 - x, \quad x \ge 0$

57. $f(x) = ax + b$

f is one-to-one; has an inverse.

$ax + b = y$

$x = \dfrac{y - b}{a}$

$y = \dfrac{x - b}{a}$

$f^{-1}(x) = \dfrac{x - b}{a}, \quad a \ne 0$

58. $f(x) = (x + a)^3 + b$

f is one-to-one; has an inverse.

$y = (x + a)^3 + b$

$y - b = (x + a)^3$

$x + a = \sqrt[3]{y - b}$

$x = \sqrt[3]{y - b} - a$

$f^{-1}(x) = \sqrt[3]{x - b} - a$

59. $f(x) = (x - 4)^2$ on $[4, \infty)$

f passes the Horizontal Line Test on $[4, \infty)$, so it is one-to-one.

60. $f(x) = |x + 2|$ on $[-2, \infty)$

f passes the Horizontal Line Test on $[-2, \infty)$, so it is one-to-one.

61. $f(x) = \dfrac{4}{x^2}$ on $(0, \infty)$

f passes the Horizontal Line Test on $(0, \infty)$, so it is one-to-one.

62. $f(x) = \cot x$ on $(0, \pi)$

f passes the Horizontal Line Test on $(0, \pi)$, so it is one-to-one.

63. $f(x) = \cos x$ on $[0, \pi]$

f passes the Horizontal Line Test on $[0, \pi]$, so it is one-to-one.

64. $f(x) = \sec x$ on $\left[0, \dfrac{\pi}{2}\right)$

f passes the Horizontal Line Test on $[0, \pi/2)$, so it is one-to-one.

65. $f(x) = (x - 3)^2$ is one-to-one for $x \ge 3$.

$(x - 3)^2 = y$

$x - 3 = \sqrt{y}$

$x = \sqrt{y} + 3$

$y = \sqrt{x} + 3$

$f^{-1}(x) = \sqrt{x} + 3, \quad x \ge 0$

(Answer is not unique.)

66. $f(x) = |x - 3|$ is one-to-one for $x \ge 3$.

$x - 3 = y$

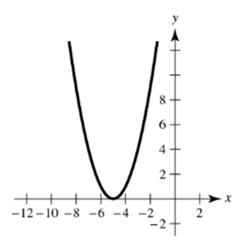
$x = y + 3$

$y = x + 3$

$f^{-1}(x) = x + 3, \quad x \ge 0$

(Answer is not unique.)

67. (a) $f(x) = (x + 5)^2$



(b) f is one-to-one on $[-5, \infty)$. (Note that f is also one-to-one on $(-\infty, -5]$.)

(c) $f(x) = (x + 5)^2 = y, \qquad x \ge -5$

$x + 5 = \sqrt{y}$

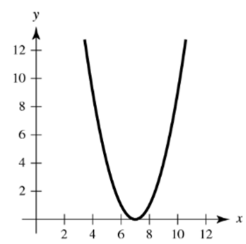
$x = \sqrt{y} - 5$

$y = \sqrt{x} - 5$

$f^{-1}(x) = \sqrt{x} - 5$

(d) Domain of f^{-1}: $x \ge 0$

68. (a) $f(x) = (7 - x)^2 = (x - 7)^2$

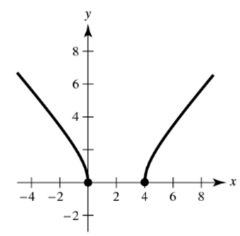


(b) f is one-to-one on $[7, \infty)$. (Note that f is also one-to-one on $(-\infty, 7]$.)

(c)
$$\begin{aligned} f(x) = (x - 7)^2 &= y, \qquad x \geq 7 \\ x - 7 &= \sqrt{y} \\ x &= 7 + \sqrt{y} \\ y &= 7 + \sqrt{x} \\ f^{-1}(x) &= 7 + \sqrt{x} \end{aligned}$$

(d) Domain of f^{-1}: $x \geq 0$

69. (a) $f(x) = \sqrt{x^2 - 4x}$

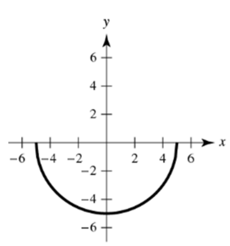


(b) f is one-to-one on $[4, \infty)$. (Note that f is also one-to-one on $(-\infty, 0]$.)

(c)
$$\begin{aligned} f(x) = \sqrt{x^2 - 4x} &= y, \ x \geq 4 \\ x^2 - 4x &= y^2 \\ x^2 - 4x + 4 &= y^2 + 4 \\ (x - 2)^2 &= y^2 + 4 \\ x - 2 &= \sqrt{y^2 + 4} \\ x &= 2 + \sqrt{y^2 + 4} \\ y &= 2 + \sqrt{x^2 + 4} \\ f^{-1}(x) &= 2 + \sqrt{x^2 + 4} \end{aligned}$$

(d) Domain of f^{-1}: $x \geq 0$

70. (a) $f(x) = -\sqrt{25 - x^2}$

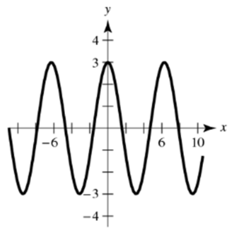


(b) f is one-to-one on $[0, 5]$. (Note that f is also one-to-one on $[-5, 0]$.)

(c)
$$\begin{aligned} f(x) = -\sqrt{25 - x^2} &= y, \qquad 0 \leq x \leq 5, -5 \leq y \leq 0 \\ 25 - x^2 &= y^2 \\ x^2 &= 25 - y^2 \\ x &= \sqrt{25 - y^2} \\ y &= \sqrt{25 - x^2} \\ f^{-1}(x) &= \sqrt{25 - x^2} \end{aligned}$$

(d) Domain of f^{-1}: $-5 \leq x \leq 0$

71. (a) $f(x) = 3\cos x$

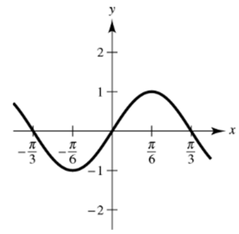


(b) f is one-to-one on $[0, \pi]$. (other answers possible)

(c) $$f(x) = 3\cos x = y$$
$$\cos x = \frac{y}{3}$$
$$x = \arccos\left(\frac{y}{3}\right)$$
$$y = \arccos\left(\frac{x}{3}\right)$$
$$f^{-1}(x) = \arccos\left(\frac{x}{3}\right)$$

(d) Domain of f^{-1}: $-3 \le x \le 3$

72. (a) $f(x) = \sin 3x$



(b) f is one-to-one on $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$. (Note that other answers are possible.)

(c) $$f(x) = \sin 3x = y, \; -\frac{\pi}{6} \le x \le \frac{\pi}{6}$$
$$3x = \arcsin y$$
$$x = \frac{1}{3}\arcsin y$$
$$f^{-1}(x) = \frac{1}{3}\arcsin x$$

(d) Domain of f^{-1}: $-1 \le x \le 1$

73. $f(x) = x^3 + 2x - 1$

$f(1) = 2 = a \Rightarrow f^{-1}(2) = 1$

74. $f(x) = 2x^5 + x^3 + 1$

$f(-1) = -2 = a \Rightarrow f^{-1}(-2) = -1$

75. $f(x) = 5\sin x, \; -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$$f\left(-\frac{\pi}{6}\right) = 5\sin\left(-\frac{\pi}{6}\right) = 5\left(-\frac{1}{2}\right) = -\frac{5}{2} = a \Rightarrow f^{-1}\left(-\frac{5}{2}\right) = -\frac{\pi}{6}$$

76. $f(x) = \cos 2x$

$f(0) = 1 = a \Rightarrow f^{-1}(1) = 0$

77. $f(x) = x^3 - \frac{4}{x}$

$f(2) = 6 = a \Rightarrow f^{-1}(6) = 2$

78. $f(x) = \sqrt{x + 4}$

$f(5) = \sqrt{5 + 4} = \sqrt{9} = 3 = a \Rightarrow f^{-1}(3) = 5$

In Exercises 79–82, use the following.

$\boldsymbol{f(x) = \frac{1}{8}x - 3}$ **and** $\boldsymbol{g(x) = x^3}$

$\boldsymbol{f^{-1}(x) = 8(x + 3)}$ **and** $\boldsymbol{g^{-1}(x) = \sqrt[3]{x}}$

79. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1)) = f^{-1}(1) = 32$

80. $(g^{-1} \circ f^{-1})(-3) = g^{-1}(f^{-1}(-3)) = g^{-1}(0) = 0$

81. $(f^{-1} \circ f^{-1})(-2) = f^{-1}(f^{-1}(-2)) = f^{-1}(8) = 88$

82. $(g^{-1} \circ g^{-1})(8) = g^{-1}(g^{-1}(8)) = g^{-1}(2) = \sqrt[3]{2}$

In Exercises 83–86, use the following.

$\boldsymbol{f(x) = x + 4}$ **and** $\boldsymbol{g(x) = 2 - x^3}$

$\boldsymbol{f^{-1}(x) = x - 4}$ **and** $\boldsymbol{g^{-1}(x) = \sqrt[3]{2 - x}}$

83. $(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x))$
$= g^{-1}(x - 4)$
$= \sqrt[3]{2 - (x - 4)}$
$= \sqrt[3]{6 - x}$

84. $(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x))$
$= f^{-1}(\sqrt[3]{2 - x})$
$= \sqrt[3]{2 - x} - 4$

85. $(f \circ g)(x) = f(g(x))$
$= f(2 - x^3)$
$= (2 - x^3) + 4$
$= 6 - x^3$

So, $(f \circ g)^{-1}(x) = \sqrt[3]{6 - x}$.

Note: $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

86. $(g \circ f)(x) = g(f(x))$
$= g(x + 4)$
$= 2 - (x + 4)^3$

So, $(g \circ f)^{-1} = \sqrt[3]{2 - x} - 4$.

Note: $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

87. (a) f is one-to-one because it passes the Horizontal Line Test.

(b) The domain of f^{-1} is the range of f: $[-2, 2]$.

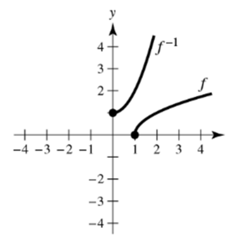
(c) $f^{-1}(2) = -4$ because $f(-4) = 2$.

88. (a) f is one-to-one because it passes the Horizontal Line Test.

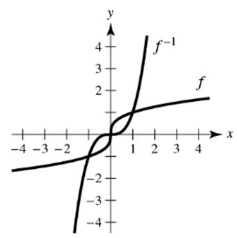
(b) The domain of f^{-1} is the range of f: $[-3, 3]$.

(c) $f^{-1}(2) \approx 1.73$ because $f(1.73) \approx 2$.

89.



90.

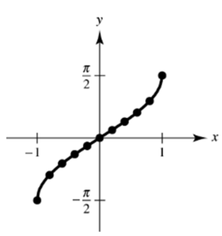


91. $y = \arcsin x$

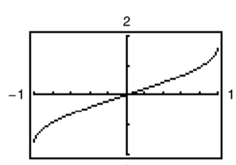
(a)

x	−1	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8	1
y	−1.571	−0.927	−0.644	−0.412	−0.201	0	0.201	0.412	0.644	0.927	1.571

(b)



(c)



(d) Symmetric about origin: $\arcsin(-x) = -\arcsin x$

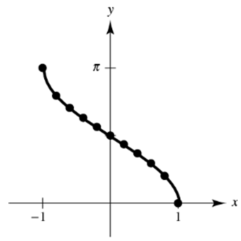
Intercept: $(0, 0)$

92. $y = \arccos x$

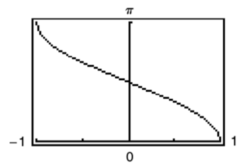
(a)

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
y	3.142	2.498	2.214	1.982	1.772	1.571	1.369	1.159	0.927	0.644	0

(b)



(c)



(d) Intercepts: $\left(0, \dfrac{\pi}{2}\right)$ and $(1, 0)$

93. $y = \arccos x$

$\left(-\dfrac{\sqrt{2}}{2}, \dfrac{3\pi}{4}\right)$ because $\cos\left(\dfrac{3\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$.

$\left(\dfrac{1}{2}, \dfrac{\pi}{3}\right)$ because $\cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$.

$\left(\dfrac{\sqrt{3}}{2}, \dfrac{\pi}{6}\right)$ because $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$.

94. No, g is not the inverse of f. $f(x) = \sin x$ is not one-to-one. The graph of g is not the graph of a function.

95. $f(x) = \arcsin(x - 1)$

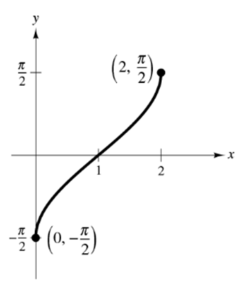
$x - 1 = \sin y$

$x = 1 + \sin y$

Domain: $[0, 2]$

Range: $[-\pi/2, \pi/2]$

$f(x)$ is the graph of $\arcsin x$ shifted right one unit.



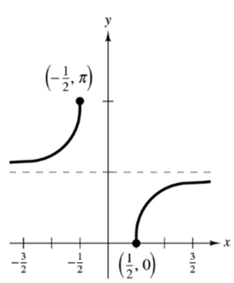
96. $f(x) = \operatorname{arcsec} 2x$

$2x = \sec y$

$x = \dfrac{1}{2}\sec y$

Domain: $(-\infty, -1/2], [1/2, \infty)$

Range: $[0, \pi/2), (\pi/2, \pi]$

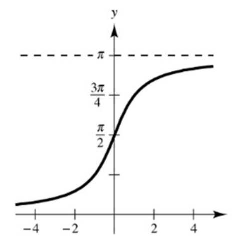


97. $f(x) = \arctan x + \dfrac{\pi}{2}$

$x = \tan\left(y - \dfrac{\pi}{2}\right)$

Domain: $(-\infty, \infty)$

Range: $[0, \pi]$



$f(x)$ is the graph of $\arctan x$ shifted $\pi/2$ unit upward.

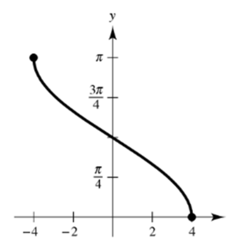
98. $f(x) = \arccos\left(\dfrac{x}{4}\right)$

$\dfrac{x}{4} = \cos y$

$x = 4\cos y$

Domain: $[-4, 4]$

Range: $[0, \pi]$



99. $\arcsin \dfrac{1}{2} = \dfrac{\pi}{6}$

100. $\arcsin 0 = 0$

101. $\arccos \dfrac{1}{2} = \dfrac{\pi}{3}$

102. $\arccos(-1) = \pi$

103. $\arctan \dfrac{\sqrt{3}}{3} = \dfrac{\pi}{6}$

104. $\operatorname{arccot}(-\sqrt{3}) = \dfrac{5\pi}{6}$

105. $\operatorname{arccsc}(-\sqrt{2}) = -\dfrac{\pi}{4}$

106. $\operatorname{arcsec} 2 = \dfrac{\pi}{3}$

107. $\arccos(0.051) \approx 1.52$

108. $\arcsin(-0.39) \approx -0.40$

109. $\operatorname{arcsec}(1.269) = \arccos\left(\dfrac{1}{1.269}\right)$

≈ 0.66

110. $\operatorname{arccsc}(-4.487) = \arcsin\left(\dfrac{-1}{4.487}\right)$

≈ -0.22

111. $\cos[\arccos(-0.1)] = -0.1$

112. $\arcsin\left(\sin \dfrac{\pi}{3}\right) = \dfrac{\pi}{3}$

113. No. Graphically, adding a constant shift the graph vertically.

114. They are not equal. For example,

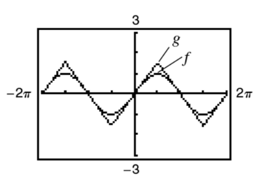
$\arctan 1 = \dfrac{\pi}{4}$ but $\dfrac{\arcsin(1)}{\arccos(1)} = \dfrac{\pi/2}{0}$, undefined.

115. The trigonometric functions are not one-to-one. So, their domains must be restricted to define the inverse trigonometric functions.

116. $\arcsin(\sin x) \neq x$ for many values of x outside

$\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

For example, $\arcsin(\sin 2\pi) = \arcsin(0) = 0 \neq 2\pi$.



117. $\arcsin(3x - \pi) = \frac{1}{2}$

$3x - \pi = \sin\left(\frac{1}{2}\right)$

$x = \frac{1}{3}\left[\sin\left(\frac{1}{2}\right) + \pi\right] \approx 1.207$

118. $\arctan(2x - 5) = -1$

$2x - 5 = \tan(-1)$

$x = \frac{1}{2}(5 + \tan(-1)) \approx 1.721$

119. $\arcsin \sqrt{2x} = \arccos \sqrt{x}$

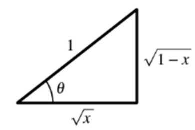
$\sqrt{2x} = \sin\left(\arccos \sqrt{x}\right)$

$\sqrt{2x} = \sqrt{1 - x},\ 0 \le x \le 1$

$2x = 1 - x$

$3x = 1$

$x = \frac{1}{3}$



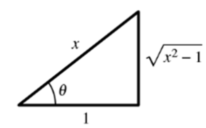
120. $\arccos x = \operatorname{arcsec} x$

$x = \cos(\operatorname{arcsec} x)$

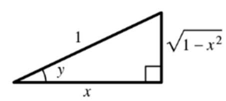
$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$



In Exercises 121–126, use the triangle.



121. $y = \arccos x$

$\cos y = x$

122. $\sin y = \sqrt{1 - x^2}$

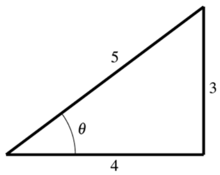
123. $\tan y = \dfrac{\sqrt{1 - x^2}}{x}$

124. $\cot y = \dfrac{x}{\sqrt{1 - x^2}}$

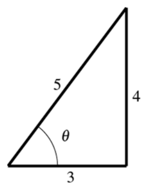
125. $\sec y = \dfrac{1}{x}$

126. $\csc y = \dfrac{1}{\sqrt{1 - x^2}}$

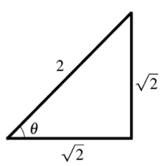
127. (a) $\sin\left(\arctan\frac{3}{4}\right) = \frac{3}{5}$



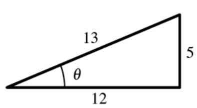
(b) $\sec\left(\arcsin\frac{4}{5}\right) = \frac{5}{3}$



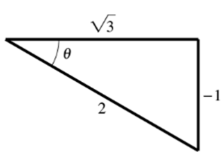
128. (a) $\tan\left(\arccos\frac{\sqrt{2}}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1$



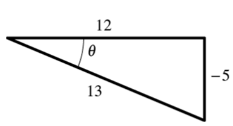
(b) $\cos\left(\arcsin\frac{5}{13}\right) = \frac{12}{13}$



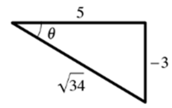
129. (a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right] = \cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$



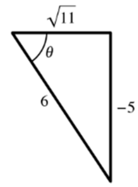
(b) $\csc\left[\arctan\left(-\frac{5}{12}\right)\right] = -\frac{13}{5}$



130. (a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right] = \frac{\sqrt{34}}{5}$



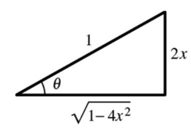
(b) $\tan\left[\arcsin\left(-\frac{5}{6}\right)\right] = -\frac{5\sqrt{11}}{11}$



131. $y = \cos(\arcsin 2x)$

$\theta = \arcsin 2x$

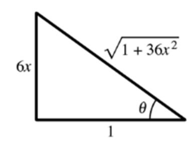
$y = \cos\theta = \sqrt{1 - 4x^2}$



132. $y = \sec(\arctan 6x)$

$\theta = \arctan 6x$

$y = \sec\theta = \sqrt{1 + 36x^2}$

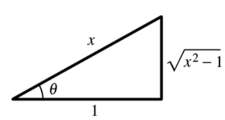


133. $y = \sin(\operatorname{arcsec} x)$

$\theta = \operatorname{arcsec} x,\ 0 \le \theta \le \pi,\ \theta \ne \frac{\pi}{2}$

$y = \sin\theta = \frac{\sqrt{x^2 - 1}}{|x|}$

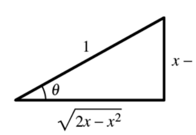
The absolute value bars on x are necessary because of the restriction $0 \le \theta \le \pi$, $\theta \ne \pi/2$, and $\sin\theta$ for this domain must always be nonnegative.



134. $y = \sec[\arcsin(x - 1)]$

$\theta = \arcsin(x - 1)$

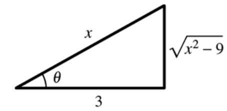
$y = \sec\theta = \frac{1}{\sqrt{2x - x^2}}$



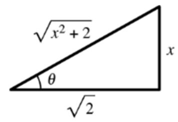
135. $y = \tan\left(\operatorname{arcsec}\frac{x}{3}\right)$

$\theta = \operatorname{arcsec}\frac{x}{3}$

$y = \tan\theta = \frac{x^2 - 9}{3}$



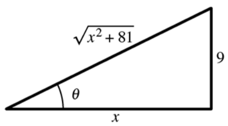
136. $y = \csc\left(\arctan \dfrac{x}{\sqrt{2}}\right)$



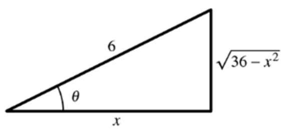
$\theta = \arctan \dfrac{x}{\sqrt{2}}$

$y = \csc\,\theta = \dfrac{\sqrt{x^2+2}}{x}$

137. $\arctan \dfrac{9}{x} = \arcsin \dfrac{9}{\sqrt{x^2+81}}$



138. $\arcsin \dfrac{\sqrt{36-x^2}}{6} = \arccos \dfrac{x}{6}$



139. (a) $\operatorname{arccsc} x = \arcsin \dfrac{1}{x}, \ |x| \ge 1$

Let $y = \operatorname{arccsc} x$.

Then for $-\dfrac{\pi}{2} \le y < 0$ and $0 < y \le \dfrac{\pi}{2}$,

$\csc y = x \Rightarrow \sin y = \dfrac{1}{x}$.

So, $y = \arcsin\left(\dfrac{1}{x}\right)$. Therefore,

$\operatorname{arccsc} x = \arcsin\left(\dfrac{1}{x}\right)$.

(b) $\arctan x + \arctan\dfrac{1}{x} = \dfrac{\pi}{2}, \ x > 0$

Let $y = \arctan x + \arctan(1/x)$.

Then $\tan y = \dfrac{\tan(\arctan x) + \tan[\arctan(1/x)]}{1 - \tan(\arctan x)\,\tan[\arctan(1/x)]}$

$= \dfrac{x + (1/x)}{1 - x(1/x)}$

$= \dfrac{x + (1/x)}{0}$ (which is undefined).

So, $y = \pi/2$. Therefore,

$\arctan x + \arctan(1/x) = \pi/2$.

140. (a) $\arcsin(-x) = -\arcsin x, \ |x| \le 1$

Let $y = \arcsin(-x)$. Then

$-x = \sin y \Rightarrow x = -\sin y \Rightarrow x = \sin(-y)$.

So, $-y = \arcsin x \Rightarrow y = -\arcsin x$.

Therefore, $\arcsin(-x) = -\arcsin x$.

(b) $\arccos(-x) = \pi - \arccos x, \ |x| \le 1$

Let $y = \arccos(-x)$. Then

$-x = \cos y \Rightarrow x = -\cos y \Rightarrow x = \cos(\pi - y)$.

So, $\pi - y = \arccos x \Rightarrow y = \pi - \arccos x$.

Therefore, $\arccos(-x) = \pi - \arccos x$.

141. (a) $\operatorname{arccot} x = y$ if and only if $\cot y = x$, $0 < y < \pi$.

For $x > 0$, $\cot y > 0$ and $0 < y < \dfrac{\pi}{2}$.

So, $\tan y = \dfrac{1}{x} > 0$ and $y = \arctan\left(\dfrac{1}{x}\right)$.

For $x = 0$, $\operatorname{arccot}(0) = \dfrac{\pi}{2}$.

For $x < 0$, $\cot y < 0$ and $\dfrac{\pi}{2} < y < \pi$.

So, $\tan y = \dfrac{1}{x} < 0$ and $\arctan\left(\dfrac{1}{x}\right) < 0$.

Therefore, you need to add π to get

$y = \pi + \arctan\left(\dfrac{1}{x}\right)$.

(b) $y = \operatorname{arcsec} x$ if and only if $\sec y = x, |x| \ge 1$,

$0 \le y \le \pi, y \ne \dfrac{\pi}{2}$.

So, $\cos y = \dfrac{1}{x}$ and $y = \arccos\left(\dfrac{1}{x}\right)$.

(c) $y = \operatorname{arccsc} x$ if and only if $\csc y = x$, $|x| \ge 1$,

$-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}, y \ne 0$.

So, $\sin y = \dfrac{1}{x}$ and $y = \arcsin\left(\dfrac{1}{x}\right)$.

142. (a) $\operatorname{arccot}(0.5) = \arctan\left(\frac{1}{0.5}\right) = \arctan(2) \approx 1.1071$

(b) $\operatorname{arcsec}(2.7) = \arccos\left(\frac{1}{2.7}\right) \approx 1.1914$

(c) $\operatorname{arccsc}(-3.9) = \arcsin\left(\frac{-1}{3.9}\right) \approx -0.2593$

(d) $\operatorname{arccot}(-0.5) = \pi + \arctan(-2.0) \approx 2.0344$

143. False. Let $f(x) = x^2$.

144. True; if f has a y-intercept.

145. False

$$\arcsin^2 0 + \arccos^2 0 = 0 + \frac{\pi}{2}^2 \neq 1$$

146. False

The range of $y = \arcsin x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

147. True

148. False. Let $f(x) = x$ or $g(x) = 1/x$.

149. Let f and g be one-to-one functions.

Let $(f \circ g)(x) = y$, then $x = (f \circ g)^{-1}(y)$. Also:

$$(f \circ g)(x) = y$$
$$f(g(x)) = y$$
$$g(x) = f^{-1}(y)$$
$$x = g^{-1}(f^{-1}(y))$$
$$x = (g^{-1} \circ f^{-1}(y))$$

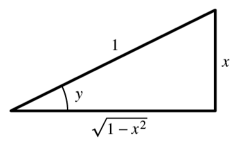
So, $(f \circ g)^{-1}(y) = (g^{-1} \circ f^{-1})(y)$ and $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

150. If f has an inverse, then f and f^{-1} are both one-to-one.

Let $(f^{-1})^{-1}(x) = y$ then $x = f^{-1}(y)$ and $f(x) = y$.

So, $(f^{-1})^{-1} = f$.

151. Let $y = \sin^{-1} x$. Then $\sin y = x$ and $\cos(\sin^{-1} x) = \cos(y) = \sqrt{1 - x^2}$, as indicated in the figure.



152. Suppose $g(x)$ and $h(x)$ are both inverses of $f(x)$. Then the graph of $f(x)$ contains the point (a, b) if and only if the graphs of $g(x)$ and $h(x)$ contain the point (b, a). Because the graphs of $g(x)$ and $h(x)$ are the same, $g(x) = h(x)$. So, the inverse of $f(x)$ is unique.

153. $\tan(\arctan x + \arctan y) = \dfrac{\tan(\arctan x + \arctan y)}{1 - \tan(\arctan x)\tan(\arctan y)} = \dfrac{x + y}{1 - xy}, xy \neq 1$

So, $\arctan x + \arctan y = \arctan\left(\frac{x + y}{1 - xy}\right), xy \neq 1$.

Let $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

$$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan\frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\cdot\frac{1}{3}\right)} = \arctan\frac{\frac{5}{6}}{1 - \frac{1}{6}} = \arctan\frac{\frac{5}{6}}{\frac{5}{6}} = \arctan 1 = \frac{\pi}{4}$$

154. $f(x) = k(2 - x - x^3)$ is one-to-one. Because

$f^{-1}(3) = -2$,

$f(-2) = 3 = k(2 - (-2) - (-2)^3) = 12k \Rightarrow k = \frac{1}{4}$.

155. $f(x) = kx + \sin x$

For $k \geq 1$, f is one-to-one, and for $k \leq -1$, f is one-to-one. Therefore, f has an inverse for $k \geq 1$ and $k \leq 1$.

156. f will be symmetric about the line $y = x$ if f is one-to-one, and equals its inverse. So assume

$$f(x_1) = f(x_2)$$
$$\frac{ax_1 + b}{cx_1 - a} = \frac{ax_2 + b}{cx_2 - a}$$
$$acx_1x_2 - a^2x_1 + bcx_2 - ab = acx_1x_2 + bcx_1 - ab$$
$$(a^2 + bc)x_2 = (a^2 + bc)x_1.$$

So, $x_1 = x_2$ if $a^2 + bc \neq 0$.

To show that $f = f^{-1}$, solve for x as follows:

$$y = \frac{ax + b}{cx - a}$$
$$ycx - ay = ax + b$$
$$(yc - a)x = b + ay$$
$$x = \frac{ay + b}{yc - a}$$
$$f^{-1}(x) = \frac{ax + b}{cx - a} = f(x)$$

So, f is symmetric about the line $y = x$ is and only if $a^2 + bc \neq 0$.

157. f is one-to-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$. So assume

$$f(x_1) = f(x_2)$$
$$\frac{ax_1 + b}{cx_1 + d} = \frac{ax_2 + b}{cx_2 + d}$$
$$acx_1x_2 + adx_1 + bcx_2 + bd = acx_1x_2 + adx_2 + bcx_1 + bd$$
$$adx_1 + bcx_2 = adx_2 + bcx_1$$
$$(ad - bc)x_1 = (ad - bc)x_2.$$

So, $x_1 = x_2$ if $ad - bc \neq 0$. To find f^{-1}, solve for x as follows.

$$y = \frac{ax + b}{cx + d}$$
$$ycx + yd = ax + b$$
$$(yc - a)x = b - yd$$
$$x = \frac{b - yd}{yc - a}$$
$$f^{-1}(x) = \frac{b - dx}{cx - a}$$

158. $y = ax^2 + bx + c$. Interchange x and y, and solve for y using the quadratic formula.

$$ay^2 + by + c - x = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4a(c - x)}}{2a}$$

Because $x \leq \frac{-b}{2a}$, use the negative sign.

$$f^{-1}(x) = \frac{-b - \sqrt{b^2 - 4ac + 4ax}}{2a}$$

Section 1.6 Exponential and Logarithmic Functions

1. $f(x) = e^x$. Domain is $(-\infty, \infty)$ and range is $(0, \infty)$. f is continuous, increasing, one-to-one, and concave upwards on its entire domain.

$\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to\infty} e^x = \infty$

2. The domain of the natural logarithmic function is $(0, \infty)$ and the range is $(-\infty, \infty)$. The function is continuous, increasing, and one-to-one, and its graph is concave downward. In addition, if a and b are positive numbers and n is rational, then $\ln(1) = 0$, $\ln(a \cdot b) = \ln a + \ln b$, $\ln(a^n) = n \ln a$, and $\ln(a/b) = \ln a - \ln b$.

3. If $\ln x = b$, then $x = e^b$ by the definition of the natural logarithmic function.

4. The functions $f(x) = e^x$ and $g(x) = \ln x$ are inverses of each other. So, $\ln e^x = g(f(x)) = x$.

5. (a) $25^{3/2} = 5^3 = 125$

(b) $81^{1/2} = 9$

(c) $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

(d) $27^{-1/3} = \dfrac{1}{27^{1/3}} = \dfrac{1}{3}$

6. (a) $64^{1/3} = 4$

(b) $5^{-4} = \dfrac{1}{5^4} = \dfrac{1}{625}$

(c) $\left(\dfrac{1}{8}\right)^{1/3} = \dfrac{1}{2}$

(d) $\left(\dfrac{1}{4}\right)^3 = \dfrac{1}{64}$

7. (a) $(5^2)(5^3) = 5^{2+3} = 5^5 = 3125$

(b) $(5^2)(5^{-3}) = 5^{2-3} = 5^{-1} = \dfrac{1}{5}$

(c) $\dfrac{5^{3x}}{25^{2x}} = \dfrac{(5^3)^x}{(25^2)^x} = \dfrac{1}{5^x}$

(d) $\left(\dfrac{1}{4}\right)^{2x}(2^{6x}) = \left[\left(\dfrac{1}{4}\right)^2\right]^x (2^6)^x = \left(\dfrac{1}{16}\right)^x (64)^x = 4^x$

8. (a) $(2^2)^3 = 2^6 = 64$

(b) $(5^4)^{1/2} = 5^2 = 25$

(c) $\left[(27^{-1})(27^{2/3})\right]^{3x} = \left[27^{-1/3}\right]^{3x} = 27^{-x} = \dfrac{1}{27^x}$

(d) $(25)^{3/2}3^2 = 5^3 3^2 = (125)9 = 1125$

9. (a) $e^3(e^4) = e^{3+4} = e^7$

(b) $(e^3)^4 = e^{12}$

(c) $(e^3)^{-2} = e^{-6} = \dfrac{1}{e^6}$

(d) $e^0 = 1$

10. (a) $\left(\dfrac{1}{e}\right)^{-2} = e^2$

(b) $(e^3)^4 = e^{12}$

(c) $\left(\dfrac{e^{-6}}{e^{-2}}\right)^2 = \left(\dfrac{e^2}{e^6}\right)^2 = \left(\dfrac{1}{e^4}\right)^2 = \dfrac{1}{e^8}$

(d) $\dfrac{1}{e^{-3}} = e^3$

11. $3^x = 81 \Rightarrow x = 4$

12. $4^x = 64 \Rightarrow x = 3$

13. $6^{x-2} = 36 \Rightarrow x - 2 = 2 \Rightarrow x = 4$

14. $5^{x+1} = 125 \Rightarrow x + 1 = 3 \Rightarrow x = 2$

15. $\left(\frac{1}{2}\right)^x = 32 \Rightarrow 2^{-x} = 32 \Rightarrow -x = 5 \Rightarrow x = -5$

16. $\left(\frac{1}{4}\right)^x = 16 \Rightarrow 4^{-x} = 16 \Rightarrow -x = 2 \Rightarrow x = -2$

17. $\left(\frac{1}{3}\right)^{x-1} = 27 \Rightarrow 3^{1-x} = 27 \Rightarrow 1 - x = 3 \Rightarrow x = -2$

18. $\left(\frac{1}{5}\right)^{2x+1} = 25 \Rightarrow 5^{-(2x+1)} = 5^2 \Rightarrow -(2x + 1) = 2$
$\Rightarrow 2x + 1 = -2 \Rightarrow x = -\frac{3}{2}$

19. $4^3 = (x + 2)^3 \Rightarrow 4 = x + 2 \Rightarrow x = 2$

20. $343 = (5x - 7)^3 \Rightarrow 7^3 = (5x - 7)^3 \Rightarrow 7 = 5x - 7$
$\Rightarrow 5x = 14 \Rightarrow x = \frac{14}{5}$

21. $x^{3/4} = 8 \Rightarrow x = 8^{4/3} = 2^4 = 16$

22. $(x+3)^{4/3} = 16 \Rightarrow x + 3 = \pm 16^{3/4}$
$\Rightarrow x + 3 = \pm 8 \Rightarrow x = 5, -11$

23. $e^x = e^{2x+1} \Rightarrow x = 2x + 1 \Rightarrow x = -1$

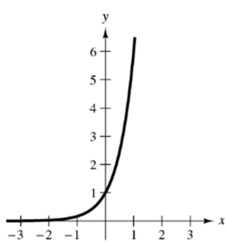
24. $e^x = 1 = e^0 \Rightarrow x = 0$

25. $e^{-2x} = e^5 \Rightarrow -2x = 5 \Rightarrow x = -\frac{5}{2}$

26. $e^{3x+1} = e^7 \Rightarrow 3x + 1 = 7 \Rightarrow x = \frac{6}{3} = 2$

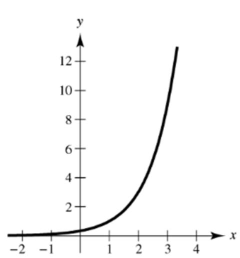
27. $y = 6^x$

x	0	1	2
y	1	6	36



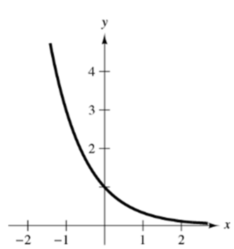
28. $y = 3^{x-1}$

x	−1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



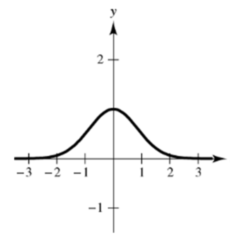
29. $y = \left(\frac{1}{3}\right)^x = 3^{-x}$

x	−2	−1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$



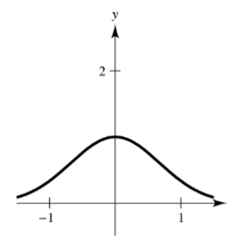
30. $y = 2^{-x^2}$

x	−2	−1	0	1	2	3
y	$\frac{1}{16}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{16}$	0.002



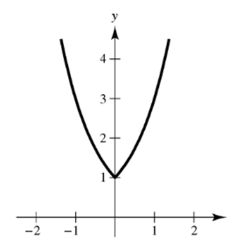
31. $f(x) = 3^{-x^2}$

x	0	±1	±2
y	1	$\frac{1}{3}$	0.0123



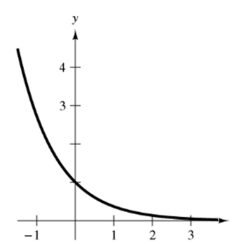
32. $f(x) = 3^{|x|}$

x	0	±1	±2
y	1	3	9



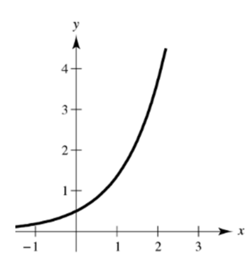
33. $y = e^{-x}$

x	−1	0	1
y	e	1	$\frac{1}{e}$



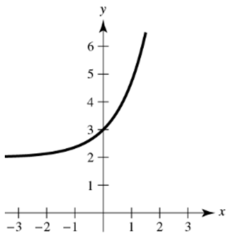
34. $y = \frac{1}{2}e^x$

x	−1	0	1	2
y	$\frac{1}{2e}$	$\frac{1}{2}$	$\frac{e}{2}$	$\frac{e^2}{2}$



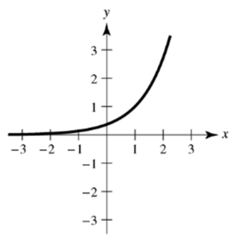
35. $y = e^x + 2$

x	-2	-1	0	1	2
y	$\frac{1}{e^2} + 2$	$\frac{1}{e} + 2$	3	$e + 2$	$e^2 + 2$



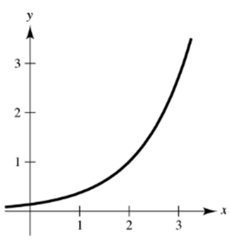
36. $y = e^{x-1}$

x	-1	0	1	2
y	$\frac{1}{e^2}$	$\frac{1}{e}$	1	e



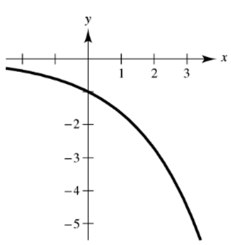
37. $h(x) = e^{x-2}$

x	0	1	2	3	4
y	e^{-2}	e^{-1}	1	e	e^2



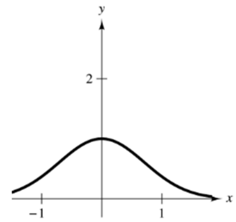
38. $g(x) = -e^{x/2}$

x	-2	0	2	4
y	$-\frac{1}{e}$	-1	$-e$	$-e^2$



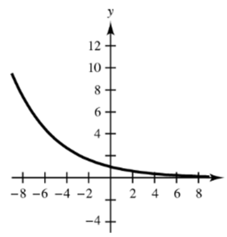
39. $y = e^{-x^2}$

x	-1	-0.5	0	0.5	1
y	$\frac{1}{e}$	$\frac{1}{e^{1/4}}$	1	$\frac{1}{e^{1/4}}$	$\frac{1}{e}$



40. $y = e^{-x/4}$

x	-8	-4	0	2	4
y	e^2	e	0	$e^{-1/2}$	e^{-1}



41. $f(x) = \frac{1}{3 + e^x}$

Because $e^x > 0, 3 + e^x > 0.$

Domain: all real numbers

42. $f(x) = \frac{1}{2 - e^x}$

$2 - e^x = 0 \Rightarrow x = \ln 2$

Domain: all $x \neq \ln 2$

43. $f(x) = \sqrt{1 - 4^x}$

$1 - 4^x \geq 0 \Rightarrow 4^x \leq 1 \Rightarrow x \ln 4 \leq \ln 1 = 0$

Domain: $x \leq 0$

44. $f(x) = \sqrt{1 + 3^{-x}}$

Because $1 + 3^{-x} > 0$ for all x, the domain is all real numbers.

45. $f(x) = \sin e^{-x}$

Domain: all real numbers

46. $f(x) = \cos(2^{1-x})$

Domain: all real numbers

47. $y = Ce^{ax}$

Graph rises from left to right

Matches (c)

48. $y = Ce^{-ax}$

Reflection in the y-axis

Matches (d)

49. $y = C(1 - e^{-ax})$

Vertical shift C units

Reflection in both the x- and y-axes

Matches (a)

50. $y = \dfrac{C}{1 + e^{-ax}}$

Matches (b)

51. $y = Ca^x$

$(0, 2): 2 = Ca^0 = C$

$(3, 54): 54 = 2a^3$

$27 = a^3$

$3 = a$

$y = 2(3^x)$

52. $y = Ca^x$

$(1, 2): 2 = Ca$

$(2, 1): 1 = Ca^2$

Dividing eliminates C: $\dfrac{2}{1} = \dfrac{Ca}{Ca^2} = \dfrac{1}{a}$

So, $a = \dfrac{1}{2}$ and $C = 4$.

$y = 4\left(\dfrac{1}{2}\right)^x$

53. $e^0 = 1$

$\ln 1 = 0$

54. $e^{-2} = 0.1353\ldots$

$\ln 0.1353\ldots = -2$

55. $e^3 = 20.0855\ldots \leftrightarrow \ln 20.0855\ldots = 3$

56. $e^{0.25} = 1.2840\ldots \leftrightarrow \ln 1.2840\ldots = 0.25$

57. $\ln 5 = 1.6094\ldots \leftrightarrow e^{1.6094\ldots} = 5$

58. $\ln 0.05 = -2.9957\ldots \leftrightarrow e^{-2.9957\ldots} = 0.05$

59. $\ln 4.15 = 1.4231\ldots$

$e^{1.4231\ldots} = 4.15$

60. $\ln 0.5 = -0.6931\ldots$

$e^{-0.6931\ldots} = \frac{1}{2}$

61. $f(x) = \ln x + 1$

Vertical shift 1 unit upward

Matches (b)

62. $f(x) = -\ln x$

Reflection in the x-axis

Matches (d)

63. $f(x) = \ln(x - 1)$

Horizontal shift 1 unit to the right

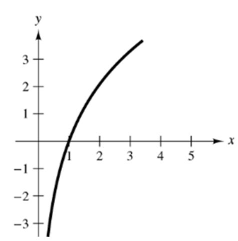
Matches (a)

64. $f(x) = -\ln(-x)$

Reflection in the y-axis and the x-axis

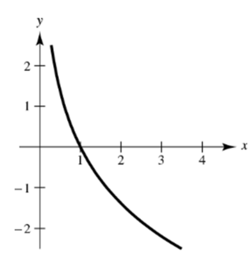
Matches (c)

65. $f(x) = 3 \ln x$



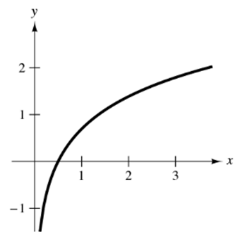
Domain: $x > 0$

66. $f(x) = -2 \ln x$



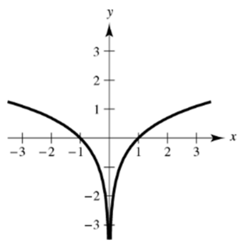
Domain: $x > 0$

67. $f(x) = \ln 2x$



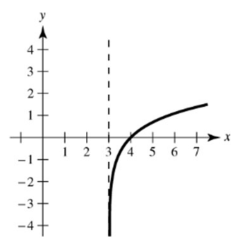
Domain: $x > 0$

68. $f(x) = \ln|x|$



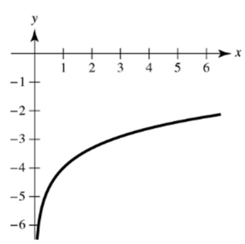
Domain: $x \neq 0$

69. $f(x) = \ln(x - 3)$



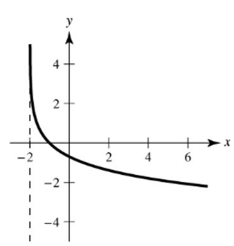
Domain: $x > 3$

70. $f(x) = \ln x - 4$



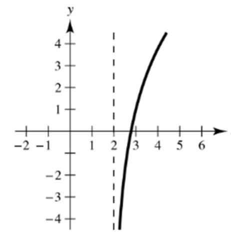
Domain: $x > 0$

71. $f(x) = -\ln(x + 2)$



Domain: $x > -2$

72. $f(x) = 4\ln(x - 2) + 1$



Domain: $x > 2$

73. 8 units upward: $e^x + 8$

Reflected in x-axis: $-(e^x + 8)$

$y = -(e^x + 8) = -e^x - 8$

74. 2 units to the left: e^{x+2}

6 units downward: $e^{x+2} - 6$

$y = e^{x+2} - 6$

75. 5 units to the right: $\ln(x - 5)$

1 unit downward: $\ln(x - 5) - 1$

$y = \ln(x - 5) - 1$

76. 3 units upward: $\ln x + 3$

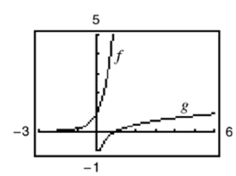
Reflected in x-axis: $\ln(-x) + 3$

$y = \ln(-x) + 3$

77. $g(x) = \ln\sqrt{x} = \frac{1}{2}\ln x$

$f(g(x)) = f\left(\frac{1}{2}\ln x\right) = e^{2(1/2 \ln x)} = e^{\ln x} = x$

$g(f(x)) = g(e^{2x}) = \frac{1}{2}\ln e^{2x} = \frac{1}{2}(2x)\ln e = x$



78. $f(g(x)) = f(\ln(x + 1))$

$= e^{\ln(x+1)} - 1$

$= (x + 1) - 1$

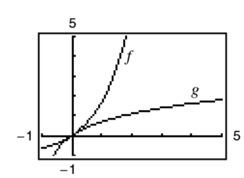
$= x$

$g(f(x)) = g(e^x - 1)$

$= \ln((e^x - 1) + 1)$

$= \ln e^x$

$= x$



79. $f(g(x)) = f\left(\frac{1}{2} + \ln\sqrt{x}\right)$

$= e^{2\left[(1/2)+\ln\sqrt{x}\right]-1}$

$= e^{2\ln x^{1/2}}$

$= e^{\ln x}$

$= x$

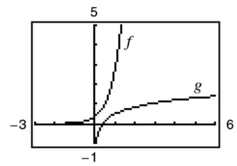
$g(f(x)) = g\left(e^{2x-1}\right)$

$= \frac{1}{2} + \ln\sqrt{e^{2x-1}}$

$= \frac{1}{2} + \frac{1}{2}\ln e^{2x-1}$

$= \frac{1}{2} + \frac{1}{2}(2x - 1)$

$= x$



80. $f(g(x)) = f\left(\ln x^3\right)$

$= f(3\ln x)$

$= e^{3\ln x/3}$

$= e^{\ln x}$

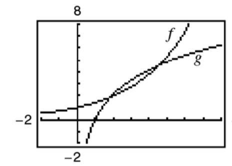
$= x$

$g(f(x)) = g\left(e^{x/3}\right)$

$= \ln\left(e^{x/3}\right)^3$

$= \ln e^x$

$= x$



81. (a) $y = e^{4x-1}$

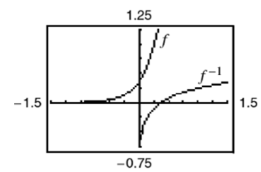
$\ln y = 4x - 1$

$\ln y + 1 = 4x$

$x = \frac{1}{4}(\ln y + 1)$

$f^{-1}(x) = \frac{1}{4}(\ln x + 1)$

(b)



(c) $f^{-1}(f(x)) = f^{-1}\left(e^{4x-1}\right) = \frac{1}{4}\left(\ln e^{4x-1} + 1\right) = \frac{1}{4}(4x - 1 + 1) = x$

$f\left(f^{-1}(x)\right) = f\left(\frac{1}{4}(\ln x + 1)\right) = e^{(\ln x+1)-1} = e^{\ln x} = x$

82. (a) $y = 3e^{-x}$

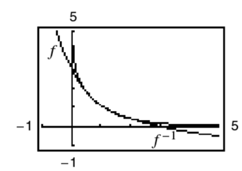
$\frac{y}{3} = e^{-x}$

$\ln\frac{y}{3} = -x$

$x = -\ln\frac{y}{3} = \ln\frac{3}{y}$

$f^{-1}(x) = \ln\frac{3}{x} = \ln 3 - \ln x$

(b)



(c) $f^{-1}(f(x)) = f^{-1}\left(3e^{-x}\right) = \ln 3 - \ln\left(3e^{-x}\right) = \ln 3 - \ln 3 - \ln e^{-x} = x$

$f\left(f^{-1}(x)\right) = f\left(\ln\frac{3}{x}\right) = 3e^{-\ln(3/x)} = 3e^{\ln(x/3)} = 3\left(\frac{x}{3}\right) = x$

83. (a) $y = 2\ln(x - 1)$

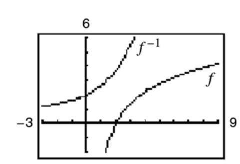
$\frac{y}{2} = \ln(x - 1)$

$e^{y/2} = x - 1$

$x = 1 + e^{y/2}$

$f^{-1}(x) = 1 + e^{x/2}$

(b)



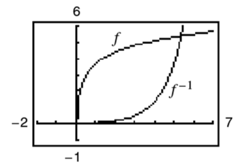
(c) $f^{-1}(f(x)) = f^{-1}(2\ln(x - 1)) = 1 + e^{\ln(x-1)} = 1 + x - 1 = x$

$f\left(f^{-1}(x)\right) = f\left(1 + x^{x/2}\right) = 2\ln\left[\left(1 + e^{x/2}\right) - 1\right] = 2\left(\frac{x}{2}\right) = x$

84. (a)

$$\begin{aligned} y &= 3 + \ln(2x) \\ y - 3 &= \ln 2x \\ e^{y-3} &= 2x \\ x &= \tfrac{1}{2}e^{y-3} \\ f^{-1}(x) &= \tfrac{1}{2}e^{x-3} \end{aligned}$$

(b)



(c) $f^{-1}(f(x)) = f^{-1}(3 + \ln(2x)) = \tfrac{1}{2}e^{3+\ln(2x)-3} = \tfrac{1}{2}(2x) = x$

$f(f^{-1}(x)) = f\left(\tfrac{1}{2}e^{x-3}\right) = 3 + \ln\left(e^{x-3}\right) = 3 + (x - 3) = x$

85. $\ln e^{x^2} = x^2$

86. $-4 \ln e^{2x-1} = -4(2x - 1) = 4 - 8x$

87. $e^{\ln(5x+2)} = 5x + 2$

88. $e^{\ln\sqrt{x}} = \sqrt{x}$

89. $-1 + \ln e^{2x} = -1 + 2x$

90. $-8 + e^{\ln x^3} = -8 + x^3$

91. (a) $\ln 6 = \ln 2 + \ln 3 \approx 1.7917$

(b) $\ln\tfrac{2}{3} = \ln 2 - \ln 3 \approx -0.4055$

(c) $\ln 81 = 4 \ln 3 \approx 4.3944$

(d) $\ln \sqrt{3} = \tfrac{1}{2} \ln 3 \approx 0.5493$

92. (a) $\ln 0.25 = \ln \tfrac{1}{4} = \ln 1 - 2 \ln 2 \approx -1.3862$

(b) $\ln 24 = 3 \ln 2 + \ln 3 \approx 3.1779$

(c) $\ln\sqrt[3]{12} = \tfrac{1}{3}(2 \ln 2 + \ln 3) \approx 0.8283$

(d) $\ln\tfrac{1}{72} = \ln 1 - (3 \ln 2 + 2 \ln 3) \approx -4.2765$

93. $\ln \dfrac{x}{4} = \ln x - \ln 4$

94. $\ln\dfrac{3}{x} = \ln 3 - \ln x$

95. $\ln \sqrt{x^5} = \ln x^{5/2} = \tfrac{5}{2} \ln x$

96. $\ln\left[\left(\sqrt{x}\right)^3\right] = \ln\left[x^{3/2}\right] = \dfrac{3}{2} \ln x$

97. $\ln \dfrac{xy}{z} = \ln x + \ln y - \ln z$

98. $\ln(xyz) = \ln x + \ln y + \ln z$

99. $\ln\left(x\sqrt{x^2 + 5}\right) = \ln x + \ln\left(x^2 + 5\right)^{1/2}$

$= \ln x + \tfrac{1}{2} \ln\left(x^2 + 5\right)$

100. $\ln\sqrt[3]{z + 1} = \ln(z + 1)^{1/3} = \tfrac{1}{3} \ln(z + 1)$

101. $\ln\sqrt{\dfrac{x - 1}{x}} = \ln\left(\dfrac{x - 1}{x}\right)^{1/2} = \dfrac{1}{2} \ln\left(\dfrac{x - 1}{x}\right)$

$= \dfrac{1}{2}\left[\ln(x - 1) - \ln x\right]$

$= \dfrac{1}{2} \ln(x - 1) - \dfrac{1}{2} \ln x$

102. $\ln z(z - 1)^2 = \ln z + \ln(z - 1)^2$

$= \ln z + 2 \ln(z - 1)$

103. $\ln 3e^2 = \ln 3 + 2 \ln e = 2 + \ln 3$

104. $\ln \dfrac{1}{e} = \ln 1 - \ln e = -1$

105. $\ln x + \ln 7 = \ln(x \cdot 7) = \ln(7x)$

106. $\ln y + \ln x^2 = \ln\left(yx^2\right)$

107. $\ln(x - 2) - \ln(x + 2) = \ln \dfrac{x - 2}{x + 2}$

108. $3 \ln x + 2 \ln y - 4 \ln z = \ln x^3 + \ln y^2 - \ln z^4$

$= \ln \dfrac{x^3y^2}{z^4}$

109. $\dfrac{1}{3}\left[2 \ln(x + 3) + \ln x - \ln\left(x^2 - 1\right)\right] = \dfrac{1}{3} \ln\dfrac{x(x + 3)^2}{x^2 - 1}$

$= \ln \sqrt[3]{\dfrac{x(x + 3)^2}{x^2 - 1}}$

110. $2\left[\ln x - \ln(x + 1) - \ln(x - 1)\right] = 2 \ln \dfrac{x}{(x + 1)(x - 1)}$

$= \ln\left(\dfrac{x}{x^2 - 1}\right)^2$

111. $2 \ln 3 - \dfrac{1}{2} \ln\left(x^2 + 1\right) = \ln 9 - \ln\sqrt{x^2 + 1} = \ln \dfrac{9}{\sqrt{x^2 + 1}}$

112. $\ln(x^2 - 4) - \ln(x + 2) + \ln 4 = \ln\left[\frac{(x^2 - 4)4}{x + 2}\right]$

$= \ln\left[\frac{4(x + 2)(x - 2)}{x + 2}\right]$

$= \ln[4(x - 2)]$

113. $e^{\ln x} = 5 \Rightarrow x = 5$

114. $e^{\ln 2x} = 20$

$2x = 20$

$x = 10$

115. $e^x = 12$

$x = \ln 12 \approx 2.485$

116. $5e^x = 36$

$e^x = \frac{36}{5}$

$x = \ln\left(\frac{36}{5}\right) \approx 1.974$

117. $9 - 2e^x = 7$

$2e^x = 2$

$e^x = 1$

$x = 0$

118. $8e^x - 12 = 7$

$8e^x = 19$

$e^x = \frac{19}{8}$

$x = \ln\left(\frac{19}{8}\right)$

≈ 0.865

119. $e^{x+4} = 21$

$x + 4 = \ln 21$

$x = \ln 21 - 4$

$x \approx -0.955$

120. $e^{x-3} = 5$

$x - 3 = \ln 5$

$x = 3 + \ln 5$

$x \approx 4.609$

121. $50e^{-x} = 30$

$e^{-x} = \frac{3}{5}$

$-x = \ln\left(\frac{3}{5}\right)$

$x = \ln\left(\frac{5}{3}\right)$

≈ 0.511

122. $100e^{-2x} = 35$

$e^{-2x} = \frac{35}{100} = \frac{7}{20}$

$-2x = \ln\left(\frac{7}{20}\right)$

$x = -\frac{1}{2}\ln\left(\frac{7}{20}\right) = \frac{1}{2}\ln\left(\frac{20}{7}\right)$

≈ 0.525

123. $\ln e^x = 1.5 \Rightarrow x = 1.5$

124. $\ln e^{2x} = 6$

$2x = 6$

$x = 3$

125. $\ln x = 2$

$x = e^2 \approx 7.389$

126. $\ln x^2 = -8$

$x^2 = e^{-8}$

$x = \pm\sqrt{e^{-8}} = \pm e^{-4} \approx \pm 0.018$

127. $\ln(x - 3) = 2$

$x - 3 = e^2$

$x = 3 + e^2 \approx 10.389$

128. $\ln 4x = 1$

$4x = e^1 = e$

$x = \frac{e}{4} \approx 0.680$

129. $\ln 2 + \ln x = 3$

$\ln 2x = 3$

$2x = e^3$

$x = \frac{1}{2}e^3$

$x \approx 10.043$

130. $\ln 3x + \ln 2 = 4$

$\ln(6x) = 4$

$6x = e^4$

$x = \frac{1}{6}e^4$

$x \approx 9.100$

131. $\ln\sqrt{x + 2} = 1$

$\sqrt{x + 2} = e^1 = e$

$x + 2 = e^2$

$x = e^2 - 2 \approx 5.389$

132. $\ln(x-2)^2 = 12$

$(x-2)^2 = e^{12}$

$x - 2 = e^6$

$x = 2 + e^6 \approx 405.429$

133. $e^{2x+1} > 3$

$2x + 1 > \ln 3$

$2x > \ln 3 - 1$

$x > \frac{1}{2}[\ln 3 - 1]$

134. $e^{1-x} < 6$

$\ln e^{1-x} < \ln 6$

$1 - x < \ln 6$

$x > 1 - \ln 6$

135. $-2 < \ln x < 0$

$e^{-2} < x < e^0 = 1$

$\dfrac{1}{e^2} < x < 1$

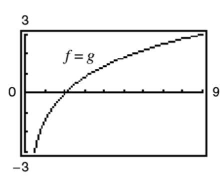
136. $1 < \ln(x^2 + 1) < 100$

$e < x^2 + 1 < e^{100}$

$e - 1 < x^2 < e^{100} - 1$

$\sqrt{e-1} < x < \sqrt{e^{100} - 1}$

137. (a)



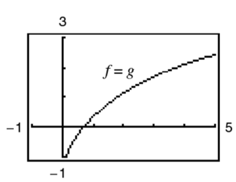
(b) $f(x) = \ln\left(\dfrac{x^2}{4}\right), \; x > 0$

$= \ln x^2 - \ln 4$

$= 2\ln x - \ln 4$

$= g(x)$

138. (a)



(b) $f(x) = \ln\sqrt{x(x^2+1)}$

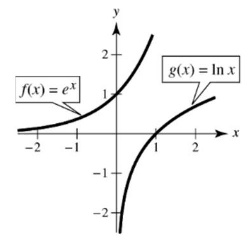
$= \frac{1}{2}\ln\left[x(x^2+1)\right]$

$= \frac{1}{2}\left[\ln x + \ln(x^2+1)\right]$

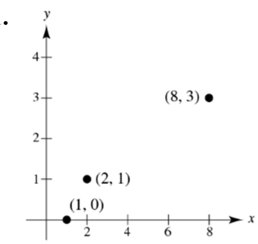
$= g(x)$

139. No; $\ln(a^b) = b\ln a$ is only true when $a > 0$ because this follows the properties of logarithms.

140. There are no real solutions of the equation $\ln x = e^x$. One way to visualize this is to observe that the graphs of $f(x) = \ln x$ and $g(x) = e^x$ do not intersect.



141.



x	1	2	8
y	0	1	3

(a) y is a logarithmic function of x: True; $y = \dfrac{\ln x}{\ln 2}$

(b) y is an exponential function of x: False

(c) x is an exponential function of y: True; $2^y = x$

(d) y is a linear function of x: False

142. The graph is that of $y_2 = e^{\ln x}$.

The domain of $y_1 = \ln(e^x)$ is $(-\infty, \infty)$.

The domain of $y_2 = e^{\ln x}$ is $x > 0$.

No, $\ln e^x \neq e^{\ln x}$ for all real values of x. They are equal for $x > 0$.

143. $\beta = \dfrac{10}{\ln 10}\ln\left(\dfrac{I}{10^{-16}}\right)$

$= \dfrac{10}{\ln 10}\left[\ln I - \ln 10^{-16}\right]$

$= \dfrac{10}{\ln 10}\left[\ln I + 16\ln 10\right]$

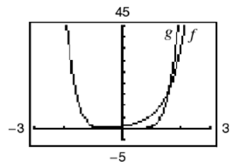
$= \dfrac{10}{\ln 10}\ln I + 160$

$= 10\log_{10} I + 160$

144. $\beta(10^{-5}) = \dfrac{10}{\ln 10}\ln 10^{-5} + 160$

$= -50 + 160 = 110$ decibels

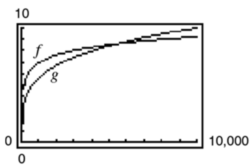
145.



The graphs intersect three times: $(-0.7899,\ 0.2429)$, $(1.6242,\ 18.3615)$ and $(6,\ 46{,}656)$.

The function $f(x) = 6^x$ grows more rapidly.

146.

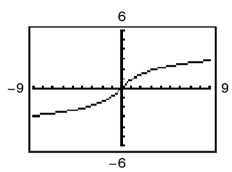


The graphs intersect twice: $(4.1771,\ 1.4296)$ and $(5503.647,\ 8.6132)$.

$g(x) = x^{1/4}$ grows more rapidly.

147. $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$

(a)



Domain: $-\infty < x < \infty$

(b)
$$\begin{aligned} f(-x) &= \ln\left(-x + \sqrt{x^2+1}\right) \\ &= \ln\left[\frac{\left(-x + \sqrt{x^2+1}\right)\left(-x - \sqrt{x^2+1}\right)}{\left(-x - \sqrt{x^2+1}\right)}\right] \\ &= \ln\left[\frac{\left(x^2\sqrt{x^2+1}\right)}{\left(-x - \sqrt{x^2+1}\right)}\right] \\ &= \ln\left[\frac{1}{\left(x + \sqrt{x^2+1}\right)}\right] \\ &= -\ln\left(x + \sqrt{x^2+1}\right) = -f(x) \end{aligned}$$

(c)
$$\begin{aligned} y &= \ln\left(x + \sqrt{x^2+1}\right) \\ e^y &= x + \sqrt{x^2+1} \\ \left(e^y - x\right)^2 &= x^2 + 1 \\ 2xe^y &= e^{2y} - 1 \\ x &= \frac{e^{2y} - 1}{2e^x} \end{aligned}$$

148. $f(x) = \ln x$
$$\begin{aligned} f\left(e^{n+1}\right) - f\left(e^n\right) &= \ln\left(e^{n+1}\right) - \ln\left(e^n\right) \\ &= (n+1)\ln e - n\ln e \\ &= (n+1) - n = 1 \end{aligned}$$

149. $n = 12$

$12! = 12 \cdot 11 \cdot 10 \cdots 3 \cdot 2 \cdot 1 = 479{,}001{,}600$

Stirlings Formula:

$$12! \approx \left(\frac{12}{e}\right)^{12}\sqrt{2\pi(12)} \approx 475{,}687{,}487$$

150. $n = 15$

$15! = 15 \cdot 14 \cdots 3 \cdot 2 \cdot 1 = 1{,}307{,}674{,}368{,}000$

Stirlings Formula:

$$\begin{aligned} 15! &\approx \left(\frac{15}{e}\right)^{15}\sqrt{2\pi(15)} \approx 1{,}300{,}430{,}722{,}200 \\ &\approx 1.3004 \times 10^{12} \end{aligned}$$

151. Let $m = \ln x$ and $n = \ln y$. Then $x = e^m$ and $y = e^n$.
$$\begin{aligned} \frac{x}{y} &= \frac{e^m}{e^n} = e^{m-n} \\ \ln\left(\frac{x}{y}\right) &= \ln\left(e^{m-n}\right) \\ \ln\left(\frac{x}{y}\right) &= (m-n)\ln e \\ \ln\left(\frac{x}{y}\right) &= m - n \\ \ln\left(\frac{x}{y}\right) &= \ln x - \ln y,\ x > 0,\ y > 0 \end{aligned}$$

152. Let $m = \ln x$.
$$\begin{aligned} x &= e^m \\ x^n &= \left(e^m\right)^n \\ \ln x^n &= \ln\left(e^{mn}\right) \\ \ln x^n &= mn \\ \ln x^n &= n\ln x \end{aligned}$$

Review Exercises for Chapter 1

1. (1) b, e, f
(ii) d
(iii) No matches
(iv) e, f
(v) a
(vi) d

2. Let $y = f(x)$ be one-to-one. Solve for x as a function of y. Interchange x and y to get $y = f^{-1}(x)$. Let the domain of f^{-1} be the range of f. Verify that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.

Example:

$f(x) = x^3$

$y = x^3$

$x = \sqrt[3]{y}$

$y = \sqrt[3]{x}$

$f^{-1}(x) = \sqrt[3]{x}$

3. (a) Let $u = v = \dfrac{\pi}{2}$.

$$\sin(u + v) = \sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \sin \pi = 0$$

$$\sin u + \sin v = \sin \frac{\pi}{2} + \sin \frac{\pi}{2} = 1 + 1 = 2$$

So, $\sin(u + v) \neq \sin u + \sin v$.

(b) Let $u = v = \pi$.

$$\cos(u + v) = \cos(\pi + \pi) = \cos 2\pi = 1$$

$$\cos u + \cos v = \cos \pi + \cos \pi = -1 - 1 = -2$$

So, $\cos(u + v) \neq \cos u + \cos v$.

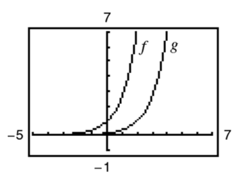
(c) Let $u = v = \dfrac{\pi}{4}$.

$$\tan(u + v) = \tan\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \tan \frac{\pi}{2} \Rightarrow \text{ undefined}$$

$$\tan u + \tan v = \tan \frac{\pi}{4} + \tan \frac{\pi}{4} = 1 + 1 = 2$$

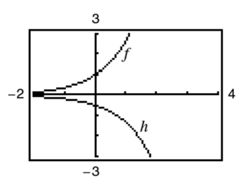
So, $\tan(u + v) \neq \tan u + \tan v$.

4. (a)



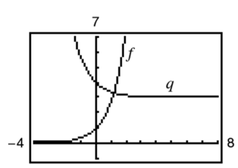
Horizontal shift 2 units to the right.

(b)



A reflection in the x-axis and a vertical shrink.

(c)



Vertical shift 3 units upward and a reflection in the y-axis.

5. $f(x) = 3\cos 2x,\ g(x) = x|x|$

(a) Domain of f: $(-\infty, \infty)$

Domain of g: $(-\infty, \infty)$

Range of f: $[-3, 3]$

Range of g: $(-\infty, \infty)$

(b) $f(-x) = f(x) \Rightarrow f$ is symmetric about the y-axis.

$g(-x) = -g(x) \Rightarrow g$ is symmetric about the origin.

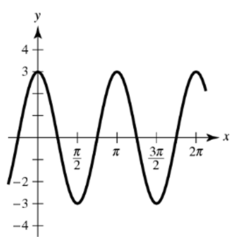
(c) $f(x) = 3\cos 2x = 0 \Rightarrow 2x = \dfrac{\pi}{2} + n\pi \Rightarrow x = \dfrac{\pi}{4} + \dfrac{n\pi}{2}$

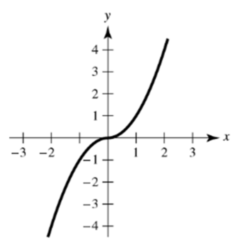
Intercepts: $\left(\dfrac{\pi}{4} + \dfrac{n\pi}{2}, 0\right)$, n is an integer, and $(0, 3)$

$g(x) = 0 \Rightarrow x = 0$

Intercept: $(0, 0)$

(d)





(e) $(f + g)(x) = 3\cos 2x + x|x|$

$(f - g)(x) = 3\cos 2x - x|x|$

$(fg)(x) = (3\cos 2x)(x|x|) = 3x|x|\cos 2\text{x}$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{(3\cos 2x)}{(x|x|)},\ x \neq 0$

(f) $f(g(x)) = f(x|x|) = 3\cos(2x|x|)$

$g(f(x)) = g(3\cos 2x) = (3\cos 2x)|3\cos 2x|$

$= 9\cos 2x|\cos 2x|$

6. $f(x) = x^2,\ g(x) = \sqrt{x}$

(a) $f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x,\ x \geq 0$

$g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

(b) Domain of $f \circ g$: $[0, \infty)$

Domain of $g \circ f$: $(-\infty, \infty)$

(c) Range of $f \circ g$: $[0, \infty)$

Range of $g \circ f$: $[0, \infty)$

(d) $f \circ g \neq g \circ f$ because their domains are different.

7. $y = 5x - 8$

$x = 0$: $y = 5(0) - 8 = -8 \Rightarrow (0, -8)$, y-intercept

$y = 0$: $0 = 5x - 8 \Rightarrow x = \frac{8}{5} \Rightarrow \left(\frac{8}{5}, 0\right)$, x-intercept

8. $y = x^2 - 8x + 12$

$x = 0$: $y = (0)^2 - 8(0) + 12 = 12 \Rightarrow (0, 12)$, y-intercept

$y = 0$: $x^2 - 8x + 12 = (x - 6)(x - 2) = 0 \Rightarrow x = 2, 6 \Rightarrow (2, 0), (6, 0)$, x-intercepts

9. $y = \dfrac{x - 3}{x - 4}$

$x = 0$: $y = \dfrac{0 - 3}{0 - 4} = \dfrac{3}{4} \Rightarrow \left(0, \dfrac{3}{4}\right)$, y-intercept

$y = 0$: $0 = \dfrac{x - 3}{x - 4} \Rightarrow x = 3 \Rightarrow (3, 0)$, x-intercept

10. $y = (x - 3)\sqrt{x + 4}$

$x = 0$: $y = (0 - 3)\sqrt{0 + 4} = -3\sqrt{4} = -3(2) = -6 \Rightarrow (0, -6)$, y-intercept

$y = 0$: $(x - 3)\sqrt{x + 4} = 0 \Rightarrow x = 3, -4 \Rightarrow (3, 0), (-4, 0)$, x-intercepts

11. $y = x^2 + 4x$ does not have symmetry with respect to either axis or the origin.

12. Symmetric with respect to y-axis because

$y = (-x)^4 - (-x)^2 + 3$

$y = x^4 - x^2 + 3.$

13. Symmetric with respect to both axes and the origin because:

$y^2 = (-x^2) - 5$	$(-y)^2 = x^2 - 5$	$(-y)^2 = (-x)^2 - 5$
$y^2 = x^2 - 5$	$y^2 = x^2 - 5$	$y^2 = x^2 - 5$

14. Symmetric with respect to the origin because:

$(-x)(-y) = -2$

$xy = -2.$

15. $y = -\dfrac{1}{2}x + 3$

y-intercept: $y = -\dfrac{1}{2}(0) + 3 = 3$

$(0, 3)$

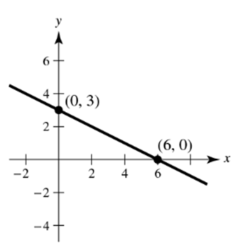
x-intercept: $-\dfrac{1}{2}x + 3 = 0$

$-\dfrac{1}{2}x = -3$

$x = 6$

$(6, 0)$

Symmetry: none



16. $y = -x^2 + 4$

y-intercept: $y = -(0)^2 + 4 = 4$

$(0, 4)$

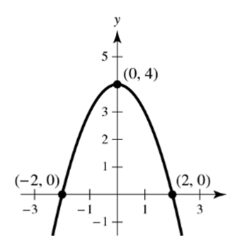
x-intercepts: $-x^2 + 4 = 0$

$(2 - x)(2 + x) = 0$

$x = \pm 2$

$(2, 0), (-2, 0)$

Symmetric with respect to the y-axis because $-(-x)^2 + 4 = -x^2 + 4.$



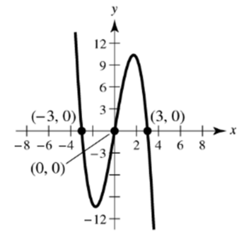
17. $y = 9x - x^3$

$9x - x^2 = x(9 - x^2) = x(3 - x)(3 + x) = 0 \Rightarrow x = 0, 3, -3$

Intercepts: $(0, 0), (3, 0), (-3, 0)$

Symmetric with respect to the origin because

$f(-x) = 9(-x) - (-x)^3 = -9x + x^3 = -(9x - x^3) = -f(x).$



18. $y^2 = 9 - x$

$y^2 + x - 9 = 0$

y-intercept: $y^2 = 9 - 0 = 9 \Rightarrow y = \pm 3$

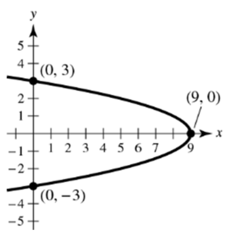
$(0, 3), (0, -3)$

x-intercept: $0^2 = 9 - x \Rightarrow x = 9$

$(9, 0)$

Symmetric with respect to the x-axis because

$(-y)^2 + x - 9 = y^2 + x - 9 = 0.$



19. $y = 2\sqrt{4 - x}$

y-intercept: $y = 2\sqrt{4 - 0} = 2\sqrt{4} = 4$

$(0, 4)$

x-intercept: $2\sqrt{4 - x} = 0$

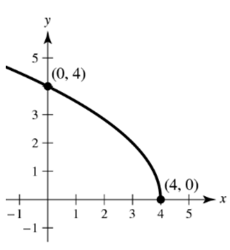
$\sqrt{4 - x} = 0$

$4 - x = 0$

$x = 4$

$(4, 0)$

Symmetry: none



20. $y = |x - 4| - 4$

y-intercept: $y = |0 - 4| - 4 = |-4| - 4 = 4 - 4 = 0$

$(0, 0)$

x-intercepts: $|x - 4| - 4 = 0$

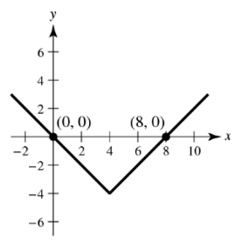
$|x - 4| = 4$

$x - 4 = 4$ or $x - 4 = -4$

$x = 8 \quad x = 0$

$(0, 0), (8, 0)$

Symmetry: none



21. $5x + 3y = -1 \Rightarrow y = \frac{1}{3}(-5x - 1)$

$x - y = -5 \Rightarrow y = x + 5$

$\frac{1}{3}(-5x - 1) = x + 5$

$-5x - 1 = 3x + 15$

$-16 = 8x$

$-2 = x$

For $x = -2$, $y = x + 5 = -2 + 5 = 3$.

Point of intersection is: $(-2, 3)$

22. $2x + 4y = 9 \Rightarrow y = \dfrac{-2x + 9}{4}$

$6x - 4y = 7 \Rightarrow y = \dfrac{6x - 7}{4}$

$$\frac{-2x + 9}{4} = \frac{6x - 7}{4}$$

$$-2x + 9 = 6x - 7$$

$$-8x = -16$$

$$x = 2$$

For $x = 2,\ y = \dfrac{6(2) - 7}{4} = \dfrac{5}{4}$

Point of intersection: $\left(2, \dfrac{5}{4}\right)$

23. $x - y = -5 \Rightarrow y = x + 5$

$x^2 - y = 1 \Rightarrow y = x^2 - 1$

$$x + 5 = x^2 - 1$$

$$0 = x^2 - x - 6$$

$$0 = (x - 3)(x + 2)$$

$$x = 3 \text{ or } x = -2$$

For $x = 3,\ y = 3 + 5 = 8.$

For $x = -2,\ y = -2 + 5 = 3.$

Points of intersection: $(3, 8),\ (-2, 3)$

24. $x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

$-x + y = 1 \Rightarrow y = x + 1$

$$1 - x^2 = (x + 1)^2$$

$$1 - x^2 = x^2 + 2x + 1$$

$$0 = 2x^2 + 2x$$

$$0 = 2x(x + 1)$$

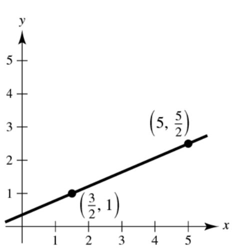
$$x = 0 \text{ or } x = -1$$

For $x = 0,\ y = 0 + 1 = 1.$

For $x = -1,\ y = -1 + 1 = 0.$

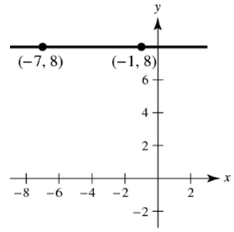
Points of intersection: $(0, 1),\ (-1, 0)$

25.



$$\text{Slope} = \frac{\left(\frac{5}{2}\right) - 1}{5 - \left(\frac{3}{2}\right)} = \frac{\frac{3}{2}}{\frac{7}{2}} = \frac{3}{7}$$

26. The line is horizontal and has slope 0.

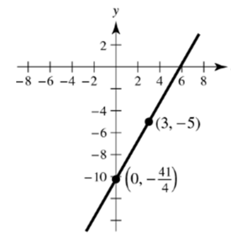


27. $y - (-5) = \frac{7}{4}(x - 3)$

$$y + 5 = \frac{7}{4}x - \frac{21}{4}$$

$$4y + 20 = 7x - 21$$

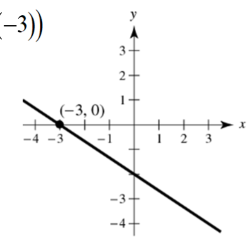
$$0 = 7x - 4y - 41$$



28. $y - 0 = -\frac{2}{3}(x - (-3))$

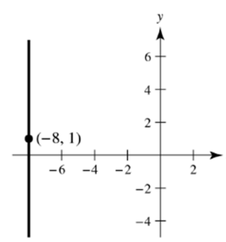
$$y = -\frac{2}{3}x - 2$$

$$2x + 3y + 6 = 0$$



29. Because m is undefined the line is vertical.

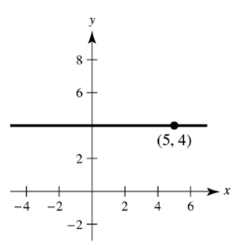
$x = -8$ or $x + 8 = 0$



30. Because $m = 0$, the line is horizontal.

$y - 4 = 0(x - 5)$

$y = 4$ or $y - 4 = 0$



31. $y - 3x = 5$

$$y = 3x + 5$$

Slope: $m = 3$

y-intercept: $(0, 5)$

32. $9 - y = x$

$y = -x + 9$

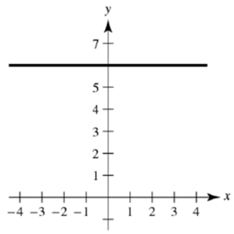
Slope: $m = -1$

y-intercept: $(0, 9)$

33. $y = 6$

Slope: 0

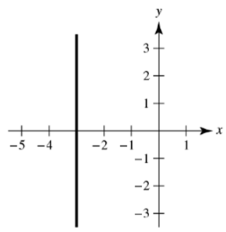
y-intercept: $(0, 6)$



34. $x = -3$

Slope: undefined

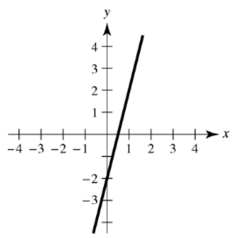
Line is vertical.



35. $y = 4x - 2$

Slope: 4

y-intercept: $(0, -2)$



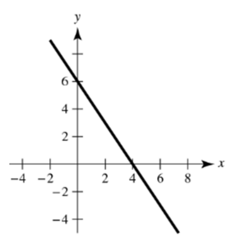
36. $3x + 2y = 12$

$2y = -3x + 12$

$y = \frac{-3}{2}x + 6$

Slope : $-\frac{3}{2}$

y-intercept: $(0, 6)$

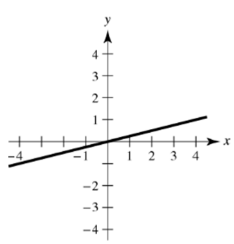


37. $m = \frac{2 - 0}{8 - 0} = \frac{1}{4}$

$y - 0 = \frac{1}{4}(x - 0)$

$y = \frac{1}{4}x$

$4y - x = 0$

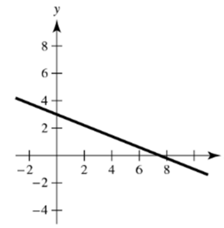


38. $m = \frac{-1 - 5}{10 - (-5)} = \frac{-6}{15} = -\frac{2}{5}$

$y - 5 = \frac{-2}{5}(x - (-5))$

$5y - 25 = -2x - 10$

$5y + 2x - 15 = 0$



39. (a) $y - 5 = \frac{7}{16}(x + 3)$

$16y - 80 = 7x + 21$

$0 = 7x - 16y + 101$

(b) $5x - 3y = 3$ has slope $\frac{5}{3}$.

$y - 5 = \frac{5}{3}(x + 3)$

$3y - 15 = 5x + 15$

$0 = 5x - 3y + 30$

(c) $3x + 4y = 8$

$4y = -3x + 8$

$y = \frac{-3}{4}x + 2$

Perpendicular line has slope $\frac{4}{3}$.

$y - 5 = \frac{4}{3}(x - (-3))$

$3y - 15 = 4x + 12$

$4x - 3y + 27 = 0$ or $y = \frac{4}{3}x + 9$

(d) Slope is undefined so the line is vertical.

$x = -3$

$x + 3 = 0$

40. (a) $$y - 4 = -\frac{2}{3}(x - 2)$$
$$3y - 12 = -2x + 4$$
$$2x + 3y - 16 = 0$$

(b) $x + y = 0$ has slope -1. Slope of the perpendicular line is 1.
$$y - 4 = 1(x - 2)$$
$$y = x + 2$$
$$0 = x - y + 2$$

(c) The slope of the line $3x - y = 0$ is 3.
The parallel line through $(2, 4)$ is
$$y - 4 = 3(x - 2)$$
$$y - 4 = 3x - 6$$
$$y - 3x + 2 = 0.$$

(d) Because the line is horizontal the slope is 0.
$$y = 4$$
$$y - 4 = 0$$

41. The slope is -850.
$V = -850t + 12{,}500$.
$V(3) = -850(3) + 12{,}500 = \$9950$

42. (a) $C = 9.25t + 13.50t + 36{,}500 = 22.75t + 36{,}500$

(b) $R = 30t$

(c) $$30t = 22.75t + 36{,}500$$
$$7.25t = 36{,}500$$
$t \approx 5034.48$ hours to break even

43. $f(x) = 5x + 4$

(a) $f(0) = 5(0) + 4 = 4$

(b) $f(5) = 5(5) + 4 = 29$

(c) $f(-3) = 5(-3) + 4 = -11$

(d) $f(t + 1) = 5(t + 1) + 4 = 5t + 9$

44. $f(x) = x^3 - 2x$

(a) $f(-3) = (-3)^3 - 2(-3) = -27 + 6 = -21$

(b) $f(2) = 2^3 - 2(2) = 8 - 4 = 4$

(c) $f(-1) = (-1)^3 - 2(-1) = -1 + 2 = 1$

(d) $$f(c - 1) = (c - 1)^3 - 2(c - 1) = c^3 - 3c^2 + 3c - 1 - 2c + 2 = c^3 - 3c^2 + c + 1$$

45. $f(x) = \begin{cases} x + 2 & x \le 1 \\ 2x - 3, & x > 1 \end{cases}$

(a) $f(-2) = (-2) + 2 = 0$

(b) $f(0) = 0 + 2 = 2$

(c) $f(1) = 1 + 2 = 3$

(d) $f(2) = 2(2) - 3 = 1$

46. $f(x) = \begin{cases} |3x - 1|, & x < 0 \\ x^2 - 1, & x \ge 0 \end{cases}$

(a) $f(-1) = |3(-1) - 1| = |-4| = 4$

(b) $f(0) = 0^2 - 1 = -1$

(c) $f(1) = 1^2 - 1 = 0$

(d) $f(a^2 + 3) = (a^2 + 3)^2 - 1 = a^4 + 6a^2 + 8$

(Note: $a^2 + 3$ is always positive.)

47. $f(x) = 4x^2$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{4(x + \Delta x)^2 - 4x^2}{\Delta x} = \frac{4\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - 4x^2}{\Delta x} = \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 4x^2}{\Delta x} = \frac{8x\Delta x + 4(\Delta x)^2}{\Delta x} = 8x + 4\Delta x, \quad \Delta x \ne 0$$

48. $f(x) = 2x - 6$
$f(1) = 2(1) - 6 = -4$

$$\frac{f(x) - f(1)}{x - 1} = \frac{(2x - 6) - (-4)}{x - 1} = \frac{2x - 6 + 4}{x - 1} = \frac{2x - 2}{x - 1} = \frac{2(x - 1)}{x - 1} = 2, \quad x \ne 1$$

49. $f(x) = x^2 + 3$
Domain: $(-\infty, \infty)$
Range: $[3, \infty)$

50. $g(x) = \sqrt{6 - x}$

Domain: $6 - x \geq 0$

$6 \geq x$

$(-\infty, 6]$

Range: $[0, \infty)$

51. $f(x) = -|x + 1|$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

52. $h(x) = \dfrac{2}{x + 1}$

Domain: all $x \neq -1$; $(-\infty, -1) \cup (-1, \infty)$

Range: all $y \neq 0$; $(-\infty, 0) \cup (0, \infty)$

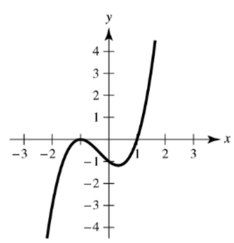
53. $f(x) = x^3 + x^2 - x - 1 = x^2(x + 1) - (x + 1)$

$= (x^2 - 1)(x + 1)$

Intercepts: $(0, -1), (1, 0), (-1, 0)$

Domain: $(-\infty, \infty)$

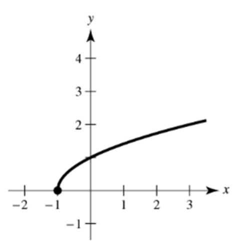
Range: $(-\infty, \infty)$



54. $g(x) = \sqrt{x + 1}$

Domain: $[-1, \infty)$

Range: $[0, \infty)$

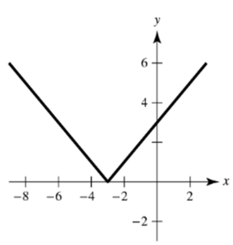


55. $h(x) = |x + 3|$

Intercepts: $(-3, 0), (0, 3)$

Domain: $(-\infty, \infty)$

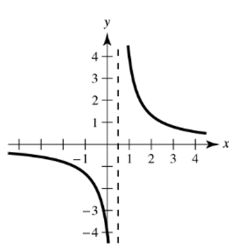
Range: $[0, \infty)$



56. $f(x) = \dfrac{4}{2x - 1}$

Domain: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

Range: $(-\infty, 0) \cup (0, \infty)$



57. $x + y^2 = 2 \Rightarrow y = \pm\sqrt{2 - x}$

y is not a function of x.

Some vertical lines intersect the graph more than once.

58. $x^2 - y = 0 \Rightarrow y = x^2$

y is a function of x.

A vertical line intersects the graph exactly once.

59. $xy + x^3 - 2y = 0$

$(x - 2)y = -x^3$

$y = \dfrac{-x^3}{x - 2}$

y is a function of x.

60. $x = 9 - y^2 \Rightarrow y = \pm\sqrt{9 - x}$

y is not a function of x.

Some y-values correspond to the same x-value.

61. $f(x) = 3x + 1, g(x) = -x$

$(f \circ g)(x) = f(g(x)) = f(-x) = -3x + 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(f(x))$

$= g(3x + 1)$

$= -(3x + 1)$

$= -3x - 1$

Domain: $(-\infty, \infty)$

$f \circ g \neq g \circ f$

62. $f(x) = \sqrt{x - 2}, g(x) = x^2$

$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 - 2}$

Domain: $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$

$(g \circ f)(x) = g(f(x))$

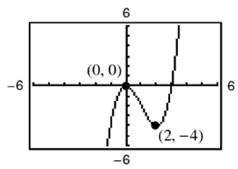
$= g\left(\sqrt{x - 2}\right)$

$= \left(\sqrt{x - 2}\right)^2$

$= x - 2$

$f \circ g \neq g \circ f$

63. $f(x) = x^3 - 3x^2$



(a) The graph of g is obtained from f by a vertical shift down 1 unit, followed by a reflection in the x-axis:

$g(x) = -[f(x) - 1] = -x^3 + 3x^2 + 1$

(b) The graph of g is obtained from f by a vertical shift upwards of 1 and a horizontal shift of 2 to the right.

$g(x) = f(x - 2) + 1 = (x - 2)^3 - 3(x - 2)^2 + 1$

64. (a) 3 (cubic), negative leading coefficient

(b) 4 (quartic), positive leading coefficient

(c) 2 (quadratic), negative leading coefficient

(d) 5, positive leading coefficient

65. $f(x) = x^4 - x^2$

$f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x)$

f is even.

$$f(x) = x^4 - x^2 = 0$$
$$x^2(x^2 - 1) = 0$$
$$x^2(x + 1)(x - 1) = 0$$

Zeros: $x = 0, -1, 1$

66. $f(x) = \sqrt{x^3 + 1}$

$f(-x) = \sqrt{(-x)^3 + 1} = \sqrt{-x^3 + 1}$

f is neither even nor odd.

$$f(x) = \sqrt{x^3 + 1} = 0$$
$$x^3 + 1 = 0$$
$$x^3 = -1$$

Zero: $x = -1$

67. $340°\left(\dfrac{\pi}{180°}\right) = \dfrac{17\pi}{9} \approx 5.934$

68. $300°\left(\dfrac{\pi}{180°}\right) = \dfrac{5\pi}{3} \approx 5.236$

69. $-480°\left(\dfrac{\pi}{180°}\right) = -\dfrac{8\pi}{3} \approx -8.378$

70. $-900°\left(\dfrac{\pi}{180°}\right) = -5\pi \approx -15.708$

71. $\dfrac{\pi}{6}\left(\dfrac{180°}{\pi}\right) = 30°$

72. $\dfrac{11\pi}{4}\left(\dfrac{180°}{\pi}\right) = 495°$

73. $-\dfrac{2\pi}{3}\left(\dfrac{180°}{\pi}\right) = -120°$

74. $-\dfrac{13\pi}{6}\left(\dfrac{180°}{\pi}\right) = -390°$

75. $\sin(-45°) = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos(-45°) = \cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan(-45°) = -\tan 45° = -1$

76. $\sin 240° = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos 240° = -\cos 60° = -\dfrac{1}{2}$

$\tan 240° = \tan 60° = \sqrt{3}$

77. $\sin \dfrac{13\pi}{6} = \sin \dfrac{\pi}{6} = \dfrac{1}{2}$

$\cos \dfrac{13\pi}{6} = \cos \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

$\tan \dfrac{13\pi}{6} = \tan \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{3}$

78. $\sin\left(-\dfrac{4\pi}{3}\right) = \sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\cos\left(-\dfrac{4\pi}{3}\right) = -\cos \dfrac{\pi}{3} = -\dfrac{1}{2}$

$\tan\left(-\dfrac{4\pi}{3}\right) = -\tan \dfrac{\pi}{3} = -\sqrt{3}$

79. $\sin 405° = \sin 45° = \dfrac{\sqrt{2}}{2}$

$\cos 405° = \cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan 405° = \tan 45° = 1$

80. $\sin 180° = 0$

$\cos 180° = -1$

$\tan 180° = 0$

81. $\tan 33° \approx 0.6494$

82. $\cot 401° = \dfrac{1}{\tan 401°} \approx 1.1504$

83. $\sec \dfrac{12\pi}{5} = \dfrac{1}{\cos(12\pi/5)} \approx 3.2361$

84. $\csc \dfrac{2\pi}{9} = \dfrac{1}{\sin(2\pi/9)} \approx 1.5557$

85. $\sin\left(-\dfrac{\pi}{9}\right) \approx -0.3420$

86. $\cos\left(-\dfrac{3\pi}{7}\right) \approx 0.2225$

87. $2 \cos \theta + 1 = 0$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

88. $2 \cos^2 \theta = 1$

$$\cos^2 \theta = \frac{1}{2}$$

$$\cos \theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

89.

$$2 \sin^2 \theta + 3 \sin \theta + 1 = 0$$

$$(2 \sin \theta - 1)(\sin \theta + 1) = 0$$

$$\sin \theta = -\frac{1}{2} \text{ or } \sin \theta = -1$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \text{ or } \theta = \frac{3\pi}{2}$$

90.

$$\cos^3 \theta = \cos \theta$$

$$\cos \theta(\cos^2 \theta - 1) = 0$$

$$\cos \theta(-\sin^2 \theta) = 0$$

$$\cos \theta = 0 \text{ or } \sin \theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or}$$

$$\theta = 0, \pi, 2\pi$$

91.

$$\sec^2 \theta - \sec \theta - 2 = 0$$

$$(\sec \theta - 2)(\sec \theta + 1) = 0$$

$$\sec \theta = 2 \text{ or } \sec \theta = -1$$

$$\cos \theta = \frac{1}{2} \text{ or } \cos \theta = -1$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \text{ or } \theta = \pi$$

92.

$$2 \sec^2 \theta + \tan^2 \theta - 5 = 0$$

$$2 \sec^2 \theta + (\sec^2 \theta - 1) - 5 = 0$$

$$3 \sec^2 \theta = 6$$

$$\sec^2 \theta = 2$$

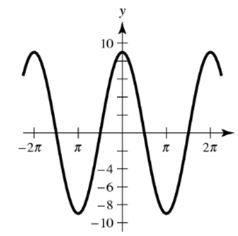
$$\sec \theta = \pm\sqrt{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

93. $y = 9 \cos x$

Period: 2π

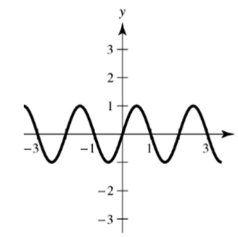
Amplitude: 9



94. $y = \sin \pi x$

Period: 2

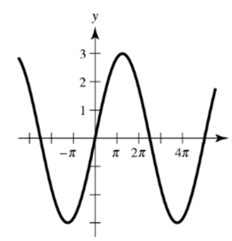
Amplitude: 1



95. $y = 3 \sin \dfrac{2x}{5}$

Period: 5π

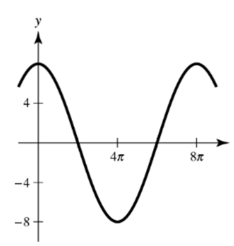
Amplitude: 3



96. $y = 8 \cos \dfrac{x}{4}$

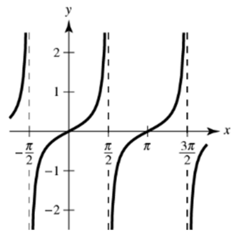
Period: 8π

Amplitude: 8



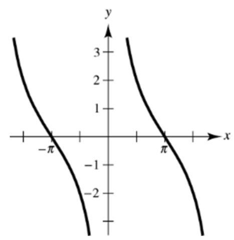
97. $y = \frac{1}{3}\tan x$

Period: π



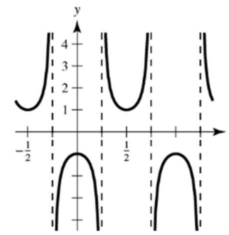
98. $y = \cot\dfrac{x}{2}$

Period: 2π



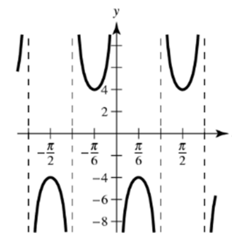
99. $y = -\sec 2\pi x$

Period: 1



100. $y = -4\csc 3x$

Period: $\dfrac{2\pi}{3}$

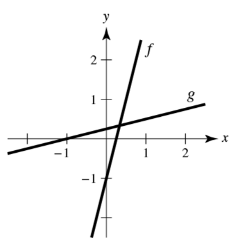


101. (a) $f(x) = 4x - 1,\ g(x) = \dfrac{x+1}{4}$

$$f(g(x)) = f\left(\frac{x+1}{4}\right) = 4\left(\frac{x+1}{4}\right) - 1 = (x+1) - 1 = x$$

$$g(f(x)) = g(4x-1) = \frac{(4x-1)+1}{4} = \frac{4x}{4} = x$$

(b)

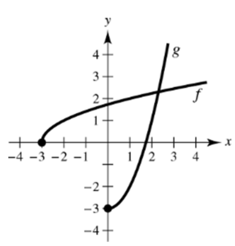


102. (a) $f(x) = \sqrt{x+3},\ g(x) = x^2 - 3, x \ge 0$

$$f(g(x)) = f(x^2 - 3) = \sqrt{(x^2-3)+3} = \sqrt{x^2} = x\ \ (x \ge 0)$$

$$g(f(x)) = g\left(\sqrt{x+3}\right) = \left(\sqrt{x+3}\right)^2 - 3 = (x+3) - 3 = x$$

(b)



103. $f(x) = x^2 + 2x + 1$

Not one-to-one; does not have an inverse.

104. $f(x) = 2 - x^3$

One-to-one; has an inverse.

105. $f(x) = \csc 3\pi x$

Not one-to-one; does not have an inverse.

106. $f(x) = 2x + \cos x$

One-to-one; has an inverse.

107. (a) $f(x) = \frac{1}{2}x - 3$

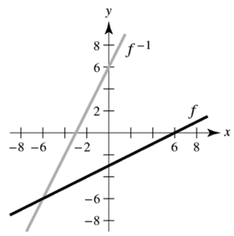
$y = \frac{1}{2}x - 3$

$2(y + 3) = x$

$2(x + 3) = y$

$f^{-1}(x) = 2x + 6$

(b)



(c) The graphs are reflections of each other in the line $y = x$.

(d) Domain f: all real numbers; Range f: all real numbers

Domain f^{-1}: all real numbers; Range f^{-1}: all real numbers

108. (a) $f(x) = 5x - 7$

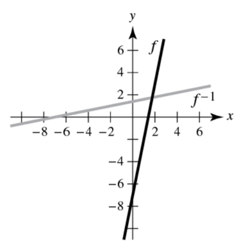
$y = 5x - 7$

$\dfrac{y + 7}{5} = x$

$\dfrac{x + 7}{5} = y$

$f^{-1}(x) = \dfrac{x + 7}{5}$

(b)



(c) The graphs are reflections of each other in the line $y = x$.

(d) Domain f: all real numbers; Range f: all real numbers

Domain f^{-1}: all real numbers; Range f^{-1}: all real numbers

109. (a) $f(x) = \sqrt{x + 1}$

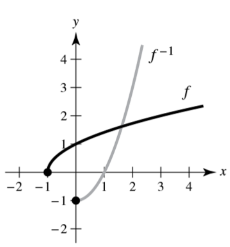
$y = \sqrt{x + 1}$

$y^2 - 1 = x$

$x^2 - 1 = y$

$f^{-1}(x) = x^2 - 1, \quad x \geq 0$

(b)



(c) The graphs are reflections of each other in the line $y = x$.

(d) Domain f: $x \geq -1$; Range f: $y \geq 0$

Domain f^{-1}: $x \geq 0$; Range f^{-1}: $y \geq -1$

110. (a) $f(x) = x^3 + 2$

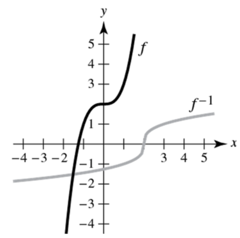
$y = x^3 + 2$

$\sqrt[3]{y - 2} = x$

$\sqrt[3]{x - 2} = y$

$f^{-1}(x) = \sqrt[3]{x - 2}$

(b)



(c) The graphs are reflections of each other in the line $y = x$.

(d) Domain f: all real numbers; Range f: all real numbers

Domain f^{-1}: all real numbers; Range f^{-1}: all real numbers

111. (a) $f(x) = \sqrt[3]{x + 1}$

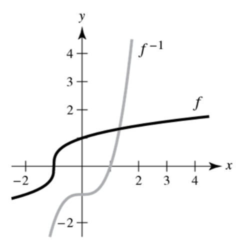
$y = \sqrt[3]{x + 1}$

$y^3 - 1 = x$

$x^3 - 1 = y$

$f^{-1}(x) = x^3 - 1$

(b)



(c) The graphs are reflections of each other in the line $y = x$.

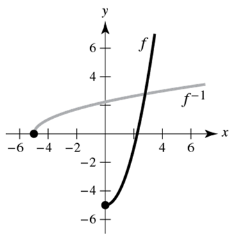
(d) Domain f: all real numbers; Range f: all real numbers

Domain f^{-1}: all real numbers; Range f^{-1}: all real numbers

112. (a)

$$f(x) = x^2 - 5, \quad x \ge 0$$
$$y = x^2 - 5$$
$$\sqrt{y+5} = x$$
$$\sqrt{x+5} = y$$
$$f^{-1}(x) = \sqrt{x+5}$$

(b)



(c) The graphs are reflections of each other in the line $y = x$.

(d) Domain $f: x \ge 0$; Range $f: y \ge -5$

Domain $f^{-1}: x \ge -5$; Range $f^{-1}: y \ge 0$

113. $f(x) = \sin 3\pi x$ is not one-to-one and an inverse does not exist.

114. $f(x) = |x+3|, x \ge -3$ is one-to-one.

$f(x) = x + 3$ because $x \ge -3$.

Therefore, the inverse function is
$y = x + 3 \Rightarrow x = y - 3 \Rightarrow f^{-1}(x) = x - 3, x \ge 0.$

115. $f(x) = x^4 + 2x^2 + 2, \; x \ge 0$

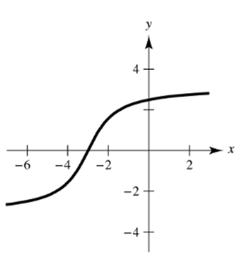
$$= \left(x^2 + 1\right)^2 + 1, \; x \ge 0$$

By the Horizontal Line Test, f is one-to-one on $[0, \infty)$.

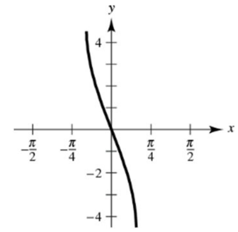
116. $f(x) = \sin \dfrac{x}{2}, \; [-\pi, \pi]$

By the Horizontal Line Test, f is one-to-one on the interval $-\pi \le x \le \pi$.

117. $f(x) = 2\arctan(x+3)$



118. $h(x) = -3\arcsin(2x)$



119. $\arctan 1 = \dfrac{\pi}{4}$ because $\tan \dfrac{\pi}{4} = 1$.

120. $\operatorname{arccsc}\left(-\dfrac{2\sqrt{3}}{3}\right) = -\dfrac{\pi}{3}$ because

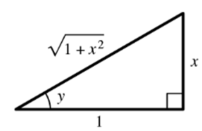
$$\csc\left(-\frac{\pi}{3}\right) = \frac{1}{\sin\left(-\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{-\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}.$$

121. $\arccos(2x + 1) = 2$

$$\cos 2 = 2x + 1$$
$$x = \frac{1}{2}(\cos 2 - 1) \approx -0.708$$

122. $\operatorname{arcsec} 2x = \arctan x$

$$2x = \sec(\arctan x)$$
$$2x = \sqrt{1 + x^2}$$
$$4x^2 = 1 + x^2, x > 0$$
$$3x^2 = 1$$
$$x = \frac{1}{\sqrt{3}}$$

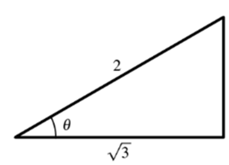


$y = \arctan x$

$\tan y = x$

123. (a) Let $\theta = \arcsin \dfrac{1}{2}$

$$\sin\theta = \frac{1}{2}.$$
$$\sin\left(\arcsin\frac{1}{2}\right) = \sin\theta = \frac{1}{2}$$



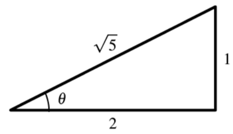
(b) Let $\theta = \arcsin \dfrac{1}{2}$

$$\sin\theta = \frac{1}{2}.$$
$$\cos\left(\arcsin\frac{1}{2}\right) = \cos\theta = \frac{\sqrt{3}}{2}$$

124. (a) Let $\theta = \operatorname{arccot} 2$

$\cot \theta = 2.$

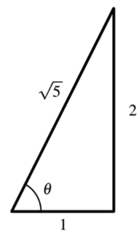
$\tan(\operatorname{arccot} 2) = \tan \theta = \dfrac{1}{2}$



(b) Let $\theta = \operatorname{arcsec} \sqrt{5}$

$\sec \theta = \sqrt{5}.$

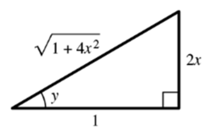
$\cos(\operatorname{arcsec} \sqrt{5}) = \cos \theta = \dfrac{1}{\sqrt{5}}$



125. Let $y = \arctan 2x$

$\tan y = 2x.$

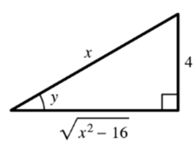
$$\sin(\arctan 2x) = \sin y = \frac{2x}{\sqrt{1 + 4x^2}}$$



126. Let $y = \operatorname{arccsc} \dfrac{x}{4}$

$\csc y = \dfrac{x}{4}.$

$$\cos\left(\operatorname{arccsc} \frac{x}{4}\right) = \cos y = \frac{\sqrt{x^2 - 16}}{x}$$



127. (a) $(3^3)(3^{-1}) = 3^2 = 9$

(b) $(3^2)^4 = 3^8 = 6561$

(c) $\dfrac{3^4}{3^{-2}} = 3^6 = 729$

(d) $\left(\dfrac{1}{3}\right)^2 9^2 = \dfrac{1}{9}(9^2) = 9$

128. (a) $(e^2)^{-2} = e^{-4} = \dfrac{1}{e^4}$

(b) $(e^6)^{1/3} = e^2$

(c) $\dfrac{e^{-1}}{e^{-2}} = \dfrac{e^2}{e^1} = e$

(d) $\left(\dfrac{e}{4}\right)^{-2} = \left(\dfrac{4}{e}\right)^2 = \dfrac{16}{e^2}$

129. $3^{x/2} = 81 = 3^4$

$\dfrac{x}{2} = 4$

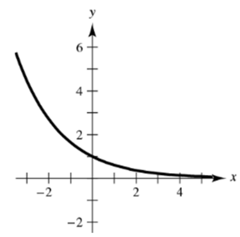
$x = 8$

130. $\left(\dfrac{1}{7}\right)^{x+1} = 49 = 7^2 = \left(\dfrac{1}{7}\right)^{-2}$

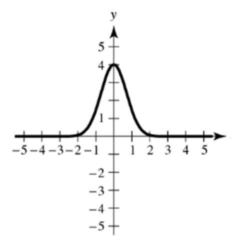
$x + 1 = -2$

$x = -3$

131. $f = e^{-x/2}$



132. $f = 4e^{-x^2}$



133. $f(x) = e^x$ matches (d).

The graph rises from left to right and the domain is all real x.

134. $f(x) = e^{-x}$ matches (a).

The graph falls from left to right and the domain is all real x.

135. $f(x) = \ln(x + 1) + 1$ matches (c).

The graph is a vertical shift upward 1 unit and a horizontal shift to the left 1 unit of $f(x) = \ln x$ and the domain is $x > -1$.

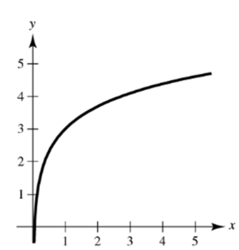
136. $f(x) = -\ln(x + 1) + 1$ matches (b).

The graph is a vertical shift upward 1 unit, a horizontal shift to the left, and a reflection in the x-axis and the domain is $x > -1$.

137. $f(x) = \ln x + 3$

Vertical shift three units upward.

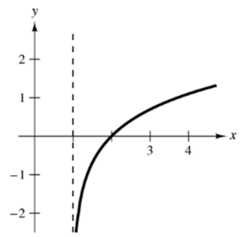
Domain: $x > 0$



138. $f(x) = \ln(x - 1)$

Horizontal shift one unit to the right.

Domain: $x > 1$



139. $\ln \sqrt[5]{\dfrac{4x^2 - 1}{4x^2 + 1}} = \dfrac{1}{5} \ln \dfrac{(2x - 1)(2x + 1)}{4x^2 + 1}$

$= \dfrac{1}{5}\left[\ln(2x - 1) + \ln(2x + 1) - \ln(4x^2 + 1)\right]$

140. $\ln\left[(x^2 + 1)(x - 1)\right] = \ln(x^2 + 1) + \ln(x - 1)$

141. $\ln 3 + \dfrac{1}{3}\ln(4 - x^2) - \ln x = \ln 3 + \ln \sqrt[3]{4 - x^2} - \ln x$

$= \ln\left(\dfrac{3\sqrt[3]{4 - x^2}}{x}\right)$

142. $3\left[\ln x - 2\ln(x^2 + 1)\right] + 2\ln 5 = 3\ln x - 6\ln(x^2 + 1) + \ln 5^2 = \ln x^3 - \ln(x^2 + 1)^6 + \ln 25 = \ln\left[\dfrac{25x^3}{(x^2 + 1)^6}\right]$

143.

$$\begin{aligned} -4 + 3e^{-2x} &= 6 \\ 3e^{-2x} &= 10 \\ e^{-2x} &= \tfrac{10}{3} \\ \ln e^{-2x} &= \ln \tfrac{10}{3} \\ -2x &= \ln \tfrac{10}{3} \\ x &= -\tfrac{1}{2}\ln \tfrac{10}{3} \approx -0.602 \end{aligned}$$

144.

$$\begin{aligned} \ln x + \ln(x - 3) &= 0 \\ \ln x(x - 3) &= 0 \\ x(x - 3) &= e^0 \\ x^2 - 3x - 1 &= 0 \\ x &= \frac{3 \pm \sqrt{13}}{2} \\ x &= \frac{3 + \sqrt{13}}{2} \end{aligned}$$

only because $\dfrac{3 - \sqrt{13}}{2} < 0$.

145. $f(x) = \sqrt{x}$, $(4, 2)$

(a) $m_1 = \dfrac{3 - 2}{9 - 4} = \dfrac{1}{5}$

The slope of the tangent line at $(4, 2)$ is greater than m_1.

(b) $m_2 = \dfrac{1 - 2}{1 - 4} = \dfrac{1}{3}$

The slope of the tangent line at $(4, 2)$ is less than m_2.

(c) $m_3 = \dfrac{2.1 - 2}{4.41 - 4} = \dfrac{0.1}{0.41} = \dfrac{10}{41} \approx 0.2439$

The slope of the tangent line at $(4, 2)$ is greater than m_3.

(d) Slope $= \dfrac{f(4 + h) - 2}{(4 + h) - 4} = \dfrac{\sqrt{4 + h} - 2}{h}$

(e) $\dfrac{\sqrt{4 + h} - 2}{h} = \dfrac{\sqrt{4 + h} - 2}{h} \cdot \dfrac{\sqrt{4 + h} + 2}{\sqrt{4 + h} + 2}$

$= \dfrac{(4 + h) - 4}{h\left[\sqrt{4 + h} + 2\right]}$

$= \dfrac{1}{\sqrt{4 + h} + 2}$

As h approaches 0, the slope approaches $\dfrac{1}{\sqrt{4} + 2} = \dfrac{1}{4} = 0.25$.

Problem Solving for Chapter 1

1. (a)
$$x^2 - 6x + y^2 - 8y = 0$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: (3, 4); Radius: 5

(b) Slope of line from (0, 0) to (3, 4) is $\frac{4}{3}$.

Slope of tangent line is $-\frac{3}{4}$. So,

$y - 0 = -\frac{3}{4}(x - 0) \Rightarrow y = -\frac{3}{4}x,$ Tangent line

(c) Slope of line from (6, 0) to (3, 4) is $\frac{4 - 0}{3 - 6} = -\frac{4}{3}$.

Slope of tangent line is $\frac{3}{4}$. So,

$y - 0 = \frac{3}{4}(x - 6) \Rightarrow y = \frac{3}{4}x - \frac{9}{2},$ Tangent line

(d)
$$-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$$
$$\frac{3}{2}x = \frac{9}{2}$$
$$x = 3$$

Intersection: $\left(3, -\frac{9}{4}\right)$

2. Let $y = mx + 1$ be a tangent line to the circle from the point (0, 1). Because the center of the circle is at $(0, -1)$ and the radius is 1 you have the following.

$$x^2 + (y + 1)^2 = 1$$
$$x^2 + (mx + 1 + 1)^2 = 1$$
$$(m^2 + 1)x^2 + 4mx + 3 = 0$$

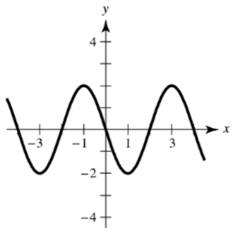
Setting the discriminant $b^2 - 4ac$ equal to zero,

$$16m^2 - 4(m^2 + 1)(3) = 0$$
$$16m^2 - 12m^2 = 12$$
$$4m^2 = 12$$
$$m = \pm\sqrt{3}$$

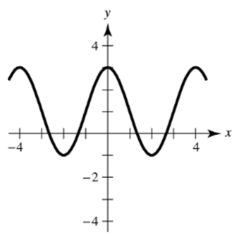
Tangent lines: $y = \sqrt{3}x + 1$ and $y = -\sqrt{3}x + 1$.

3. The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. So, the total time is $T = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4}$ hours.

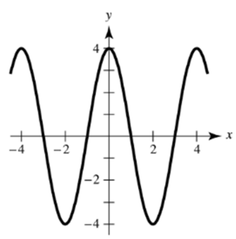
4. (a) $f(x+1)$



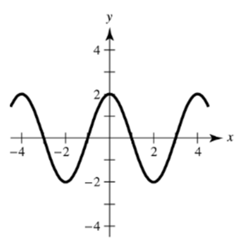
(b) $f(x)+1$



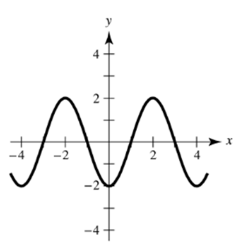
(c) $2f(x)$



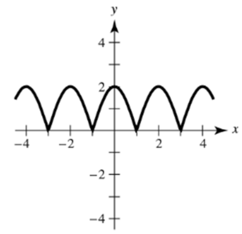
(d) $f(-x)$



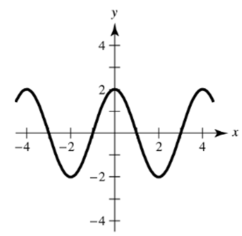
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$

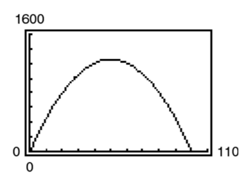


5. (a) $x + 2y = 100 \Rightarrow y = \dfrac{100 - x}{2}$

$$A(x) = xy = x\left(\frac{100 - x}{2}\right) = -\frac{x^2}{2} + 50x$$

Domain: $0 < x < 100$ or $(0, 100)$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m.

(c) $A(x) = -\frac{1}{2}(x^2 - 100x)$

$= -\frac{1}{2}(x^2 - 100x + 2500) + 1250$

$= -\frac{1}{2}(x - 50)^2 + 1250$

$A(50) = 1250\ \text{m}^2$ is the maximum.

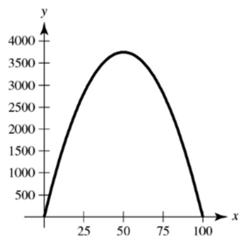
$x = 50$ m, $y = 25$ m

6. (a) $4y + 3x = 300 \Rightarrow y = \dfrac{300 - 3x}{4}$

$$A(x) = x(2y) = x\left(\frac{300 - 3x}{2}\right) = \frac{-3x^2 + 300x}{2}$$

Domain: $0 < x < 100$

(b)



Maximum of 3750 ft^2 at $x = 50$ ft, $y = 37.5$ ft.

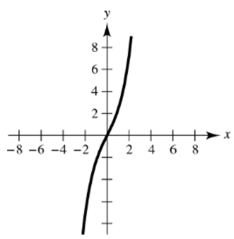
(c) $A(x) = -\frac{3}{2}(x^2 - 100x)$

$$= -\tfrac{3}{2}(x^2 - 100x + 2500) + 3750$$

$$= -\tfrac{3}{2}(x - 50)^2 + 3750$$

$A(50) = 3750$ square feet is the maximum area, where $x = 50$ ft and $y = 37.5$ ft.

7. $f(x) = e^x - e^{-x}$



$$y = e^x - e^{-x}$$

$$ye^x = e^{2x} - 1$$

$$(e^x)^2 - ye^x - 1 = 0 \qquad \text{(Quadratic in } e^x\text{)}$$

$$e^x = \frac{y \pm \sqrt{y^2 + 4}}{2}$$

$$e^x = \frac{y + \sqrt{y^2 + 4}}{2} \qquad \text{(Use positive solution.)}$$

$$e^y = \frac{x + \sqrt{x^2 + 4}}{2}$$

$$f^{-1}(x) = y = \ln\left[\frac{x + \sqrt{x^2 + 4}}{2}\right] \qquad \text{Inverse}$$

8. $f(x) = y = \dfrac{1}{1-x}$

(a) Domain: all $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

(b) $f(f(x)) = f\left(\dfrac{1}{1-x}\right) = \dfrac{1}{1-\left(\dfrac{1}{1-x}\right)} = \dfrac{1}{\dfrac{1-x-1}{1-x}}$

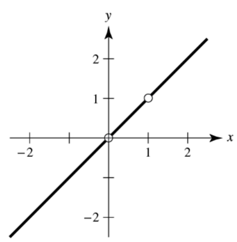
$$= \frac{1-x}{-x} = \frac{x-1}{x}$$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(c) $f(f(f(x))) = f\left(\dfrac{x-1}{x}\right) = \dfrac{1}{1-\left(\dfrac{x-1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(d) The graph is not a line. It has holes at (0, 0) and (1, 1).



9.

$$d_1d_2 = 1$$

$$\left[(x+1)^2 + y^2\right]\left[(x-1)^2 + y^2\right] = 1$$

$$(x+1)^2(x-1)^2 + y^2\left[(x+1)^2 + (x-1)^2\right] + y^4 = 1$$

$$(x^2-1)^2 + y^2\left[2x^2 + 2\right] + y^4 = 1$$

$$x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 = 1$$

$$(x^4 + 2x^2y^2 + y^4) - 2x^2 + 2y^2 = 0$$

$$(x^2 + y^2)^2 = 2(x^2 - y^2)$$

Let $y = 0$. Then $x^4 = 2x^2 \Rightarrow x = 0$ or $x^2 = 2$.

So, $(0, 0)$, $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$ are on the curve.