Solution and Answer Guide

GUSTAFSON/HUGHES, COLLEGE ALGEBRA 2023, 9780357723654; CHAPTER 1: EQUATIONS AND INEQUALITIES

TABLE OF CONTENTS

END OF SECTION EXERCISE SOLUTIONS

EXERCISES 1.1

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Remove parentheses and simplify.

   $-4(2x+1)-2(x-5)+3x$

   Solution
   $-4(2x+1)-2(x-5)+3x = -8x-4-2x+10+3x = -7x+6$

2. Factor completely and simplify. $5x^2-17x+6$

Solution

$5x^2 - 17x + 6 = (5x - 2)(x - 3)$

3. Substitute −6 for x in the equation $3x - 2(x + 1) = 2x + 4$. Is the equation that results true or false?

Solution

$3(-6) - 2(-6 + 1) = 2(-6) + 4$

$-18 + 12 - 2 = -12 + 4$

$-8 = -8$, which is a true statement.

4. Identify the LCD of $\frac{2x}{5}$, $\frac{x - 1}{2}$, and $\frac{3}{4}$.

Solution

The LCD of 5, 2, and 4 is 20.

5. Identify the LCD of $\frac{2}{x - 8}$ and $\frac{3x - 1}{x^2 - 9x + 8}$.

Solution

$x^2 - 9x + 8 = (x - 8)(x - 1)$

The LCD between $x - 8$ and $(x - 8)(x - 1)$ is $(x - 8)(x - 1)$.

6. Multiply and simplify.

$$(x - 3)(x + 3)\left(\frac{4}{x - 3} - \frac{2}{x + 2}\right)$$

Solution

$$\left[(x - 3)(x + 2)\left(\frac{4}{x - 3}\right)\right] - \left[(x - 3)(x + 2)\left(\frac{2}{x + 2}\right)\right]$$

$$\left[\cancel{(x - 3)}(x + 2)\left(\frac{4}{\cancel{x - 3}}\right)\right] - \left[(x - 3)\cancel{(x + 2)}\left(\frac{2}{\cancel{x + 2}}\right)\right]$$

$$= \left[(x + 2)(4)\right] - \left[(x - 3)(2)\right]$$

$$= 4x + 8 - (2x - 6) = 4x + 8 - 2x + 6$$

$$= 2x + 14$$

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. If a number satisfies an equation, it is called a __________ or a __________ of the equation.

Solution
root, solution

8. If an equation is true for all values of its variable, it is called an __________.

Solution
identity

9. A contradiction is an equation that is true for __________ values of its variable.

Solution
no

10. A __________ equation is true for some values of its variable and is not true for others.

Solution
conditional

11. An equation of the form $ax + b = 0$ is called a __________ equation.

Solution
linear

12. If an equation contains rational expressions, it is called a __________ equation.

Solution
rational

13. A conditional linear equation has __________ root.

Solution
one

14. The __________ of a fraction can never be 0.

Solution
denominator

Practice

Find any restrictions on the values of *x* in each equation.

15. $2x + 8 = -17$

Solution
$2x + 5 = -17$
no restrictions

16. $\frac{1}{2}x - 7 = 12$

Solution
$$\frac{1}{2}x - 7 = 14$$

no restrictions

17. $\frac{1}{x} = 10$

Solution

$$\frac{1}{x} = 12$$
$$x \neq 0$$

18. $\frac{4}{x - 2} = 9x$

Solution

$$\frac{3}{x - 2} = 9x$$
$$x \neq 2$$

19. $\frac{8}{x - 6} = \frac{1}{x + 2}$

Solution

$$\frac{8}{x - 6} = \frac{5}{x + 2}$$
$$x - 6 \neq 0 \quad x + 2 \neq 0$$
$$x \neq 6 \quad x \neq -2$$
$$x \neq 6, x \neq -2$$

20. $\frac{x}{x - 3} = -\frac{3}{x + 4}$

Solution

$$\frac{x}{x - 3} = -\frac{4}{x + 4}$$
$$x - 3 \neq 0 \quad x + 4 \neq 0$$
$$x \neq 3 \quad x \neq -4$$
$$x \neq 3, x \neq -4$$

21. $\frac{x}{x - 3} = \frac{5x}{x^2 - 16}$

Solution

$$\frac{1}{x - 3} = \frac{5x}{x^2 - 16}$$
$$\frac{1}{x - 3} = \frac{5x}{(x + 4)(x - 4)}$$

$$x - 3 \neq 0 \quad x + 4 \neq 0 \quad x - 4 \neq 0$$
$$x \neq 3 \quad x \neq -4 \quad x \neq 4$$
$$x \neq 3, x \neq -4, x \neq 4$$

22. $\dfrac{x + 5}{x^2 - 3x - 4} = \dfrac{5}{x} + 2$

Solution

$$\frac{1}{x^2 - 3x - 4} = \frac{5}{x} + 2$$
$$\frac{1}{(x + 1)(x - 4)} = \frac{5}{x} + 2$$
$$x + 1 \neq 0 \quad x - 4 \neq 0 \quad x \neq 0$$
$$x \neq -1 \quad x \neq 4$$
$$x \neq -1, x \neq 4, x \neq 0$$

Solve each equation, if possible. Classify each one as an identity, a conditional equation, or a contradiction.

23. $2x + 5 = 15$

Solution

$$2x + 5 = 15$$
$$2x + 5 - 5 = 15 - 5$$
$$2x = 10$$
$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

conditional equation

24. $3x + 2 = x + 8$

Solution

$$3x + 2 = x + 8$$
$$3x - x + 2 = x - x + 8$$
$$2x + 2 = 8$$
$$2x + 2 - 2 = 8 - 2$$
$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

conditional equation

25. $2(n + 2) - 5 = 2n$

Solution

$$
\begin{aligned}
2(n + 2) - 5 &= 2n \\
2n + 4 - 5 &= 2n \\
2n - 1 &= 2n \\
2n - 2n - 1 &= 2n - 2n \\
-1 &\neq 0
\end{aligned}
$$

no solution contradiction

26. $3(m + 2) = 2(m + 3) + m$

Solution

$$
\begin{aligned}
3(m + 2) &= 2(m + 3) + m \\
3m + 6 &= 2m + 6 + m \\
3m + 6 &= 3m + 6
\end{aligned}
$$

all real numbers identity

27. $\dfrac{x + 7}{2} = 7$

Solution

$$
\begin{aligned}
\frac{x + 7}{2} &= 7 \\
2 \cdot \frac{x + 7}{2} &= 2(7) \\
x + 7 &= 14 \\
x + 7 - 7 &= 14 - 7 \\
x &= 7
\end{aligned}
$$

conditional equation

28. $\dfrac{x}{2} - 7 = 14$

Solution

$$
\begin{aligned}
\frac{x}{2} - 7 &= 14 \\
\frac{x}{2} - 7 + 7 &= 14 + 7 \\
\frac{x}{2} &= 21 \\
2 \cdot \frac{x}{2} &= 2(21) \\
x &= 42
\end{aligned}
$$

conditional equation

29. $2(a + 1) = 3(a - 2) - a$

Solution

$$\begin{aligned} 2(a+1) &= 3(a-2) - a \\ 2a + 2 &= 3a - 6 - a \\ 2a + 2 &= 2a - 6 \\ 2a - 2a + 2 &= 2a - 2a - 6 \\ 2 &\neq -6 \end{aligned}$$

no solution; contradiction

30. $x^2 = (x + 4)(x - 4) + 16$

Solution

$$\begin{aligned} x^2 &= (x+4)(x-4) + 16 \\ x^2 &= x^2 - 16 + 16 \\ x^2 &= x^2 \end{aligned}$$

all real numbers; identity

31. $3(x - 3) = \dfrac{6x - 18}{2}$

Solution

$$\begin{aligned} 3(x-3) &= \frac{6x-18}{2} \\ 3x - 9 &= \frac{6x-18}{2} \\ 2(3x-9) &= 2 \cdot \frac{6x-18}{2} \\ 6x - 18 &= 6x - 18 \end{aligned}$$

all real numbers; identity

32. $x(x + 2) = (x + 1)^2$

Solution

$$\begin{aligned} x(x+2) &= (x+1)^2 \\ x^2 + 2x &= (x+1)(x+1) \\ x^2 + 2x &= x^2 + 2x + 1 \\ x^2 - x^2 + 2x &= x^2 - x^2 + 2x + 1 \\ 2x &= 2x + 1 \\ 2x - 2x &= 2x - 2x + 1 \\ 0 &\neq 1 \end{aligned}$$

no solution; contradiction

33. $\dfrac{3}{b-3} = 1$

Solution

$$\begin{aligned}
\frac{3}{b-3} &= 1 \\
(b-3)\cdot\frac{3}{b-3} &= (b-3)(1) \\
3 &= b-3 \\
3+3 &= b-3+3 \\
6 &= b
\end{aligned}$$

conditional equation

34. $x^2 - 8x + 15 = (x-3)(x+5)$

Solution

$$\begin{aligned}
x^2 - 8x + 15 &= (x-3)(x+5) \\
x^2 - 8x + 15 &= x^2 + 2x - 15 \\
x^2 - x^2 - 8x + 15 &= x^2 - x^2 + 2x - 15 \\
-8x + 15 &= 2x - 15 \\
-8x + 8x + 15 &= 2x + 8x - 15 \\
15 &= 10x - 15 \\
15 + 15 &= 10x - 15 + 15 \\
30 &= 10x \\
\frac{30}{10} &= \frac{10x}{10} \\
3 &= x
\end{aligned}$$

conditional equation

35. $2x^2 + 5x - 3 = (2x-1)(x+3)$

Solution

$$\begin{aligned}
2x^2 + 5x - 3 &= (2x-1)(x+3) \\
2x^2 + 5x - 3 &= 2x^2 + 5x - 3
\end{aligned}$$

all real numbers; identity

36. $2x^2 + 5x - 3 = 2x\left(x + \dfrac{19}{2}\right)$

Solution

$$2x^2 + 5x - 3 = 2x\left(x + \frac{19}{2}\right)$$
$$2x^2 + 5x - 3 = 2x^2 + 19x$$
$$2x^2 - 2x^2 + 5x - 3 = 2x^2 - 2x^2 + 19x$$
$$5x - 3 = 19x$$
$$5x - 5x - 3 = 19x - 5x$$
$$-3 = 14x$$
$$\frac{-3}{14} = \frac{14x}{14}$$
$$-\frac{3}{14} = x$$

conditional equation

Solve each linear equation.

37. $4x - 11 = 9$

Solution

$$4x - 11 = 9$$
$$4x = 20$$
$$x = 5$$

38. $8y + 16 = -8$

Solution

$$8y + 16 = -8$$
$$8y = -24$$
$$y = -3$$

39. $16 = 3z - 2$

Solution

$$16 = 3z - 2$$
$$18 = 3z$$
$$6 = z$$

40. $-20 = 5x + 10$

Solution

$$-20 = 5x + 10$$
$$-30 = 5x$$
$$-6 = x$$

41. $2x + 7 = 10 - x$

Solution

$$\begin{aligned} 2x + 7 &= 10 - x \\ 3x + 7 &= 10 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

42. $9a - 3 = 15 + 3a$

Solution

$$\begin{aligned} 9a - 3 &= 15 + 3a \\ 6a - 3 &= 15 \\ 6a &= 18 \\ a &= 3 \end{aligned}$$

43. $4y - 1 = -2y + 19$

Solution

$$\begin{aligned} 4y - 1 &= -2y + 19 \\ 6y - 1 &= 19 \\ 6y &= 20 \\ y &= \frac{10}{3} \end{aligned}$$

44. $-2 - 7x = 1 + 2x$

Solution

$$\begin{aligned} -2 - 7x &= 1 + 2x \\ -2 - 9x &= 1 \\ -9x &= 3 \\ x &= -\frac{1}{3} \end{aligned}$$

45. $5(2y - 9) = 3y - 4$

Solution

$$\begin{aligned} 5(2y - 9) &= 3y - 4 \\ 10y - 45 &= 3y - 4 \\ 7y - 45 &= -4 \\ 7y &= 41 \\ y &= \frac{41}{7} \end{aligned}$$

46. $13x + 1 = -(5 - x)$

Solution

$$\begin{aligned} 13x + 1 &= -(5 - x) \\ 13x + 1 &= -5 + x \\ 12x + 1 &= -5 \\ 12x &= -6 \\ x &= -\frac{1}{2} \end{aligned}$$

47. $5(x - 2) = 2(x + 4)$

Solution

$$\begin{aligned} 5(x - 2) &= 2(x + 4) \\ 5x - 10 &= 2x + 8 \\ 3x - 10 &= 8 \\ x &= 6 \end{aligned}$$

48. $5(r - 4) = -5(r - 4)$

Solution

$$\begin{aligned} 5(r - 4) &= -5(r - 4) \\ 5r - 20 &= -5r + 20 \\ 10r - 20 &= 20 \\ 10r &= 40 \\ r &= 4 \end{aligned}$$

49. $7(2x + 5) - 6(x + 8) = 7$

Solution

$$\begin{aligned} 7(2x + 5) - 6(x + 8) &= 7 \\ 14x + 35 - 6x - 48 &= 7 \\ 8x - 13 &= 7 \\ 8x &= 20 \\ x &= \frac{20}{8} = \frac{5}{2} \end{aligned}$$

50. $6(x - 5) - 4(x + 2) = -1$

Solution

$$\begin{aligned} 6(x - 5) - 4(x + 2) &= -1 \\ 6x - 30 - 4x - 8 &= -1 \\ 2x - 38 &= -1 \\ 2x &= 37 \\ x &= \frac{37}{2} \end{aligned}$$

51. $3(x - 2) - (x + 5) = 7 + 4(x - 3)$

Solution

$$\begin{aligned} 3(x - 2) - (x + 5) &= 7 + 4(x - 3) \\ 3x - 6 - x - 5 &= 7 + 4x - 12 \\ 2x - 11 &= 4x - 5 \\ -2x - 11 &= -5 \\ -2x &= 6 \\ x &= -3 \end{aligned}$$

52. $8 - 2(y + 1) = 3(y + 6) - (y - 2)$

Solution

$$\begin{aligned} 8 - 2(y + 1) &= 3(y + 6) - (y - 2) \\ 8 - 2y - 2 &= 3y + 18 - y + 2 \\ 6 - 2y &= 2y + 20 \\ 6 - 4y &= 20 \\ -4y &= 14 \\ y &= -\frac{7}{2} \end{aligned}$$

53. $(t + 1)(t - 1) = (t + 2)(t - 3) + 4$

Solution

$$\begin{aligned} (t + 1)(t - 1) &= (t + 2)(t - 3) + 4 \\ t^2 - 1 &= t^2 - t - 6 + 4 \\ -1 &= -t - 2 \\ t - 1 &= -2 \\ t &= -1 \end{aligned}$$

54. $(x - 2)(x - 3) = (x + 3)(x + 4)$

Solution

$$\begin{aligned} (x - 2)(x - 3) &= (x + 3)(x + 4) \\ x^2 - 5x + 6 &= x^2 + 7x + 12 \\ -5x + 6 &= 7x + 12 \\ -12x + 6 &= 12 \\ -12x &= 6 \\ x &= -\frac{6}{12} = -\frac{1}{2} \end{aligned}$$

Solve the linear equations containing fractions.

55. $\frac{5}{3}z - 8 = 7$

Solution

$$\frac{5}{3}z - 8 = 7$$
$$\frac{5}{3}z = 15$$
$$3 \cdot \frac{5}{3}z = 3(15)$$
$$5z = 45$$
$$z = 9$$

56. $\frac{4}{3}y + 12 = -4$

Solution

$$\frac{4}{3}y + 12 = -4$$
$$\frac{4}{3}y = -16$$
$$3 \cdot \frac{4}{3}y = 3(-16)$$
$$4y = -48$$
$$y = -12$$

57. $\frac{z}{5} + 2 = 4$

Solution

$$\frac{z}{5} + 2 = 4$$
$$\frac{z}{5} = 2$$
$$5 \cdot \frac{z}{5} = 5(2)$$
$$z = 10$$

58. $\frac{3p}{7} - p = -4$

Solution

$$\frac{3p}{7} - p = -4$$
$$7\left(\frac{3p}{7} - p\right) = 7(-4)$$
$$3p - 7p = -28$$
$$-4p = -28$$
$$p = 7$$

59. $\frac{7}{2}x + 5 = x + \frac{15}{2}$

Solution

$$\begin{aligned} \frac{7}{2}x + 5 &= x + \frac{15}{2} \\ 2\left(\frac{7}{2}x + 5\right) &= 2\left(x + \frac{15}{2}\right) \\ 7x + 10 &= 2x + 15 \\ 5x + 10 &= 15 \\ 5x &= 5 \\ x &= 1 \end{aligned}$$

60. $\frac{x}{2} - \frac{1}{5} = \frac{1}{2} + \frac{x}{3}$

Solution

$$\begin{aligned} \frac{x}{2} - \frac{1}{5} &= \frac{1}{2} + \frac{x}{3} \\ 30\left(\frac{x}{2} - \frac{1}{5}\right) &= 30\left(\frac{1}{2} + \frac{x}{3}\right) \quad \text{(multiply by common denominator)} \\ 15x - 6 &= 15 + 10x \\ 5x - 6 &= 15 \\ 5x &= 21 \\ x &= \frac{21}{5} \end{aligned}$$

61. $\frac{3x - 2}{3} = 2x + \frac{7}{3}$

Solution

$$\begin{aligned} \frac{3x - 2}{3} &= 2x + \frac{7}{3} \\ 3 \cdot \frac{3x - 2}{3} &= 3\left(2x + \frac{7}{3}\right) \\ 3x - 2 &= 6x + 7 \\ -3x - 2 &= 7 \\ -3x &= 9 \\ x &= -3 \end{aligned}$$

62. $2x - \frac{7}{6} + \frac{x}{6} = \frac{4x + 3}{6}$

Solution

$$\begin{aligned} 2x - \frac{7}{6} + \frac{x}{6} &= \frac{4x + 3}{6} \\ 6\left(2x - \frac{7}{6} + \frac{x}{6}\right) &= 6 \cdot \frac{4x + 3}{6} \\ 12x - 7 + x &= 4x + 3 \\ 13x - 7 &= 4x + 3 \\ 9x - 7 &= 3 \\ 9x &= 10 \\ x &= \frac{10}{9} \end{aligned}$$

63. $\frac{3x + 1}{20} = \frac{1}{2}$

Solution

$$\begin{aligned} \frac{3x + 1}{20} &= \frac{1}{2} \\ 20 \cdot \frac{3x + 1}{20} &= 20 \cdot \frac{1}{2} \\ 3x + 1 &= 10 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

64. $2(2x + 1) - \frac{3x}{2} = \frac{-3(4 + x)}{2}$

Solution

$$\begin{aligned} 2(2x + 1) - \frac{3x}{2} &= \frac{-3(4 + x)}{2} \\ 2\left[2(2x + 1) - \frac{3x}{2}\right] &= 2 \cdot \frac{-3(4 + x)}{2} \\ 4(2x + 1) - 3x &= -3(4 + x) \\ 8x + 4 - 3x &= -12 - 3x \\ 5x + 4 &= -12 - 3x \\ 8x + 4 &= -12 \\ 8x &= -16 \\ x &= -2 \end{aligned}$$

65. $\frac{3 + x}{3} + \frac{x + 7}{2} = 4x + 1$

Solution

$$\frac{3+x}{3}+\frac{x+7}{2}=4x+1$$
$$6\left(\frac{3+x}{3}+\frac{x+7}{2}\right)=6(4x+1)$$
$$2(3+x)+3(x+7)=24x+6$$
$$6+2x+3x+21=24x+6$$
$$5x+27=24x+6$$
$$-19x+27=6$$
$$-19x=-21$$
$$x=\frac{21}{19}$$

66. $\frac{3}{2}(3x-2)-10x-4=0$

Solution

$$\frac{3}{2}(3x-2)-10x-4=0$$
$$2\left[\frac{3}{2}(3x-2)-10x-4\right]=2(0)$$
$$3(3x-2)-20x-8=0$$
$$9x-6-20x-8=0$$
$$-11x-14=0$$
$$-11x=14$$
$$x=-\frac{14}{11}$$

67. $\frac{a(a-3)+5}{7}=\frac{(a-1)^2}{7}$

Solution

$$\frac{a(a-3)+5}{7}=\frac{(a-1)^2}{7}$$
$$7\left[\frac{a(a-3)+5}{7}\right]=7\left[\frac{(a-1)^2}{7}\right]$$
$$a(a-3)+5=(a-1)(a-1)$$
$$a^2-3a+5=a^2-2a+1$$
$$-3a+5=-2a+1$$
$$5=a+1$$
$$4=a$$

68. $\dfrac{(y+2)^2}{3} = y + 2 + \dfrac{y^2}{3}$

Solution

$$\frac{(y+2)^2}{3} = y + 2 + \frac{y^2}{3}$$
$$3\left[\frac{(y+2)^2}{3}\right] = 3\left(y + 2 + \frac{y^2}{3}\right)$$
$$(y+2)^2 = 3y + 6 + y^2$$
$$y^2 + 4y + 4 = y^2 + 3y + 6$$
$$4y + 4 = 3y + 6$$
$$y + 4 = 6$$
$$y = 2$$

Solve each rational equation. Check for false or extraneous solutions.

69. $\dfrac{4}{x} - \dfrac{2}{5} = \dfrac{6}{x}$

Solution

$$\frac{4}{x} - \frac{2}{5} = \frac{6}{x}$$
$$5x\left(\frac{4}{x} - \frac{2}{5}\right) = 5x \cdot \frac{6}{x}$$
$$20 - 2x = 30$$
$$-2x = 10$$
$$x = -5$$

70. $\dfrac{3}{x} + \dfrac{1}{2} = \dfrac{4}{x}$

Solution

$$\frac{3}{x} + \frac{1}{2} = \frac{4}{x}$$
$$2x\left(\frac{3}{x} + \frac{1}{2}\right) = 2x \cdot \frac{4}{x}$$
$$6 + x = 8$$
$$x = 2$$

71. $\dfrac{1}{4x} + \dfrac{1}{3} = -\dfrac{2}{3x} + \dfrac{1}{2}$

Solution

$$\begin{aligned}
\frac{1}{4x} + \frac{1}{3} &= -\frac{2}{3x} + \frac{1}{2} \\
12x\left(\frac{1}{4x} + \frac{1}{3}\right) &= 12x\left(-\frac{2}{3x} + \frac{1}{2}\right) \quad \text{(multiply by common denominator)} \\
3 + 4x &= -8 + 6x \\
3 - 2x &= -8 \\
-2x &= -11 \\
x &= \frac{11}{2}
\end{aligned}$$

72. $\dfrac{2}{5x} + \dfrac{1}{3} = \dfrac{2}{x} - \dfrac{3}{5}$

Solution

$$\begin{aligned}
\frac{2}{5x} + \frac{1}{3} &= \frac{2}{x} - \frac{3}{5} \\
15x\left(\frac{2}{5x} + \frac{1}{3}\right) &= 15x\left(\frac{2}{x} - \frac{3}{5}\right) \quad \text{(multiply by common denominator)} \\
6 + 5x &= 30 - 9x \\
6 + 14x &= 30 \\
14x &= 24 \\
x &= \frac{12}{7}
\end{aligned}$$

73. $\dfrac{2}{x+1} + \dfrac{1}{3} = \dfrac{1}{x+1}$

Solution

$$\begin{aligned}
\frac{2}{x+1} + \frac{1}{3} &= \frac{1}{x+1} \\
3(x+1)\left(\frac{2}{x+1} + \frac{1}{3}\right) &= 3(x+1) \cdot \frac{1}{x+1} \\
6 + 1(x+1) &= 3(1) \\
6 + x + 1 &= 3 \\
x + 7 &= 3 \\
x &= -4
\end{aligned}$$

74. $\dfrac{3}{x-2} + \dfrac{1}{x} = \dfrac{3}{x-2}$

Solution

$$\frac{3}{x-2}+\frac{1}{x}=\frac{3}{x-2}$$
$$x(x-2)\left(\frac{3}{x-2}+\frac{1}{x}\right)=x(x-2)\cdot\frac{3}{x-2}$$
$$3x+1(x-2)=3x$$
$$3x+x-2=3x$$
$$4x-2=3x$$
$$x=2$$

The answer does not check. ⇒ no solution

75. $\dfrac{9t+6}{t(t+3)}=\dfrac{7}{t+3}$

Solution

$$\frac{9t+6}{t(t+3)}=\frac{7}{t+3}$$
$$t(t+3)\left[\frac{9t+6}{t(t+3)}\right]=t(t+3)\cdot\frac{7}{t+3}$$
$$9t+6=7t$$
$$2t+6=0$$
$$2t=-6$$
$$t=-3$$

The answer does not check. ⇒ no solution

76. $x+\dfrac{2(-2x+1)}{3x+5}=\dfrac{3x^2}{3x+5}$

Solution

$$x+\frac{2(-2x+1)}{3x+5}=\frac{3x^2}{3x+5}$$
$$(3x+5)\left[x+\frac{2(-2x+1)}{3x+5}\right]=(3x+5)\cdot\frac{3x^2}{3x+5}$$
$$x(3x+5)+2(-2x+1)=3x^2$$
$$3x^2+5x-4x+2=3x^2$$
$$x+2=0$$
$$x=-2$$

77. $\dfrac{2}{(a-7)(a+2)}=\dfrac{4}{(a+3)(a+2)}$

Solution

$$\begin{aligned}
\frac{2}{(a-7)(a+2)} &= \frac{4}{(a+3)(a+2)} \\
(a-7)(a+2)(a+3)\cdot\frac{2}{(a-7)(a+2)} &= (a-7)(a+2)(a+3)\cdot\frac{4}{(a+3)(a+2)} \\
2(a+3) &= 4(a-7) \\
2a+6 &= 4a-28 \\
-2a &= -34 \\
a &= 17
\end{aligned}$$

78. $\frac{2}{n-2}+\frac{1}{n+1}=\frac{1}{n^2-n-2}$

Solution

$$\begin{aligned}
\frac{2}{n-2}+\frac{1}{n+1} &= \frac{1}{n^2-n-2} \\
\frac{2}{n-2}+\frac{1}{n+1} &= \frac{1}{(n-2)(n+1)} \\
(n-2)(n+1)\left(\frac{2}{n-2}+\frac{1}{n+1}\right) &= (n-2)(n+1)\cdot\frac{1}{(n-2)(n+1)} \\
2(n+1)+1(n-2) &= 1 \\
2n+2+n-2 &= 1 \\
3n &= 1 \\
n &= \frac{1}{3}
\end{aligned}$$

79. $\frac{3}{x^2-16}=\frac{2}{x-4}+\frac{3}{x+4}$

Solution

$$\begin{aligned}
\frac{3}{x^2-16} &= \frac{2}{x-4}+\frac{3}{x+4} \\
\frac{3}{(x+4)(x-4)} &= \frac{2}{x-4}+\frac{3}{x+4} \\
(x+4)(x-4)\left[\frac{3}{(x+4)(x-4)}\right] &= (x-4)(x+4)\left[\frac{2}{x-4}+\frac{3}{x+4}\right] \\
3 &= 2(x+4)+3(x-4) \\
3 &= 2x+8+3x-12 \\
3 &= 5x-4 \\
7 &= 5x \\
\frac{7}{5} &= x
\end{aligned}$$

80. $\dfrac{2}{x-6} = \dfrac{5}{x+6} - \dfrac{x}{x^2-36}$

Solution

$$\begin{aligned}
\frac{2}{x-6} &= \frac{5}{x+6} - \frac{x}{x^2-36} \\
\frac{2}{x-6} &= \frac{5}{x+6} - \frac{x}{(x-6)(x+6)} \\
(x+6)(x-6)\left(\frac{2}{x-6}\right) &= (x+6)(x-6)\left[\frac{5}{x+6} - \frac{x}{(x-6)(x+6)}\right] \\
2(x+6) &= 5(x-6) - x \\
2x+12 &= 5x-30-x \\
2x+12 &= 4x-30 \\
-2x+12 &= -30 \\
-2x &= -42 \\
x &= 21
\end{aligned}$$

81. $\dfrac{2x+3}{x^2+5x+6} + \dfrac{3x-2}{x^2+x-6} = \dfrac{5x-2}{x^2-4}$

Solution

$$\begin{aligned}
\frac{2x+3}{x^2+5x+6} + \frac{3x-2}{x^2+x-6} &= \frac{5x-2}{x^2-4} \\
\frac{2x+3}{(x+3)(x+2)} + \frac{3x-2}{(x+3)(x-2)} &= \frac{5x-2}{(x+2)(x-2)} \\
(x-2)(2x+3) + (x+2)(3x-2) &= (x+3)(5x-2) \\
2x^2 - x - 6 + 3x^2 + 4x - 4 &= 5x^2 + 13x - 6 \quad \text{\{multiply by common denominator\}} \\
5x^2 + 3x - 10 &= 5x^2 + 13x - 6 \\
3x - 10 &= 13x - 6 \\
-10x &= 4 \\
x &= -\frac{4}{10} = -\frac{2}{5}
\end{aligned}$$

82. $\dfrac{3x}{x^2+x} - \dfrac{2x}{x^2+5x} = \dfrac{x+2}{x^2+6x+5}$

Solution

$$\frac{3x}{x^2+x}-\frac{2x}{x^2+5x}=\frac{x+2}{x^2+6x+5}$$
$$\frac{3x}{x(x+1)}-\frac{2x}{x(x+5)}=\frac{x+2}{(x+5)(x+1)}$$
$$\frac{3}{x+1}-\frac{2}{x+5}=\frac{x+2}{(x+5)(x+1)}$$
$$3(x+5)-2(x+1)=x+2 \quad \text{\{multiply by common denominator\}}$$
$$3x+15-2x-2=x+2$$
$$x+13=x+2$$
$$13\neq 2 \Rightarrow \text{no solution}$$

83. $\dfrac{3x+5}{x^3+8}+\dfrac{3}{x^2-4}=\dfrac{2(3x-2)}{(x-2)(x^2-2x+4)}$

Solution

$$\frac{3x+5}{x^3+8}+\frac{3}{x^2-4}=\frac{2(3x-2)}{(x-2)(x^2-2x+4)}$$
$$\frac{3x+5}{(x+2)(x^2-2x+4}+\frac{3}{(x+2)(x-2)}=\frac{2(3x-2)}{(x-2)(x^2-2x+4)}$$
$$(x-2)(3x+5)+(x^2-2x+4)(3)=2(x+2)(3x-2) \quad \text{\{multiply by common denominator\}}$$
$$3x^2-x-10+3x^2-6x+12=6x^2+8x-8$$
$$6x^2-7x+2=6x^2+8x-8$$
$$-15x=-10$$
$$x=\frac{-10}{-15}=\frac{2}{3}$$

84. $\dfrac{1}{n+8}-\dfrac{3n-4}{5n^2+42n+16}=\dfrac{1}{5n+2}$

Solution

$$\frac{1}{n+8}-\frac{3n-4}{5n^2+42n+16}=\frac{1}{5n+2}$$
$$\frac{1}{n+8}-\frac{3n-4}{(5n+2)(n+8)}=\frac{1}{5n+2}$$
$$(5n+2)(1)-(3n-4)=n+8 \quad \text{\{multiply by common denominator\}}$$
$$5n+2-3n+4=n+8$$
$$2n+6=n+8$$
$$n=2$$

85. $\dfrac{1}{11-n}-\dfrac{2(3n-1)}{-7n^2+74n+33}=\dfrac{1}{7n+3}$

Solution

$$\frac{1}{11 - n} - \frac{2(3n - 1)}{-7n^2 + 74n + 33} = \frac{1}{7n + 3}$$
$$\frac{-1}{n - 11} + \frac{2(3n - 1)}{7n^2 - 74n - 33} = \frac{1}{7n + 3}$$
$$\frac{-1}{n - 11} + \frac{6n - 2}{(7n + 3)(n - 11)} = \frac{1}{7n + 3}$$
$$-(7n + 3) + 6n - 2 = (n - 11)1 \quad \{\text{multiply by common denominator}\}$$
$$-7n - 3 + 6n - 2 = n - 11$$
$$-n - 5 = n - 11$$
$$-2n = -6$$
$$n = 3$$

86. $\dfrac{4}{a^2 - 13a - 48} - \dfrac{2}{a^2 - 18a + 32} = \dfrac{1}{a^2 + a - 6}$

Solution

$$\frac{4}{a^2 - 13a - 48} - \frac{2}{a^2 - 18a + 32} = \frac{1}{a^2 + a - 6}$$
$$\frac{4}{(a - 16)(a + 3)} - \frac{2}{(a - 16)(a - 2)} = \frac{1}{(a + 3)(a - 2)}$$
$$4(a - 2) - 2(a + 3) = 1(a - 16) \quad \{\text{multiply by common denominator}\}$$
$$4a - 8 - 2a - 6 = a - 16$$
$$2a - 14 = a - 16$$
$$a = -2$$

87. $\dfrac{5}{y + 4} + \dfrac{2}{y + 2} = \dfrac{6}{y + 2} - \dfrac{1}{y^2 + 6y + 8}$

Solution

$$\frac{5}{y + 4} + \frac{2}{y + 2} = \frac{6}{y + 2} - \frac{1}{y^2 + 6y + 8}$$
$$\frac{5}{y + 4} = \frac{4}{y + 2} - \frac{1}{(y + 2)(y + 4)}$$
$$5(y + 2) = 4(y + 4) - 1 \quad \{\text{multiply by common denominator}\}$$
$$5y + 10 = 4y + 16 - 1$$
$$5y + 10 = 4y + 15$$
$$y = 5$$

88. $\dfrac{6}{2a - 6} - \dfrac{3}{3 - 3a} = \dfrac{1}{a^2 - 4a + 3}$

Solution

$$\frac{6}{2a-6}-\frac{3}{3-3a}=\frac{1}{a^2-4a+3}$$

$$\frac{6}{2(a-3)}-\frac{3}{3(1-a)}=\frac{1}{(a-3)(a-1)}$$

$$\frac{3}{a-3}+\frac{1}{a-1}=\frac{1}{(a-3)(a-1)}$$

$$3(a-1)+1(a-3)=1 \quad \text{\{multiply by common denominator\}}$$

$$3a-3+a-3=1$$

$$4a-6=1$$

$$4a=7$$

$$a=\frac{7}{4}$$

89. $\dfrac{3y}{6-3y}+\dfrac{2y}{2y+4}=\dfrac{8}{4-y^2}$

Solution

$$\frac{3y}{6-3y}+\frac{2y}{2y+4}=\frac{8}{4-y^2}$$

$$\frac{3y}{3(2-y)}+\frac{2y}{2(y+2)}=\frac{8}{(2+y)(2-y)}$$

$$\frac{y}{2-y}+\frac{y}{2+y}=\frac{8}{(2+y)(2-y)}$$

$$y(2+y)+y(2-y)=8 \quad \text{\{multiply by common denominator\}}$$

$$2y+y^2+2y-y^2=8$$

$$4y=8$$

$y=2 \Rightarrow$ The solution does not check, so the equation has no solution.

90. $\dfrac{3+2a}{a^2+6+5a}-\dfrac{2-3a}{a^2-6+a}=\dfrac{5a-2}{a^2-4}$

Solution

$$\frac{3+2a}{a^2+6+5a}-\frac{2-3a}{a^2-6+a}=\frac{5a-2}{a^2-4}$$

$$\frac{2a+3}{(a+2)(a+3)}+\frac{3a-2}{(a+3)(a-2)}=\frac{5a-2}{(a+2)(a-2)}$$

$$(a-2)(2a+3)+(a+2)(3a-2)=(a+3)(5a-2) \quad \text{\{multiply by common denominator\}}$$

$$2a^2-a-6+3a^2+4a-4=5a^2+13a-6$$

$$-10a=4$$

$$a=\frac{4}{-10}=-\frac{2}{5}$$

91. $\frac{a}{a+2} - 1 = -\frac{3a+2}{a^2+4a+4}$

Solution

$$\frac{a}{a+2} - 1 = \frac{3a+2}{a^2+4a+4}$$
$$\frac{a}{a+2} - 1 = -\frac{3a+2}{(a+2)(a+2)}$$
$$(a+2)(a+2)\left[\frac{a}{a+2} - 1\right] = (a+2)(a+2) \cdot \left[-\frac{3a+2}{(a+2)(a+2)}\right]$$
$$a(a+2) - (a+2)(a+2) = -(3a+2)$$
$$a^2 + 2a - (a^2 + 4a + 4) = -3a - 2$$
$$a^2 + 2a - a^2 - 4a - 4 = -3a - 2$$
$$-2a - 4 = -3a - 2$$
$$a = 2$$

92. $\frac{x-1}{x+3} + \frac{x-2}{x-3} = \frac{1-2x}{3-x}$

Solution

$$\frac{x-1}{x+3} + \frac{x-2}{x-3} = \frac{1-2x}{3-x}$$
$$\frac{x-1}{x+3} + \frac{x-2}{x-3} = \frac{2x-1}{x-3}$$
$$(x-3)(x-1) + (x+3)(x-2) = (x+3)(2x-1) \quad \{\text{miltiply by common denominator}\}$$
$$x^2 - 4x + 3 + x^2 + x - 6 = 2x^2 + 5x - 3$$
$$2x^2 - 3x - 3 = 2x^2 + 5x - 3$$
$$-8x = 0$$
$$x = \frac{0}{-8} = 0$$

Solve each formula for the specified variable.

93. $f = ma$; m

Solution

$$f = ma$$
$$\frac{f}{a} = \frac{ma}{a}$$
$$\frac{f}{a} = m$$

94. $P = 2l + 2w$; w

Solution

$$P = 2l + 2w$$
$$P - 2l = 2w$$
$$\frac{P - 2l}{2} = \frac{2w}{2}$$
$$\frac{P - 2l}{2} = w$$

95. $ax + b = 0$; x

Solution

$$ax + b = 0$$
$$ax = -b$$
$$x = -\frac{b}{a}$$

96. $V = \frac{1}{3}\pi r^2 h$; h

Solution

$$V = \frac{1}{3}\pi r^2 h$$
$$3V = 3 \cdot \frac{1}{3}\pi r^2 h$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$
$$\frac{3V}{\pi r^2} = h$$

97. $V = \frac{1}{3}\pi r^2 h$; r^2

Solution

$$V = \frac{1}{3}\pi r^2 h$$
$$3V = 3 \cdot \frac{1}{3}\pi r^2 h$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$
$$\frac{3V}{\pi h} = r^2$$

98. $z = \dfrac{x - \mu}{\sigma}$; μ

Solution

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ z\sigma &= \frac{x - \mu}{\sigma} \cdot \sigma \\ z\sigma &= x - \mu \\ \mu + z\sigma &= x \\ \mu &= x - z\sigma \end{aligned}$$

99. $P_n = L + \dfrac{si}{f}$; s

Solution

$$\begin{aligned} P_n &= L + \frac{si}{f} \\ P_n - L &= \frac{si}{f} \\ f(P_n - L) &= f \cdot \frac{si}{f} \\ f(P_n - L) &= si \\ \frac{f(P_n - L)}{i} &= \frac{si}{i} \\ \frac{f(P_n - L)}{i} &= s \end{aligned}$$

100. $P_n = L + \dfrac{si}{f}$; f

Solution

$$\begin{aligned} P_n &= L + \frac{si}{f} \\ P_n - L &= \frac{si}{f} \\ f(P_n - L) &= f \cdot \frac{si}{f} \\ f(P_n - L) &= si \\ \frac{f(P_n - L)}{P_n - L} &= \frac{si}{P_n - L} \\ f &= \frac{si}{P_n - L} \end{aligned}$$

101. $F = \dfrac{mMg}{r^2}$; m

Solution

$$\begin{aligned} F &= \frac{mMg}{r^2} \\ Fr^2 &= \frac{mMg}{r^2} \cdot r^2 \\ Fr^2 &= mMg \\ \frac{Fr^2}{Mg} &= \frac{mMg}{Mg} \\ \frac{Fr^2}{Mg} &= m \end{aligned}$$

102. $\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q}$; f

Solution

$$\begin{aligned} \frac{1}{f} &= \frac{1}{p} + \frac{1}{q} \\ fpq \cdot \frac{1}{f} &= fpq\left(\frac{1}{p} + \frac{1}{q}\right) \\ pq &= fq + fp \\ pq &= f(q + p) \\ \frac{pq}{q + p} &= \frac{f(q + p)}{q + p} \\ \frac{pq}{q + p} &= f \end{aligned}$$

103. $\dfrac{x}{a} + \dfrac{y}{b} = 1$; y

Solution

$$\begin{aligned} \frac{x}{a} + \frac{y}{b} &= 1 \\ \frac{y}{b} &= 1 - \frac{x}{a} \\ b \cdot \frac{y}{b} &= b\left(1 - \frac{x}{a}\right) \\ y &= b\left(1 - \frac{x}{a}\right) \end{aligned}$$

104. $\dfrac{x}{a} - \dfrac{y}{b} = 1$; a

Solution

$$\begin{aligned}
\frac{x}{a} - \frac{y}{b} &= 1 \\
ab\left(\frac{x}{a} - \frac{y}{b}\right) &= ab \cdot 1 \\
bx - ay &= ab \\
bx &= ab + ay \\
bx &= a(b + y) \\
\frac{bx}{b + y} &= \frac{a(b + y)}{b + y} \\
\frac{bx}{b + y} &= a
\end{aligned}$$

105. $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$; r

Solution

$$\begin{aligned}
\frac{1}{r} &= \frac{1}{r_1} + \frac{1}{r_2} \\
rr_1r_2 \cdot \frac{1}{r} &= rr_1r_2\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \\
r_1r_2 &= rr_2 + rr_1 \\
r_1r_2 &= r(r_2 + r_1) \\
\frac{r_1r_2}{r_2 + r_1} &= \frac{r(r_2 + r_1)}{r_2 + r_1} \\
\frac{r_1r_2}{r_2 + r_1} &= r
\end{aligned}$$

106. $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$; r_1

Solution

$$\begin{aligned}
\frac{1}{r} &= \frac{1}{r_1} + \frac{1}{r_2} \\
rr_1r_2 \cdot \frac{1}{r} &= rr_1r_2\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \\
r_1r_2 &= rr_2 + rr_1 \\
r_1r_2 - rr_1 &= rr_2 \\
r_1(r_2 + r) &= rr_2 \\
\frac{r_1(r_2 - r)}{r_2 - r} &= \frac{rr_2}{r_2 - r} \\
r_1 &= \frac{rr_2}{r_2 - r}
\end{aligned}$$

107. $n\ l = a + (n - 1)d;$

Solution

$$\begin{aligned} l &= a + (n-1)d \\ l &= a + nd - d \\ l - a + d &= nd \\ \frac{l-a+d}{d} &= \frac{nd}{d} \\ \frac{l-a+d}{d} &= n \end{aligned}$$

108. $l = a + (n - 1)d$; d

Solution

$$\begin{aligned} l &= a + (n-1)d \\ l - a &= (n-1)d \\ \frac{l-a}{n-1} &= \frac{(n-1)d}{n-1} \\ \frac{l-a}{n-1} &= d \end{aligned}$$

109. $a = (n - 2)\frac{180}{n}$; n

Solution

$$\begin{aligned} a &= (n-2)\frac{180}{n} \\ an &= (n-2)\frac{180}{n} \cdot n \\ an &= (n-2)180 \\ an &= 180n - 360 \\ 360 &= 180n - an \\ 360 &= n(180 - a) \\ \frac{360}{180-a} &= n \end{aligned}$$

110. $S = \frac{a - lr}{1 - r}$; a

Solution

$$\begin{aligned} S &= \frac{a-lr}{1-r} \\ S(1-r) &= \frac{a-lr}{1-r}(1-r) \\ S(1-r) &= a - lr \\ S - Sr + lr &= a \end{aligned}$$

111. $R = \dfrac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$; r_1

Solution

$$
\begin{aligned}
R &= \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} \\
R &= \frac{r_1 r_2 r_3 (1)}{r_1 r_2 r_3 \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)} \\
R &= \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} \\
R(r_2 r_3 + r_1 r_3 + r_1 r_2) &= r_1 r_2 r_3 \\
R r_2 r_3 + R r_1 r_3 + R r_1 r_2 &= r_1 r_2 r_3 \\
R r_1 r_3 + R r_1 r_2 - r_1 r_2 r_3 &= -R r_2 r_3 \\
r_1 (R r_3 + R r_2 - r_2 r_3) &= -R r_2 r_3 \\
r_1 &= \frac{-R r_2 r_3}{R r_3 + R r_2 - r_2 r_3}
\end{aligned}
$$

112. $R = \dfrac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$; r_3

Solution

$$
\begin{aligned}
R &= \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} \\
R &= \frac{r_1 r_2 r_3 (1)}{r_1 r_2 r_3 \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)} \\
R &= \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} \\
R(r_2 r_3 + r_1 r_3 + r_1 r_2) &= r_1 r_2 r_3 \\
R r_2 r_3 + R r_1 r_3 + R r_1 r_2 &= r_1 r_2 r_3 \\
R r_2 r_3 + R r_1 r_3 - r_1 r_2 r_3 &= -R r_1 r_2 \\
r_3 (R r_2 + R r_1 - r_1 r_2) &= -R r_1 r_2 \\
r_3 &= \frac{-R r_1 r_2}{R r_2 + R r_1 - r_1 r_2}
\end{aligned}
$$

Fix It

In exercises 113 and 114, identify the step the first error is made and fix it.

113. Solve the linear equation for x:

$$5(2x + 1) - 8x = -2(x - 5) + 7x$$

Solution
Step 4 was incorrect.

Step 1: $10x + 5 - 8x = -2x + 10 + 7x$

Step 2: $(10x - 8x) + 5 = (-2x + 7x) + 10$

Step 3: $2x + 5 = 5x + 10$

Step 4: $-3x = 5$

Step 5: $x = -\frac{5}{3}$

114. Write the values of x that make the denominator zero and then solve the rational equation:

$$\frac{5}{x - 5} + \frac{3}{x + 7} = \frac{60}{(x - 5)(x + 7)}$$

Solution
Step 5 was incorrect. Recall from Step 1, that 5 and –7 would cause the denominators to be zero. Therefore, there is no solution because 5 is an extraneous solution.

Discovery and Writing

115. Explain the difference between an identity and a contradiction. Give examples of each.

Solution
Answers may vary.

116. Share a strategy that can be used to identify the restrictions on a variable in a rational equation.

Solution
Answers may vary.

117. Explain why a conditional linear equation always has exactly one root.

Solution
Answers may vary.

118. Define an extraneous solution and explain how such a solution occurs.

Solution
Answers may vary.

Critical Thinking

Determine if the statement is true or false. If the statement is false, then correct it and make it true.

119. The equation $4x + 5(x - 3) = 9x - 15$ is a contradiction.

Solution

$$4x + 5(x - 3) = 9x - 15$$
$$4x + 5x - 15 = 9x - 15$$
$$9x - 15 = 9x - 15$$

False. The equation is an identity.

120. The equation $4x + 5(x - 3) = 9x - 15$ is an identity.

Solution

$$4x + 5(x - 3) = 9x + 15$$
$$4x + 5x - 15 = 9x + 15$$
$$9x - 15 = 9x + 15$$
$$-15 \neq 15$$

False. The equation is a contradiction.

121. $\sqrt{7}$, -4.5, and π would be included in the solution set for $-2x - 8 = -(2x + 8)$.

Solution

$$-2x - 8 = -(2x + 8)$$
$$-2x - 8 = -2x - 8$$

True. The equation is an identity, so all real numbers are solutions.

122. The equation $x + 188{,}424 = x + 188{,}425$ has an infinite number of solutions.

Solution

$$x + 188{,}424 = x + 188{,}425$$

False. The equation is a contradiction, so it has no solution.

123. The solution set of $\dfrac{1}{\left(\frac{1}{x-3}\right)} + \dfrac{1}{\left(\frac{1}{x-4}\right)} = 7$ is $\{7\}$.

Solution

$$\frac{1}{\left(\frac{1}{x-3}\right)} + \frac{1}{\left(\frac{1}{x-4}\right)} = 7$$
$$(x - 3) + (x - 4) = 7$$
$$2x - 7 = 7$$
$$2x = 14$$
$$x = 7 \text{ True.}$$

124. If $y_1 = \dfrac{1}{x - \pi}$ and $y_2 = 1$, then $y_1 = y_2$ when $x = 2\pi$.

Solution

$$y_1 = \frac{1}{x - \pi} = 1 = y_2$$

$$(x - \pi) \cdot \left(\frac{1}{x - \pi}\right) = (x - \pi) \cdot 1$$

$$1 = x - \pi$$

$$\pi + 1 = x$$

False. $y_1 = y_2$ when $x = \pi + 1$.

Exercises 1.2

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Write an algebraic expression that represents the average of the four microbiology test scores 50, 75, 100 and x.

   Solution

   $$\frac{50 + 75 + 100 + x}{4}$$

2. Rita watched 6 times as many movies on Netflix as Emma last year. If Emma watched x movies last year, write an expression for the number of movies Rita watched.

   Solution

   $6x$

3. If the width of a rectangle is represented by x and the length is represented by $2x + 30$, write a simplified expression representing its perimeter.

   Solution

   Width = x
   Length = $2x + 30$
   P = 2 (Length)+ 2 (width)
   P = $2(2x + 30) + 2x$
   P = $4x + 60 + 2x$
   P = $6x + 60$

4. Jackson wins $50,000. If he invests x dollars of it in Amazon stock, write an expression for the amount remaining which he invests in an annuity.

   Solution

   $50{,}000 - x$

5. If the original price x of a North Face jacket is discounted 12%, write an expression for the selling price of the jacket.

Solution

$\boxed{\text{Original}} - \boxed{\text{12\% of original}}$

$x - 0.12x$

6. If it takes Olivia x hours to complete a job, write an expression that represents the part of the job she completes in one hour.

Solution

$\frac{1}{x}$

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. To average n scores, __________ the scores and divide by n.

Solution
add

8. The formula for the __________ of a rectangle is $P = 2l + 2w$.

Solution
perimeter

9. The simple annual interest earned on an investment is the product of the interest rate and the __________ invested.

Solution
amount

10. The number of units manufactured at which the cost on two machines is equal is called the __________.

Solution
break point

11. Distance traveled is the product of the __________ and the __________.

Solution
rate, time

12. 5% of 30 liters is __________ liters.

Solution
$0.05(30) = 1.5$

Practice

Solve each problem.

13. **Algebra scores** A student has completed all assignments in college algebra except for taking the comprehensive final exam. The student's current test scores are shown in the table.

Test 1	Test 2	Test 3	Test 4	Online Homework	Final Exam
60	78	80	90	88	?

If the student's online homework average for the semester is weighted as a test grade and the final exam grade is weighted as two test grades, what must the student score on the final exam to have an 80 average in the course?

Solution

Let x = the score on the final exam. Since the final is weighted as two test grades, it counts as two test grades.

$$\begin{aligned} \frac{\boxed{\text{Sum of scores}}}{7} &= 80 \\ \frac{60 + 78 + 80 + 90 + 88 + 2x}{7} &= 80 \\ \frac{2x + 396}{7} &= 80 \\ 2x + 396 &= 560 \\ 2x &= 164 \\ x &= 82 \end{aligned}$$

His score on the final exam needs to be 82.

14. **Psychology scores** Mandy has completed all assignments in psychology except for taking her comprehensive final exam. Her current scores are shown in the table.

Test 1	Test 2	Test 3	Test 4	Test 5	Final Exam
70	88	93	85	88	?

If the final exam score is weighted as a test grade and also replaces her lowest test score, what must she make on the final exam to have a 90 average in the course?

Solution

Let x = the score on the final exam. Replace the lowest test score (70) with x also.

$$\begin{aligned} \frac{\boxed{\text{Sum of scores}}}{6} &= 90 \\ \frac{x + 88 + 93 + 85 + 88 + x}{6} &= 90 \\ \frac{2x + 354}{6} &= 90 \\ 2x + 354 &= 540 \\ 2x &= 186 \\ x &= 93 \end{aligned}$$

His score on the final exam needs to be 93.

15. **Test scores** A student scored 5 points higher on his midterm and 13 points higher on his final than he did on his first exam. If his mean (average) score was 90, what was his score on the first exam?

Solution

Let x = the score on the first exam. Then $x + 5$ = the score on the midterm, and $x + 13$ = the score on the final.

$$\begin{aligned} \frac{\boxed{\text{Sum of scores}}}{3} &= 90 \\ \frac{x + x + 5 + x + 13}{3} &= 90 \\ \frac{3x + 18}{3} &= 90 \\ 3x + 18 &= 270 \\ 3x &= 252 \\ x &= 84 \end{aligned}$$

His score on the first exam was 84.

16. **Test scores** Rashida took four tests in chemistry class. On each successive test, her score improved by 3 points. If her mean score was 69.5, what did she score on the first test?

Solution

Let x = the score on the first exam. Then her score on the following tests were $x + 3$, $x + 6$ and $x + 9$.

$$\begin{aligned} \frac{\boxed{\text{Sum of scores}}}{4} &= 69.5 \\ \frac{x + x + 3 + x + 6 + x + 9}{4} &= 69.5 \\ \frac{4x + 18}{4} &= 69.5 \\ 4x + 18 &= 278 \\ 4x &= 260 \\ x &= 65 \end{aligned}$$

Her score on the first exam was 65%.

17. **Teacher certification** On the Illinois certification test for teachers specializing in learning disabilities, a teacher earned the scores shown in the accompanying table. What was the teacher's score in program development?

Human development with special needs	82
Assessment	90
Program development and instruction	?
Professional knowledge and legal issues	78
AVERAGE SCORE	86

Solution

Let x = the program development score.

$$\begin{aligned}\frac{\boxed{\text{Sum of scores}}}{4} &= 86\\ \frac{82 + 90 + x + 78}{4} &= 86\\ \frac{x + 250}{4} &= 86\\ x + 250 &= 344\\ x &= 94\end{aligned}$$

The program development score was 94.

18. **Golfing** Par on a golf course is 72. If a golfer shot rounds of 76, 68, and 70 in a tournament, what will she need to shoot on the final round to average par?

Solution

Let x = the score on the final round.

$$\begin{aligned}\frac{\boxed{\text{Sum of scores}}}{4} &= 72\\ \frac{76 + 68 + 70 + x}{4} &= 72\\ \frac{x + 214}{4} &= 72\\ x + 214 &= 288\\ x &= 74\end{aligned}$$

She needs to shoot 74 on the final round.

19. **Replacing locks** A locksmith at Pop-A-Lock charges \$40 plus \$28 for each lock installed. How many locks can be replaced for \$236?

Solution

Let x = the number of locks replaced.

$$\begin{aligned}40 + 28 \cdot \boxed{\begin{matrix}\text{Number}\\ \text{of locks}\end{matrix}} &= 236\\ 40 + 28x &= 236\\ 28x &= 196\\ x &= 7\end{aligned}$$

7 locks can be changed for \$236.

20. **Delivering ads** A University of Florida student earns \$20 per day delivering advertising brochures door-to-door, plus \$1.50 for each person he interviews. How many people did he interview on a day when he earned \$56?

Solution

Let x = the number of interviews.

$$\begin{aligned} 20 + 0.75 \cdot \boxed{\text{Number of interviews}} &= 56 \\ 20 + 0.75x &= 56 \\ 0.75x &= 36 \\ x &= 48 \end{aligned}$$

He interviewed 48 people.

21. **Electronic LED billboard** An electronic LED billboard in Times Square is 26 feet taller than it is wide. If its perimeter is 92 feet, find the dimensions of the billboard.

Solution

Let x = the width

Then $x + 26$ = the height.

$$\begin{aligned} \boxed{\text{Perimeter}} &= 92 \\ 2x + 2(x + 26) &= 92 \\ 2x + 2x + 52 &= 92 \\ 4x &= 40 \\ x &= 10 \end{aligned}$$

The dimensions are 10 ft by 36 ft.

22. **Hockey rink** A National Hockey League rink is 115 feet longer than it is wide. If the perimeter of the rink is 570 feet, find the dimensions of the rink?

Solution

Let x = the width

Then $x + 115$ = the height.

$$\begin{aligned} \boxed{\text{Perimeter}} &= 570 \\ 2x + 2(x + 115) &= 570 \\ 2x + 2x + 230 &= 570 \\ 4x &= 340 \\ x &= 85 \end{aligned}$$

The dimensions are 85 ft by 200 ft.

23. **Debit card** The width of your Visa bank debit card is 1.586 inches times its height. Find the dimensions of the debit card if its perimeter is 10.990 inches.

Solution

Let h = *height*

Let $1.586h$ = *width*

$\boxed{\textit{Perimeter}} = 10.99$

$10.99 = 2h + 2(1.586h)$

$10.99 = 5.172h$

2.125 inches = *height*

1.586(2.125) = 3.370 *inches* = *width*

24. **International airport runway** To accommodate the largest jets, an international airport runway is 1500 m more than 50 times its width. If the perimeter of the runway is 11,160 m, determine the length and width of the runway.

Solution

Let w = *width*

Let $50w + 1500$ = *length*

$\boxed{\textit{Perimeter}} = 11{,}160$

$11{,}160 = 2(50w + 1500) + 2w$

$11{,}160 = 100w + 3000 + 2w$

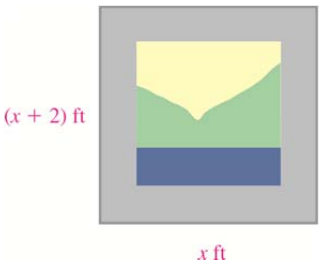
$11{,}160 = 102w + 3000$

$8160 = 102w$

$80\ m = \textit{width}$

$50(80) + 1500 = 5500\ m = \textit{length}$

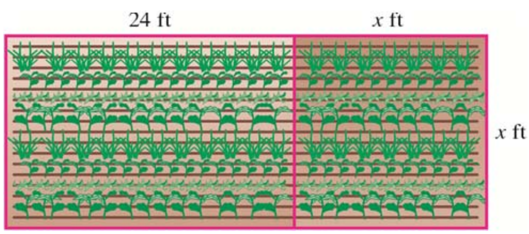
25. **Width of a picture frame** A picture frame with width x feet and height $(x + 2)$ feet was built with 14 feet of framing material. Find x its width.



Solution

$$\begin{aligned} \boxed{\text{Perimeter}} &= 14 \\ x + (x + 2) + x + (x + 2) &= 14 \\ 4x + 4 &= 14 \\ 4x &= 10 \\ x &= \frac{5}{2} = 2\frac{1}{2} \Rightarrow \text{The width is } 2\frac{1}{2} \text{ feet.} \end{aligned}$$

26. **Fencing a garden** A gardener fences in a rectangular region that is formed by adjoining a rectangle of length 24 feet and width x feet to a square measuring x feet on each side. When he does, he needs twice as much fencing as he did just for the square region. How much fencing will he need?



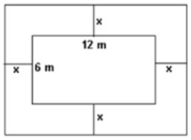
Solution

$$\boxed{\text{Total Fence Length}} = 2 \cdot \boxed{\text{Square Fence Length}}$$
$$x + (x + 24) + x + (x + 24) = 2 \cdot (x + x + x + x)$$
$$4x + 48 = 8x$$
$$48 = 4x$$
$$x = 12$$

The total fencing required is $4x + 48 = 4(12) + 48 = 96$ feet.

27. **Swimming pool** A rectangular swimming pool measures 12 meters by 6 meters and is surrounded by a 116 meter rectangular wooden fence. If the fence forms a border around the pool of uniform width, determine the width of the border.

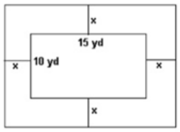
Solution
Let x = the width of the border.



$$\boxed{\text{Perimeter of fence}} = 116$$
$$2(2x + 6) + 2(2x + 12) = 116$$
$$4x + 12 + 4x + 24 = 116$$
$$8x + 36 = 116$$
$$8x = 80$$
$$x = 10$$

28. **Aquarium** A rectangular glass aquarium has a length of 15 yards, a width of 10 yards, and is placed inside a rectangular room for viewing at a museum. If the room has a perimeter of 146 yards and forms a walkway of uniform width that surrounds the aquarium, determine the width of the walkway.

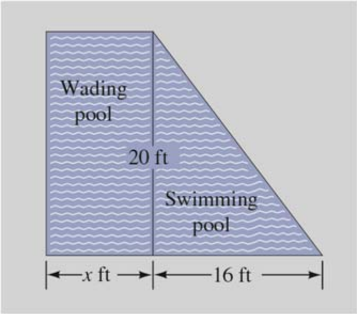
Solution
Let x = the width of the border.



$$\boxed{\text{Perimeter of room}} = 146$$
$$2(2x + 10) + 2(2x + 15) = 146$$
$$4x + 20 + 4x + 30 = 146$$
$$8x + 50 = 146$$
$$8x = 96$$
$$x = 12$$

The walkway has a width of 12 yards.

29. **Wading pool dimensions** A larger swimming pool is created by adjoining a right triangular-shaped swimming pool, with legs 16 ft and 20 ft, to a rectangular-shaped wading pool that is x ft by 20 ft. Find the dimensions of the wading pool. (*Hint:* The area of a triangle $= \frac{1}{2}bh$, and the area of a rectangle $= lw$.)



Solution

$$\boxed{\text{Total Area}} = 2 \cdot \boxed{\text{Triangular Area}}$$

$$20x + \frac{1}{2}(16)(20) = 2 \cdot \frac{1}{2}(16)(20)$$

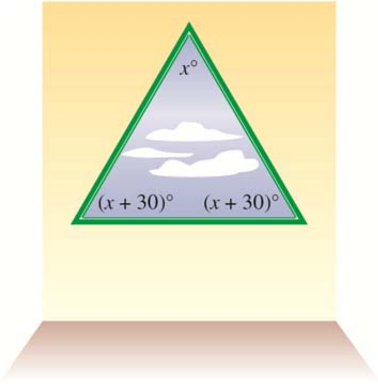
$$20x + 160 = 320$$

$$20x = 160$$

$$x = 8$$

The dimensions are 8 feet by 20 feet.

30. **House construction** A builder wants to install a triangular window with angles, $x°$, $(x + 30)°$, and $(x + 30)°$. What angles will he have to cut to make the window fit? (*Hint:* The sum of the angles in a triangle equals 180°.)



Solution

$$\boxed{\text{Sum of angles}} = 180$$

$$x + x + 30 + x + 30 = 180$$

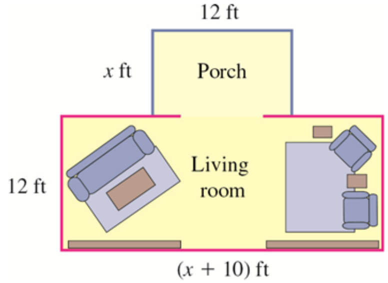
$$3x + 60 = 180$$

$$3x = 120$$

$$x = 40$$

The angles measure 40°, 70° and 70°.

31. **Length of a living room** If a carpenter adjoins a rectangular-shaped porch, 12 ft by x ft, to rectangular-shaped living room, 12 ft by $(x + 10)$ ft, the living area will be increased by 50%. Find the length of the living room.



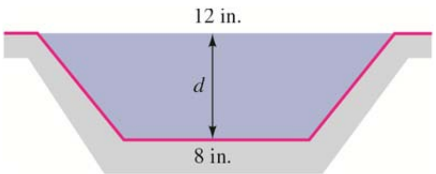
Solution

$$\boxed{\text{New Area}} = \boxed{\text{Old Area}} + 0.50 \cdot \boxed{\text{Old Area}}$$

$$\begin{aligned} 12(x+10) + 12x &= 12(x+10) + 0.50 \cdot 12(x+10) \\ 12x + 120 + 12x &= 12x + 120 + 6x + 60 \\ 12x + 120 &= 18x + 180 \\ 6x &= 60 \\ x &= 10 \end{aligned}$$

$\Rightarrow$ The length of the living room is $x + 10 = 20$ feet.

32. **Depth of water in a trough** A trapezoidal-shaped trough, with bases 8 inches and 12 inches, has a cross-sectional area of 54 square inches. If the depth of the trough is d inches, find its depth. (*Hint:* Area of a trapezoid $= \frac{1}{2}h(b_1 + b_2)$.)



Solution

$$\begin{aligned} \boxed{\text{Area}} &= 54 \\ \frac{1}{2}d(12 + 8) &= 54 \\ 10d &= 54 \\ d &= 5.4 \end{aligned}$$

$\Rightarrow$ The depth is 5.4 inches

33. **Investment** Jeffrey invested \$16,000 in two accounts paying 4% and 6% annual interest. If the total interest earned in one year was \$815, how much did he invest at each rate?

Solution

Let x = the amount invested at 4%. Then $16000 - x$ = the amount invested at 6%.

$$\boxed{\text{Interest at 4\%}} + \boxed{\text{Interest at 6\%}} = \boxed{\text{Total interest}}$$

$$\begin{aligned} 0.04x + 0.06(16000 - x) &= 815 \\ 0.04x + 960 - 0.06x &= 815 \\ -0.02x &= -145 \\ x &= 7250 \end{aligned}$$

\$7250 was invested at 4% and \$8750 was invested at 6%.

34. **Investment** An executive invests \$22,000, some at 7% and the rest at 6% annual interest. If he receives an annual return of \$1420, how much is invested at each rate?

Solution

Let x = the amount invested at 7%. Then $22000 - x$ = the amount invested at 6%.

$$\boxed{\text{Interest at 7\%}} + \boxed{\text{Interest at 6\%}} = \boxed{\text{Total interest}}$$

$$\begin{aligned} 0.07x + 0.06(22000 - x) &= 1420 \\ 0.07x + 1320 - 0.06x &= 1420 \\ 0.01x &= 100 \\ x &= 10000 \end{aligned}$$

\$10,000 was invested at 7% and \$12,000 was invested at 6%.

35. **Equity funds** You invest part of \$25,000 in a stock fund that earns 11% interest and the remainder in a stock fund that incurred at 4% loss. If the total interest earned in one year was \$2,000, how much was invested in stock?

Solution

Let x be the amount that earns 11%. Then $25,000 - x$ = the amount that incurs 4% loss.

$$\boxed{\text{Interest gained at 11\%}} - \boxed{\text{amount lost at 4\%}} = \boxed{\text{Total interest}}$$

$$\begin{aligned} 0.11x - 0.04(25,000 - x) &= 2000 \\ 0.11x - 1000 + 0.04x &= 2000 \\ 0.15x - 1000 &= 2000 \\ 0.15x &= 3000 \\ x &= 20,000 \end{aligned}$$

\$20,000 was invested at 11% and \$5000 was invested at a 4% loss.

36. **Mutual funds** You invest part of \$45,000 in a mutual fund that earns 15% interest and the remainder in a mutual fund that incurred at 2% loss. If the total interest earned in one year was \$5050, how much was invested in mutual fund?

Solution

Let x be the amount that earns 15%. Then $45,000 - x$ = the amount that incurs 2% loss.

$$\boxed{\text{Interest gained at 15\%}} - \boxed{\text{amount lost at 2\%}} = \boxed{\text{Total interest}}$$

$$\begin{aligned}0.15x - 0.02(45{,}000 - x) &= 5050\\ 0.15x - 900 + 0.02x &= 5050\\ 0.17x - 900 &= 5050\\ 0.17x &= 5950\\ x &= 35{,}000\end{aligned}$$

\$35,000 was invested at 15% and \$10,000 was invested at a 2% loss.

37. **Financial planning** After inheriting some money, a woman wants to invest enough to have an annual income of \$5000. If she can invest \$20,000 at 9% annual interest, how much more will she have to invest at 7% to achieve her goal? (See the table.)

Type	Rate	Amount	Income
9% investment	0.09	20,000	.09(20,000)
7% investment	0.07	x	$.07x$

Solution

Let x = the amount invested at 7%.

$$\boxed{\text{Interest at 7\%}} + \boxed{\text{Interest at 9\%}} = \boxed{\text{Total interest}}$$

$$\begin{aligned}0.07x + 0.09(20000) &= 5000\\ 0.07x + 1800 &= 5000\\ 0.07x &= 3200\\ x &\approx 45714.29\end{aligned}$$

She needs to invest \$45,714.29 at 7% to reach her goal.

38. **Investment** A woman invests \$37,000, part at 8% and the rest at $9\frac{1}{2}\%$ annual interest. If the $9\frac{1}{2}\%$ investment provides \$452.50 more income than the 8% investment, how much is invested at each rate?

Solution

Let x = the amount invested at 8%. Then $37{,}000 - x$ = the amount invested at $9\frac{1}{2}\%$.

$$\boxed{\text{Interest at } 9\tfrac{1}{2}\%} = \boxed{\text{Interest at 8\%}} + 452.50$$

$$\begin{aligned}0.095(37{,}000 - x) &= 0.08x + 452.50\\ 3515 - 0.095x &= 0.08x + 452.50\\ 3062.50 &= 0.175x\\ 17500 &= x\end{aligned}$$

\$17,500 is invested at 8% and \$19,500 is invested at $9\frac{1}{2}\%$.

39. **Investment** Equal amounts are invested at 6%, 7%, and 8% annual interest. If the three investments yield a total of \$2037 annual interest, find the total investment.

Solution

Let x = the amount invested at each rate.

$$\boxed{\text{Interest at 6\%}} + \boxed{\text{Interest at 7\%}} + \boxed{\text{Interest at 8\%}} = \boxed{\text{Total Interest}}$$

$$\begin{aligned} 0.06x + 0.07x + 0.08x &= 2037 \\ 0.21x &= 2037 \\ x &= 9700 \end{aligned}$$

$9,700 was invested at each rate, for a total investment of $29,100.

40. **Investment** Ali invested equal amounts of money at 5%, 7%, 9%, and 11% annual interest. If the four investments yielded a total of $1440 annual interest, find the total amount invested.

Solution

Let x = the amount invested at each rate.

$$\boxed{\text{Interest at 5\%}} + \boxed{\text{Interest at 7\%}} + \boxed{\text{Interest at 9\%}} + \boxed{\text{Interest at 11\%}} = \boxed{\text{Total Interest}}$$

$$\begin{aligned} 0.05x + 0.07x + 0.09x + 0.11x &= 1440 \\ 0.32x &= 1440 \\ x &= 4500 \end{aligned}$$

$4500 was invested at each rate, for a total investment of $18,000.

41. **Ticket sales** A full-price ticket for a college basketball game costs $2.50, and a student ticket costs $1.75. If 585 tickets were sold, and the total receipts were $1,217.25, how many tickets were student tickets?

Solution

Let x = the number of full-price tickets sold. Then $585 - x$ = the number of student tickets sold.

$$2.50 \cdot \boxed{\begin{array}{c}\text{\# of}\\ \text{full-price}\end{array}} + 1.75 \cdot \boxed{\begin{array}{c}\text{\# of}\\ \text{student}\end{array}} = 1217.25$$

$$\begin{aligned} 2.50x + 1.75(585 - x) &= 1217.25 \\ 2.50x + 1023.75 - 1.75x &= 1217.25 \\ 0.75x &= 193.50 \\ x &= 258 \Rightarrow \text{There were 327 student tickets sold.} \end{aligned}$$

42. **Ticket sales** Of the 800 tickets sold to a movie, 480 were full-price tickets costing $7 each. If the gate receipts were $4960, what did a student ticket cost?

Solution

Let x = the cost of a student ticket.

$$\boxed{\begin{array}{c}\text{Cost of}\\ \text{full-price}\end{array}} \cdot \boxed{\begin{array}{c}\text{\# of}\\ \text{full-price}\end{array}} + \boxed{\begin{array}{c}\text{Cost of}\\ \text{student}\end{array}} \cdot \boxed{\begin{array}{c}\text{\# of}\\ \text{student}\end{array}} = 4960$$

$$\begin{aligned} 480(7) + x(800 - 480) &= 4960 \\ 3360 + 320x &= 4960 \\ 320x &= 1600 \\ x &= 5 \Rightarrow \text{A student tickets cost \$5.} \end{aligned}$$

43. **Beachfront condo stay** The cost per night to stay in a two-bedroom beachfront condo in Orange Beach, AL, is $377. This includes a 16% tax. What is the nightly cost?

Solution

Let x be the original nightly cost in the condo.

$$\boxed{\text{Original nightly cost}} + \boxed{\text{tax}} = \boxed{\text{total nightly cost}}$$

$$\begin{aligned} x + 0.16x &= 377 \\ 1.16x &= 377 \\ x &= 325 \end{aligned}$$

The original nightly cost in the condo was $325.

44. **Beachfront condo stay** The cost per night to stay in a three-bedroom beachfront condo in Myrtle Beach, SC, is $295. This includes a 18% tax. What is the nightly cost?

Solution

Let x be the original nightly cost in the condo.

$$\boxed{\text{Original nightly cost}} + \boxed{\text{tax}} = \boxed{\text{total nightly cost}}$$

$$\begin{aligned} x + 0.18x &= 295 \\ 1.18x &= 295 \\ x &= 250 \end{aligned}$$

The original nightly cost in the condo was $250.

45. **Discount** An iPad Air is on sale for $413.08. What was the original price of the iPad if it was discounted 8%?

Solution

Let p = the original price.

$$\boxed{\begin{array}{c}\text{Original}\\\text{price}\end{array}} - \boxed{\text{Discount}} = \boxed{\begin{array}{c}\text{New}\\\text{price}\end{array}}$$

$$\begin{aligned} p - 0.08p &= 413.08 \\ 0.92p &= 413.08 \\ p &= 449 \end{aligned}$$

The original price was $449.

46. **Discount** After being discounted 20%, a weather radio sells for $63.96. Find the original price.

Solution

Let p = the original price.

$$\boxed{\begin{array}{c}\text{Original}\\\text{price}\end{array}} - \boxed{\text{Discount}} = \boxed{\begin{array}{c}\text{New}\\\text{price}\end{array}}$$

$$\begin{aligned} p - 0.20p &= 63.96 \\ 0.80p &= 63.96 \\ p &= 79.95 \end{aligned}$$

The original price was $79.95.

47. **Markup** A business owner increases the wholesale cost of a kayak by 70% and sells it for $365.50. Find the wholesale cost.

Solution
Let w = the wholesale cost.

$$\boxed{\text{wholesale cost}} + \boxed{\text{Markup}} = \boxed{\text{Selling price}}$$

$$\begin{aligned} w + 0.70w &= 365.50 \\ 1.70w &= 365.50 \\ w &= 215 \end{aligned}$$

The wholesale cost is $215.

48. **Markup** A merchant increases the wholesale cost of a surfboard by 30% to determine the selling price. If the surfboard sells for $588.90, find the wholesale cost.

Solution
Let w = the wholesale cost.

$$\boxed{\text{wholesale cost}} + \boxed{\text{Markup}} = \boxed{\text{Selling price}}$$

$$\begin{aligned} w + 0.30w &= 588.90 \\ 1.30w &= 588.90 \\ w &= 453 \end{aligned}$$

The wholesale cost is $453.

49. **Break-point analysis** A machine to mill a brass plate has a setup cost of $600 and a unit cost of $3 for each plate manufactured. A bigger machine has a setup cost of $800 but a unit cost of only $2 for each plate manufactured. Find the break point.

Solution
Let x = # of plates for equal costs.

$$\boxed{\text{cost of 1st machine}} = \boxed{\text{cost of 2nd machine}}$$

$$\begin{aligned} 600 + 3x &= 800 + 2x \\ x &= 200 \end{aligned}$$

The break point is 200 plates.

50. **Break-point analysis** A machine to manufacture fasteners has a setup cost of $1200 and a unit cost of $0.005 for each fastener manufactured. A newer machine has a setup cost of $1500 but a unit cost of only $0.0015 for each fastener manufactured. Find the break point.

Solution
Let x = # of fasteners for equal costs.

$$\boxed{\text{cost of 1st machine}} = \boxed{\text{cost of 2nd machine}}$$
$$1200 + 0.005x = 1500 + 0.0015x$$
$$0.0035x = 300$$
$$x \approx 85714$$
The break point is about 85,714 fasteners.

51. **Computer sales** A computer store has fixed costs of $8925 per month and a unit cost of $850 for every computer it sells. If the store can sell all the computers it can get for $1275 each, how many must be sold for the store to break even? (*Hint:* The break-even point occurs when costs equal income.)

Solution
Let x = # of computer to break even.

$$\boxed{\text{Income}} = \boxed{\text{Expenses}}$$
$$1275x = 8925 + 850x$$
$$425x = 8925$$
$$x = 21$$
21 computers need to be sold to break even.

52. **Restaurant management** A restaurant has fixed costs of $137.50 per day and an average unit cost of $4.75 for each meal served. If a typical meal costs $6, how many customers must eat at the restaurant each day for the owner to break even?

Solution
Let x = # of meals to break even.

$$\boxed{\text{Income}} = \boxed{\text{Expenses}}$$
$$6x = 137.50 + 4.75x$$
$$1.25x = 137.50$$
$$x = 110$$
More than 110 meals need to be sold to make a profit.

53. **Roofing houses** Kyle estimates that it will take him 7 days to roof his house. A professional roofer estimates that it will take him 4 days to roof the same house. How long will it take if they work together?

Solution
Let x = days for both working together.

$$\boxed{\text{Man in 1 day}} + \boxed{\text{Roofer in 1 day}} = \boxed{\text{Total in 1 day}}$$
$$\frac{1}{7} + \frac{1}{4} = \frac{1}{x}$$
$$28x\left(\frac{1}{7} + \frac{1}{4}\right) = 28x\left(\frac{1}{x}\right)$$
$$4x + 7x = 28$$

$$11x = 28$$
$$x = \frac{28}{11} = 2\frac{6}{11}$$

They can roof the house in $2\frac{6}{11}$ days.

54. **Sealing asphalt** One crew can seal a parking lot in 8 hours and another in 10 hours. How long will it take to seal the parking lot if the two crews work together?

Solution
Let x = hours for both working together.

$$\boxed{\begin{array}{c}\text{Crew 1}\\\text{in 1 hour}\end{array}} + \boxed{\begin{array}{c}\text{Crew 2}\\\text{in 1 hour}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\\text{1 hour}\end{array}}$$

$$\frac{1}{8} + \frac{1}{10} = \frac{1}{x}$$
$$40x\left(\frac{1}{8} + \frac{1}{10}\right) = 40x\left(\frac{1}{x}\right)$$
$$5x + 4x = 40$$
$$9x = 40$$
$$x = \frac{40}{9} = 4\frac{4}{9}$$

They can seal the parking lot in $4\frac{4}{9}$ hours.

55. **Mowing lawns** Julie can mow a lawn with a lawn tractor in 2 hours, and her husband can mow the same lawn with a push mower in 4 hours. How long will it take to mow the lawn if they work together?

Solution
Let x = hours for both working together.

$$\boxed{\begin{array}{c}\text{Woman}\\\text{in 1 hour}\end{array}} + \boxed{\begin{array}{c}\text{Man in}\\\text{1 hour}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\\text{1 hour}\end{array}}$$

$$\frac{1}{2} + \frac{1}{4} = \frac{1}{x}$$
$$4x\left(\frac{1}{2} + \frac{1}{4}\right) = 4x\left(\frac{1}{x}\right)$$
$$2x + x = 4$$
$$3x = 4$$
$$x = \frac{4}{3} = 1\frac{1}{3}$$

They can seal the parking lot in $1\frac{1}{3}$ hours.

56. **Filling swimming pools** A garden hose can fill a swimming pool in 3 days, and a larger hose can fill the pool in 2 days. How long will it take to fill the pool if both hoses are used?

Solution

Let x = days for both houses to fill the pool.

$$\boxed{\begin{array}{c}\text{1st hose}\\\text{in 1 day}\end{array}} + \boxed{\begin{array}{c}\text{2nd hose}\\\text{in 1 day}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\\text{1 day}\end{array}}$$

$$\begin{aligned}\frac{1}{3}+\frac{1}{2} &= \frac{1}{x}\\ 6x\left(\frac{1}{3}+\frac{1}{2}\right) &= 6x\left(\frac{1}{x}\right)\\ 2x+3x &= 6\\ 5x &= 6\\ x &= \frac{6}{5} = 1\frac{1}{5}\end{aligned}$$

The pool can be filled in $1\frac{1}{5}$ days.

57. **Filling swimming pools** An empty swimming pool can be filled in 10 hours. When full, the pool can be drained in 19 hours. How long will it take to fill the empty pool if the drain is left open?

Solution

Let x = hours for pool to fill with drain open.

$$\boxed{\begin{array}{c}\text{Pipe in}\\\text{1 hour}\end{array}} - \boxed{\begin{array}{c}\text{Drain in}\\\text{1 hour}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\\text{1 hour}\end{array}}$$

$$\begin{aligned}\frac{1}{10}-\frac{1}{19} &= \frac{1}{x}\\ 190x\left(\frac{1}{10}-\frac{1}{19}\right) &= 190x\left(\frac{1}{x}\right)\\ 19x-10x &= 190\\ 9x &= 190\\ x &= \frac{190}{9} = 21\frac{1}{9}\end{aligned}$$

The pool can be filled in $21\frac{1}{9}$ hours.

58. **Preparing seafood** Kadek stuffs shrimp in his job as a seafood chef. He can stuff 1000 shrimp in 6 hours. When his sister helps him, they can stuff 1000 shrimp in 4 hours. If Kadek gets sick, how long will it take his sister to stuff 500 shrimp?

Solution

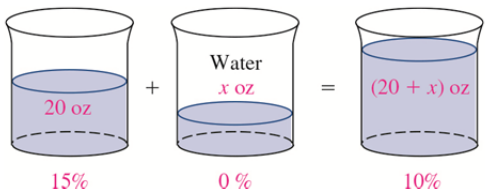
Let x = hours for sister to stuff 1000 shrimp.

$$\boxed{\text{Sam in 1 hour}} + \boxed{\text{Sister in 1 hour}} = \boxed{\text{Total in 1 hour}}$$

$$\begin{aligned} \frac{1}{6} + \frac{1}{x} &= \frac{1}{4} \\ 24x\left(\frac{1}{6} + \frac{1}{x}\right) &= 24x\left(\frac{1}{4}\right) \\ 4x - 24 &= 6x \\ 24 &= 2x \\ 12 &= x \end{aligned}$$

She can stuff 1,000 shrimp in 12 hours, so she can stuff 500 shrimp in 6 hours.

59. **Diluting solutions** How much water should be added to 20 ounces of a 15% solution of alcohol to dilute it to a 10% solution?



Solution

Let x = the ounces of water added.

$$\boxed{\text{Oz of alc. at start}} + \boxed{\text{Oz of alc. added}} = \boxed{\text{Oz of alc. at end}}$$

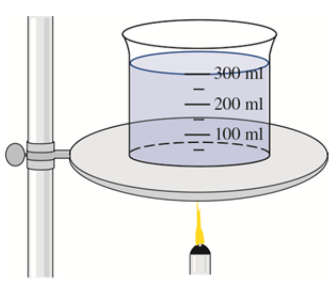
$$\begin{aligned} 0.15(20) + 0(x) &= 0.10(24 + x) \\ 3 &= 2 + 0.1x \\ 1 &= 0.1x \\ \frac{1}{0.1} &= x \\ 10 &= x \end{aligned}$$

10 oz of water should be added.

60. **Increasing concentrations** The beaker shown below contains a 2% saltwater solution.

a. How much water must be boiled away to increase the concentration of the salt solution from 2% to 3%?

b. Where on the beaker would the new water level be?



Solution

Let x = the ml of water removed.

$$\boxed{\begin{array}{c}\text{ml of salt}\\ \text{at start}\end{array}} - \boxed{\begin{array}{c}\text{ml of salt}\\ \text{removed}\end{array}} = \boxed{\begin{array}{c}\text{ml of salt}\\ \text{at end}\end{array}}$$

$$\begin{aligned} 0.02(300) - 0(x) &= 0.03(300 - x) \\ 6 &= 9 - 0.03x \\ 0.03x &= 3 \\ x &= \frac{3}{0.03} \\ x &= 100 \end{aligned}$$

a. 100 ml of water should be boiled away.

b. The new level will be at the 200-ml mark.

61. **Winterizing cars** A car radiator has a 6-liter capacity. If the liquid in the radiator is 40% antifreeze, how much liquid must be replaced with pure antifreeze tobring the mixture up to a 50% solution?

Solution

Let x = the liters of liquid replaced with pure antifreeze.

$$\boxed{\begin{array}{c}\text{Liters of}\\ \text{a.f. at start}\end{array}} - \boxed{\begin{array}{c}\text{Liters of}\\ \text{a.f. removed}\end{array}} + \boxed{\begin{array}{c}\text{Liters of}\\ \text{a.f. replaced}\end{array}} = \boxed{\begin{array}{c}\text{Liters of}\\ \text{a.f. at end}\end{array}}$$

$$\begin{aligned} 0.40(6) - 0.40x + x &= 0.50(6) \\ 2.4 + 0.6x &= 3 \\ 0.06x &= 0.6 \\ x &= 1 \Rightarrow \text{1 liter should be replaced with pure antifreeze.} \end{aligned}$$

62. **Mixing milk** If a bottle holding 3 liters of milk contains $3\frac{1}{2}\%$ butterfat, how much skimmed milk must be added to dilute the milk to 2% butterfat?

Solution

Let x = the liters of skimmed milk added.

$$\boxed{\begin{array}{c}\text{Liters of butterfat}\\ \text{at start}\end{array}} + \boxed{\begin{array}{c}\text{Liters of}\\ \text{butterfat added}\end{array}} = \boxed{\begin{array}{c}\text{Liters of butterfat}\\ \text{at end}\end{array}}$$

$$\begin{aligned} 0.035(3) + 0(x) &= 0.02(3 + x) \\ 0.105 + 0 &= 0.06 + 0.02x \\ 0.045 &= 0.02x \\ 2.25 &= x \Rightarrow \text{2.25 liters of skimmed milk should be added.} \end{aligned}$$

63. **Preparing solutions** A nurse has 1 liter of a solution that is 20% alcohol. How much pure alcohol must he add to bring the solution up to a 25% concentration?

Solution
Let x = the liters of pure alcohol added.

$$\boxed{\begin{array}{c}\text{Liters of}\\\text{alcohol at start}\end{array}} + \boxed{\begin{array}{c}\text{Liters of}\\\text{alcohol added}\end{array}} = \boxed{\begin{array}{c}\text{Liters of}\\\text{alcohol at end}\end{array}}$$

$$\begin{aligned} 0.20(1) + x &= 0.25(1 + x) \\ 0.20 + x &= 0.25 + 0.25x \\ 0.75x &= 0.05 \\ x &= \frac{0.05}{0.75} = \frac{1}{15} \Rightarrow \frac{1}{15} \text{ of a liters of pure alcohol should be added.} \end{aligned}$$

64. **Diluting solutions** If there are 400 cubic centimeters of a chemical in 1 liter of solution, how many cubic centimeters of water must be added to dilute it to a 25% solution? (*Hint:* 1000 cc = 1 liter.)

Solution
Let x = the cubic centimeters of water added.

$$\boxed{\begin{array}{c}\text{Cubic centimeters of}\\\text{chemical at start}\end{array}} + \boxed{\begin{array}{c}\text{Cubic centimeters of}\\\text{chemical added}\end{array}} = \boxed{\begin{array}{c}\text{Cubic centimeters of}\\\text{chemical at end}\end{array}}$$

$$\begin{aligned} 400 + 0 &= 0.25(1000 + x) \\ 400 &= 250 + 0.25x \\ 150 &= 0.25x \\ 600 &= x \Rightarrow \text{600 cubic centimeters of water should be added.} \end{aligned}$$

65. **Cleaning swimming pools** A swimming pool contains 15,000 gallons of water. How many gallons of chlorine must be added to "shock the pool" and bring the water to a $\frac{3}{100}\%$ solution?

Solution
Let x = the gallons of pure chlorine added.

$$\boxed{\begin{array}{c}\text{Gallons of}\\\text{chlorine at start}\end{array}} + \boxed{\begin{array}{c}\text{Gallons of}\\\text{chlorine added}\end{array}} = \boxed{\begin{array}{c}\text{Gallons of}\\\text{chlorine at end}\end{array}}$$

$$\begin{aligned} 0(15000) + x &= 0.0003(15000 + x) \\ x &= 4.5 + 0.0003x \\ 0.9997x &= 4.5 \\ x &\approx 4.5 \Rightarrow \text{About 4.5 gallons of pure chlorine should be added.} \end{aligned}$$

66. **Mixing fuels** An automobile engine can run on a mixture of gasoline and a substitute fuel. If gas costs \$3.50 per gallon and the substitute fuel costs \$2 per gallon, what percent of a mixture must be substitute fuel to bring the cost down to \$2.75 per gallon?

Solution
Let x = the percentage of substitute fuel used. Then 1 – x = the percentage of gasoline used.

$$\boxed{\begin{matrix}\text{Percentage of}\\ \text{gasoline used}\end{matrix}} \cdot \boxed{\begin{matrix}\text{Cost per gallon}\\ \text{of gasoline}\end{matrix}} + \boxed{\begin{matrix}\text{Percentage of}\\ \text{fuel used}\end{matrix}} \cdot \boxed{\begin{matrix}\text{Cost per gallon}\\ \text{of fuel}\end{matrix}} = \boxed{\begin{matrix}\text{Cost per gallon}\\ \text{of mixture}\end{matrix}}$$

$$\begin{aligned} (1 - x)(3.50) + x(2) &= 2.75 \\ 3.5 - 3.5x + 2x &= 2.75 \\ -1.5x &= -0.75 \\ x &= 0.5 = 50\% \end{aligned}$$

The substitute fuel should be 25% of the mixture.

67. **Evaporation** How many liters of water must evaporate to turn 12 liters of a 24% salt solution into a 36% solution?

Solution
Let x = the liters of water evaporated.

$$\boxed{\begin{matrix}\text{Liters of}\\ \text{salt at start}\end{matrix}} - \boxed{\begin{matrix}\text{Liters of}\\ \text{salt evaporated}\end{matrix}} = \boxed{\begin{matrix}\text{Liters of}\\ \text{salt at end}\end{matrix}}$$

$$\begin{aligned} 0.24(12) - 0(x) &= 0.36(12 - x) \\ 2.88 - 0 &= 4.32 - 0.36x \\ 0.36x &= 1.44 \\ x &= 4 \Rightarrow \text{4 liters of water should be evaporated.} \end{aligned}$$

68. **Increasing concentrations** A beaker contains 320 ml of a 5% saltwater solution. How much water should be boiled away to increase the concentration to 6%?

Solution
Let x = the ml of water boiled away.

$$\boxed{\begin{matrix}\text{ml of salt}\\ \text{at start}\end{matrix}} - \boxed{\begin{matrix}\text{ml of salt}\\ \text{removed}\end{matrix}} = \boxed{\begin{matrix}\text{ml of salt}\\ \text{at end}\end{matrix}}$$

$$\begin{aligned} 0.05(320) - 0(x) &= 0.06(320 - x) \\ 16 - 0 &= 19.2 - 0.06x \\ 0.06x &= 3.2 \\ x &= \frac{3.2}{0.06} = \frac{320}{6} = \frac{160}{3} = 53\frac{1}{3} \Rightarrow 53\frac{1}{3} \text{ ml of water should be boiled away.} \end{aligned}$$

69. **Lowering fat** How many pounds of extra-lean hamburger that is 7% fat must be mixed with 30 pounds of hamburger that is 15% fat to obtain a mixture that is 10% fat?

Solution
Let x = the pounds of extra-lean hamburger used.

$$\boxed{\begin{array}{c}\text{Pounds of fat}\\\text{in hamburger}\end{array}} + \boxed{\begin{array}{c}\text{Pounds of fat}\\\text{in lean hamburger}\end{array}} = \boxed{\begin{array}{c}\text{Pounds of fat}\\\text{in mixture}\end{array}}$$

$$\begin{aligned} 0.15(30) + 0.07(x) &= 0.10(30 + x) \\ 4.5 + 0.07x &= 3 + 0.1x \\ 1.5 &= 0.03x \\ 50 &= x \end{aligned}$$

50 pounds of the extra-lean hamburger should be used.

70. **Dairy foods** How many gallons of cream that is 22% butterfat must be mixed with milk that is 2% butterfat to get 20 gallons of milk containing 4% butterfat?

Solution
Let x = the gallons of cream used. Then $20 - x$ = the gallons of milk used

$$\boxed{\begin{array}{c}\text{Gallons of fat}\\\text{in cream}\end{array}} + \boxed{\begin{array}{c}\text{Gallons of fat}\\\text{in milk}\end{array}} = \boxed{\begin{array}{c}\text{Gallons of fat}\\\text{in mixture}\end{array}}$$

$$\begin{aligned} 0.22(x) + 0.02(20 - x) &= 0.04(20) \\ 0.22(x) + 0.4 - 0.02x &= 0.8 \\ 0.2x &= 0.4 \\ x &= 2 \end{aligned}$$

2 gallons of cream should be used.

71. **Mixing solutions** How many gallons of a 5% alcohol solution must be mixed with 90 gallons of 1% solution to obtain a 2% solution?

Solution
Let x = the gallons of 5% solution used.

$$\boxed{\begin{array}{c}\text{Gallons of alc.}\\\text{in 5\% solution}\end{array}} + \boxed{\begin{array}{c}\text{Gallons of alc.}\\\text{in 1\% solution}\end{array}} = \boxed{\begin{array}{c}\text{Gallons of alc.}\\\text{in 2\% solution}\end{array}}$$

$$\begin{aligned} 0.05(x) + 0.01(90) &= 0.02(x + 90) \\ 0.05x + 0.9 &= 0.02x + 1.8 \\ 0.03x &= 0.9 \\ x &= 30 \end{aligned}$$

30 gallons of the 5% solution should be used.

72. **Preparing medicines** A doctor prescribes an ointment that is 2% hydrocortisone. A pharmacist has 1% and 5% concentrations in stock. How much of each should the pharmacist use to make a 1-ounce tube?

Solution

Let x = the ounces of 1% cream used. Then 1 – x = the ounces of 5% cream used.

$$\boxed{\begin{array}{c}\text{Ounces of h.c.}\\\text{in 1\% cream}\end{array}} + \boxed{\begin{array}{c}\text{Ounces of h.c.}\\\text{in 5\% cream}\end{array}} = \boxed{\begin{array}{c}\text{Ounces of h.c.}\\\text{in final cream}\end{array}}$$

$$\begin{aligned} 0.01x + 0.05(1 - x) &= 0.02(1) \\ 0.01x + 0.05 - 0.05x &= 0.02 \\ -0.04x &= -0.03 \\ x &= 0.75 \end{aligned}$$

0.75 ounces of the 1% cream should be used with 0.25 ounces of the 5% cream.

73. **Feeding cattle** A cattleman wants to mix 2400 pounds of cattle feed that is to be 14% protein. Barley (11.7% protein) will make up 25% of the mixture. The remaining 75% will be made up of oats (11.8% protein) and soybean meal (44.5% protein). How many pounds of each will he use?

Solution

Since the mixture is to be 25% barley, there will be 0.25(2400) = 600 pounds of barley used.

Thus, the other 1800 pounds will be either oats or soybean meal.

Let x = the number of pounds of oats used. Then 1800 – x = the number of pounds of meal used.

$$\boxed{\begin{array}{c}\text{Pounds of protein}\\\text{from barley}\end{array}} + \boxed{\begin{array}{c}\text{Pounds of protein}\\\text{from oats}\end{array}} + \boxed{\begin{array}{c}\text{Pounds of protein}\\\text{from soybean meal}\end{array}} = \boxed{\begin{array}{c}\text{Total pounds}\\\text{of protein}\end{array}}$$

$$\begin{aligned} 0.117(600) + 0.118x + 0.445(1800 - x) &= 0.14(2400) \\ 70.2 + 0.118x + 801 - 0.445x &= 336 \\ 871.2 - 0.327x &= 336 \\ -0.327x &= -535.2 \\ x &\approx 1637 \end{aligned}$$

The farmer should use 600 pounds of barley, 1,637 pounds of oats and 163 pounds of soybean meal.

74. **Feeding cattle** If the cattleman in Exercise 73 wants only 20% of the mixture to be barley, how many pounds of each should he use?

Solution

Since the mixture is to be 20% barley, there will be 0.20(2400) = 480 pounds of barley used.

Thus, the other 1920 pounds will be either oats or soybean meal.

Let x = the number of pounds of oats used. Then 1920 – x = the number of pounds of meal used.

$$\boxed{\begin{array}{c}\text{Pounds of protein}\\\text{from barley}\end{array}} + \boxed{\begin{array}{c}\text{Pounds of protein}\\\text{from oats}\end{array}} + \boxed{\begin{array}{c}\text{Pounds of protein}\\\text{from soybean meal}\end{array}} = \boxed{\begin{array}{c}\text{Total pounds}\\\text{of protein}\end{array}}$$

$$0.117(480) + 0.118x + 0.445(1920 - x) = 0.14(2400)$$

$$\begin{aligned} 56.16 + 0.118x + 854.4 - 0.445x &= 336 \\ 910.56 - 0.327x &= 336 \\ -0.327x &= -574.56 \\ x &\approx 1757 \end{aligned}$$

The farmer should use 480 pounds of barley, 1,757 pounds of oats and 163 pounds of soybean meal.

75. **Driving rates** Javed drove to Daytona Beach, Florida, in 5 hours. When he returned, there was less traffic, and the trip took only 3 hours. If Javed averaged 26 mph faster on the return trip, how fast did he drive each way?

Solution

Let r = his first rate. Then $r + 26$ = his return rate.

$$\boxed{\text{Distance to city}} = \boxed{\text{Return distance}}$$

$$\begin{aligned} 5r &= 3(r + 26) \\ 5r &= 3r + 78 \\ 2r &= 78 \\ r &= 39 \Rightarrow \text{He drove 39 mph going and 65 mph returning.} \end{aligned}$$

76. **Distance problem** Allison drove home at 60 mph, but her brother Austin, who left at the same time, could drive at only 48 mph. When Allison arrived, Austin still had 60 miles to go. How far did Allison drive?

Solution

Let t = the time Allison and Austin travel.

$$\boxed{\text{Distance Allison travels}} = \boxed{\text{Distance Austin travels}} + 60$$

$$\begin{aligned} 60t &= 48t + 60 \\ 12t &= 60 \\ t &= 5 \Rightarrow \text{They traveled for 5 hours, so Allison traveled 300 miles.} \end{aligned}$$

77. **Distance problem** Two cars leave Hinds Community College traveling in opposite directions. One car travels at 60 mph and the other at 64 mph. In how many hours will they be 310 miles apart?

Solution

Let t = the time the cars travel.

$$\boxed{\text{Distance 1st car travels}} + \boxed{\text{Distance 2nd car travels}} = \boxed{\text{Total Distance}}$$

$$\begin{aligned} 60t + 64t &= 310 \\ 124t &= 310 \\ t &= 2.5 \Rightarrow \text{They will be 310 miles apart after 2.5 hours.} \end{aligned}$$

78. **Bank robbery** Some bank robbers leave town, speeding at 70 mph. Ten minutes later, the police give chase, traveling at 78 mph. How long, after the robbery, will it take the police to overtake the robbers?

Solution

Let t = the hours the robbers travel. Then $t - \frac{10}{60} = t - \frac{1}{6}$ = the hours the police travel.

$$\boxed{\text{Distance robbers travel}} = \boxed{\text{Distance police travel}}$$

$$\begin{aligned} 70t &= 78\left(t - \frac{1}{6}\right) \\ 70t &= 78t - 13 \\ -8t &= -13 \\ t &= \frac{13}{8} = 1\frac{5}{8} \end{aligned}$$

The police will catch up $1\frac{5}{8}$ hours after the robbery.

79. **Jogging problem** Two cross-country runners are 440 yards apart and are running toward each other, one at 8 mph and the other at 10 mph. In how many seconds will they meet?

Solution
Let t = the time the runners run.

$$\boxed{\text{Distance 1st runs}} + \boxed{\text{Distance 2nd runs}} = \boxed{\text{Distance between them (in miles)}}$$

$$\begin{aligned} 8t + 10t &= \frac{440}{1760} \\ 18t &= \frac{1}{4} \\ t &= \frac{1}{72}\text{ hour} = \frac{1}{72}(60\text{ minutes}) = \frac{5}{6}\text{ minute} = 50\text{ seconds} \end{aligned}$$

They will meet after 50 seconds.

80. **Driving rates** One morning, Justin drove 5 hours before stopping to eat lunch. After lunch, he increased his speed by 10 mph. If he completed a 430-mile trip in 8 hours of driving time, how fast did he drive in the morning?

Solution
Let r = the rate before lunch. Then $r + 10$ = the rate after lunch.

$$\boxed{\begin{array}{c}\text{Distance}\\\text{before lunch}\end{array}} + \boxed{\begin{array}{c}\text{Distance}\\\text{after lunch}\end{array}} = \boxed{\text{Total Distance}}$$

$$\begin{aligned} 5r + 3(r + 10) &= 430 \\ 5r + 3r + 30 &= 430 \\ 8r &= 400 \\ r &= 50 \Rightarrow \text{He drove 50 mph before lunch.} \end{aligned}$$

81. **Boating problem** A Johnson motorboat goes 5 miles upstream in the same time it requires to go 7 miles downstream. If the river flows at 2 mph, find the speed of the boat in still water.

Solution
Let r = the speed of the boat in still water.
Then the speed of the boat is $r + 2$ downstream and $r - 2$ upstream.

$$\boxed{\text{Time upstream}} = \boxed{\text{Time downstream}} \qquad \{\text{Note: Time} = \text{Distance} \div \text{Rate}\}$$

$$\begin{aligned} \frac{5}{r - 2} &= \frac{7}{r + 2} \\ (r + 2)(r - 2)\frac{5}{r - 2} &= (r + 2)(r - 2)\frac{7}{r + 2} \\ 5(r + 2) &= 7(r - 2) \\ 5r + 10 &= 7r - 14 \\ 24 &= 2r \\ 12 &= r \Rightarrow \text{The speed of the boat is 12 mph.} \end{aligned}$$

82. **Wind velocity** A plane can fly 340 mph in still air. If it can fly 200 miles downwind in the same amount of time it can fly 140 miles upwind, find the velocity of the wind.

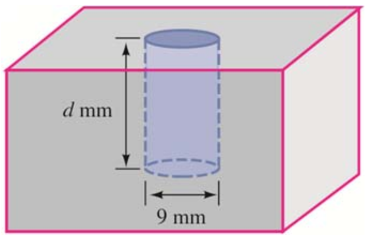
Solution
Let w = the speed of the wind.
Then the speed of the plane is $340 + w$ downwind and $340 - w$ upwind.

$$\boxed{\text{Time upwind}} = \boxed{\text{Time downwind}}$$

$$\begin{aligned} \frac{140}{340 - w} &= \frac{200}{340 + w} \\ (340 + w)(340 - w)\frac{140}{340 - w} &= (340 + w)(340 - w)\frac{200}{340 + w} \\ 140(340 + w) &= 200(340 - w) \\ 47{,}600 + 140w &= 68{,}000 - 200w \\ 340w &= 20{,}400 \\ w &= 60 \Rightarrow \text{The speed of the wind is 60 mph.} \end{aligned}$$

Use a calculator to help solve each problem.

83. **Machine tool design** 712.51 cubic millimeters of material was removed by drilling the blind hole as shown in the illustration. Find the depth of the hole. (*Hint:* The volume of a cylinder is given by $V = \pi r^2 h$.)

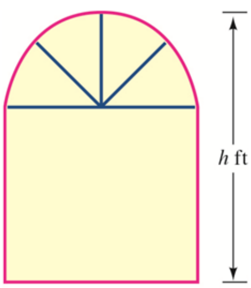


Solution

$$\begin{aligned} V &= \pi r^2 h \\ 712.51 &= \pi(4.5)^2 d \\ \frac{712.51}{\pi(4.5)^2} &= d \\ 11.2 &\approx d \end{aligned}$$

The hole is about 11.2 millimeters deep.

84. **Architecture** The Norman window with dimensions as shown is a rectangle topped by a semicircle. If the area of the window is 68.2 square feet, find its height h.



Solution

Since the diameter of the semicircle is 6 feet, the radius of the semicircle is 3 feet.

$$\boxed{\text{Area of rectangle}} + \boxed{\text{Area of semicircle}} = \boxed{\text{Total area}}$$

$$\begin{aligned} 6(h-3) + \tfrac{1}{2}\pi(3)^2 &= 68.2 \\ 6h - 18 + 4.5\pi &= 68.2 \\ 6h &= 68.2 + 18 - 4.5\pi \\ 6h &\approx 72.0628 \\ h &\approx 12 \end{aligned}$$

The height of the window is about 12 feet.

Discovery and Writing

85. Consider the strategy you use to solve investment and uniform motion problems. Describe any similarities you observe in these problem types.

    Solution
    Answers may vary.

86. Which type of application was hardest for you to solve? Why? What strategy or approach works best for you when approaching solving this problem?

    Solution
    Answers may vary.

87. Explain why the solution to an application problem should be checked in the original wording of the problem and not in the equation obtained from the words.

    Solution
    Answers may vary.

EXERCISES 1.3

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Simplify the radical. $\sqrt{48}$

    Solution

    $\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$

2. Subtract and simplify. $\sqrt{8} - 2\sqrt{50}$

    Solution

    $$\begin{aligned} \sqrt{8} - 2\sqrt{50} &= \sqrt{4 \cdot 2} - 2\sqrt{50} \\ &= 2\sqrt{2} - 2\sqrt{25 \cdot 2} \\ &= 2\sqrt{2} - 2 \cdot 5\sqrt{2} \\ &= 2\sqrt{2} - 10\sqrt{2} \\ &= -8\sqrt{2} \end{aligned}$$

3. Multiply. $(3 + 2x)(2 - 7x)$

    Solution

    $(3 + 2x)(2 - 7x) = 6 - 21x + 4x - 14x^2 = -14x^2 - 17x + 6$

4. Multiply. $\left(5 + 2\sqrt{7}\right)\left(5 - 2\sqrt{7}\right)$

Solution

$\left(5 + 2\sqrt{7}\right)\left(5 - 2\sqrt{7}\right) = 25 - 10\sqrt{7} + 10\sqrt{7} - 4 \cdot 7 = 25 - 28 = -3$

5. Rationalize the denominator and simplify. $\dfrac{4}{\sqrt{6}}$

Solution

$\dfrac{4}{\sqrt{6}}\left(\dfrac{\sqrt{6}}{\sqrt{6}}\right) = \dfrac{4\sqrt{6}}{6} = \dfrac{2\sqrt{6}}{3}$

6. Rationalize the denominator and simplify. $\dfrac{3 + 2\sqrt{2}}{4 - \sqrt{2}}$

Solution

$\dfrac{3 + 2\sqrt{2}}{4 - \sqrt{2}}\left(\dfrac{4 + \sqrt{2}}{4 + \sqrt{2}}\right) = \dfrac{12 + 3\sqrt{2} + 4\sqrt{2} + 2 \cdot 2}{16 + 4\sqrt{2} - 4\sqrt{2} - 2} = \dfrac{16 + 11\sqrt{2}}{14}$

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. $\sqrt{3}$, $\sqrt{-9}$ and $\sqrt{-12}$ are examples of __________ numbers.

Solution
imaginary

8. In the complex number $a + bi$, a is the __________ part, and b is the __________ part.

Solution
real, imaginary

9. If $a = 0$ and $b \neq 0$ in the complex number $a + bi$, the number is an __________ number.

Solution
imaginary

10. If $b = 0$ in the complex number $a + bi$, the number is a __________ number.

Solution
real

11. The complex conjugate of $2 + 5i$ is __________.

Solution

$2 - 5i$

12. By definition, $|a + bi| =$ __________.

Solution

$\sqrt{a^2 + b^2}$

13. The absolute value of a complex number is a __________ number.

Solution

real

14. The product of two complex conjugates is a __________ number.

Solution

real

Practice

Simplify the imaginary numbers.

15. $\sqrt{-144}$

Solution

$\sqrt{-144} = \sqrt{-1}\sqrt{144} = 12i$

16. $-\sqrt{-225}$

Solution

$-\sqrt{-225} = -\sqrt{-1}\sqrt{225} = -15i$

17. $-\sqrt{-128}$

Solution

$-\sqrt{-128} = -\sqrt{-1}\sqrt{64}\sqrt{2} = -8i\sqrt{2}$

18. $\sqrt{-108}$

Solution

$\sqrt{-108} = \sqrt{-1}\sqrt{36}\sqrt{3} = 6i\sqrt{3}$

19. $-2\sqrt{-24}$

Solution

$-2\sqrt{-24} = -2\sqrt{-1}\sqrt{24} = -2i \cdot 2\sqrt{6} = -4i\sqrt{6}$

20. $7\sqrt{-48}$

Solution

$7\sqrt{-48} = 7\sqrt{-1}\sqrt{48} = 7i \cdot 4\sqrt{3} = 28i\sqrt{3}$

21. $\sqrt{-25}\sqrt{-4}$

Solution

$\sqrt{-25}\sqrt{-4} = \sqrt{-1}\sqrt{25}\sqrt{-1}\sqrt{4} = 10i^2 = 10 \cdot -1 = -10$

22. $3\sqrt{-8}\sqrt{-1}$

Solution

$3\sqrt{-8}\sqrt{-1} = 3\sqrt{-1}\sqrt{8}\sqrt{-1} = i^2 \cdot 3 \cdot 2\sqrt{2} = -6\sqrt{2}$

23. $\dfrac{\sqrt{-16}}{\sqrt{-9}}$

Solution

$\dfrac{\sqrt{-16}}{\sqrt{-9}} = \dfrac{\sqrt{-1}\sqrt{16}}{\sqrt{-1}\sqrt{9}} = \dfrac{4i}{3i} = \dfrac{4}{3}$

24. $\dfrac{6\sqrt{-1}}{\sqrt{-64}}$

Solution

$\dfrac{6\sqrt{-1}}{\sqrt{-64}} = \dfrac{6i}{\sqrt{-1}\sqrt{64}} = \dfrac{6i}{8i} = \dfrac{3}{4}$

25. $\sqrt{-\dfrac{50}{9}}$

Solution

$\sqrt{-\dfrac{50}{9}} = \sqrt{-1} \cdot \dfrac{\sqrt{50}}{\sqrt{9}} = i \cdot \dfrac{5\sqrt{2}}{3} = \dfrac{5\sqrt{2}}{3}i$

26. $-\sqrt{-\dfrac{72}{25}}$

Solution

$-\sqrt{-\dfrac{72}{25}} = -\sqrt{-1} \cdot \dfrac{\sqrt{72}}{\sqrt{25}} = -i \cdot \dfrac{6\sqrt{2}}{5} = -\dfrac{6\sqrt{2}}{5}i$

27. $-7\sqrt{-\frac{3}{8}}$

Solution

$$-7\sqrt{-\frac{3}{8}} = -7\sqrt{-1}\cdot\frac{\sqrt{3}}{\sqrt{8}} = -7i\cdot\frac{\sqrt{3}}{\sqrt{8}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -7i\cdot\frac{\sqrt{6}}{\sqrt{16}} = -\frac{7\sqrt{6}}{4}i$$

28. $5\sqrt{-\frac{5}{27}}$

Solution

$$5\sqrt{-\frac{5}{27}} = 5\sqrt{-1}\cdot\frac{\sqrt{5}}{\sqrt{27}} = 5i\cdot\frac{\sqrt{5}}{\sqrt{27}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = 5i\cdot\frac{\sqrt{15}}{\sqrt{81}} = \frac{5\sqrt{15}}{9}i$$

Determine the real and imaginary part of each complex number.

29. $6 - 11i$

Solution

real part is 6; imaginary part is −11

30. $\frac{3}{5} + 6i$

Solution

real part is $\frac{3}{5}$; imaginary part is 6

31. $\sqrt{5} - \frac{2}{3}i$

Solution

real part is $\sqrt{5}$; imaginary part is $\frac{-2}{3}$

32. $9 - \pi i$

Solution

real part is 9; imaginary part is $-\pi$

33. $-\frac{4}{5}$

Solution

real part is $\frac{-4}{5}$; imaginary part is 0

34. $6i$

Solution

real part is 0; imaginary part is 6

Find the values of *x* and *y*.

35. $x + (x + y)i = 3 + 8i$

Solution

Equate real parts: $\boxed{x = 3}$

$x + y = 8$

Equate imaginary parts: $3 + y = 8$

$\boxed{y = 5}$

36. $x + 5i = y - yi$

Solution

Equate imaginary parts: $5 = -y$

$\boxed{-5 = y}$

Equate real parts: $5 = y$

$\boxed{x = -5}$

37. $3x - 2yi = 2 + (x + y)i$

Solution

Equate real parts: $3x = 2$

$\boxed{x = \frac{2}{3}}$

Equate imaginary parts:

$-2y = x + y$

$-3y = x$

$y = -\frac{1}{3}x$

$y = -\frac{1}{3} \cdot \frac{2}{3}$

$\boxed{y = -\frac{2}{9}}$

38. $\begin{cases} 2 + (x + y)i = 2 - i \\ x + 3i = 2 + 3i \end{cases}$

Solution

Equate real parts: $\boxed{x = 2}$

Equate imaginary parts:

$x + y = -1$

$2 + y = -1$

$\boxed{y = -3}$

Perform all operations. Give all answers in *a* + *bi* form.

39. $\sqrt{-4} + \sqrt{-100}$

Solution

$\sqrt{-4} + \sqrt{-100} = \sqrt{-1}\sqrt{4} + \sqrt{-1}\sqrt{100} = 2i + 10i = 0 + 12i$

40. $\sqrt{-9} - \sqrt{-121}$

Solution

$\sqrt{-9} - \sqrt{-121} = \sqrt{-1}\sqrt{9} - \sqrt{-1}\sqrt{121} = 3i - 11i = 0 - 8i$

41. $2\sqrt{-72} - \sqrt{-18}$

Solution

$2\sqrt{-72} - \sqrt{-18} = 2\sqrt{-1}\sqrt{72} - \sqrt{-1}\sqrt{18} = 2i \cdot 6\sqrt{2} - 3i\sqrt{2} = 0 + 9i\sqrt{2}$

42. $\sqrt{-27} + 3\sqrt{-75}$

Solution

$\sqrt{-27} + 3\sqrt{-75} = \sqrt{-1}\sqrt{27} + 3\sqrt{-1}\sqrt{75} = 3i\sqrt{3} + 3i \cdot 5\sqrt{3} = 0 + 18i\sqrt{3}$

43. $\dfrac{-6 + \sqrt{-4}}{2}$

Solution

$\dfrac{-6 + \sqrt{-4}}{2} = \dfrac{-6 + \sqrt{-1}\sqrt{4}}{2} = \dfrac{-6 + 2i}{2} = -3 + i$

44. $\dfrac{-8 - \sqrt{-100}}{-6}$

Solution

$\dfrac{-8 - \sqrt{-100}}{-6} = \dfrac{-8 - \sqrt{-1}\sqrt{100}}{-6}\dfrac{-8 - 10i}{-6} = \dfrac{4}{3} + \dfrac{5}{3}i$

45. $\dfrac{-12 - \sqrt{-18}}{3}$

Solution

$\dfrac{-12 - \sqrt{-18}}{-3} = \dfrac{-12 - \sqrt{-1}\sqrt{18}}{3} = \dfrac{-12 - 3i}{3} = -4 - i\sqrt{2}$

46. $\dfrac{5 + \sqrt{-200}}{20}$

Solution

$\dfrac{5 + \sqrt{-200}}{20} = \dfrac{5 + \sqrt{-1}\sqrt{200}}{20} = \dfrac{5 + 10i\sqrt{2}}{20} = \dfrac{1}{4} + \dfrac{\sqrt{2}}{2}i$

47. $(2 - 7i) + (3 + i)$

Solution

$$\begin{aligned}(2 - 7i) + (3 + i) &= 2 - 7i + 3 + i \\ &= 5 - 6i\end{aligned}$$

48. $(-7 + 2i) + (2 - 8i)$

Solution

$$\begin{aligned}(-7 + 2i) + (2 + 8i) &= -7 + 2i + 2 - 8i \\ &= -5 - 6i\end{aligned}$$

49. $(5 - 6i) - (7 + 4i)$

Solution

$$\begin{aligned}(5 - 6i) - (7 + 4i) &= 5 - 6i - 7 - 4i \\ &= -2 - 10i\end{aligned}$$

50. $(11 + 2i) - (13 - 5i)$

Solution

$$\begin{aligned}(11 + 2i) - (13 - 5i) &= 11 + 2i - 13 + 5i \\ &= -2 + 7i\end{aligned}$$

51. $(14i + 2) + \left(2 - \sqrt{-16}\right)$

Solution

$$(14i + 2) + \left(2 - \sqrt{-16}\right) = (14i + 2) + (2 - 4i) = 14i + 2 + 2 - 4i = 4 + 10i$$

52. $\left(5 + \sqrt{-64}\right) - (23i - 32)$

Solution

$$\left(5 + \sqrt{-64}\right) - (23i - 32) = (5 + 8i) - (23i - 32) = 5 + 8i - 23i + 32 = 37 - 15i$$

53. $\left(3 + \sqrt{-4}\right) - \left(2 + \sqrt{-9}\right)$

Solution

$$\left(3 + \sqrt{-4}\right) - \left(2 + \sqrt{-9}\right) = (3 + 2i) - (2 + 3i) = 3 + 2i - 2 - 3i = 1 - i$$

54. $\left(7 - \sqrt{-25}\right) + \left(-8 + \sqrt{-1}\right)$

Solution

$$\left(7 - \sqrt{-25}\right) + \left(-8 + \sqrt{-1}\right) = (7 - 5i) + (-8 + i) = 7 - 5i - 8 + i = -1 - 4i$$

55. $(4 + 7i) + (8 - 2i) - (5 + 4i)$

Solution

$(4 + 7i) + (8 - 2i) - (5 + 4i) = 4 + 7i + 8 - 2i - 5 - 4i = 7 + i$

56. $(5 - 7i) - (4 - 2i) + (8 + i)$

Solution

$(5 - 7i) - (4 - 2i) + (8 + i) = 5 - 7i - 4 + 2i + 8 + i = 9 - 4i$

57. $\left(3 + \sqrt{-16}\right) - \left(4 - \sqrt{-36}\right) + \left(5 - \sqrt{-144}\right)$

Solution

$$\left(3 + \sqrt{-16}\right) - \left(4 - \sqrt{-36}\right) + \left(5 - \sqrt{-144}\right) = (3 + 4i) - (4 - 6i) + (5 - 12i)$$
$$= 3 + 4i - 4 + 6i + 5 - 12i = 4 - 2i$$

58. $\left(-1 + \sqrt{-1}\right) - \left(2 + \sqrt{-81}\right) + \left(8 - \sqrt{-121}\right)$

Solution

$$\left(-1 + \sqrt{-1}\right) - \left(2 + \sqrt{-81}\right) + \left(8 - \sqrt{-121}\right) = (-1 + i) - (2 + 9i) + (8 - 11i)$$
$$= -1 + i - 2 - 9i + 8 - 11i = 5 - 19i$$

59. $-5(3 + 5i)$

Solution

$-5(3 + 5i) = -15 - 25i$

60. $5(2 - i)$

Solution

$5(2 - i) = 10 - 5i$

61. $7i(4 - 8i)$

Solution

$7i(4 - 8i) = 28i - 56i^2 = 28i - 56(-1) = 28i + 56 = 56 + 28i$

62. $-2i(3 - 7i)$

Solution

$-2i(3 - 7i) = -6i + 14i^2 = -6i + 14(-1) = -6i - 14 = -14 - 6i$

63. $(2 + 3i)(3 + 5i)$

Solution

$(2 + 3i)(3 + 5i) = 6 + 19i + 15(-1) = 6 + 19i - 15 = -9 + 19i$

64. $(5 - 7i)(2 + i)$

Solution

$(5 - 7i)(2 + i) = 10 - 9i - 7i^2 = 10 - 9i - 7(-1) = 10 - 9i + 7 = 17 - 9i$

65. $(2 + 3i)^2$

Solution

$(2 + 3i)^2 = (2 + 3i)(2 + 3i) = 4 + 12i + 9i^2 = 4 + 12i + 9(-1) = 4 + 12i - 9 = -5 + 12i$

66. $(3 - 4i)^2$

Solution

$(3 - 4i)^2 = (3 - 4i)(3 - 4i) = 9 - 24i + 16i^2 = 9 - 24i + 16(-1) = 9 - 24i - 16 = -7 - 24i$

67. $(11 + \sqrt{-25})(2 - \sqrt{-36})$

Solution

$$\begin{aligned}(11 + \sqrt{-25})(2 - \sqrt{-36}) = (11 + 5i)(2 - 6i) = 22 - 56i - 30i^2 &= 22 - 56i - 30(-1) \\ &= 22 - 56i + 30 = 52 - 56i\end{aligned}$$

68. $(6 + \sqrt{-49})(6 - \sqrt{-49})$

Solution

$(6 + \sqrt{-49})(6 - \sqrt{-49}) = (6 + 7i)(6 - 7i) = 36 - 49i^2 = 36 - 49(-1) = 36 + 49 = 85 + 0i$

69. $(\sqrt{-16} + 3)(2 + \sqrt{-9})$

Solution

$$\begin{aligned}(\sqrt{-16} + 3)(2 + \sqrt{-9}) = (4i + 3)(2 + 3i) = 6 + 17i + 12i^2 &= 6 + 17i + 12(-1) \\ &= 6 + 17i - 12 = -6 + 17i\end{aligned}$$

70. $(12 - \sqrt{-4})(-7 + \sqrt{-25})$

Solution

$$\left(12 - \sqrt{-4}\right)\left(-7 + \sqrt{-25}\right) = \left(12 - 2i\right)\left(-7 + 5i\right) = -84 + 74i - 10i^2 = -84 + 74i - 10\left(-1\right)$$
$$= -84 + 74i + 10$$
$$= -74 + 74i$$

71. $\dfrac{1}{-i}$

Solution

$$\frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{1} = 0 + i$$

72. $\dfrac{3}{i}$

Solution

$$\frac{3}{i} = \frac{3}{i} \cdot \frac{i}{i} = \frac{3i}{i^2} = \frac{3i}{-1} = 0 - 3i$$

73. $\dfrac{-4}{3i}$

Solution

$$\frac{-4}{3i} = \frac{-4}{3i} \cdot \frac{i}{i} = \frac{-4i}{3i^2} = \frac{-4i}{-3} = 0 + \frac{4}{3}i$$

74. $\dfrac{10}{7i}$

Solution

$$\frac{10}{7i} = \frac{10}{7i} \cdot \frac{i}{i} = \frac{10i}{7i^2} = \frac{10i}{-7} = 0 - \frac{10}{7}i$$

75. $\dfrac{1}{2 + i}$

Solution

$$\frac{1}{2 + i} = \frac{1\left(2 - i\right)}{\left(2 + i\right)\left(2 - i\right)} = \frac{2 - i}{2^2 - i^2} = \frac{2 - i}{4 - \left(-1\right)} = \frac{2 - i}{5} = \frac{2}{5} - \frac{1}{5}i$$

76. $\dfrac{-2}{3 - i}$

Solution

$$\frac{-2}{3 - i} = \frac{-2\left(3 + i\right)}{\left(3 - i\right)\left(3 + i\right)} = \frac{-2\left(3 + i\right)}{3^2 - i^2} = \frac{-2\left(3 + i\right)}{9 - \left(-1\right)} = \frac{-2\left(3 + i\right)}{10} = \frac{-\left(3 + i\right)}{5} = -\frac{3}{5} - \frac{1}{5}i$$

77. $\dfrac{2i}{7 + i}$

Solution

$$\frac{2i}{7+i} = \frac{2i(7-i)}{(7+i)(7-i)} = \frac{14i - 2i^2}{7^2 - i^2} = \frac{14i - 2(-1)}{49 - (-1)} = \frac{14i + 2}{50} = \frac{7i + 1}{25} = \frac{1}{25} + \frac{7}{25}i$$

78. $\dfrac{-3i}{2 + 5i}$

Solution

$$\frac{-3i}{2+5i} = \frac{-3i(2+5i)}{(2+5i)(2-5i)} = \frac{-6i + 15i^2}{2^2 - (5i)^2} = \frac{-6i - 15}{4 - 25i^2} = \frac{-6i + 15}{29} = -\frac{15}{29} - \frac{6}{29}i$$

79. $\dfrac{2 + i}{3 - i}$

Solution

$$\frac{2+i}{3-i} = \frac{(2+i)(3+i)}{(3-i)(3+i)} = \frac{6 + 5i + i^2}{9 - i^2} = \frac{5 + 5i}{10} = \frac{5}{10} + \frac{5}{10}i = \frac{1}{2} + \frac{1}{2}i$$

80. $\dfrac{3 - i}{1 + i}$

Solution

$$\frac{3-i}{1+i} = \frac{(3-i)(1-i)}{(1+i)(1-i)} = \frac{3 - 4i + i^2}{1 - i^2} = \frac{2 - 4i}{2} = \frac{2}{2} - \frac{4}{2}i = 1 - 2i$$

81. $\dfrac{4 - 5i}{2 + 3i}$

Solution

$$\frac{4-5i}{2+3i} = \frac{(4-5i)(2-3i)}{(2+3i)(2-3i)} = \frac{8 - 22i + 15i^2}{4 - 9i^2} = \frac{-7 - 22i}{13} = -\frac{7}{13} - \frac{22}{13}i$$

82. $\dfrac{34 + 2i}{2 - 4i}$

Solution

$$\frac{34+2i}{2-4i} = \frac{(34+2i)(2+4i)}{(2-4i)(2+4i)} = \frac{68 + 140i + 8i^2}{4 - 16i^2} = \frac{60 + 140i}{20} = \frac{60}{20} + \frac{140}{20}i = 3 + 7i$$

83. $\dfrac{5 - \sqrt{-16}}{-8 + \sqrt{-4}}$

Solution

$$\frac{5 - \sqrt{-16}}{-8 + \sqrt{-4}} = \frac{5 - 4i}{-8 + 2i} = \frac{(5 - 4i)(-8 - 2i)}{(-8 - 2i)(-8 - 2i)} = \frac{-40 + 22i + 8i^2}{64 - 4i^2} = \frac{-48 + 22i}{68} = -\frac{48}{68} + \frac{22}{68}i$$
$$= -\frac{12}{17} + \frac{11}{34}i$$

84. $\dfrac{3 - \sqrt{-9}}{2 - \sqrt{-1}}$

Solution

$$\frac{3 - \sqrt{-9}}{2 - \sqrt{-1}} = \frac{3 - 3i}{2 - i} = \frac{(3 - 3i)(2 + i)}{(2 - i)(2 + i)} = \frac{6 - 3i - 3i^2}{4 - i^2} = \frac{9 - 3i}{5} = \frac{9}{5} - \frac{3}{5}i$$

85. $\dfrac{2 + i\sqrt{3}}{3 + i}$

Solution

$$\frac{2 + i\sqrt{3}}{3 + i} = \frac{(2 + i\sqrt{3})(3 - i)}{(3 + i)(3 - i)} = \frac{6 - 2i + 3i\sqrt{3} - i^2\sqrt{3}}{9 - i^2} = \frac{6 + \sqrt{3} + (3\sqrt{3} - 2)i}{10}$$
$$= \frac{6 + \sqrt{3}}{10} + \frac{3\sqrt{3} - 2}{10}i$$

86. $\dfrac{3 + i}{4 - i\sqrt{2}}$

Solution

$$\frac{3 + i}{4 - i\sqrt{2}} = \frac{(3 + i)(4 + i\sqrt{2})}{(4 - i\sqrt{2})(4 + i\sqrt{2})} = \frac{12 + 3i\sqrt{2} + 4i + i^2\sqrt{2}}{16 - 2i^2} = \frac{12 - \sqrt{2} + (3\sqrt{2} + 4)i}{18}$$
$$= \frac{12 - \sqrt{2}}{18} + \frac{4 + 3\sqrt{2}}{18}i$$

Simplify each expression.

87. i^9

Solution

$$i^9 = i^8 i = (i^4)^2 i = 1^2 i = i$$

88. i^{26}

Solution

$i^{26} = i^{24}i^2 = \left(i^4\right)^6 i^2 = 1^6 i^2 = 1(-1) = -1$

89. i^{38}

Solution

$i^{38} = i^{36}i^2 = \left(i^4\right)^9 i^2 = 1^9 i^2 = i^2 = -1$

90. i^{99}

Solution

$i^{99} = i^{96}i^3 = \left(i^4\right)^{24} i^3 = 1^{24} i^3 = i^3 = -i$

91. i^{87}

Solution

$i^{87} = i^{84}i^3 = \left(i^4\right)^{21} i^3 = 1^{21} i^3 = i^3 = -i$

92. i^{44}

Solution

$i^{44} = \left(i^4\right)^{11} = 1^{11} = 1$

93. i^{100}

Solution

$i^{100} = \left(i^4\right)^{25} = 1^{25} = 1$

94. i^{201}

Solution

$i^{201} = i^{200}i = \left(i^4\right)^{50} i = 1^{50} i = i$

95. i^{-6}

Solution

$i^{-6} = \dfrac{1}{i^6} = \dfrac{1 \cdot i^2}{i^6 \cdot i^2} = \dfrac{i^2}{i^8} = \dfrac{i^2}{1} = i^2 = -1$

96. i^0

Solution

$i^0 = 1$

97. i^{-10}

Solution

$$i^{-10} = \frac{1}{i^{10}} = \frac{1 \cdot i^2}{i^{10} \cdot i^2} = \frac{i^2}{i^{12}} = \frac{i^2}{1} = i^2 = -1$$

98. i^{-31}

Solution

$$i^{-31} = \frac{1}{i^{31}} = \frac{1 \cdot i}{i^{31} \cdot i} = \frac{i}{i^{32}} = \frac{i}{1} = i$$

99. $\frac{1}{i^3}$

Solution

$$\frac{1}{i^3} = \frac{1 \cdot i}{i^3 \cdot i} = \frac{i}{i^4} = \frac{i}{1} = i = i$$

100. $\frac{3}{i^5}$

Solution

$$\frac{3}{i^5} = \frac{3 \cdot i^3}{i^5 \cdot i^3} = \frac{3i^3}{i^8} = \frac{3i^3}{1} = 3i^3 = -3i$$

101. $\frac{-4}{i^{10}}$

Solution

$$\frac{-4}{i^{10}} = \frac{-4 \cdot i^2}{i^{10} \cdot i^2} = \frac{-4i^2}{i^{12}} = \frac{-4i^2}{1} = -4(-1) = 4$$

102. $\frac{-10}{i^{24}}$

Solution

$$\frac{-10}{i^{24}} = \frac{-10}{1} = -10$$

Write without absolute value symbols.

103. $|3 + 4i|$

Solution

$|3 + 4i| = \sqrt{3^2 + 4^4} = \sqrt{9 + 16} = \sqrt{25} = 5$

104. $|5 + 12i|$

Solution

$|5 + 12i| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

105. $|2 + 3i|$

Solution

$|2 + 3i| = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}$

106. $|5 - i|$

Solution

$|5 - i| = \sqrt{5^2 + (-1)^2} = \sqrt{24 + 1} = \sqrt{26}$

107. $\left|-7 + \sqrt{-49}\right|$

Solution

$\left|-7 + \sqrt{-49}\right| = |-7 + 7i| = \sqrt{(-7)^2 + 7^2} = \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$

108. $\left|-2 - \sqrt{-16}\right|$

Solution

$\left|-2 - \sqrt{-16}\right| = |-2 - 4i| = \sqrt{(-2)^2 + (-4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

109. $\left|\frac{1}{2} + \frac{1}{2}i\right|$

Solution

$\left|\frac{1}{2} + \frac{1}{2}i\right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

110. $\left|\frac{1}{2} - \frac{1}{4}i\right|$

Solution

$$\left|\frac{1}{2} - \frac{1}{4}i\right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{4}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{16}} = \sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{4}$$

111. $|-6i|$

Solution

$$|-6i| = |0 - 6i| = \sqrt{0^2 + (-6)^2} = \sqrt{0 + 36} = \sqrt{36} = 6$$

112. $|5i|$

Solution

$$|5i| = |0 + 5i| = \sqrt{0^2 + 5^2} = \sqrt{0 + 25} = \sqrt{25} = 5$$

113. $\left|\frac{2}{1 + i}\right|$

Solution

$$\left|\frac{2}{1 + i}\right| = \left|\frac{2(1 - i)}{(1 + i)(1 - i)}\right| = \left|\frac{2(1 - i)}{1 - i^2}\right| = \left|\frac{2(1 - i)}{2}\right| = |1 - i| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

114. $\left|\frac{3}{3 + i}\right|$

Solution

$$\begin{aligned}\left|\frac{3}{3 + i}\right| = \left|\frac{3(3 - i)}{(3 + i)(3 - i)}\right| = \left|\frac{3(3 - i)}{9 - i^2}\right| = \left|\frac{3(3 - i)}{10}\right| = \left|\frac{9 - 3i}{10}\right| &= \sqrt{\left(\frac{9}{10}\right)^2 + \left(-\frac{3}{10}\right)^2} \\ &= \sqrt{\frac{81}{100} + \frac{9}{100}} \\ &= \sqrt{\frac{90}{100}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}\end{aligned}$$

115. $\left|\frac{-3i}{2 + i}\right|$

Solution

$$\left|\frac{-3i}{2+i}\right| = \left|\frac{-3i(2-i)}{(2+i)(2-i)}\right| = \left|\frac{-3i(2-i)}{4-i^2}\right| = \left|\frac{-3i(2-i)}{5}\right| = \left|\frac{-6i+3i^2}{5}\right|$$

$$= \left|-\frac{3}{5}-\frac{6}{5}i\right|$$

$$= \sqrt{\left(-\frac{3}{5}\right)^2+\left(-\frac{6}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25}+\frac{36}{25}}$$

$$= \sqrt{\frac{45}{25}} = \sqrt{\frac{45}{5}} = \frac{3\sqrt{5}}{5}$$

116. $\left|\frac{5i}{i-2}\right|$

Solution

$$\left|\frac{5i}{i-2}\right| = \left|\frac{5i}{-2+i}\right| = \left|\frac{5i(-2-i)}{(-2+i)(-2-i)}\right|\left|\frac{5i(-2-i)}{4=i^2}\right| = \left|\frac{5i(-2-i)}{5}\right| = \left|\frac{-10i-5i^2}{5}\right|$$

$$= |1-2i|$$

$$= \sqrt{1+(-2)^2}$$

$$= \sqrt{1+4} = \sqrt{5}$$

117. $\left|\frac{i+2}{i-2}\right|$

Solution

$$\left|\frac{i+2}{i-2}\right| = \left|\frac{(i+2)(i+2)}{(i-2)(i+2)}\right| = \left|\frac{i^2+4i+4}{i^2-4}\right| = \left|\frac{3+4i}{5}\right| = \left|\frac{3}{5}+\frac{4}{5}i\right| = \sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25}+\frac{16}{25}}$$

$$= \sqrt{\frac{25}{25}} = \sqrt{1} = 1$$

118. $\left|\frac{2+i}{2-i}\right|$

Solution

$$\left|\frac{2+i}{2-i}\right| = \left|\frac{(2+i)(2+i)}{(2-i)(2-i)}\right| = \left|\frac{4+4i+i^2}{4^2-i^2}\right| = \left|\frac{3+4i}{5}\right| = \left|\frac{3}{5}+\frac{4}{5}i\right| = \sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25}+\frac{16}{25}}$$

$$= \sqrt{\frac{25}{25}} = \sqrt{1} = 1$$

Factor each expression over the set of complex numbers.

119. $x^2 + 4$

Solution

$$x^2 + 4 = x^2 - (-4) = x^2 - (2i^2) = (x + 2i)(x - 2i)$$

120. $16a^2 + 9$

Solution

$$16a^2 + 9 = (4a)^2 - (-9) = (4a)^2 - (3i)^2 = (4a + 3i)(4a - 3i)$$

121. $25p^2 + 36q^2$

Solution

$$25p^2 + 36q^2 = (5p)^2 - (-36q^2) = (5p)^2 - (6qi)^2 = (5p + 6qi)(5p - 6qi)$$

122. $100r^2 + 49s^2$

Solution

$$100r^2 + 49s^2 = (10r)^2 - (-49s^2) = (10r)^2 - (7si)^2 = (10r + 7si)(10r - 7si)$$

123. $2y^2 + 8z^2$

Solution

$$2y^2 + 8z^2 = 2(y^2 + 4z^2) = 2\left[y^2 - (-4z^2)\right] = 2\left[y^2 - (2zi)^2\right] = 2(y + 2zi)(y - 2zi)$$

124. $12b^2 + 75c^2$

Solution

$$12b^2 + 75c^2 = 3(4b^2 + 25c^2) = 3\left[(2b)^2 - (-25c^2)\right] = 3\left[(2b)^2 - (5ci)^2\right] = 3(2b + 5ci)(2b - 5ci)$$

125. $50m^2 + 2n^2$

Solution

$$50m^2 + 2n^2 = 2\left(25m^2 + n^2\right) = 2\left[\left(5m\right)^2 - \left(-n^2\right)\right] = 2\left[\left(5m\right)^2 - \left(ni\right)^2\right] = 2\left(5m + ni\right)\left(5m - ni\right)$$

126. $64a^4 + 4b^2$

Solution

$$64a^4 + 4b^2 = 4\left(16a^4 + b^2\right) = 4\left[\left(4a^2\right)^2 - \left(-b^2\right)\right] = 4\left[\left(4a^2\right)^2 - \left(bi\right)^2\right] = 4\left(4a^2 + bi\right)\left(4a^2 - bi\right)$$

Fix It

In exercises 127 and 128, identify the step the first error is made and fix it.

127. Multiply and write in standard form: $\left(6 - \sqrt{-9}\right)\left(5 + \sqrt{-49}\right)$

Solution
Step 4 was incorrect.

Step 1: $\left(6 - 3i\right)\left(5 + 7i\right)$

Step 2: $6\left(5\right) + 6\left(7i\right) + \left(-3i\right)\left(5\right) + \left(-3i\right)\left(7i\right)$

Step 3: $30 + 42i - 15i - 21i^2$

Step 4: $30 + 27i + 21$

Step 5: $9 + 27i$

128. Divide and write in standard form: $\dfrac{11 - 10i}{1 - 4i}$

Solution
Step 5 was incorrect.

Step 1: $\dfrac{11 - 10i}{1 - 4i} \cdot \dfrac{1 + 4i}{1 + 4i}$

Step 2: $\dfrac{11\left(1\right) + 11\left(4i\right) + \left(-10i\right)\left(1\right) + \left(-10i\right)\left(4i\right)}{1\left(1\right) + 1\left(4i\right) + \left(-4i\right)\left(1\right) + \left(-4i\right)\left(4i\right)}$

Step 3: $\dfrac{11 + 44i - 10i + 40}{1 + 4i - 4i + 16}$

Step 4: $\dfrac{51 + 34i}{17}$

Step 5: $3 + 2i$

Applications

In electronics, the formula *V* = *IR* is called Ohm's Law. It gives the relationship in a circuit between the voltage *V* (in volts), the current *I* (in amperes), and the resistance *R* (in ohms).

129. **Electronics** Find V when $I = 3 - 2i$ amperes and $R = 3$ ohms.

Solution

$$V = IR = (3 - 2i)(3 + 6i) = 9 + 18i - 6i - 12i^2 = 9 + 12i + 12 = 21 + 12i$$

130. **Electronics** Find R when $I = 2 - 3i$ amperes and $V = 21 + i$ volts.

Solution

$$R = \frac{V}{I} = \frac{21 + i}{2 - 3i} = \frac{(21 + i)(2 + 3i)}{(2 - 3i)(2 + 3i)} = \frac{42 + 63i + 2i + 3i^2}{4 - 9i^2} = \frac{39 + 65i}{13} = 3 + 5i$$

131. **Electronics** The impedance Z in an AC (alternating current) circuit is a measure of how much the circuit impedes (hinders) the flow of current through it. The impedance is related to the voltage V and the current I by the following formula.

$V = IZ$

If a circuit has a current of $(0.5 + 2.0i)$ amps and an impedance of $(0.4 - 3.0i)$ ohms, find the voltage.

Solution

$$V = IZ = (0.5 + 2.0i)(0.4 - 3.0i) = 0.2 - 1.5i + 0.8i - 6i^2 = 0.2 - 0.7i + 6 = 6.2 - 0.7i$$

132. **Fractals** Complex numbers are fundamental in the creation of the intricate geometric shape shown below, called a *fractal*.

The process of creating this image is based on the following sequence of steps, which begins by picking any complex number, which we will call z.

1. Square z, and then add that result to z.
2. Square the result from step 1, and then add it to z.
3. Square the result from step 2, and then add it to z.

If we begin with the complex number i, what is the result after performing steps 1, 2, and 3?

Solution

1. $i^2 + i = -1 + i$
2. $(-1 + i)^2 + i = (-1 + i)(-1 + i) + i = 1 - i - i + i^2 + i = -i$
3. $(-i)^2 + i = i^2 + i = -1 + i$

Discovery and Writing

133. Show that the addition of two complex numbers is commutative by adding the complex numbers $a + bi$ and $c + di$ in both orders and observing that the sums are equal.

Solution

$$\begin{aligned}(a + bi) + (c + di) &= a + bi + c + di \\ &= a + c + bi + di \\ &= (a + c) + (b + d)i\end{aligned}$$

$$\begin{aligned}(c + di) + (a + bi) &= c + di + a + bi \\ &= c + a + di + bi \\ &= (c + a) + (d + b)i \\ &= (a + c) + (b + d)i\end{aligned}$$

134. Show that the multiplication of two complex numbers is commutative by multiplying the complex numbers $a + bi$ and $c + di$ in both orders and observing that the products are equal.

Solution

$$\begin{aligned}(a + bi)(c + di) &= ac + adi + bci + bdi^2 \\ &= ac + (ad + bc)i - bd \\ &= (ac - bd) + (ad + bc)i\end{aligned}$$

$$\begin{aligned}(c + di)(a + bi) &= ac + bci + adi + bdi^2 \\ &= ac + (bc + ad)i - bd \\ &= (ac - bd) + (ad + bc)i\end{aligned}$$

135. Show that the addition of complex numbers is associative.

Solution

$$\begin{aligned}\left[(a + bi) + (c + di)\right] + (e + fi) &= a + bi + c + di + e + fi \\ &= a + c + e + bi + di + fi \\ &= (a + c + e) + (b + d + f)i\end{aligned}$$

$$\begin{aligned}(a + bi) + \left[(c + di)\right] + (e + fi) &= a + bi + c + di + e + fi \\ &= a + c + e + bi + di + fi \\ &= (a + c + e) + (b + d + f)i\end{aligned}$$

136. Explain how to determine whether two complex numbers are equal.

Solution
Answers will vary.

137. Define the complex conjugate of a complex number.

Solution
Answers will vary.

138. Explain how to divide two complex numbers.

Solution
Answers will vary.

Critical Thinking

Determine if the statement is true or false. If the statement is false, then correct it and make it true.

139. $\sqrt{-300} = -10\sqrt{3}$

Solution

False. $\sqrt{-300} = \sqrt{-1}\sqrt{100}\sqrt{3} = 10i\sqrt{3}$

140. $\sqrt[3]{-125} = 5i$

Solution

False. $\sqrt[3]{-125} = -5$

141. $\dfrac{\pi}{i} = -\pi i$

Solution

True. $\dfrac{\pi}{i} = \dfrac{\pi}{i} \cdot \dfrac{i}{i} = \dfrac{\pi i}{i^2} = -\pi i$

142. $(2 + 3i)^3 = 8 - 27i$

Solution

False. $(2 + 3i)^3 = (2 + 3i)(2 + 3i)(2 + 3i) = (-5 + 12i)(2 + 3i) = -46 + 9i$

143. $4444i^{4444} = 4444$

Solution

True. $4444i^{4444} = 4444\left(i^4\right)^{1111} = 4444 \cdot 1^{1111} = 4444$

144. $\sqrt{-10}\sqrt{-7} = \sqrt{70}$

Solution

False. $\sqrt{-10}\sqrt{-7} = \sqrt{-1}\sqrt{10}\sqrt{-1}\sqrt{7} = i\sqrt{10} \cdot i\sqrt{7} = i^2\sqrt{70} = -\sqrt{70}$

145. $(5 - 6i)(5 + 6i)(2 - i)(2 + i)$ is a real number.

Solution

True. $(5 - 6i)(5 + 6i)$ is a real number and $(2 - i)(2 + i)$ is a real number, so their product is too.

146. $81x^2 + 100y^2$ can be factored.

Solution

True. $81x^2 + 100y^2 = (9x)^2 - \left(-10y^2\right) = (9x)^2 - (10yi)^2 = (9x + 10yi)(8x - 10yi)$

EXERCISES 1.4

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Factor the trinomial. $24x^2 - 2x - 15$

   Solution

   $24x^2 - 2x - 15 = (4x + 3)(6x - 5)$

2. Factor the perfect square trinomial. $x^2 - 18x + 81$

   Solution

   $x^2 - 18x + 81 = (x - 9)(x - 9) = (x - 9)^2$

3. Solve $5x - 7 = -2\sqrt{2}$ for x.

   Solution

   $$5x - 7 = -2\sqrt{2}$$
   $$5x = 7 - 2\sqrt{2}$$
   $$x = \frac{7 - 2\sqrt{2}}{5}$$

4. If you take half of $\frac{4}{5}$ and square it, what do you get?

   Solution

   $$\left(\frac{1}{2} \cdot \frac{4}{5}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

5. Simplify.

   a. $\dfrac{4 + \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$

   b. $\dfrac{4 - \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$

   Solution

   a. $\dfrac{4 + \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)} = \dfrac{4 + \sqrt{16 + 20}}{2} = \dfrac{4 + 6}{2} = 5$

   b. $\dfrac{4 - \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)} = \dfrac{4 - \sqrt{16 + 20}}{2} = \dfrac{4 - 6}{2} = -1$

6. Simplify. $\frac{18 + 5\sqrt{27}}{9}$

Solution

$$\frac{18 + 5\sqrt{27}}{9} = \frac{18 + 5 \cdot 3\sqrt{3}}{9} = \frac{18 + 15\sqrt{3}}{9} = \frac{6 + 5\sqrt{3}}{9}$$

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. A quadratic equation is an equation that can be written in the form __________, where $a \neq 0$.

Solution

$ax^2 + bx + c = 0$

8. If a and b are real numbers and __________, then $a = 0$ or $b = 0$.

Solution

$ab = 0$

9. The equation $x^2 = c$ has two roots. They are $x =$ __________ and $x =$ __________.

Solution

$\sqrt{c}, \ -\sqrt{c}$

10. The Quadratic Formula is __________ $a \neq 0$.

Solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

11. If a, b, and c are real numbers and if $b^2 - 4ac = 0$, the two roots of the quadratic equation are repeated __________.

Solution

rational numbers

12. If a, b, and c are real numbers and $b^2 - 4ac < 0$, the two roots of the quadratic equation are __________.

Solution

nonreal complex numbers

Practice

Solve each equation by factoring.

13. $x^2 + 4x = 0$

Solution

$$x^2 + 4x = 0$$
$$x(x + 4) = 0$$
$x = 0$ **or** $x + 4 = 0$
$x = 0$ $\quad x = -4$

14. $x^2 = 15x$

Solution

$$x^2 = 15x$$
$$x^2 - 15x = 0$$
$$x(x - 15) = 0$$
$x = 0$ **or** $x - 15 = 0$
$x = 0$ $\quad x = 15$

15. $3x^2 + 21x = 0$

Solution

$$3x^2 + 21x = 0$$
$$3x(x + 7) = 0$$
$3x = 0$ **or** $x + 7 = 0$
$x = 0$ $\quad x = -7$

16. $30x = 6x^2$

Solution

$$30x = 6x^2$$
$$0 = 6x^2 - 30x$$
$$0 = 6x(x - 5)$$
$6x = 0$ **or** $x - 5 = 0$
$x = 0$ $\quad x = 5$

17. $x^2 - 144 = 0$

Solution

$$x^2 - 144 = 0$$
$$(x + 12)(x - 12) = 0$$
$$x + 12 = 0 \quad \textbf{or} \quad x - 12 = 0$$
$$x = -12 \qquad\qquad x = 12$$

18. $4x^2 - 49 = 0$

Solution

$$4x^2 - 49 = 0$$
$$(2x + 7)(2x - 7) = 0$$
$$2x + 7 = 0 \quad \textbf{or} \quad 2x - 7 = 0$$
$$2x = -7 \qquad\qquad 2x = 7$$
$$x = \frac{-7}{2} \qquad\qquad x = \frac{7}{2}$$

19. $x^2 - x - 6 = 0$

Solution

$$x^2 - x - 6 = 0$$
$$(x + 2)(x - 3) = 0$$
$$x + 2 = 0 \quad \textbf{or} \quad x - 3 = 0$$
$$x = -2 \qquad\qquad x = 3$$

20. $x^2 + 8x + 15 = 0$

Solution

$$x^2 + 8x + 15 = 0$$
$$(x + 5)(x + 3) = 0$$
$$x + 5 = 0 \quad \textbf{or} \quad x + 3 = 0$$
$$x = -5 \qquad\qquad x = -3$$

21. $2x^2 + x - 10 = 0$

Solution

$$2x^2 + x - 10 = 0$$
$$(2x + 5)(x - 2) = 0$$
$$2x + 5 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$2x = -5 \qquad\qquad x = 2$$
$$x = -\tfrac{5}{2} \qquad\qquad x = 2$$

22. $3x^2 + 4x - 4 = 0$

Solution

$$3x^2 + 4x - 4 = 0$$
$$(3x - 2)(x + 2) = 0$$
$$3x - 2 = 0 \quad \textbf{or} \quad x + 2 = 0$$
$$3x = 2 \qquad x = -2$$
$$x = \tfrac{2}{3} \qquad x = -2$$

23. $5x^2 - 13x + 6 = 0$

Solution

$$5x^2 - 13x + 6 = 0$$
$$(5x - 3)(x - 2) = 0$$
$$5x - 3 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$5x = 3 \qquad x = 2$$
$$x = \tfrac{3}{5} \qquad x = 2$$

24. $2x^2 + 5x - 12 = 0$

Solution

$$2x^2 + 5x - 12 = 0$$
$$(2x - 3)(x + 4) = 0$$
$$2x - 3 = 0 \quad \textbf{or} \quad x + 4 = 0$$
$$2x = 3 \qquad x = -4$$
$$x = \tfrac{3}{2} \qquad x = -4$$

25. $15x^2 + 16x = 15$

Solution

$$15x^2 + 16x = 15$$
$$15x^2 + 16x - 15 = 0$$
$$(3x + 5)(5x - 3) = 0$$
$$3x + 5 = 0 \quad \textbf{or} \quad 5x - 3 = 0$$
$$3x = -5 \qquad 5x = 3$$
$$x = -\tfrac{5}{3} \qquad x = \tfrac{3}{5}$$

26. $6x^2 - 25x = -25$

Solution

$$6x^2 - 25x = -25$$
$$6x^2 - 25x + 25 = 0$$
$$(3x - 5)(2x - 5) = 0$$
$$3x - 5 = 0 \quad \textbf{or} \quad 2x - 5 = 0$$
$$3x = 5 \qquad 2x = 5$$
$$x = \tfrac{5}{3} \qquad x = \tfrac{5}{2}$$

27. $12x^2 + 9 = 24x$

Solution

$$12x^2 + 9 = 24x$$
$$12x^2 - 24x + 9 = 0$$
$$3(4x^2 - 8x + 3) = 0$$
$$(2x - 1)(2x - 3) = 0$$
$$2x - 1 = 0 \quad \textbf{or} \quad 2x - 3 = 0$$
$$2x = 1 \qquad 2x = 3$$
$$x = \tfrac{1}{2} \qquad x = \tfrac{3}{2}$$

28. $24x^2 + 6 = 24x$

Solution

$$24x^2 + 6 = 24x$$
$$24x^2 - 24x + 6 = 0$$
$$6(4x^2 - 4x + 1) = 0$$
$$(2x - 1)(2x - 1) = 0$$
$$2x - 1 = 0 \quad \textbf{or} \quad 2x - 1 = 0$$
$$2x = 1 \qquad 2x = 1$$
$$x = \tfrac{1}{2} \qquad x = \tfrac{1}{2}$$

Use the Square Root Property to solve each equation.

29. $x^2 = 9$

Solution

$$x^2 = 9$$
$$x = \sqrt{9} \quad \textbf{or} \quad x = -\sqrt{9}$$
$$x = 3 \qquad x = -3$$

30. $x^2 = \frac{1}{64}$

Solution

$$x^2 = 64$$

$$x = \sqrt{64} \quad \textbf{or} \quad x = -\sqrt{64}$$

$$x = \tfrac{1}{8} \qquad\qquad x = -\tfrac{1}{8}$$

31. $x^2 = -169$

Solution

$$x^2 = -169$$

$$x = \sqrt{-169} \quad \textbf{or} \quad x = -\sqrt{-169}$$

$$x = 13i \qquad\qquad x = -13i$$

32. $x^2 = -81$

Solution

$$x^2 = -81$$

$$x = \sqrt{-81} \quad \textbf{or} \quad x = -\sqrt{-81}$$

$$x = 9i \qquad\qquad x = -9i$$

33. $y^2 - 50 = 0$

Solution

$$y^2 - 50 = 0$$

$$y^2 = 50$$

$$y = \sqrt{50} \quad \textbf{or} \quad y = -\sqrt{50}$$

$$y = 5\sqrt{2} \qquad\qquad y = -5\sqrt{2}$$

34. $x^2 - 75 = 0$

Solution

$$x^2 - 75 = 0$$

$$x^2 = 75$$

$$x = \sqrt{75} \quad \textbf{or} \quad x = -\sqrt{75}$$

$$x = 5\sqrt{3} \qquad\qquad x = -5\sqrt{3}$$

35. $y^2 + 54 = 0$

Solution

$$y^2 + 54 = 0$$
$$y^2 = -54$$
$$y = \sqrt{-54} \quad \textbf{or} \quad y = -\sqrt{-54}$$
$$y = i\sqrt{9}\sqrt{6} \qquad y = -i\sqrt{9}\sqrt{6}$$
$$y = 3i\sqrt{6} \qquad y = -3i\sqrt{6}$$

36. $x^2 + 125 = 0$

Solution

$$x^2 + 125 = 0$$
$$x^2 = -125$$
$$x = \sqrt{-125} \quad \textbf{or} \quad x = -\sqrt{-125}$$
$$x = i\sqrt{25}\sqrt{5} \qquad x = -i\sqrt{25}\sqrt{5}$$
$$x = 5i\sqrt{5} \qquad x = -5i\sqrt{5}$$

37. $2x^2 = 40$

Solution

$$2x^2 = 40$$
$$x^2 = 20$$
$$x = \sqrt{20} \quad \textbf{or} \quad x = -\sqrt{20}$$
$$x = 2\sqrt{5} \qquad x = -2\sqrt{5}$$

38. $5x^2 = 400$

Solution

$$5x^2 = 400$$
$$x^2 = 80$$
$$x = \sqrt{80} \quad \textbf{or} \quad x = -\sqrt{80}$$
$$x = 4\sqrt{5} \qquad x = -4\sqrt{5}$$

39. $2x^2 = -90$

Solution

$$2x^2 = -90$$
$$x^2 = -45$$

$$x = \sqrt{-45} \quad \textbf{or} \quad x = -\sqrt{-45}$$
$$x = i\sqrt{9}\sqrt{5} \qquad x = -i\sqrt{9}\sqrt{5}$$
$$x = 3i\sqrt{5} \qquad x = -3i\sqrt{5}$$

40. $5x^2 = -200$

Solution

$$5x^2 = -200$$
$$x^2 = -40$$
$$x = \sqrt{-40} \quad \textbf{or} \quad x = -\sqrt{-40}$$
$$x = i\sqrt{4}\sqrt{10} \qquad x = -i\sqrt{4}\sqrt{10}$$
$$x = 2i\sqrt{10} \qquad x = -2i\sqrt{10}$$

41. $4x^2 = 7$

Solution

$$4x^2 = 7$$
$$x^2 = \tfrac{7}{4}$$
$$x = \sqrt{\tfrac{7}{4}} \quad \textbf{or} \quad x = -\sqrt{\tfrac{7}{4}}$$
$$x = \tfrac{\sqrt{7}}{2} \qquad x = -\tfrac{\sqrt{7}}{2}$$

42. $16x^2 = 11$

Solution

$$16x^2 = 11$$
$$x^2 = \tfrac{11}{16}$$
$$x = \sqrt{\tfrac{11}{16}} \quad \textbf{or} \quad x = -\sqrt{\tfrac{11}{16}}$$
$$x = \tfrac{\sqrt{11}}{4} \qquad x = -\tfrac{\sqrt{11}}{4}$$

43. $9x^2 = -7$

Solution

$$9x^2 = -7$$
$$x^2 = -\frac{7}{9}$$
$$x = \sqrt{-\frac{7}{9}} \quad \textbf{or} \quad x = -\sqrt{-\frac{7}{9}}$$
$$x = i\frac{\sqrt{7}}{\sqrt{9}} \qquad x = -i\frac{\sqrt{7}}{\sqrt{9}}$$
$$x = \frac{\sqrt{7}}{3}i \qquad x = -\frac{\sqrt{7}}{3}i$$

44. $25x^2 = -11$

Solution

$$25x^2 = -11$$
$$x^2 = -\frac{11}{25}$$
$$x = \sqrt{-\frac{11}{25}} \quad \textbf{or} \quad x = -\sqrt{-\frac{11}{25}}$$
$$x = i\frac{\sqrt{11}}{\sqrt{25}} \qquad x = -i\frac{\sqrt{11}}{\sqrt{25}}$$
$$x = \frac{\sqrt{11}}{5}i \qquad x = -\frac{\sqrt{11}}{5}i$$

45. $2x^2 - 13 = 0$

Solution

$$2x^2 - 13 = 0$$
$$2x^2 = 13$$
$$x^2 = \frac{13}{2}$$
$$x = \sqrt{\frac{13}{2}} \quad \textbf{or} \quad x = -\sqrt{\frac{13}{2}}$$
$$x = \frac{\sqrt{13}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \qquad x = -\frac{\sqrt{13}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{\sqrt{26}}{2} \qquad x = -\frac{\sqrt{26}}{2}$$

46. $-3x^2 = -11$

Solution

$$-3x^2 = -11$$
$$x^2 = \frac{11}{3}$$
$$x = \sqrt{\frac{11}{3}} \quad \textbf{or} \quad x = -\sqrt{\frac{11}{3}}$$
$$x = \frac{\sqrt{11}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \qquad x = -\frac{\sqrt{11}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$x = \frac{\sqrt{33}}{3} \qquad x = -\frac{\sqrt{33}}{3}$$

47. $2x^2 + 15 = 0$

Solution

$$2x^2 + 15 = 0$$
$$2x^2 = -15$$
$$x^2 = -\frac{15}{2}$$

$$x = \sqrt{-\frac{15}{2}} \quad \textbf{or} \quad x = -\sqrt{-\frac{15}{2}}$$

$$x = i\frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \qquad x = -i\frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = \frac{\sqrt{30}}{2}i \qquad x = -\frac{\sqrt{30}}{2}i$$

48. $-5x^2 = 11$

Solution

$$-5x^2 = 11$$

$$x^2 = -\frac{11}{5}$$

$$x = \sqrt{-\frac{11}{5}} \quad \textbf{or} \quad x = -\sqrt{-\frac{11}{2}}$$

$$x = i\frac{\sqrt{11}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \qquad x = -i\frac{\sqrt{11}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$x = \frac{\sqrt{55}}{5}i \qquad x = -\frac{\sqrt{55}}{5}i$$

49. $(x + 1)^2 - 8 = 0$

Solution

$$(x - 1)^2 - 8 = 0$$

$$(x - 1)^2 = 8$$

$$x - 1 = \sqrt{8} \quad \textbf{or} \quad x - 1 = -\sqrt{8}$$

$$x - 1 = \sqrt{4}\sqrt{2} \qquad x - 1 = -\sqrt{4}\sqrt{2}$$

$$x = 1 + 2\sqrt{2} \qquad x = 1 - 2\sqrt{2}$$

50. $(y + 2)^2 - 98 = 0$

Solution

$$(y + 2)^2 - 98 = 0$$

$$(y + 2)^2 = 98$$

$$y + 2 = \sqrt{98} \quad \textbf{or} \quad y + 2 = -\sqrt{98}$$

$$y + 2 = \sqrt{49}\sqrt{2} \qquad y + 2 = -\sqrt{49}\sqrt{2}$$

$$y = -2 + 7\sqrt{2} \qquad y = -2 - 7\sqrt{2}$$

51. $(x + 1)^2 + 12 = 0$

Solution

$$(x+1)^2+12=0$$

$$(x+1)^2=-12$$

$x+1=\sqrt{-12}$ **or** $x+1=-\sqrt{-12}$

$x=-1+i\sqrt{4}\sqrt{3}$ $\quad$ $x=-1-i\sqrt{4}\sqrt{3}$

$x=-1+2i\sqrt{3}$ $\quad$ $x=-1-2i\sqrt{3}$

52. $(y+2)^2+120=0$

Solution

$$(y+2)^2+120=0$$

$$(y+2)^2=-120$$

$y+2=\sqrt{-200}$ **or** $y+2=-\sqrt{-200}$

$y=-2+i\sqrt{4}\sqrt{30}$ $\quad$ $y=-2-i\sqrt{4}\sqrt{30}$

$y=-2+2i\sqrt{30}$ $\quad$ $y=-2-2i\sqrt{30}$

53. $(2x+1)^2=27$

Solution

$$(2x+1)^2=27$$

$2x+1=\sqrt{27}$ **or** $2x+1=-\sqrt{27}$

$2x+1=3\sqrt{3}$ $\quad$ $2x+1=-3\sqrt{3}$

$2x=-1+3\sqrt{3}$ $\quad$ $2x=-1-3\sqrt{3}$

$x=\frac{-1+3\sqrt{3}}{2}$ $\quad$ $x=\frac{-1-3\sqrt{3}}{2}$

54. $(5y+2)^2-48=0$

Solution

$$(5y+2)^2-48=0$$

$$(5y+2)^2=48$$

$5y+2=\sqrt{48}$ **or** $5y+2=-\sqrt{48}$

$5y+2=4\sqrt{3}$ $\quad$ $5y+2=-4\sqrt{3}$

$5y=-2+4\sqrt{3}$ $\quad$ $5y=-2-4\sqrt{3}$

$y=\frac{-2+4\sqrt{3}}{5}$ $\quad$ $y=\frac{-2-4\sqrt{3}}{5}$

55. $(5x + 1)^2 = -8$

Solution

$$(5x + 1)^2 = -8$$

$$5x + 1 = \sqrt{-8} \quad \textbf{or} \quad 5x + 1 = -\sqrt{-8}$$

$$5x = -1 + i\sqrt{4}\sqrt{2} \qquad 5x = -1 - i\sqrt{4}\sqrt{2}$$

$$x = -\frac{1}{5} + \frac{2i\sqrt{2}}{5} \qquad x = -\frac{1}{5} - \frac{2i\sqrt{2}}{5}$$

56. $(7y + 2)^2 + 48 = 0$

Solution

$$(7y + 2)^2 + 48 = 0$$

$$(7y + 2)^2 = -48$$

$$7y + 2 = \sqrt{-48} \quad \textbf{or} \quad 7y + 2 = -\sqrt{-48}$$

$$7y = -2 + i\sqrt{16}\sqrt{3} \qquad 7y = -2 - i\sqrt{16}\sqrt{3}$$

$$y = -\frac{2}{7} + \frac{4i\sqrt{3}}{7} \qquad y = -\frac{2}{7} - \frac{4i\sqrt{3}}{7}$$

57. $5(10x - 1)^2 = 25$

Solution

$$5(10x - 1)^2 = 25$$

$$(10x - 1)^2 = 5$$

$$10x - 1 = \sqrt{5} \quad \textbf{or} \quad 10x - 1 = -\sqrt{5}$$

$$10x = 1 + \sqrt{5} \qquad 10x = 1 - \sqrt{5}$$

$$x = \frac{1 + \sqrt{5}}{10} \qquad x = \frac{1 - \sqrt{5}}{10}$$

58. $7(3x + 4)^2 = 14$

Solution

$$7(3x + 4)^2 = 14$$

$$(3x + 4)^2 = 2$$

$$3x + 4 = \sqrt{2} \quad \textbf{or} \quad 3x + 4 = -\sqrt{2}$$

$$3x = -4 + \sqrt{2} \qquad 3x = -4 - \sqrt{2}$$

$$x = \frac{-4 + \sqrt{2}}{3} \qquad x = \frac{-4 - \sqrt{2}}{3}$$

59. $-3(8x + 11)^2 = 9$

Solution

$$-3(8x + 11)^2 = 9$$
$$(8x + 11)^2 = -3$$

$8x + 11 = \sqrt{-3}$ **or** $8x + 11 = -\sqrt{-3}$

$8x = -11 + i\sqrt{3}$ $\qquad$ $8x = -11 - i\sqrt{3}$

$x = \dfrac{-11 + i\sqrt{3}}{8}$ $\qquad$ $x = \dfrac{-11 - i\sqrt{3}}{8}$

$x = \dfrac{-11}{8} + \dfrac{i\sqrt{3}}{8}$ $\qquad$ $x = \dfrac{-11}{8} - \dfrac{i\sqrt{3}}{8}$

60. $-4(6x - 5)^2 + 3 = 19$

Solution

$$-4(6x - 5)^2 + 3 = 19$$
$$-4(6x - 5)^2 = 16$$
$$(6x - 5)^2 = -4$$

$6x - 5 = \sqrt{-4}$ **or** $6x - 5 = -\sqrt{-4}$

$6x - 5 = 2i$ $\qquad$ $6x - 5 = -2i$

$6x = 5 + 2i$ $\qquad$ $6x = 5 + 2i$

$x = \dfrac{5 + 2i}{6}$ $\qquad$ $x = \dfrac{5 - 2i}{6}$

$x = \dfrac{5}{6} + \dfrac{1}{3}i$ $\qquad$ $x = \dfrac{5}{6} - \dfrac{1}{3}i$

Complete the square to make each a perfect-square trinomial.

61. $x^2 + 6x$

Solution

$$x^2 + 6x + \left[\tfrac{1}{2}(6)\right]^2 = x^2 + 6x + 3^2$$
$$= x^2 + 6x + 9$$

62. $x^2 + 8x$

Solution

$$x^2 + 8x + \left[\tfrac{1}{2}(8)\right]^2 = x^2 + 8x + 4^2$$
$$= x^2 + 8x + 16$$

63. $x^2 - 4x$

Solution

$$\begin{aligned} x^2 - 4x + \left[\tfrac{1}{2}(-4)\right]^2 &= x^2 - 4x + (-2)^2 \\ &= x^2 - 4x + 4 \end{aligned}$$

64. $x^2 - 12x$

Solution

$$\begin{aligned} x^2 - 12x + \left[\tfrac{1}{2}(-12)\right]^2 &= x^2 - 12x + (-6)^2 \\ &= x^2 - 12x + 36 \end{aligned}$$

65. $a^2 + 5a$

Solution

$$\begin{aligned} a^2 + 5a + \left[\tfrac{1}{2}(5)\right]^2 &= a^2 + 5a + \left(\frac{5}{2}\right)^2 \\ &= a^2 + 5a + \frac{25}{4} \end{aligned}$$

66. $t^2 + 9t$

Solution

$$\begin{aligned} t^2 + 9t + \left[\tfrac{1}{2}(9)\right]^2 &= t^2 + 9t + \left(\frac{9}{2}\right)^2 \\ &= t^2 + 9t + \frac{81}{4} \end{aligned}$$

67. $r^2 - 11r$

Solution

$$r^2 - 11r + \left[\tfrac{1}{2}(-11)\right]^2 = r^2 - 11r + \left(\tfrac{-11}{2}\right)^2 = r^2 - 11r + \tfrac{121}{4}$$

68. $s^2 - 7s$

Solution

$$s^2 - 7s + \left[\tfrac{1}{2}(-7)\right]^2 = s^2 - 7s + \left(\tfrac{-7}{2}\right)^2 = s^2 - 7s + \tfrac{49}{4}$$

69. $y^2 + \frac{3}{4}y$

Solution

$$y^2 + \frac{3}{4}y + \left[\frac{1}{2}\left(\frac{3}{4}\right)\right]^2 = y^2 + \frac{3}{4}y + \left(\frac{3}{8}\right)^2$$
$$= y^2 + \frac{3}{4}y + \frac{9}{64}$$

70. $p^2 + \frac{3}{2}p$

Solution

$$p^2 + \frac{3}{2}p + \left[\frac{1}{2}\left(\frac{3}{4}\right)\right]^2 = p^2 + \frac{3}{2}p + \left(\frac{3}{4}\right)^2$$
$$= p^2 + \frac{3}{2}p + \frac{9}{16}$$

71. $q^2 - \frac{1}{5}q$

Solution

$$q^2 - \frac{1}{5}q + \left[\frac{1}{2}\left(-\frac{1}{5}\right)\right]^2 = q^2 - \frac{1}{5}q + \left(\frac{-1}{10}\right)^2 = q^2 - \frac{1}{5}q + \frac{1}{100}$$

72. $m^2 - \frac{2}{3}m$

Solution

$$m^2 - \frac{2}{3}m + \left[\frac{1}{2}\left(-\frac{2}{3}\right)\right]^2 = m^2 - \frac{2}{3}m + \left(\frac{-1}{3}\right)^2 = m^2 - \frac{2}{3}m + \frac{1}{9}$$

73. $x^2 + 12x = -8$

Solution

$$x^2 + 12x = -8$$
$$x^2 + 12x + 36 = -8 + 36$$
$$(x + 6)^2 = 28$$

$x + 6 = \sqrt{28}$ **or** $x + 6 = -\sqrt{28}$

$x + 6 = 2\sqrt{7}$ $\quad$ $x + 6 = -2\sqrt{7}$

$x = -6 + 2\sqrt{7}$ $\quad$ $x = -6 - 2\sqrt{7}$

74. $x^2 - 6x = -1$

Solution

$$x^2 - 6x = -1$$
$$x^2 - 6x + 9 = -1 + 9$$
$$(x-3)^2 = 8$$
$$x - 3 = \sqrt{8} \quad \textbf{or} \quad x - 3 = -\sqrt{8}$$
$$x - 3 = 2\sqrt{2} \qquad x - 3 = -2\sqrt{2}$$
$$x = 3 + 2\sqrt{2} \qquad x = 3 - 2\sqrt{2}$$

75. $x^2 - 10x + 37 = 0$

Solution

$$x^2 - 10x + 37 = 0$$
$$x^2 - 10x = -37$$
$$x^2 - 10x + 25 = -37 + 25$$
$$(x-5)^2 = -12$$
$$x - 5 = \sqrt{-12} \quad \textbf{or} \quad x - 5 = -\sqrt{-12}$$
$$x = 5 + i\sqrt{4}\sqrt{3} \qquad x = 5 - i\sqrt{4}\sqrt{3}$$
$$x = 5 + 2i\sqrt{3} \qquad x = 5 - 2i\sqrt{3}$$

76. $a^2 + 16a + 82 = 0$

Solution

$$a^2 + 16a + 82 = 0$$
$$a^2 + 16a = -82$$
$$a^2 + 16a + 64 = -82 + 64$$
$$(a+8)^2 = -18$$
$$a + 8 = \sqrt{-18} \quad \text{or} \quad a + 8 = -\sqrt{-18}$$
$$a = -8 + i\sqrt{9}\sqrt{2} \qquad a = -8 - i\sqrt{9}\sqrt{2}$$
$$a = -8 + 3i\sqrt{2} \qquad a = -8 - 3i\sqrt{2}$$

77. $x^2 + 5 = -5x$

Solution

$$x^2 + 5 = -5x$$
$$x^2 + 5x = -5$$

$$x^2 + 5x + \frac{25}{4} = -5 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{5}{4}$$

$$x + \frac{5}{2} = \sqrt{\frac{5}{4}} \quad \textbf{or} \quad x + \frac{5}{2} = -\sqrt{\frac{5}{4}}$$

$$x + \frac{5}{2} = \frac{\sqrt{5}}{2} \qquad x + \frac{5}{2} = -\frac{\sqrt{5}}{2}$$

$$x = \frac{-5 + \sqrt{5}}{2} \qquad x = \frac{-5 - \sqrt{5}}{2}$$

78. $x^2 + 1 = -4x$

Solution

$$x^2 + 1 = -4x$$

$$x^2 + 4x = -1$$

$$x^2 + 4x + 4 = -1 + 4$$

$$(x + 2)^2 = 3$$

$$x + 2 = \sqrt{3} \quad \textbf{or} \quad x + 2 = -\sqrt{3}$$

$$x = -2 + \sqrt{3} \qquad x = -2 - \sqrt{3}$$

79. $y^2 + 11y = -49$

Solution

$$y^2 + 11y = -49$$

$$y^2 + 11y + \frac{121}{4} = -\frac{196}{4} + \frac{121}{4}$$

$$\left(y + \frac{11}{2}\right)^2 = -\frac{75}{4}$$

$$y + \frac{11}{2} = \sqrt{-\frac{75}{4}} \quad \textbf{or} \quad y + \frac{11}{2} = -\sqrt{-\frac{75}{4}}$$

$$y = -\frac{11}{2} + i\frac{\sqrt{75}}{\sqrt{4}} \qquad y = -\frac{11}{2} - i\frac{\sqrt{75}}{\sqrt{4}}$$

$$y = -\frac{11}{2} + \frac{5\sqrt{3}}{2}i \qquad y = -\frac{11}{2} - \frac{5\sqrt{3}}{2}i$$

80. $x^2 - 5x = -22$

Solution

$$x^2 - 5x = -22$$
$$x^2 - 5x + \frac{25}{4} = -\frac{88}{4} + \frac{25}{4}$$
$$\left(x - \frac{5}{2}\right)^2 = -\frac{63}{4}$$
$$x - \frac{5}{2} = \sqrt{-\frac{63}{4}} \quad \textbf{or} \quad x - \frac{5}{2} = -\sqrt{-\frac{63}{4}}$$
$$x = \frac{5}{2} + i\frac{\sqrt{63}}{\sqrt{4}} \qquad x = \frac{5}{2} - i\frac{\sqrt{63}}{\sqrt{4}}$$
$$x = \frac{5}{2} + \frac{3\sqrt{7}}{2}i \qquad x = \frac{5}{2} - \frac{3\sqrt{7}}{2}i$$

81. $2x^2 - 20x = -49$

Solution

$$2x^2 - 20x = -49$$
$$x^2 - 10x = -\frac{49}{2}$$
$$x^2 - 10x + 25 = -\frac{49}{2} + \frac{50}{2}$$
$$(x - 5)^2 = \frac{1}{2}$$
$$x - 5 = \sqrt{\frac{1}{2}} \quad \textbf{or} \quad x - 5 = -\sqrt{\frac{1}{2}}$$
$$x - 5 = \frac{1}{\sqrt{2}} \qquad x - 5 = -\frac{1}{\sqrt{2}}$$
$$x - \frac{10}{2} = \frac{\sqrt{2}}{2} \qquad x - \frac{10}{2} = -\frac{\sqrt{2}}{2}$$
$$x = \frac{10+\sqrt{2}}{2} \qquad x = \frac{10-\sqrt{2}}{2}$$

82. $4x^2 + 8x = 7$

Solution

$$4x^2 + 8x = 7$$
$$x^2 + 2x = \frac{7}{4}$$
$$x^2 + 2x + 1 = \frac{7}{4} + \frac{4}{4}$$
$$(x - 1)^2 = \frac{11}{4}$$

$x + 1 = \sqrt{\frac{11}{4}}$ **or** $x + 1 = -\sqrt{\frac{11}{4}}$

$x + 1 = \frac{\sqrt{11}}{2}$ $\quad$ $x + 1 = -\frac{\sqrt{11}}{2}$

$x + \frac{2}{2} = \frac{\sqrt{11}}{2}$ $\quad$ $x + \frac{2}{2} = -\frac{\sqrt{11}}{2}$

$x = \frac{-2+\sqrt{11}}{2}$ $\quad$ $x = \frac{-2-\sqrt{11}}{2}$

83. $3x^2 = 1 - 4x$

Solution

$$3x^2 = 1 - 4x$$
$$3x^2 + 4x = 1$$
$$x^2 + \frac{4}{3}x = \frac{1}{3}$$
$$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{1}{3} + \frac{4}{9}$$
$$\left(x + \frac{2}{3}\right)^2 = \frac{7}{9}$$

$x + \frac{2}{3} = \sqrt{\frac{7}{9}}$ **or** $x + \frac{2}{3} = -\sqrt{\frac{7}{9}}$

$x + \frac{2}{3} = \frac{\sqrt{7}}{3}$ $\quad$ $x + \frac{2}{3} = -\frac{\sqrt{7}}{3}$

$x = \frac{-2 + \sqrt{7}}{3}$ $\quad$ $x = \frac{-2 - \sqrt{7}}{3}$

84. $3x^2 + 4x = 5$

Solution

$$3x^2 + 4x = 5$$
$$x^2 + \frac{4}{3}x = \frac{5}{3}$$
$$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{5}{3} + \frac{4}{9}$$
$$\left(x + \frac{2}{3}\right)^2 = \frac{19}{9}$$

$x + \frac{2}{3} = \sqrt{\frac{19}{9}}$ **or** $x + \frac{2}{3} = -\sqrt{\frac{19}{9}}$

$x + \frac{2}{3} = \frac{\sqrt{19}}{3}$ $\quad$ $x + \frac{2}{3} = -\frac{\sqrt{19}}{3}$

$x = \frac{-2 + \sqrt{19}}{3}$ $\quad$ $x = \frac{-2 - \sqrt{19}}{3}$

85. $2x^2 = 3x + 1$

Solution

$$2x^2 = 3x + 1$$
$$2x^2 - 3x = 1$$
$$x^2 - \frac{3}{2}x = \frac{1}{2}$$
$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$
$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$x - \frac{3}{4} = \sqrt{\frac{17}{16}}$ **or** $x - \frac{3}{4} = -\sqrt{\frac{17}{16}}$

$x - \frac{3}{4} = \frac{\sqrt{17}}{4}$ $\qquad$ $x - \frac{3}{4} = -\frac{\sqrt{17}}{4}$

$x = \frac{3 + \sqrt{17}}{4}$ $\qquad$ $x = \frac{3 - \sqrt{17}}{4}$

86. $2x^2 + 5x = 14$

Solution

$$2x^2 + 5x = 14$$
$$x^2 + \frac{5}{2}x = 7$$
$$x^2 + \frac{5}{2}x + \frac{25}{16} = \frac{112}{16} + \frac{25}{16}$$
$$\left(x + \frac{5}{2}\right)^2 = \frac{137}{16}$$

$x + \frac{5}{4} = \sqrt{\frac{137}{16}}$ **or** $x + \frac{5}{4} = -\sqrt{\frac{137}{16}}$

$x + \frac{5}{4} = \frac{\sqrt{137}}{4}$ $\qquad$ $x + \frac{5}{4} = -\frac{\sqrt{137}}{4}$

$x = \frac{-5 + \sqrt{137}}{4}$ $\qquad$ $x = \frac{-5 - \sqrt{137}}{4}$

Use the Quadratic Formula to solve each equation.

87. $9x^2 = 18x - 14$

Solution

$9x^2 = 18x - 14 \Rightarrow 9x^2 - 18x + 14 = 0 \Rightarrow a = 9, b = -18, c = 14$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(9)(14)}}{2(9)} = \frac{18 \pm \sqrt{324 - 504}}{18}$$

$$= \frac{18 \pm \sqrt{-180}}{18} = \frac{18 \pm 6i\sqrt{5}}{18} = \frac{6(3 \pm i\sqrt{5})}{18} = \frac{3 \pm i\sqrt{5}}{3} = 1 \pm \frac{\sqrt{5}}{3}i$$

88. $7z^2 = -14z - 13$

Solution

$7z^2 = -14z - 13 \Rightarrow 7z^2 + 14z + 13 = 0 \Rightarrow a = 7, b = 14, c = 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm \sqrt{(14)^2 - 4(7)(13)}}{2(7)} = \frac{-14 \pm \sqrt{196 - 364}}{14}$$

$$= \frac{-14 \pm \sqrt{-168}}{14} = \frac{-14 \pm 2i\sqrt{42}}{14} = \frac{2(-7 \pm i\sqrt{42})}{14} = \frac{-7 \pm i\sqrt{42}}{7} = -1 \pm \frac{\sqrt{42}}{7}i$$

89. $2x^2 = 14x - 30i$

Solution

$2x^2 = 14x - 30 \Rightarrow 2x^2 - 14x + 30 = 0 \Rightarrow a = 2, b = -14, c = 30$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(2)(30)}}{2(2)} = \frac{14 \pm \sqrt{196 - 240}}{4}$$

$$= \frac{14 \pm \sqrt{-44}}{4} = \frac{14 \pm 2i\sqrt{11}}{4} = \frac{2(7 \pm i\sqrt{11})}{4} = \frac{7 \pm i\sqrt{11}}{2} = \frac{7}{2} \pm \frac{\sqrt{11}}{2}i$$

90. $5x^2 + x = -5$

Solution

$5x^2 + x = -5 \Rightarrow 5x^2 + x + 5 = 0 \Rightarrow a = 5, b = 1, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{(1)^2 - 4(5)(5)}}{2(5)} = \frac{-1 \pm \sqrt{1 - 100}}{10}$$

$$= \frac{-1 \pm \sqrt{-99}}{10} = \frac{-1 \pm 3i\sqrt{11}}{10} = -\frac{1}{10} \pm \frac{3\sqrt{11}}{10}i$$

91. $3x^2 = -5x - 1$

Solution

$3x^2 = -5x - 1 \Rightarrow 3x^2 + 5x + 1 = 0 \Rightarrow a = 3, b = 5, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(1)}}{2(3)} = \frac{-5 \pm \sqrt{25 - 12}}{6} = \frac{-5 \pm \sqrt{13}}{6}$$

92. $2x^2 = 5x + 11$

Solution

$2x^2 = 5x + 11 \Rightarrow 2x^2 - 5x - 11 = 0 \Rightarrow a = 2, b = -5, c = -11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-11)}}{2(2)} = \frac{5 \pm \sqrt{25 + 88}}{4} = \frac{5 \pm \sqrt{113}}{4}$$

93. $x^2 + 1 = -7x$

Solution

$x^2 + 1 = -7x \Rightarrow x^2 + 7x + 1 = 0 \Rightarrow a = 1, b = 7, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(1)}}{2(1)} = \frac{-7 \pm \sqrt{49 - 4}}{2} = \frac{-7 \pm \sqrt{45}}{2}$$

$$= \frac{-7 \pm 3\sqrt{5}}{2}$$

94. $13x^2 + 1 = -10x$

Solution

$13x^2 + 1 = -10x \Rightarrow 13x^2 + 10x + 1 = 0 \Rightarrow a = 13, b = 10, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(10) \pm \sqrt{(10)^2 - 4(13)(1)}}{2(13)} = \frac{-10 \pm \sqrt{100 - 52}}{26} = \frac{-10 \pm \sqrt{48}}{26}$$

$$x = \frac{-10 \pm \sqrt{48}}{26} = \frac{-10 \pm 4\sqrt{3}}{26} = \frac{2(-5 \pm 2\sqrt{3})}{26} = \frac{-5 \pm 2\sqrt{3}}{13}$$

95. $3x^2 + 6x = -1$

Solution

$3x^2 + 6x = -1 \Rightarrow 3x^2 + 6x + 1 = 0 \Rightarrow a = 3, b = 6, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(6) \pm \sqrt{(6)^2 - 4(3)(1)}}{2(3)} = \frac{-6 \pm \sqrt{36 - 12}}{6} = \frac{-6 \pm \sqrt{24}}{6}$$

$$x = \frac{-6 \pm \sqrt{24}}{6} = \frac{-6 \pm 2\sqrt{6}}{6} = \frac{2(-3 \pm \sqrt{6})}{6} = \frac{-3 \pm \sqrt{6}}{3}$$

96. $2x(x + 3) = -1$

Solution

$2x(x+3) = -1 \Rightarrow 2x^2 + 6x + 1 = 0 \Rightarrow a = 2, b = 6, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(1)}}{2(2)} = \frac{-6 \pm \sqrt{36 - 8}}{4} = \frac{-6 \pm \sqrt{28}}{4}$$

$$x = \frac{-6 \pm \sqrt{28}}{4} = \frac{-6 \pm 2\sqrt{7}}{4} = \frac{-3 \pm \sqrt{7}}{2}$$

97. $7x^2 = 2x + 2$

Solution

$7x^2 = 2x + 2 \Rightarrow 7x^2 - 2x - 2 = 0 \Rightarrow a = 7, b = -2, c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(-2)}}{2(7)} = \frac{2 \pm \sqrt{4 + 56}}{14} = \frac{2 \pm \sqrt{60}}{14}$$

$$x = \frac{2 \pm \sqrt{60}}{14} = \frac{2 \pm 2\sqrt{15}}{14} = \frac{1 \pm \sqrt{15}}{7}$$

98. $5x\left(x + \frac{1}{5}\right) = 3$

Solution

$5x\left(x + \frac{1}{5}\right) = 3 \Rightarrow 5x^2 + x - 3 = 0 \Rightarrow a = 5, b = 1, c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1) \pm \sqrt{(1)^2 - 4(5)(-3)}}{2(5)} = \frac{-1 \pm \sqrt{1 + 60}}{10} = \frac{-1 \pm \sqrt{61}}{10}$$

99. $x^2 + 2x + 2 = 0$

Solution

$x^2 + 2x + 2 = 0 \Rightarrow a = 1, b = 2, c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i$$

100. $a^2 + 4a + 8 = 0$

Solution

$a^2 + 4a + 8 = 0 \Rightarrow a = 1, b = 4, c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(8)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 32}}{2} = \frac{-4 \pm \sqrt{-16}}{2} = \frac{-4 \pm 4i}{2}$$

$$= -2 \pm 2i$$

101. $y^2 + 4y + 5 = 0$

Solution

$y^2 + 4y + 5 = 0 \Rightarrow a = 1, b = 4, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2}$$
$$= -2 \pm i$$

102. $x^2 + 2x + 5 = 0$

Solution

$x^2 + 2x + 5 = 0 \Rightarrow a = 1, b = 2, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2}$$
$$= -1 \pm 2i$$

103. $x^2 - 2x = -5$

Solution

$x^2 - 2x = -5 \Rightarrow x^2 - 2x + 5 = 0 \Rightarrow a = 1, b = -2, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)} = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$
$$= 1 \pm 2i$$

104. $z^2 - 3z = -8$

Solution

$z^2 - 3z = -8 \Rightarrow z^2 - 3z + 8 = 0 \Rightarrow a = 1, b = -3, c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(8)}}{2(1)} = \frac{3 \pm \sqrt{9 - 32}}{2} = \frac{3 \pm \sqrt{-23}}{2}$$
$$= \frac{3}{2} \pm \frac{\sqrt{23}}{2}i$$

105. $x^2 - \frac{2}{3}x = -\frac{2}{9}$

Solution

$$x^2 - \frac{2}{3}x = -\frac{2}{9} \Rightarrow x^2 - \frac{2}{3}x + \frac{2}{9} = 0 \Rightarrow 9x^2 - 6x + 2 = 0 \Rightarrow a = 9, b = -6, c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(2)}}{2(9)} = \frac{6 \pm \sqrt{36 - 72}}{18} = \frac{6 \pm \sqrt{-36}}{18}$$

$$= \frac{6 \pm 6i}{18} = \frac{1}{3} \pm \frac{1}{3}i$$

106. $x^2 + \frac{5}{4} = x$

Solution

$$x^2 + \frac{5}{4} = x \Rightarrow x^2 - x + \frac{5}{4} = 0 \Rightarrow 4x^2 - 4x + 5 = 0 \Rightarrow a = 4, b = -4, c = 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(5)}}{2(4)} = \frac{4 \pm \sqrt{16 - 80}}{8} = \frac{4 \pm \sqrt{-64}}{8}$$

$$= \frac{4 \pm 8i}{8} = \frac{1}{2} \pm i$$

Solve each formula for the indicated variable.

107. $h = \frac{1}{2}gt^2; t$

Solution

$$h = \frac{1}{2}gt^2$$

$$2h = gt^2$$

$$\frac{2h}{g} = t^2$$

$$\pm\sqrt{\frac{2h}{g}} = t$$

$$\pm\sqrt{\frac{2h}{g}} \cdot \frac{\sqrt{g}}{\sqrt{g}} = t$$

$$\pm\sqrt{\frac{2hg}{g}} = t$$

108. $x^2 + y^2 = r^2; x$

Solution

$$x^2 + y^2 = r^2$$

$$x^2 = r^2 - y^2$$

$$x = \pm\sqrt{r^2 - y^2}$$

109. $h = 64t - 16t^2; t$

Solution

$$h = 64t - 16t^2$$

$$16t^2 - 64t + h = 0; a = 16, b = -64, c = h$$

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-64) \pm \sqrt{(-64)^2 - 4(16)h}}{2(16)} \\ &= \frac{64 \pm \sqrt{4096 - 64h}}{32} \\ &= \frac{64 \pm \sqrt{64(64 - h)}}{32} \\ &= \frac{64 \pm 8\sqrt{64 - h}}{32} = \frac{8 \pm \sqrt{64 - h}}{4} \end{aligned}$$

110. $y = 16x^2 - 4; x$

Solution

$$\begin{aligned} y &= 16x^2 - 4 \\ y + 4 &= 16x^2 \\ \frac{y + 4}{16} &= x^2 \\ \pm\sqrt{\frac{y + 4}{16}} &= x \\ \pm\sqrt{\frac{y + 4}{4}} &= x \end{aligned}$$

111. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; y$

Solution

$$\begin{aligned} \frac{x^2}{a^2} + \frac{y^2}{b^2} &= 1 \\ \frac{y^2}{b^2} &= 1 - \frac{x^2}{a^2} \\ \frac{y^2}{b^2} &= \frac{a^2 - x^2}{a^2} \\ y^2 &= \frac{b^2(a^2 - x^2)}{a^2} \end{aligned}$$

$$y = \pm\sqrt{\frac{b^2\left(a^2 - x^2\right)}{a^2}}$$
$$y = \pm\frac{b\sqrt{a^2 - x^2}}{a}$$

112. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1;\ x$

Solution

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{a^2} = 1 + \frac{y^2}{b^2}$$
$$\frac{x^2}{a^2} = \frac{b^2 + y^2}{b^2}$$
$$x^2 = \frac{a^2\left(b^2 + y^2\right)}{b^2}$$
$$x = \pm\sqrt{\frac{a^2\left(b^2 + y^2\right)}{b^2}}$$
$$x = \pm\frac{a\sqrt{\left(b^2 + y^2\right)}}{b}$$

113. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1;\ a$

Solution

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$$a^2b^2\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = a^2b^2(1)$$
$$b^2x^2 - a^2y^2 = a^2b^2$$
$$b^2x^2 = a^2b^2 + a^2y^2$$
$$b^2x^2 = a^2\left(b^2 + y^2\right)$$
$$\frac{b^2x^2}{b^2 + y^2} = a^2$$
$$\pm\sqrt{\frac{b^2x^2}{b^2 + y^2}} = a$$
$$\pm\frac{bx\sqrt{b^2 + y^2}}{b^2 + y^2} = a$$

114. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$; b

Solution

$$\begin{aligned}
\frac{x^2}{a^2} - \frac{y^2}{b^2} &= 1 \\
a^2b^2\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) &= a^2b^2(1) \\
b^2x^2 - a^2y^2 &= a^2b^2 \\
b^2x^2 - a^2b^2 &= a^2y^2 \\
b^2\left(x^2 - a^2\right) &= a^2y^2 \\
b^2 &= \frac{a^2y^2}{x^2 - a^2} \\
b &= \pm\sqrt{\frac{a^2y^2}{x^2 - a^2}} \\
b &= \pm\frac{ay\sqrt{x^2 - a^2}}{x^2 - a^2}
\end{aligned}$$

115. $x^2 + xy - y^2 = 0$; x

Solution

$x^2 + xy - y^2 = 0 \Rightarrow a = 1, b = y, c = -y^2$

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(y) \pm \sqrt{(y)^2 - 4(1)(-y^2)}}{2(1)} \\
&= \frac{-y \pm \sqrt{y^2 + 4y^2}}{2} \\
&= \frac{-y \pm \sqrt{5y^2}}{2} = \frac{-y \pm y\sqrt{5}}{2}
\end{aligned}$$

116. $x^2 - 3xy + y^2 = 0$; y

Solution

$x^2 - 3xy + y^2 = 0 \Rightarrow y^2 - 3xy + x^2 = 0$

$a = 1, b = -3x, c = x^2$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-3x) \pm \sqrt{(-3x)^2 - 4(1)(-x^2)}}{2(1)} \\ &= \frac{3x \pm \sqrt{9x^2 - 4x^2}}{2} \\ &= \frac{3x \pm \sqrt{5x^2}}{2} = \frac{3x \pm x\sqrt{5}}{2} \end{aligned}$$

Use the discriminant to determine the number and type of roots. Do not solve the equation.

117. $x^2 + 6x + 9 = 0$

Solution

$x^2 + 6x + 9 = 0 \Rightarrow a = 1, b = 6, c = 9$

$b^2 - 4ac = 6^2 - 4(1)(9) = 36 - 36 = 0$

one repeated rational number

118. $-3x^2 + 2x = 21$

Solution

$-3x^2 + 2x = 21 \Rightarrow -3x^2 + 2x - 21 = 0$

$a = -3, b = 2, c = -21$

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(-3)(-21) \\ &= 4 - 252 = -248 \end{aligned}$$

two different nonreal complex numbers

119. $3x^2 - 2x + 5 = 0$

Solution

$3x^2 - 2x + 5 = 0 \Rightarrow a = 3, b = -2, c = 5$

$$\begin{aligned} b^2 - 4ac &= (-2)^2 - 4(3)(5) \\ &= 4 - 60 = -56 \end{aligned}$$

two different nonreal complex numbers

120. $9x^2 + 42x + 49 = 0$

Solution

$9x^2 + 42x + 49 = 0$

$a = 9, b = 42, c = 49$

$$\begin{aligned} b^2 - 4ac &= (42)^2 - 4(9)(49) \\ &= 1764 - 1764 = 0 \end{aligned}$$

one repeated rational number

121. $10x^2 + 29x = 21$

Solution

$10x^2 + 29x = 21 \Rightarrow 10x^2 + 29x - 21 = 0$

$a = 10, b = 29, c = -21$

$b^2 - 4ac = (29)^2 - 4(10)(-21)$

$= 841 + 840 = 1681$

two different rational numbers

122. $10x^2 + x = 21$

Solution

$10x^2 + x = 21 \Rightarrow 10x^2 + x - 21 = 0$

$a = 10, b = 1, c = -21$

$b^2 - 4ac = (1)^2 - 4(10)(-21)$

$= 1 + 840 = 841$

two different rational numbers

123. $x^2 - 5x + 2 = 0$

Solution

$x^2 - 5x + 2 = 0 \Rightarrow a = 1, b = -5, c = 2$

$b^2 - 4ac = (-5)^2 - 4(1)(2) = 25 - 8 = 17$

two different irrational numbers

124. $-8x^2 - 2x = 13$

Solution

$-8x^2 - 2x = 13 \Rightarrow -8x^2 - 2x - 13 = 0$

$a = -8, b = -2, c = -13$

$b^2 - 4ac = (-2)^2 - 4(-8)(-13)$

$= 4 - 416 = -412$

two different nonreal complex numbers

125. Find two values of k such that $x^2 + kx + 3k - 5 = 0$ will have two roots that are equal.

Solution

$x^2 + kx + 3k - 5 = 0$

$a = 1, b = k, c = 3k - 5$

Set the discriminant equal to 0:

$$b^2 - 4ac = 0$$
$$k^2 - 4(1)(3k - 5) = 0$$
$$k^2 - 4(3k - 5) = 0$$
$$k^2 - 12k + 20 = 0$$
$$(k - 2)(k - 10) = 0$$
$k = 2$ or $k = 10$

126. For what value(s) of b will the solutions of $x^2 - 2bx + b^2 = 0$ be equal?

Solution

$$x^2 + 2bx + b^2 = 0$$
$a = 1$, $b = -2b$, $c = b^2$

Set the discriminant equal to 0:

$$b^2 - 4ac = 0$$
$$(-2b)^2 - 4(1)(b^2) = 0$$
$$4b^2 - 4b^2 = 0$$
$$0 = 0$$
True for all values of b

Change each rational equation to quadratic form and solve it by the most efficient method.

127. $x + 1 = \dfrac{12}{x}$

Solution

$$x + 1 = \frac{12}{x}$$
$$x(x + 1) = x\left(\frac{12}{x}\right)$$
$$x^2 + x = 12$$
$$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$
$x + 4 = 0$ **or** $x - 3 = 0$

$x = -4 \qquad x = 0$

128. $x - 2 = \dfrac{15}{x}$

Solution

$$
\begin{aligned}
x - 2 &= \frac{15}{x} \\
x(x - 2) &= x\left(\frac{15}{x}\right) \\
x^2 - 2x &= 15 \\
x^2 - 2x - 15 &= 0 \\
(x + 3)(x - 5) &= 0
\end{aligned}
$$

$$
x + 3 = 0 \quad \textbf{or} \quad x - 5 = 0
$$

$$
x = -3 \qquad\qquad x = 5
$$

129. $8x - \frac{3}{x} = 10$

Solution

$$
\begin{aligned}
8x - \frac{3}{x} &= 10 \\
x\left(8x - \frac{3}{x}\right) &= x(10) \\
8x^2 - 3 &= 10x \\
8x^2 - 10x - 3 &= 0 \\
(4x + 1)(2x - 3) &= 0
\end{aligned}
$$

$$
4x + 1 = 0 \quad \textbf{or} \quad 2x - 3 = 0
$$

$$
4x = -1 \qquad\qquad 2x = 3
$$

$$
x = -\tfrac{1}{4} \qquad\qquad x = \tfrac{3}{2}
$$

130. $15x - \frac{4}{x} = 4$

Solution

$$
\begin{aligned}
15x - \frac{4}{x} &= 4 \\
x\left(15x - \frac{4}{x}\right) &= x(4) \\
15x^2 - 4 &= 4x \\
15x^2 - 4x - 4 &= 0 \\
(5x + 2)(3x - 2) &= 0
\end{aligned}
$$

$$
5x + 2 = 0 \quad \textbf{or} \quad 3x - 2 = 0
$$

$$
5x = -2 \qquad\qquad 3x = 2
$$

$$
x = -\tfrac{2}{5} \qquad\qquad x = \tfrac{2}{3}
$$

131. $\frac{5}{x} = \frac{4}{x^2} - 6$

Solution

$$\frac{5}{x} = \frac{4}{x^2} - 6$$
$$x^2\left(\frac{5}{x}\right) = x^2\left(\frac{4}{x^2} - 6\right)$$
$$5x = 4 - 6x^2$$
$$6x^2 + 5x - 4 = 0$$
$$(3x + 4)(2x - 1) = 0$$
$$3x + 4 = 0 \quad \textbf{or} \quad 2x - 1 = 0$$
$$3x = -4 \qquad 2x = 1$$
$$x = -\tfrac{4}{3} \qquad x = \tfrac{1}{2}$$

132. $\frac{6}{x^2} + \frac{1}{x} = 12$

Solution

$$\frac{6}{x^2} + \frac{1}{x} = 12$$
$$x^2\left(\frac{6}{x^2} + \frac{1}{x}\right) = x^2(12)$$
$$6 + x = 12x^2$$
$$0 = 12x^2 - x - 6$$
$$0 = (3x + 2)(4x - 3)$$
$$3x + 2 = 0 \quad \textbf{or} \quad 4x - 3 = 0$$
$$3x = -2 \qquad 4x = 3$$
$$x = -\tfrac{2}{3} \qquad x = \tfrac{3}{4}$$

133. $x\left(30 - \frac{13}{x}\right) = \frac{10}{x}$

Solution

$$x\left(30 - \frac{13}{x}\right) = \frac{10}{x}$$
$$30x - 13 = \frac{10}{x}$$
$$x(30x - 13) = x\left(\frac{10}{x}\right)$$

$$30x^2 - 13x = 10$$
$$30x^2 - 13x = 10$$
$$30x^2 - 13x - 10 = 0$$
$$(5x + 2)(6x - 5) = 0$$
$$5x + 2 = 0 \quad \textbf{or} \quad 6x - 5 = 0$$
$$5x = -2 \qquad 6x = 5$$
$$x = -\tfrac{2}{5} \qquad x = \tfrac{5}{6}$$

134. $x\left(20 - \dfrac{17}{x}\right) = \dfrac{10}{x}$

Solution

$$x\left(20 - \frac{17}{x}\right) = \frac{10}{x}$$
$$20x - 17 = \frac{10}{x}$$
$$x(20x - 17) = x\left(\frac{10}{x}\right)$$
$$20x^2 - 17x = 10$$
$$20x^2 - 17x - 10 = 0$$
$$(5x + 2)(4x - 5) = 0$$
$$5x + 2 = 0 \quad \textbf{or} \quad 4x - 5 = 0$$
$$5x = -2 \qquad 4x = 5$$
$$x = -\tfrac{2}{5} \qquad x = \tfrac{5}{4}$$

135. $\dfrac{1}{x} + \dfrac{3}{x + 2} = 2$

Solution

$$\frac{1}{x} + \frac{3}{x + 2} = 2$$
$$x(x + 2)\left(\frac{1}{x} + \frac{3}{x + 2}\right) = x(x + 2)(2)$$
$$1(x + 2) + 3x = 2x(x + 2)$$
$$x + 2 + 3x = 2x^2 + 4x$$
$$0 = 2x^2 - 2$$
$$0 = 2(x + 1)(x - 1)$$
$$x + 1 = 0 \quad \textbf{or} \quad x - 1 = 0$$
$$x = -1 \qquad x = 1$$

136. $\frac{1}{x-1} + \frac{1}{x-4} = \frac{5}{4}$

Solution

$$\begin{aligned}
\frac{1}{x-1} + \frac{1}{x-4} &= \frac{5}{4} \\
4(x-1)(x-4)\left(\frac{1}{x-1} + \frac{1}{x-4}\right) &= 4(x-1)(x-4)\frac{5}{4} \\
4(x-4) + 4(x-1) &= 5(x-1)(x-4) \\
4x - 16 + 4x - 4 &= 5x^2 - 25x + 20 \\
0 &= 5x^2 - 33x + 40 \\
0 &= (5x-8)(x-5)
\end{aligned}$$

$$\begin{aligned}
5x - 8 &= 0 \quad \textbf{or} \quad & x - 5 &= 0 \\
5x &= 8 & x &= 5 \\
x &= \tfrac{8}{5} & x &= 5
\end{aligned}$$

137. $\frac{1}{x+1} + \frac{5}{2x-4} = 1$

Solution

$$\begin{aligned}
\frac{1}{x+1} + \frac{1}{2x-4} &= 1 \\
(x+1)(2x-4)\left(\frac{1}{x+1} + \frac{5}{2x-4}\right) &= (x+1)(2x-4)1 \\
1(2x-4) + 5(x+1) &= (x+1)(2x-4) \\
2x - 4 + 5x + 5 &= 2x^2 - 2x - 4 \\
0 &= 2x^2 - 9x - 5 \\
0 &= (2x+1)(x-5)
\end{aligned}$$

$$\begin{aligned}
2x + 1 &= 0 \quad \textbf{or} \quad & x - 5 &= 0 \\
2x &= -1 & x &= 5 \\
x &= -\tfrac{1}{2} & x &= 5
\end{aligned}$$

138. $\frac{x(2x+1)}{x-2} = \frac{10}{x-2}$

Solution

$$\begin{aligned}
\frac{x(2x+1)}{x-2} &= \frac{10}{x-2} \\
(x-2)\frac{x(2x+1)}{x-2} &= (x-2)\frac{10}{x-2}
\end{aligned}$$

$$x(2x + 1) = 10$$
$$2x^2 + x = 10$$
$$2x^2 + x - 10 = 0$$
$$(2x + 5)(x - 2) = 0$$
$$2x + 5 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$2x = -5 \qquad x = 2$$
$$x = -\tfrac{5}{2} \qquad x = 2$$

Since $x = 2$ does not check, the only solution is $x = -\frac{5}{2}$.

139. $x + 1 + \dfrac{x + 2}{x - 1} = \dfrac{3}{x - 1}$

Solution

$$x + 1 + \frac{x + 2}{x - 1} = \frac{3}{x - 1}$$
$$(x - 1)\left(\frac{x + 1}{1} + \frac{x + 2}{x - 1}\right) = (x - 1)\frac{3}{x - 1}$$
$$(x - 1)(x + 1) + x + 2 = 3$$
$$x^2 - 1 + x + 2 = 3$$
$$x^2 + x - 2 = 0$$
$$(x + 2)(x - 1) = 0$$
$$x + 2 = 0 \quad \textbf{or} \quad x - 1 = 0$$
$$x = -2 \qquad x = 1$$

Since $x = 1$ does not check, the only solution is $x = -2$.

140. $\dfrac{1}{4 - y} = \dfrac{1}{4} + \dfrac{1}{y + 2}$

Solution

$$\frac{1}{4 - y} = \frac{1}{4} + \frac{1}{y + 2}$$
$$4(4 - y)(y + 2)\left(\frac{1}{4 - y}\right) = 4(4 - y)(y + 2)\left(\frac{1}{4} + \frac{1}{y + 2}\right)$$
$$4(y + 2)(1) = (4 - y)(y + 2)(1) + 4(4 - y)(1)$$
$$4y + 8 = -y^2 + 2y + 8 + 16 - 4y$$
$$y^2 + 6y - 16 = 0$$
$$(y + 8)(y - 2) = 0$$
$$y + 8 = 0 \quad \textbf{or} \quad y - 2 = 0$$
$$y = -8 \qquad y = 2$$

141. $\frac{4 + a}{2a} = \frac{a - 2}{3}$

Solution

$$\frac{4 + a}{2a} = \frac{a - 2}{3}$$

$$6a\left(\frac{4 + a}{2a}\right) = 6a\left(\frac{a - 2}{3}\right)$$

$$12 + 3a = 2a^2 - 4a$$

$$0 = 2a^2 - 7a - 12$$

$$a = 2,\ b = -7,\ c = -12$$

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-12)}}{2(2)} = \frac{7 \pm \sqrt{145}}{4}$$

142. $\frac{(a - 2)(a + 4)}{10} = \frac{a(a - 3)}{5}$

Solution

$$\frac{(a - 2)(a + 4)}{10} = \frac{a(a - 3)}{5}$$

$$10\left[\frac{(a - 2)(a + 4)}{10}\right] = 10\left[\frac{a(a - 3)}{5}\right]$$

$$(a - 2)(a + 4) = 2a(a - 3)$$

$$a^2 + 2a - 8 = 2a^2 - 6a$$

$$0 = a^2 - 8a + 8$$

$$a = 1,\ b = -8,\ c = 8$$

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(8)}}{2(1)} = \frac{8 \pm \sqrt{32}}{2} = \frac{8 \pm 4\sqrt{2}}{2} = 4 \pm 2\sqrt{2}$$

143. $x + \frac{36}{x} = 0$

Solution

$$x + \frac{36}{x} = 0$$

$$x\left(x + \frac{36}{x}\right) = x \cdot 0$$

$$x^2 + 36 = 0$$

$$x^2 = -36$$

$$x = \pm\sqrt{-36}$$

$$x = \pm 6i$$

144. $x + \frac{5}{x} = 2$

Solution

$$\begin{aligned} x + \frac{5}{x} &= 2 \\ x\left(x + \frac{5}{x}\right) &= x \cdot 2 \\ x^2 + 5 &= 2x \\ x^2 - 2x &= -5 \\ x^2 - 2x + 1 &= -5 + 1 \\ (x - 1)^2 &= -4 \\ x - 1 &= \pm\sqrt{-4} \\ x &= 1 \pm 2i \end{aligned}$$

Fix It

In exercises 145 and 146, identify the step the first error is made and fix it.

145. Solve by completing the square: $x^2 - 6x + 34 = 0$

Solution
Step 4 was incorrect.

Step 1: $x^2 - 6x = -34$

Step 2: $x^2 - 6x + 9 = -34 + 9$

Step 3: $(x - 3)^2 = -25$

Step 4: $x - 3 = \pm 5i$

Step 5: $x = 3 + 5i$

146. Solve $x^2 = 6x - 2$ by using the quadratic formula. To do so, identify *a*, *b*, and *c*. Then substitute into the formula and simplify.

Solution
Step 3 was incorrect.

Step 1: $a = 1$; $b = -6$; $c = 2$

Step 2: $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$

Step 3: $x = \frac{6 \pm \sqrt{24}}{2}$

Step 4: $x = \frac{6 \pm 2\sqrt{6}}{2}$

Step 5: $x = 3 \pm \sqrt{6}$

Discovery and Writing

147. Explain why the Zero-Factor Theorem is true.

Solution
Answers may vary.

148. Explain how to complete the square on $x^2 - 17x$.

Solution
Answers may vary.

149. If r_1 and r_2 are the roots of $ax^2 + bx + c = 0$, show that $r_1 + r_2 = -\frac{b}{a}$.

Solution

If r_1 and r_2 are the roots of $ax^2 + bx + c = 0$, then their values are

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$
$$r_1 + r_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$$

150. If r_1 and r_2 are the roots of $ax^2 + bx + c = 0$, show that $r_1 r_2 = \frac{c}{a}$.

Solution

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$
$$r_1 r_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \cdot \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{\left(-b + \sqrt{b^2 - 4ac}\right)\left(-b - \sqrt{b^2 - 4ac}\right)}{4a^2}$$
$$= \frac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{4a^2} = \frac{b^2 - \left(b^2 - 4ac\right)}{4a^2} = \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

In Exercises 151 and 152, a stone is thrown straight upward, higher than the top of a tree. The stone is even with the top of the tree at time t_1 on the way up and at time t_2 on the way down. If the height of the tree is h feet, both t_1 and t_2 are solutions of $h = v_0 t - 16t^2$.

151. Show that the tree is $16t_1t_2$ feet tall.

Solution

Rewrite the equation as $16t^2 - v_0 t + h = 0$ and solve for t using the quadratic formula.

$a = 16, b = -v_0, c = h$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-v_0) \pm \sqrt{(-v_0)^2 - 4(16)(h)}}{2(16)} = \frac{v_0 \pm \sqrt{v_0^2 - 64h}}{32}$$

Since t_1 and t_2 are the solutions to the equation, we have

$t_1 = \frac{v_0 - \sqrt{v_0^2 - 64h}}{32}$ and $t_2 = \frac{v_0 + \sqrt{v_0^2 - 64h}}{32}$. Calculate $16t_1t_2$:

$$16t_1t_2 = 16 \cdot \frac{v_0 - \sqrt{v_0^2 - 64h}}{32} \cdot \frac{v_0 + \sqrt{v_0^2 - 64h}}{32} = 16 \cdot \frac{v_0^2 - (v_0^2 - 64h)}{1024}$$
$$= \frac{16.64h}{1024} = \frac{1024h}{1024} = h$$

Thus, $h = 16t_1t_2$.

152. Show that v_0 is $16(t_1 + t_2)$ feet per second.

Solution

Proceed as in #135 to calculate t_1 and t_2. Then

$$16(t_1 + t_2) = 16\left(\frac{v_0 - \sqrt{v_0^2 - 64h}}{32} + \frac{v_0 + \sqrt{v_0^2 - 64h}}{32}\right) = 16\left(\frac{2v_0}{32}\right) = \frac{32v_0}{32} = v_0.$$

Thus, $v_0 = 16(t_1 + t_2)$.

Critical Thinking

In Exercises 153–156, match each quadratic equation on the left with the easiest strategy to use to solve it on the right.

153. $6x^2 + 76 = 0$	a. Factoring
154. $6x^2 + 35x - 6 = 0$	b. Square Root Property
155. $x^2 - 6x = 6$	c. Completing the Square
156. $6x^2 = 6x + 1$	d. Quadratic Formula

Solution

153. b

154. a

155. c

156. d

Determine if the statement is true or false. If the statement is false, then correct it and make it true.

157. $1492x^2 + 1984x - 1776 = 0$ has real number solutions.

Solution

$1492x^2 + 1984x - 1776 = 0$

$a = 1492, b = 1984, c = -1776$

$$\begin{aligned} b^2 - 4ac &= (1984)^2 - 4(1492)(-1776) \\ &= 3{,}936{,}256 + 10{,}599{,}168 \\ &= 14{,}535{,}424 \end{aligned}$$

The solutions are real numbers. True.

158. $2004x^2 + 10x + 1994 = 0$ has real number solutions.

Solution

$2004x^2 + 10x + 1994 = 0$

$a = 2004, b = 10, c = 1994$

$$\begin{aligned} b^2 - 4ac &= (10)^2 - 4(2004)(1994) \\ &= 100 - 15{,}983{,}904 \\ &= -15{,}983{,}804 \end{aligned}$$

The solutions are real numbers. False.

Exercises 1.5

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. If the length of a rectangle is 15 feet and its width is $\frac{12}{5}$ feet, what is the area of the rectangle?

Solution

$15 \cdot \frac{12}{5} = 36ft^2$

2. Write and algebraic expression that represents the area of a rectangle if its width is x and its length is $(5x - 2)$.

Solution

$x(5x - 2) = 5x^2 - 2x$

3. The shorter leg of a right triangle measures 10 yards and the longer leg measures 20 yards. Find the hypotenuse of the right triangle.

Solution

$10^2 + 20^2 = x^2$

$100 + 400 = x^2$

$500 = x^2$

$x = \sqrt{500}$ *or* $x = -\sqrt{500}$ (however, x can't be negative)

$x = \sqrt{100 \cdot 5}$

$x = 10\sqrt{5}$ *yards*

4. Distance equals rate times time, $d = rt$. a) What does r equal? b) What does t equal?

Solution

a. $r = \dfrac{d}{t}$

b. $t = \dfrac{d}{t}$

5. Solve for t: $-16t^2 + 96t = 0$

Solution

$-16t^2 + 96t = 0$

$-16t(t - 6) = 0$

$-16t = 0$ or $t - 6 = 0$

$t = 0$ $t = 6$

6. What would be the most efficient method to use to solve an application problem in which the equation $-4.9t^2 + 28.6t + 67.3 = 0$ occurs?

Solution
The quadratic formula

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. The formula for the area of a rectangle is __________.

Solution
$A = lw$

8. The __________ Theorem states that the sum of the squares of the lengths of a right triangle equals the square of the length of the hypotenuse.

Solution
$d = rt$, Pythagorean

Practice

Solve each problem.

9. **Geometric problem** A rectangle is 4 feet longer than it is wide. If its area is 32 square feet, find its dimensions.

Solution
Let w = the width of the rectangle. Then $w + 4$ = the length.

$$\boxed{\text{Width}} \cdot \boxed{\text{Length}} = \boxed{\text{Area}}$$
$$w(w+4) = 32$$
$$w^2 + 4w = 32$$
$$w^2 + 4w - 32 = 0$$
$$(w+8)(w-4) = 0$$
$$w + 8 = 0 \quad \textbf{or} \quad w - 4 = 0$$
$$w = -8 \qquad\qquad w = 4$$

Since the width cannot be negative, the only reasonable solution is $w = 4$. The dimensions are 4 feet by 8 feet.

10. **Geometric problem** A rectangle is five times as long as it is wide. If the area is 125 square feet, find its perimeter.

Solution
Let w = the width of the rectangle. Then $5w$ = the length.

$$\boxed{\text{Width}} \cdot \boxed{\text{Length}} = \boxed{\text{Area}}$$
$$w \cdot 5w = 125$$
$$5w^2 = 125$$
$$w^2 = 25$$
$$w = \sqrt{25} \quad \textbf{or} \quad w = -\sqrt{25}$$
$$w = 5 \qquad\qquad w = -5$$

Since the width cannot be negative, the only reasonable solution is $w = 5$. The dimensions are 5 feet by 25 feet, and the perimeter is 60 feet.

11. **Jumbotron** The length of a rectangular jumbotron is 88 feet more than its width. If the jumbotron has an area of 11,520 square feet, find its dimensions.

Solution
Let w = the width of the screen. Then $w + 88$ = the length.

$$\boxed{\text{Width}} \cdot \boxed{\text{Length}} = \boxed{\text{Area}}$$
$$w(w+88) = 11520$$
$$w^2 + 88w = 11520$$

$$w^2 + 88w - 11520 = 0 \Rightarrow a = 1, b = 88, c = -11520$$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(88) \pm \sqrt{(88)^2 - 4(1)(-11520)}}{2(1)} = \frac{-(88) \pm \sqrt{53824}}{2}$$

$w = 72$ or $w = -160$; Since the screen cannot have a negative width, the solution is $w = 72$ and the dimensions of the screen are 160 ft by 72 ft.

12. **IMAX screen** A large movie screen is in the Panasonic IMAX theater at Darling Harbor, Sydney, Australia. The rectangular screen has an area of 11,349 square feet. Find the dimensions of the screen if it is 20 feet longer than it is wide.

Solution

Let w = the width of the screen. Then $w + 20$ = the length.

$$\boxed{\text{Width}} \cdot \boxed{\text{Length}} = \boxed{\text{Area}}$$

$$w(w + 20) = 11349$$

$$w^2 + 20w = 11349$$

$$w^2 + 20w - 11349 = 0 \Rightarrow a = 1, b = 20, c = -11349$$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(20) \pm \sqrt{(20)^2 - 4(1)(-11349)}}{2(1)} = \frac{-20 \pm \sqrt{45796}}{2}$$

$w = 97$ or $w = -117$; Since the screen cannot have a negative width, the solution is $w = 97$ and the dimensions of the screen are 117 ft by 97 ft.

13. **Geometric problem** The side of a square is 4 centimeters shorter than the side of a second square. If the sum of their areas is 106 square centimeters, find the length of one side of the larger square.

Solution

Let s = the side of the second square. Then $s - 4$ = the side of the first square.

$$\boxed{\begin{array}{c}\text{Area of}\\ \text{first}\end{array}} + \boxed{\begin{array}{c}\text{Area of}\\ \text{second}\end{array}} = 106$$

$$(s - 4)^2 + s^2 = 106$$

$$s^2 - 8s + 16 + s^2 = 106$$

$$2s^2 - 8s - 90 = 0$$

$$2(s^2 - 4s - 45) = 0$$

$$2(s + 5)(s - 9) = 0$$

$s + 5 = 0$ **or** $s - 9 = 0$

$s = -5$ $\quad$ $s = 9$

Since the side cannot be negative, the only reasonable solution is $s = 9$. The larger square has a side of length 9 cm.

14. **Geometric problem** If two opposite sides of a square are increased by 10 meters and the other sides are decreased by 8 meters, the area of the rectangle that is formed is 63 square meters. Find the area of the original square.

Solution

Let s = the side of the original square. Then the new rectangle has dimensions of $s + 10$ and $s - 8$.

$$\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$$
$$(s + 10)(s - 8) = 63$$
$$s^2 - 2s - 80 = 63$$
$$s^2 + 2s - 143 = 0$$
$$(s + 13)(s - 11) = 0$$
$$s + 13 = 0 \quad \textbf{or} \quad s - 11 = 0$$
$$s = -13 \qquad\qquad s = 11$$

Since the side cannot be negative, the only reasonable solution is $s = 11$. The original area was $11^2 = 121 \text{ m}^2$

15. **Geometric problem** Find the dimensions of a rectangle whose area is 180 cm^2 and whose perimeter is 54 cm.

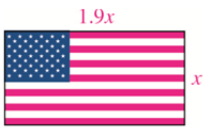
Solution

$$P = 2l + 2w, \text{ so } l = \frac{P - 2w}{2} = \frac{54 - 2w}{2}.$$

$$\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$$
$$\frac{54 - 2w}{2} \cdot w = 180$$
$$(54 - 2w)w = 360$$
$$54w - 2w^2 = 360$$
$$0 = 2w^2 - 54w + 360$$
$$0 = 2(w^2 - 27w + 180)$$
$$0 = 2(w - 12)(w - 15)$$
$$w - 12 = 0 \quad \textbf{or} \quad w - 15 = 0$$
$$w = 12 \qquad\qquad w = 15$$

The dimensions are 12 cm by 15 cm.

16. **Flags** In 1912, an order by President Taft fixed the width and length of the U.S. flag in the ratio of 1 to 1.9. If 100 square feet of cloth are to be used to make a U.S. flag, estimate its dimensions to the nearest $\frac{1}{4}$ foot.



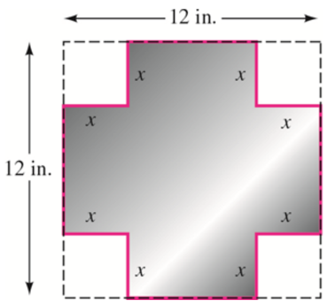
Solution

Let the dimensions be x and $1.9x$.

$$\boxed{\text{Width}} \cdot \boxed{\text{Length}} = \boxed{\text{Area}}$$
$$x(1.9x) = 100$$
$$1.9x^2 = 100$$
$$x^2 = \frac{100}{1.9}$$
$$x = \pm\sqrt{\frac{100}{1.9}}$$
$$x \approx \pm 7.25$$
$$1.9x \approx 1.9(7.25) \approx 13.75$$

Since the dimensions cannot be negative, the only reasonable solution $7\frac{1}{4}$ ft by $13\frac{3}{4}$ ft.

17. **Metal fabrication** A piece of tin, 12 inches on a side, is to have four equal squares cut from its corners, as in the illustration. If the edges are then to be folded up to make a box with a floor area of 64 square inches, find the depth of the box.



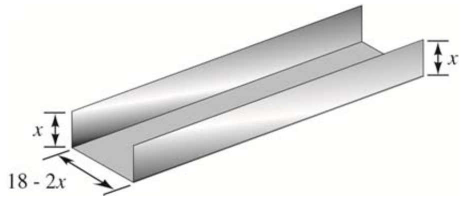
Solution

The floor area of the box is a square with a side of length $12 - 2x$.

$$\boxed{\text{Floor area}} = 64$$
$$(12 - 2x)^2 = 64$$
$$144 - 48x + 4x^2 = 64$$
$$4x^2 - 48x + 80 = 64$$
$$4(x^2 - 12x + 12) = 0$$
$$4(x - 2)(x - 10) = 0$$
$$x - 2 = 0 \quad \textbf{or} \quad x - 10 = 0$$
$$x = 2 \qquad\qquad x = 10$$

The solution $x = 10$ does not make sense in the problem, so the depth is 2 inches.

18. **Making gutters** A piece of sheet metal, 18 inches wide, is bent to form the gutter shown in the illustration. If the cross-sectional area is 36 square inches, find the depth of the gutter.



Solution

$$\boxed{\text{Cross-sectional area}} = 36$$
$$x(18 - 2x) = 36$$
$$18x - 2x^2 = 36$$
$$0 = 2x^2 - 18x + 36$$
$$0 = 2(x^2 - 9x + 18)$$
$$0 = 2(x - 3)(x - 6)$$
$$x - 3 = 0 \quad \textbf{or} \quad x - 6 = 0$$
$$x = 3 \qquad\qquad x = 6$$

Both solutions are valid, so the depth of the gutter is either 3 inches or 6 inches.

19. **Parking lot** A rectangular parking lot at PetSmart is 480 feet by 550 feet. Determine the diagonal of the parking lot.

Solution
Using the Pythagorean Theorem:

$$\mathit{height}^2 + \mathit{width}^2 = \mathit{diagonal}^2$$
$$480^2 + 550^2 = \mathit{diagonal}^2$$
$$532{,}900 = x^2$$
$$x = \sqrt{532{,}900} \quad \text{or} \quad x = -\sqrt{532{,}900}$$
$$x = 730 \qquad\qquad x = -730$$

Since lengths are positive, the answer is $x = 730$ feet.

20. **Photograph** The diagonal of a square photograph of Katy Perry measures $8\sqrt{2}$ inches. Find the length of one of its sides.

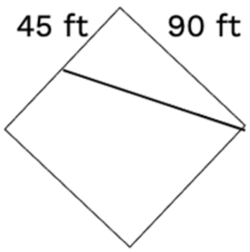
Solution
Using the Pythagorean Theorem:

$$\mathit{side}^2 + \mathit{side}^2 = \mathit{diagonal}^2$$
$$x^2 + x^2 = \left(8\sqrt{2}\right)^2$$
$$2x^2 = 128$$
$$x^2 = 64$$
$$x = 8 \quad \text{or} \quad x = -8$$

Since lengths are positive, the answer is $x = 8$ *inches*

21. **Baseball** A baseball diamond is a square, 90 feet on a side. A shortstop for the Los Angeles Dodgers fields a grounder at a point halfway between second base and third base. How far will he have to throw the ball to make an out at first base?

Solution



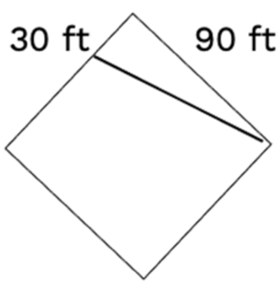
A right triangle is formed in which the longer leg is 90 feet and the shorter leg is 45 feet (half of 90 feet). The hypotenuse will represent the distance that the ball will be thrown.

$$90^2 + 45^2 = x^2$$
$$10{,}125 = x^2$$
$$x = \sqrt{10{,}125} \quad \text{or} \quad x = -\sqrt{10{,}125}$$
$$x = 45\sqrt{5} \quad \text{or} \quad x = -45\sqrt{5}$$

Since lengths are positive, the answer is $x = 45\sqrt{5}$ feet

22. **Baseball** A baseball diamond is a square, 90 feet on a side. A shortstop for the Chicago Cubs fields a grounder at a point one third of the way between second base and third base. How far will he have to throw the ball to make an out at first base?

Solution



A right triangle is formed in which the longer leg is 90 feet and the shorter leg is 30 feet (one-third of 90 feet). The hypotenuse will represent the distance that the ball will be thrown.

$$90^2 + 30^2 = x^2$$
$$9{,}000 = x^2$$
$$x = \sqrt{9000} \quad \text{or} \quad x = -\sqrt{9000}$$
$$x = 30\sqrt{10} \quad \text{or} \quad x = -30\sqrt{10}$$

Since lengths are positive, the answer is $x = 30\sqrt{10}$ feet

23. **Geometric problem** The base of a triangle is onethird as long as its height. If the area of the triangle is 24 square meters, how long is its base?

Solution

Let h = the height of the triangle. Then $\frac{1}{3}h$ = the base of the triangle.

$$\frac{1}{2} \cdot \boxed{\text{Base}} \cdot \boxed{\text{Height}} = \boxed{\text{Area}}$$
$$\frac{1}{2} \cdot \frac{1}{3}h \cdot h = 24$$
$$\frac{1}{6}h^2 = 24$$
$$h^2 = 144$$
$$h = \sqrt{144} \quad \textbf{or} \quad h = -\sqrt{144}$$
$$h = 12 \qquad h = -12$$

Since the height cannot be negative, the only reasonable solution is $h = 12$. The base has a length of 4 meters.

24. **Geometric problem** The base of a triangle is onehalf as long as its height. If the area of the triangle is 100 square yards, find its height.

Solution

Let h = the height of the triangle. Then $\frac{1}{2}h$ = the base of the triangle.

$$\frac{1}{2} \cdot \boxed{\text{Base}} \cdot \boxed{\text{Height}} = \boxed{\text{Area}}$$
$$\frac{1}{2} \cdot \frac{1}{2}h \cdot h = 100$$
$$\frac{1}{4}h^2 = 100$$
$$h^2 = 400$$
$$h = \sqrt{400} \quad \textbf{or} \quad h = -\sqrt{400}$$
$$h = 20 \qquad h = -20$$

Since the height cannot be negative, the only reasonable solution is $h = 20$. The base has a length of 20 yards.

25. **Right triangle** If one leg of a right triangle is 14 meters shorter than the other leg, and the hypotenuse is 26 meters, find the length of the two legs.

Solution

Let the legs have lengths x and $x - 14$.

$$x^2 + (x - 14)^2 = 26^2$$
$$x^2 + x^2 - 28x + 196 = 676$$
$$2x^2 - 28x - 480 = 0$$
$$2(x^2 - 14x - 240) = 0$$
$$2(x - 24)(x + 10) = 0$$

$$x - 24 = 0 \quad \textbf{or} \quad x + 10 = 0$$
$$x = 24 \qquad\qquad x = -10$$

Since lengths are positive, the answer is $x = 24$, and the legs have length 10 meters and 24 meters.

26. **Right triangle** If one leg of a right triangle is five times the other leg, and the hypotenuse is $10\sqrt{26}$ centimeters, find the length of the two legs.

Solution

Let the legs have lengths x and $5x$.

$$x^2 + (5x)^2 = \left(10\sqrt{26}\right)^2$$
$$x^2 + 25x^2 = 2600$$
$$26x^2 = 2600$$
$$x^2 = 100$$
$$x = \sqrt{100} \quad \textbf{or} \quad x = -\sqrt{100}$$
$$x = 10 \qquad\qquad x = -10$$

Since lengths are positive, the answer is $x = 10$, and the legs have length 10 cm and 50 cm.

27. **Manufacturing** A manufacturer of television sets for a news studio received an order for sets with a 46-inch screen (measured along the diagonal). If the televisions are $17\frac{1}{2}$ inches wider than they are high, find the dimensions of the screen to the nearest tenth of an inch.

Solution

Let x = the height of the screen. Then $x + 17.5$ = the width. Use the Pythagorean Theorem:

$$\text{height}^2 + \text{width}^2 = \text{diagonal}^2$$
$$x^2 + (x + 17.5)^2 = 46^2$$
$$x^2 + x^2 + 35x + 306.25 = 2116$$
$$2x^2 + 35x - 1809.75 = 0$$
$$a = 2,\ b = 35,\ c = -1809.75$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-35 \pm \sqrt{35^2 - 4(2)(-1809.75)}}{2(2)}$$
$$= \frac{-35 \pm \sqrt{15703}}{4} \approx \frac{-35 \pm 125.312}{4}$$

The only positive solution is 22.6. The dimensions are 22.6 inches by 40.1 inches.

28. **Finding dimensions** An oriental rug is 2 feet longer than it is wide. If the diagonal of the rug is 12 feet, to the nearest tenth of a foot, find its dimensions.

Solution

Let x = the width of the rug. Then $x + 2$ = the length. Use the Pythagorean Theorem:

$$\begin{aligned} \text{width}^2 + \text{height}^2 &= \text{diagonal}^2 \\ x^2 + (x+2)^2 &= 12^2 \\ x^2 + x^2 + 4x + 4 &= 144 \\ 2x^2 + 4x - 140 &= 0 \end{aligned}$$

$$a = 2,\ b = 4,\ c = -140$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{4^2 - 4(2)(-140)}}{2(2)} \\ &= \frac{-4 \pm \sqrt{1136}}{4} \approx \frac{-4 \pm 33.705}{4} \end{aligned}$$

The only positive solution is 7.4. The dimensions are 7.4 feet by 9.4 feet.

29. **Cycling rates** A cyclist rides from DeKalb to Rockford, a distance of 40 miles. His return trip takes 2 hours longer, because his speed decreases by 10 mph. How fast does he ride each way?

Solution

Let r = the cyclist's rate from DeKalb to Rockford. Then his return rate is $r - 10$.

$$\begin{aligned} \boxed{\text{Return time}} &= \boxed{\text{First time}} + 2 \\ \frac{40}{r-10} &= \frac{40}{r} + 2 \\ r(r-10)\frac{40}{r-10} &= r(r-10)\left(\frac{40}{r} + 2\right) \\ 40r &= 40(r-10) + 2r(r+10) \\ 40r &= 40r - 400 + 2r^2 - 20r \\ 0 &= 2r^2 - 20r - 400 \\ 0 &= 2(r-20)(r+10) \end{aligned}$$

$$r - 20 = 0 \quad \textbf{or} \quad r + 10 = 0$$
$$r = 20 \qquad\qquad r = -10$$

	Rate	Time	Dist.
First trip	r	$\frac{40}{r}$	40
Return trip	$r - 10$	$\frac{40}{r-10}$	40

Since $r = -10$ does not make sense, the solution is $r = 20$. The cyclist rides 20 mph going and 10 mph returning.

30. **Travel times** Jake drives a moped from one town to another, a distance of 120 kilometers. He drives 10 kilometers per hour faster on the return trip, cutting 1 hour off the time. How fast does he drive each way?

Solution

Let r = the farmer's first rate. Then his return rate is $r + 10$.

$$\boxed{\text{Return time}} = \boxed{\text{First time}} - 1$$

$$\frac{120}{r+10} = \frac{120}{r} - 1$$

$$r(r+10)\frac{120}{r+10} = r(r+10)\left(\frac{120}{r} - 1\right)$$

$$120r = 120(r+10) - r(r+10)$$

$$120r = 120r + 1200 - r^2 - 10r$$

$$r^2 + 10r - 1200 = 0$$

$$(r-30)(r+40) = 0$$

$$r - 30 = 0 \quad \textbf{or} \quad r + 40 = 0$$

$$r = 30 \qquad\qquad r = -40$$

	Rate	Time	Dist.
First trip	r	$\frac{120}{r}$	120
Return trip	$r + 10$	$\frac{120}{r+10}$	120

Since $r = -40$ does not make sense, the solution is $r = 30$. The farmer drives 30 kph going and 40 kph returning.

31. **Uniform motion problem** If the speed were increased by 10 mph, a 420-mile trip would take 1 hour less time. How long will the trip take at the slower speed?

Solution

Let r = the slower rate. Then the faster rate is $r + 10$.

$$\boxed{\text{Faster time}} = \boxed{\text{Slower time}} - 1$$

$$\frac{420}{r+10} = \frac{420}{r} - 1$$

$$r(r+10)\frac{420}{r+10} = r(r+10)\left(\frac{420}{r} - 1\right)$$

$$420r = 420(r+10) - r(r+10)$$

$$420r = 420r + 4200 - r^2 - 10r$$

$$r^2 + 10r - 4200 = 0$$

$$(r-60)(r+70) = 0$$

$$r - 60 = 0 \quad \textbf{or} \quad r + 70 = 0$$

$$r = 60 \qquad\qquad r = -70$$

	Rate	Time	Dist.
Slower trip	r	$\frac{420}{r}$	420
Faster trip	$r + 10$	$\frac{420}{r+10}$	420

Since $r = -70$ does not make sense, the solution is $r = 60$. The slower speed results in a trip of length 7 hours.

32. **Uniform motion problem** By increasing her usual speed by 25 kilometers per hour, a bus driver decreases the time on a 25-kilometer trip by 10 minutes. Find the usual speed.

Solution

Let r = the driver's slower rate. Then her faster rate is $r + 25$.

$$\boxed{\text{Faster time}} = \boxed{\text{Slower time}} - \frac{10}{60}$$

$$\frac{25}{r+25} = \frac{25}{r} - \frac{1}{6}$$

$$6r(r+25)\frac{25}{r+25} = 6r(r+25)\left(\frac{25}{r} - \frac{1}{6}\right)$$

$$150r = 150(r+25) - r(r+25)$$

$$150r = 150r + 3750 - r^2 - 25r$$

$$r^2 + 25r - 3750 = 0$$

$$(r-50)(r+75) = 0$$

$$r - 50 = 0 \quad \textbf{or} \quad r + 75 = 0$$

$$r = 50 \qquad\qquad r = -75$$

	Rate	Time	Dist.
Slower trip	r	$\frac{25}{r}$	25
Faster trip	$r + 25$	$\frac{25}{r+25}$	25

Since $r = -75$ does not make sense, the solution is $r = 50$. The driver's usual speed is 50 kph.

33. **Falling coins** An object falls $16t^2$ feet in t seconds. If a penny is dropped from the top of the Sears Tower in Chicago, from a height of 1454 feet, how long will it take for the penny to hit the ground? Round to one decimal place.

Solution

Set $s = 1454$:

$$s = 16t^2$$

$$1454 = 16t^2$$

$$\frac{1454}{16} = t^2$$

$$t = \sqrt{\frac{1454}{16}} \quad \textbf{or} \quad t = -\sqrt{\frac{1454}{16}}$$

$$t \approx 9.5 \qquad\qquad t \approx -9.5$$

$t = -9.5$ does not make sense, so it takes it about 9.5 seconds to hit the ground.

34. **Stunt act** According to the *Guinness Book of World Records,* stuntman Dan Koko fell a distance of 312 feet into an airbag after jumping from the Vegas World Hotel and Casino. If Dan fell $16t^2$ feet in t seconds, to the nearest tenth of a second, how long did he fall?

Solution

Set $d = 312$:

$$d = 16t^2$$
$$312 = 16t^2$$
$$\frac{312}{16} = t^2$$
$$t = \sqrt{\frac{312}{16}} \quad \textbf{or} \quad t = -\sqrt{\frac{312}{16}}$$
$$t \approx 4.4 \qquad t \approx -4.4$$

$t = -4.4$ does not make sense, so the fall lasted about 4.4 seconds.

35. **Tybee Island lighthouse** Kylie accidentally drops her car keys from the top of the Tybee Island lighthouse, 44.2 meters high. If the keys fall $4.9t^2$ meters per second, where t represents time in seconds, how long will it take her keys to hit the ground? Round to the nearest second.

Solution

Set $h = 0$:

$$h = -4.9t^2 + 44.2$$
$$0 = -4.9t^2 + 44.2$$
$$-4.9t^2 + 44.2 = 0$$
$$-4.9t^2 = -44.2$$
$$t^2 = \frac{44.2}{4.9}$$
$$t = \sqrt{\frac{44.2}{4.9}} \quad \text{or} \quad t = -\sqrt{\frac{44.2}{4.9}}$$
$$t \approx 3 \quad \text{or} \quad t \approx -3$$

$t = -3$ does not make sense, so it takes about 3 seconds for the keys to hit the ground.

36. **Brooklyn Bridge** Brad accidentally drops his water bottle from the Brooklyn Bridge, 84.28 meters high. If the bottle falls $4.9t^2$ meters per second, where t represents time in seconds, how long will it take the water bottle to land in New York City's East River? Round to one decimal place.

Solution

Set $h = 0$:

$$h = -4.9t^2 + 84.28$$
$$0 = -4.9t^2 + 84.28$$

$$-4.9t^2 + 84.28 = 0$$
$$-4.9t^2 = -84.28$$
$$t^2 = \frac{84.28}{4.9}$$
$$t = \sqrt{\frac{84.28}{4.9}} \quad \text{or} \quad t = -\sqrt{\frac{84.28}{4.9}}$$
$$t \approx 4.1 \quad \text{or} \quad t \approx -4.1$$

$t = -4.1$ does not make sense, so it takes about 4.1 seconds for the keys to hit the ground.

37. **Cliff jumping** Jeff jumped off of a cliff into the ocean while vacationing in Maui, Hawaii. His height h above the water in feet after t seconds is represented by the equation $h = -16t^2 + 5t + 30$. After how many seconds did he hit the water? Round to the nearest tenth.

Solution

Set $h = 0$:

$$h = -16t^2 + 5t + 30$$
$$0 = -16t^2 + 5t + 30$$
$$t = \frac{-5 \pm \sqrt{25 - 4(-16)(30)}}{2(-16)}$$
$$t = \frac{-5 \pm \sqrt{1945}}{-32}$$
$$t \approx 1.5 \quad \text{or} \quad t \approx -1.2$$

$t = -1.2$ doesn't make sense, so it takes about 1.5 seconds for Jeff to hit the water.

38. **Platform diving** An athlete dives from a 32 feet platform with an initial velocity of 7 feet per second. The formula $h = -16t^2 + 7t + 32$ can be used to determine the height of the diver in feet after t seconds. When did the athlete reach the pool water? Round to two decimal places.

Solution

Set $h = 0$:

$$h = -16t^2 + 7t + 32$$
$$0 = -16t^2 + 7t + 32$$
$$t = \frac{-7 \pm \sqrt{49 - 4(-16)(32)}}{2(-16)}$$
$$t = \frac{-7 \pm \sqrt{2097}}{-32}$$
$$t \approx 1.65 \quad \text{or} \quad t \approx -1.21$$

$t = -1.2$ doesn't make sense, so it takes about 1.65 seconds for the diver to hit the water.

39. **Ballistics** The height of a projectile fired upward with an initial velocity of 400 feet per second is given by the formula $h = -16t^2 + 400t$, where h is the height in feet and t is the time in seconds. Find the time required for the projectile to return to earth.

Solution

Set $h = 0$:

$$h = -16t^2 + 400t$$
$$0 = -16t^2 + 400t$$
$$16t^2 - 400t = 0$$
$$16t(t - 25) = 0$$
$$16t = 0 \quad \text{or} \quad t - 25 = 0$$
$$t = 0 \qquad\qquad t = 25$$

$t = 0$ represents when the projectile was fired, so it returns to earth after 25 seconds.

40. **Ballistics** The height of an object tossed upward with an initial velocity of 104 feet per second is given by the formula $h = -16t^2 + 104t$, where h is the height in feet and t is the time in seconds. Find the time required for the object to return to its point of departure.

Solution

Set $h = 0$:

$$h = -16t^2 + 104t$$
$$0 = -16t^2 + 104t$$
$$16t^2 - 104t = 0$$
$$8t(2t - 13) = 0$$
$$8t = 0 \quad \text{or} \quad 2t - 13 = 0$$
$$t = 0 \qquad\qquad t = \tfrac{13}{2} = 6.5$$

$t = 0$ represents when the projectile was fired, so it returns after 6.5 seconds.

41. **Ballistics** The height of an object thrown upward with an initial velocity of 32 feet per second is given by the formula $h = -16t^2 + 32t$, where t is the time in seconds. How long will it take the object to reach a height of 16 feet?

Solution

Set $h = 16$:

$$h = -16t^2 + 32t$$
$$16 = -16t^2 + 32t$$
$$16t^2 - 32t + 16 = 0$$
$$16(t - 1)(t - 1) = 0$$
$$t - 1 = 0 \quad \textbf{or} \quad t - 1 = 0$$
$$t = 1 \qquad\qquad t = 1$$

It takes the object 1 second to reach a height of 16 feet.

42. **Cruise ship anchor** A cruise ship drops anchor in a harbor. The formula $h = -16t^2 + 50$ can be used to determine the height h in feet of the anchor at time t in seconds after it is released. When is the anchor 5 feet above the water? Round to the nearest tenth.

Solution

Set $h = 5$:

$$h = -16t^2 + 50$$
$$5 = -16t^2 + 50$$
$$0 = -16t^2 + 45$$
$$16t^2 = 45$$
$$t^2 = 2.8125$$

$t \approx 1.7$ or $t \approx -1.7$

$t = -1.7$ doesn't make sense, so it takes about 1.7 seconds for the anchor to be 5 feet above water.

43. **Water balloon toss** A water balloon is tossed from the window of college student's dormitory. The equation $h = -4.9t^2 + 5.2t + 10.1$ can be used to determine the height h in meters of the water balloon at time t in seconds after it is released. When will the water balloon hit the ground? Round to the nearest tenth.

Solution

Set $h = 0$:

$$h = -4.9t^2 + 5.2t + 10.1$$
$$0 = -4.9t^2 + 5.2t + 10.1$$
$$t = \frac{-5.2 \pm \sqrt{(5.2)^2 - 4(-4.9)(10.1)}}{2(-4.9)}$$
$$t = \frac{-5.2 \pm \sqrt{225}}{-9.8}$$

$t \approx 2.1$ or $t \approx -1$

$t = -1$ doesn't make sense, so it takes about 2.1 seconds for the water balloon to hit the ground.

44. **Beachball toss** A beachball is tossed from the top of a swimming pool sliding board. The formula $h = -4.9t^2 + 8.1t + 5.275$ can be used to determine the height h in meters of the beachball at time t in seconds after it is released. When will the beachball hit the pool water? Round to the nearest tenth.

Solution

Set $h = 0$:

$$h = -4.9t^2 + 8.1t + 5.275$$
$$0 = -4.9t^2 + 8.1t + 5.275$$
$$t = \frac{-8.1 \pm \sqrt{(8.1)^2 - 4(-4.9)(5.275)}}{4(-4.9)}$$
$$t = \frac{-8.1 \pm \sqrt{169}}{-9.8}$$

$t \approx 2.2 \quad \text{or} \quad t \approx -0.5$

$t = -0.5$ doesn't make sense, so it takes about 2.2 seconds for the beachball to hit the pool water.

45. **Setting fares** A bus company has 3000 passengers daily, paying a 25¢ fare. For each nickel increase in fare, the company projects that it will lose 80 passengers. What fare increase will produce $994 in daily revenue?

Solution

Let x = the number of nickel increases. The new fare = 25 + 5x (in cents), while the number of passengers = 3000 – 80x.

$$\boxed{\begin{array}{c}\text{Number of}\\ \text{Passengers}\end{array}} \cdot \boxed{\text{Fare}} = \boxed{\text{Revenue}}$$

$$\begin{aligned}(3000 - 80x)(25 + 5x) &= 99400\\ 75000 + 13000x - 400x^2 &= 99400\\ 400x^2 - 13000x + 24400 &= 0\\ 200(2x^2 - 65x + 122) &= 0\\ 200(2x - 61)(x - 2) &= 0\end{aligned}$$

$$2x - 61 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$x = 30.5 \qquad\qquad x = 2$$

Since you cannot have half of a nickel increase, $x = 30.5$ does not make sense. Thus, there should be 2 nickel increases, for a fare increase of 10 cents.

46. **Jazz concerts** A jazz group on tour has been drawing average crowds of 500 persons. It is projected that for every $1 increase in the $12 ticket price, the average attendance will decrease by 50. At what ticket price will nightly receipts be $5600?

Solution

Let x = the number of dollar increases. The new price = 12 + x, while the number attending = 500 – 50x.

$$\boxed{\begin{array}{c}\text{Number of}\\ \text{attending}\end{array}} \cdot \boxed{\text{Price}} = \boxed{\text{Revenue}}$$

$$\begin{aligned}(500 - 50x)(12 + x) &= 5600\\ 6000 - 100x - 50x^2 &= 5600\\ 50x^2 + 100x - 400 &= 0\\ 50(x^2 + 2x - 8) &= 0\\ 50(x + 4)(x - 2) &= 0\end{aligned}$$

$$x + 4 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$x = -4 \qquad\qquad x = 2$$

Since you cannot have a negative number of increases, $x = -4$ does not make sense. Thus, there should be an increase of 2 dollars, for a ticket price of $14.

47. **Concert receipts** Tickets for the annual symphony orchestra pops concert cost $15, and the average attendance at the concerts has been 1200 persons. Management projects that for each 50¢ decrease in ticket price, 40 more patrons will attend. How many people attended the concert if the receipts were $17,280?

Solution

Let x = the number of $0.50 decreases. The new price = 15 − 0.5x, while the number attending = 1200 + 40x.

$$\boxed{\text{Number of attending}} \cdot \boxed{\text{Price}} = \boxed{\text{Revenue}}$$

$$\begin{aligned}(1200 + 40x)(15 - 0.5x) &= 17280 \\ 18000 - 20x^2 &= 17280 \\ 20x^2 &= 720 \\ x^2 &= 36\end{aligned}$$

$$x = \sqrt{36} \quad \textbf{or} \quad x = -\sqrt{36}$$

$$x = 6 \qquad\qquad x = -6$$

$x = -6$ does not make sense. Thus, there should be six 50-cent decreases, for a ticket price of $12 and an attendance of 1440 people.

48. **Projecting demand** The *Vilas County News* earns a profit of $20 per year for each of its 3000 subscribers. Management projects that the profit per subscriber would increase by 1¢ for each additional subscriber over the current 3000. How many subscribers are needed to bring a total profit of $120,000?

Solution

Let x = the number of subscribers over 3000. The new profit = 20 + 0.01x, while the number subscribing = 3000 + x.

$$\boxed{\text{Number subscribing}} \cdot \boxed{\text{Profit}} = \boxed{\text{Total profit}}$$

$$\begin{aligned}(3000 + x)(20 + 0.01x) &= 120000 \\ 60000 + 50x + 0.01x^2 &= 120000 \\ 0.01x^2 + 50x - 60000 &= 0 \\ x^2 + 5000x - 6000000 &= 0 \\ (x + 6000)(x - 1000) &= 0\end{aligned}$$

$$x + 6000 = 0 \quad \textbf{or} \quad x - 1000 = 0$$

$$x = -6000 \qquad\qquad x = 1000$$

$x = -6000$ does not make sense. The increase should be 1000, for a total of 4000.

49. **Investment problems** Morgan and Kyung each have a bank CD. Morgan's is $1000 larger than Kyung's, but the interest rate is 1% less. Last year Morgan received interest of $280, and Kyung received $240. Find the rate of interest for each CD.

Solution

Let x = Chloe's principal.

$$\boxed{\text{Morgan's rate}} = \boxed{\text{Chloe's rate}} - 0.01$$

$$\frac{280}{x+100} = \frac{240}{x} - 0.01$$

$$x(x+1000)\frac{280}{x+1000} = x(x+1000)\left(\frac{240}{x} - 0.01\right)$$

$$280x = 240(x+1000) - 0.01x(x+1000)$$

$$0.01x^2 + 50x - 240{,}000 = 0$$

$$x^2 + 5000x - 24{,}000{,}000 = 0$$

$$(x-3000)(x+8000) = 0$$

$x - 3000 = 0$ **or** $x + 8000 = 0$

$x = 3000$ $\qquad x = -8000 \Rightarrow$ $x = -8000$ does not make sense. The principal amounts were \$3000 and \$4000. The interest rates were 8% for Chloe and 7% for Morgan.

	I	P	r
Chloe	240	x	$\frac{240}{x}$
Morgan	280	$x + 1000$	$\frac{280}{x+1000}$

50. **Investment problem** Safa and Laura have both invested some money. Safa invested \$3000 more than Laura and at a 2% higher interest rate. If Safa received \$800 annual interest and Laura received \$400, how much did Safa invest?

Solution

Let x = Laura's principal.

$$\boxed{\text{Scott's rate}} = \boxed{\text{Laura's rate}} + 0.02$$

$$\frac{800}{x+3000} = \frac{400}{x} + 0.02$$

$$x(x+3000)\frac{800}{x+3000} = x(x+3000)\left(\frac{400}{x} + 0.02\right)$$

$$800x = 400(x+3000) + 0.02x(x+3000)$$

$$800x = 400x + 1{,}200{,}000 + 0.02x^2 + 60x$$

$$0.02x^2 - 340x + 1{,}200{,}000 = 0$$

$$x^2 + 17{,}000x + 60{,}000{,}000 = 0$$

$$(x-5000)(x-12{,}000) = 0$$

$x - 5000 = 0$ **or** $x - 12{,}000 = 0$

$x = 5000$ $\qquad x = 12{,}000 \Rightarrow$ Laura invested either \$5000 or \$12,000, so Scott invested either \$8000 or \$15,000.

	I	P	r
Laura	400	x	$\frac{400}{x}$
Scott	800	$x + 3000$	$\frac{800}{x+3000}$

51. **Buying microwave ovens** Some mathematics professors would like to purchase a $150 microwave oven for the department workroom. If four of the professors don't contribute, everyone's share will increase by $10. How many professors are in the department?

Solution

Let x = the total number of professors.

$$\boxed{\text{New share with lower number}} = \boxed{\text{Original share}} + 10$$

$$\frac{150}{x-4} = \frac{150}{x} + 10$$

$$x(x-4)\frac{150}{x-4} = x(x-4)\left(\frac{150}{x} + 10\right)$$

$$150x = 150(x-4) + 10x(x-4)$$

$$150x = 150x - 600 + 10x^2 - 40x$$

$$0 = 10x^2 - 40x - 600$$

$$0 = x^2 - 4x - 60$$

$$0 = (x-10)(x+6)$$

$x - 10 = 0$ **or** $x + 6 = 0$

$x = 10$ $\quad$ $x = -6$

$x = -6$ does not make sense, so there are 10 professors in the department.

52. **Digital cameras** A merchant could sell one model of digital cameras at list price for $180. If he had three more cameras, he could sell each one for $10 less and still receive $180. Find the list price of each camera.

Solution

Let x = the actual number of cameras.

$$\boxed{\text{Actual price per camera}} - 10 = \boxed{\text{New price per camera}}$$

$$\frac{180}{x} - 10 = \frac{180}{x+3}$$

$$x(x+3)\left(\frac{180}{x} - 10\right) = x(x+3)\frac{180}{x+3}$$

$$180(x+3) - 10x(x+3) = 180x$$

$$10x^2 + 30x - 540 = 0$$

$$x^2 + 3x - 54 = 0$$

$$(x-6)(x+9) = 0$$

$$x - 6 = 0 \quad \textbf{or} \quad x + 9 = 0$$
$$x = 6 \qquad\qquad x = -9$$

$x = -9$ does not make sense, so there are 6 cameras, with each costing $30.

53. **Filling storage tanks** Two pipes are used to fill a water storage tank. The first pipe can fill the tank in 4 hours, and the two pipes together can fill the tank in 2 hours less time than the second pipe alone. How long would it take for the second pipe to fill the tank?

Solution

Let x = time for the second pipe to fill tank.

$$\boxed{\begin{array}{c}\text{First in}\\ \text{1 hour}\end{array}} + \boxed{\begin{array}{c}\text{Second in}\\ \text{1 hour}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\ \text{1 hour}\end{array}}$$

$$\begin{aligned}
\frac{1}{4} + \frac{1}{x} &= \frac{1}{x-2}\\
4x(x-2)\left(\frac{1}{4} + \frac{1}{x}\right) &= 4x(x-2)\cdot\frac{1}{x-2}\\
x(x-2) + 4(x-2) &= 4x\\
x^2 - 2x + 4x - 8 &= 4x\\
x^2 - 2x - 8 &= 0\\
(x-4)(x+2) &= 0
\end{aligned}$$

$$x - 4 = 0 \quad \textbf{or} \quad x + 2 = 0$$
$$x = 4 \qquad\qquad x = -2$$

Since $x = -2$ does not make sense, the solution is $x = 4$. It takes the second pipe 4 hours to fill the tank alone.

54. **Filling swimming pools** A hose can fill a swimming pool in 6 hours. Another hose needs 3 more hours to fill the pool than the two hoses combined. How long would it take the second hose to fill the pool?

Solution

Let x = time for the both hoses to fill pool.

$$\boxed{\begin{array}{c}\text{First in}\\ \text{1 hour}\end{array}} + \boxed{\begin{array}{c}\text{Second in}\\ \text{1 hour}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\ \text{1 hour}\end{array}}$$

$$\begin{aligned}
\frac{1}{6} + \frac{1}{x+3} &= \frac{1}{x}\\
6x(x+3)\left(\frac{1}{6} + \frac{1}{x+3}\right) &= 6x(x+3)\cdot\frac{1}{x}\\
x(x+3) + 6x &= 6(x+3)\\
x^2 + 3x + 6x &= 6x + 18\\
x^2 + 3x - 18 &= 0\\
(x-3)(x+6) &= 0
\end{aligned}$$

$$x - 3 = 0 \quad \textbf{or} \quad x + 6 = 0$$
$$x = 3 \qquad\qquad x = -6$$

Since $x = -6$ does not make sense, the solution is $x = 3$. It takes the second hose 6 hours to fill the pool alone.

55. **Mowing lawns** Kristy can mow a lawn in 1 hour less time than her brother Steven. Together they can finish the job in 5 hours. How long would it take Kristy if she worked alone?

Solution

Let x = time for the Steven to mow lawn.

$$\boxed{\begin{array}{c}\text{Steven in}\\ \text{1 hour}\end{array}} + \boxed{\begin{array}{c}\text{Kristy in}\\ \text{1 hour}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\ \text{1 hour}\end{array}}$$

$$\begin{aligned}
\frac{1}{x} + \frac{1}{x-1} &= \frac{1}{5}\\
5x(x-1)\left(\frac{1}{x} + \frac{1}{x-1}\right) &= 5x(x-1)\cdot\frac{1}{5}\\
5(x-1) + 5x &= x(x-1)\\
5x - 5 + 5x &= x^2 - x\\
0 &= x^2 - 11x + 5
\end{aligned}$$

$a = 1, b = -11, c = 5$

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
&= \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(5)}}{2(1)}\\
&= \frac{11 \pm \sqrt{101}}{2} \approx 10.5 \text{ or } 0.5
\end{aligned}$$

$x = 0.5$ does not make sense, so Kristy could mow the lawn in about 9.5 hours alone.

56. **Cleaning the garage** Working together, Sarah and Heidi can clean the garage in 2 hours. If they work alone, it takes Heidi 3 hours longer than it takes Sarah. How long would it take Heidi to clean the garage alone?

Solution

Let x = time for the Sarah to clean it.

$$\boxed{\begin{array}{c}\text{Sarah in}\\ \text{1 hour}\end{array}} + \boxed{\begin{array}{c}\text{Heidi in}\\ \text{1 hour}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\ \text{1 hour}\end{array}}$$

$$\begin{aligned}
\frac{1}{x} + \frac{1}{x+3} &= \frac{1}{2}\\
2x(x+3)\left(\frac{1}{x} + \frac{1}{x+3}\right) &= 2x(x+3)\cdot\frac{1}{2}\\
2(x+3) + 2x &= x(x+3)\\
2x + 6 + 2x &= x^2 + 3x\\
0 &= x^2 - x - 6\\
0 &= (x-3)(x+2)
\end{aligned}$$

$$x - 3 = 0 \quad \textbf{or} \quad x + 2 = 0$$
$$x = 3 \qquad\qquad x = -2$$

$x = -2$ does not make sense. It would take Heidi 6 hours to clean the garage alone.

57. **Planting windscreens** A farmer intends to construct a windscreen by planting trees in a quarter-mile row. His daughter points out that 44 fewer trees will be needed if they are planted 1 foot farther apart. If her dad takes her advice, how many trees will be needed? A row starts and ends with a tree. (*Hint:* 1 mile = 5280 feet.)

Solution

The number of trees is the length of the row divided by the space between the trees, plus 1. Let x = the original spacing.

$$\boxed{\text{Original number}} - 44 = \boxed{\text{New number}}$$
$$\frac{1320}{x} + 1 - 44 = \frac{1320}{x + 1} + 1$$
$$\frac{1320}{x} - 44 = \frac{1320}{x + 1}$$
$$x(x + 1)\left(\frac{1320}{x} - 44\right) = x(x + 1)\frac{1320}{x + 1}$$
$$1320(x + 1) - 44x(x + 1) = 1320x$$
$$44x^2 + 44x - 1320 = 0$$
$$x^2 + x - 30 = 0$$
$$(x - 5)(x + 6) = 0$$
$$x - 5 = 0 \quad \textbf{or} \quad x + 6 = 0$$
$$x = 5 \qquad\qquad x = -6$$

$x = -6$ does not make sense. The original spacing was 5 feet, resulting in 265 trees, so the new spacing will require 221 trees.

58. **Angle between spokes** If a wagon wheel had 10 more spokes, the angle between spokes would decrease by 6°. How many spokes does the wheel have?

Solution

Let x = the actual number of spokes.

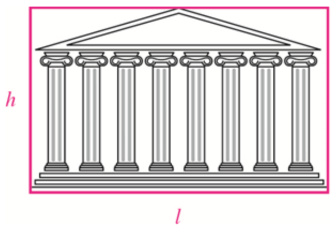
$$\boxed{\text{Actual angles between spokes}} - 6 = \boxed{\text{New angle between spokes}}$$
$$\frac{360}{x} - 6 = \frac{360}{x + 10}$$
$$x(x + 10)\left(\frac{360}{x} - 6\right) = x(x + 10)\frac{360}{x + 10}$$
$$360(x + 10) - 6x(x + 10) = 360x$$
$$6x^2 + 60x - 3600 = 0$$
$$x^2 + 10x - 600 = 0$$
$$(x - 20)(x + 30) = 0$$

$$x - 20 = 0 \quad \textbf{or} \quad x + 30 = 0$$
$$x = 20 \qquad\qquad x = -30$$

$x = -30$ does not make sense, so there are 20 spokes.

59. **Architecture A golden rectangle** is one of the most visually appealing of all geometric forms. The front of the Parthenon, built in Athens in the 5th century B.C. and shown in the illustration, is a golden rectangle. In a golden rectangle, the length l and the height h of the rectangle must satisfy the following equation. If a rectangular billboard is to have a height of 15 feet, how long should it be if it is to form a golden rectangle? Round to the nearest tenth of a foot.

$$\frac{l}{h} = \frac{h}{l - h}$$



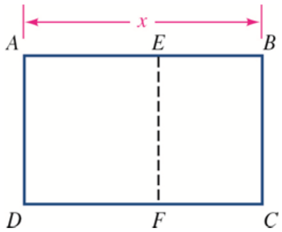
Solution

Let $h = 15$:

$$\frac{l}{h} = \frac{h}{l - h}$$
$$\frac{l}{15} = \frac{15}{l - 15}$$
$$15(l - 15) \cdot \frac{l}{15} = 15(l - 15) \cdot \frac{15}{l - 15}$$
$$l(l - 15) = 15^2$$
$$l^2 - 15l - 225 = 0$$
$$a = 1, b = -15, c = -225$$
$$l = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(225)}}{2(1)}$$
$$= \frac{15 \pm \sqrt{1125}}{2} \approx \frac{15 \pm 33.541}{2}$$

The only positive solution is $l = 24.3$ ft.

60. **Golden ratio** Rectangle $ABCD$, shown here, will be a **golden rectangle** if $\frac{AB}{AD} = \frac{BC}{BE}$ where $AE = AD$. Let $AE = 1$ and find the ratio of AB to AD.



Solution

If $AE = 1$, then $AD = BC = 1$ and $BE = x - 1$.

$$\frac{AB}{AD} = \frac{BC}{BE}$$

$$\frac{x}{1} = \frac{1}{x-1}$$

$$(x-1)\cdot\frac{x}{1} = (x-1)\cdot\frac{1}{x-1}$$

$$x^2 - x = 1$$

$$x^2 - x - 1 = 0$$

$$a = 1, b = -1, c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

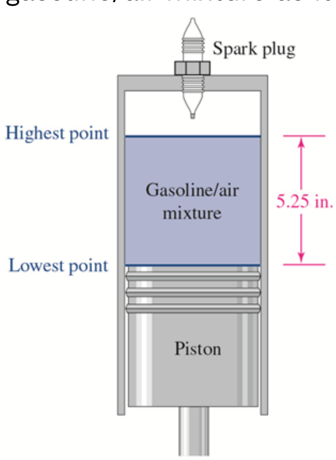
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{5}}{2} \approx \frac{1 \pm 2.236}{2}$$

The only positive solution is $x = 1.618$.

The ratio is about $\frac{1.618}{1}$, or 1.618 to 1.

61. **Automobile engines** As the piston shown moves upward, it pushes a cylinder of a gasoline/air mixture that is ignited by the spark plug. The formula that gives the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h the height. Find the radius of the piston (to the nearest hundredth of an inch) if it displaces 47.75 cubic inches of gasoline/air mixture as it moves from its lowest to its highest point.



Solution

$$\begin{aligned} V &= \pi r^2 h \\ 47.75 &= \pi r^2 (5.25) \\ \frac{47.75}{5.25\pi} &= r^2 \\ \pm\sqrt{\frac{47.75}{5.25\pi}} &= r \\ \pm 1.70 &\approx r \end{aligned}$$

The radius is about 1.70 in.

62. **History** One of the important cities of the ancient world was Babylon. Greek historians wrote that the city was square-shaped. Its area numerically exceeded its perimeter by about 124. Find its dimensions in miles. (Round to the nearest tenth.)

Solution

Let x = the length of a side of the city.

$$\boxed{\text{Area}} = \boxed{\text{Perimeter}} + 124$$

$$x^2 = 4x + 124$$

$$x^2 - 4x - 124 = 0$$

$$a = 1,\ b = -4,\ c = -124$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-124)}}{2(1)} \\ &= \frac{4 \pm \sqrt{512}}{2} \approx 13.3 \text{ or } -9.3 \end{aligned}$$

The dimensions were 13.3 mi by 13.3 mi.

Discovery and Writing

63. Summarize the general strategy used to solve application problems in this section.

Solution

Answers may vary.

64. Describe why it is important to check your solutions to an application problem.

Solution

Answers may vary.

65. Which of the preceding application problems did you find the hardest? Why?

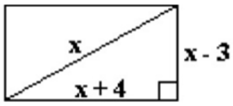
Solution

Answers may vary.

66. Is it possible for a rectangle to have a width that is 3 units shorter than its diagonal and a length that is 4 units longer than its diagonal?

Solution

Let x = the length of the diagonal.



Using the Pythagorean Theorem:

$$(x+4)^2 + (x-3)^2 = x^2$$
$$x^2 + 8x + 16 + x^2 - 6x + 9 = x^2$$
$$x^2 + 2x + 25 = 0$$

$a = 1, b = 2, c = 25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(1)(25)}}{2(1)} = \frac{-2 \pm \sqrt{-96}}{2}$$

This does not equal a real number, so it is not possible.

Exercises 1.6

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Factor completely: $4x^4 - 36x^2$

Solution

$$4x^4 - 36x^2 = 4x^2(x^2 - 9) = 4x^2(x+3)(x-3)$$

2. Factor completely: $2y^3 - 3y^2 - 35y$

Solution

$$2y^3 - 3y^2 - 35y = y(2y^2 - 3y - 35) = y(y-5)(2y+7)$$

3. Factor by grouping: $z^3 - 5z^2 - 2z + 10$

Solution

$$z^3 - 5z^2 - 2z + 10 = z^2(z-5) - 2(z-5) = (z^2 - 2)(z-5)$$

4. Simplify: $(-32)^{\frac{3}{5}}$

   Solution

   $(-32)^{\frac{3}{5}} = \left(\sqrt[5]{-32}\right)^3 = (-2)^3 = -8$

5. If $u = x^{\frac{1}{3}}$ what is u^2?

   Solution

   If $u = x^{\frac{1}{3}}$, then $u^2 = \left(x^{1/3}\right)^2 = x^{2/3}$

6. Perform the operation and simplify: $(\sqrt{3x - 1} + 6)^2$

   Solution

   $\left(\sqrt{3x - 1} + 6\right)^2 = \left(\sqrt{3x - 1} + 6\right)\left(\sqrt{3x - 1} + 6\right) = 3x - 1 + 12\sqrt{3x - 1} + 36$

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. Equal powers of equal real numbers are __________.

   Solution
   equal

8. If a and b are real numbers and $a = b$ then a^2 = __________.

   Solution
   b^2

9. False solutions that don't satisfy the equation are called __________ solutions.

   Solution
   extraneous

10. Radical equations contain radicals with variables in their__________.

    Solution
    radicands

Practice

Use factoring to solve each equation.

11. $x^3 + 9x^2 + 20x = 0$

Solution

$$x^3 + 9x^2 + 20x = 0$$
$$x(x^2 + 9x + 20) = 0$$
$$x(x + 5)(x + 4) = 0$$

$x = 0$ **or** $x + 5 = 0$ **or** $x + 4 = 0$

$x = 0$ $\quad x = -5$ $\quad x = -4$

12. $x^3 + 4x^2 - 21x = 0$

Solution

$$x^3 + 4x^2 - 21x = 0$$
$$x(x^2 + 4x - 21) = 0$$
$$x(x + 7)(x - 3) = 0$$

$x = 0$ **or** $x + 7 = 0$ **or** $x - 3 = 0$

$x = 0$ $\quad x = -7$ $\quad x = 3$

13. $6a^3 - 5a^2 - 4a = 0$

Solution

$$6a^3 - 5a^2 - 4a = 0$$
$$a(6a^2 - 5a - 4) = 0$$
$$a(2a + 1)(3a - 4) = 0$$

$a = 0$ **or** $2a + 1 = 0$ **or** $3a - 4 = 0$

$a = 0$ $\quad 2a = -1$ $\quad 3a = 4$

$a = 0$ $\quad a = -\frac{1}{2}$ $\quad a = \frac{4}{3}$

14. $8b^3 - 10b^2 + 3b = 0$

Solution

$$8b^3 - 10b^2 + 3b = 0$$
$$b(8b^2 - 10b + 3) = 0$$
$$b(4b - 3)(2b - 1) = 0$$

$b = 0$ **or** $4b - 3 = 0$ **or** $2b - 1 = 0$

$b = 0$ $\quad 4b = 3$ $\quad 2b = 1$

$b = 0$ $\quad b = \frac{3}{4}$ $\quad b = \frac{1}{2}$

15. $y^4 - 26y^2 + 25 = 0$

Solution

$$y^4 - 26y^2 + 25 = 0$$
$$(y^2 - 25)(y^2 - 1) = 0$$
$$y^2 - 25 = 0 \quad \textbf{or} \quad y^2 - 1 = 0$$
$$y^2 = 25 \qquad y^2 = 1$$
$$y = \pm 5 \qquad y = \pm 1$$

16. $y^4 - 13y^2 + 36 = 0$

Solution

$$y^4 - 13y^2 + 36 = 0$$
$$(y^2 - 4)(y^2 - 9) = 0$$
$$y^2 - 4 = 0 \quad \textbf{or} \quad y^2 - 9 = 0$$
$$y^2 = 4 \qquad y^2 = 9$$
$$y = \pm 2 \qquad y = \pm 3$$

17. $2y^4 - 46y^2 = -180$

Solution

$$2y^4 - 46y^2 = -180$$
$$2(y^4 - 23y^2 + 90) = 0$$
$$2(y^2 - 18)(y^2 - 5) = 0$$
$$y^2 - 18 = 0 \quad \textbf{or} \quad y^2 - 5 = 0$$
$$y^2 = 18 \qquad y^2 = 5$$
$$y = \pm\sqrt{18} \qquad y = \pm\sqrt{5}$$
$$y = \pm 3\sqrt{2} \qquad y = \pm\sqrt{5}$$

18. $2x^4 - 102x^2 = -196$

Solution

$$2x^4 - 102x^2 = -196$$
$$2(x^4 - 51x^2 + 98) = 0$$
$$2(x^2 - 49)(x^2 - 2) = 0$$
$$x^2 - 49 = 0 \quad \textbf{or} \quad x^2 - 2 = 0$$
$$x^2 = 49 \qquad x^2 = 2$$
$$x = \pm 7 \qquad x = \pm\sqrt{2}$$

19. $x^4 = 8x^2 + 9$

Solution

$$x^4 = 8x^2 + 9$$
$$x^4 - 8x^2 - 9 = 0$$
$$(x^2 - 9)(x^2 + 1) = 0$$
$$x^2 - 9 = 0 \quad \textbf{or} \quad x^2 + 1 = 0$$
$$x^2 = 9 \qquad x^2 = -1$$
$$x = \pm\sqrt{9} \qquad x = \pm\sqrt{-1}$$
$$x = \pm 3 \qquad x = \pm i$$

20. $x^4 - 12x^2 = 64$

Solution

$$x^4 - 12x^2 = 64$$
$$x^4 - 12x^2 - 64 = 0$$
$$(x^2 - 16)(x^2 + 4) = 0$$
$$x^2 - 16 = 0 \quad \textbf{or} \quad x^2 + 4 = 0$$
$$x^2 = 16 \qquad x^2 = -4$$
$$x = \pm\sqrt{16} \qquad x = \pm\sqrt{-4}$$
$$x = \pm 4 \qquad x = \pm 2i$$

21. $4y^4 + 7y^2 - 36 = 0$

Solution

$$4y^4 + 7y^2 - 36 = 0$$
$$(4y^2 - 9)(y^2 + 4) = 0$$
$$4y^2 - 9 = 0 \quad \textbf{or} \quad y^2 + 4 = 0$$
$$y^2 = \tfrac{9}{4} \qquad y^2 = -4$$
$$y = \pm\sqrt{\tfrac{9}{4}} \qquad y = \pm\sqrt{-4}$$
$$y = \pm\tfrac{3}{2} \qquad y = \pm 2i$$

22. $9y^4 + 56y^2 - 225 = 0$

Solution

$$9y^4 + 56y^2 - 225 = 0$$
$$(9y^2 - 25)(y^2 + 9) = 0$$

$$9y^2 - 25 = 0 \quad \textbf{or} \quad y^2 + 9 = 0$$
$$y^2 = \frac{25}{9} \qquad y^2 = -9$$
$$y = \pm\sqrt{\frac{25}{9}} \qquad y = \pm\sqrt{-9}$$
$$y = \pm\frac{5}{3} \qquad y = \pm 3i$$

Solve each equation by factoring or by making an appropriate substitution.

23. $2x^3 + 3x^2 - 4x = 6$

Solution

$$2x^3 + 3x^2 - 4x = 6$$
$$2x^3 + 3x^2 - 4x - 6 = 0$$
$$x^2(2x + 3) - 2(2x + 3) = 0$$
$$(2x + 3)(x^2 - 2) = 0$$
$$2x + 3 = 0 \quad \text{or} \quad x^2 - 2 = 0$$
$$2x = -3 \qquad x^2 = 2$$
$$x = \frac{-3}{2} \qquad x = \pm\sqrt{2}$$

24. $3x^3 - x^2 = 12x - 4$

Solution

$$3x^3 - x^2 = 12x - 4$$
$$3x^3 - x^2 - 12x + 4 = 0$$
$$x^2(3x - 1) - 4(3x - 1) = 0$$
$$(3x - 1)(x^2 - 4) = 0$$
$$3x^3 - x^2 = 12x - 4$$
$$3x^3 - x^2 - 12x + 4 = 0$$
$$x^2(3x - 1) - 4(3x - 1) = 0$$
$$(3x - 1)(x^2 - 4) = 0$$
$$3x - 1 = 0 \quad \text{or} \quad x^2 - 4 = 0$$
$$3x = 1 \quad \text{or} \quad x^2 = 4$$
$$x = \frac{1}{3} \quad \text{or} \quad x = \pm 2$$

25. $x^3 - x^2 + 9x - 9 = 0$

Solution

$$x^3 - x^2 + 9x - 9 = 0$$
$$x^2(x - 1) + 9(x - 1) = 0$$
$$(x - 1)(x^2 + 9) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x^2 - 9 = 0$$
$$x = 1 \quad \text{or} \quad x^2 = -9$$
$$x = \pm\sqrt{-9}$$
$$x = \pm\, 3i$$

26. $x^3 = -5x^2 - 8x - 40$

Solution

$$x^3 = -5x^2 - 8x - 40$$
$$x^3 + 5x^2 + 8x + 40 = 0$$
$$x^2(x + 5) + 8(x + 5) = 0$$
$$(x + 5)(x^2 + 8) = 0$$
$$x + 5 = 0 \quad \text{or} \quad x^2 + 8 = 0$$
$$x = -5 \qquad x^2 = -8$$
$$x = \pm\, 2i\sqrt{2}$$

27. $x^4 - 37x^2 + 36 = 0$

Solution

$$x^4 - 37x^2 + 36 = 0$$
$$(x^2 - 36)(x^2 - 1) = 0$$
$$x^2 - 36 = 0 \quad \textbf{or} \quad x^2 - 1 = 0$$
$$x^2 = 36 \qquad x^2 = 1$$
$$x = \pm 6 \qquad x = \pm 1$$

28. $x^4 - 50x^2 + 49 = 0$

Solution

$$x^4 - 50x^2 + 49 = 0$$
$$(x^2 - 49)(x^2 - 1) = 0$$
$$x^2 - 49 = 0 \quad \textbf{or} \quad x^2 - 1 = 0$$
$$x^2 = 49 \qquad x^2 = 1$$
$$x = \pm 7 \qquad x = \pm 1$$

29. $2m^{2/3} + 3m^{1/3} - 2 = 0$

Solution

$$2m^{2/3} + 3m^{1/3} - 2 = 0$$
$$\left(2m^{1/3} - 1\right)\left(m^{1/3} + 2\right) = 0$$

$2m^{1/3} - 1 = 0$ **or** $m^{1/3} + 2 = 0$

$m^{1/3} = \frac{1}{2}$ $\quad$ $m^{1/3} = -2$

$\left(m^{1/3}\right)^3 = \left(\frac{1}{2}\right)^3$ $\quad$ $\left(m^{1/3}\right)^3 = (-2)^3$

$m = \frac{1}{8}$ $\quad$ $m = -8$

Both answers check.

30. $6t^{2/5} + 11t^{1/5} + 3 = 0$

Solution

$$6t^{2/5} + 11t^{1/5} + 3 = 0$$
$$\left(2t^{1/5} + 3\right)\left(3t^{1/5} + 1\right) = 0$$

$2t^{1/5} + 3 = 0$ **or** $3t^{1/5} + 1 = 0$

$t^{1/5} = -\frac{3}{2}$ $\quad$ $t^{1/5} = -\frac{1}{3}$

$\left(t^{1/5}\right)^5 = \left(-\frac{3}{2}\right)^5$ $\quad$ $\left(t^{1/5}\right)^5 = \left(-\frac{1}{3}\right)^5$

$t = -\frac{243}{32}$ $\quad$ $t = -\frac{1}{243}$

Both answers check.

31. $x - 13x^{1/2} + 12 = 0$

Solution

$$x - 13x^{1/2} + 12 = 0$$
$$\left(x^{1/2} - 12\right)\left(x^{1/2} - 1\right) = 0$$

$x^{1/2} - 12 = 0$ **or** $x^{1/2} - 1 = 0$

$x^{1/2} = 12$ $\quad$ $x^{1/2} = 1$

$\left(x^{1/2}\right)^2 = (12)^2$ $\quad$ $\left(x^{1/2}\right)^2 = (1)^2$

$x = 144$ $\quad$ $x = 1$

Both answers check.

32. $p + p^{1/2} - 20 = 0$

Solution

$$p + p^{1/2} - 20 = 0$$
$$\left(p^{1/2} + 5\right)\left(p^{1/2} - 4\right) = 0$$

$p^{1/2} + 5 = 0$ **or** $p^{1/2} - 4 = 0$

$p^{1/2} = -5$ $\quad$ $p^{1/2} = 4$

$\left(p^{1/2}\right)^2 = (-5)^2$ $\quad$ $\left(p^{1/2}\right)^2 = (4)^2$

$p = 25$ $\quad$ $p = 16$

$p = 25$ does not check and is extraneous.

33. $6p + p^{1/2} = 1$

Solution

$$6p + p^{1/2} = 1$$
$$6p + p^{1/2} - 1 = 0$$
$$\left(2p^{1/2} + 1\right)\left(3p^{1/2} - 1\right) = 0$$

$2p^{1/2} + 1 = 0$ **or** $3p^{1/2} - 1 = 0$

$$p^{1/2} = -\tfrac{1}{2} \qquad p^{1/2} = \tfrac{1}{3}$$
$$\left(p^{1/2}\right)^2 = \left(-\tfrac{1}{2}\right)^2 \qquad \left(p^{1/2}\right)^2 = \left(\tfrac{1}{3}\right)^2$$
$$p = \tfrac{1}{4} \qquad p = \tfrac{1}{9}$$

$p = \frac{1}{4}$ does not check and is extraneous.

34. $3r - r^{1/2} = 2$

Solution

$$3r - r^{1/2} = 2$$
$$3r - r^{1/2} - 2 = 0$$
$$\left(3r^{1/2} + 2\right)\left(r^{1/2} - 1\right) = 0$$

$3r^{1/2} + 2 = 0$ **or** $r^{1/2} - 1 = 0$

$$r^{1/2} = -\tfrac{2}{3} \qquad r^{1/2} = 1$$
$$\left(r^{1/2}\right)^2 = \left(-\tfrac{2}{3}\right)^2 \qquad \left(r^{1/2}\right)^2 = \left(1\right)^2$$
$$r = \tfrac{4}{9} \qquad r = 1$$

$r = \frac{4}{9}$ does not check and is extraneous.

35. $2t^{1/3} + 3t^{1/6} - 2 = 0$

Solution

$$2t^{1/3} + 3t^{1/6} - 2 = 0$$
$$\left(2t^{1/6} - 1\right)\left(t^{1/6} + 2\right) = 0$$

$2t^{1/6} - 1 = 0$ **or** $t^{1/6} + 2 = 0$

$$t^{1/6} = \tfrac{1}{2} \qquad t^{1/6} = -2$$
$$\left(t^{1/6}\right)^6 = \left(\tfrac{1}{2}\right)^6 \qquad \left(t^{1/6}\right)^6 = \left(-2\right)^6$$
$$t = \tfrac{1}{64} \qquad t = 61$$

$r = 64$ does not check and is extraneous.

36. $z^3 - 7z^{3/2} - 8 = 0$

Solution

$$z^3 - 7z^{3/2} - 8 = 0$$
$$\left(z^{3/2} + 1\right)\left(z^{3/2} - 8\right) = 0$$

$z^{3/2} + 1 = 0$ **or** $z^{3/2} - 8 = 0$

$$z^{3/2} = -1 \qquad z^{3/2} = 8$$
$$\left(z^{3/2}\right)^2 = \left(-1\right)^2 \qquad \left(z^{3/2}\right)^2 = \left(8\right)^2$$
$$z^3 = 1 \qquad z^3 = 64$$
$$z = 1 \qquad z = 4$$

$z = 1$ does not check and is extraneous.

37. $x^{-2} - 10x^{-1} + 16 = 0$

Solution

$$x^{-2} - 10x^{-1} + 16 = 0$$
$$(x^{-1} - 8)(x^{-1} - 2) = 0$$

$$x^{-1} - 8 = 0 \quad \textbf{or} \quad x^{-1} - 2 = 0$$
$$x^{-1} = 8 \qquad x^{-1} = 2$$
$$(x^{-1})^{-1} = (8)^{-1} \qquad (x^{-1})^{-1} = (2)^{-1}$$
$$x = \tfrac{1}{8} \qquad x = \tfrac{1}{2}$$

Both answers check.

38. $2y^{-2} + 9y^{-1} - 5 = 0$

Solution

$$2y^{-2} + 9y^{-1} - 5 = 0$$
$$(2y^{-1} - 1)(y^{-1} + 5) = 0$$

$$2y^{-1} - 1 = 0 \quad \textbf{or} \quad y^{-1} + 5 = 0$$
$$y^{-1} = \tfrac{1}{2} \qquad y^{-1} = -5$$
$$(y^{-1})^{-1} = \left(\tfrac{1}{2}\right)^{-1} \qquad (y^{-1})^{-1} = (-5)^{-1}$$
$$y = 2 \qquad x = -\tfrac{1}{5}$$

Both answers check.

39. $z^{3/2} - z^{1/2} = 0$

Solution

$$z^{3/2} - z^{1/2} = 0$$
$$z^{1/2}(z^{2/2} - 1) = 0$$
$$z^{1/2}(z - 1) = 0$$

$$z^{1/2} = 0 \quad \textbf{or} \quad z - 1 = 0$$
$$(z^{1/2})^2 = 0^2 \qquad z = 1$$
$$z = 0 \qquad z = 1$$

Both answers check.

40. $r^{5/2} - r^{3/2} = 0$

Solution

$$r^{5/2} - r^{3/2} = 0$$
$$r^{3/2}(r^{2/2} - 1) = 0$$
$$r^{3/2}(r - 1) = 0$$

$$\begin{aligned} r^{3/2} &= 0 \quad \textbf{or} \quad r - 1 = 0 \\ \left(r^{3/2}\right)^2 &= 0^2 \qquad\qquad r = 1 \\ r^3 &= 0 \qquad\qquad\quad r = 1 \\ & \qquad\qquad\qquad\quad\; r = 1 \\ r &= 0 \end{aligned}$$

Both answers check.

Find all real solutions of each equation.

41. $\sqrt{x - 2} - 3 = 2$

Solution

$$\begin{aligned} \sqrt{x - 2} - 3 &= 2 \\ \sqrt{x - 2} &= 5 \\ \left(\sqrt{x - 2}\right)^2 &= 5^2 \\ x - 2 &= 25 \\ x &= 27 \end{aligned}$$

The solution checks.

42. $\sqrt{a - 3} - 5 = 0$

Solution

$$\begin{aligned} \sqrt{a - 3} - 5 &= 0 \\ \sqrt{a - 3} &= 5 \\ \left(\sqrt{a - 3}\right)^2 &= 5^2 \\ a - 3 &= 25 \\ a &= 28 \end{aligned}$$

The solution checks.

43. $3\sqrt{x + 1} = \sqrt{6}$

Solution

$$\begin{aligned} 3\sqrt{x + 1} &= \sqrt{6} \\ \left(3\sqrt{x + 1}\right)^2 &= \left(\sqrt{6}\right)^2 \\ 9(x + 1) &= 6 \\ 9x + 9 &= 6 \\ 9x &= -3 \\ x &= -\tfrac{1}{3} \end{aligned}$$

The solution checks.

44. $\sqrt{x + 3} = 2\sqrt{x}$

Solution

$$\begin{aligned}\sqrt{x + 3} &= 2\sqrt{x}\\ \left(\sqrt{x + 3}\right)^2 &= \left(2\sqrt{x}\right)^2\\ x + 3 &= 4x\\ 3 &= 3x\\ 1 &= x\end{aligned}$$

The solution checks.

45. $\sqrt{5a - 2} = \sqrt{a + 6}$

Solution

$$\begin{aligned}\sqrt{5a - 2} &= \sqrt{a + 6}\\ \left(\sqrt{5a - 2}\right)^2 &= \left(\sqrt{a + 6}\right)^2\\ 5a - 2 &= a + 6\\ 4a &= 8\\ a &= 2\end{aligned}$$

The solution checks.

46. $\sqrt{16x + 4} = \sqrt{x + 4}$

Solution

$$\begin{aligned}\sqrt{16x + 4} &= \sqrt{x + 4}\\ \left(\sqrt{16x + 4}\right)^2 &= \left(\sqrt{x + 4}\right)^2\\ 16x + 4 &= x + 4\\ 15x &= 0\\ x &= 0\end{aligned}$$

The solution checks.

47. $2\sqrt{x^2 + 3} = \sqrt{-16x - 3}$

Solution

$$\begin{aligned}2\sqrt{x^2 + 3} &= \sqrt{-16x - 3}\\ \left(2\sqrt{x^2 + 3}\right)^2 &= \left(\sqrt{-16x - 3}\right)^2\\ 4\left(x^2 + 3\right) &= -16x - 3\\ 4x^2 + 12 &= -16x - 3\\ 4x^2 + 16x + 15 &= 0\\ (2x + 3)(2x + 5) &= 0\end{aligned}$$

$$2x + 3 = 0 \quad \textbf{or} \quad 2x + 5 = 0$$
$$x = -\tfrac{3}{2} \qquad\qquad x = -\tfrac{5}{2}$$

Both solutions check.

48. $\sqrt{x^2 + 1} = \dfrac{\sqrt{-7x + 11}}{\sqrt{6}}$

Solution

$$\sqrt{x^2 + 1} = \frac{\sqrt{-7x + 11}}{\sqrt{6}}$$
$$\left(\sqrt{x^2 + 1}\right)^2 = \left(\frac{\sqrt{-7x + 11}}{\sqrt{6}}\right)^2$$
$$x^2 + 1 = \frac{-7x + 11}{6}$$
$$6x^2 + 6 = -7x + 11$$
$$6x^2 + 7x - 5 = 0$$
$$(2x - 1)(3x + 5) = 0$$
$$2x - 1 = 0 \quad \textbf{or} \quad 3x + 5 = 0$$
$$x = \tfrac{1}{2} \qquad\qquad x = -\tfrac{5}{3}$$

Both solutions check.

49. $\sqrt{x^2 + 21} = x + 3$

Solution

$$\sqrt{x^2 + 21} = x + 3$$
$$\left(\sqrt{x^2 + 21}\right)^2 = (x + 3)^2$$
$$x^2 + 21 = x^2 + 6x + 9$$
$$21 = 6x + 9$$
$$12 = 6x$$
$$2 = x$$

The solution checks.

50. $\sqrt{5 - x^2} = -(x + 1)$

Solution

$$\sqrt{5 - x^2} = -(x + 1)$$
$$\left(\sqrt{5 - x^2}\right)^2 = \left[-(x + 1)\right]^2$$
$$5 - x^2 = (x + 1)^2$$

$$5 - x^2 = x^2 + 2x + 1$$
$$0 = 2x^2 + 2x - 4$$
$$0 = 2(x + 2)(x - 1)$$
$$x + 2 = 0 \quad \textbf{or} \quad x - 1 = 0$$
$$x = -2 \qquad\qquad x = 1$$

$x = 1$ does not check and is extraneous.

51. $\sqrt{x + 37} = x - 5$

Solution

$$\sqrt{x + 37} = x - 5$$
$$\left(\sqrt{x + 37}\right)^2 = (x - 5)^2$$
$$x + 37 = x^2 - 10x + 25$$
$$0 = x^2 - 11x - 12$$
$$0 = (x - 12)(x + 1)$$
$$x - 12 = 0 \quad \textbf{or} \quad x + 1 = 0$$
$$x = 12 \qquad\qquad x = -1$$

$x = -1$ does not check and is extraneous.

52. $\sqrt{10 + x} - x - 4 = 0$

Solution

$$\sqrt{10 + x} - x - 4 = 0$$
$$\sqrt{10 + x} = x + 4$$
$$\left(\sqrt{10 + x}\right)^2 = (x + 4)^2$$
$$10 + x = x^2 + 8x + 16$$
$$0 = x^2 + 7x + 6$$
$$0 = (x + 6)(x + 1)$$
$$x + 6 = 0 \quad \textbf{or} \quad x + 1 = 0$$
$$x = -6 \qquad\qquad x = -1$$

$x = -6$ does not check and is extraneous.

53. $\sqrt{3z + 1} = z - 1$

Solution

$$\sqrt{3z + 1} = z - 1$$
$$\left(\sqrt{3z + 1}\right)^2 = (z - 1)^2$$
$$3z + 1 = z^2 - 2z + 1$$

$$0 = z^2 - 5z$$
$$0 = z(z - 5)$$
$$z = 0 \quad \textbf{or} \quad z - 5 = 0$$
$$z = 0 \qquad\qquad z = 5$$
$z = 0$ does not check and is extraneous.

54. $\sqrt{y + 2} = 4 - y$

Solution

$$\sqrt{y + 2} = 4 - y$$
$$\left(\sqrt{y + 2}\right)^2 = (4 - y)^2$$
$$y + 2 = 16 - 8y + y^2$$
$$0 = y^2 - 9y + 14$$
$$0 = (y - 2)(y - 7)$$
$$y - 2 = 0 \quad \textbf{or} \quad y - 7 = 0$$
$$y = 0 \qquad\qquad y = 7$$
$y = 7$ does not check and is extraneous.

55. $x - \sqrt{7x - 12} = 0$

Solution

$$x - \sqrt{7x - 12} = 0$$
$$x = \sqrt{7x - 12}$$
$$x^2 = \left(\sqrt{7x - 12}\right)^2$$
$$x^2 = 7x - 12$$
$$x^2 - 7x + 12 = 0$$
$$(x - 4)(x - 3) = 0$$
$$x - 4 = 0 \quad \textbf{or} \quad x - 3 = 0$$
$$x = 4 \qquad\qquad x = 3$$
Both solution checks.

56. $x - \sqrt{4x - 4} = 0$

Solution

$$x - \sqrt{4x - 4} = 0$$
$$x = \sqrt{4x - 4}$$
$$x^2 = \left(\sqrt{4x - 4}\right)^2$$
$$x^2 = 4x - 4$$

$$x^2 - 4x + 4 = 0$$
$$(x - 2)(x - 2) = 0$$
$$x - 2 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$x = 2 \qquad\qquad x = 2$$

The solution checks.

57. $x + 4 = \dfrac{\sqrt{6x + 6}}{5} + 3$

Solution

$$x + 4 = \frac{\sqrt{6x + 6}}{5} + 3$$
$$x + 1 = \sqrt{\frac{6x + 6}{5}}$$
$$(x + 1)^2 = \left(\sqrt{\frac{6x + 6}{5}}\right)^2$$
$$x^2 + 2x + 1 = \frac{6x + 6}{5}$$
$$5x^2 + 10x + 5 = 6x + 6$$
$$5x^2 + 4x - 1 = 0$$
$$(5x - 1)(x + 1) = 0$$
$$5x - 1 = 0 \quad \textbf{or} \quad x + 1 = 0$$
$$x = \tfrac{1}{5} \qquad\qquad x = -1$$

Both solutions check.

58. $\dfrac{\sqrt{8x + 43}}{3} - 1 = x$

Solution

$$\frac{\sqrt{8x + 43}}{3} - 1 = x$$
$$\sqrt{\frac{8x + 43}{3}} = x + 1$$
$$\left(\sqrt{\frac{8x + 43}{3}}\right)^2 = (x + 1)^2$$
$$\frac{8x + 43}{3} = x^2 + 2x + 1$$
$$8x + 43 = 3x^2 + 6x + 3$$
$$0 = 3x^2 - 2x - 40$$
$$0 = (3x + 10)(x - 4)$$

$3x + 10 = 0$ **or** $x - 4 = 0$

$x = -\frac{10}{3}$ $\quad$ $x = 4$

$x = -\frac{10}{3}$ does not check and is extraneous.

59. $\sqrt{\frac{x^2 - 1}{x - 2}} = 2\sqrt{2}$

Solution

$$\sqrt{\frac{x^2 - 1}{x - 2}} = 2\sqrt{2}$$

$$\left(\sqrt{\frac{x^2 - 1}{x - 2}}\right)^2 = \left(2\sqrt{2}\right)^2$$

$$\frac{x^2 - 1}{x - 2} = 8$$

$$x^2 - 1 = 8x - 16$$

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$x - 3 = 0$ **or** $x - 5 = 0$

$x = 3$ $\quad$ $x = 5$

Both solutions check.

60. $\frac{\sqrt{x^2 - 1}}{\sqrt{3x - 5}} = \sqrt{2}$

Solution

$$\frac{\sqrt{x^2 - 1}}{\sqrt{3x - 5}} = \sqrt{2}$$

$$\left(\frac{\sqrt{x^2 - 1}}{\sqrt{3x - 5}}\right)^2 = \left(\sqrt{2}\right)^2$$

$$\frac{x^2 - 1}{3x - 5} = 2$$

$$x^2 - 1 = 6x - 10$$

$$x^2 - 6x + 9 = 0$$

$$(x - 3)(x - 3) = 0$$

$x - 3 = 0$ **or** $x - 3 = 0$

$x = 3$ $\quad$ $x = 3$

The solutions check.

61. $\sqrt{2p + 1} - 1 = \sqrt{p}$

Solution

$$\sqrt{2p + 1} - 1 = \sqrt{p}$$
$$\left(\sqrt{2p + 1} - 1\right)^2 = \left(\sqrt{p}\right)^2$$
$$2p + 1 - 2\sqrt{2p + 1} + 1 = p$$
$$p + 2 = 2\sqrt{2p + 1}$$
$$(p + 2)^2 = \left(2\sqrt{2p + 1}\right)^2$$
$$p^2 + 4p + 4 = 4(2p + 1)$$
$$p^2 + 4p + 4 = 8p + 4$$
$$p^2 - 4p = 0$$
$$p(p - 4) = 0$$
$$p = 0 \quad \textbf{or} \quad p - 4 = 0$$
$$p = 0 \qquad\qquad p = 4$$

Both solutions check.

62. $\sqrt{r} + \sqrt{r + 2} = 2$

Solution

$$\sqrt{r} + \sqrt{r + 2} = 2$$
$$\sqrt{r} = 2 - \sqrt{r + 2}$$
$$\left(\sqrt{r}\right)^2 = \left(2 - \sqrt{r + 2}\right)^2$$
$$r = 4 - 4\sqrt{r + 2} + r + 2$$
$$4\sqrt{r + 2} = 6$$
$$\left(4\sqrt{r + 2}\right)^2 = 6^2$$
$$16(r + 2) = 36$$
$$16r + 32 = 36$$
$$16r = 4$$
$$r = \frac{4}{16} = \frac{1}{4}$$

The solutions check.

63. $\sqrt{x + 3} = \sqrt{2x + 8} - 1$

Solution

$$\begin{aligned}
\sqrt{x+3} &= \sqrt{2x+8} - 1 \\
\left(\sqrt{x+3}\right)^2 &= \left(\sqrt{2x+8} - 1\right)^2 \\
x+3 &= 2x+8-2\sqrt{2x+8}+1 \\
2\sqrt{2x+8} &= x+6 \\
\left(2\sqrt{2x+8}\right)^2 &= \left(x+6\right)^2 \\
4\left(2x+8\right) &= x^2+12x+36 \\
8x+32 &= x^2+12x+36 \\
0 &= x^2+4x+4 \\
0 &= \left(x+2\right)\left(x+2\right)
\end{aligned}$$

$x+2=0$ **or** $x+2=0$

$x=-2$ $\quad\quad x=-2$

The solution checks.

64. $\sqrt{x+2}+1=\sqrt{2x+5}$

Solution

$$\begin{aligned}
\sqrt{x+2}+1 &= \sqrt{2x+5} \\
\left(\sqrt{x+2}+1\right)^2 &= \left(\sqrt{2x+5}\right)^2 \\
x+2+2\sqrt{x+2}+1 &= 2x+5 \\
2\sqrt{x+2} &= x+2 \\
\left(2\sqrt{x+2}\right)^2 &= \left(x+2\right)^2 \\
4\left(x+2\right) &= x^2+4x+4 \\
4x+8 &= x^2+4x+4 \\
0 &= x^2-4 \\
0 &= \left(x+2\right)\left(x-2\right)
\end{aligned}$$

$x+2=0$ **or** $x-2=0$

$x=-2$ $\quad\quad x=2$

Both solutions check.

65. $\sqrt{y+8}-\sqrt{y-4}=-2$

Solution

$$\sqrt{y+8}-\sqrt{y-4}=-2$$
$$\sqrt{y+8}=\sqrt{y-4}-2$$
$$\left(\sqrt{y+8}\right)^2=\left(\sqrt{y-4}-2\right)^2$$
$$y+8=y-4-4\sqrt{y-4}+4$$
$$4\sqrt{y-4}=-8$$
$$\left(4\sqrt{y-4}\right)^2=(-8)^2$$
$$16(y-4)=64$$
$$16y-64=64$$
$$16y=128$$
$$y=8$$

The solution does not check. $\Rightarrow$ No solution.

66. $\sqrt{z+5}-2=\sqrt{z-3}$

Solution

$$\sqrt{z+5}-2=\sqrt{z-3}$$
$$\left(\sqrt{z+5}-2\right)^2=\left(\sqrt{z-3}\right)^2$$
$$z+5-2(2)\sqrt{z+5}+4=z-3$$
$$12=4\sqrt{z+5}$$
$$12^2=\left(4\sqrt{z+5}\right)^2$$
$$144=16(z+5)$$
$$144=16z+80$$
$$64=16z$$
$$4=z$$

The solution checks.

67. $\sqrt{2b+3}-\sqrt{b+1}=\sqrt{b-2}$

Solution

$$\sqrt{2b+3}-\sqrt{b+1}=\sqrt{b-2}$$
$$\left(\sqrt{2b+3}-\sqrt{b+1}\right)^2=\left(\sqrt{b-2}\right)^2$$
$$2b+3-2\sqrt{(2b+3)(b+1)}+b+1=b-2$$
$$3b+4-2\sqrt{2b^2+5b+3}=b-2$$
$$2b+6=2\sqrt{2b^2+5b+3}$$

$$\begin{aligned}
(2b+6)^2 &= \left(2\sqrt{2b^2+5b+3}\right)^2 \\
4b^2+24b+36 &= 4\left(2b^2+5b+3\right) \\
4b^2+24b+36 &= 8b^2+20b+12 \\
0 &= 4b^2-4b-24 \\
0 &= 4(b-3)(b+2)
\end{aligned}$$

$$b-3=0 \quad \textbf{or} \quad b+2=0$$
$$b=3 \qquad\qquad b=-2$$

$b=-2$ does not check, so it is an extraneous solution.

68. $\sqrt{a+1}+\sqrt{3a}=\sqrt{5a+1}$

Solution

$$\begin{aligned}
\sqrt{a+1}+\sqrt{3a} &= \sqrt{5a+1} \\
\left(\sqrt{a+1}+\sqrt{3a}\right)^2 &= \left(\sqrt{5a+1}\right)^2 \\
a+1+2\sqrt{3a(a+1)}+3a &= 5a+1 \\
4a+1+2\sqrt{3a^2+3a} &= 5a+1 \\
2\sqrt{3a^2+3a} &= a \\
\left(2\sqrt{3a^2+3a}\right)^2 &= a^2 \\
4\left(3a^2+3a\right) &= a^2 \\
12a^2+12a &= a^2 \\
11a^2+12a &= 0 \\
a(11a+12) &= 0
\end{aligned}$$

$$a=0 \quad \textbf{or} \quad 11a+12=0$$
$$a=0 \qquad\qquad a=-\tfrac{12}{11}$$

$a=-\frac{12}{11}$ does not check, so it is an extraneous solution.

69. $\sqrt{\sqrt{b}+\sqrt{b+8}}=2$

Solution

$$\begin{aligned}
\sqrt{\sqrt{b}+\sqrt{b+8}} &= 2 \\
\left(\sqrt{\sqrt{b}+\sqrt{b+8}}\right)^2 &= 2^2 \\
\sqrt{b}+\sqrt{b+8} &= 4 \\
\sqrt{b+8} &= 4-\sqrt{b}
\end{aligned}$$

$$\begin{aligned}
\left(\sqrt{b+8}\right)^2 &= \left(4-\sqrt{b}\right)^2 \\
b+8 &= 16-8\sqrt{b}+b \\
8\sqrt{b} &= 8 \\
\sqrt{b} &= 1 \\
\left(\sqrt{b}\right)^2 &= 1^2 \\
b &= 1 \Rightarrow \text{The solution checks.}
\end{aligned}$$

70. $\sqrt{\sqrt{x+19}-\sqrt{x-2}} = \sqrt{3}$

Solution

$$\begin{aligned}
\sqrt{\sqrt{x+19}-\sqrt{x-2}} &= \sqrt{3} \\
\left(\sqrt{\sqrt{x+19}-\sqrt{x-2}}\right)^2 &= \left(\sqrt{3}\right)^2 \\
\sqrt{x+19}-\sqrt{x-2} &= 3 \\
\sqrt{x+19} &= 3+\sqrt{x-2} \\
\left(\sqrt{x+19}\right)^2 &= \left(3+\sqrt{x-2}\right)^2 \\
x+19 &= 9+6\sqrt{x-2}+x-2 \\
12 &= 6\sqrt{x-2} \\
2 &= \sqrt{x-2} \\
2^2 &= \left(\sqrt{x-2}\right)^2 \\
4 &= x-2 \\
6 &= x \Rightarrow \text{The solutions checks.}
\end{aligned}$$

71. $\sqrt[3]{7x+1} = 4$

Solution

$$\begin{aligned}
\sqrt[3]{7x+1} &= 4 \\
\left(\sqrt[3]{7x+1}\right)^3 &= 4^3 \\
7x+1 &= 64 \\
7x &= 63 \\
x &= 9
\end{aligned}$$

The solution checks.

72. $\sqrt[3]{11a - 40} = 5$

Solution

$$\begin{aligned} \sqrt[3]{11a - 40} &= 5 \\ \left(\sqrt[3]{11a - 40}\right)^3 &= 5^3 \\ 11a - 40 &= 125 \\ 11a &= 165 \\ a &= 15 \end{aligned}$$

The solution checks.

73. $\sqrt[3]{x^3 + 7} = x + 1$

Solution

$$\begin{aligned} \sqrt[3]{x^3 + 7} &= x + 1 \\ \left(\sqrt[3]{x^3 + 7}\right)^3 &= (x + 1)^3 \\ x^3 + 7 &= x^3 + 3x^2 + 3x + 1 \\ 0 &= 3x^2 + 3x - 6 \\ 0 &= 3(x + 2)(x - 1) \end{aligned}$$

$x + 2 = 0$ **or** $x - 1 = 0$

$x = -2$ $\quad$ $x = 1$

Both solutions check.

74. $\sqrt[3]{x^3 - 7} + 1 = x$

Solution

$$\begin{aligned} \sqrt[3]{x^3 - 7} + 1 &= x \\ \sqrt[3]{x^3 - 7} &= x - 1 \\ \left(\sqrt[3]{x^3 - 7}\right)^3 &= (x - 1)^3 \\ x^3 - 7 &= x^3 - 3x^2 + 3x - 1 \\ 3x^2 - 3x - 6 &= 0 \\ 3(x - 2)(x + 1) &= 0 \end{aligned}$$

$x - 2 = 0$ **or** $x + 1 = 0$

$x = 2$ $\quad$ $x = -1$

Both solutions check.

75. $\sqrt[3]{8x^3 + 61} = 2x + 1$

Solution

$$\sqrt[3]{8x^3 + 61} = 2x + 1$$
$$\left(\sqrt[3]{8x^3 + 61}\right)^3 = (2x + 1)^3$$
$$8x^3 + 61 = 8x^3 + 12x^2 + 6x + 1$$
$$0 = 12x^2 + 6x - 60$$
$$0 = 6(2x + 5)(x - 2)$$
$$2x + 5 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$x = -\tfrac{5}{2} \qquad\qquad x = 2$$

Both solutions check.

76. $\sqrt[3]{8x^3 - 37} = 2x + 1$

Solution

$$\sqrt[3]{8x^3 - 37} = 2x + 1$$
$$\left(\sqrt[3]{8x^3 + 37}\right)^3 = (2x + 1)^3$$
$$8x^3 - 37 = 8x^3 - 12x^2 + 6x + 1$$
$$12x^2 - 6x - 36 = 0$$
$$6(2x + 3)(x - 2) = 0$$
$$2x + 3 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$x = -\tfrac{3}{2} \qquad\qquad x = 2$$

Both solutions check.

77. $\sqrt[4]{30t + 25} = 5$

Solution

$$\sqrt[4]{30t + 25} = 5$$
$$\left(\sqrt[4]{30t + 25}\right)^4 = 5^4$$
$$30t + 25 = 626$$
$$30t = 600$$
$$t = 20$$

The solution checks.

78. $\sqrt[4]{3z+1} = 2$

Solution

$$\begin{aligned} \sqrt[4]{3z+1} &= 2 \\ \left(\sqrt[4]{3z+1}\right)^4 &= 2^4 \\ 3z+1 &= 16 \\ 3z &= 15 \\ z &= 5 \end{aligned}$$

The solution checks.

79. $\sqrt[5]{2x-11} = \sqrt[5]{14}$

Solution

$$\begin{aligned} \sqrt[5]{2x-11} &= \sqrt[5]{14} \\ \left(\sqrt[5]{2x-11}\right)^5 &= \left(\sqrt[5]{14}\right)^5 \\ 2x-11 &= 14 \\ 2x &= 25 \\ x &= \tfrac{25}{2} \end{aligned}$$

The solution checks.

80. $\sqrt[5]{x^2-24} = 1$

Solution

$$\begin{aligned} \sqrt[5]{x^2-24} &= 1 \\ \left(\sqrt[5]{x^2-24}\right)^5 &= (1)^5 \\ x^2-24 &= 1 \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

Both solutions check.

Fix It

In exercises 81 and 82, identify the step the first error is made and fix it.

81. Solve the radical equation: $\sqrt{2x-1} - x = -2$

Solution

Step 3 was incorrect.

Step 1: $\sqrt{2x-1} = x - 2$

Step 2: $\left(\sqrt{2x - 1}\right)^2 = (x - 2)^2$

Step 3: $2x - 1 = x^2 - 4x + 4$

Step 4: $0 = x^2 - 6x + 5$

Step 5: $0 = (x - 5)(x - 1)$

Step 6: $x = 5, \ -1$, however, $x = -1$ is an extraneous solution, so $x = 5$

82. Solve for x by making a substitution: $x^{2/3} - 7x^{1/3} = 8$

Solution
Step 5 was incorrect.

Step 1: $x^{2/3} - 7x^{1/3} - 8 = 0$

Step 2: Let $u = x^{1/3}$

Step 3: $u^2 - 7u - 8 = 0$

Step 4: $(u - 8)(u + 1) = 0$

Step 5: $u = 8$ or $u = -1$

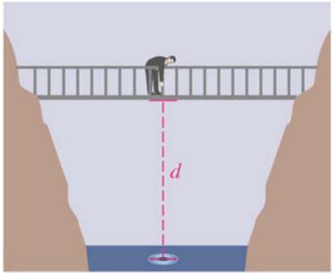
Step 6: $x^{1/3} = 8$ or $x^{1/3} = -1$

Step 7: $x = 512$ or $x = -1$

Applications

83. **Height of a bridge** The distance d (in feet) that an object will fall in t seconds is given by the following formula. To find the height of a bridge above a river, a man drops a stone into the water. (See the illustration.) If it takes the stone 5 seconds to hit the water, how high is the bridge?

$$t = \sqrt{\frac{d}{16}}$$



Solution

$$t = \sqrt{\frac{d}{16}}$$

$$5 = \sqrt{\frac{d}{16}}$$

$$5^2 = \left(\sqrt{\frac{d}{16}}\right)^2$$

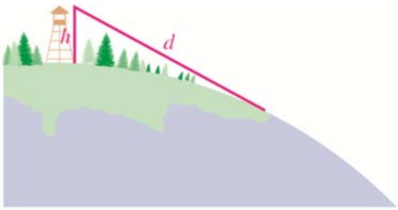
$$25 = \frac{d}{16}$$

$400 = d \Rightarrow$ The bridge is 400 feet high.

84. **Horizon distance** The higher a lookout tower, the farther an observer can see. (See the illustration.) The distance d (called the **horizon distance**, measured in miles) is related to the height h of the observer (measured in feet) by the following formula.

$$d = \sqrt{1.5h}$$

How tall must a tower be for the observer to see 30 miles?



Solution

$$d = \sqrt{1.5h}$$

$$30 = \sqrt{1.5h}$$

$$30^2 = \left(\sqrt{1.5h}\right)^2$$

$$900 = 1.5h$$

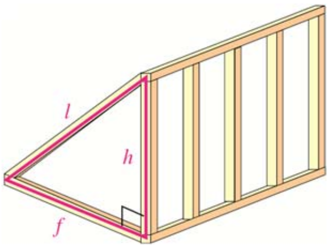
$$600 = h$$

The tower must be 600 feet tall.

85. **Carpentry** During construction, carpenters often brace walls, as shown in the illustration. The appropriate length of the brace is given by the following formula.

$$l = \sqrt{f^2 + h^2}$$

If a carpenter nails a 10-foot brace to the wall 6 feet above the floor, how far from the base of the wall should he nail the brace to the floor?



Solution

$$l = \sqrt{f^2 + h^2}$$
$$10 = \sqrt{f^2 + 6^2}$$
$$10^2 = \left(\sqrt{f^2 + 36}\right)^2$$
$$100 = f^2 + 36$$
$$64 = f^2$$
$$\pm 8 = f$$

He should nail the brace to the floor 8 feet from the wall.

86. **Windmills** The power generated by a windmill is related to the velocity of the wind by the following formula where P is the power (in watts) and v is the velocity of the wind (in mph).

$$v = \sqrt[3]{\frac{P}{0.02}}$$

To the nearest 10 watts, find the power generated when the velocity of the wind is 31 mph.

Solution

$$v = \sqrt[3]{\frac{P}{0.02}}$$
$$31 = \sqrt[3]{\frac{P}{0.02}}$$
$$31^3 = \left(\sqrt[3]{\frac{P}{0.02}}\right)^3$$
$$29791 = \frac{P}{0.02}$$
$$29791(0.02) = p$$
$$600 \approx P$$

The power generated is about 600 watts.

87. **Diamonds** The *effective rate of interest r* earned by an investment is given by the following formula where P is the initial investment that grows to value A after n years.

$$r = \sqrt[n]{\frac{A}{P}} - 1$$

If a diamond buyer got $4000 for a 1.03-carat diamond that he had purchased 4 years earlier and earned an annual rate of return of 6.5% on the investment, what did he originally pay for the diamond?

Solution

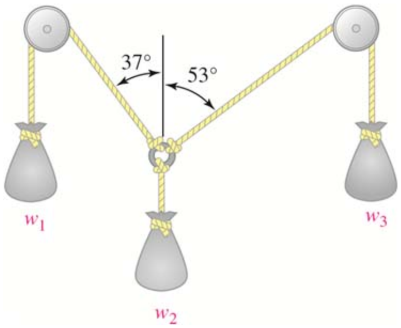
$$\begin{aligned} r &= \sqrt[n]{\frac{A}{P}} - 1 \\ 0.065 &= \sqrt[4]{\frac{4000}{P}} - 1 \\ 1.065 &= \sqrt[4]{\frac{4000}{P}} \\ (1.065)^4 &= \left(\sqrt[4]{\frac{4000}{P}}\right)^4 \\ 1.286466 &\approx \frac{4000}{P} \\ 1.286466P &\approx 4000 \\ P &\approx 3109 \end{aligned}$$

The original price was about $3109.

88. **Theater productions** The ropes, pulleys, and sandbags shown in the illustration are part of a mechanical system used to raise and lower scenery for a stage play. For the scenery to be in the proper position, the following formula must apply:

$$w_2 = \sqrt{w_1^2 + w_3^2}$$

If $w_2 = 12.5$ lb and $w_3 = 7.5$ lb, find w_1.



Solution

$$\begin{aligned} w_2 &= \sqrt{w_1^2 + w_3^2} \\ 12.5 &= \sqrt{w_1^2 + (7.5)^2} \\ (12.5)^2 &= \left(\sqrt{w_1^2 + 56.25}\right)^2 \\ 156.25 &= w_1^2 + 56.25 \\ 100 &= w_1^2 \\ \pm\sqrt{100} &= w_1 \\ w_1 &= 10 \text{ lb} \end{aligned}$$

Discovery and Writing

89. Explain the Power Property of Real Numbers.

Solution
Answers may vary.

90. Describe what it means for an equation to be quadratic in form.

Solution
Answers may vary.

91. Identify two methods that can be used to solve the equation $x^4 - 6x^2 - 7 = 0$. Compare and contrast the two methods.

Solution
Answers may vary.

92. Outline a strategy that can be used to solve radical equations.

Solution
Answers may vary.

93. Explain why squaring both sides of an equation might introduce extraneous roots.

Solution
Answers may vary.

94. Can cubing both sides of an equation introduce extraneous roots? Explain.

Solution
Answers may vary.

Critical Thinking

Determine if the statement is true or false. If the statement is false, then correct it and make it true.

95. Factoring can be used to solve $x^4 + 6x^3 + 5 = 0$.

Solution

False. The equation is not quadratic in form. $x^4 + 6x^2 + 5 = 0$ can be solved by factoring.

96. The first step used to solve the equation $4x^4 = 2x^2$ is to divide both sides of the equation by $2x^2$.

Solution
False. The first step is to rewrite the equation so that zero is on one side of the equation.

97. To solve the equation $5y^{\frac{2}{3}} - 4y^{\frac{1}{3}} - 1 = 0$, we can make the substitution $u = y^{\frac{1}{3}}$.

Solution
True.

98. The equation $x^{\frac{1}{8}} + 7x^{\frac{1}{4}} + 12 = 0$ is quadratic in form.

Solution
True.

99. If $\sqrt{\sqrt{\sqrt{x}}} = 2$ then $x = 16$.

Solution

False. $\sqrt{\sqrt{\sqrt{x}}} = 2$

$$\sqrt{\sqrt{x}} = 4$$
$$\sqrt{x} = 16$$
$$x = 256$$

100. To solve the radical equation $\sqrt{z} - \sqrt{z + 2} = 2$, we square each term individually.

Solution
False. Isolate one of the radicals first, then square both sides of the equation.

101. To solve the radical equation $\sqrt{x + 1} - \sqrt{2x + 3} = -1$, the first step is to square both sides.

Solution
False. Isolate one of the radicals first, then square both sides of the equation.

102. IF $\sqrt[999]{x^2} = -1$, then $x = \pm i$.

Solution

True. $\sqrt[999]{x^2} = -1$

$$\left(\sqrt[999]{x^2}\right)^{999} = (-1)^{999}$$

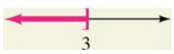
$$x^2 = -1 \Rightarrow x = \pm i$$

Exercises 1.7

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Express the graph shown on the number line using interval notation.



Solution

$(-\infty, 3]$

2. Express the graph shown on the number line using interval notation.

Solution

$(-5, 2]$

3. Solve the linear equation: $2(x - 5) = 5 = 3(x - 3)$

Solution

$$2(x - 5) = 5 = 3(x - 3)$$
$$2x - 10 = 5 + 3x - 9$$
$$2x - 10 = -4 + 3x$$
$$-x - 10 = -4$$
$$-x = 6$$
$$x = -6$$

4. Solve the quadratic equation: $3x^2 - 7x - 6 = 0$

Solution

$$3x^2 + 7x - 6 = 0$$
$$(3x - 2)(x + 3) = 0$$
$3x - 2 = 0$ **or** $x + 3 = 0$

$3x = 2$ **or** $x = -3$

$$x = \frac{2}{3}$$

5. Is −3 a solution of $(4x - 1)(x + 2) > 0$?

Solution

Is $[(4(-3) - 1)](-3 + 2) > 0$?

$-13 \cdot -1 > 0$

$13 > 0$ True

Yes, −3 is a solution.

6. Is −3 a solution of $\frac{-3}{x + 3} \leq 0$?

Solution

Is $\frac{-3}{-3 + 3} \leq 0$?

No, since $\frac{-3}{0}$ is undefined.

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. If $x > y$, then x lies to the __________ of y on a number line.

Solution

right

8. $a < b$, __________, $a > b$.

Solution

$a = b$

9. If $a < b$ and $b < c$, then __________.

Solution

$a < c$

10. If $a < b$ then $a + c <$ __________.

Solution

$b + c$

11. If $a < b$ then $a - c <$ __________.

Solution

$b - c$

12. If $a < b$ and $c > 0$, then ac __________ bc.

Solution

$<$

13. If $a < b$ and $c < 0$, then ac __________ bc.

Solution
$>$

14. If $a < b$ and $c < 0$, then $\frac{a}{c}$ __________ $\frac{b}{c}$.

Solution
$>$

15. $3x - 5 < 12$ and $ax + c > 0 (a \neq 0)$ are examples of __________ inequalities.

Solution
linear

16. $ax^2 + bx - c \geq 0\ a \neq 0$ and $3x^2 - 6x < 0$ are examples of __________ inequalities.

Solution
quadratic

17. If two inequalities have the same solution set, they are called __________ inequalities.

Solution
equivalent

18. An inequality that contains a fraction with a polynomial numerator and denominator is called a __________ inequality.

Solution
rational

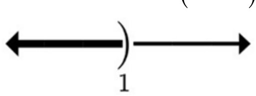
Practice

Solve each linear inequality and graph its solution set on a number line. Write the solution set in interval notation.

19. $3x + 2 < 5$

Solution

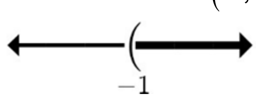
$$3x + 2 < 5$$
$$3x < 3$$
$$x < 1 \Rightarrow (-\infty, 1)$$



20. $-2x + 4 < 6$

Solution

$$-2x + 4 < 6$$
$$2x < 2$$
$$x > -1 \Rightarrow (-1, \infty)$$



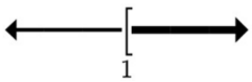
21. $3x + 2 \geq 5$

Solution

$3x + 2 \geq 5$

$3x \geq 3$

$x \geq 1 \Rightarrow [1, \infty)$



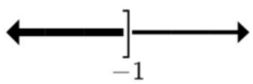
22. $-2x + 4 \geq 6$

Solution

$-2x + 4 \geq 6$

$-2x \geq 2$

$x \leq -1 \Rightarrow (-\infty, -1]$



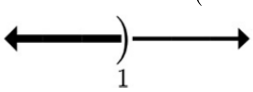
23. $-5x + 3 > -2$

Solution

$-5x + 3 > -2$

$-5x > -5$

$x < 1 \Rightarrow (-\infty, 1)$



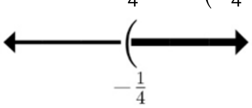
24. $4x - 3 > -4$

Solution

$4x - 3 > -4$

$4x > -1$

$x > -\frac{1}{4} \Rightarrow \left(-\frac{1}{4}, \infty\right)$



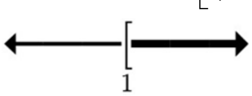
25. $-5x + 3 \leq -2$

Solution

$-5x + 3 \leq -2$

$-5x \leq -5$

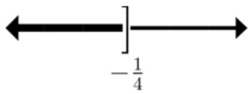
$x \geq 1 \Rightarrow [1, \infty)$



26. $4x - 3 \le -4$

Solution

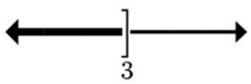
$$4x - 3 \le -4$$
$$4x \le -1$$
$$x \le -\tfrac{1}{4} \Rightarrow \left(-\infty, \ -\tfrac{1}{4}\right)$$



27. $2(x - 3) \le -2(x - 3)$

Solution

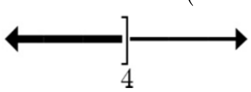
$$2(x - 3) \le -2(x - 3)$$
$$2x - 6 \le -2x + 6$$
$$4x \le 12$$
$$x \le 3 \Rightarrow (-\infty, 3]$$



28. $3(x + 2) \le 2(x + 5)$

Solution

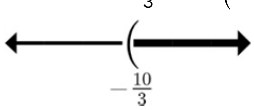
$$3(x + 2) \le 2(x + 5)$$
$$3x + 6 \le 2x + 10$$
$$x \le 4 \Rightarrow (-\infty, 4]$$



29. $\frac{3}{5}x + 4 > 2$

Solution

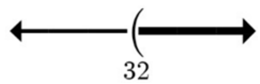
$$\frac{3}{5}x + 4 > 2$$
$$5\left(\frac{3}{5}x + 4\right) > 5(2)$$
$$3x + 20 > 10$$
$$3x > -10$$
$$x > -\tfrac{10}{3} \Rightarrow \left(-\tfrac{10}{3}, \infty\right)$$



30. $\frac{1}{4}x - 3 > 5$

Solution

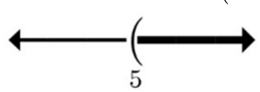
$$\begin{aligned}
\frac{1}{4}x - 3 &> 5 \\
4\left(\frac{1}{4}x - 3\right) &> 4(5) \\
x - 12 &> 20 \\
x &> 32 \Rightarrow (32, \infty)
\end{aligned}$$



31. $\frac{x + 3}{4} < \frac{2x - 4}{3}$

Solution

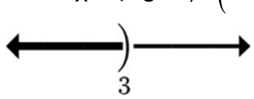
$$\begin{aligned}
\frac{x + 3}{4} &< \frac{2x - 4}{3} \\
12 \cdot \frac{x + 3}{4} &< 12 \cdot \frac{2x - 4}{3} \\
3(x + 3) &< 4(2x - 4) \\
3x + 9 &< 8x - 16 \\
-5x &< -25 \\
x &> 5 \Rightarrow (5, \infty)
\end{aligned}$$



32. $\frac{x + 2}{5} > \frac{x - 1}{2}$

Solution

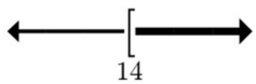
$$\begin{aligned}
\frac{x + 2}{5} &> \frac{x - 1}{2} \\
10 \cdot \frac{x + 2}{5} &> 10 \cdot \frac{x - 1}{2} \\
2(x + 2) &> 5(x - 1) \\
2x + 4 &> 5x - 5 \\
-3x &> -9 \\
x &< 3 \Rightarrow (-\infty, 3)
\end{aligned}$$



33. $\dfrac{6(x - 4)}{5} \geq \dfrac{3(x + 2)}{4}$

Solution

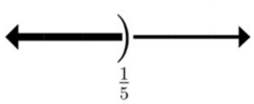
$$\begin{aligned}
\frac{6(x - 4)}{5} &\geq \frac{3(x + 2)}{4} \\
20 \cdot \frac{6x - 24}{5} &\geq 20 \cdot \frac{3x + 6}{4} \\
4(6x - 24) &\geq 5(3x + 6) \\
24x - 96 &\geq 15x + 30 \\
9x &\geq 126 \\
x &\geq 14 \Rightarrow [14, \infty)
\end{aligned}$$



34. $\dfrac{3(x + 3)}{2} < \dfrac{2(x + 7)}{3}$

Solution

$$\begin{aligned}
\frac{3(x + 3)}{2} &< \frac{2(x + 7)}{3} \\
6 \cdot \frac{3x + 9}{2} &< 6 \cdot \frac{2x + 14}{3} \\
3(3x + 9) &< 2(2x + 14) \\
9x + 27 &< 4x + 28 \\
5x &< 1 \\
x &< \tfrac{1}{5} \Rightarrow \left(-\infty, \tfrac{1}{5}\right)
\end{aligned}$$

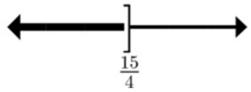


35. $\dfrac{5}{9}(a + 3) - a \geq \dfrac{4}{3}(a - 3) - 1$

Solution

$$\begin{aligned}
\frac{5}{9}(a + 3) - a &\geq \frac{4}{3}(a - 3) - 1 \\
9\left[\frac{5}{9}(a + 3) - a\right] &\geq 9\left[\frac{4}{3}(a - 3) - 1\right] \\
5(a + 3) - 9a &\geq 12(a - 3) - 9 \\
5a + 15 - 9a &\geq 12a - 36 - 9
\end{aligned}$$

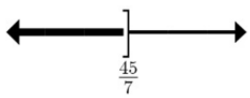
$$\begin{aligned} -16a &\geq -60 \\ a &\leq \frac{-60}{-16} \\ a &\leq \tfrac{15}{4} \Rightarrow \left(-\infty, \tfrac{15}{4}\right) \end{aligned}$$



36. $\dfrac{2}{3}y - y \leq -\dfrac{3}{2}(y - 5)$

Solution

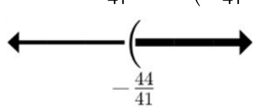
$$\begin{aligned} \frac{2}{3}y - y &\leq -\frac{3}{2}(y - 5) \\ 6\left(\frac{2}{3}y - y\right) &\leq 6\left[-\frac{3}{2}(y - 5)\right] \\ 4y - 6y &\leq -9(y - 5) \\ -2y &\leq -9y + 45 \\ 7y &\leq 45 \\ y &\leq \tfrac{45}{7} \Rightarrow \left(-\infty, \tfrac{45}{7}\right) \end{aligned}$$



37. $\dfrac{2}{3}a - \dfrac{3}{4}a < \dfrac{3}{5}\left(a + \dfrac{2}{3}\right) + \dfrac{1}{3}$

Solution

$$\begin{aligned} \frac{2}{3}a - \frac{3}{4}a &< \frac{3}{5}\left(a + \frac{2}{3}\right) + \frac{1}{3} \\ 60\left(\frac{2}{3}a - \frac{3}{4}a\right) &< 60\left[\frac{3}{5}\left(a + \frac{2}{3}\right) + \frac{1}{3}\right] \\ 40a - 45a &< 36\left(a + \frac{2}{3}\right) + 20 \\ -5a &< 36a + 24 + 20 \\ -41a &< 44 \\ a &> -\tfrac{44}{41} \Rightarrow \left(-\tfrac{44}{41}, \infty\right) \end{aligned}$$



Solve each compound inequality and graph its solution set on a number line. Write the solution set in interval notation.

38. $\frac{1}{4}b + \frac{2}{3}b - \frac{1}{2} > \frac{1}{2}(b + 1) + b$

Solution

$$\frac{1}{4}b + \frac{2}{3}b - \frac{1}{2} > \frac{1}{2}(b + 1) + b$$

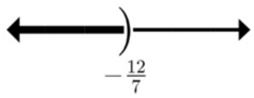
$$12\left(\frac{1}{4}b + \frac{2}{3}b - \frac{1}{2}\right) > 12\left[\frac{1}{2}(b + 1) + b\right]$$

$$3b + 8b - 6 > 6(b + 1) + 12b$$

$$11b - 6 > 6b + 6 + 12b$$

$$-7b > 12$$

$$b < -\tfrac{12}{7} \Rightarrow \left(-\infty, -\tfrac{12}{7}\right)$$



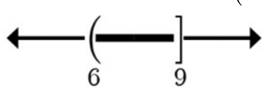
39. $4 < 2x - 8 \leq 10$

Solution

$$4 < 2x - 8 \leq 10$$

$$12 < 2x \leq 18$$

$$12 < x \leq 9 \Rightarrow (6, 9]$$



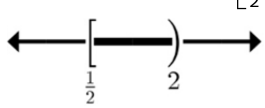
40. $3 \leq 2x + 2 < 6$

Solution

$$3 \leq 2x + 2 < 6$$

$$1 \leq 2x < 4$$

$$\tfrac{1}{2} \leq x < 2 \Rightarrow \left[\tfrac{1}{2}, 2\right)$$



41. $9 \geq \frac{x - 4}{2} > 2$

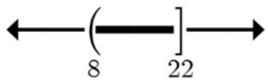
Solution

$$9 \geq \frac{x - 4}{2} > 2$$

$$18 \geq x - 4 > 2$$

$$22 \geq x > 8$$

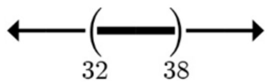
$$8 < x \leq 22 \Rightarrow (8, 22]$$



42. $5 < \dfrac{x - 2}{6} < 6$

Solution

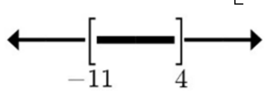
$$\begin{aligned} 5 &< \frac{x-2}{6} < 6 \\ 30 &< x - 2 < 36 \\ 32 &< \quad x \quad < 38 \Rightarrow (32, 38] \end{aligned}$$



43. $0 \le \dfrac{4 - x}{3} \le 5$

Solution

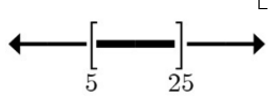
$$\begin{aligned} 0 &\le \frac{4-x}{3} \le 5 \\ 0 &\le 4 - x \le 15 \\ -4 &\le \quad -x \quad \le 11 \\ 4 &\ge \quad x \quad \ge -11 \\ -11 &\le \quad x \quad \le 4 \Rightarrow [-11, 4] \end{aligned}$$



44. $0 \ge \dfrac{5 - x}{2} \ge -10$

Solution

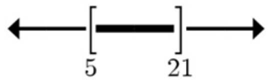
$$\begin{aligned} 0 &\ge \frac{5-x}{2} \ge -10 \\ 0 &\ge 5 - x \ge -20 \\ -5 &\ge \quad -x \quad \ge -25 \\ 5 &\le \quad x \quad \le 25 \Rightarrow [5, 25] \end{aligned}$$



45. $-2 \ge \dfrac{1 - x}{2} \ge -10$

Solution

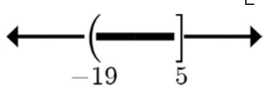
$$\begin{aligned} -2 &\ge \frac{1-x}{2} \ge -10 \\ -4 &\ge 1-x \ge -20 \\ -5 &\ge -x \ge -21 \\ 5 &\le x \le 21 \Rightarrow [5,\ 21] \end{aligned}$$



46. $-2 \le \dfrac{1-x}{2} < 10$

Solution

$$\begin{aligned} -2 &\le \frac{1-x}{2} < 10 \\ -4 &\le 1-x < 20 \\ -5 &\le -x < 19 \\ 5 &\ge x > -19 \\ -19 &< x \le 5 \Rightarrow [-19,\ 5] \end{aligned}$$



47. $-3x > -2x > -x$

Solution

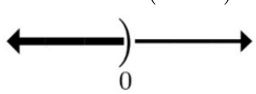
$$-3x > -2x > -x$$

$$-3x > -2x \quad \textbf{and} \quad -2x > -x$$

$$-x > 0 \qquad\qquad -x > 0$$

$$x < 0 \qquad\qquad x < 0$$

$$x < 0 \Rightarrow (-\infty,\ 0)$$



48. $-3x < -2x < -x$

Solution

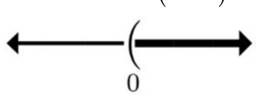
$$-3x < -2x < -x$$

$$-3x < -2x \quad \textbf{and} \quad -2x < -x$$

$$-x < 0 \qquad\qquad -x < 0$$

$$x > 0 \qquad\qquad x > 0$$

$$x > 0 \Rightarrow (0,\ \infty)$$



49. $x < 2x < 3x$

Solution

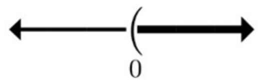
$$x < 2x < 3x$$

$x < 2x$ **and** $2x < 3x$

$-x < 0$ $\quad$ $-x < 0$

$x > 0$ $\quad$ $x > 0$

$$x > 0 \Rightarrow (0, \infty)$$



50. $x > 2x > 3x$

Solution

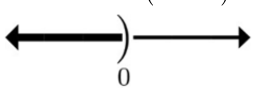
$$x > 2x > 3x$$

$x > 2x$ **and** $2x > 3x$

$-x > 0$ $\quad$ $-x > 0$

$x < 0$ $\quad$ $x < 0$

$$x < 0 \Rightarrow (-\infty, 0)$$



51. $2x + 1 < 3x - 2 < 12$

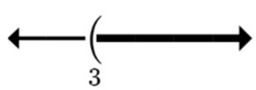
Solution

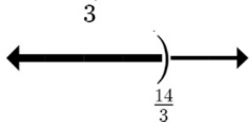
$$2x + 1 < 3x - 2 < 12$$

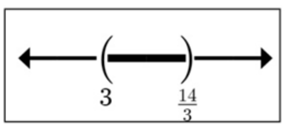
$2x + 1 < 3x - 2$ **and** $3x - 2 < 12$

$-x < -3$ $\quad$ $3x < 14$

$x > 3$ $\quad$ $x < \frac{14}{3}$

$x > 3$ 

$x < \frac{14}{3}$ 

$x > 3$
and
$x < \frac{14}{3}$ 

Solution set: $\left(3, \frac{14}{3}\right)$

52. $2 - x < 3x + 5 < 18$

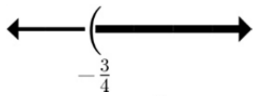
Solution

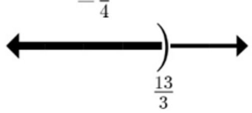
$$2 - x < 3x + 5 < 18$$

$2 - x < 3x + 5$ **and** $3x + 5 < 18$

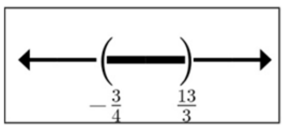
$-4x < 3$ $\qquad$ $3x < 13$

$x > -\frac{3}{4}$ $\qquad$ $x < \frac{13}{3}$

$x > -\frac{3}{4}$ 

$x < \frac{13}{3}$ 

$x > -\frac{3}{4}$
and
$x < \frac{13}{3}$



Solution set: $\left(-\frac{3}{4}, \frac{13}{3}\right)$

53. $2 + x < 3x - 2 < 5x + 2$

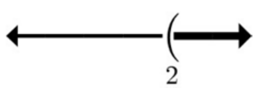
Solution

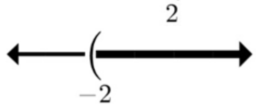
$$2 + x < 3x - 2 < 5x + 2$$

$2 + x < 3x - 2$ **and** $3x - 2 < 5x + 2$

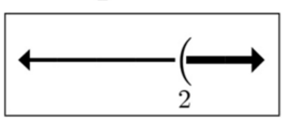
$-2x < -4$ $\qquad$ $-2x < 4$

$x > 2$ $\qquad$ $x > -2$

$x > 2$ 

$x < -2$ 

$x > 2$
and
$x > -2$



Solution set: $(2, \infty)$

54. $x > 2x + 3 > 4x - 7$

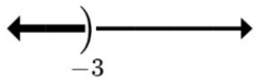
Solution

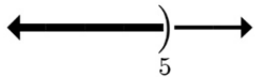
$$x > 2x + 3 > 4x - 7$$

$x > 2x + 3$ **and** $2x + 3 > 4x - 7$

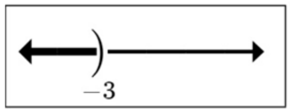
$-x > 3$ $\qquad$ $-2x > -10$

$x < -3$ $\qquad$ $x < 5$

$x < -3$ 

$x < 5$ 

$x < -3$
and
$x < 5$



Solution set: $(-\infty, -3)$

55. $3 + x > 7x - 2 > 5x - 10$

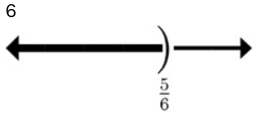
Solution

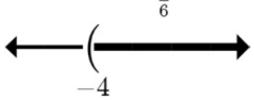
$$3 + x > 7x - 2 > 5x - 10$$

$3 + x > 7x - 2$ **and** $7x - 2 > 5x - 10$

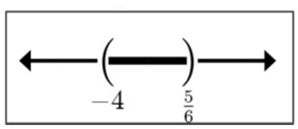
$-6x > -5$ $\quad\quad$ $2x > -8$

$x < \frac{5}{6}$ $\quad\quad$ $x > -4$

$x < \frac{5}{6}$ 

$x > -4$ 

$x < \frac{5}{6}$
and
$x > -4$



Solution set: $\left(-4, \frac{5}{6}\right)$

56. $2 - x < 3x + 1 < 10x$

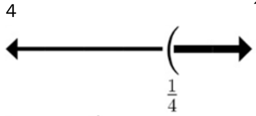
Solution

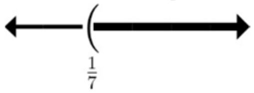
$$2 - x < 3x + 1 < 10x$$

$2 - x < 3x + 1$ **and** $3x + 1 < 10x$

$-4x < -1$ $\quad\quad$ $-7x < -1$

$x > \frac{1}{4}$ $\quad\quad$ $x > \frac{1}{7}$

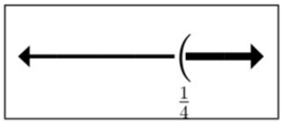
$x > \frac{1}{4}$ 

$x > \frac{1}{7}$ 

$x > \frac{1}{4}$

and

$x > \frac{1}{7}$



Solution set: $\left(\frac{1}{4}, \infty\right)$

57. $x \le x + 1 \le 2x + 3$

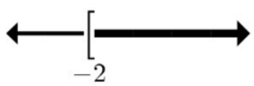
Solution

$$x \le x + 1 \le 2x + 3$$

$x \le x + 1$	**and**	$x + 1 \le 2x + 3$
$0 \le 1$		$-x \le 2$
true for all real numbers x		$x \ge -2$

$0 \le 1$

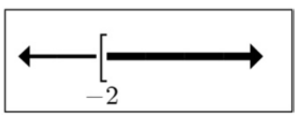
$x \ge -2$



$0 \le 1$

and

$x \ge -2$



Solution set: $[-2, \infty)$

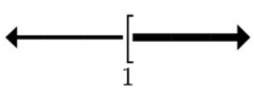
58. $-x \ge -2x + 1 \ge -3x + 1$

Solution

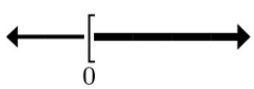
$$-x \ge -2x + 1 \ge -3x + 1$$

$-x \ge -2x + 1$	**and**	$-2x + 1 \ge -3x + 1$
$x \ge 1$		$x \ge 0$

$x \ge 1$



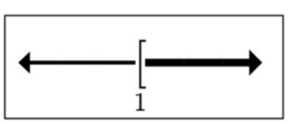
$x \ge 0$



$x \ge 1$

and

$x \ge 0$



Solution set: $[1, \infty)$

59. $x^2 + 7x + 12 < 0$

Solution

$x^2 + 7x + 12 < 0$

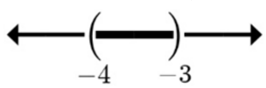
$(x + 3)(x + 4) < 0$

factors $= 0$: $x = -3$, $x = -4$

intervals: $(-\infty, -4)$, $(-4, -3)$, $(-3, \infty)$

interval	test number	value of $x^2 + 7x + 12$
$(-\infty, -4)$	-5	$+2$
$(-4, -3)$	-3.5	-0.25
$(-3, \infty)$	0	$+12$

Solution set: $(-4, -3)$



60. $x^2 - 13x + 12 \le 0$

Solution

$x^2 - 13x + 12 \le 0$

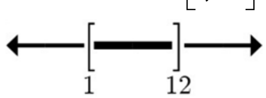
$(x - 12)(x - 1) \le 0$

factors $= 0$: $x = 12$, $x = 1$

intervals: $(-\infty, 1)$, $(1, 12)$, $(12, \infty)$

interval	test number	value of $x^2 - 13x + 12$
$(-\infty, 1)$	0	$+12$
$(1, 12)$	2	-10
$(12, \infty)$	13	$+12$

Solution set: $[1, 12]$



61. $x^2 - 5x + 6 \geq 0$

Solution

$x^2 - 5x + 6 \geq 0$

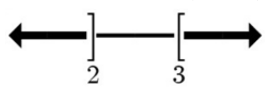
$(x - 3)(x - 2) \geq 0$

factors $= 0$: $x = 3, x = 2$

intervals: $(-\infty, 2), (2, 3), (3, \infty)$

interval	test number	value of $x^2 - 5x + 6$
$(-\infty, 2)$	0	+6
$(2, 3)$	2.5	−0.25
$(3, \infty)$	4	+2

Solution set: $(-\infty, 2) \cup (3, \infty)$



62. $6x^2 + 5x - 6 > 0$

Solution

$6x^2 + 5x - 6 > 0$

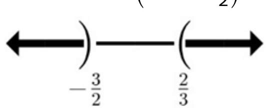
$(2x + 3)(3x - 2) > 0$

factors $= 0$: $x = -\frac{3}{2}, x = \frac{2}{3}$

intervals: $\left(-\infty, -\frac{3}{2}\right), \left(-\frac{3}{2}, \frac{2}{3}\right), \left(\frac{2}{3}, \infty\right)$

interval	test number	value of $6x^2 + 5x - 6$
$\left(-\infty, -\frac{3}{2}\right)$	−2	+8
$\left(-\frac{3}{2}, \frac{2}{3}\right)$	0	−6
$\left(\frac{2}{3}, \infty\right)$	1	+5

Solution: $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{2}{3}, \infty\right)$



63. $x^2 + 5x + 6 < 0$

Solution

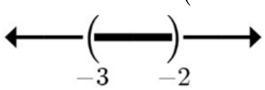
$$x^2 + 5x + 6 < 0$$
$$(x + 3)(x + 2) < 0$$

factors $= 0$: $x = -3$, $x = -2$

intervals: $(-\infty, -3)$, $(-3, -2)$, $(-2, \infty)$

interval	test number	value of $x^2 + 5x + 6$
$(-\infty, -3)$	-4	$+2$
$(-3, -2)$	-2.5	-0.25
$(-2, \infty)$	0	$+6$

Solution set: $(-3, -2)$



64. $x^2 + 9x + 20 \geq 0$

Solution

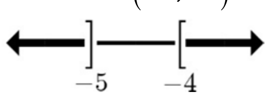
$$x^2 + 9x + 20 \geq 0$$
$$(x + 4)(x + 5) \geq 0$$

factors $= 0$: $x = -4$, $x = -5$

intervals: $(-\infty, -5)$, $(-5, -4)$, $(-4, \infty)$

interval	test number	value of $x^2 + 9x + 20$
$(-\infty, -5)$	-6	$+2$
$(-5, -4)$	-4.5	-0.25
$(-4, \infty)$	0	$+20$

Solution: $(-\infty, -5) \cup (-4, \infty)$



65. $2x^2 - 5x - 3 \geq 0$

Solution

$2x^2 - 5x - 3 \geq 0$

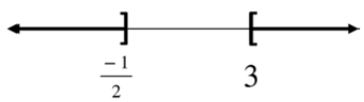
$(2x + 1)(x - 3) \geq 0$

factors $= 0$: $x = -\frac{1}{2}, x = 3$

intervals: $\left(-\infty, -\frac{1}{2}\right), \left(-\frac{1}{2}, 3\right), (3, \infty)$

interval	test number	value of $2x^2 - 5x - 3$
$\left(-\infty, -\frac{1}{2}\right)$	−1	4
$\left(-\frac{1}{2}, 3\right)$	0	−34
$(3, \infty)$	4	9

Solution: $\left(-\infty, -\frac{1}{2}\right) \cup [3, \infty)$



66. $3x^2 + 5x - 2 < 0$

Solution

$3x^2 + 5x - 2 < 0$

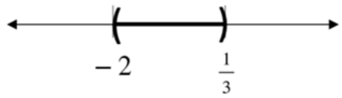
$(3x - 1)(x + 2) < 0$

factors $= 0$: $x = \frac{1}{3}, x = -2$

intervals: $(-\infty, -2), \left(-2, \frac{1}{3}\right), \left(\frac{1}{3}, \infty\right)$

interval	test number	value of $3x^2 + 5x - 2$
$(-\infty, -2)$	−3	10
$\left(-2, \frac{1}{3}\right)$	0	−2
$\left(\frac{1}{3}, \infty\right)$	1	6

Solution: $\left(-2, \frac{1}{3}\right)$



67. $6x^2 + 5x + 1 \geq 0$

Solution

$6x^2 + 5x + 1 \geq 0$

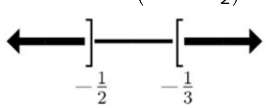
$(2x + 1)(3x + 1) \geq 0$

factors $= 0$: $x = -\frac{1}{2}, x = -\frac{1}{3}$

intervals: $\left(-\infty, -\frac{1}{2}\right), \left(-\frac{1}{2}, -\frac{1}{3}\right), \left(-\frac{1}{3}, \infty\right)$

interval	test number	value of $6x^2 + 5x + 1$
$\left(-\infty, -\frac{1}{2}\right)$	−1	+2
$\left(-\frac{1}{2}, -\frac{1}{3}\right)$	−0.4	−0.04
$\left(-\frac{1}{3}, \infty\right)$	0	+1

Solution: $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{3}, \infty\right)$



68. $x^2 + 9x + 20 < 0$

Solution

$x^2 + 9x + 20 < 0$

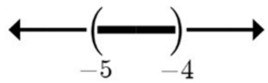
$(x + 5)(x + 4) < 0$

factors $= 0$: $x = -5, x = -4$

intervals: $(-\infty, -5), (-5, -4), (-4, \infty)$

interval	test number	value of $x^2 + 9x + 20$
$(-\infty, -5)$	−6	+2
$(-5, -4)$	−4.5	−0.25
$(-4, \infty)$	0	+20

Solution: $(-5, -4)$



69. $6x^2 - 5x < -1$

Solution

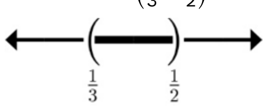
$$6x^2 - 5x < -1$$
$$6x^2 - 5x + 1 < 0$$
$$(2x - 1)(3x - 1) < 0$$

factors $= 0$: $x = \frac{1}{2}, x = \frac{1}{3}$

intervals: $\left(-\infty, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

interval	test number	value of $6x^2 - 5x + 1$
$\left(-\infty, \frac{1}{3}\right)$	0	+1
$\left(\frac{1}{3}, \frac{1}{2}\right)$	0.4	−0.04
$\left(\frac{1}{2}, \infty\right)$	1	+2

Solution: $\left(\frac{1}{3}, \frac{1}{2}\right)$



70. $2x^2 \geq 3 - x$

Solution

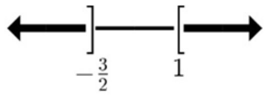
$$2x^2 \geq 3 - x$$
$$2x^2 + x - 3 \geq 0$$
$$(2x + 3)(x - 1) \geq 0$$

factors $= 0$: $x = -\frac{3}{2}, x = 1$

intervals: $\left(-\infty, -\frac{3}{2}\right), \left(-\frac{3}{2}, 1\right), (1, \infty)$

interval	test number	value of $2x^2 + x - 3$
$\left(-\infty, -\frac{3}{2}\right)$	−2	+3
$\left(-\frac{3}{2}, 1\right)$	0	−3
$(1, \infty)$	2	+7

Solution: $\left(-\infty, -\frac{3}{2}\right) \cup (1, \infty)$



71. $4x^2 - 4x + 1 > 0$

Solution

$4x^2 - 4x + 1 > 0$

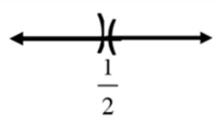
$(2x - 1)(2x - 1) > 0$

factors $= 0$: $x = \frac{1}{2}$

intervals: $\left(-\infty, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

interval	test number	value of $4x^2 - 4x + 1$
$\left(-\infty, \frac{1}{2}\right)$	0	1
$\left(\frac{1}{2}, \infty\right)$	1	1

Solution: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$



72. $9x^2 + 24x > -16$

Solution

$9x^2 + 24x > -16$

$9x^2 + 24x + 16 > 0$

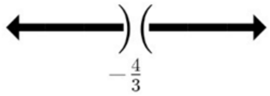
$(3x + 4)(3x + 4) > 0$

factors $= 0$: $x = -\frac{4}{3}, x = -\frac{4}{3}$

intervals: $\left(-\infty, -\frac{4}{3}\right), \left(-\frac{4}{3}, \infty\right)$

interval	test number	value of $9x^2 + 24x + 16$
$\left(-\infty, -\frac{4}{3}\right)$	−2	+4
$\left(-\frac{4}{3}, \infty\right)$	0	+16

Solution: $\left(-\infty, -\frac{4}{3}\right) \cup \left(-\frac{4}{3}, \infty\right)$



73. $9x^2 \le 24x - 16$

Solution

$$9x^2 \le 24x - 16$$
$$9x^2 - 24x + 16 \le 0$$
$$(3x - 4)(3x - 4) \le 0$$

factors $= 0$: $x = \frac{4}{3}, x = \frac{4}{3}$

intervals: $\left(-\infty, \frac{4}{3}\right), \left(\frac{4}{3}, \infty\right)$

interval	test number	value of $9x^2 - 24x + 16$
$\left(-\infty, \frac{4}{3}\right)$	0	+16
$\left(\frac{4}{3}, \infty\right)$	2	+4

Solution: $x = \frac{4}{3}$, or $\left[\frac{4}{3}, \frac{4}{3}\right]$

74. $25x^2 + 20x \le -4$

Solution

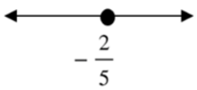
$$25x^2 + 20x + 4 \le 0$$
$$(5x + 2)(5x + 2) \le 0$$

factors $= 0$: $x = -\frac{2}{5}$

intervals: $\left(-\infty, -\frac{2}{5}\right), \left(-\frac{2}{5}, \infty\right)$

interval	test number	value of $25x^2 + 20x + 4$
$\left(-\infty, -\frac{2}{5}\right)$	–1	49
$\left(-\frac{2}{5}, \infty\right)$	0	4

Solution: $\left(-\frac{2}{5}, -\frac{2}{5}\right)$



75. $x^2 \geq 2x - 1$

Solution

$x^2 - 2x + 1 \geq 0$

$(x - 1)(x - 1) \geq 0$

factors $= 0$: $x = 1$

intervals: $(-\infty, 1), (1, \infty)$

interval	test number	value of $x^2 - 2x + 1$
$(-\infty, 1)$	0	1
$(1, \infty)$	2	1

Solution: $(-\infty, \infty)$

76. $-x^2 + 6x \leq 9$

Solution

$0 \leq x^2 - 6x + 9$

$0 \leq (x - 3)(x - 3)$

factors $= 0$: $x = 3$

intervals: $(-\infty, 3), (3, \infty)$

interval	test number	value of $x^2 - 6x + 9$
$(-\infty, 3)$	0	9
$(3, \infty)$	4	1

Solution: $(-\infty, \infty)$

77. $x^2 - 3 \geq 0$

Solution

$x^2 - 3 \geq 0$

$x^2 - 3 = 0$

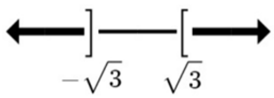
$x^2 = 3$

$x = \pm\sqrt{3}$

intervals: $\left(-\infty, -\sqrt{3}\right), \left(-\sqrt{3}, \sqrt{3}\right), \left(\sqrt{3}, \infty\right)$

interval	test number	value of $x^2 - 3$
$\left(-\infty, -\sqrt{3}\right)$	−2	+1
$\left(-\sqrt{3}, \sqrt{3}\right)$	0	−3
$\left(\sqrt{3}, \infty\right)$	2	+1

Solution: $\left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$



78. $x^2 - 7 \leq 0$

Solution

$x^2 - 7 \leq 0$

$x^2 - 7 = 0$

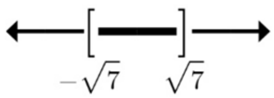
$x^2 = 7$

$x^2 = \pm\sqrt{7}$

intervals: $\left(-\infty, -\sqrt{7}\right), \left(-\sqrt{7}, \sqrt{7}\right), \left(\sqrt{7}, \infty\right)$

interval	test number	value of $x^2 - 7$
$\left(-\infty, -\sqrt{7}\right)$	−3	+2
$\left(-\sqrt{7}, \sqrt{7}\right)$	0	−7
$\left(\sqrt{7}, \infty\right)$	3	+2

Solution: $\left(-\sqrt{7}, \sqrt{7}\right)$



79. $x^2 - 11 < 0$

Solution

$x^2 - 11 < 0$

$x^2 - 11 = 0$

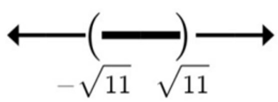
$x^2 = 11$

$x = \pm\sqrt{11}$

intervals: $\left(-\infty, -\sqrt{11}\right), \left(-\sqrt{11}, \sqrt{11}\right), \left(\sqrt{11}, \infty\right)$

interval	test number	value of $x^2 - 11$
$\left(-\infty, -\sqrt{11}\right)$	-4	$+5$
$\left(-\sqrt{11}, \sqrt{11}\right)$	0	-11
$\left(\sqrt{11}, \infty\right)$	4	$+5$

Solution: $\left(-\sqrt{11}, \sqrt{11}\right)$



80. $x^2 - 20 > 0$

Solution

$x^2 - 20 > 0$

$x^2 - 20 = 0$

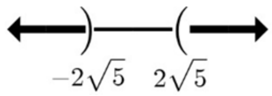
$x^2 = 20$

$x = \pm\sqrt{20} = \pm 2\sqrt{5}$

intervals: $\left(-\infty, -2\sqrt{5}\right), \left(-2\sqrt{5}, 2\sqrt{5}\right), \left(2\sqrt{5}, \infty\right)$

interval	test number	value of $x^2 - 20$
$\left(-\infty, -2\sqrt{5}\right)$	-5	$+5$
$\left(-2\sqrt{5}, 2\sqrt{5}\right)$	0	-20
$\left(2\sqrt{5}, \infty\right)$	5	$+5$

Solution: $\left(-\infty, -2\sqrt{5}\right) \cup \left(2\sqrt{5}, \infty\right)$



Solve the rational inequality and graph its solution set on a number line. Write the solution set in interval notation.

81. $\dfrac{x+3}{x-2} < 0$

Solution

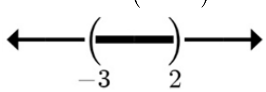
$\dfrac{x+3}{x-2} < 0$

factors $= 0$: $x = -3$, $x = 2$

intervals: $(-\infty, -3)$, $(-3, 2)$, $(2, \infty)$

interval	test number	sign of $\frac{x+3}{x-2}$
$(-\infty, -3)$	−4	+
$(-3, 2)$	0	−
$(2, \infty)$	3	+

Solution: $(-3, 2)$



82. $\dfrac{x+3}{x-2} > 0$

Solution

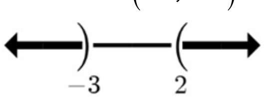
$\dfrac{x+3}{x-2} > 0$

factors $= 0$: $x = -3$, $x = 2$

intervals: $(-\infty, -3)$, $(-3, 2)$, $(2, \infty)$

interval	test number	sign of $\frac{x+3}{x-2}$
$(-\infty, -3)$	−4	+
$(-3, 2)$	0	−
$(2, \infty)$	3	+

Solution: $(-\infty, -3) \cup (2, \infty)$



83. $\dfrac{x^2+x}{x-1} > 0$

Solution

$$\frac{x^2 + x}{x^2 - 1} > 0$$

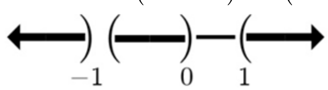
$$\frac{x(x + 1)}{(x + 1)(x - 1)} > 0$$

factors $= 0$: $x = 0, x = -1, x = 1$

intervals: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$

interval	test number	sign of $\frac{x^2+x}{x^2-1}$
$(-\infty, -1)$	-2	+
$(-1, 0)$	$-\frac{1}{2}$	+
$(0, 1)$	$\frac{1}{2}$	−
$(1, \infty)$	2	+

Solution: $(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$



84. $\frac{x^2 - 4}{x^2 - 9} < 0$

Solution

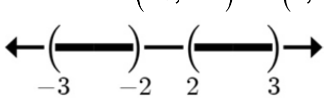
$$\frac{x^2 - 4}{x^2 - 9} < 0 \Rightarrow \frac{(x + 2)(x - 2)}{(x + 3)(x - 3)} < 0$$

factors $= 0$: $x = \pm 2, x = \pm 3$

intervals: $(-\infty, -3), (-3, -2), (-2, 2), (2, 3), (3, \infty)$

interval	test number	sign of $\frac{x^2+4}{x^2-9}$
$(-\infty, -3)$	-4	+
$(-3, -2)$	-2.5	−
$(-2, 2)$	0	+
$(2, 3)$	2.5	−
$(3, \infty)$	4	+

Solution: $(-3, -2) \cup (2, 3)$



85. $\dfrac{x^2 + 5x + 6}{x^2 + x - 6} \geq 0$

Solution

$\dfrac{x^2 + 5x + 6}{x^2 + x - 6} \geq 0 \Rightarrow \dfrac{(x + 3)(x + 2)}{(x + 3)(x - 2)} \geq 0$

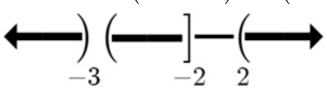
factors $= 0$: $x = -3, x = \pm 2$

intervals: $(-\infty, -3), (-3, -2), (-2, 2), (2, \infty)$

interval	test number	sign of $\frac{x^2+5x+6}{x^2+x-6}$
$(-\infty, -3)$	−4	+
$(-3, -2)$	−2.5	+
$(-2, 2)$	0	−
$(2, \infty)$	3	+

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $(-\infty, -3) \cup (-3, -2) \cup (2, \infty)$



86. $\dfrac{x^2 + 10x + 25}{x^2 - x - 12} \leq 0$

Solution

$\dfrac{x^2 + 10x + 25}{x^2 - x - 12} \leq 0 \Rightarrow \dfrac{(x + 5)(x + 5)}{(x + 3)(x - 4)} \leq 0$

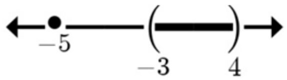
factors $= 0$: $x = -3, x = 4, x = -5$

intervals: $(-\infty, -5), (-5, -3), (-3, 4), (4, \infty)$

interval	test number	sign of $\frac{x^2+10x+25}{x^2-x-12}$
$(-\infty, -5)$	−6	+
$(-5, -3)$	−4	+
$(-3, 4)$	0	−
$(4, \infty)$	5	+

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $(-5, -5) \cup (-3, 4)$



87. $\dfrac{6x^2 - x - 1}{x^2 + 4x + 4} > 0$

Solution

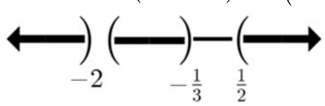
$\dfrac{6x^2 - x - 1}{x^2 + 4x + 4} > 0 \Rightarrow \dfrac{(2x - 1)(3x + 1)}{(x + 2)(x + 2)} > 0$

factors $= 0$: $x = \frac{1}{2}, x = -\frac{1}{3}, x = -2$

intervals: $(-\infty, -2), \left(-2, -\frac{1}{3}\right), \left(-\frac{1}{3}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

interval	test number	sign of $\frac{6x^2 - x - 1}{x^2 + 4x + 4}$
$(-\infty, -2)$	−3	+
$\left(-2, -\frac{1}{3}\right)$	−1	+
$\left(-\frac{1}{3}, \frac{1}{2}\right)$	0	−
$\left(\frac{1}{2}, \infty\right)$	1	+

Solution: $(-\infty, -2) \cup \left(-2, -\frac{1}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$



88. $\dfrac{6x^2 - 3x - 3}{x^2 - 2x - 8} < 0$

Solution

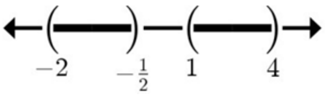
$\dfrac{6x^2 - 3x - 3}{x^2 - 2x - 8} < 0 \Rightarrow \dfrac{3(2x + 1)(x - 1)}{(x + 2)(x - 4)} < 0$

factors $= 0$: $x = -\frac{1}{2}, x = 1, x = -2, x = 4$

intervals: $(-\infty, -2), \left(-2, -\frac{1}{2}\right), \left(-\frac{1}{2}, 1\right), (1, 4), (4, \infty)$

interval	test number	sign of $\frac{6x^2 - 3x - 3}{x^2 - 2x - 8}$
$(-\infty, -2)$	−3	+
$\left(-2, -\frac{1}{2}\right)$	−1	−
$\left(-\frac{1}{2}, 1\right)$	0	+
$(1, 4)$	2	−
$(4, \infty)$	5	+

Solution: $\left(-2, -\frac{1}{2}\right) \cup (1, 4)$



89. $\frac{3}{x} > 2$

Solution

$$\frac{3}{x} > 2$$

$$\frac{3}{x} - 2 > 0$$

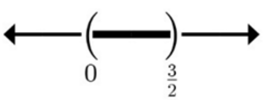
$$\frac{3 - 2x}{x} > 0$$

factors $= 0$: $x = \frac{3}{2}, x = 0$

intervals: $(-\infty, 0), \left(0, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$

interval	test number	sign of $\frac{3-2x}{x}$
$(-\infty, 0)$	−1	–
$\left(0, \frac{3}{2}\right)$	1	+
$\left(\frac{3}{2}, \infty\right)$	2	–

Solution: $\left(0, \frac{3}{2}\right)$



90. $\frac{3}{x} < 2$

Solution

$$\frac{3}{x} < 2$$

$$\frac{3}{x} - 2 < 0$$

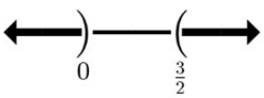
$$\frac{3 - 2x}{x} < 0$$

factors $= 0$: $x = \frac{3}{2}, x = 0$

intervals: $(-\infty, 0), \left(0, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$

interval	test number	sign of $\frac{3-2x}{x}$
$(-\infty, 0)$	-1	$-$
$\left(0, \frac{3}{2}\right)$	1	+
$\left(\frac{3}{2}, \infty\right)$	2	$-$

Solution: $(-\infty, 0) \cup \left(\frac{3}{2}, \infty\right)$



91. $\dfrac{20}{x} \le 10$

Solution

$$\frac{20}{x} \le 10$$

$$\frac{20}{x} - 10 \le 0$$

$$\frac{20 - 10x}{x} \le 0$$

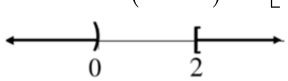
factors = 0: $x = 0, x = 2$

intervals: $(-\infty, 0), (0, 2), (2, \infty)$

interval	test number	sign of $\frac{20 - 10x}{x}$
$(-\infty, 0)$	-1	$-$
$(0, 2)$	1	+
$(2, \infty)$	3	$-$

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $(-\infty, 0) \cup [2, \infty)$



92. $\dfrac{21}{x} \ge 7$

Solution

$$\frac{21}{x} \ge 7$$

$$\frac{21}{x} - 7 \ge 0$$

$$\frac{21 - 7x}{x} \ge 0$$

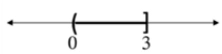
factors = 0: $x = 0, x = 3$

intervals: $(-\infty, 0), (0, 3), (3, \infty)$

interval	test number	sign of $\frac{21 - 7x}{x}$
$(-\infty, 0)$	–1	–
$(0, 3)$	1	+
$(3, \infty)$	4	–

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $(0, 3]$



93. $\frac{4}{x - 4} > 2$

Solution

$$\frac{4}{x - 4} > 2$$

$$\frac{4}{x - 4} - 2 > 0$$

$$\frac{4 - 2x + 8}{x - 4} > 0$$

$$\frac{12 - 2x}{x - 4} > 0$$

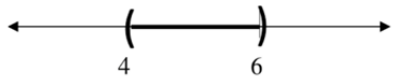
factors = 0: $x = 4, x = 6$

intervals: $(-\infty, 4), (4, 6), (6, \infty)$

interval	test number	sign of $\frac{12 - 2x}{x - 4}$
$(-\infty, 4)$	0	–
$(4, 6)$	5	+
$(6, \infty)$	7	–

Do not include endpoints which make the numerator or denominator equal to 0.

Solution: $(4,\ 6)$



94. $\dfrac{-15}{x+2} > 5$

Solution

$$\frac{-15}{x+2} > 5$$
$$\frac{-15}{x+2} - 5 > 0$$
$$\frac{-15-5x-10}{x+2} > 0$$
$$\frac{-25-5x}{x+2} > 0$$

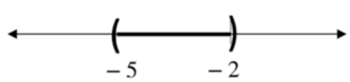
factors $= 0$: $x = -2,\ x = -5$

intervals: $(-\infty,\ -5),\ (-5,\ -2),\ (-2,\ \infty)$

Interval	Test number	Sign of $\dfrac{-25-5x}{x+2}$
$(-\infty,\ -5)$	−6	−
$(-5,\ -2)$	−3	+
$(-2,\ \infty)$	0	−

Do not include endpoints which make the numerator or denominator equal to 0.

Solution: $(-5,\ -2)$



95. $\dfrac{3}{x-2} \le 5$

Solution

$$\frac{3}{x-2} \le 5$$
$$\frac{3}{x-2} - 5 \le 0$$
$$\frac{3}{x-2} - \frac{5(x-2)}{x-2} \le 0$$

$$\frac{3 - 5x + 10}{x - 2} \le 0$$
$$\frac{13 - 5x}{x - 2} \le 0$$

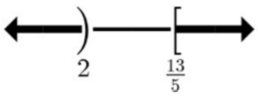
factors $= 0$: $x = \frac{13}{5}, x = 2$

intervals: $(-\infty, 2), \left(2, \frac{13}{5}\right), \left(\frac{13}{5}, \infty\right)$

interval	test number	sign of $\frac{13-5x}{x-2}$
$(-\infty, 2)$	0	–
$\left(2, \frac{13}{5}\right)$	$\frac{11}{5}$	+
$\left(\frac{13}{5}, \infty\right)$	3	–

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $(-\infty, 2) \cup \left[\frac{13}{5}, \infty\right)$



96. $\frac{3}{x + 2} \le 4$

Solution

$$\frac{3}{x + 2} \le 4$$
$$\frac{3}{x + 2} - 4 \le 0$$
$$\frac{3}{x + 2} - \frac{4(x + 2)}{x + 2} \le 0$$
$$\frac{3 - 4x - 8}{x + 2} \le 0$$
$$\frac{-4x - 5}{x + 2} \le 0$$

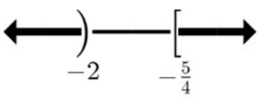
factors $= 0$: $x = -2, x = -\frac{5}{4}$

intervals: $(-\infty, -2), \left(-2, -\frac{5}{4}\right), \left(-\frac{5}{4}, \infty\right)$

interval	test number	sign of $\frac{-4x-5}{x+2}$
$(-\infty, -2)$	–3	–
$\left(-2, -\frac{5}{4}\right)$	$-\frac{7}{4}$	+
$\left(-\frac{5}{4}, \infty\right)$	0	–

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $\left(-\infty,\ -2\right) \cup \left[-\frac{5}{4},\ \infty\right)$



97. $\dfrac{2x}{x - 1} \leq 3$

Solution

$$\frac{2x}{x - 1} \leq 3$$
$$\frac{2x}{x - 1} - 3 \leq 0$$
$$\frac{2x - 3x + 3}{x - 1} \leq 0$$
$$\frac{-x + 3}{x - 1} \leq 0$$

factors $= 0$: $x = 1, x = 3$

intervals: $\left(-\infty,\ 1\right), \left(1,\ 3\right), \left(3,\ \infty\right)$

interval	test number	sign of $\frac{-x + 3}{x - 1}$
$\left(-\infty,\ 1\right)$	0	−
$\left(1,\ 3\right)$	2	+
$\left(3,\ \infty\right)$	4	−

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $\left(-\infty,\ 1\right) \cup \left[3\ \cdot\ \infty\right)$

98. $\dfrac{-2x + 3}{x + 3} \geq 4$

Solution

$$\frac{-2x + 3}{x + 3} \geq 4$$
$$\frac{-2x + 3}{x + 3} - 4 \geq 0$$
$$\frac{-2x + 3 - 4x - 12}{x + 3} \geq 0$$

$$\frac{-6x - 9}{x + 3} \geq 0$$

factors = 0: $x = -3, x = -\frac{3}{2}$

intervals: $(-\infty, -3)(-\infty, -3), \left(-3, -\frac{3}{2}\right), \left(-\frac{3}{2}, \infty\right)$

interval	test number	sign of $\frac{-6x - 9}{x + 3}$
$(-\infty, -3)$	−4	−
$\left(-3, -\frac{3}{2}\right)$	−2	+
$\left(-\frac{3}{2}, \infty\right)$	0	−

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $\left(-3, -\frac{3}{2}\right]$

99. $\frac{6}{x^2 - 1} < 1$

Solution

$$\frac{6}{x^2 - 1} < 1$$

$$\frac{6}{x^2 - 1} - 1 < 0$$

$$\frac{6}{x^2 - 1} - \frac{x^2 - 1}{x^2 - 1} < 0$$

$$\frac{7 - x^2}{x^2 - 1} < 0$$

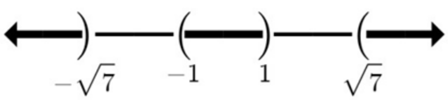
$$\frac{7 - x^2}{(x + 1)(x - 1)} < 0$$

factors = 0: $x = \pm\sqrt{7}, x = \pm 1$

intervals: $(-\infty, -\sqrt{7}), (-\sqrt{7}, -1), (-1, 1), (1, \sqrt{7}), (\sqrt{7}, \infty)$

interval	test number	sign of $\frac{7 - x^2}{x^2 - 1}$
$\left(-\infty, -\sqrt{7}\right)$	−3	−
$\left(-\sqrt{7}, -1\right)$	−2	+
$\left(-1, 1\right)$	0	−
$\left(1, \sqrt{7}\right)$	2	+
$\left(\sqrt{7}, \infty\right)$	3	−

Solution: $\left(-\infty, -\sqrt{7}\right) \cup \left(-1, 1\right) \cup \left(\sqrt{7}, \infty\right)$



100. $\dfrac{6}{x^2 - 1} > 1$

Solution

$$\frac{6}{x^2 - 1} > 1$$

$$\frac{6}{x^2 - 1} - 1 > 0$$

$$\frac{6}{x^2 - 1} - \frac{x^2 - 1}{x^2 - 1} > 0$$

$$\frac{7 - x^2}{x^2 - 1} > 0$$

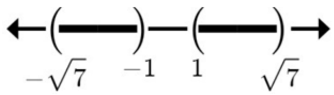
$$\frac{7 - x^2}{(x + 1)(x - 1)} > 0$$

factors $= 0$: $x = \pm\sqrt{7}$, $x = \pm 1$

intervals: $\left(-\infty, -\sqrt{7}\right)$, $\left(-\sqrt{7}, -1\right)$, $\left(-1, 1\right)$, $\left(1, \sqrt{7}\right)$, $\left(\sqrt{7}, \infty\right)$

interval	test number	sign of $\frac{7 - x^2}{x^2 - 1}$
$\left(-\infty, -\sqrt{7}\right)$	−3	−
$\left(-\sqrt{7}, -1\right)$	−2	+
$\left(-1, 1\right)$	0	−
$\left(1, \sqrt{7}\right)$	2	+
$\left(\sqrt{7}, \infty\right)$	3	−

Solution: $\left(-\sqrt{7},\ -1\right) \cup \left(1,\ \sqrt{7}\right)$



Fix It

In exercises 101 and 102, identify the step the first error is made and fix it.

101. Solve the linear inequality: $-3x + 4(x - 1) < 5(x + 1)$. Write the solution set using interval notation.

Solution
Step 4 was not correct.

Step 1: $-3x + 4x - 4 < 5x + 5$

Step 2: $x - 4 < 5x + 5$

Step 3: $-4x < 9$

Step 4: $x > -\dfrac{9}{4}$

Step 5: $\left(-\dfrac{9}{4}, \infty\right)$

102. Solve the rational inequality: $\dfrac{6}{x} \geq 3$. Write the solution set using interval notation.

Solution
Step 5 was not correct.

Step 1: $\dfrac{6}{x} - 3 \geq 0$

Step 2: $\dfrac{6 - 3x}{x} \geq 0$

Step 3: We establish three intervals: $(-\infty,\ 0)$, $(0,\ 2)$, $(2, \infty)$

Step 4: The numbers in the interval $(0,\ 2)$ satisfy the inequality.

Step 5: The solution set is $(0,\ 2]$

Applications

Solve each problem.

103. **Golfing lessons** Macy decides to take golfing lessons. If her new set of golf clubs cost \$250 and private lessons are \$60 per hour lesson, what is the maximum number of lessons she can take if the total spent for lessons and purchasing clubs is at most \$970?

Solution
Let x = the number of lessons.

$$\begin{aligned} \boxed{\text{Total cost}} &\le 970 \\ 250 + 60x &\le 970 \\ 60x &\le 720 \\ x &\le 12 \end{aligned}$$

She can take at most 12 lessons.

104. **Surfing lessons** Dylan and Dusty plan to take weekly surfing lessons together. If the 2-hour lessons are $40 per person and they plan to spend $200 each on new surfboards, what is the maximum number of lessons the two can take if the total amount spent for lessons and surfboards is at most $960?

Solution
Let x = the number of lessons.

$$\begin{aligned} \boxed{\text{Total cost}} &\le 960 \\ 400 + 80x &\le 960 \\ 80x &\le 560 \\ x &\le 7 \end{aligned}$$

She can take at most 7 lessons.

105. **Long distance** A long-distance telephone call costs 40¢ for the first three minutes and 10¢ for each additional minute. At most how many minutes can a person talk and not exceed $2?

Solution
Let x = the number of minutes after 3 minutes. The total cost = 40 + 10x cents.

$$\begin{aligned} \boxed{\text{Total cost}} &< 200 \\ 40 + 10x &< 200 \\ 10x &< 160 \\ x &< 16 \end{aligned}$$

A person can talk for up to 16 minutes after the initial 3 minutes, for a total of up to 19 minutes for less than $2.

106. **Buying a computer** A student who can afford to spend up to $2000 sees the ad shown in the illustration. If she buys a touch-screen laptop, how many games can she buy?

Solution

Let x = the number of games. Then the total cost $= 1695.95 + 19.95x$.

$$\begin{aligned} \boxed{\text{Total cost}} &< 2000 \\ 1695.95 + 19.95x &< 2000 \\ 19.95x &< 304.05 \\ x &< 15.2 \end{aligned}$$

She can buy up to 15 games.

107. **Musical items** Andy can spend up to \$275 on a guitar and some music books. If he can buy a guitar for \$150 and music books for \$9.75, what is the greatest number of music books that he can buy?

Solution

Let x = the number of books. Then the total cost = $150 + 9.75x$.

$$\begin{aligned} \boxed{\text{Total cost}} &< 275 \\ 150 + 9.75x &< 275 \\ 9.75x &< 125 \\ x &< 12.8 \end{aligned}$$

He can buy up to 12 books.

108. **Buying a digital camera** Audrey wants to spend less than \$600 for a digital camera and some batteries. If the camera of her choice costs \$425 and batteries cost \$7.50 each, how many batteries can she buy?

Solution

Let x = the number of DVDs. Then the total cost = $425 + 7.50x$.

$$\begin{aligned} \boxed{\text{Total cost}} &< 600 \\ 425 + 7.50x &< 600 \\ 7.50x &< 175 \\ x &< 23.3 \end{aligned}$$

She can buy up to 23 DVDs.

109. **Buying a refrigerator** Madeline, who has \$1200 to spend, wants to buy a refrigerator. Refer to the following table and write an inequality that shows how much she can pay p for the refrigerator.

State sales tax	6.5%
City sales tax	0.25%

Solution

Let p = the price of the refrigerator. Then the total cost = $p + 0.065p + 0.0025p$.

$$\begin{aligned} \boxed{\text{Total cost}} &< 1200 \\ p + 0.065p + 0.0025p &< 1200 \\ 1.0675p &< 1200 \\ p &< 1124.122 \\ p &\le \$1124.12 \end{aligned}$$

110. **Renting a rototiller** The cost of renting a rototiller is \$17.50 for the first hour and \$8.95 for each additional hour. How long, to the nearest hour, can a person have the rototiller if the cost must be less than \$75?

Solution

Let x = the number of hours after the first. Then the total cost = 17.50 + 8.95x.

$$\begin{aligned} \boxed{\text{Total cost}} &< 75 \\ 17.50 + 8.95x &< 75 \\ 8.95x &< 57.50 \\ x &< 6.4 \end{aligned}$$

A person could have the rototiller for up to 6 hours after the first hour, for a total of up to 7 hours.

111. **Profit** Profit occurs when revenue exceeds cost. If the revenue R in dollars from producing and selling x Hugo Boss polo shirts is $R = 26x$ dollars and the cost C is $C = 6x + 3660$ dollars, what production level produces a profit?

Solution

$$\begin{aligned} R &> C \\ 26x &> 6x + 3660 \\ 26x &> 3660 \\ x &> 183 \end{aligned}$$

112. **Profit** The revenue R in dollars of producing and selling x Yankee candles is $R = 19x$ and the cost C is $C = 3x + 2800$. At what production level will revenue exceed cost and the company obtain a profit?

Solution

$$\begin{aligned} R &> C \\ 19x &> 3x + 2800 \\ 16x &> 2800 \\ x &> 175 \end{aligned}$$

113. **Real estate taxes** A city council has proposed the following two methods of taxing real estate:

Method 1	\$2200 + 4% of assessed value
Method 2	\$1200 + 6% of assessed value

For what range of assessments a would the first method benefit the taxpayer?

Solution

Let a = the assessed value. Find when Method 1 < Method 2:

$$\begin{aligned} 2000 + 0.04a &< 1200 + 0.06a \\ 1000 &< 0.02a \\ 50000 &< a \end{aligned}$$

The first method will benefit the taxpayer when $a > \$50,000$.

114. **Medical plans** A college provides its employees with a choice of the two medical plans shown in the following table. For what size hospital bills is Plan 2 better for the employee than Plan 1? (*Hint:* The cost to the employee includes both the deductible payment and the employee's coinsurance payment.)

Plan 1	Plan 2
Employee pays \$100	Employee pays \$200
Plan pays 70% of the rest	Plan pays 80% of the rest

Solution

Let b = the hospital bill. Find when Cost of Plan 1 > Cost of Plan 2:

$$\begin{aligned} 100 + 0.30(b - 100) &> 200 + 0.20(b - 200) \\ 100 + 0.30b - 30 &> 200 + 0.20b - 40 \\ 0.1b &> 90 \\ b &> 900 \end{aligned}$$

Plan 2 is better for bills over \$900.

115. **Medical plans** To save costs, the college in Exercise 96 raised the employee deductible, as shown in the following table. For what size hospital bills is Plan 2 better for the employee than Plan 1? (*Hint:* The cost to the employee includes both the deductible payment and the employee's coinsurance payment.)

Plan 1	Plan 2
Employee pays \$200	Employee pays \$400
Plan pays 70% of the rest	Plan pays 80% of the rest

Solution

Let b = the hospital bill. Find when Cost of Plan 1 > Cost of Plan 2:

$$\begin{aligned} 200 + 0.30(b - 200) &> 400 + 0.20(b - 400) \\ 200 + 0.03b - 60 &> 400 + 0.02b - 80 \\ 0.1b &> 180 \\ b &> 1800 \end{aligned}$$

Plan 2 is better for bills over \$1,800.

116. **Geometry** The perimeter of a rectangle is to be between 180 inches and 200 inches. Find the range of values for its length l when its width is 40 inches.

Solution

Let P = the perimeter. Then the length is equal to $\frac{P - 2w}{2}$, or $\frac{P - 80}{2}$.

$$\begin{array}{ccccc} 180 & < & P & < & 200 \\ 180 - 80 & < & P - 80 & < & 200 - 80 \\ 100 & < & P - 80 & < & 120 \\ \frac{100}{2} & < & \frac{P-80}{2} & < & \frac{120}{2} \\ 50 & < & \text{length} & < & 60 \end{array}$$

The length is between 50 and 60 inches.

117. **Geometry** The perimeter of an equilateral triangle is to be between 50 centimeters and 60 centimeters. Find the range of lengths of one side *s*.

Solution

Let P = the perimeter. Then the length of one side is equal to $\frac{P}{3}$.

$$\begin{array}{ccccc} 50 & < & P & < & 60 \\ \frac{50}{3} & < & \frac{P}{3} & < & \frac{60}{3} \\ 16\frac{2}{3} & < & \text{lenght} & < & 20 \end{array}$$

The length of a side is between $16\frac{2}{3}$ and 20 cm.

118. **Geometry** The perimeter of a square is to be from 25 meters to 60 meters. Find the range of values for its area *A*.

Solution

Let P = the perimeter. Then the length of one side is equal to $\frac{P}{4}$, so the area is equal to

$$A = s^2 = \left(\frac{P}{4}\right)^2$$

$$\begin{array}{ccccc} 25 & < & P & < & 60 \\ \frac{25}{4} & < & \frac{P}{4} & < & \frac{60}{4} \\ \left(\frac{25}{4}\right)^2 & < & \left(\frac{P}{4}\right)^2 & < & 15^2 \\ \frac{625}{16} & < & \text{Area} & < & 225 \end{array}$$

The area is between $\frac{625}{16}$ m^2 and 225 m^2.

119. **Projectile height** If a Nerf sports bash ball is projected from ground level with an initial velocity of 160 feet per second, its height *s* in feet *t* seconds after being projected is given by the equation $s = -16t^2 + 160t$. When will the height of the bash ball exceed 144 feet?

Solution

$$\begin{aligned} -16t^2 + 160t &> 144 \\ -16t^2 + 160t - 144 &> 0 \\ -16\left(t^2 - 10t + 9\right) &> 0 \\ t^2 - 10t + 9 &< 0 \\ (t - 1)(t - 9) &< 0 \end{aligned}$$

factors $= 0$: $t = 1, t = 9$

intervals: $(-\infty, 1), (1, 9), (9, \infty)$

interval	test number	value of $t^2 - 10t + 9$
$(-\infty, 1)$	0	+9
$(1, 9)$	2	−7
$(9, \infty)$	10	+9

Solution: $(1, 9)$. It will exceed 144 ft between 1 and 9 seconds.

120. **Projectile height** If a jumbo hypercharged pop sky ball is projected from ground level with an initial velocity of 192 feet per second, its height s in feet t seconds after being projected is given by the equation $s = -16t^2 + 240t$. When will the height of the pop sky ball exceed 576 feet?

Solution

$$-16t^2 + 240t > 576$$
$$-16t^2 + 240t - 576 > 0$$
$$-16(t^2 - 15t + 36) > 0$$
$$t^2 - 15t + 36 < 0$$
$$(t - 3)(t - 12) < 0$$

factors $= 0$: $t = 3, t = 12$

intervals: $(-\infty, 3), (3, 12), (12, \infty)$

interval	test number	value of $t^2 - 15t + 36$
$(-\infty, 3)$	0	+36
$(3, 12)$	4	−8
$(12, \infty)$	13	+10

Solution: $(3, 12)$. It will exceed 576 ft between 3 and 12 seconds.

Discovery and Writing

121. The techniques used for solving linear equations and linear inequalities are similar, yet different. Explain.

Solution
Answers may vary.

122. When graphing the solution set of an inequality, what does a bracket indicate on the number line? What does a parenthesis indicate on the number line?

Solution
Answers may vary.

123. What is a quadratic inequality? Give two examples.

Solution
Answers may vary.

124. What is a rational inequality? Give two examples.

Solution
Answers may vary.

Critical Thinking

Determine if the statement is true or false. If the statement is false, then correct it and make it true.

125. The solution set of the inequality $x^2 - 100 \leq 0$ is $(-\infty, 10]$.

Solution
False.

$x^2 - 100 \leq 0$

$(x + 10)(x - 10) \leq 0$

factors $= 0$: $x = -10$, $x = 10$

intervals: $(-\infty, -10)$, $(-10, 10)$, $(10, \infty)$

interval	test number	value of $x^2 - 100$
$(-\infty, -10)$	−11	+21
$(-10, 10)$	0	−100
$(10, \infty)$	11	+21

Solution: $[-10, 10]$

126. The solution set of $x^2 > 0$ is all real numbers.

Solution

False. The solution set of $x^2 > 0$ is all real numbers except 0.

127. To solve the rational inequality $\frac{1}{x - 10} < 2$, the first step is multiply both sides by $x - 10$ to clear the inequality of fractions.

Solution
False. The first step is top subtract 2 from both sides of the equation.

128. The solution set of the inequality $\frac{100}{x} \geq 10$ is [0,10].

Solution

False. $\frac{100}{x} \geq 10$

$\frac{100}{x} - 10 \geq 0$

$\frac{100 - 10x}{x} \geq 0$

factors $= 0$: $x = 0, x = 10$

intervals: $(-\infty, 0), (0, 10), (10, \infty)$

interval	test number	sign of $\frac{100-10x}{x}$
$(-\infty, 0)$	-1	$-$
$(0, 10)$	1	$+$
$(10, \infty)$	11	$-$

Solution: $(0, 10]$

Exercises 1.8

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Identify two real numbers whose absolute value is 12.

Solution

$12, -12$

2. Tell whether the statement is true or false. $-|-7| < -|-6|$

Solution

$-|-7| < -|-6|$

$-7 < -6$

True

3. Solve each equation.

a. $\frac{3x - 1}{4} = 8$

b. $\frac{3x - 1}{4} = -8$

Solution

a. $\dfrac{3x - 1}{4} = 8$

$3x - 1 = 32$

$3x = 33$

$x = 11$

b. $\dfrac{3x - 1}{4} = -8$

$3x - 1 = -32$

$3x = -31$

$3x = \dfrac{-31}{3}$

4. Solve each equation.

a. $6x + 2 = 2x - 5$

b. $6x + 2 = -(2x - 5)$

Solution

a. $6x + 2 = 2x - 5$

$4x + 2 = -5$

$4x = -7$

$x = -\dfrac{7}{4}$

b. $6x + 2 = -(2x - 5)$

$6x + 2 = -2x + 5$

$8x + 2 = 5$

$8x = 3$

$x = \dfrac{3}{8}$

5. Solve and write the solution set in interval notation. $-9 \leq 2x - 5 \leq 9$

Solution

$-9 \leq 2x - 5 \leq 9$

$-4 \leq 2x \leq 14$

$-2 \leq x \leq 7$

$[-2, 7]$

6. Solve and write the solution set in interval notation. $2x - 5 \le -9$ or $2x - 5 \ge 9$

Solution

$2x - 5 \le -9$ *or* $2x - 5 \ge 9$

$2x \le -4$ *or* $2x \ge 14$

$x \le -2$ *or* $x \ge 7$

$(-\infty, -2] \cup [7, \infty)$

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. If $x \ge 0$, then $|x| =$ __________.

Solution

x

8. If $x < 0$, then $|x| =$ __________.

Solution

$-x$

9. $|x| = k$ is equivalent to __________.

Solution

$x = k$ or $x = -k$

10. $|a| = |b|$ is equivalent to $a = b$ or __________.

Solution

$a = -b$

11. $|x| < k$ is equivalent to __________.

Solution

$-k < x < k$

12. $|x| > k$ is equivalent to __________.

Solution

$x < -k$ or $x > k$

13. $|x| \ge k$ is equivalent to __________.

Solution

$x \le -k$ or $x \ge k$

14. $\sqrt{a^2}$ = ___________.

Solution

$|a|$

Practice

Write each expression without absolute value symbols.

15. $|6|$

Solution

$|7| = 7$

16. $|-15|$

Solution

$|-9| = 9$

17. $|0|$

Solution

$|0| = 0$

18. $|3 - 5|$

Solution

$|3 - 5| = |-2| = 2$

19. $|5| - |-3|$

Solution

$|5| - |-3| = 5 - 3 = 2$

20. $|-3| + |5|$

Solution

$|-3| + |5| = 3 + 5 = 8$

21. $|\pi - 2|$

Solution

$|\pi - 2| = +(\pi - 2) = \pi - 2$

22. $|\pi - 4|$

Solution

$|\pi - 4| = -(\pi - 4) = 4 - \pi$

23. $|x - 5|$ and $x \geq 5$

Solution

$x \geq 5 \Rightarrow |x - 5| = x - 5$

24. $|x - 5|$ and $x \leq 5$

Solution

$x \leq 5 \Rightarrow |x - 5| = -(x - 5) = 5 - x$

25. $|x^3|$

Solution

$$|x^3| = \begin{cases} x^3 & \text{if } x \geq 0 \\ -x^3 & \text{if } x < 0 \end{cases}$$

26. $|2x|$

Solution

$$|2x| = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$$

Solve each equation with one absolute value.

27. $|x + 2| = 2$

Solution

$$|x + 2| = 2$$

$$x + 2 = 2 \quad \textbf{or} \quad x + 2 = -2$$

$$x = 0 \qquad\qquad x = -4$$

28. $|2x + 5| = 3$

Solution

$$|2x + 5| = 3$$

$$2x + 5 = 3 \quad \textbf{or} \quad 2x + 5 = -3$$

$$2x = -2 \qquad\qquad 2x = -8$$

$$x = -1 \qquad\qquad x = -4$$

29. $|3x - 1| - 7 = -2$

Solution

$$|3x - 1| - 7 = -2$$
$$|3x - 1| = 5$$

$3x - 1 = 5$ **or** $3x - 1 = -5$

$3x = 6 \qquad 3x = -4$

$x = 2 \qquad x = -\frac{4}{3}$

30. $|7x - 5| + 5 = 8$

Solution

$$|7x - 5| + 5 = 8$$
$$|7x - 5| = 3$$

$7x - 5 = 3$ **or** $7x - 5 = -3$

$7x = 8 \qquad 7x = 2$

$x = \frac{8}{7} \qquad x = \frac{2}{7}$

31. $\left|\frac{3x - 4}{2}\right| = 5$

Solution

$$\left|\frac{3x - 4}{2}\right| = 5$$

$\frac{3x - 4}{2} = 5$ **or** $\frac{3x - 4}{2} = -5$

$3x - 4 = 10 \qquad 3x - 4 = -10$

$3x = 14 \qquad 3x = -6$

$x = \frac{14}{3} \qquad x = -2$

32. $\left|\frac{10x + 1}{2}\right| = \frac{9}{2}$

Solution

$$\left|\frac{10x + 1}{2}\right| = \frac{9}{2}$$

$$\frac{10x+1}{2} = \frac{9}{2} \quad \textbf{or} \quad \frac{10x+1}{2} = -\frac{9}{2}$$
$$10x+1 = 9 \qquad 10x+1 = -9$$
$$10x = 8 \qquad 10x = -10$$
$$x = \frac{8}{10} = \frac{4}{5} \qquad x = -1$$

33. $\left|\frac{2x-4}{5}\right| + 7 = 9$

Solution

$$\left|\frac{2x-4}{5}\right| + 6 = 8$$
$$\left|\frac{2x-4}{5}\right| = 2$$
$$\frac{2x-4}{5} = 2 \quad \textbf{or} \quad \frac{2x-4}{5} = -2$$
$$2x-4 = 10 \qquad 2x-4 = -10$$
$$2x = 14 \qquad 2x = -6$$
$$x = 7 \qquad x = -3$$

34. $\left|\frac{3x+11}{7}\right| - 15 = -14$

Solution

$$\left|\frac{3x+11}{7}\right| - 15 = -14$$
$$\left|\frac{3x+11}{7}\right| = 1$$
$$\frac{3x+11}{7} = 1 \quad \textbf{or} \quad \frac{3x+11}{7} = -1$$
$$3x+11 = 7 \qquad 3x+11 = -7$$
$$3x = -4 \qquad 3x = -18$$
$$x = -\frac{4}{3} \qquad x = -6$$

35. $\left|\frac{x-3}{4}\right| = -5$

Solution

$$\left|\frac{x-3}{4}\right| = -2$$

An absolute value can never equal a negative number. no solution

36. $\left|\frac{x+5}{2}\right| + 11 = 10$

Solution

$$\left|\frac{x+5}{2}\right| + 3 = 2$$

$$\left|\frac{x+5}{2}\right| = -1$$

An absolute value can never equal a negative number. no solution

37. $\left|\frac{x-5}{3}\right| = 0$

Solution

$$\left|\frac{x-5}{3}\right| = 0$$

$$\frac{x-5}{3} = 0 \quad \textbf{or} \quad \frac{x-5}{3} = -0$$

$$x - 5 = 0 \qquad x - 5 = 0$$

$$x = 5 \qquad x = 5$$

38. $\left|\frac{x+7}{9}\right| = 0$

Solution

$$\left|\frac{x+7}{9}\right| = 0$$

$$\frac{x+7}{9} = 0 \quad \textbf{or} \quad \frac{x+7}{9} = -0$$

$$x + 7 = 0 \qquad x + 7 = 0$$

$$x = -7 \qquad x = -7$$

39. $\left|\frac{4x-2}{x}\right| = 3$

Solution

$$\left|\frac{4x-2}{x}\right| = 3$$

$$\frac{4x-2}{x} = 3 \quad \textbf{or} \quad \frac{4x-2}{x} = -3$$
$$4x-2 = 3x \qquad 4x-2 = -3x$$
$$x = 2 \qquad 7x = 2$$
$$x = 2 \qquad x = \tfrac{2}{7}$$

40. $\left|\frac{2(x-3)}{3x}\right| = 6$

Solution

$$\left|\frac{2(x-3)}{3x}\right| = 6$$

$$\frac{2x-6}{3x} = 6 \quad \textbf{or} \quad \frac{2x-6}{3x} = -6$$
$$2x-6 = 18x \qquad 2x-6 = -18x$$
$$-16x = 6 \qquad 20x = 6$$
$$x = -\tfrac{3}{8} \qquad x = \tfrac{3}{10}$$

41. $|x| = x$

Solution

$|x| = x$

True for all $x \geq 0$.

42. $|x| + x = 4$

Solution

$$|x| + x = 2$$
$$|x| = -x + 2$$
$$x = -x + 2 \quad \textbf{or} \quad x = (-x + 2)$$
$$2x = 2 \qquad x = x - 2$$
$$x = 1 \qquad 0 = -2 \Rightarrow \text{not true}$$

Solve each equation with two absolute values.

43. $|x + 3| = |x|$

Solution

$$|x + 3| = |x|$$

$$x + 3 = x \quad \textbf{or} \quad x + 3 = -x$$
$$0 = 3 \qquad\qquad 2x = -3$$
$$\text{not true} \qquad\qquad x = -\tfrac{3}{2}$$

44. $|x + 5| = |5 - x|$

Solution

$$|x + 5| = |5 - x|$$
$$x + 5 = 5 - x \quad \textbf{or} \quad x + 5 = -(5 - x)$$
$$2x = 0 \qquad\qquad x + 5 = -5 + x$$
$$x = 0 \qquad\qquad 0 = -10$$
$$\text{not true}$$

45. $|x - 3| = |2x + 3|$

Solution

$$|x - 3| = |2x + 3|$$
$$x - 3 = 2x + 3 \quad \textbf{or} \quad x - 3 = -(2x + 3)$$
$$-x = 6 \qquad\qquad x - 3 = -2x - 3$$
$$x = -6 \qquad\qquad 3x = 0$$
$$x = 0$$

46. $|x - 2| = |3x + 8|$

Solution

$$|x - 2| = |3x + 8|$$
$$x - 2 = 3x + 8 \quad \textbf{or} \quad x - 2 = -(3x + 8)$$
$$-2x = 10 \qquad\qquad x - 2 = -3x - 8$$
$$x = -5 \qquad\qquad 4x = -6$$
$$x = -\tfrac{3}{2}$$

47. $|x + 2| = |x - 2|$

Solution

$$|x + 2| = |x - 2|$$
$$x + 2 = x - 2 \quad \textbf{or} \quad x + 2 = -(x - 2)$$
$$0 = -4 \qquad\qquad x + 2 = -x + 2$$
$$\text{not ture} \qquad\qquad 2x = 0$$
$$x = 0$$

48. $|2x - 3| = |3x - 5|$

Solution

$$|2x - 3| = |3x - 5|$$

$$2x - 3 = 3x - 5 \quad \textbf{or} \quad 2x - 3 = -(3x - 5)$$

$$-x = -2 \qquad 2x - 3 = -3x + 5$$

$$x = 2 \qquad 5x = 8$$

$$x = \tfrac{8}{5}$$

49. $\left|\frac{x + 3}{2}\right| = |2x - 3|$

Solution

$$\left|\frac{x + 3}{2}\right| = |2x - 3|$$

$$\frac{x + 3}{2} = 2x - 3 \quad \textbf{or} \quad \frac{x + 3}{2} = -(2x - 3)$$

$$x + 3 = 4x - 6 \qquad \frac{x + 3}{2} = -2x + 3$$

$$-3x = -9 \qquad x + 3 = -4x + 6$$

$$x = 3 \qquad 5x = 3$$

$$x = \tfrac{3}{5}$$

50. $\left|\frac{x - 2}{3}\right| = |6 - x|$

Solution

$$\left|\frac{x - 2}{3}\right| = |6 - x|$$

$$\frac{x - 2}{2} = 6 - x \quad \textbf{or} \quad \frac{x - 2}{2} = -(6 - x)$$

$$\frac{x - 2}{3} = 6 - x \qquad \frac{x - 2}{3} = -6 + x$$

$$x - 2 = 18 - 3x \qquad x - 2 = -18 + 3x$$

$$4x = 20 \qquad -2x = -16$$

$$x = 5 \qquad x = 8$$

51. $\left|\frac{3x - 1}{2}\right| = \left|\frac{2x + 3}{3}\right|$

Solution

$$\left|\frac{3x-1}{2}\right| = \left|\frac{2x+3}{3}\right|$$

$$\begin{aligned} \frac{3x-1}{2} &= \frac{2x+3}{3} & \textbf{or} \quad \frac{3x-1}{2} &= -\frac{2x+3}{3} \\ 6\left(\frac{3x-1}{2} = \frac{2x+3}{3}\right) && 6\left(\frac{3x-1}{2} = -\frac{2x+3}{3}\right) \\ 3(3x-1) &= 2(2x+3) & 3(3x-1) &= -2(2x+3) \\ 9x-3 &= 4x+6 & 9x-3 &= -4x-6 \\ 5x &= 9 & 13x &= -3 \\ x &= \tfrac{9}{5} & x &= -\tfrac{3}{13} \end{aligned}$$

52. $\left|\frac{5x+2}{3}\right| = \left|\frac{x-1}{4}\right|$

Solution

$$\left|\frac{5x+2}{3}\right| = \left|\frac{x-1}{4}\right|$$

$$\begin{aligned} \frac{5x+2}{3} &= \frac{x-1}{4} & \textbf{or} \quad \frac{5x+2}{3} &= -\frac{x-1}{4} \\ 12\left(\frac{5x+2}{3} = \frac{x-1}{4}\right) && 12\left(\frac{5x+2}{3} = -\frac{x-1}{4}\right) \\ 4(5x+2) &= 3(x-1) & 4(5x+2) &= -3(x-1) \\ 20x+8 &= 3x-3 & 20x+8 &= -3x+3 \\ 17x &= -11 & 23x &= -5 \\ x &= -\tfrac{11}{17} & x &= -\tfrac{5}{23} \end{aligned}$$

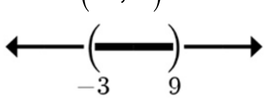
Solve each absolute value inequality. Express the solution set in interval notation, and graph it.

53. $|x-3| < 6$

Solution

$$\begin{aligned} |x-3| &< 6 \\ -6 < x-3 &< 6 \\ -3 < x &< 9 \end{aligned}$$

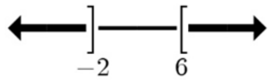
$(-3, 9)$



54. $|x - 2| \geq 4$

Solution

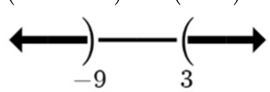
$$|x - 2| \geq 4$$
$$x - 2 \geq 4 \quad \textbf{or} \quad x - 2 \leq -4$$
$$x \geq 6 \qquad\qquad x \leq -2$$
$$(-\infty, \ -2] \cup [6, \infty)$$



55. $|x + 3| > 6$

Solution

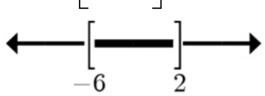
$$|x + 3| > 6$$
$$x + 3 > 6 \quad \textbf{or} \quad x + 3 < -6$$
$$x > 3 \qquad\qquad x < -9$$
$$(-\infty, \ -9) \cup (3, \infty)$$



56. $|x + 2| \leq 4$

Solution

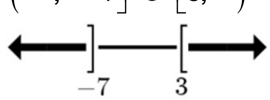
$$|x + 2| \leq 4$$
$$-4 \leq x + 2 \leq 4$$
$$-6 \leq x \leq 2$$
$$[-6, 2]$$



57. $|2x + 4| \geq 10$

Solution

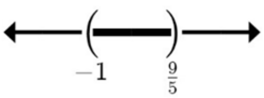
$$|2x + 4| \geq 10$$
$$2x + 4 \geq 10 \quad \textbf{or} \quad 2x + 4 \leq -10$$
$$2x \geq 6 \qquad\qquad 2x \leq -14$$
$$x \geq 3 \qquad\qquad x \leq -7$$
$$(-\infty, \ -7] \cup [3, \infty)$$



58. $|5x - 2| < 7$

Solution

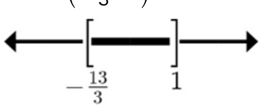
$$|5x - 2| < 7$$
$$-7 < 5x - 2 < 7$$
$$-5 < 5x < 9$$
$$-1 < x < \frac{9}{5}$$
$$\left(-1, \frac{9}{5}\right)$$



59. $|3x + 5| + 1 \leq 9$

Solution

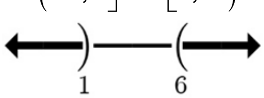
$$|3x + 5| + 1 \leq 9$$
$$|3x + 5| \leq 8$$
$$-8 \leq 3x + 5 \leq 8$$
$$-13 \leq 3x \leq 3$$
$$-\frac{13}{3} \leq x \leq 1$$
$$\left(-\frac{13}{3}, 1\right)$$



60. $|2x - 7| - 3 > 2$

Solution

$$|2x - 7| - 3 > 2$$
$$|2x - 7| > 5$$
$$2x - 7 > 5 \quad \textbf{or} \quad 2x - 7 < -5$$
$$2x > 12 \qquad 2x < 2$$
$$x > 6 \qquad x < 1$$
$$(-\infty, 1] \cup [6, \infty)$$



61. $|x + 3| > 0$

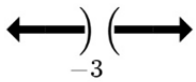
Solution

$$|x + 3| > 0$$

$$x + 3 > 0 \quad \textbf{or} \quad x + 3 < -0$$

$$x > -3 \qquad\qquad x < -3$$

$$(-\infty, \ -3] \cup [-3, \infty)$$



62. $|x - 3| \le 0$

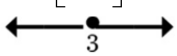
Solution

$$|x - 3| \le 0$$

$$-0 \le x - 3 \le 0$$

$$3 \le x \le 3$$

$$[3, 3]$$



63. $\left|\dfrac{5x + 2}{3}\right| < 1$

Solution

$$\left|\frac{5x+2}{3}\right| < 1$$

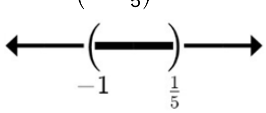
$$-1 < \frac{5x+2}{3} < 1$$

$$-3 < 5x + 2 < 3$$

$$-5 < 5x < 1$$

$$-1 < x < \frac{1}{5}$$

$$\left(-1, \frac{1}{5}\right)$$



64. $\left|\dfrac{3x + 2}{4}\right| > 2$

Solution

$$\left|\frac{3x+2}{4}\right| > 2$$

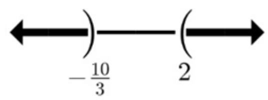
$$\frac{3x+2}{4} > 2 \quad \textbf{or} \quad \frac{3x+2}{4} < -2$$

$$3x + 2 > 8 \qquad 3x + 2 < -8$$

$$3x > 6 \qquad 3x < -10$$

$$x > 2 \qquad x < -\frac{10}{3}$$

$$\left(-\infty, -\frac{10}{3}\right) \cup (2, \infty)$$



65. $3\left|\dfrac{3x - 1}{2}\right| > 5$

Solution

$$3\left|\frac{3x-1}{2}\right| > 5$$

$$\left|\frac{3x-1}{2}\right| > \frac{5}{3}$$

$$\frac{3x-1}{2} > \frac{5}{3} \qquad \frac{3x-1}{2} < -\frac{5}{3}$$

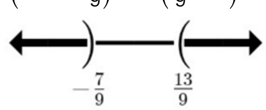
$$6 \cdot \frac{3x-1}{2} > 6 \cdot \frac{5}{3} \qquad 6 \cdot \frac{3x-1}{2} < 6\left(-\frac{5}{3}\right)$$

$$9x - 3 > 10 \quad \textbf{or} \quad 9x - 3 < -10$$

$$9x > 13 \qquad 9x < -7$$

$$x > \frac{13}{9} \qquad x < -\frac{7}{9}$$

$$\left(-\infty. -\frac{7}{9}\right) \cup \left(\frac{13}{9}, \infty\right)$$



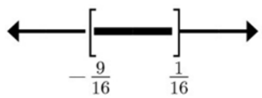
66. $2\left|\dfrac{8x + 2}{5}\right| \leq 1$

Solution

$$2\left|\frac{8x+2}{5}\right| \leq 1$$

$$\left|\frac{8x+2}{5}\right| \leq \frac{1}{2}$$

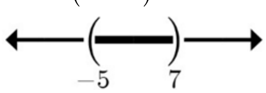
$$-\frac{1}{2} \le \frac{8x+2}{5} \le \frac{1}{2}$$
$$10\left(-\frac{1}{2}\right) \le 10 \cdot \frac{8x+2}{5} \le 10 \cdot \frac{1}{2}$$
$$-5 \le 16x + 4 \le 5$$
$$-9 \le 16x \le 1$$
$$-\frac{9}{16} \le x \le \frac{1}{16}$$
$$\left[-\frac{9}{16}, \frac{1}{16}\right]$$



67. $\dfrac{|x-1|}{-2} > -3$

Solution

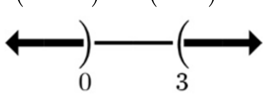
$$\frac{|x-1|}{-2} > -3$$
$$|x-1| < 6$$
$$-6 < x - 1 < 6$$
$$-5 < x < 7$$
$$(-5, 7)$$



68. $\dfrac{|2x-3|}{-3} < -1$

Solution

$$\frac{|2x-3|}{-3} < -1$$
$$|2x-3| > 3$$
$$2x - 3 > 3 \quad \textbf{or} \quad 2x - 3 < -3$$
$$2x > 6 \qquad 2x < 0$$
$$x > 3 \qquad x < 0$$
$$(-\infty, 0) \cup (3, \infty)$$



Solve each compound inequality with absolute value. Express the solution set in interval notation, and graph it.

69. $0 < |2x + 1| < 3$

Solution

$$0 < |2x + 1| < 3$$

$0 < |2x + 1|$ **and** $|2x + 1| < 3$

(1) $|2x + 1| > 0$

$2x + 1 > 0$ **or** $2x + 1 < -0$

$2x > -1$ $\quad$ $2x < -1$

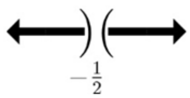
$x > -\frac{1}{2}$ $\quad$ $x < -\frac{1}{2}$

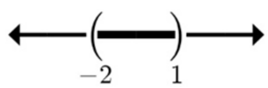
(2) $|2x + 1| < 3$

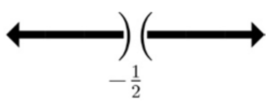
$-3 < 2x + 1 < 3$

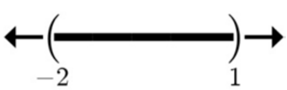
$-4 < 2x < 2$

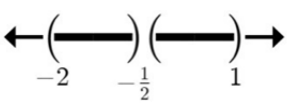
$-2 < x < 1$

(1) 

(2) 

(1) 

(2) 

(1) and (2)  $\Rightarrow \left(-2, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 1\right)$

70. $0 < |2x - 3| < 1$

Solution

$$0 < |2x - 3| < 1$$

$0 < |2x - 3|$ **and** $|2x - 3| < 1$

(1) $|2x - 3| > 0$

$2x - 3 > 0$ **or** $2x - 3 < -0$

$2x > 3$ $\quad$ $2x < 3$

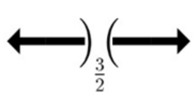
$x > \frac{3}{2}$ $\quad$ $x < \frac{3}{2}$

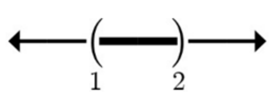
(2) $|2x - 3| < 1$

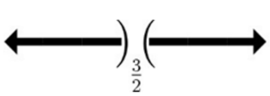
$-1 < 2x - 3 < 1$

$2 < 2x < 4$

$1 < x < 2$

(1) 

(2) 

(1) 

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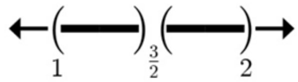
(1) and (2) 

$\Rightarrow \left(1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 2\right)$

71. $8 > |3x - 1| > 3$

Solution

$$8 > |3x - 1| > 3$$

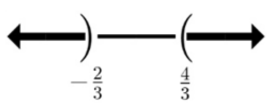
$|3x - 1| > 3$ **and** $8 > |3x - 1|$

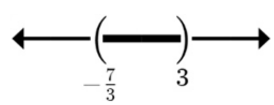
(**1**) $|3x - 1| > 3$ (**2**) $|3x - 1| < 8$

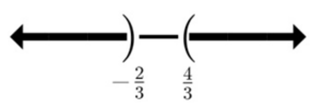
$3x - 1 > 3$ **or** $3x - 1 < -3$ $\quad\quad$ $-8 < 3x - 1 < 8$

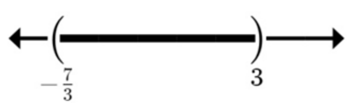
$3x > 4$ $\quad$ $3x < -2$ $\quad\quad$ $-7 < 3x < 9$

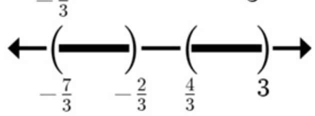
$x > \frac{4}{3}$ $\quad$ $x < -\frac{2}{3}$ $\quad\quad$ $-\frac{7}{3} < x < 3$

(1) 

(2) 

(1) 

(2) 

(1) and (2) 

$\Rightarrow \left(-\frac{7}{3}, -\frac{2}{3}\right) \cup \left(\frac{4}{3}, 3\right)$

72. $8 > |4x - 1| > 5$

Solution

$$8 > |4x - 1| > 5$$

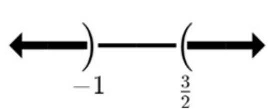
$|4x - 1| > 5$ **and** $8 > |4x - 1|$

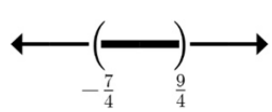
(**1**) $|4x - 1| > 5$ (**2**) $|4x - 1| < 8$

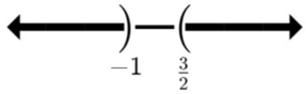
$4x - 1 > 5$ **or** $4x - 1 < -5$ $\quad\quad$ $-8 < 4x - 1 < 8$

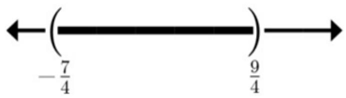
$4x > 6$ $\quad$ $4x < -4$ $\quad\quad$ $-7 < 4x < 9$

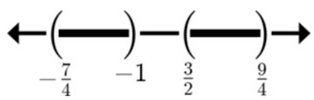
$x > \frac{3}{2}$ $\quad$ $x < -1$ $\quad\quad$ $-\frac{7}{4} < x < \frac{9}{4}$

(1) 

(2) 

(1) 

(2) 

(1) and (2) 

$\Rightarrow \left(-\frac{7}{4}, -1\right) \cup \left(\frac{3}{2}, \frac{9}{4}\right)$

73. $2 < \left|\dfrac{x-5}{3}\right| < 4$

Solution

$$2 < \left|\frac{x-5}{3}\right| < 4$$

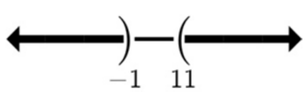
$$2 < \left|\frac{x-5}{3}\right| \quad \textbf{and} \quad \left|\frac{x-5}{3}\right| < 4$$

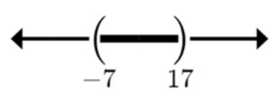
$$(1)\ \left|\frac{x-5}{3}\right| > 2 \qquad\qquad (2)\ \left|\frac{x-5}{3}\right| < 4$$

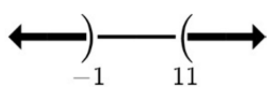
$$\frac{x-5}{3} > 2 \quad \textbf{or} \quad \frac{x-5}{3} < -2 \qquad\qquad -4 < \frac{x-5}{3} < 4$$

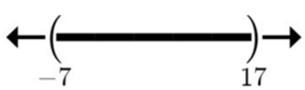
$$x - 5 > 6 \qquad x - 5 < -6 \qquad\qquad -12 < x - 5 < 12$$

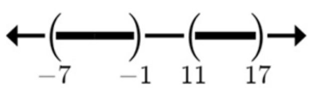
$$x > 11 \qquad x < -1 \qquad\qquad -7 < x < 17$$

(1) 

(2) 

(1) 

(2) 

(1) and (2) 

$\Rightarrow (-7, -1) \cup (11, 17)$

74. $3 < \left|\dfrac{x-3}{2}\right| < 5$

Solution

$$3 < \left|\frac{x-3}{2}\right| < 5$$

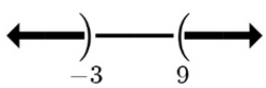
$$3 < \left|\frac{x-3}{2}\right| \quad \textbf{and} \quad \left|\frac{x-3}{2}\right| < 5$$

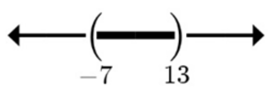
$$(1)\ \left|\frac{x-3}{2}\right| > 3 \qquad\qquad (2)\ \left|\frac{x-3}{2}\right| < 5$$

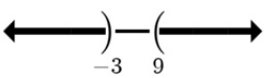
$$\frac{x-3}{2} > 3 \quad \textbf{or} \quad \frac{x-3}{2} < -3 \qquad\qquad -5 < \frac{x-3}{2} < 5$$

$$x - 3 > 6 \qquad x - 3 < -6 \qquad\qquad -10 < x - 3 < 10$$

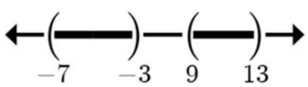
$$x > 9 \qquad x < -3 \qquad\qquad -7 < x < 13$$

(1) 

(2) 

(1) 

(2) 

(1) and (2) 

$\Rightarrow (-7, -3) \cup (9, 13)$

75. $10 > \left|\frac{x-2}{2}\right| > 4$

Solution

$$10 > \left|\frac{x-2}{2}\right| > 4$$

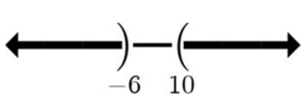
$$\left|\frac{x-2}{2}\right| > 4 \quad \textbf{and} \quad 10 > \left|\frac{x-2}{2}\right|$$

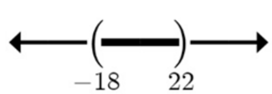
$$(1)\ \left|\frac{x-2}{2}\right| > 4 \qquad\qquad (2)\ \left|\frac{x-2}{2}\right| < 10$$

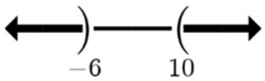
$$\frac{x-2}{2} > 4 \quad \textbf{or} \quad \frac{x-2}{2} < -4 \qquad\qquad -10 < \frac{x-2}{2} < 10$$

$$x - 2 > 8 \qquad x - 2 > -8 \qquad\qquad -20 < x - 2 < 20$$

$$x > 10 \qquad x < -6 \qquad\qquad -18 < x < 22$$

(1) 

(2) 

(1) 

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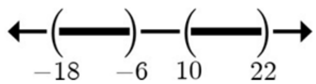
(1) and (2) 

$\Rightarrow (-18, -6) \cup (10, 22)$

76. $5 \geq \left|\dfrac{x+2}{3}\right| > 1$

Solution

$$5 \geq \left|\frac{x+2}{3}\right| > 1$$

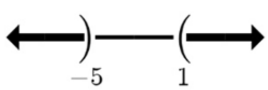
$$\left|\frac{x+2}{3}\right| > 1 \quad \textbf{and} \quad 5 \geq \left|\frac{x+2}{3}\right|$$

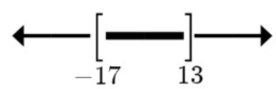
$$(\mathbf{1})\ \left|\frac{x+2}{3}\right| > 1 \qquad\qquad (\mathbf{2})\ \left|\frac{x+2}{3}\right| \leq 5$$

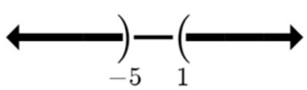
$$\frac{x+2}{3} > 1 \quad \textbf{or} \quad \frac{x+2}{3} < -1 \qquad\qquad -5 \leq \frac{x+2}{3} \leq 5$$

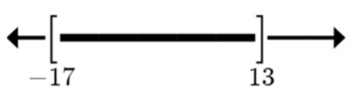
$$x+2 > 3 \qquad x+2 < -3 \qquad\qquad -15 \leq x+2 \leq 15$$

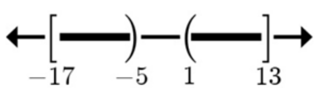
$$x > 1 \qquad x < -5 \qquad\qquad -17 \leq x \leq 13$$

(1) 

(2) 

(1) 

(2) 

(1) and (2) 

$\Rightarrow (-17, -5) \cup (1, 13)$

77. $2 \leq \left|\dfrac{x+1}{3}\right| < 3$

Solution

$$2 \le \left|\frac{x+1}{3}\right| < 3$$

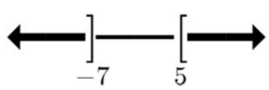
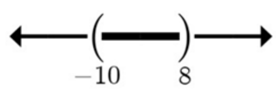
$$2 \le \left|\frac{x+1}{3}\right| \quad \textbf{and} \quad \left|\frac{x+1}{3}\right| < 3$$

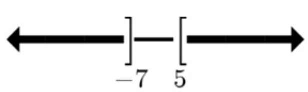
$$(1)\ \left|\frac{x+1}{3}\right| \ge 2 \qquad\qquad (2)\ \left|\frac{x+1}{3}\right| < 3$$

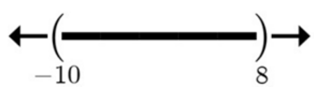
$$\frac{x+1}{3} \ge 2 \quad \textbf{or} \quad \frac{x+1}{3} \le -2 \qquad\qquad -3 < \frac{x+1}{3} < 3$$

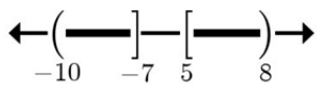
$$x+1 \ge 6 \qquad x+1 \le -6 \qquad\qquad -9 < x+1 < 9$$

$$x \ge 5 \qquad x \le -7 \qquad\qquad -10 < x < 8$$

(1)  **(2)** 

(1) 

(2) 

(1) and (2)  $\Rightarrow (-10, -7) \cup (5, 8)$

78. $8 > \left|\frac{3x+1}{2}\right| > 2$

Solution

$$8 > \left|\frac{3x+1}{2}\right| > 2$$

$$\left|\frac{3x+1}{2}\right| > 2 \quad \textbf{and} \quad 8 > \left|\frac{3x+1}{2}\right|$$

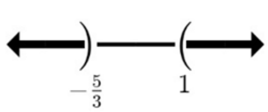
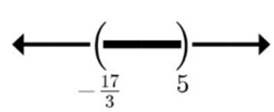
$$(1)\ \left|\frac{3x+1}{2}\right| > 2 \qquad\qquad (2)\ \left|\frac{3x+1}{2}\right| < 8$$

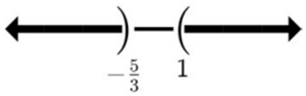
$$\frac{3x+1}{2} > 2 \quad \textbf{or} \quad \frac{3x+1}{2} < -2 \qquad\qquad -8 < \frac{3x+1}{2} < 8$$

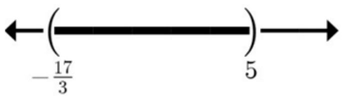
$$3x+1 > 4 \qquad 3x+1 < -4 \qquad\qquad -16 < 3x+1 < 16$$

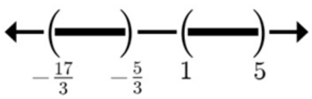
$$3x > 3 \qquad 3x < -5 \qquad\qquad -17 < 3x < 15$$

$$x > 1 \qquad x \le -\tfrac{5}{3} \qquad\qquad -\tfrac{17}{3} < x < 5$$

(1)  **(2)** 

(1) 

(2) 

(1) and (2) 

$\Rightarrow \left(-\frac{17}{3}, -\frac{5}{3}\right) \cup (1, 5)$

Solve each inequality and express the solution using interval notation.

79. $|x + 1| \geq |x|$

Solution

$$\begin{aligned} |x + 1| &\geq |x| \\ \sqrt{(x + 1)^2} &\geq \sqrt{x^2} \\ (x + 1)^2 &\geq x^2 \\ x^2 + 2x + 1 &\geq x^2 \\ 2x &\geq -1 \\ x &\geq -\tfrac{1}{2} \end{aligned}$$

Solution: $\left[-\frac{1}{2}, \infty\right)$

80. $|x + 1| < |x + 2|$

Solution

$$\begin{aligned} |x + 1| &< |x + 2| \\ \sqrt{(x + 1)^2} &< \sqrt{(x + 2)^2} \\ (x + 1)^2 &< (x + 2)^2 \\ x^2 + 2x + 1 &< x^2 + 4x + 4 \\ -2x &< 3 \\ x &> -\tfrac{3}{2} \end{aligned}$$

Solution: $\left[-\frac{3}{2}, \infty\right)$

81. $|2x + 1| < |2x - 1|$

Solution

$$
\begin{aligned}
|2x + 1| &< |2x - 1| \\
\sqrt{(2x + 1)^2} &< \sqrt{(2x - 1)^2} \\
(2x + 1)^2 &< (2x - 1)^2 \\
4x^2 + 4x + 1 &< 4x^2 - 4x + 1 \\
8x &< 0 \\
x &< 0
\end{aligned}
$$

Solution: $(-\infty, 0)$

82. $|3x - 2| \geq |3x + 1|$

Solution

$$
\begin{aligned}
|3x - 2| &\geq |3x + 1| \\
\sqrt{(3x - 2)^2} &\geq \sqrt{(3x + 1)^2} \\
(3x - 2)^2 &\geq (3x + 1)^2 \\
9x^2 - 12x + 4 &\geq 9x^2 + 6x + 1 \\
-18x &\geq -3 \\
x &\leq \tfrac{1}{6}
\end{aligned}
$$

Solution: $\left(-\infty, \tfrac{1}{6}\right]$

83. $|x + 1| < |x|$

Solution

$$
\begin{aligned}
|x + 1| &< |x| \\
\sqrt{(x + 1)^2} &< \sqrt{x^2} \\
(x + 1)^2 &< x^2 \\
x^2 + 2x + 1 &< x^2 \\
2x &< -1 \\
x &< -\tfrac{1}{2}
\end{aligned}
$$

Solution: $\left(-\infty, -\tfrac{1}{2}\right)$

84. $|x + 2| \le |x + 1|$

Solution

$$\begin{aligned} |x + 2| &\le |x + 1| \\ \sqrt{(x + 2)^2} &\le \sqrt{(x + 1)^2} \\ (x + 2)^2 &\le (x + 1)^2 \\ x^2 + 4x + 4 &\le x^2 + 2x + 1 \\ 2x &\le -3 \\ x &\le -\tfrac{3}{2} \end{aligned}$$

Solution: $\left(-\infty, \ -\tfrac{3}{2}\right)$

85. $|2x + 1| \ge |2x - 1|$

Solution

$$\begin{aligned} |2x + 1| &\ge |2x - 1| \\ \sqrt{(2x + 1)^2} &\ge \sqrt{(2x - 1)^2} \\ (2x + 1)^2 &\ge (2x - 1)^2 \\ 4x^2 + 4x + 1 &\ge 4x^2 - 4x + 1 \\ 8x &\ge 0 \\ x &\ge 0 \end{aligned}$$

Solution: $[0, \infty)$

86. $|3x - 2| < |3x + 1|$

Solution

$$\begin{aligned} |3x - 2| &< |3x + 1| \\ \sqrt{(3x - 2)^2} &< \sqrt{(3x + 1)^2} \\ (3x - 2)^2 &< (3x + 1)^2 \\ 9x^2 - 12x + 4 &< 9x^2 + 6x + 1 \\ -18x &< -3 \\ x &> \tfrac{1}{6} \end{aligned}$$

Solution: $\left(\tfrac{1}{6}, \infty\right)$

Fix It

In exercises 87 and 88, identify the step the first error is made and fix it.

87. Solve the absolute value equation: $\dfrac{|4x - 5|}{3} + 4 = 8.$

Solution
Step 3 was incorrect.

Step 1: $\dfrac{|4x - 5|}{3} = 4$

Step 2: $|4x - 5| = 12$

Step 3: $4x - 5 = 12$ *or* $4x - 5 = -12$

Step 4: $4x = 17$ *or* $4x = -7$

Step 5: $x = \dfrac{17}{4}$ *or* $x = -\dfrac{7}{4}$

88. Solve the absolute value inequality: $|4x - 5| - 7 < -5$. Write the solution set using interval notation.

Solution
Step 2 was incorrect.

Step 1: $|4x - 5| < 2$

Step 2: $-2 < 4x - 5 < 2$

Step 3: $3 < 4x < 7$

Step 4: $\dfrac{3}{4} < x < \dfrac{7}{4}$

Step 5: $\left(\dfrac{3}{4}, \dfrac{7}{4}\right)$

Applications

89. **Finding temperature ranges** The temperatures on a summer day satisfy the inequality $|t - 78°| \le 8°$, where t is the temperature in degrees Fahrenheit. Express this range without using absolute value symbols.

Solution

$$\left|t - 78^\circ\right| \le 8^\circ$$
$$-8^\circ \le t - 78^\circ \le 8^\circ$$
$$70^\circ \le t \le 86^\circ$$

90. **Finding operating temperatures** A tablet has an operating temperature of $\left|t - 40^\circ\right| < 80^\circ$, where t is the temperature in degrees Fahrenheit. Express this range without using absolute value symbols.

Solution

$$\left|t - 40^\circ\right| \le 80^\circ$$
$$-80^\circ < t - 40^\circ < 80^\circ$$
$$-40^\circ < t < 120^\circ$$

91. **Range of camber angles** The specifications for a certain car state that the camber angle c of its wheels should be $0.6^\circ \pm 0.5^\circ$. Express this range with an inequality containing an absolute value.

Solution

$$0.6^\circ + 0.5^\circ = 1.1^\circ$$
$$0.6^\circ - 0.5^\circ = 1.1^\circ$$
$$0.1^\circ \le c \le 1.1^\circ$$
$$0.6^\circ - 0.5^\circ \le c \le 0.6^\circ + 0.5^\circ$$
$$-0.5^\circ \le c - 0.6^\circ \le 0.5^\circ$$
$$\left|c - 0.6^\circ\right| \le 0.5^\circ$$

92. **Tolerance of a sheet of steel** A sheet of steel is to be 0.25 inch thick, with a tolerance of 0.015 inch. Express this specification with an inequality containing an absolute value.

Solution

$$0.25 + 0.015 = 0.265$$
$$0.25 - 0.015 = 0.235$$
$$0.235 \le x \le 0.265$$
$$0.25 - 0.015 \le x \le 0.25 + 0.015$$
$$-0.015 \le x - 0.25 \le 0.015$$
$$\left|x - 0.25\right| \le 0.015 \text{ in.}$$

93. **Humidity level** A Steinway piano should be placed in an environment where the relative humidity h is between 38% and 72%. Express this range with an inequality containing an absolute value.

Solution

$$\frac{38 + 72}{2} = \frac{110}{2} = 55$$

$$38 = 55 - 17$$

$$72 = 55 + 17$$

$$\begin{aligned} 38 < \quad h \quad &< 72 \\ 55 - 17 < \quad h \quad &< 55 + 17 \\ -17 < h - 55 &< 17 \\ |h - 55| &< 17 \end{aligned}$$

94. **Light bulbs** A light bulb is expected to last h hours, where $|h - 1500| \le 200$. Express this range without using absolute value symbols.

Solution

$$\begin{aligned} |h - 1500| &\le 200 \\ -200 \le h - 1500 &\le 200 \\ 1300 \le \quad h \quad &\le 1700 \end{aligned}$$

95. **Error analysis** In a lab, students measured the percent of copper p in a sample of copper sulfate. The students know that copper sulfate is actually 25.46% copper by mass. They are to compare their results to the actual value and find the amount of *experimental error.*

a. Which measurements shown in the illustration satisfy the absolute value inequality $|p - 25.46| \le 1.00$?

b. What can be said about the amount of error for each of the trials listed in part a?

Lab 4 Section A
Title:
"Percent copper (CU) in copper sulfate ($CuSO_4 \cdot 5H_2O$)"

Results

	% Copper
Trial #1:	22.91%
Trial #2:	26.45%
Trial #3	26.49%
Trial #4:	24.76%

Solution

$$\begin{aligned} |p - 25.46| &\le 1.00 \\ -1.00 \le p - 25.46 &\le 1.00 \\ 24.46 \le \quad p \quad &\le 26.46 \end{aligned}$$

a. 24.76% and 26.45% are within the range.

b. The error is less than 1%.

96. **Error analysis** See Exercise 95.

a. Which measurements satisfy the absolute value inequality $|p - 25.46| > 1.00$?

b. What can be said about the amount of error for each of the trials listed in part a?

Solution

$$|p - 25.46| > 1.00$$

$$p - 25.46 > 1.00 \quad \textbf{or} \quad p - 25.46 < -1.00$$

$$p > 26.46 \qquad\qquad p < 24.46$$

a. 22.91% and 26.49% are within the range.

b. The error is More than 1%.

97. **Physical therapist income** The yearly income range in dollars of a physical therapist can be modeled by the inequality $\left|\frac{x-93{,}500}{2}\right| \le 13{,}250.$ What is the income range? Write the answer using interval notation. This is according to ZipRecruiter.

Solution

$$\left|\frac{x - 93{,}500}{2}\right| \le 13{,}250$$

$$-13{,}250 \le \frac{x - 93{,}500}{2} \le 13{,}250$$

$$-26{,}500 \le x - 93{,}500 \le 26{,}500$$

$$67{,}000 \le x \le 120{,}000$$

$$[67{,}000,\ 120{,}000]$$

98. **Plumber income** The yearly income range in dollars of a plumber can be modeled by the inequality $\left|\frac{x-51{,}500}{4}\right| \le 4{,}375.$ What is the income range? Write the answer using interval notation. This is according to ZipRecruiter.

Solution

$$\left|\frac{x - 51{,}500}{4}\right| \le 4375$$

$$-4375 \le \frac{x - 51{,}500}{4} \le 4375$$

$$-17{,}500 \le x - 51{,}500 \le 17{,}500$$

$$34{,}000 \le x \le 69{,}000$$

$$[34{,}000,\ 69{,}000]$$

Discovery and Writing

99. Explain how to find the absolute value of a number.

Solution

Answers may vary.

100. Explain why the equation $|x| + 9 = 0$ has no solution.

Solution
Answers may vary.

101. If $k > 0$, explain the differences between the solution sets of $|x| < k$ and $|x| > k$.

Solution
Answers may vary.

102. If $k < 0$, explain why the solution set of $|x| < k$ has no solution.

Solution
Answers may vary.

103. If $k < 0$, explain why the solution set of $|x| > k$ is all real numbers.

Solution
Answers may vary.

104. Explain how to solve an inequality with two absolute values.

Solution
Answers may vary.

Critical Thinking

Determine if the statement is true or false. If the statement is false, then correct it and make it true.

105. Absolute value equations always have two solutions.

Solution
False. Absolute value equations can have zero, one, or two solutions.

106. $|-x| = x$

Solution
False. $|-x| = x$ only when $x \geq 0$.

107. The solution set of $|x| \geq 5$ is $[5, \infty)$.

Solution
False. $|x| \geq 5$

$x \geq 5$ **or** $x \leq -5$

$(-\infty, -5] \cup [5, \infty)$

108. $|a + b| \geq |a| + |b|$

Solution

False. $|a + b| \leq |a| + |b|$.

109. $|x| + 555 < 554$ has no solution.

Solution

True. $|x| + 555 < 554$

$|x| < -1$

This inequality is never true.

110. The solution set of $|x| + 555 > 554$ is all real numbers.

Solution

True. $|x| + 555 > 554$

$|x| > -1$

This inequality is always true.

Chapter Review Solutions

Exercises

Find the restrictions on *x*, if any.

1. $4x - 9 = 11$

Solution

$3x + 7 = 4$

no restrictions on x

2. $x + \frac{1}{x} = 2$

Solution

$x + \frac{1}{x} = 2$

restrictions: $x \neq 0$

3. $\frac{4x}{(x - 1)} = 8$

Solution

$\frac{1}{x - 1} = 4$

restrictions: $x \neq 1$

4. $\frac{1}{x-2} = \frac{2}{x-3}$

Solution

$\frac{1}{x-2} = \frac{2}{x-3}$

restrictions: $x \neq 2, x \neq 3$

Solve each equation and classify it as an identity, a conditional equation, or a contradiction.

5. $3(9x+4) = 28$

Solution

$$\begin{aligned} 3(9x+4) &= 28 \\ 27x + 12 &= 28 \\ 27x &= 16 \\ x &= \tfrac{16}{27} \end{aligned}$$

conditional equation

6. $\frac{3}{2}a = 7(a+11)$

Solution

$$\begin{aligned} \frac{3}{2}a &= 7(a+11) \\ 2 \cdot \frac{3}{2}a &= 2 \cdot 7(a+11) \\ 3a &= 14a + 154 \\ -11a &= 154 \\ a &= -\tfrac{154}{11} = -14 \end{aligned}$$

conditional equation

7. $8(3x-5) - 4(x+3) = 12$

Solution

$$\begin{aligned} 8(3x-5) - 4(x+3) &= 12 \\ 24x - 40 - 4x - 12 &= 12 \\ 20x - 52 &= 12 \\ 20x &= 64 \\ x &= \tfrac{64}{20} = \tfrac{16}{5} \Rightarrow \text{conditional equation} \end{aligned}$$

8. $\dfrac{x+3}{x+4}+\dfrac{x+3}{x+2}=2$

Solution

$$\begin{aligned}\frac{x+3}{x+4}+\frac{x+3}{x+2}&=2\\(x+4)(x+2)\left(\frac{x+3}{x+4}+\frac{x+3}{x+2}\right)&=(x+4)(x+2)\cdot 2\\(x+2)(x+3)+(x+4)(x+3)&=\left(x^2+6x+8\right)\cdot 2\\x^2+5x+6+x^2+7x+12&=2x^2+12x+16\\2x^2+12x+18&=2x^2+12x+16\\18&\neq 16 \Rightarrow \text{no solution, contradiction}\end{aligned}$$

9. $\dfrac{3}{x-1}=\dfrac{1}{2}$

Solution

$$\begin{aligned}\frac{3}{x-1}&=\frac{1}{2}\\2(x-1)\cdot\frac{3}{x-1}&=2(x-1)\cdot\frac{1}{2}\\6&=x-1\\7&=x\end{aligned}$$

conditional equation

10. $\dfrac{8x^2+72x}{9+x}=8x$

Solution

$$\begin{aligned}\frac{8x^2+72x}{9+x}&=8x\\(9+x)\cdot\frac{8x^2+72x}{9+x}&=(9+x)\cdot 8x\\8x^2+72x&=72x+8x^2\end{aligned}$$

all real numbers except −9, indentity

11. $\dfrac{3x}{x-1}-\dfrac{5}{x+3}=3$

Solution

$$\begin{aligned} \frac{3x}{x-1} - \frac{5}{x+3} &= 3 \\ (x-1)(x+3)\left(\frac{3x}{x-1} - \frac{5}{x+3}\right) &= (x-1)(x+3) \cdot 3 \\ 3x(x+3) - 5(x-1) &= (x^2 + 2x - 3) \cdot 3 \\ 3x^2 + 9x - 5x + 5 &= 3x^2 + 6x - 9 \\ 4x + 5 &= 6x - 9 \\ -2x &= -14 \\ x &= 7 \Rightarrow \text{ conditional equation} \end{aligned}$$

12. $x + \dfrac{1}{2x-3} = \dfrac{2x^2}{2x-3}$

Solution

$$\begin{aligned} x + \frac{1}{2x-3} &= \frac{2x^2}{2x-3} \\ (2x-3)\left(x + \frac{1}{2x-3}\right) &= (2x-3) \cdot \frac{2x^2}{2x-3} \\ (2x-3)x + 1 &= 2x^2 \\ 2x^2 - 3x + 1 &= 2x^2 \\ -3x &= -1 \\ x &= \tfrac{1}{3} \Rightarrow \text{ conditional equation} \end{aligned}$$

13. $\dfrac{4}{x^2 - 13x - 48} - \dfrac{1}{x^2 + x - 6} = \dfrac{2}{x^2 - 18x + 32}$

Solution

$$\begin{aligned} \frac{4}{x^2 - 13x - 48} - \frac{1}{x^2 + x - 6} &= \frac{2}{x^2 - 18x + 32} \\ \frac{4}{(x-16)(x+3)} - \frac{1}{(x+3)(x-2)} &= \frac{2}{(x-16)(x-2)} \\ 4(x-2) - (x-16) &= 2(x+3) \qquad \{\text{multiply by common denominator}\} \\ 4x - 8 - x + 16 &= 2x + 6 \\ 3x + 8 &= 2x + 6 \\ x &= -2 \Rightarrow \text{ conditional equation} \end{aligned}$$

14. $\frac{a-1}{a+3} + \frac{2a-1}{3-a} = \frac{2-a}{a-3}$

Solution

$$\begin{aligned}
\frac{a-1}{a+3} + \frac{2a-1}{3-a} &= \frac{2-a}{a-3} \\
\frac{a-1}{a+3} + \frac{1-2a}{a-3} &= \frac{2-a}{a-3} \\
(a-1)(a-3) + (1-2a)(a+3) &= (2-a)(a+3) \quad \{\text{multiply by common denominator}\} \\
a^2 - 3a - a + 3 + a + 3 - 2a^2 - 6a &= 2a + 6 - a^2 - 3a \\
-a^2 - 9a + 6 &= -a^2 - a + 6 \\
-9a + 6 &= -a + 6 \\
0 &= 8a \\
0 &= a \Rightarrow \text{conditional equation}
\end{aligned}$$

Solve each formula for the indicated variable.

15. $C = \frac{5}{9}(F - 32);\ F$

Solution

$$\begin{aligned}
C &= \frac{5}{9}(F - 32) \\
\frac{9}{5}C &= \frac{9}{5} \cdot \frac{5}{9}(F - 32) \\
\frac{9}{5}C &= F - 32 \\
\frac{9}{5}C + 32 &= F
\end{aligned}$$

16. $P_n = 1 + \frac{si}{f};\ f$

Solution

$$\begin{aligned}
P_n &= l + \frac{si}{f} \\
P_n - l &= \frac{si}{f} \\
f(P_n - l) &= f \cdot \frac{si}{f} \\
f(P_n - l) &= si \\
\frac{f(P_n - l)}{P_n - l} &= \frac{si}{P_n - l} \\
f &= \frac{si}{P_n - l}
\end{aligned}$$

17. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}; f_1$

Solution

$$\begin{aligned}
\frac{1}{f} &= \frac{1}{f_1} + \frac{1}{f_2} \\
ff_1f_2 \cdot \frac{1}{f} &= ff_1f_2\left(\frac{1}{f_1} + \frac{1}{f_2}\right) \\
f_1f_2 &= ff_2 + ff_1 \\
f_1f_2 - ff_1 &= ff_2 \\
f_1(f_2 - f) &= ff_2 \\
\frac{f_1(f_2 - f)}{f_2 - f} &= \frac{ff_2}{f_2 - f} \\
f_1 &= \frac{ff_2}{f_2 - f}
\end{aligned}$$

18. $S = \frac{a - lr}{1 - r}; l$

Solution

$$\begin{aligned}
S &= \frac{a - lr}{1 - r} \\
S(1 - r) &= \frac{a - lr}{1 - r}(1 - r) \\
S(1 - r) &= a - lr \\
S - Sr &= a - lr \\
lr &= a - S + Sr \\
\frac{lr}{r} &= \frac{a - S + Sr}{r} \\
l &= \frac{a - S + Sr}{r}
\end{aligned}$$

19. **Test scores** Gabrielle took four tests in an English class. On each successive test, her score improved by 4 points. If her mean score was 66%, what did she score on the first test?

Solution

Let x = the score on the first exam. Then his scores on the following tests were $x + 4$, $x + 8$ and $x + 12$.

$$\begin{aligned}
\frac{\boxed{\text{Sum of scores}}}{4} &= 66 \\
\frac{x + x + 4 + x + 8 + x + 12}{4} &= 66 \\
\frac{4x + 24}{4} &= 66 \\
4x + 24 &= 264 \\
4x &= 240 \\
x &= 60
\end{aligned}$$

His score on the first test was 60%.

20. **Fencing a garden** A homeowner has 100 feet of fencing to enclose a rectangular garden. If the garden is to be 5 feet longer than it is wide, find its dimensions.

Solution

Let w = the width. Then $w + 5$ = the length.

$$\begin{aligned}
\boxed{\text{Perimeter}} &= 100 \\
2w + 2(w + 5) &= 100 \\
2w + 2w + 10 &= 100 \\
4w + 10 &= 100 \\
4w &= 90 \\
w &= 22.5
\end{aligned}$$

The dimensions are 22.5 ft by 27.5 ft.

21. **Travel** Two shoppers leave a shopping center by car traveling in opposite directions. If one car averages 45 mph and the other 50 mph, how long will it take for the cars to be 285 miles apart?

Solution

Let t = the time the cars travel.

$$\begin{aligned}
\boxed{\begin{array}{c}\text{Distance 1st}\\ \text{car travels}\end{array}} + \boxed{\begin{array}{c}\text{Distance 2nd}\\ \text{car travels}\end{array}} &= \boxed{\text{Total distance}} \\
45t + 50t &= 285 \\
95t &= 285 \\
t &= 3 \Rightarrow \text{They will be 285 miles apart after 3 hours.}
\end{aligned}$$

22. **Travel** Two taxis leave an airport and travel in the same direction. If the average speed of one taxi is 40 mph and the average speed of the other taxi is 46 mph, how long will it take before the cars are 3 miles apart?

Solution

Let t = the time the cars travel.

$$\boxed{\begin{array}{c}\text{Distance 1st}\\ \text{car travels}\end{array}} - \boxed{\begin{array}{c}\text{Distance 2nd}\\ \text{car travels}\end{array}} = \boxed{\text{Distance between them}}$$

$$\begin{aligned} 46t - 40t &= 3 \\ 6t &= 3 \\ t &= 0.5 \Rightarrow \text{They will be 3 miles apart after 0.5 hours.} \end{aligned}$$

23. **Preparing a solution** A liter of fluid is 50% alcohol. How much water must be added to dilute it to a 20% solution?

Solution

Let x = the liters of water added.

$$\boxed{\begin{array}{c}\text{Liters of}\\ \text{alcohol at start}\end{array}} + \boxed{\begin{array}{c}\text{Liters of}\\ \text{alcohol added}\end{array}} = \boxed{\begin{array}{c}\text{Liters of}\\ \text{alcohol at end}\end{array}}$$

$$\begin{aligned} 0.50(1) + 0 &= 0.20(1 + x) \\ 0.50 &= 0.20 + 0.20x \\ 0.30 &= 0.20x \\ 1.5 &= x \Rightarrow \text{1.5 liters of water should be added.} \end{aligned}$$

24. **Washing windows** Scott can wash 37 windows in 3 hours, and Bill can wash 27 windows in 2 hours. How long will it take the two of them to wash 100 windows?

Solution

Let x = hours for both working together.

$$\boxed{\begin{array}{c}\text{Number Scott}\\ \text{washes in 1 hour}\end{array}} \cdot \boxed{\begin{array}{c}\text{Number}\\ \text{of hours}\end{array}} + \boxed{\begin{array}{c}\text{Number Bill}\\ \text{washes in 1 hour}\end{array}} \cdot \boxed{\begin{array}{c}\text{Number}\\ \text{of hours}\end{array}} = \boxed{\text{100 windows}}$$

$$\begin{aligned} \frac{37}{3}x + \frac{27}{2}x &= 100 \\ 6\left(\frac{37}{3}x + \frac{27}{2}x\right) &= 6(100) \\ 74x + 81x &= 600 \\ 155x &= 600 \\ x &= \frac{600}{155} \approx 3.9 \end{aligned}$$

They can wash 100 windows together in about 3.9 hours

25. **Filling a tank** A tank can be filled in 9 hours by one pipe and in 12 hours by another. How long will it take both pipes to fill the empty tank?

Solution

Let x = hours for both pipes to fill the tank.

$$\boxed{\begin{array}{c}\text{1st pipe}\\\text{in 1 hour}\end{array}} + \boxed{\begin{array}{c}\text{2nd pipe}\\\text{in 1 hour}\end{array}} = \boxed{\begin{array}{c}\text{Total in}\\\text{1 hour}\end{array}}$$

$$\begin{aligned}\frac{1}{9} + \frac{1}{12} &= \frac{1}{x}\\ 36x\left(\frac{1}{9} + \frac{1}{12}\right) &= 36x\left(\frac{1}{x}\right)\\ 4x + 3x &= 36\\ 7x &= 36\\ x &= \frac{36}{7} = 5\frac{1}{7}\end{aligned}$$

The tank can be filled in $5\frac{1}{7}$ hours.

26. **Producing brass** How many ounces of pure zinc must be alloyed with 20 ounces of brass that is 30% zinc and 70% copper to produce brass that is 40% zinc?

Solution

Let x = the ounces of pure zinc added.

$$\boxed{\begin{array}{c}\text{Ounces of}\\\text{zinc at start}\end{array}} + \boxed{\begin{array}{c}\text{Ounces of}\\\text{zinc added}\end{array}} = \boxed{\begin{array}{c}\text{Ounces of}\\\text{zinc at end}\end{array}}$$

$$\begin{aligned}0.30(20) + x &= 0.40(20 + x)\\ 6 + x &= 8 + 0.40x\\ 0.60x &= 2\\ 6x &= 20\\ x &= \frac{20}{6} = 3\frac{1}{3}\end{aligned}$$

$3\frac{1}{3}$ ounces of zinc should be added.

27. **Lending money** A bank lends $10,000, part of it at 11% annual interest and the rest at 14%. If the annual income is $1265, how much was loaned at each rate?

Solution

Let x = the amount invested at 11%. Then $10{,}000 - x$ = the amount invested at 14%.

$$\boxed{\begin{array}{c}\text{Interest}\\\text{at 11\%}\end{array}} + \boxed{\begin{array}{c}\text{Interest}\\\text{at 14\%}\end{array}} = \boxed{\begin{array}{c}\text{Total}\\\text{interest}\end{array}}$$

$$\begin{aligned}0.11x + 0.14(10{,}000 - x) &= 1{,}265\\ 0.11x + 1{,}400 - 0.14x &= 1{,}265\\ -0.03x &= -135\\ x &= 4{,}500\end{aligned}$$

$4,500 was invested at 11% and $5,500 was invested at 14%.

28. **Producing oriental rugs** An oriental rug manufacturer can use one loom with a setup cost of \$750 that can weave a rug for \$115. Another loom, with a setup cost of \$950, can produce a rug for \$95. How many rugs are produced if the costs are the same on each loom?

Solution

Let x = # of rugs for equal costs.

$$\boxed{\text{Cost of 1st loom}} = \boxed{\text{Cost of 2nd loom}}$$

$$750 + 115x = 950 + 95x$$

$$20x = 200$$

$$x = 10$$

The costs are the same on either loom for 10 rugs.

Perform all operations and express all answers in *a* + *bi* form.

29. $3\sqrt{-300}$

Solution

$$3\sqrt{-300} = 3\sqrt{-1}\sqrt{100}\sqrt{3} = 30i\sqrt{3}$$

30. $-\sqrt{\dfrac{-45}{4}}$

Solution

$$-\sqrt{\frac{-45}{4}} = -\sqrt{-1} \cdot \frac{\sqrt{9}\sqrt{5}}{\sqrt{4}} = -\frac{3\sqrt{5}}{2}i$$

31. $(2 - 3i) + (-4 + 2i)$

Solution

$$(2 - 3i) + (-4 + 2i) = 2 - 3i - 4 + 2i = -2 - i$$

32. $(3 - \sqrt{-36}) + (\sqrt{-16} + 2)$

Solution

$$(3 - \sqrt{-36}) + (\sqrt{-16} + 2) = (3 - 6i) + (4i + 2) = 3 - 6i + 4i + 2 = 5 - 2i$$

33. $(2 - 3i) - (4 + 2i)$

Solution

$$(2 - 3i) - (4 + 2i) = 2 - 3i - 4 - 2i = -2 - 5i$$

34. $(5 - 11i)(5 + 11i)$

Solution

$(5 - 11i)(5 + 11i) = 25 + 55i - 55i - 121i^2 = 25 - 121(-1) = 146 = 146 + 0i$

35. $(8 - 3i)^2$

Solution

$(8 - 3i)^2 = (8 - 3i)(8 - 3i) = 64 - 24i - 24i + 9i^2 = 64 - 48i + 9(-1) = 55 - 48i$

36. $(3 + \sqrt{-9})(2 - \sqrt{-25})$

Solution

$(3 + \sqrt{-9})(2 - \sqrt{-25}) = (3 + 3i)(2 - 5i) = 6 - 9i - 15i^2 = 6 - 9i + 15 = 21 - 9i$

37. $\frac{3}{i}$

Solution

$\frac{3}{i} = \frac{3i}{ii} = \frac{3i}{i^2} = \frac{3i}{-1} = 0 - 3i$

38. $-\frac{5}{6i}$

Solution

$-\frac{5}{6i} = -\frac{5 \cdot i}{6i \cdot i} = -\frac{5i}{6i^2} = -\frac{5i}{-6} = 0 + \frac{5}{6}i$

39. $\frac{3}{1 + i}$

Solution

$\frac{3}{1 + i} = \frac{3(1 - i)}{(1 + i)(1 - i)} = \frac{3(1 - i)}{1^2 - i^2} = \frac{3 - 3i}{2} = \frac{3}{2} - \frac{3}{2}i$

40. $\frac{2i}{2 - i}$

Solution

$\frac{2i}{2 - i} = \frac{2i(2 + i)}{(2 - i)(2 + i)} = \frac{4i + 2i^2}{2^2 - i^2} = \frac{-2 + 4i}{5} = -\frac{2}{5} + \frac{4}{5}i$

41. $\dfrac{3+i}{3-i}$

Solution

$$\frac{3+i}{3-i} = \frac{(3+i)(3+i)}{(3-i)(3+i)} = \frac{9+6i+i^2}{3^2-i^2} = \frac{8+6i}{10} = \frac{8}{10}+\frac{6}{10}i = \frac{4}{5}+\frac{3}{5}i$$

42. $\dfrac{3-2i}{1+i}$

Solution

$$\frac{3-2i}{1+i} = \frac{(3-2i)(1-i)}{(1+i)(1-i)} = \frac{3-5i+2i^2}{1^2-i^2} = \frac{1-5i}{2} = \frac{1}{2}-\frac{5}{2}i$$

43. Simplify: i^{53}.

Solution

$$i^{53} = i^{52}i = \left(i^4\right)^{13}i = 1^{13}i = 0+i$$

44. Simplify: i^{103}.

Solution

$$i^{103} = i^{100}i^3 = \left(i^4\right)^{25}i^3 = 1^{25}i^3 = 0-i$$

45. $-\dfrac{2}{i^3}$

Solution

$$-\frac{2}{i^3} = -\frac{2\cdot i}{i^3\cdot i} = -\frac{2i}{i^4} = -\frac{2i}{1} = 0-2i$$

46. $|3-i|$

Solution

$$|3-i| = \sqrt{3^2+(-1)^2} = \sqrt{9+1}$$
$$= \sqrt{10}+0i$$

47. $\left|\dfrac{1+i}{1-i}\right|$

Solution

$$\left|\frac{1+i}{1-i}\right| = \left|\frac{(1+i)(1+i)}{(1-i)(1+i)}\right| = \left|\frac{1+2i+i^2}{1^2-i^2}\right| = \left|\frac{2i}{2}\right| = |0+i| = \sqrt{0^2+1^2} = 1$$

48. Factor $64r^2 + 9s^2$ over the set of complex numbers.

Solution

$$64r^2 + 9s^2 = 64r^2 - \left(-9s^2\right) = \left(8r\right)^2 - \left(3si\right)^2 = \left(8r + 3si\right)\left(8r - 3si\right)$$

Solve each equation by factoring.

49. $2x^2 - x - 6 = 0$

Solution

$$2x^2 - x - 6 = 0$$
$$\left(2x + 3\right)\left(x - 2\right) = 0$$
$$2x + 3 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$2x = -3 \qquad x = 2$$
$$x = -\tfrac{3}{2} \qquad x = 2$$

50. $12x^2 + 13x = 4$

Solution

$$12x^2 + 13x = 4$$
$$12x^2 + 13x - 4 = 0$$
$$\left(4x - 1\right)\left(3x + 4\right) = 0$$
$$4x - 1 = 0 \quad \textbf{or} \quad 3x + 4 = 0$$
$$4x = 1 \qquad 3x = -4$$
$$x = \tfrac{1}{4} \qquad x = -\tfrac{4}{3}$$

51. $5x^2 - 8x = 0$

Solution

$$5x^2 - 8x = 0$$
$$x\left(5x - 8\right) = 0$$
$$x = 0 \quad \textbf{or} \quad 5x - 8 = 0$$
$$x = 0 \qquad 5x = 8$$
$$x = 0 \qquad x = \tfrac{8}{5}$$

52. $27x^2 = 30x - 8$

Solution

$$27x^2 = 30x - 8$$
$$27x^2 - 30x + 8 = 0$$
$$(9x - 4)(3x - 2) = 0$$
$$9x - 4 = 0 \quad \textbf{or} \quad 3x - 2 = 0$$
$$9x = 4 \qquad 3x = 2$$
$$x = \tfrac{4}{9} \qquad x = \tfrac{2}{3}$$

Solve each equation by using the Square Root Property.

53. $3x^2 = 24$

Solution

$$2x^2 = 16$$
$$x^2 = 8$$
$$\sqrt{x^2} = \pm\sqrt{8}$$
$$x = \pm 2\sqrt{2}$$

54. $12x^2 = -60$

Solution

$$12x^2 = -60$$
$$x^2 = -5$$
$$\sqrt{x^2} = \pm\sqrt{-5}$$
$$x = \pm i\sqrt{5}$$

55. $(4z - 5)^2 = 32$

Solution

$$(4z - 5)^2 = 32$$
$$\sqrt{(4z - 5)^2} = \pm\sqrt{32}$$
$$4z - 5 = \pm 4\sqrt{2}$$
$$4z = 5 \pm 4\sqrt{2}$$
$$z = \frac{5 \pm 4\sqrt{2}}{4}$$

56. $(5x - 7)^2 = -45$

Solution

$$(5x - 7)^2 = -45$$
$$\sqrt{(5x - 7)^2} = \pm\sqrt{-45}$$
$$5x - 7 = \pm 3i\sqrt{5}$$
$$5x = 7 \pm 3i\sqrt{5}$$
$$x = \frac{7 \pm 3i\sqrt{5}}{5} = \frac{7}{5} \pm \frac{3\sqrt{5}}{5}i$$

Solve each equation by completing the square.

57. $x^2 - 8x + 15 = 0$

Solution

$$x^2 - 8x + 15 = 0$$
$$x^2 - 8x = -15$$
$$x^2 - 8x + 16 = -15 + 16$$
$$(x - 4)^2 = 1$$
$$x - 4 = \sqrt{1} \quad \textbf{or} \quad x - 4 = -\sqrt{1}$$
$$x - 4 = 1 \qquad x - 4 = -1$$
$$x = 5 \qquad x = 3$$

58. $3x^2 + 18x = -24$

Solution

$$3x^2 + 18x = -24$$
$$\frac{3x^2 + 18x}{3} = \frac{-24}{3}$$
$$x^2 + 6x = -8$$
$$x^2 + 6x + 9 = -8 + 9$$
$$(x + 3)^2 = 1$$
$$x + 3 = \sqrt{1} \quad \textbf{or} \quad x + 3 = -\sqrt{1}$$
$$x + 3 = 1 \qquad x + 3 = -1$$
$$x = -2 \qquad x = -4$$

59. $5x^2 - x - 1 = 0$

Solution

$$5x^2 - x - 1 = 0$$
$$5x^2 - x = 1$$
$$x^2 - \frac{1}{5}x = \frac{1}{5}$$
$$x^2 - \frac{1}{5}x + \frac{1}{100} = \frac{1}{5} + \frac{1}{100}$$
$$\left(x - \frac{1}{10}\right)^2 = \frac{21}{100}$$

$$x - \frac{1}{10} = \sqrt{\frac{21}{100}} \quad \textbf{or} \quad x - \frac{1}{10} = -\sqrt{\frac{21}{100}}$$
$$x - \frac{1}{10} = \frac{\sqrt{21}}{10} \qquad x - \frac{1}{10} = -\frac{\sqrt{21}}{10}$$
$$x = \frac{1 + \sqrt{21}}{10} \qquad x = \frac{1 - \sqrt{21}}{10}$$

60. $5x^2 - x = 0$

Solution

$$5x^2 - x = 0$$
$$x^2 - \frac{1}{5}x = 0$$
$$x^2 - \frac{1}{5}x + \frac{1}{100} = 0 + \frac{1}{100}$$
$$\left(x - \frac{1}{10}\right)^2 = \frac{1}{100}$$

$$x - \frac{1}{10} = \sqrt{\frac{1}{100}} \quad \textbf{or} \quad x - \frac{1}{10} = -\sqrt{\frac{1}{100}}$$
$$x - \frac{1}{10} = \frac{1}{10} \qquad x - \frac{1}{10} = -\frac{1}{10}$$
$$x = \frac{2}{10} = \frac{1}{5} \qquad x = \frac{0}{10} = 0$$

61. Solve: $3x^2 - 2x + 1 = 0.$

Solution

$$3x^2 - 2x + 1 = 0$$
$$3x^2 - 2x = -1$$
$$x^2 - \frac{2}{3}x = -\frac{1}{3}$$
$$x^2 - \frac{2}{3}x + \frac{1}{9} = -\frac{1}{3} + \frac{1}{9}$$
$$\left(x - \frac{1}{3}\right)^2 = -\frac{2}{9}$$

$x - \frac{1}{3} = \sqrt{-\frac{2}{9}}$ **or** $x - \frac{1}{3} = -\sqrt{-\frac{2}{9}}$

$x - \frac{1}{3} = \frac{\sqrt{2}}{3}i$ $\quad$ $x - \frac{1}{3} = -\frac{\sqrt{2}}{3}i$

$x = \frac{1}{3} + \frac{\sqrt{2}}{3}i$ $\quad$ $x = \frac{1}{3} - \frac{\sqrt{2}}{3}i$

Use the Quadratic Formula to solve each equation.

62. $x^2 + 5x - 14 = 0$

Solution

$x^2 + 5x - 14 = 0 \Rightarrow a = 1, b = 5, c = -14$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-14)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 56}}{2} = \frac{-5 \pm \sqrt{81}}{2}$$
$$= \frac{-5 \pm 9}{2}$$

$x = \frac{-5 + 9}{2} = \frac{4}{2} = 2$ or $x = \frac{-5 - 9}{2} = \frac{-14}{2} = -7$

63. $3x^2 - 25x = 18$

Solution

$3x^2 - 25x = 18 \Rightarrow 3x^2 - 25x - 18 = 0 \Rightarrow a = 3, b = -25, c = -18$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(3)(-18)}}{2(3)} = \frac{25 \pm \sqrt{625 + 216}}{6}$$
$$= \frac{25 \pm \sqrt{841}}{6} = \frac{25 \pm 29}{6}$$

$x = \frac{25 + 29}{6} = \frac{54}{6} = 9$ or $x = \frac{25 - 29}{6} = \frac{-4}{6} = -\frac{2}{3}$

64. $5x^2 = 1 - x$

Solution

$5x^2 = 1 - x \Rightarrow 5x^2 + x - 1 = 0 \Rightarrow a = 5, b = 1, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1) \pm \sqrt{(1)^2 - 4(5)(-1)}}{2(5)} = \frac{-1 \pm \sqrt{1 + 20}}{10} = \frac{-1 \pm \sqrt{21}}{10}$$

65. $5 = a^2 + 2a$

Solution

$5 = a^2 + 2a \Rightarrow a^2 + 2a - 5 = 0 \Rightarrow a = 1, b = 2, c = -5$

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-5)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 20}}{2} = \frac{-2 \pm \sqrt{24}}{2}$$

$$= \frac{-2 \pm 2\sqrt{6}}{2} = -1 \pm \sqrt{6}$$

66. Solve: $3x^2 + 4 = 2x$.

Solution

$3x^2 + 4 = 2x \Rightarrow 3x^2 - 2x + 4 = 0 \Rightarrow a = 3, b = -2, c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)} = \frac{2 \pm \sqrt{4 - 48}}{6} = \frac{2 \pm \sqrt{-44}}{6}$$

$$= \frac{2}{6} \pm \frac{2\sqrt{11}}{6}i = \frac{1}{3} \pm \frac{\sqrt{11}}{3}i$$

67. Calculate the discriminant associated with the equation $6x^2 + 5x + 1 = 0$.

Solution

$6x^2 + 5x + 1 = 0$

$a = 6, b = 5, c = 1$

$b^2 - 4ac = (5)^2 - 4(6)(1) = 25 - 24 = 1$

68. Determine the number and nature of the roots of the equation in Exercise 67.

$3x^2 + 18x = -24$

Solution

two different rational numbers

69. Find the value of k that will make the roots of $kx^2 + 4x + 12 = 0$ equal.

Solution

$kx^2 + 4x + 12 = 0$

$a = k, b = 4, c = 12$

Set the discriminant equal to 0:

$$\begin{aligned} b^2 - 4ac &= 0 \\ 4^2 - 4(k)(12) &= 0 \\ 16 - 48k &= 0 \\ -48k &= -16 \\ k &= \frac{1}{3} \end{aligned}$$

70. Find the values of k that will make the roots of $4y^2 + (k + 2)y = 1 - k$ equal.

Solution

$$\begin{aligned} 4y^2 + (k + 2)y &= 1 - k \\ 4y^2 + (k + 2)y - 1 + k &= 0 \end{aligned}$$

$a = 4, b = k + 2, c = -1 + k$

Set the discriminant equal to 0:

$$\begin{aligned} b^2 - 4ac &= 0 \\ (k + 2)^2 - 4(4)(-1 + k) &= 0 \\ k^2 + 4k + 4 + 16 - 16k &= 0 \\ k^2 - 12k + 20 &= 0 \\ (k - 10)(k - 2) &= 0 \end{aligned}$$

$k - 10 = 0$ **or** $k - 2 = 0$

$k = 10 \qquad k = 2$

71. $\frac{3x}{2} - \frac{2x}{x - 1} = x - 3$

Solution

$$\begin{aligned} \frac{3x}{2} - \frac{2x}{x - 1} &= x - 3 \\ 2(x - 1)\left(\frac{3x}{2} - \frac{2x}{x - 1}\right) &= 2(x - 1)(x - 3) \\ (x - 1) \cdot 3x - 2(2x) &= 2(x^2 - 4x + 3) \\ 3x^2 - 3x - 4x &= 2x^2 - 8x + 6 \end{aligned}$$

$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x + 3 = 0 \quad \textbf{or} \quad x - 2 = 0$$
$$x = -3 \qquad\qquad x = 2$$

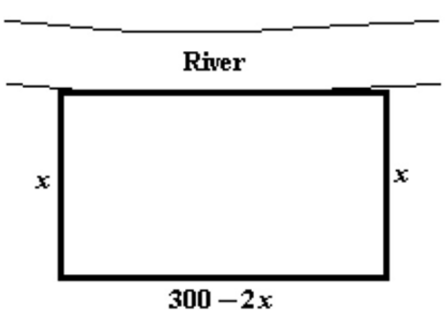
72. Solve: $\dfrac{4}{a - 4} + \dfrac{4}{a - 1} = 5.$

Solution

$$\frac{4}{a - 4} + \frac{4}{a - 1} = 5$$
$$(a - 4)(a - 1)\left(\tfrac{4}{a-4} + \tfrac{4}{a-1}\right) = (a - 4)(a - 1)5$$
$$4(a - 1) + 4(a - 4) = 5(a^2 - 5a + 4)$$
$$4a - 4 + 4a - 16 = 5a^2 - 25a + 20$$
$$0 = 5a^2 - 33a + 40$$
$$0 = (5a - 8)(a - 5)$$
$$5a - 8 = 0 \quad \textbf{or} \quad a - 5 = 0$$
$$5a = 8 \qquad\qquad a = 5$$
$$a = \tfrac{8}{5} \qquad\qquad a = 5$$

73. **Fencing a field** A farmer wishes to enclose a rectangular garden with 300 yards of fencing. A river runs along one side of the garden, so no fencing is needed there. Find the dimensions of the rectangle if the area is 10,450 square yards.

Solution
Let x = one side of the garden.



$$\text{Area} = 10450$$
$$x(300 - 2x) = 10450$$
$$-2x^2 + 300x = 10450$$
$$0 = 2x^2 - 300x + 10450$$

$$0 = 2(x^2 - 150x + 5225)$$
$$0 = 2(x - 95)(x - 55)$$
$$x - 95 = 0 \quad \textbf{or} \quad x - 55 = 0$$
$$x = 95 \qquad\qquad x = 55$$

The dimensions are 95 yards by 110 yards or 55 yards by 190 yards.

74. **Flying rates** A jet plane, flying 120 mph faster than a propeller-driven plane, travels 3520 miles in 3 hours less time than the propeller plane requires to fly the same distance. How fast does each plane fly?

Solution

Let r = the rate of the propeller-driven plane. Then the rate of the jet plane is $r + 120$.

$$\boxed{\text{Jet time}} = \boxed{\text{Propeller time}} - 3$$
$$\frac{3520}{r + 120} = \frac{3520}{r} - 3$$
$$r(r + 120)\frac{3520}{r + 120} = r(r + 120)\left(\frac{3520}{r} - 3\right)$$
$$3520r = 3520(r + 120) - 3r(r + 120)$$
$$3520r = 3520r + 422400 - 3r^2 - 360r$$
$$3r^2 + 360r + 422{,}400 = 0$$
$$3(r - 320)(r + 440) = 0$$
$$r - 320 = 0 \quad \textbf{or} \quad r + 440 = 0$$
$$r = 320 \qquad\qquad r = -440$$

	Rate	Time	Dist.
Propeller	r	$\frac{3520}{r}$	3520
Jet	$r + 120$	$\frac{3520}{r+120}$	3520

Since $r = -440$ does not make sense, the solution is $r = 320$. The prop. plane's rate is 320 mph, while the jet plane's rate is 440 mph.

75. **Flight of a ball** A ball thrown into the air reaches a height h (in feet) according to the formula $h = -16t^2 + 64t$, where t is the time elapsed since the ball was thrown. Find the shortest time it will take the ball to reach a height of 48 feet.

Solution

Set $h = 48$:

$$h = -16t^2 + 64t$$
$$48 = -16t^2 + 64t$$
$$16t^2 - 64t + 48 = 0$$
$$16(t - 1)(t - 3) = 0$$

$t - 1 = 0$ **or** $t - 3 = 0$

$t = 1$ $\quad$ $t = 3$

The shortest time required for the ball to reach a height of 48 feet is 1 second.

76. **Width of a walk** A bricklayer built a walk of uniform width around a rectangular pool. If the area of the walk is 117 square feet and the dimensions of the pool are 16 feet by 20 feet, how wide is the walk?

Solution

Let x = the width of the walk. Then the total dimensions are $16 + 2x$ by $20 + 2x$.

$$\boxed{\text{Total area}} - \boxed{\text{Area of pool}} = \boxed{\text{Area of walk}}$$

$$(16 + 2x)(20 + 2x) - (16)(20) = 117$$

$$320 + 72x + 4x^2 - 320 = 117$$

$$4x^2 + 72x - 117 = 0$$

$$(2x + 39)(2x - 3) = 0$$

$2x + 39 = 0$ **or** $2x - 3 = 0$

$x = -\frac{39}{2}$ $\quad$ $x = \frac{3}{2}$

Since $x = -\frac{39}{2}$ does not make sense, the only solution is $x = \frac{3}{2}$. The walk is $1\frac{1}{2}$ feet wide.

Solve each equation.

77. $x^3 + 4x^2 - 12x = 0$

Solution

$$x^3 + 4x^2 - 12x = 0$$

$$x(x^2 + 4x - 12) = 0$$

$$x(x + 6)(x - 2) = 0$$

$x = 0$ **or** $x + 6 = 0$ **or** $x - 2 = 0$

$x = 0$ $\quad$ $x = -6$ $\quad$ $x = 2$

78. $3x^3 + 4x^2 - 4x = 0$

Solution

$$3x^3 + 4x^2 - 4x = 0$$

$$x(3x^2 + 4x - 4) = 0$$

$$x(3x - 2)(x + 2) = 0$$

$x = 0$ **or** $3x - 2 = 0$ **or** $x + 2 = 0$

$x = 0$ $\quad$ $x = \frac{2}{3}$ $\quad$ $x = -2$

79. $x^4 - 2x^2 + 1 = 0$

Solution

$$x^4 - 2x^2 + 1 = 0$$
$$(x^2 - 1)(x^2 - 1) = 0$$
$$x^2 - 1 = 0 \quad \textbf{or} \quad x^2 - 1 = 0$$
$$x^2 = 0 \qquad x^2 = 1$$
$$x = \pm 1 \qquad x = \pm 1$$

80. $x^4 - 36 = -35x^2$

Solution

$$x^4 - 36 = -35x^2$$
$$x^4 + 35x^2 - 36 = 0$$
$$(x^2 + 36)(x^2 - 1) = 0$$
$$x^2 + 36 = 0 \quad \textbf{or} \quad x^2 - 1 = 0$$
$$x^2 = -36 \qquad x^2 = 1$$
$$x = \pm 6i \qquad x = \pm 1$$

81. $a - a^{1/2} - 6 = 0$

Solution

$$a - a^{1/2} - 6 = 0$$
$$(a^{1/2} + 2)(a^{1/2} - 3) = 0$$
$$a^{1/2} + 2 = 0 \quad \textbf{or} \quad a^{1/2} - 3 = 0$$
$$a^{1/2} = -2 \qquad a^{1/2} = 3$$
$$(a^{1/2})^2 = (-2)^2 \qquad (a^{1/2})^2 = (3)^2$$
$$a = 4 \qquad a = 9$$

$a = 4$ does not check and is extraneous.

82. $x^{2/3} + x^{1/3} - 6 = 0$

Solution

$$x^{2/3} + x^{1/3} - 6 = 0$$
$$(x^{1/3} - 2)(x^{1/3} + 3) = 0$$
$$x^{1/3} - 2 = 0 \quad \textbf{or} \quad x^{1/3} + 3 = 0$$
$$x^{1/3} = 2 \qquad x^{1/3} = -3$$
$$(x^{1/3})^3 = (2)^3 \qquad (x^{1/3})^3 = (-3)^3$$
$$x = 8 \qquad x = -27$$

Both answers check.

83. $6y^{-2} + 13y^{-1} - 5 = 0$

Solution

$$6y^{-2} + 13y^{-1} - 5 = 0$$
$$(3y^{-1} - 1)(2y^{-1} + 5) = 0$$

$$3y^{-1} - 1 = 0 \quad \textbf{or} \quad 2y^{-1} + 5 = 0$$
$$y^{-1} = \tfrac{1}{3} \qquad\qquad y^{-1} = -\tfrac{5}{2}$$
$$(y^{-1})^{-1} = \left(\tfrac{1}{3}\right)^{-1} \qquad\qquad (y^{-1})^{-1} = \left(-\tfrac{5}{2}\right)^{-1}$$
$$y = 3 \qquad\qquad y = -\frac{2}{5}$$

Both answers check.

84. $\sqrt{5x - 11} - 5 = -3$

Solution

$$\sqrt{5x - 11} - 5 = -3$$
$$\sqrt{5x - 11} = 2$$
$$\left(\sqrt{5x - 11}\right)^2 = 2^2$$
$$5x - 11 = 4$$
$$x = 3$$

The solution checks.

85. $\sqrt{x - 1} + x = 7$

Solution

$$\sqrt{x - 1} + x = 7$$
$$\sqrt{x - 1} = 7 - x$$
$$\left(\sqrt{x - 1}\right)^2 = (7 - x)^2$$
$$x - 1 = 49 - 14x + x^2$$
$$0 = x^2 - 15x + 50$$
$$0 = (x - 5)(x - 10)$$
$$x - 5 = 0 \quad \textbf{or} \quad x - 10 = 0$$
$$x = 5 \qquad\qquad x = 10$$

$x = 10$ does not check and is extraneous

86. $\sqrt{a+9}-\sqrt{a}=3$

Solution

$$\begin{aligned}
\sqrt{a+9}-\sqrt{a} &= 3\\
\sqrt{a+9} &= 3+\sqrt{a}\\
\left(\sqrt{a+9}\right)^2 &= \left(3+\sqrt{a}\right)^2\\
a+9 &= 9+6\sqrt{a}+a\\
0 &= 6\sqrt{a}\\
0^2 &= \left(6\sqrt{a}\right)^2\\
0 &= 36a \Rightarrow a=0
\end{aligned}$$

The solution checks.

87. $\sqrt{5-x}+\sqrt{5+x}=4$

Solution

$$\begin{aligned}
\sqrt{5-x}+\sqrt{5+x} &= 4\\
\sqrt{5+x} &= 4-\sqrt{5-x}\\
\left(\sqrt{5+x}\right)^2 &= \left(4-\sqrt{5-x}\right)^2\\
5+x &= 16-8\sqrt{5-x}+5-x\\
8\sqrt{5-x} &= 16-2x\\
\left(8\sqrt{5-x}\right)^2 &= \left(16-2x\right)^2\\
64\left(5-x\right) &= 256-64x+4x^2\\
320-64x &= 4x^2-64x+256\\
0 &= 4x^2-64\\
0 &= 4\left(x+4\right)\left(x-4\right)
\end{aligned}$$

$x+4=0$ **or** $x-4=0$

$x=-4 \qquad x=4$

Both solutions check.

88. $\sqrt{y+5}+\sqrt{y}=1$

Solution

$$\begin{aligned}
\sqrt{y+5}+\sqrt{y} &= 1\\
\sqrt{y+5} &= 1-\sqrt{y}\\
\left(\sqrt{y+5}\right)^2 &= \left(1-\sqrt{y}\right)^2
\end{aligned}$$

$$\begin{aligned} y + 5 &= 1 - 2\sqrt{y} + y \\ 2\sqrt{y} &= -4 \\ \left(2\sqrt{y}\right)^2 &= (-4)^2 \\ 4y &= 16 \\ y &= 4 \end{aligned}$$

The solution does not check. $\Rightarrow$ No solution.

89. $\sqrt[3]{4x - 9} + 3 = 2$

Solution

$$\begin{aligned} \sqrt[3]{4x - 9} + 3 &= 2 \\ \sqrt[3]{4x - 9} &= -1 \\ \left(\sqrt[3]{4x - 9}\right)^3 &= (-1)^3 \\ 4x - 9 &= -1 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

The solution checks.

90. $\sqrt[4]{x - 2} + 3 = 5$

Solution

$$\begin{aligned} \sqrt[4]{x - 2} + 3 &= 5 \\ \sqrt[4]{x - 2} &= 2 \\ \left(\sqrt[4]{x - 2}\right)^4 &= (2)^4 \\ x - 2 &= 16 \\ x &= 18 \end{aligned}$$

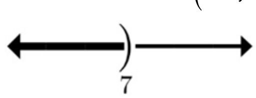
The solution checks.

Solve each inequality; graph the solution set and write the answer in interval notation.

91. $2x - 9 < 5$

Solution

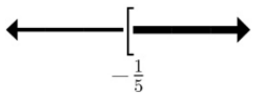
$$\begin{aligned} 2x - 9 &< 5 \\ 2x &< 14 \\ x &< 7 \Rightarrow (-\infty, 7) \end{aligned}$$



92. $5x + 3 \geq 2$

Solution

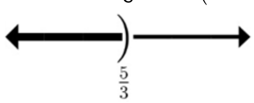
$$
\begin{aligned}
5x + 3 &\geq 2 \\
5x &\geq -1 \\
x &\geq -\tfrac{1}{5} \Rightarrow \left[-\tfrac{1}{5}, \infty\right)
\end{aligned}
$$



93. $\dfrac{5(x - 1)}{2} < x$

Solution

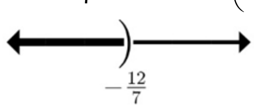
$$
\begin{aligned}
\frac{5(x - 1)}{2} &< x \\
5(x - 1) &< 2x \\
5x - 5 &< 2x \\
3x &< 5 \\
x &< \tfrac{5}{3} \Rightarrow \left(-\infty, \tfrac{5}{3}\right)
\end{aligned}
$$



94. $\dfrac{1}{4}x + \dfrac{2}{3}x - x > \dfrac{1}{2} + \dfrac{1}{2}(x + 1)$

Solution

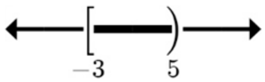
$$
\begin{aligned}
\frac{1}{4}x + \frac{2}{3}x - x &> \frac{1}{2} + \frac{1}{2}(x + 1) \\
12\left(\frac{1}{4}x + \frac{2}{3}x - x\right) &> 12\left(\frac{1}{2} + \frac{1}{2}(x + 1)\right) \\
3x + 8x - 12x &> 6 + 6(x + 1) \\
-x &> 6 + 6x + 6 \\
-12 &> 7x \\
-\frac{12}{7} &> x \Rightarrow \left(-\infty, -\frac{12}{7}\right)
\end{aligned}
$$



95. $0 \le \dfrac{3 + x}{2} < 4$

Solution

$$0 \le \frac{3 + x}{2} < 4$$
$$0 \le 3 + x < 8$$
$$-3 \le x < 5 \Rightarrow [-3, 5)$$



96. $2 + a < 3a - 2 \le 5a + 2$

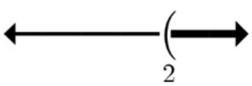
Solution

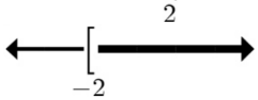
$$2 + a < 3a - 2 \le 5a + 2$$

$2 + a < 3a - 2$ **and** $3a - 2 \le 5a + 2$

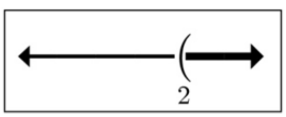
$4 < 2a$ $-4 \le 2a$

$a > 2$ $a \ge -2$

$a > 2$ 

$a \ge -2$ 

$a > 2$
and
$a \ge -2$



Solution set: $(2, \infty)$

97. $(x + 2)(x - 4) > 0$

Solution

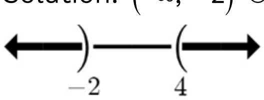
$(x + 2)(x - 4) > 0$

factors $= 0$: $x = -2$, $x = 4$

intervals: $(-\infty, -2)$, $(-2, 4)$, $(4, \infty)$

interval	test number	value of $(x + 2)(x - 4)$
$(-\infty, -2)$	−3	+7
$(-2, 4)$	0	−8
$(4, \infty)$	5	+7

Solution: $(-\infty, -2) \cup (4, \infty)$



98. $(x - 1)(x + 4) < 0$

Solution

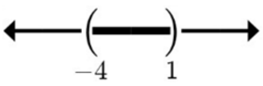
$(x - 1)(x + 4) < 0$

factors $= 0$: $x = 1$, $x = -4$

intervals: $(-\infty, -4)$, $(-4, 1)$, $(1, \infty)$

interval	test number	value of $(x - 1)(x + 4)$
$(-\infty, -4)$	−5	+6
$(-4, 1)$	0	−4
$(1, \infty)$	2	+8

Solution: $(-4, 1)$



99. $x^2 - 2x - 3 \le 0$

Solution

$x^2 - 2x - 3 \le 0$

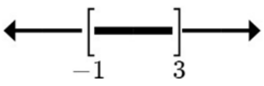
$(x - 3)(x + 1) \le 0$

factors $= 0$: $x = 3$, $x = -1$

intervals: $(-\infty, -1)$, $(-1, 3)$, $(3, \infty)$

interval	test number	value of $x^2 - 2x - 3$
$(-\infty, -1)$	−2	+5
$(-1, 3)$	0	−3
$(3, \infty)$	4	+5

Solution: $[-1, 3]$



100. $2x^2 + x > 3$

Solution

$2x^2 + x - 3 > 0$

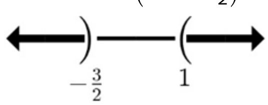
$(2x + 3)(x - 1) > 0$

factors $= 0$: $x = -\frac{3}{2}$, $x = 1$

intervals: $\left(-\infty, -\frac{3}{2}\right)$, $\left(-\frac{3}{2}, 1\right)$, $(1, \infty)$

interval	test number	value of $2x^2 + x - 3$
$\left(-\infty, -\frac{3}{2}\right)$	−2	+3
$\left(-\frac{3}{2}, 1\right)$	0	−3
$(1, \infty)$	2	+7

Solution: $\left(-\infty, -\frac{3}{2}\right) \cup (1, \infty)$



101. $\dfrac{x + 2}{x - 3} \geq 0$

Solution

$\dfrac{x + 2}{x - 3} \geq 0$

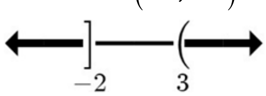
factors $= 0$: $x = -2$, $x = 3$

intervals: $(-\infty, -2)$, $(-2, 3)$, $(3, \infty)$

interval	test number	sign of $\dfrac{x + 2}{x - 3}$
$(-\infty, -2)$	−3	+
$(-2, 3)$	0	−
$(3, \infty)$	4	+

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $(-\infty, -2) \cup (3, \infty)$



102. $\frac{x - 1}{x + 4} \le 0$

Solution

$\frac{x - 1}{x + 4} \le 0$

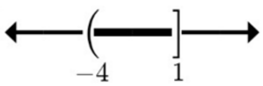
factors $= 0$: $x = 1$, $x = -4$

intervals: $(-\infty, -4)$, $(-4, 1)$, $(1, \infty)$

interval	test number	sign of $\frac{x - 1}{x + 4}$
$(-\infty, -4)$	−5	+
$(-4, 1)$	0	−
$(1, \infty)$	2	+

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $(-4, 1]$



103. $\frac{x^2 + x - 2}{x - 3} \ge 0$

Solution

$\frac{x^2 + x - 2}{x - 3} \ge 0$

$\frac{(x + 2)(x - 1)}{x - 3} \ge 0$

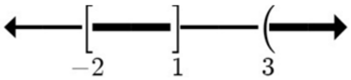
factors $= 0$: $x = -2$, $x = 1$, $x = 3$

intervals: $(-\infty, -2)$, $(-2, 1)$, $(1, 3)$, $(3, \infty)$

interval	test number	sign of $\frac{x^2 + x - 2}{x - 3}$
$(-\infty, -2)$	−3	−
$(-2, 1)$	0	+
$(1, 3)$	2	−
$(3, \infty)$	4	+

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $[-2, 1] \cup (3, \infty)$



104. $\frac{5}{x} < 2$

Solution

$$\frac{5}{x} < 2$$

$$\frac{5}{x} - 2 < 0$$

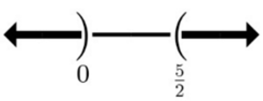
$$\frac{5 - 2x}{x} < 0$$

factors $= 0$: $x = \frac{5}{2}$, $x = 0$

intervals: $(-\infty, 0)$, $\left(0, \frac{5}{2}\right)$, $\left(\frac{5}{2}, \infty\right)$

interval	test number	value of $\frac{5-2x}{x}$
$(-\infty, 0)$	–1	–7
$\left(0, \frac{5}{2}\right)$	1	+3
$\left(\frac{5}{2}, \infty\right)$	3	$-\frac{1}{3}$

Solution: $(-\infty, 0) \cup \left(\frac{5}{2}, \infty\right)$



Solve each equation or inequality.

105. $|x + 1| = 6$

Solution

$$|x + 1| = 6$$

$x + 1 = 6$ **or** $x + 1 = -6$

$x = 5$ $\qquad$ $x = -7$

106. $\left|\frac{3x + 11}{7}\right| - 1 = 0$

Solution

$$\left|\frac{3x+11}{7}\right| - 1 = 0$$

$$\left|\frac{3x+11}{7}\right| = 1$$

$$\frac{3x+11}{7} = 1 \quad \textbf{or} \quad \frac{3x+11}{7} = -1$$

$$3x + 11 = 7 \qquad 3x + 11 = -7$$

$$3x = -4 \qquad 3x = -18$$

$$x = -\tfrac{4}{3} \qquad x = -6$$

107. $\left|\frac{2a-6}{3a}\right| - 6 = 0$

Solution

$$\left|\frac{2a-6}{3a}\right| - 6 = 0$$

$$\left|\frac{2a-6}{3a}\right| = 6$$

$$\frac{2a-6}{3a} = 6 \quad \textbf{or} \quad \frac{2a-6}{3a} = -6$$

$$2a - 6 = 18a \qquad 2a - 6 = -18a$$

$$-16a = 6 \qquad 20a = 6$$

$$a = -\tfrac{6}{16} \qquad a = \tfrac{6}{20}$$

$$a = -\tfrac{3}{8} \qquad a = \tfrac{3}{10}$$

108. $|2x - 1| = |2x + 1|$

Solution

$$|2x - 1| = |2x + 1|$$

$$2x - 1 = 2x + 1 \quad \textbf{or} \quad 2x - 1 = -(2x + 1)$$

$$0 = 2 \qquad 2x - 1 = -2x - 1$$

never true

$$4x = 0$$

$$x = 0$$

109. $|3x - 11| + 16 = 5$

Solution

$$|3x - 11| + 16 = 5$$

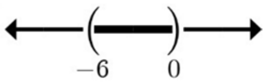
$$|3x - 11| = -11$$

An absolute value can never equal a negative number. no solution

110. $|x + 3| < 3$

Solution

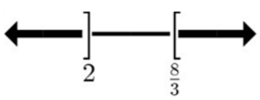
$$|x + 3| < 3$$
$$-3 < x + 3 < 3$$
$$-6 < x < 0$$
$$(-6, 0)$$



111. $|3x - 7| \geq 1$

Solution

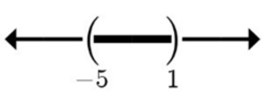
$$|3x - 7| \geq 1$$
$$3x - 7 \geq 1 \quad \textbf{or} \quad 3x - 7 \leq -1$$
$$3x \geq 8 \qquad 3x \leq 6$$
$$x \geq \tfrac{8}{3} \qquad x \leq 2$$
$$(-\infty, 2] \cup \left[\tfrac{8}{3}, \infty\right)$$



112. $\left|\dfrac{x + 2}{3}\right| + 5 < 6$

Solution

$$\left|\tfrac{x+2}{3}\right| + 5 < 6$$
$$\left|\tfrac{x+2}{3}\right| < 1$$
$$-1 < \tfrac{x+2}{3} < 1$$
$$-3 < x + 2 < 3$$
$$-5 < x < 1$$
$$(-5, 1)$$



113. $\left|\dfrac{x-3}{4}\right| > 8$

Solution

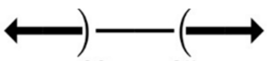
$$\left|\frac{x-3}{4}\right| > 8$$

$\frac{x-3}{4} > 8$ **or** $\frac{x-3}{4} < -8$

$x - 3 > 32 \qquad x - 3 < -32$

$x > 35 \qquad x < -29$

$(-\infty, -29) \cup (35, \infty)$



114. $1 > |2x + 3| < 4$

Solution

$$1 < |2x + 3| < 4$$

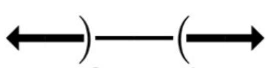
$1 < |2x + 3|$ **and** $|2x + 3| < 4$

(1) $|2x + 3| > 1$ **(2)** $|2x + 3| < 4$

$2x + 3 > 1$ **or** $2x + 3 < -1$ $\qquad -4 < 2x + 3 < 4$

$2x > -2 \qquad 2x < -4 \qquad -7 < 2x < 1$

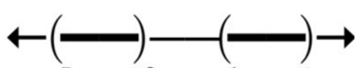
$x > -1 \qquad x < -2 \qquad -\frac{7}{2} < x < \frac{1}{2}$

(1) 

(2) 

(1) 

(2) 

(1) and (2) 

$\Rightarrow \left(-\frac{7}{2}, -2\right) \cup \left(-1, \frac{1}{2}\right)$

115. $0 < |3x - 4| < 7$

Solution

$$0 < |3x - 4| < 7$$

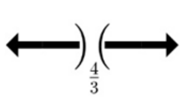
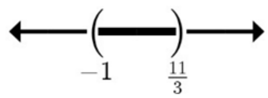
$$0 < |3x - 4| \quad \textbf{and} \quad |3x - 4| < 7$$

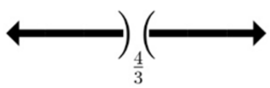
$$(\mathbf{1})\ |3x - 4| > 0 \qquad (\mathbf{2})\ |3x - 4| < 7$$

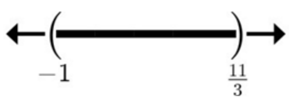
$$3x - 4 > 0 \ \textbf{ or } \ 3x - 4 < -0 \qquad -7 < 3x - 4 < 7$$

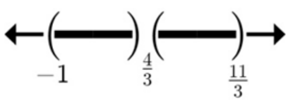
$$3x > 4 \qquad 3x < 4 \qquad -3 < 3x < 11$$

$$x > \tfrac{4}{3} \qquad x < \tfrac{4}{3} \qquad -1 < x < \tfrac{11}{3}$$

(1)  **(2)** 

(1) 

(2) 

(1) and (2) 

$$\Rightarrow \left(-1, \tfrac{4}{3}\right) \cup \left(\tfrac{4}{3}, \tfrac{11}{3}\right)$$

Chapter Test Solutions

Find all restrictions on *x*.

1. $\dfrac{x}{x(x - 1)} = 2$

Solution

$$\frac{x}{x(x - 1)} = 2$$

restrictions: $x \neq 0$, $x \neq 1$

2. $\dfrac{4}{3x - 2} + 3 = 7$

Solution

$$\frac{4}{3x - 2} + 3 = 7$$

restrictions: $x \neq \frac{2}{3}$

Solve each equation.

3. $7(2a + 5) - 7 = 6(a + 8)$

Solution

$$\begin{aligned} 7(2a + 5) - 7 &= 6(a + 8) \\ 14a + 35 - 7 &= 6a + 48 \\ 8a &= 20 \\ a &= \frac{20}{8} = \frac{5}{2} \end{aligned}$$

4. $\frac{1}{a - 2} - \frac{1}{5a} = \frac{3}{2a}$

Solution

$$\begin{aligned} \frac{1}{a - 2} - \frac{1}{5a} &= \frac{3}{2a} \\ 10a(a - 2)\left(\frac{1}{a - 2} - \frac{1}{5a}\right) &= 10a(a - 2) \cdot \frac{3}{2a} \\ 10a(1) - 2(a - 2) &= 15(a - 2) \\ 10a - 2a + 4 &= 15a - 30 \\ 34 &= 7a \\ \frac{34}{7} &= a \end{aligned}$$

5. Solve for x: $z = \frac{x - \mu}{\sigma}$.

Solution

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ az &= a \cdot \frac{x - \mu}{\sigma} \\ za &= x - \mu \\ za + \mu &= x \end{aligned}$$

6. Solve for a: $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$.

Solution

$$\begin{aligned}
\frac{1}{a} &= \frac{1}{b} + \frac{1}{c} \\
abc \cdot \frac{1}{a} &= abc\left(\frac{1}{b} + \frac{1}{c}\right) \\
bc &= ac + ab \\
bc &= a(c + b) \\
\frac{bc}{c + b} &= \frac{a(c + b)}{c + b} \\
\frac{bc}{c + b} &= a
\end{aligned}$$

7. **Test scores** A student's average on three tests is 75. If the final is to count as two one-hour tests, what grade must the student make to bring the average up to 80?

Solution
Let x = the score on the final exam. Note: This score is counted twice.

$$\begin{aligned}
\frac{\boxed{\text{Sum of scores}}}{5} &= 80 \\
\frac{75 + 75 + 75 + x + x}{5} &= 80 \\
\frac{2x + 225}{5} &= 80 \\
2x + 225 &= 400 \\
2x &= 175 \\
x &= 87.5
\end{aligned}$$

The student needs to score 87.5.

8. **Investment** A woman invested part of \$20,000 at 6% interest and the rest at 7%. If her annual interest is \$1260, how much did she invest at 6%?

Solution
Let x = the amount invested at 6%. Then 20,000 − x = the amount invested at 7%.

$$\begin{aligned}
\boxed{\begin{array}{c}\text{Interest}\\ \text{at 6\%}\end{array}} + \boxed{\begin{array}{c}\text{Interest}\\ \text{at 7\%}\end{array}} &= \boxed{\begin{array}{c}\text{Total}\\ \text{Interest}\end{array}} \\
0.06x + 0.07(20{,}000 - x) &= 1{,}260 \\
0.06x + 1{,}400 - 0.07x &= 1{,}260 \\
-0.01x &= -140 \\
x &= 14{,}000
\end{aligned}$$

\$14,000 was invested at 6%.

Simplify the imaginary numbers.

9. $3\sqrt{-96}$

Solution

$$3\sqrt{-96} = 3\sqrt{-1}\sqrt{16}\sqrt{6} = 12i\sqrt{6}$$

10. $\sqrt{-\frac{18}{5}}$

Solution

$$\sqrt{-\frac{18}{5}} = \sqrt{-\frac{18 \cdot 5}{5 \cdot 5}} = \sqrt{-\frac{90}{25}} = \frac{\sqrt{-1}\sqrt{9}\sqrt{10}}{\sqrt{25}} = \frac{3\sqrt{10}}{5}i$$

Perform each operation and write all answers in *a* + *bi* form.

11. $(4 - 5i) - (-3 + 7i)$

Solution

$$\begin{aligned}(4 - 5i) - (-3 + 7i) &= 4 - 5i + 3 - 7i \\ &= 7 - 12i\end{aligned}$$

12. $(4 - 5i)(3 - 7i)$

Solution

$$\begin{aligned}(4 - 5i)(3 - 7i) &= 12 - 43i + 35i^2 \\ &= 12 - 43i - 35 \\ &= -23 - 43i\end{aligned}$$

13. $\frac{2}{2 - i}$

Solution

$$\frac{2}{2 - i} = \frac{2(2 + i)}{(2 - i)(2 + i)} = \frac{4 + 2i}{2^2 - i^2} = \frac{4 + 2i}{5} = \frac{4}{5} + \frac{2}{5}i$$

14. $\frac{1 + i}{1 - i}$

Solution

$$\frac{1 + i}{1 - i} = \frac{(1 + i)(1 + i)}{(1 - i)(1 + i)} = \frac{1 + 2i + i^2}{1^2 - i^2} = \frac{2i}{2} = 0 + i$$

Simplify each expression.

15. i^{13}

Solution

$i^{13} = i^{12}i = \left(i^4\right)^3 i = 1^3 i = i$

16. $7i^4$

Solution

$1i^4 = 1 \cdot 1 = 1$

Solve each equation.

17. $4x^2 - 8x + 3 = 0$

Solution

$$4x^2 - 8x + 3 = 0$$
$$(2x - 3)(2x - 1) = 0$$
$$2x - 3 = 0 \quad \textbf{or} \quad 2x - 1 = 0$$
$$2x = 3 \qquad 2x = 1$$
$$x = \tfrac{3}{2} \qquad x = \tfrac{1}{2}$$

18. $2b^2 - 12 = -5b$

Solution

$$2b^2 - 12 = -5b$$
$$2b^2 + 5b - 12 = 0$$
$$(2b - 3)(b + 4) = 0$$
$$2b - 3 = 0 \quad \textbf{or} \quad b + 4 = 0$$
$$2b = 3 \qquad b = -4$$
$$b = \tfrac{3}{2} \qquad b = -4$$

19. $5x^2 = -135$

Solution

$$5x^2 = -135$$
$$x^2 = -27$$
$$x = \pm\sqrt{-27}$$
$$x = \pm 3i\sqrt{3}$$

20. Use completing the square to solve $x^2 - 14x = 23$.

Solution

$$x^2 - 14x = 23$$
$$x^2 - 14x + 49 = 23 + 49$$
$$(x - 7)^2 = 72$$

$x - 7 = \sqrt{72}$ **or** $x - 7 = -\sqrt{72}$

$x - 7 = 6\sqrt{2}$ $\quad x - 7 = -6\sqrt{2}$

$x = 7 + 6\sqrt{2}$ $\quad x = 7 - 6\sqrt{2}$

21. Use the Quadratic Formula to solve $3x^2 - 5x - 9 = 0$.

Solution

$3x^2 - 5x - 9 = 0 \Rightarrow a = 3, b = -5, c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-9)}}{2(3)} = \frac{5 \pm \sqrt{25 + 108}}{6} = \frac{5 \pm \sqrt{133}}{6}$$

22. $\dfrac{3}{x^2 - 5x - 14} = \dfrac{4}{x^2 + 5x + 6}$

Solution

$$\frac{3}{x^2 - 5x - 14} = \frac{4}{x^2 + 5x + 6}$$
$$\frac{3}{(x - 7)(x - 2)} = \frac{4}{(x + 2)(x + 3)}$$
$$(x - 7)(x + 2)(x + 3)\frac{3}{(x - 7)(x + 2)} = (x - 7)(x + 2)(x + 3)\frac{4}{(x + 2)(x + 3)}$$
$$3(x + 3) = 4(x - 7)$$
$$3x + 9 = 4x - 28$$
$$37 = x$$

23. Find k such that $x^2 + (k + 1)x + k + 4 = 0$ will have two equal roots.

Solution

$x^2 + (k + 1)x + k + 4 = 0$

$a = 1, b = k + 1, c = k + 4$

Set the discriminant equal to 0:

$$b^2 - 4ac = 0$$
$$(k+1)^2 - 4(1)(k+4) = 0$$
$$k^2 + 2k + 1 - 4k - 16 = 0$$
$$k^2 - 2k - 15 = 0$$
$$(k-5)(k+3) = 0$$
$$k - 5 = 0 \quad \textbf{or} \quad k + 3 = 0$$
$$k = 5 \qquad\qquad k = -3$$

24. **Height of a projectile** The height h (in feet) of a projectile shot up into the air, at time t (in seconds), is given by the formula $h = -16t^2 + 128t$. Find the time t required for the projectile to return to its starting point.

Solution

Set $h = 0$:

$$h = -16t^2 + 128t$$
$$0 = -16t^2 + 128t$$
$$0 = -16t^2(t - 8)$$
$$-16t = 0 \quad \textbf{or} \quad t - 8 = 0$$
$$t = 0 \qquad\qquad t = 8$$

The projectile will return after 8 seconds.

Find each absolute value.

25. $|5 - 12i|$

Solution

$$|5 - 12i| = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

26. $\left|\dfrac{1}{3+i}\right|$

Solution

$$\left|\frac{1}{3+i}\right| = \left|\frac{1(3-i)}{(3+i)(3-i)}\right| = \left|\frac{3-i}{3^2 - i^2}\right| = \left|\frac{3-i}{10}\right| = \left|\frac{3}{10} - \frac{1}{10}i\right| = \sqrt{\left(\frac{3}{10}\right)^2 + \left(-\frac{1}{10}\right)^2}$$
$$= \sqrt{\frac{9}{100} + \frac{1}{100}}$$
$$= \sqrt{\frac{10}{100} + \frac{\sqrt{10}}{10}}$$

Solve each equation.

27. $z^4 - 13z^2 + 36 = 0$

Solution

$$z^4 - 13z^2 + 36 = 0$$
$$\left(z^4 - 4\right)\left(z^2 - 9\right) = 0$$

$$z^2 - 4 = 0 \quad \textbf{or} \quad z^2 - 9 = 0$$
$$z^2 = 4 \qquad\qquad z^2 = 9$$
$$z = \pm 2 \qquad\qquad z = \pm 3$$

28. $2p^{2/5} - p^{1/5} - 1 = 0$

Solution

$$2p^{2/5} - p^{1/5} - 1 = 0$$
$$\left(2p^{1/5} + 1\right)\left(p^{1/5} - 1\right) = 0$$

$$2p^{1/5} + 1 = 0 \quad \textbf{or} \quad p^{1/5} - 1 = 0$$
$$p^{1/5} = -\tfrac{1}{2} \qquad\qquad p^{1/5} = 1$$
$$\left(p^{1/5}\right)^5 = \left(-\tfrac{1}{2}\right)^5 \qquad\qquad \left(p^{1/5}\right)^5 = \left(1\right)^5$$
$$p = -\tfrac{1}{32} \qquad\qquad p = 1$$

Both answers check.

29. $\sqrt{x + 5} = 12$

Solution

$$\sqrt{x + 5} = 12$$
$$\left(\sqrt{x + 5}\right)^2 = 12^2$$
$$x + 5 = 144$$
$$x = 139$$

The answer checks.

30. $\sqrt{2z + 3} = 1 - \sqrt{z + 1}$

Solution

$$\sqrt{2z + 3} = 1 - \sqrt{z + 1}$$
$$\left(\sqrt{2z + 3}\right)^2 = \left(1 - \sqrt{z + 1}\right)^2$$
$$2z + 3 = 1 - 2\sqrt{z + 1} + z + 1$$
$$2\sqrt{z + 1} = -z - 1$$
$$\left(2\sqrt{z + 1}\right)^2 = \left(-z - 1\right)^2$$
$$4\left(z + 1\right) = z^2 + 2z + 1$$
$$4z + 4 = z^2 + 2z + 1$$

$$0 = z^2 - 2z - 3$$
$$0 = (z + 1)(z - 3)$$
$$z + 1 = 0 \quad \textbf{or} \quad z - 3 = 0$$
$$z = -1 \qquad\qquad z = 3$$

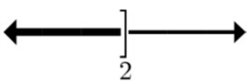
The answer $z = 3$ is extraneous.

Solve each inequality; graph the solution set and write the answer using interval notation.

31. $5x - 3 \le 7$

Solution

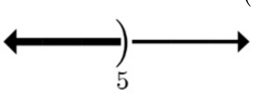
$$5x - 3 \le 7$$
$$5x \le 10$$
$$x \le 2 \Rightarrow (-\infty, 2]$$



32. $\dfrac{x + 3}{4} > \dfrac{2x - 4}{3}$

Solution

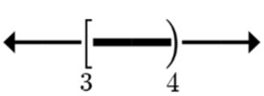
$$\frac{x + 3}{4} > \frac{2x - 4}{3}$$
$$12 \cdot \tfrac{x+3}{4} > 12 \cdot \tfrac{2x-4}{3}$$
$$3(x + 3) > 4(2x - 4)$$
$$3x + 9 > 8x - 16$$
$$-5x > -25$$
$$x < 5 \Rightarrow (-\infty, 5)$$



33. $5 \le 2x - 1 < 7$

Solution

$$5 \le 2x - 1 < 7$$
$$6 \le 2x < 8$$
$$3 \le x < 4 \Rightarrow [3, 4)$$



34. $1 + x < 3x - 3 < 4x - 2$

Solution

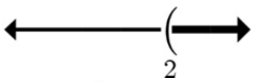
$$1 + x < 3x - 3 < 4x - 2$$

$1 + x < 3x - 3$ **and** $3x - 3 < 4x - 2$

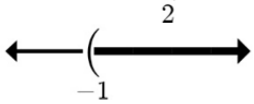
$-2x < -4$ $\qquad$ $-x < 1$

$x > 2$ $\qquad$ $x > -1$

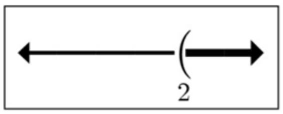
$x > 2$



$x > -1$



$x > 2$
and
$x > -1$



Solution: $(2, \infty)$

35. $x^2 - 7x - 8 \geq 0$

Solution

$x^2 - 7x - 8 \geq 0$

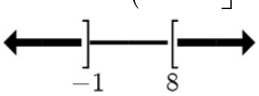
$(x + 1)(x - 8) \geq 0$

factors $= 0$: $x = -1$, $x = 8$

intervals: $(-\infty, -1)$, $(-1, 8)$, $(8, \infty)$

interval	test number	value of $x^2 - 7x - 8$
$(-\infty, -1)$	−2	+10
$(-1, 8)$	0	−8
$(8, \infty)$	9	+10

Solution: $(-\infty, -1] \cup [8, \infty)$



36. $\dfrac{x + 2}{x - 1} \leq 0$

Solution

$\dfrac{x + 2}{x - 1} \leq 0$

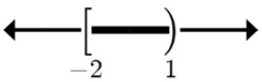
factors $= 0$: $x = -2$, $x = 1$

intervals: $(-\infty, -2)$, $(-2, 1)$, $(1, \infty)$

interval	test number	sign of $\frac{x+2}{x-1}$
$(-\infty,\ -2)$	-3	$+$
$(-2,\ 1)$	0	$-$
$(1,\ \infty)$	2	$+$

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $[-2,\ 1)$



Solve each equation.

37. $\left|\dfrac{3x+2}{2}\right| = 4$

Solution

$$\left|\frac{3x+2}{2}\right| = 4$$

$$\begin{aligned} \tfrac{3x+2}{2} &= 4 \quad \textbf{or} & \tfrac{3x+2}{2} &= -4 \\ 3x+2 &= 8 & 3x+2 &= -8 \\ 3x &= 6 & 3x &= -10 \\ x &= 2 & x &= -\tfrac{10}{3} \end{aligned}$$

38. $|x+3| = |x-3|$

Solution

$$|x+3| = |x-3|$$

$$\begin{aligned} x+3 &= x-3 \quad \textbf{or} & x+3 &= -(x-3) \\ 0 &= -6 & x+3 &= -x+3 \\ &\text{not true} & 2x &= 0 \\ & & x &= 0 \end{aligned}$$

Solve each inequality; graph the solution set and write the answer using interval notation.

39. $|2x-5| > 2$

Solution

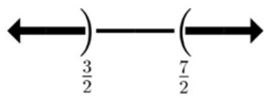
$$|2x - 5| > 2$$

$$2x - 5 > 2 \quad \textbf{or} \quad 2x - 5 < -2$$

$$2x > 7 \qquad\qquad 2x < 3$$

$$x > \tfrac{7}{2} \qquad\qquad x < \tfrac{3}{2}$$

$$\left(-\infty, \tfrac{3}{2}\right) \cup \left(\tfrac{7}{2}, \infty\right)$$



40. $\left|\dfrac{2x + 3}{3}\right| \le 5$

Solution

$$\left|\frac{2x + 3}{3}\right| \le 5$$

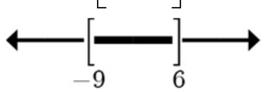
$$-5 \le \tfrac{2x+3}{3} \le 5$$

$$-15 \le 2x + 3 \le 15$$

$$-18 \le 2x \le 12$$

$$-9 \le x \le 6$$

$$[-9, 6]$$



Cumulative Review Exercises

Consider the set $\left\{-5, -3, -2, 0, 1, \sqrt{2}, 2, \frac{5}{2}, 5, 6, 11\right\}$.

1. Which numbers are even integers?

   Solution
   even integers: −2, 0, 2, 6

2. Which numbers are prime numbers?

   Solution
   prime numbers: 2, 5, 11

Write each inequality as an interval and graph it.

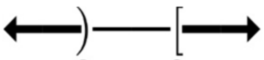
3. $-4 \le x < 7$

   Solution
   $-4 \le x < 7 \Rightarrow [-4, 7)$

4. $x \geq 2$ or $x < 0$

Solution

$x \geq 2$ or $x < 0 \Rightarrow (-\infty, 0) \cup [2, \infty)$



Determine which property of the real numbers justifies each expression.

5. $(a + b) + c = c + (a + b)$

Solution

Commutative Property of Addition

6. If $x < 3$ and $3 < y$, then $x < y$.

Solution

Transitive Property

Simplify each expression. Assume that all variables represent positive numbers. Give all answers with positive exponents.

7. $(81a^4)^{1/2}$

Solution

$$(81a^4)^{1/2} = \left[(9a^2)^2\right]^{1/2} = 9a^2$$

8. $81(a^4)^{1/2}$

Solution

$$81(a^4)^{1/2} = 81\left[(a^2)^2\right]^{1/2} = 81a^2$$

9. $(a^{-3}b^{-2})^{-2}$

Solution

$$(a^{-3}b^{-2})^{-2} = (a^{-3})^{-2}(b^{-2})^{-2} = a^6b^4$$

10. $\left(\dfrac{4x^4}{12x^2y}\right)^{-2}$

Solution

$$\left(\frac{4x^4}{12x^2y}\right)^{-2} = \left(\frac{12x^2y}{4x^4}\right)^2 = \left(\frac{3y}{x^2}\right)^2 = \frac{9y^2}{x^4}$$

11. $\left(\frac{4x^0y^2}{x^2y}\right)^{-2}$

Solution

$$\left(\frac{4x^0y^2}{x^2y}\right)^{-2} = \left(\frac{x^2y}{4x^0y^2}\right)^2 = \left(\frac{x^2}{4y}\right)^2 = \frac{x^4}{16y^2}$$

12. $\left(\frac{4x^{-5}y^2}{6x^{-2}y^{-3}}\right)^2$

Solution

$$\left(\frac{4x^{-5}y^2}{6x^{-2}y^{-3}}\right)^2 = \left(\frac{2y^5}{3x^3}\right)^2 = \frac{4y^{10}}{9x^6}$$

13. $\left(a^{1/2}b\right)^2\left(ab^{1/2}\right)^2$

Solution

$$\left(a^{1/2}b\right)^2\left(ab^{1/2}\right)^2 = \left(ab^2\right)\left(a^2b\right) = a^3b^3$$

14. $\left(a^{1/2}b^{1/2}c\right)^2$

Solution

$$\left(a^{1/2}b^{1/2}c\right)^2 = abc^2$$

Rationalize each denominator and simplify.

15. $\frac{3}{\sqrt{3}}$

Solution

$$\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

16. $\frac{2}{\sqrt[3]{4x}}$

Solution

$$\frac{2}{\sqrt[3]{4x}} = \frac{2\sqrt[3]{2x^2}}{\sqrt[3]{4x}\sqrt[3]{2x^2}} = \frac{2\sqrt[3]{2x^2}}{\sqrt[3]{8x^3}} = \frac{2\sqrt[3]{2x^2}}{2x} = \frac{\sqrt[3]{2x^2}}{x}$$

17. $\dfrac{3}{y - \sqrt{3}}$

Solution

$$\frac{3}{y - \sqrt{3}} = \frac{3\left(y + \sqrt{3}\right)}{\left(y - \sqrt{3}\right)\left(y + \sqrt{3}\right)} = \frac{3\left(y + \sqrt{3}\right)}{y^2 - \left(\sqrt{3}\right)^2} = \frac{3\left(y + \sqrt{3}\right)}{y^2 - 3}$$

18. $\dfrac{3x}{\sqrt{x} - 1}$

Solution

$$\frac{3x}{\sqrt{x} - 1} = \frac{3x\left(\sqrt{x} + 1\right)}{\left(\sqrt{x} - 1\right)\left(\sqrt{x} + 1\right)} = \frac{3x\left(\sqrt{x} + 1\right)}{\left(\sqrt{x}\right)^2 - 1^2} = \frac{3x\left(\sqrt{x} + 1\right)}{x - 1}$$

Simplify each expression and combine like terms.

19. $\sqrt{75} - 3\sqrt{5}$

Solution

$$\sqrt{75} - 3\sqrt{5} = \sqrt{25}\sqrt{3} - 3\sqrt{5} = 5\sqrt{3} - 3\sqrt{5}$$

20. $\sqrt{18} + \sqrt{8} - 2\sqrt{2}$

Solution

$$\sqrt{18} + \sqrt{8} - 2\sqrt{2} = \sqrt{9}\sqrt{2} + \sqrt{4}\sqrt{2} - 2\sqrt{2} = 3\sqrt{2} + 2\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$$

21. $\left(\sqrt{2} - \sqrt{3}\right)^2$

Solution

$$\left(\sqrt{2} - \sqrt{3}\right)^2 = \left(\sqrt{2} - \sqrt{3}\right)\left(\sqrt{2} - \sqrt{3}\right) = \sqrt{4} - 2\sqrt{6} + \sqrt{9} = 5 - 2\sqrt{6}$$

22. $\left(3 - \sqrt{5}\right)\left(3 + \sqrt{5}\right)$

Solution

$$\left(3 - \sqrt{5}\right)\left(3 + \sqrt{5}\right) = 9 - \sqrt{25} = 9 - 5 = 4$$

Perform the operations and simplify when necessary.

23. $\left(3x^2 - 2x + 5\right) - 3\left(x^2 + 2x - 1\right)$

Solution

$$\left(3x^2 - 2x + 5\right) - 3\left(x^2 + 2x - 1\right) = 3x^2 - 2x + 5 - 3x^2 - 6x + 3 = -8x + 8$$

24. $5x^2\left(2x^2 - x\right) + x\left(x^2 - x^3\right)$

Solution

$$5x^2\left(2x^2 - x\right) + x\left(x^2 - x^3\right) = 10x^4 - 5x^3 + x^3 - x^4 = 9x^4 - 4x^3$$

25. $\left(3x - 5\right)\left(2x + 7\right)$

Solution

$$\left(3x - 5\right)\left(2x + 7\right) = 6x^2 + 21x - 10x - 35 = 6x^2 + 11x - 35$$

26. $\left(z + 2\right)\left(z^2 - z + 2\right)$

Solution

$$\left(z + 2\right)\left(z^2 - z + 2\right) = z^3 - z^2 + 2z + 2z^2 - 2z + 4 = z^3 + z^2 + 4$$

27. $3x + 2\overline{\smash{)}6x^3 + x^2 + x + 2}$

Solution

$$\begin{array}{rl}
 & 2x^2 - x + 1 \\
3x + 2 & \overline{\smash{)}6x^3 + x^2 + x + 2} \\
 & \underline{6x^3 + 4x^2} \\
 & \quad -3x^2 + x \\
 & \quad \underline{-3x^2 - 2x} \\
 & \qquad\qquad 3x + 2 \\
 & \qquad\qquad \underline{3x + 2} \\
 & \qquad\qquad\qquad 0
\end{array}$$

28. $x^2 + 2\overline{\smash{)}3x^4 + 7x^2 - x + 2}$

Solution

$$\begin{array}{r|l} & 3x^2 + 1 + \frac{-x}{x^2+2} \\ x^2 + 2 & 3x^4 + 0x^3 + 7x^2 - x + 2 \\ & \underline{3x^4 \qquad\quad + 6x^2} \\ & \qquad\qquad x^2 - x + 2 \\ & \qquad\qquad \underline{x^2 \qquad + 2} \\ & \qquad\qquad\quad -x \end{array}$$

Factor each polynomial.

29. $3t^2 - 6t$

Solution

$3t^2 - 6t = 3t(t - 2)$

30. $3x^2 - 10x - 8$

Solution

$3x^2 - 10x - 8 = (3x + 2)(x - 4)$

31. $x^8 - 2x^4 + 1$

Solution

$$\begin{aligned} x^8 - 2x^4 + 1 = (x^4 - 1)(x^4 - 1) &= (x^2 + 1)(x^2 - 1)(x^2 + 1)(x^2 - 1) \\ &= (x^2 + 1)^2(x + 1)(x - 1)(x + 1)(x - 1) \\ &= (x^2 + 1)^2(x + 1)^2(x - 1)^2 \end{aligned}$$

32. $x^6 - 1$

Solution

$x^6 - 1 = (x^3)^2 - 1^2 = (x^3 + 1)(x^3 - 1) = (x + 1)(x^2 - x + 1)(x - 1)(x^2 + x + 1)$

Perform the operations and simplify.

33. $\dfrac{x^2 - 4}{x^2 + 5x + 6} \cdot \dfrac{x^2 - 2x - 15}{x^2 + 3x - 10}$

Solution

$$\frac{x^2 - 4}{x^2 + 5x + 6} \cdot \frac{x^2 - 2x - 15}{x^2 + 3x - 10} = \frac{(x + 2)(x - 2)}{(x + 2)(x + 3)} \cdot \frac{(x - 5)(x + 3)}{(x + 5)(x - 2)} = \frac{x - 5}{x + 5}$$

34. $\dfrac{6x^3 + x^2 - x}{x + 2} \div \dfrac{3x^2 - x}{x^2 + 4x + 4}$

Solution

$$\frac{6x^3 + x^2 - x}{x + 2} \div \frac{3x^2 - x}{x^2 + 4x + 4} = \frac{x\left(6x^2 + x - 1\right)}{x + 2} \cdot \frac{x^2 + 4x + 4}{3x^2 - x}$$
$$= \frac{x\left(2x + 1\right)\left(3x - 1\right)}{x + 2} \cdot \frac{\left(x + 2\right)\left(x + 2\right)}{x\left(3x - 1\right)} = \left(2x + 1\right)\left(x + 2\right)$$

35. $\dfrac{2}{x + 3} + \dfrac{5x}{x - 3}$

Solution

$$\frac{2}{x + 3} + \frac{5x}{x - 3} = \frac{2\left(x - 3\right)}{\left(x + 3\right)\left(x - 3\right)} + \frac{5x\left(x + 3\right)}{\left(x - 3\right)\left(x + 3\right)} = \frac{2x - 6}{\left(x + 3\right)\left(x - 3\right)} + \frac{5x^2 + 15x}{\left(x + 3\right)\left(x - 3\right)}$$
$$= \frac{5x^2 + 17x - 6}{\left(x + 3\right)\left(x - 3\right)}$$

36. $\dfrac{x - 2}{x + 3}\left(\dfrac{x + 3}{x^2 - 4} - 1\right)$

Solution

$$\frac{x - 2}{x + 3}\left(\frac{x + 3}{x^2 - 4} - 1\right) = \frac{x - 2}{x + 3}\left(\frac{x + 3}{x^2 - 4} - \frac{x^2 - 4}{x^2 - 4}\right) = \frac{x - 2}{x + 3}\left(\frac{-x^2 + x + 7}{\left(x + 2\right)\left(x - 2\right)}\right) = \frac{-x^2 + x + 7}{\left(x + 3\right)\left(x + 2\right)}$$

37. $\dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{ab}}$

Solution

$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{ab}} = \frac{ab\left(\frac{1}{a} + \frac{1}{b}\right)}{ab\left(\frac{1}{ab}\right)} = \frac{b + a}{1} = b + a$$

38. $\dfrac{x^{-1} - y^{-1}}{x - y}$

Solution

$$\frac{x^{-1} - y^{-1}}{x - y} = \frac{\frac{1}{x} - \frac{1}{y}}{x - y} = \frac{xy\left(\frac{1}{x} - \frac{1}{y}\right)}{xy\left(x - y\right)} = \frac{y - x}{xy\left(x - y\right)} = -\frac{1}{xy}$$

Solve each equation.

39. $\dfrac{3x}{x+5} = \dfrac{x}{x-5}$

Solution

$$\begin{aligned} \frac{3x}{x+5} &= \frac{x}{x-5} \\ 3x(x-5) &= x(x+5) \\ 3x^2 - 15x &= x^2 + 5x \\ 2x^2 - 20x &= 0 \\ 2x(x-10) &= 0 \end{aligned}$$

$$2x = 0 \quad \textbf{or} \quad x - 10 = 0$$
$$x = 0 \qquad\qquad x = 10$$

40. $8(2x-3) - 3(5x+2) = 4$

Solution

$$\begin{aligned} 8(2x-3) - 3(5x+2) &= 4 \\ 16x - 24 - 15x - 6 &= 4 \\ x &= 34 \end{aligned}$$

Solve each formula for the indicated variable.

41. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$; R

Solution

$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ RR_1R_2 \cdot \frac{1}{R} &= RR_1R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \\ R_1R_2 &= RR_2 + RR_1 \\ R_1R_2 &= R(R_2 + R_1) \\ \frac{R_1R_2}{R_2 + R_1} &= \frac{R(R_2 + R_1)}{R_2 + R_1} \\ \frac{R_1R_2}{R_2 + R_1} &= R \end{aligned}$$

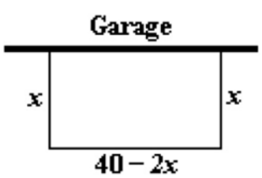
42. $S = \dfrac{a - lr}{1 - r}$; r

Solution

$$\begin{aligned} S &= \frac{a - lr}{1 - r} \\ S(1 - r) &= \frac{a - lr}{1 - r}(1 - r) \\ S(1 - r) &= a - lr \\ S - Sr &= a - lr \\ S - a &= Sr - lr \\ S - a &= r(S - l) \\ \frac{S - a}{S - l} &= r \end{aligned}$$

43. **Gardening** A gardener wishes to enclose her rectangular raspberry patch with 40 feet of fencing. The raspberry bushes are planted along the garage, so no fencing is needed on that side. Find the dimensions if the total area is to be 192 square feet.

Solution



$$\begin{aligned} \text{Area} &= 192 \\ x(40 - 2x) &= 192 \\ 40x - 2x^2 &= 192 \\ 0 &= 2x^2 - 40x + 192 \\ 0 &= 2(x - 8)(x - 12) \end{aligned}$$

$x - 8 = 0$ **or** $x - 12 = 0$

$x = 8$ $\quad$ $x = 12$

If $x = 8$, then the dimensions are 8 feet by 24 feet.

If $x = 12$, then the dimensions are 12 feet by 16 feet.

44. **Financial planning** A college student invested part of a $25,000 inheritance at 7% interest and the rest at 6%. If his annual interest is $1670, how much did he invest at 6%?

Solution

Let x = the amount invested at 6%. Then 25,000 − x = the amount invested at 7%.

$$\boxed{\begin{array}{c}\text{Interest}\\ \text{at 6\%}\end{array}} + \boxed{\begin{array}{c}\text{Interest}\\ \text{at 7\%}\end{array}} = \boxed{\begin{array}{c}\text{Total}\\ \text{interest}\end{array}}$$

$$\begin{aligned} 0.06x + 0.07(25{,}000 - x) &= 1{,}670 \\ 0.06x + 1{,}750 - 0.07x &= 1{,}670 \\ -0.01x &= -80 \\ x &= 8{,}000 \Rightarrow \$8{,}000 \text{ was invested at 6\%.} \end{aligned}$$

Perform the operations. If the result is not real, express the answer in *a* + *bi* form.

45. $\dfrac{2 + i}{2 - i}$

Solution

$$\frac{2 + i}{2 - i} = \frac{(2 + i)(2 + i)}{(2 - i)(2 + i)} = \frac{4 + 4i + i^2}{4 - i^2} = \frac{3 + 4i}{5} = \frac{3}{5} + \frac{4}{5}i$$

46. $\dfrac{i(3 - i)}{(1 + i)(1 + i)}$

Solution

$$\frac{i(3 - i)}{(1 + i)(1 + i)} = \frac{i(3 - i)(1 - i)(1 - i)}{(1 + i)(1 - i)(1 + i)(1 - i)} = \frac{(3i - i^2)(1 - 2i + i^2)}{(1 - i^2)(1 - i^2)}$$

$$= \frac{(1 + 3i)(-2i)}{1 - 2i^2 + i^4} = \frac{-2i - 6i^2}{4} = \frac{6 - 2i}{4} = \frac{3}{2} - \frac{1}{2}i$$

47. $|3 + 4i|$

Solution

$$|3 + 4i| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

48. $\dfrac{5}{i^7} + 5i$

Solution

$$\frac{5}{i^7} + 5i = \frac{5i}{i^7 i} + 5i - \frac{5i}{i^8} + 5i = \frac{5i}{(i^4)^2} + 5i = \frac{5i}{1^2} + 5i = 5i + 5i = 10i = 0 + 10i$$

Solve each equation.

49. $15x^2 - 16x - 7 = 0$

Solution

$$15x^2 - 16x - 7 = 0$$
$$(5x - 7)(3x + 1) = 0$$
$$5x - 7 = 0 \quad \textbf{or} \quad 3x + 1 = 0$$
$$x = \tfrac{7}{5} \qquad\qquad x = -\tfrac{1}{3}$$

50. $(7x - 4)^2 = -8$

Solution

$$(7x - 4)^2 = -8$$
$$\sqrt{(7x - 4)^2} = \pm\sqrt{-8}$$
$$(7x - 4) = \pm 2i\sqrt{2}$$
$$7x = 4 \pm 2i\sqrt{2}$$
$$x = \frac{4}{7} \pm \frac{2\sqrt{2}}{7}i$$

51. $\frac{x + 3}{x - 1} - \frac{6}{x} = 1$

Solution

$$\frac{x + 3}{x - 1} - \frac{6}{x} = 1$$
$$x(x - 1)\left(\frac{x + 3}{x - 1} - \frac{6}{x}\right) = x(x - 1)(1)$$
$$x(x + 3) - 6(x - 1) = x^2 - x$$
$$x^2 + 3x - 6x + 6 = x^2 - x$$
$$-2x = -6$$
$$x = 3$$

52. $x^4 + 36 = 13x^2$

Solution

$$x^4 + 36 = 13x^2$$
$$x^4 - 13x^2 + 36 = 0$$
$$(x^2 - 4)(x^2 - 9) = 0$$
$$x^2 - 4 = 0 \quad \textbf{or} \quad x^2 - 9 = 0$$
$$x^2 = 4 \qquad\qquad x^2 = 9$$
$$x = \pm 2 \qquad\qquad x = \pm 3$$

53. $\sqrt{y+2} + \sqrt{11-y} = 5$

Solution

$$\begin{aligned}
\sqrt{y+2} + \sqrt{11-y} &= 5 \\
\sqrt{y+2} - 5 &= -\sqrt{11-y} \\
\left(\sqrt{y+2} - 5\right)^2 &= \left(-\sqrt{11-y}\right)^2 \\
y + 2 - 10\sqrt{y+2} + 25 &= 11 - y \\
-10\sqrt{y+2} &= -2y - 16 \\
\left(-10\sqrt{y+2}\right)^2 &= \left(-2y - 16\right)^2 \\
100\left(y+2\right) &= 4y^2 + 64y + 256 \\
100y + 200 &= 4y^2 + 64y + 256 \\
0 &= 4y^2 - 36y + 56 \\
0 &= 4\left(y-2\right)\left(y-7\right)
\end{aligned}$$

$y - 2 = 0$ **or** $y - 7 = 0$

$y = 2$ $\quad$ $y = 7$

Both solutions check.

54. $z^{2/3} - 13z^{1/3} + 36 = 0$

Solution

$$\begin{aligned}
z^{2/3} - 13z^{1/3} + 36 &= 0 \\
\left(z^{1/3} - 4\right)\left(z^{1/3} - 9\right) &= 0
\end{aligned}$$

$z^{1/3} - 4 = 0$ **or** $z^{1/3} - 9 = 0$

$z^{1/3} = 4$ $\quad$ $z^{1/3} = 9$

$\left(z^{1/3}\right)^3 = 4^3$ $\quad$ $\left(z^{1/3}\right)^3 = 9^3$

$z = 64$ $\quad$ $z = 729$

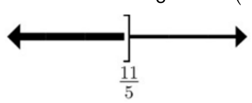
Both solutions check.

Solve each inequality; graph the solution set and write the answer using interval notation.

55. $5x - 7 \le 4$

Solution

$$\begin{aligned}
5x - 7 &\le 4 \\
5x &\le 11 \\
x &\le \tfrac{11}{5} \Rightarrow \left(-\infty, \tfrac{11}{5}\right]
\end{aligned}$$



56. $x^2 - 8x + 15 > 0$

Solution

$x^2 - 8x + 15 > 0$

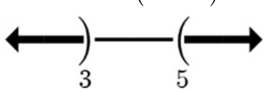
$(x - 3)(x - 5) > 0$

factors $= 0$: $x = 3$, $x = 5$

intervals: $(-\infty, 3)$, $(3, 5)$, $(5, \infty)$

interval	test number	value of $x^2 - 8x + 15$
$(-\infty, 3)$	0	+15
$(3, 5)$	4	1
$(5, \infty)$	6	+3

Solution: $(-\infty, 3) \cup (5, \infty)$



57. $\dfrac{x^2 + 4x + 3}{x - 2} \geq 0$

Solution

$\dfrac{x^2 + 4x + 3}{x - 2} \geq 0$

$\dfrac{(x + 3)(x + 1)}{x - 2} \geq 0$

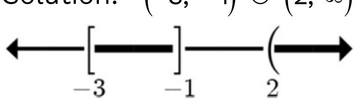
factors $= 0$: $x = -3$, $x = -1$, $x = 2$

intervals: $(-\infty, -3)$, $(-3, -1)$, $(-1, 2)$, $(2, \infty)$

interval	test number	sign of $\dfrac{x^2 + 4x + 3}{x - 2}$
$(-\infty, -3)$	−4	–
$(-3, -1)$	−2	+
$(-1, 2)$	0	–
$(2, \infty)$	3	+

Include endpoints which make the numerator equal to 0. Do not include endpoints which make the denominator equal to 0.

Solution: $(-3, -1) \cup (2, \infty)$



58. $\frac{9}{x} > x$

Solution

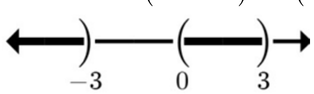
$$\frac{9}{x} > x$$
$$\frac{9}{x} - x > 0$$
$$\frac{9 - x^2}{x} > 0$$
$$\frac{(3 + x)(3 - x)}{x} > 0$$

factors $= 0$: $x = -3$, $x = 3$, $x = 0$

intervals: $(-\infty, -3)$, $(-3, 0)$, $(0, 3)$, $(3, \infty)$

interval	test number	sign of $\frac{9 - x^2}{x}$
$(-\infty, -3)$	-4	+
$(-3, 0)$	-1	–
$(0, 3)$	1	+
$(3, \infty)$	4	–

Solution: $(-\infty, -3) \cup (0, 3)$



59. $|2x - 3| \geq 5$

Solution

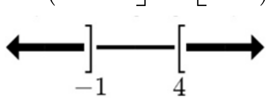
$$|2x - 3| \geq 5$$

$2x - 3 \geq 5$ **or** $2x - 3 \leq -5$

$2x \geq 8$ $\qquad$ $2x \leq -2$

$x \geq 4$ $\qquad$ $x \leq -1$

$$(-\infty, -1] \cup [4, \infty)$$



60. $\left|\frac{3x - 5}{2}\right| < 2$

Solution

$$\left|\frac{3x-5}{2}\right| < 2$$

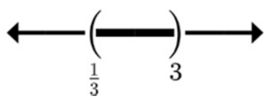
$$-2 < \frac{3x-5}{2} < 2$$

$$-4 < 3x-5 < 4$$

$$1 < 3x < 9$$

$$\frac{1}{3} < x < 3$$

$$\left(\frac{1}{3}, 3\right)$$



Group Activity Solutions

Health and Body Mass Index (BMI)

Real-World Example of a Quadratic Equation

The American Heart Association recommends that we know our "numbers." In addition to cholesterol, blood pressure, and glucose numbers, our Body Mass Index (BMI) is an important number to know. It is a measure of the level of body fat. A high BMI is related to a greater risk of developing heart disease, osteoarthritis, diabetes, stroke, and certain cancers.

Group Activity

BMI is calculated according the following formula.

$$\text{BMI} = \frac{703w}{h^2} \qquad w \text{ is weight in pounds; } h \text{ is height in inches}$$

- $\text{BMI} < 18.5 \Rightarrow$ underweight
- $18.5 \le \text{BMI} \le 24.9 \Rightarrow$ normal
- $25.0 \le \text{BMI} \le 29.9 \Rightarrow$ overweight
- $\text{BMI} \ge 30 \Rightarrow$ obese

a. The approximate weight and BMI of seven people are shown in the table. Determine the approximate height of each in inches. Round to the nearest inch.

Person	**BMI**	**Weight in Pounds**
A	17.9	118
B	30.8	240
C	21.0	130
D	26.0	166
E	28.8	224
F	22.1	150
G	21.9	120

b. Based on the height calculated in part **a** for Person A, a healthy BMI would be 20. How much weight should Person A strive to gain to reach that BMI number?

c. Based on the height calculated in part **a** for Person E, a healthy BMI would be 24. How much weight should Person E strive to lose to reach that BMI number?

Solution

a. We are given $\text{BMI} = \dfrac{703w}{h^2}$

Solve the equation for height.

$$\text{BMI} \cdot h^2 = 703w$$

$$h^2 = \sqrt{\frac{703w}{BMI}}$$

$h = \pm\sqrt{\dfrac{703w}{BMI}}$, however, negative heights will not make sense.

So, $h = \sqrt{\dfrac{703w}{BMI}}$

Person	BMI	w	$h = \sqrt{\dfrac{703w}{BMI}}$
A	17.9	118	68 inches
B	30.8	240	74 inches
C	21	130	66 inches
D	26	166	67 inches
E	28.8	224	74 inches
F	22.1	150	69 inches
G	21.9	120	62 inches

b. Solve the BMI equation for weight: $w = \dfrac{BMI \cdot h^2}{703}$

For person A, $h = 68$. If $BMI = 20$ then:

$$w = \frac{20 \cdot 68^2}{703} = 132 \textit{ pounds.}$$

Person A currently weighs 118 pounds and would need to gain 14 pounds to reach a BMI of 20.

c. For person E, $h = 74$. If $BMI = 27$ then:

$$w = \frac{24 - 74^2}{703} = 187 \textit{ pounds}.$$

Person E currently weighs 224 pounds and would need to lose 37 pounds to reach a BMI of 24.