

# 1 Review and Applications of Basic Mathematics

## Exercise 1.1

a.  $10 + 10 \times 0 = 10 + 0 = \underline{10}$

b.  $2 \times 2 + 4 - 8 = 4 + 4 - 8 = \underline{0}$

c.  $(10 + 10) \times 0 = 20 \times 0 = \underline{0}$

d.  $2 \times (2 + 4) - 8 = 2 \times 6 - 8 = 12 - 8 = \underline{4}$

e.  $0 + 3 \times 3 - 3^2 + 10 = 0 + 9 - 9 + 10 = \underline{10}$

f.  $12 - 2 \times 5 + 2^2 \times 0 = 12 - 10 + 4 \times 0 = 12 - 10 + 0 = \underline{2}$

g.  $0 + 3 \times 3 - (3^2 + 10) = 0 + 9 - 19 = \underline{-10}$

h.  $(12 - 2) \times (5 + 2^2) \times 0 = 10 \times 9 \times 0 = \underline{0}$

i.  $\frac{2^2 - 4}{(4 - 2)^2} = \frac{4 - 4}{2^2} = \frac{0}{4} = \underline{0}$

j.

$$\frac{(2 - 4)^2}{5 - 2^2} = \frac{(-2)^2}{5 - 4} = \frac{4}{1} = \underline{4}$$

1.

1.  $20 - 4 \times 2 - 8 = 20 - 8 - 8 = \underline{4}$

2.  $18 \div 3 + 6 \times 2 = 6 + 12 = \underline{18}$

3.  $(20 - 4) \times 2 - 8 = 16 \times 2 - 8 = 32 - 8 = \underline{24}$

4.  $18 \div (3 + 6) \times 2 = 18 \div 9 \times 2 = 2 \times 2 = \underline{4}$

5.  $20 - (4 \times 2 - 8) = 20 - (8 - 8) = \underline{20}$

6.  $(18 \div 3 + 6) \times 2 = (6 + 6) \times 2 = \underline{24}$

7.  $54 - 36 \div 4 + 2^2 = 54 - 9 + 4 = \underline{49}$

8.  $(5 + 3)^2 - 3^2 \div 9 + 3 = 8^2 - 9 \div 9 + 3 = 64 - 1 + 3 = \underline{66}$

9.  $(54 - 36) \div (4 + 2)^2 = 18 \div 6^2 = 18 \div 36 = \underline{0.5}$

10.  $5 + (3^2 - 3)^2 \div (9 + 3) = 5 + (9 - 3)^2 \div 12 = 5 + 36 \div 12 = 5 + 3 = \underline{8}$

11.  $\frac{8^2 - 4^2}{(4 - 2)^3} = \frac{64 - 16}{2^3} = \frac{48}{8} = \underline{6}$

12.  $\frac{(8 - 4)^2}{4 - 2^3} = \frac{4^2}{4 - 8} = \frac{16}{-4} = \underline{-4}$

13.  $3(6+4)^2 - 5(17-20)^2 = 3 \times 10^2 - 5(-3)^2 = 3 \times 100 - 5 \times 9 = 300 - 45 = \underline{\underline{255}}$
14.  $(4 \times 3 - 2)^2 \div (4 - 3 \times 2^2) = (12 - 2)^2 \div (4 - 3 \times 4) = 10^2 \div (4 - 12) = 100 \div (-8) = \underline{\underline{-12.5}}$
15.  $[(20 + 8 \times 5) - 7 \times (-3)] \div 9 = (20 + 40 + 21) \div 9 = 81 \div 9 = \underline{\underline{9}}$
16.  $5 \left[ 19 + (5^2 - 16)^2 \right]^2 = 5 \left[ 19 + (25 - 16)^2 \right]^2 = 5(19 + 81)^2 = 5 \times 100^2 = \underline{\underline{50,000}}$
17.  $\$100(1 + 0.06 \times \frac{45}{365}) = \$100(1 + 0.00739726) = \underline{\underline{\$100.74}}$
18.  $\frac{\$200}{1 + 0.09 \times \frac{4}{12}} = \frac{\$200}{1 + 0.03} = \frac{\$200}{1.03} = \underline{\underline{\$194.17}}$
19.  $\frac{\$500}{(1 + 0.05)^2} = \frac{\$500}{1.05^2} = \frac{\$500}{1.1025} = \underline{\underline{\$453.51}}$
20.  $\$1000(1 + 0.02)^3 = \$1000(1.02^3) = \$1000(1.061208) = \underline{\underline{\$1061.21}}$
21.  $\$100 \left[ \frac{(1 + 0.04)^2 - 1}{0.04} \right] = \$100 \left( \frac{1.04^2 - 1}{0.04} \right) = \$100 \left( \frac{0.0816}{0.04} \right) = \underline{\underline{\$204.00}}$
22.  $\$300 \left[ \frac{1 - \frac{1}{(1 + 0.03)^2}}{0.03} \right] = \$300 \left( \frac{1 - \frac{1}{1.0609}}{0.03} \right) = \$300 \left( \frac{1 - 0.942596}{0.03} \right) = \underline{\underline{\$574.04}}$

### Concept Questions (Section 1.2)

1. You must retain at least one more figure than you require in the answer. To achieve four-figure accuracy in the answer, you must retain a minimum of five figures in the values used in the calculations. B)
2. We want six-figure accuracy in the answer. Therefore, values used in the calculations must be accurate to at least seven figures. B)
3. We want seven-figure accuracy in the answer. Therefore, values used in the calculations must retain at least eight figures. C)
4. To be accurate to the nearest 0.01%, an interest rate greater than 10% must have four-figure accuracy. Therefore, five figures must be retained in numbers used in the calculations. C)

### Exercise 1.2

- a.  $\frac{1}{10} = \underline{\underline{0.10}} = \underline{\underline{10\%}}$
- b.  $\frac{2}{5} = \underline{\underline{0.40}} = \underline{\underline{40\%}}$
- c.  $\frac{1}{4} = \underline{\underline{0.25}} = \underline{\underline{25\%}}$

- d.  $\frac{3}{4} = \underline{0.75} = \underline{75\%}$
- e.  $1\frac{1}{2} = \underline{1.50} = \underline{150\%}$
- f.  $2\frac{1}{3} = \underline{2.3333} = \underline{233.33\%}$
- g.  $\frac{10}{5} = \underline{2.00} = \underline{200\%}$
- h.  $5\frac{2}{3} = \underline{5.6667} = \underline{566.67\%}$
- i.  $0.25 \times 80 = \underline{20}$
- j.  $0.20 \times 120 = \underline{24}$
- k. Money in Savings =  $0.20 \times \$1000 = \$200$   
 Money in TFSA =  $0.50 \times \$200 = \underline{\$100}$
1.  $\frac{7}{8} = \underline{0.87500} = \underline{87.500\%}$
2.  $\frac{65}{104} = \underline{0.62500} = \underline{62.500\%}$
3.  $\frac{47}{20} = \underline{2.3500} = \underline{235.00\%}$
4.  $-\frac{9}{16} = \underline{-0.56250} = \underline{-56.250\%}$
5.  $\frac{-35}{25} = \underline{-1.4000} = \underline{-140.00\%}$
6.  $1\frac{7}{25} = \underline{1.2800} = \underline{128.00\%}$
7.  $\frac{25}{1000} = \underline{0.025000} = \underline{2.5000\%}$
8.  $\frac{1000}{25} = \underline{40.000} = \underline{4000.0\%}$
9.  $2\frac{2}{100} = \underline{2.0200} = \underline{202.00\%}$
10.  $-1\frac{11}{32} = \underline{-1.3438} = \underline{-134.38\%}$
11.  $\frac{37.5}{50} = \underline{0.75000} = \underline{75.000\%}$
12.  $\frac{22.5}{-12} = \underline{-1.8750} = \underline{-187.50\%}$
13.  $\frac{5}{6} = \underline{0.8\bar{3}} = \underline{83.\bar{3}\%}$
14.  $-\frac{8}{3} = \underline{-2.\bar{6}} = \underline{-266.\bar{6}\%}$
15.  $7\frac{7}{9} = \underline{7.\bar{7}} = \underline{777.\bar{7}\%}$
16.  $1\frac{1}{11} = \underline{1.0\bar{9}} = \underline{109.0\bar{9}\%}$
17.  $\frac{10}{9} = \underline{1.\bar{1}} = \underline{111.\bar{1}\%}$
18.  $-\frac{4}{900} = \underline{-0.00\bar{4}} = \underline{-0.\bar{4}\%}$

19.  $-\frac{7}{270} = -\underline{\underline{0.0259}} = -\underline{\underline{2.592\%}}$
20.  $\frac{37}{27} = \underline{\underline{1.370}} = \underline{\underline{137.037\%}}$
21.  $11.3845 \approx \underline{\underline{11.38}}$
22.  $9.6455 \approx \underline{\underline{9.646}}$
23.  $0.5545454 \approx \underline{\underline{0.5545}}$
24.  $1000.49 \approx \underline{\underline{1000}}$
25.  $1.0023456 \approx \underline{\underline{1.002}}$
26.  $0.030405 \approx \underline{\underline{0.03041}}$
27.  $40.09515 \approx \underline{\underline{40.10}}$
28.  $0.0090909 \approx \underline{\underline{0.009091}}$
29.  $\frac{1}{6} = \underline{\underline{0.16667}} = \underline{\underline{16.667\%}}$
30.  $\frac{7}{6} = \underline{\underline{1.1667}} = \underline{\underline{116.67\%}}$
31.  $\frac{1}{60} = \underline{\underline{0.016667}} = \underline{\underline{1.6667\%}}$
32.  $2\frac{5}{9} = \underline{\underline{2.5556}} = \underline{\underline{255.56\%}}$
33.  $\frac{250}{365} = \underline{\underline{0.68493}} = \underline{\underline{68.493\%}}$
34.  $\frac{15}{365} = \underline{\underline{0.041096}} = \underline{\underline{4.1096\%}}$
35.  $\frac{0.11}{12} = \underline{\underline{0.0091667}} = \underline{\underline{0.91667\%}}$
36.  $\frac{0.095}{12} = \underline{\underline{0.0079167}} = \underline{\underline{0.79167\%}}$
37.  $\$92\left(1 + 0.095 \times \frac{112}{365}\right) = \$92 \times 1.02915 = \underline{\underline{\$94.68}}$
38.  $\$100\left(1 + 0.11 \times \frac{5}{12}\right) = \$100 \times 1.04583 = \underline{\underline{\$104.58}}$
39.  $\$454.76\left(1 - 0.105 \times \frac{11}{12}\right) = \$454.76 \times 0.903750 = \underline{\underline{\$410.99}}$
40.  $\frac{1}{(1+0.22)^3} = 0.550707 = \underline{\underline{0.55}}$
41.  $\$1447\left(1 + \frac{0.18}{2}\right)^3\left(1 + \frac{0.21}{3}\right)^2 = \$1447 (1.295029) (1.1449) = \underline{\underline{\$2145.44}}$
42.  $\frac{\$790.84}{1 + 0.13 \times \frac{311}{365}} = \frac{\$790.84}{1.110767} = \underline{\underline{\$711.98}}$
43.  $\frac{\$3490}{1 + 0.125 \times \frac{91}{365}} = \frac{\$3490}{1.031164} = \underline{\underline{\$3384.52}}$

44.  $\frac{\$10,000}{1 - 0.10 \times \frac{182}{365}} = \frac{\$10,000}{0.95013699} = \underline{\underline{\$10,524.80}}$
45.  $\$650\left(1 + \frac{0.105}{2}\right)^2 = \$650(1.0525)^2 = \underline{\underline{\$720.04}}$
46.  $\$950.75\left(1 - \frac{0.095}{4}\right)^2 = \$950.75(0.97625)^2 = \underline{\underline{\$906.13}}$
47.  $\frac{\$15,400}{\left(1 + \frac{0.13}{12}\right)^6} = \frac{\$15,400}{1.0108333^6} = \underline{\underline{\$14,435.88}}$
48.  $\frac{\$550}{\left(1 + \frac{0.115}{2}\right)^4} = \frac{\$550}{1.05750^4} = \underline{\underline{\$439.79}}$
49.  $0.\overline{33} \times \$1527 = \underline{\underline{\$509.00}}$
50.  $0.0275 \times \$2.75 = \underline{\underline{\$0.08}}$
51.  $2.50 \times \$25 = \underline{\underline{\$62.50}}$
52.  $0.00025 \times \$200 = \underline{\underline{\$0.05}}$
53.  $0.005 \times \$30 = \underline{\underline{\$0.15}}$
54. Off-peak hours =  $\frac{12}{24} \times 100 = \underline{\underline{50\%}}$   
 Mid-peak hours =  $\frac{6}{24} \times 100 = \underline{\underline{25\%}}$   
 On-peak hours =  $\frac{6}{24} \times 100 = \underline{\underline{25\%}}$
55.  $0.12 \times (0.055 \times \$458,000) = \underline{\underline{\$3,022.80}}$
56. Money available to be spent on entertainment is  $100 - (53+42) = 5\%$   
 In dollars,  $0.05 \times \$14,775 = \underline{\underline{\$738.75}}$   
 They can spend \$738.75 on entertainment.
57. Sales of in-store products =  $0.36 \times \$102,300 = \$36,828$   
 HST collected on in-store products =  $0.13 \times \$36,828 = \underline{\underline{\$4,787.64}}$
58. Shots scored from 2-point zone =  $0.545454 \times 33 = 18$   
 Shots scored from 3-point distance =  $0.46667 \times 15 = 7$   
 Foul shots scored =  $0.793 \times 29 = 23$   
 Total points scored =  $18(2) + 7(3) + 23(1) = \underline{\underline{80}}$

$$\begin{aligned}
59. \quad & \frac{\$6600(1+0.085 \times \frac{153}{365})}{1+0.125 \times \frac{82}{365}} = \frac{\$6600(1.035630)}{1.028082} = \underline{\underline{\$6648.46}} \\
60. \quad & \frac{\$780(1+\frac{0.0825}{2})^5}{(1+\frac{0.10}{12})^8} = \frac{\$780(1.22398)}{1.06864} = \underline{\underline{\$893.38}} \\
61. \quad & \$1000 \left[ \frac{(1+\frac{0.09}{12})^7 - 1}{\frac{0.09}{12}} \right] = \$1000 \left( \frac{0.05369613}{0.0075} \right) = \underline{\underline{\$7159.48}} \\
62. \quad & \frac{\$350}{\frac{0.0975}{12}} \left[ 1 - \frac{1}{(1+\frac{0.0975}{12})^5} \right] = \frac{\$350}{0.008125} (0.03965324) = \$1708.14 \\
63. \quad & \frac{\frac{\$9500}{(1+\frac{0.075}{4})^5 - 1}}{\frac{0.075}{4}} = \frac{\$9500}{\frac{0.09733216}{0.01875000}} = \frac{\$9500}{5.191049} = \$1830.07 \\
64. \quad & \$45 \left[ \frac{1 - \frac{1}{(1+\frac{0.0837}{2})^4}}{\frac{0.0837}{2}} \right] + \frac{\$1000}{(1+\frac{0.0837}{2})^4} = \$45 \left( \frac{1 - \frac{1}{1.178205}}{0.04185} \right) + \frac{\$1000}{1.178205} \\
& = \$45 \left( \frac{0.151251}{0.04185} \right) + \$848.75 \\
& = \$162.635 + \$848.749 \\
& = \underline{\underline{\$1011.38}}
\end{aligned}$$

65. Seats not sold to season-ticket holders = 100% – 67.5% = 32.5%

Number of seats not sold to season-ticket holders = 0.325 x 19289 = 6,269 seats

Rounded to the nearest 100, 6300 seats were not sold to season-ticket holders.

66. Percentage of impurities = 100% – 99.95% = 0.05%

Amount of impurities = 0.0005 x 31.16 g = 0.01558 g = 15.58 mg

67. Portion of commission retained = 0.60 x 4.8% = 2.88%

Income is 2.88% of sales = 0.0288 x \$5,225,000 = \$150,480

That is, \$150,480 = 0.0288 x Sales

Stan's commission was \$150,480.

68. If 18% of \$128,500 is lower than \$29,210 then that will be the contribution.

0.18 x \$128,500 = \$23,130

Maximum RRSP contribution is \$23,130 since it is lower than \$29,210.

69. Sodium intake from other foods = 100% - 35% = 65%

0.65 x 2300 mg = 1495 mg = 1.495 grams

### Exercise 1.3

1. Regular weekly earnings =  $\frac{\$58,800}{52} = \$1130.77$   
Equivalent hourly rate =  $\frac{\$1130.77}{35} = \$32.31$   
Overtime hourly rate =  $1.5(\$32.31) = \$48.47$   
Gross pay for 39-hour week =  $\$1130.77 + 4(\$48.47) = \underline{\$1324.65}$
2. Regular biweekly earnings =  $\frac{\$37,500}{26} = \$1442.31$   
Equivalent hourly rate =  $\frac{\$1442.31}{2 \times 37.5} = \$19.23$   
Gross earnings =  $\$1442.31 + 9(1.5)\$19.23 = \underline{\$1701.92}$
3. Regular biweekly earnings =  $\frac{\$54,600}{26} = \$2100.00$   
Equivalent hourly wage =  $\frac{\$2100.00}{2 \times 40} = \$26.25$   
Hasad worked 3 hours of overtime in the first week and 6.5 hours in the second week.  
Gross pay =  $\$2100.00 + 9.5(1.5)\$26.25 = \underline{\$2474.06}$
4. Annual earnings =  $52(40)\$31.50 = \$65,520$   
Equivalent semimonthly earnings =  $\frac{\$65,520}{24} = \underline{\$2730.00}$
5. Regular hours worked =  $7.5 + 7.5 + 6 + 6 + 7.5 = 34.5$   
Overtime hours worked =  $4.5 + 1 + 1.5 = 7$   
Gross earnings =  $34.5(\$17.70) + 7(1.5)(\$17.70) = \underline{\$796.50}$
6. Total hours worked = 51.5 of which 8 hours were worked on a statutory holiday.  
Overtime hours worked =  $51.5 - (40 + 8) = 3.5$   
Regular earnings =  $40(\$34.50) = \$1380.00$   
Overtime pay =  $3.5(1.5)\$34.50 = \$181.13$   
Holiday pay =  $8(\$34.50) = \$276.00$   
Holiday premium =  $8(2)\$34.50 = \$552.00$   
Gross earnings =  $\underline{\$2389.13}$
7. Output in excess of quota =  $4 + 6 + 7 + 8 + 10 = 35$  shirts  
Total pay =  $40(\$7.50) + 35(\$3.00) = \underline{\$405.00}$
8. Weight packed per day =  $7.5(250)(0.500\text{kg}) = 937.5$  kg.  
Earnings per day =  $7.5(\$8.25) + (937.5 - 500)(\$0.18) = \underline{\$140.63}$
9. October earnings =  $(\# \text{ renewals}) \times \$20 + (\# \text{ new policies}) \times \$35 + 0.055(\text{Total premiums})$   
 $= 126(\$20) + 37(\$35) + 0.055(\$14,375 + \$47,880)$   
 $= \underline{\$7239.03}$

10. Annual sales =  $12(\$11,000) = \$132,000$   
 Hillary's earnings =  $0.21(\$132,000) + 0.07(\$132,000 - \$100,000) = \underline{\underline{\$29,960}}$
11. Estimated earnings from Supreme Audio & Video =  $\$2000 + 0.04(\$55,000) = \underline{\underline{\$4200}}$   
 Estimated earnings from Buy-Right =  $\$1500 + 0.03(\$25,000) + 0.06(\$55,000 - \$25,000)$   
 $= \underline{\underline{\$4050}}$
12. a. Earnings will be the greater of  
 $\$600$  or  $0.11(\text{Sales}) = 0.11(\$5636) = \underline{\underline{\$619.96}}$   
 b. The salesperson will earn the \$600 from sales if  
 $0.11(\text{Sales}) = \$600$   
 $\text{Sales} = \frac{\$600}{0.11}$   
 That is, if Sales =  $\frac{\$600}{0.11} = \underline{\underline{\$5454.55}}$  per week
13. Gross earnings =  $0.033(\$50,000) + 0.044(\$50,000) + 0.055(\$40,000)$   
 $= \underline{\underline{\$6050.00}}$

14. a. Earnings =  $\$2000 + 0.022(\$227,000 - \$150,000) = \underline{\$3694.00}$   
 b. Average earnings =  $\$2000 + 0.022(\$235,000 - \$150,000) = \$3870.00$   
 For a straight commission rate to generate the same monthly earnings,  

$$\text{Commission rate} = \frac{\$3870}{\$235,000} \times 100\% = 1.65\%$$
15. a. Earnings =  $0.05(\$20,000) + 0.075(\$20,000) + 0.10(\$14,880) = \underline{\$3988.00}$   
 b. For the same earnings from a single straight commission rate,  

$$\text{Commission rate} \times \$54,880 = \$3988.00$$

$$\text{Commission rate} = \frac{\$3988}{\$54,880} \times 100\% = \underline{7.27\%}$$
16. Commission earned =  $\$630.38 - \$300 = \$330.38$   
 Hence,  

$$0.03(\text{Sales subject to commission}) = \$330.38$$

$$\text{Sales subject to commission} = \frac{\$330.38}{0.03} = \$11,012.67$$

$$\text{Total sales} = \$11,012.67 + \$20,000 = \underline{\$31,012.67}$$
17. Commission earned in August =  $\$3296.97 - \$1500.00 = \$1796.97$   
 Hence,  

$$\text{Sales subject to commission} = (\$151,342 - \$100,000) = \$51,342$$

$$\text{Commission rate} = \frac{\$1796.97}{\$51,342} \times 100\% = \underline{3.50\%}$$
18. Commission earned on first \$90,000 of sales was  
 $0.04(\$40,000) + 0.05(\$50,000) = \$4100$   
 Commission earned on sales in excess of \$90,000 was  $\$5350 - \$4100 = \$1250$   
 That is,  

$$0.06(\text{Sales exceeding } \$90,000) = \$1250$$

$$\text{Sales exceeding } \$90,000 = \frac{\$1250}{0.06} = \$20,833.33$$

$$\text{Total sales for the month} = \$90,000 + \$20,833.33 = \underline{\$110,833.33}$$
19. Required monthly commission =  $\$4000 - \$2000 = \$2000$   
 Commission income on first \$50,000 of monthly sales is  
 $0.03(\$50,000 - \$25,000) = \$750$   
 The combined commission and bonus rate on sales exceeding \$50,000 is  $3\% + 3\% = 6\%$ .  
 Hence,  

$$0.06(\text{Sales exceeding } \$50,000) = \$2000 - \$750$$

$$\text{Sales exceeding } \$50,000 = \frac{\$1250}{0.06} = \$20,833.33$$

$$\text{Required monthly sales} = \$50,000 + \$20,833.33 = \underline{\$70,833.33}$$



## Concept Questions (Section 1.4)

1. You should calculate a weighted average when some of the values being averaged are more important or occur more frequently than other values.
2. The weighted average will equal the simple average when the items being averaged all have the same weighting factor. This will happen when each of the values being averaged has the same importance, or occurs the same number of times.
3. If you invest the same amount of money in each investment, each rate of return has the same importance. The portfolio's rate of return will then equal the simple average of the individual rates of return.

## Exercise 1.4

1. Weight each number of TV sets per household by the number of homes with that number of TVs. The weighted average number of TVs per household in the survey sample is

$$\frac{(4 \times 4) + (22 \times 3) + (83 \times 2) + (140 \times 1) + (5 \times 0)}{254} = \underline{1.53}$$

Based on the survey, we estimate the average number of TVs per household to be 1.53.

2. The weighted average cost per share is

$$\frac{1000(\$15.63) + 500(\$19.00) + 300(\$21.75)}{1800} = \underline{\$17.59}$$

- 3 We should weight each "goals against" figure by the number of games in which that number was scored.

$$\text{GAA} = \frac{1(0) + 2(1) + 3(2) + 4(3) + 7(4) + 2(6) + 1(10)}{20} = \underline{3.50}$$

4. The amount of sales subject to each commission rate should be used as the weighting factor.

- a. The average commission rate will be

$$\frac{\$30,000(3\%) + \$20,000(4\%) + \$10,000(6\%)}{\$60,000} = \underline{3.83\%}$$

- b. The average commission rate will be:

$$\frac{\$30,000(3\%) + \$20,000(4\%) + \$50,000(6\%)}{\$100,000} = \underline{4.70\%}$$

5. Babe Ruth's weighted average slugging percentage is

$$\frac{714(4) + 136(3) + 506(2) + 2873(1) + 4170(0)}{8399} \times 100 = \frac{7149}{8399} \times 100 = 0.85117 \times 100 = \underline{85.12\%}$$

6. The weighted average cost of a hotel room is

$$\frac{\$158(4) + \$199(2) + \$239(1) + \$130(5) + \$118(2)}{4+2+1+5+2} = \frac{\$2155}{14} = \underline{\underline{\$153.93}}$$

7. The weighted average interest rate that will be charged on the new \$117,500 balance is

$$\frac{\$87,500(3.2\%) + \$30,000(2.5\%)}{\$117,500} = \underline{3.02\%}$$

8. The weighted grade point average is

$$\text{GPA} = \frac{5(2.3) + 3(2.7) + 4(3.3) + 2(1.7) + 3(3.0) + 4(2.0)}{5 + 3 + 4 + 2 + 3 + 4} = \frac{53.2}{21} = \underline{2.53}$$

9. Weight each score by the number of students who obtained that score. The weighted average score is

$$\frac{2(10) + 6(9) + 9(8) + 7(7) + 3(6) + 2(5) + 0(4) + 1(3) + 0(2) + 0(1)}{30} = \underline{7.53}$$

10. Weight each semester's GPA by the number of credits on which the respective GPA was obtained. The cumulative GPA is

$$\frac{6(3.5) + 9(3.0) + 12(2.75) + 7.5(3.2)}{6 + 9 + 12 + 7.5} = \frac{105.0}{34.5} = \underline{3.04}$$

11. Note that the age of receivables (rather than the dollar amount of receivables) is to be averaged. The relative importance of each of the three age classifications is determined by the dollar amount in each category. Hence, the weighting factors are the respective dollar amounts of receivables. The (weighted) average age of accounts receivable is

$$\frac{\$12,570(30) + \$6850(60) + \$1325(90)}{\$12,570 + \$6850 + \$1325} = \frac{\$907,350}{\$20,745} = \underline{43.74 \text{ days}}$$

12. The rate of return for the entire portfolio is the weighted average return on the five securities in the portfolio. Each rate of return should be weighted by the fraction of the money invested in the respective security. The rate of return on the portfolio is

$$\frac{0.15(14\%) + 0.20(10\%) + 0.10(-13\%) + 0.35(12\%) + 0.20(27\%)}{1.00} = \underline{12.40\%}$$

13. a. The weighted average cost of units purchased during the year is

$$\frac{300(\$10.86) + 1000(\$10.47) + 500(\$10.97)}{1800} = \underline{\$10.67}$$

- b. The weighted average cost of the beginning inventory and units purchased during the year is

$$\frac{156(\$10.55) + 1800(\$10.674)}{1956} = \underline{\$10.66}$$

- c. Value of ending inventory =  $239 \times \text{Weighted average cost}$   
 $= 239(\$10.66)$   
 $= \underline{\$2547.74}$

14. The weighted average price increase was

$$\frac{0.30(10\%) + 0.20(-5\%) + 0.50(15\%)}{1.00} = \underline{\underline{9.50\%}}$$

| Menu category | Menu price (as % of input cost) | % of total revenue |
|---------------|---------------------------------|--------------------|
| Appetizers    | 300                             | 10                 |
| Entrees       | 200                             | 50                 |
| Desserts      | 225                             | 15                 |
| Beverage      | 250                             | 25                 |

a. The weighted average menu price (as % of input cost) is

$$\frac{0.10(300\%) + 0.50(200\%) + 0.15(225\%) + 0.25(250\%)}{0.10 + 0.50 + 0.15 + 0.25} = \underline{\underline{226.25\%}} \text{ of input costs}$$

On average, Menu prices = 2.2625(Input costs)

b. We can find the average input cost as a percentage of revenue (menu prices) by rearranging the equation in part a:

$$\text{Input costs} = \frac{\text{Menu prices}}{2.2625} = 0.44199(\text{Menu prices})$$

On average, input costs are 44.20% of revenue.

| Period       | Balance                        | No. of days |
|--------------|--------------------------------|-------------|
| 1st to 7th   | \$35,000                       | 7           |
| 8th to 24th  | \$35,000 + \$10,000 = \$45,000 | 17          |
| 25th to 31st | \$45,000 – \$20,000 = \$25,000 | 7           |

The weighted average balance on the loan was

$$\frac{7(\$35,000) + 17(\$45,000) + 7(\$25,000)}{7 + 17 + 7} = \underline{\underline{\$38,225.81}}$$

| Period              | No. of months | Number of employees |
|---------------------|---------------|---------------------|
| Jan. 1 to Mar. 31   | 3             | 14                  |
| Apr. 1 to Apr. 30   | 1             | 14 + 7 = 21         |
| May 1 to May 31     | 1             | 21 + 8 = 29         |
| June 1 to Aug. 31   | 3             | 29 + 11 = 40        |
| Sept. 1 to Sept. 30 | 1             | 40 – 6 = 34         |
| Oct. 1 to Dec. 31   | 3             | 34 – 14 = 20        |

17. We want the average number of people working over the course of the year. The given figures for the number of employees added or laid off at various times are used to determine the cumulative number of people employed.

Each number in the third column must be weighted by the number of months in the second column. The average number employed was

$$\frac{3(14) + 1(21) + 1(29) + 3(40) + 1(34) + 3(20)}{12} = \underline{25.50}$$

18. The given figures for the amount invested from time to time are used to determine the cumulative investment
- | <u>Period</u>       | <u>No. of months</u> | <u>Cumulative investment</u> |
|---------------------|----------------------|------------------------------|
| Sept. 1 to Sept. 30 | 1                    | \$57,000                     |
| Oct. 1 to Oct. 31   | 1                    | 72,000                       |
| Nov. 1 to Jan. 31   | 3                    | 99,000                       |
| Feb. 1 to Feb. 28   | 1                    | 76,000                       |
| Mar. 1 to Apr. 30   | 2                    | 63,000                       |
| May 1 to Aug. 31    | 4                    | 57,000                       |
- The (weighted) average investment was

$$\frac{[1(57) + 1(72) + 3(99) + 1(76) + 2(63) + 4(57)] \times \$1000}{12} = \underline{\$71,333.33}$$

19. Each number of shares in the third column must be weighted by the number of months in the second column. The (weighted) average number of shares outstanding was
- | <u>Period</u>     | <u>No. of months</u> | <u>Number of shares outstanding (millions)</u> |
|-------------------|----------------------|------------------------------------------------|
| Jan. 1 to Feb. 28 | 2                    | 5                                              |
| Mar. 1 to May 31  | 3                    | 5 + 1 = 6                                      |
| June 1 to Oct. 31 | 5                    | 6 + 0.5 = 6.5                                  |
| Nov. 1 to Dec. 31 | 2                    | 6.5 + 0.75 = 7.25                              |

$$\frac{[2(5) + 3(6) + 5(6.5) + 2(7.25)] \times 1 \text{ million}}{12} = 6.25 \text{ million} = \underline{6,250,000}$$

20. a. Each cost in the third column must be weighted by the amount of the ingredient in the Deluxe Nut Combo. The (weighted) average cost is
- | <u>Ingredient</u> | <u>Weight (kg)</u> | <u>Cost per Kg</u> |
|-------------------|--------------------|--------------------|
| Peanuts           | 5                  | \$2.95             |
| Cashews           | 2                  | \$9.50             |
| Almonds           | 1                  | \$11.50            |
| Sunflower seeds   | 0.5                | \$2.75             |
| Raisins           | 0.4                | \$3.60             |
| Smarties          | 0.3                | \$6.40             |
- $$\frac{5(\$2.95) + 2(\$9.50) + 1(\$11.50) + 0.5(\$2.75) + 0.4(\$3.60) + 0.3(\$6.40)}{5 + 2 + 1 + 0.5 + 0.4 + 0.3} = \$5.433/\text{kg}$$

Lien's average cost is \$0.54 per 100 g

b. The retail price is  $1.50(\$0.543) = \underline{\$0.81 \text{ per } 100\text{g}}$ .

21. The rating for each factor must be weighted by the percentage of respondents who selected that rating.

| Factor | Not at all Important<br>(1) | Somewhat Important<br>(2) | Important<br>(3) | Extremely Important<br>(4) |
|--------|-----------------------------|---------------------------|------------------|----------------------------|
| Price  | 19%                         | 24%                       | 28%              | 29%                        |

|            |     |     |     |     |
|------------|-----|-----|-----|-----|
| Service    | 13% | 30% | 39% | 18% |
| Quality    | 0%  | 43% | 21% | 36% |
| Promotions | 11% | 32% | 45% | 12% |

The weighted average ratings (out of 4) for the factors are:

$$\text{Price} = \frac{19(1) + 24(2) + 28(3) + 29(4)}{100} = 2.67$$

$$\text{Service} = \frac{13(1) + 30(2) + 39(3) + 18(4)}{100} = 2.62$$

$$\text{Quality} = \frac{0(1) + 43(2) + 21(3) + 36(4)}{100} = \underline{2.93}$$

$$\text{Promotions} = \frac{11(1) + 32(2) + 45(3) + 12(4)}{100} = 2.58$$

The factor with the highest weighted average rating is quality = 2.93 out of 4.

### Exercise 1.5

| 1. | <u>Quarter</u> | <u>Sales – Purchases</u> | <u>GST Remittance (Refund)</u> |
|----|----------------|--------------------------|--------------------------------|
|    | 1              | \$155,365                | <u>\$7768.25</u>               |
|    | 2              | (340,305)                | <u>(17,015.25)</u>             |
|    | 3              | 408,648                  | <u>20,432.40</u>               |
|    | 4              | 164,818                  | <u>8240.90</u>                 |

| 2. | <u>Month</u> | <u>Sales – Purchases</u> | <u>HST Remittance (Refund)</u> |
|----|--------------|--------------------------|--------------------------------|
|    | March        | \$(77,760)               | <u>\$(10,108.80)</u>           |
|    | April        | (8255)                   | <u>(1073.15)</u>               |
|    | May          | 136,515                  | <u>17,746.95</u>               |
|    | June         | 114,875                  | <u>14,933.75</u>               |

3. The GST charged in each case will be  
 $0.05(\$39,500) = \$1975.00$ 
  - a. With no PST in Alberta, the total amount paid will be  
 $\$39,500 + \$1975.00 = \underline{\$41,475.00}$
  - b. PST in Saskatchewan =  $0.06(\$39,500) = \$2370.00$   
Total amount =  $\$39,500 + \$1975.00 + \$2370.00 = \underline{\$43,845.00}$
  - c. PST in Quebec =  $0.09975(\$39,500) = \$3940.13$   
Total amount =  $\$39,500 + \$1975.00 + \$3940.13 = \underline{\$45,415.13}$
4. Cost in Manitoba =  $\$1000 + \text{GST} + \text{PST} = \$1000 + 0.05(\$1000) + 0.07(\$1000) = \$1120.00$

$$\begin{aligned}
 \text{Cost in PEI} &= \$1000 + \text{HST} = \$1000 + 0.15(\$1000) \\
 &= \$1000 + \$150 \\
 &= \$1150.00
 \end{aligned}$$

The consumer will pay  $\$1150.00 - \$1120.00 = \underline{\$30.00}$  more in PEI.

5. a. The HST reported for a \$39.45 (pre-tax) item is  $0.13(\$39.45) = \underline{\$5.13}$ .
- b. The HST inclusive price is  $\$39.45 + \$5.13 = \$44.58$ . If \$50 cash is paid, change will be based on the rounded price of \$44.60. Therefore, change will be  $\$50 - \$44.60 = \underline{\$5.40}$ .

6. The HST rate in New Brunswick is 15%.

For each \$100 of pre-tax price, the tax-inclusive price must include HST of \$15. That is, a tax-inclusive price of \$115 includes \$15 of HST. The HST is, therefore,

$$\frac{\$15}{\$115} \times 100\% = 13.043\% \text{ of the HST-inclusive price}$$

Consequently, a \$495 tax-inclusive price includes

$$\text{HST} = 0.13043 \times \$495 = \underline{\$64.56}$$

$$7. \text{ Property tax} = \frac{\text{Mill rate}}{1000} \times \text{Assessed value} = \frac{16.8629}{1000} \times \$227,000 = \underline{\$3827.88}$$

8. a. 0.1 mill = \$0.10 per \$1000 of assessed value = \$0.01 per \$100 of assessed value

$$b. \text{ Property tax increase} = \frac{0.1}{1000} \times \$200,000 = \underline{\$20.00}$$

$$9. \text{ Total taxes} = \frac{15.0294}{1000} \times \$143,000 + \frac{4.6423}{1000} \times \$467,000$$

$$= \$2149.204 + \$2167.954$$

$$= \underline{\$4317.16}$$

$$10. a. \text{ Current year's taxes} = \frac{\$1.52193}{\$100} \times \$298,000 = \$4535.35$$

$$\text{Previous year's taxes} = \frac{\$1.56324}{\$100} \times \$285,000 = \$4455.23$$

$$\text{Change in property taxes} = \underline{\$80.12 \text{ increase}}$$

- b. For the current year's taxes to remain at \$4455.23,

$$\frac{\text{New tax rate}}{\$100} \times \$298,000 = \$4455.23$$

$$\frac{\$4455.23}{\$298,000} \times \$100 =$$

$$\text{New tax rate} = \underline{\$1.49504 \text{ per } \$100 \text{ of assessed value}}$$

$$11. a. \text{ Tax increase} = \frac{\text{Mill rate increase}}{1000} \times \text{Assessed value}$$

$$\$2,430,000 = \frac{\text{Mill rate increase}}{1000} \times \$6,780,000,000$$

$$\text{Mill rate increase} = \frac{\$2430}{\$6,780,000} \times 1000 = 0.3584$$

$$\text{Next year's mill rate} = 7.1253 + 0.3584 = \underline{7.4837}$$

- b. Next year's assessment = 1.05(\$6.78 billion) = \$7.119 billion

$$\text{Next year's budget} = \text{Current year's taxes} + \$2,430,000$$

$$= \frac{7.1253}{1000} \times \$6.78 \text{ billion} + \$2,430,000$$

$$= \$50,739,534$$

Next year's school mill rate applied to next year's assessment must generate

enough tax revenue to meet next year's budget. That is,

$$\begin{aligned} \$50,739,534 &= \frac{\text{New mill rate}}{1000} \times \$7.119 \text{ billion} \\ \text{New mill rate} &= \frac{\$50,739,534}{\$7,119,000,000} \times 1000 = \underline{\underline{7.1273}} \end{aligned}$$

$$\begin{aligned} 12. \text{ Current budget} &= \text{Last year's budget} + \$750,000 \\ &= \frac{\$0.94181}{\$100} \times \$1,563,000,000 + \$750,000 \\ &= \$15,470,490.30 \end{aligned}$$

$$\text{Current assessment} = \$1563 \text{ million} + \$97 \text{ million} = \$1660 \text{ million}$$

$$\begin{aligned} \text{Hence, } \$15,470,490.30 &= \frac{\text{New tax rate}}{\$100} \times \$1660 \text{ million} \\ \frac{\$15,470,490.30 \times \$100}{\$1,660,000,000} &= \underline{\underline{\$0.93196}} \\ \text{New tax rate} &= \underline{\underline{\$0.93196}} \end{aligned}$$

That is, the tax rate would have to be \$0.93196 per \$100 of assessed value.

### Review Problems

$$1. \quad a. \quad (2^3 - 3)^2 - 20 \div (2 + 2^3) = (8 - 3)^2 - 20 \div (2 + 8) = 25 - 20 \div 10 = 25 - 2 = \underline{\underline{23}}$$

$$\begin{aligned} b. \quad 4(2 \times 3^2 - 2^3)^2 \div (10 - 4 \times 5) &= 4(2 \times 9 - 8)^2 \div (10 - 20) \\ &= 4 \times 10^2 \div (-10) \\ &= 4 \times 100 \div (-10) \\ &= 400 \div (-10) \\ &= \underline{\underline{-40}} \end{aligned}$$

$$c. \quad \$213.85 \left(1 - 0.095 \times \frac{5}{12}\right) = \$213.85(1 - 0.0395833) = \underline{\underline{\$205.39}}$$

$$d. \quad \frac{\$2315}{1 + 0.0825 \times \frac{77}{365}} = \frac{\$2315}{1.0174041} = \underline{\underline{\$2275.40}}$$

$$e. \quad \$325.75 \left(1 + \frac{0.105}{4}\right)^2 = \$325.75(1.053189) = \underline{\underline{\$343.08}}$$

$$f. \quad \frac{\$710}{\left(1 + \frac{0.0925}{2}\right)^3} = \frac{\$710}{1.145266} = \underline{\underline{\$619.94}}$$

$$\begin{aligned} g. \quad \$885.75 \left(1 + 0.0775 \times \frac{231}{365}\right) - \frac{\$476.50}{1 + 0.0775 \times \frac{49}{365}} &= \$885.75(1.049048) - \frac{\$476.50}{1.010404} \\ &= \$929.194 - \$471.594 \\ &= \underline{\underline{\$457.60}} \end{aligned}$$

$$h. \$859 \left(1 + \frac{0.0825}{12}\right)^3 + \frac{\$682}{\left(1 + \frac{0.0825}{12}\right)^2} = \$859(1.020767) + \frac{\$682}{1.013797} \\ = \$876.839 + \$672.719 \\ = \underline{\underline{\$1549.56}}$$

$$2. a. 96 - (6 - 4^2) \times 7 - 2 = 96 - (-10)7 - 2 = 96 + 70 - 2 = \underline{\underline{164}}$$

$$b. 81 \div (5^2 - 16) - 4(2^3 - 13) = 81 \div 9 - 4(-5) = 9 + 20 = \underline{\underline{29}}$$

$$c. \frac{\$827.69}{1 + 0.125 \times \frac{273}{365}} + \$531.49 \left(1 + 0.125 \times \frac{41}{365}\right) = \frac{\$827.69}{1.093493} + \$531.49(1.014041) \\ = \$756.923 + \$538.953 \\ = \underline{\underline{\$1295.88}}$$

$$d. \$550.45 \left(1 + 0.0875 \times \frac{195}{365}\right) - \frac{\$376.29}{1 + 0.0875 \times \frac{99}{365}} = \$550.45(1.046747) - \frac{\$376.29}{1.023733} \\ = \$576.182 - \$367.567 \\ = \underline{\underline{\$208.62}}$$

$$e. \$1137 \left(1 + \frac{0.0975}{12}\right)^2 + \frac{\$2643}{\left(1 + \frac{0.0975}{12}\right)^3} = \$1137(1.016316) + \frac{\$2643}{1.024574} \\ = \$1155.551 + \$2579.609 \\ = \underline{\underline{\$3735.16}}$$

$$3. 0.62 \times \$99 = \underline{\underline{\$61.38}}$$

$$4. 0.80 \times \$156.25 = \underline{\underline{\$125.00}}$$

$$5. 0.0075 \times \$133.\overline{33} = \underline{\underline{\$1.00}}$$

$$6. \text{Two hours} = 2(60) = 120 \text{ minutes} \\ 0.125 \times 120 = \underline{\underline{15 \text{ minutes}}}$$

$$7. \text{Actual profit} = 0.90 \times \$23,400 = \underline{\underline{\$21,060}}$$

$$8. \text{Price increase} = 0.35 \times \$2.20 = \$0.77$$

$$\text{Selling price} = \$2.20 + \$0.77 = \underline{\underline{\$2.97}} \text{ per share}$$

$$9. a. \text{Gross biweekly earnings} = \frac{\$56,600}{26} = \$2176.92 \\ \frac{\$2176.92}{2 \times 37.5} = \underline{\underline{\$29.03}}$$

$$\text{Equivalent hourly wage} = \frac{\$2176.92}{2 \times 37.5} = \underline{\underline{\$29.03}}$$

$$b. \text{Total remuneration} = \$2176.92 + 4.5(1.5)\$29.03 = \underline{\underline{\$2372.87}}$$

- $$\frac{\$61,000}{26} = \$2346.15$$
10. Gross biweekly earnings =  $\frac{\$2346.15}{75} = \$31.28$   
 Equivalent hourly rate =  $\$2346.15 + 33(1.5)\$31.28 = \underline{\underline{\$3894.51}}$   
 Gross pay =
11. Total hours worked = 41 hours  
 Overtime hours worked = 1.5 (on Wednesday) + 0.5 (on Friday) = 2 hours  
 Regular hours worked = 41 – 2 (hrs of overtime) – 3 (hrs on stat holiday) = 36 hours  
 Regular earnings = 36(\$42.50) = \$1530.00  
 Overtime pay = 2(1.5)\$42.50 = \$127.50  
 Holiday pay = 7.5(\$42.50) = \$318.75  
 Holiday premium = 3(2)\$42.50 = \$255.00  
 Gross earnings = \$2231.25
12. Gross earnings = \$1000 + 0.08(\$10,000) + 0.10(\$38,670 – \$30,000) = \$2667
13. Commission earnings = Commission rate (Sales – \$40,000)  
 \$3188.35 – \$1000 = Commission rate (\$88,630 – \$40,000)  
 $\frac{\$2188.35}{\$48,630} \times 100\% = \underline{\underline{4.50\%}}$
14. a. Commission = Commission rate x Base  
 = 0.04(\$200,000) + 0.025(\$89,000)  
 = \$8000 + \$2225  
 = \$10,225.00
- b. Average commission rate =  $\frac{\text{Total commission}}{\text{Selling price}} \times 100\% = \frac{\$10,225}{\$289,000} \times 100\% = \underline{\underline{3.54\%}}$
15. Average change in revenue for the year =  $\frac{22(5\%) + 18(2\%) + 32(-9\%) + 28(-2\%)}{22 + 18 + 32 + 28}$   
 $= \frac{-198\%}{100} = \underline{\underline{-1.98\%}}$
16. Rate of return on the portfolio = Weighted average rate of return  
 $\frac{\$16,800(-4.3\%) + \$25,600(-1.1\%) + \$31,000(8.2\%)}{\$16,800 + \$25,600 + \$31,000}$   
 = 2.10%
17. Rate of return on entire portfolio  
 = Weighted average rate of return  
 $\frac{\$5000(30\%) + 20,000(-3\%) + \$8000(-15\%) + \$25,000(13\%) + \$4500(45\%)}{\$5000 + \$20,000 + \$8000 + \$25,000 + \$4500}$   
 = 7.96%
- 18.

| <u>Period</u>     | <u>No. of months</u> | <u>Cumulative investment</u>      |
|-------------------|----------------------|-----------------------------------|
| Jan. 1 to Feb. 28 | 2                    | \$96,400                          |
| Mar. 1 to Mar. 31 | 1                    | $\$96,400 - \$14,200 = \$82,200$  |
| Apr. 1 to July 31 | 4                    | $\$82,200 - \$21,800 = \$60,400$  |
| Aug. 1 to Oct. 31 | 3                    | $\$60,400 + \$23,700 = \$84,100$  |
| Nov. 1 to Dec. 31 | 2                    | $\$84,100 + \$19,300 = \$103,400$ |

Average investment during the year

$$\begin{aligned}
 & \frac{2(\$96,400) + 1(\$82,200) + 4(\$60,400) + 3(\$84,100) + 2(\$103,400)}{12} \\
 & = \\
 & = \underline{\underline{\$81,308.33}}
 \end{aligned}$$

19.

| <u>Period</u>      | <u>No. of months</u> | <u>Number of employees</u> |
|--------------------|----------------------|----------------------------|
| July 1 to Aug. 31  | 2                    | 7                          |
| Sept. 1 to Oct. 31 | 2                    | $7 + 6 = 13$               |
| Nov. 1 to Nov. 30  | 1                    | $13 + 18 = 31$             |
| Dec. 1 to Feb. 28  | 3                    | $31 + 23 = 54$             |
| Mar. 1 to Mar. 31  | 1                    | $54 - 11 = 43$             |
| Apr. 1 to Apr. 30  | 1                    | $43 - 20 = 23$             |
| May 1 to June 30   | 2                    | $23 - 16 = 7$              |

The (weighted) average number of employees was

$$\frac{2(7) + 2(13) + 1(31) + 3(54) + 1(43) + 1(23) + 2(7)}{12} = 26.1$$