

NOT FOR SALE

Chapter 1: Review of Real Numbers

Prep Test

1. $\frac{5}{12} + \frac{7}{30} = \frac{25}{60} + \frac{14}{60} = \frac{39}{60} = \frac{3 \cdot 13}{3 \cdot 20} = \frac{13}{20}$

2. $\frac{8}{15} - \frac{7}{20} = \frac{32}{60} - \frac{21}{60} = \frac{11}{60}$

3. $\frac{5}{6} \cdot \frac{4}{15} = \frac{5 \cdot 2 \cdot 2}{3 \cdot 2 \cdot 5 \cdot 3} = \frac{2}{9}$

4. $\frac{4}{15} \div \frac{2}{5} = \frac{4}{15} \cdot \frac{5}{2} = \frac{2 \cdot 2 \cdot 5}{5 \cdot 3 \cdot 2} = \frac{2}{3}$

5. $8.000 + 29.340 + 7.065 = 44.405$

6. $92.00 - 18.37 = 73.63$

7. $2.19(3.4) = 7.446$

8. $32.436 \div 0.6 = 324.36 \div 6 = 54.06$

9. a) $-6 > -8$ Yes
b) $-10 > -8$ No
c) $0 > -8$ Yes
d) $8 > -8$ Yes

10. $\frac{1}{2} = 0.5$ C
 $\frac{7}{10} = 0.7$ D
 $\frac{3}{4} = 0.75$ A
 $\frac{89}{100} = 0.89$ B

Section 1.1

Concept Check

1.a) integers: 0, -3

b) rational numbers: $-\frac{15}{2}$, 0, -3, $2.\overline{33}$

c) irrational numbers:

$$\pi, 4.232232223\dots, \frac{\sqrt{5}}{4}, \sqrt{7}$$

d) real numbers: all

2.a) integers: -17

b) rational numbers: -17.03412 , $\frac{27}{91}$, $6.\overline{12}$

c) irrational numbers: $\frac{3}{\pi}$, $-1.010010001\dots$

d) real numbers: all

3. $\frac{2}{3}$

4. $-\pi$

5. $-1.\overline{45}$

6. 0

7. 5.6

8. -6

9. 0

10. -41

11. -54

12. -42

Objective A Exercises

13. Replace y with each element in the set to determine if the inequality is true or false.

$-6 > -4$ False

$-4 > -4$ False

$7 > -4$ True

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14. Replace x with each element in the set to determine if the inequality is true or false.

$$\begin{aligned} -6 < -3 & \text{ True} \\ -3 < -3 & \text{ False} \\ 3 < -3 & \text{ False} \end{aligned}$$

15. Replace w with each element in the set to determine if the inequality is true or false.

$$\begin{aligned} -2 &\leq -1 & \text{ True} \\ -1 &\leq -1 & \text{ True} \\ 0 &\leq -1 & \text{ False} \\ 1 &\leq -1 & \text{ False} \end{aligned}$$

16. Replace p with each element in the set to determine if the inequality is true or false.

$$\begin{aligned} -10 &\geq 0 & \text{ False} \\ -5 &\geq 0 & \text{ False} \\ 0 &\geq 0 & \text{ True} \\ 5 &\geq 0 & \text{ True} \end{aligned}$$

17. Replace b with each element in the set then evaluate the expression.

$$\begin{aligned} -(-9) &= 9 \\ -(0) &= 0 \\ -(9) &= -9 \end{aligned}$$

18. Replace a with each element in the set then evaluate the expression.

$$\begin{aligned} -(-3) &= 3 \\ -(-2) &= 2 \\ -(0) &= 0 \end{aligned}$$

19. Replace c with each element in the set then evaluate the expression.

$$\begin{aligned} |-4| &= 4 \\ |0| &= 0 \\ |4| &= 4 \end{aligned}$$

20. Replace q with each element in the set then evaluate the expression.

$$\begin{aligned} |-3| &= 3 \\ |0| &= 0 \\ |7| &= 7 \end{aligned}$$

21. $-x > 0$ whenever the value of x is a negative real number.

22. Since $|y|$ will always give us a nonnegative result there is no real number y for which $-|y| > 0$.

Objective B Exercises

23. $\{-2, -1, 0, 1, 2, 3, 4\}$

24. $\{-3, -2, -1\}$

25. $\{2, 4, 6, 8, 10, 12\}$

26. $\{1, 3, 5, 7, 9, 11, 13\}$

27. $\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$

28. $\{-20, -16, -12, -8, -4\}$

29. $\{x \mid x > 4, x \in \text{integers}\}$

30. $\{x \mid x < -2, x \in \text{integers}\}$

31. $\{x \mid x \geq -2\}$

32. $\{x \mid x \leq 2\}$

33. $\{x \mid 0 < x < 1\}$

34. $\{x \mid -2 < x < 5\}$

35. $\{x \mid 1 \leq x \leq 4\}$

36. $\{x \mid 0 \leq x \leq 2\}$

37. Yes

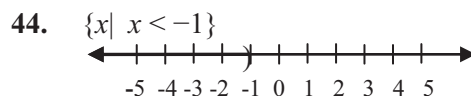
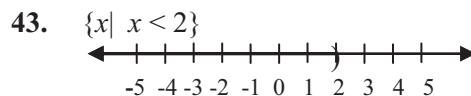
38. No

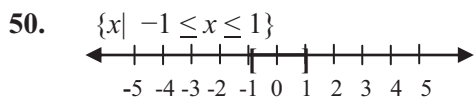
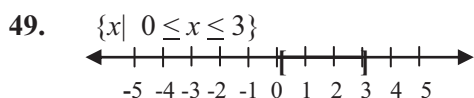
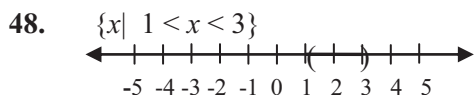
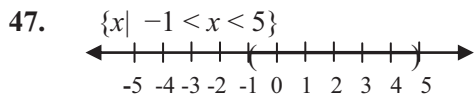
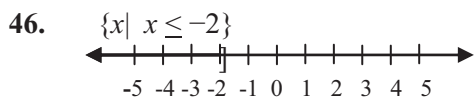
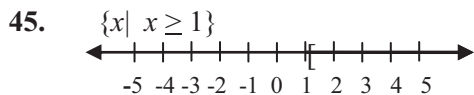
39. No

40. Yes

41. No

42. Yes





51. $(-2, 4)$

52. $(0, 3)$

53. $[-1, 5]$

54. $[0, 3]$

55. $(-\infty, 1)$

56. $(-\infty, 6]$

57. $[-2, \infty)$

58. $[3, \infty)$

59. $\{x \mid 0 < x < 8\}$

60. $\{x \mid -2 < x < 4\}$

61. $\{x \mid -5 \leq x \leq 7\}$

62. $\{x \mid 3 \leq x \leq 4\}$

63. $\{x \mid -3 \leq x < 6\}$

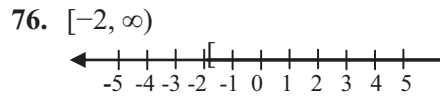
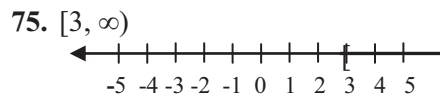
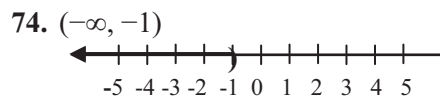
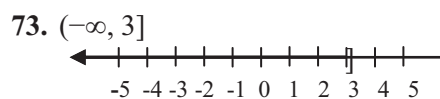
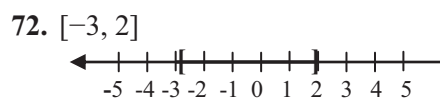
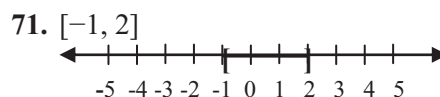
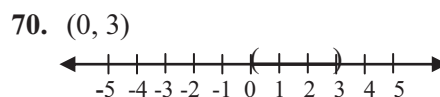
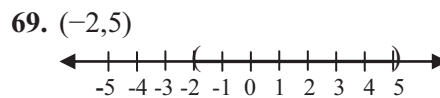
64. $\{x \mid 4 < x \leq 5\}$

65. $\{x \mid x \leq 4\}$

66. $\{x \mid x < -2\}$

67. $\{x \mid x > 5\}$

68. $\{x \mid x \geq -2\}$



Objective C Exercises

77. $A \cup B = \{1, 2, 4, 6, 9\}$

78. $A \cup B = [-1, 0, 1, 2]$

79. $A \cup B = \{2, 3, 5, 8, 9, 10\}$

80. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

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81. $A \cup B = \{-4, -2, 0, 2, 4, 8\}$

82. $A \cup B = \{-3, -2, -1, 0, 1\}$

83. $A \cup B = \{1, 2, 3, 4, 5\}$

84. $A \cup B = \{0, 1, 2, 3, 4, 5\}$

85. $A \cap B = \{6\}$

86. $A \cap B = \{0\}$

87. $A \cap B = \{5, 10, 20\}$

88. $A \cap B = \{1, 9\}$

89. $A \cap B = \emptyset$

90. $A \cap B = \emptyset$

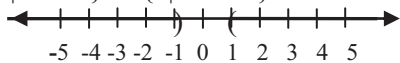
91. $A \cap B = \{4, 6\}$

92. $A \cap B = \{-5, 0, 7\}$

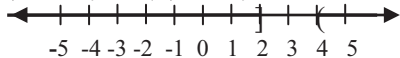
93. $[5, \infty) \cap (0, 5)$

94. $\{x \mid x \geq -1\} \cup \{x \mid x < 6\}$

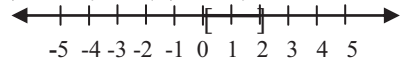
95. $\{x \mid x > 1\} \cup \{x \mid x < -1\}$



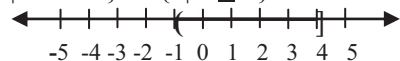
96. $\{x \mid x \leq 2\} \cup \{x \mid x > 4\}$



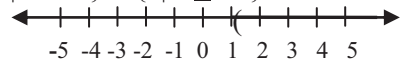
97. $\{x \mid x \leq 2\} \cap \{x \mid x \geq 0\}$



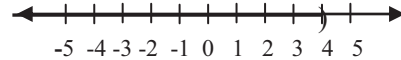
98. $\{x \mid x > -1\} \cap \{x \mid x \leq 4\}$



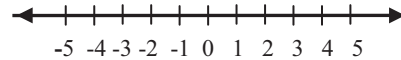
99. $\{x \mid x > 1\} \cap \{x \mid x \geq -2\}$



100. $\{x \mid x < 4\} \cup \{x \mid x \leq 0\}$



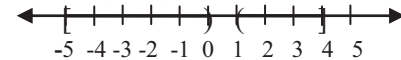
101. $\{x \mid x > -3\} \cup \{x \mid x < 1\}$



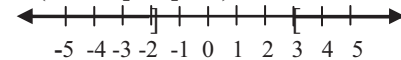
102. $\{x \mid x > -2\} \cup \{x \mid x < -4\}$

\emptyset

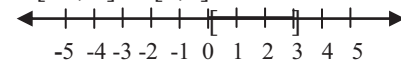
103. $[-5, 0) \cup (1, 4]$



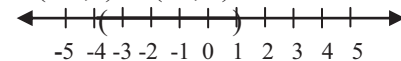
104. $(-\infty, -2] \cup [3, \infty)$



105. $[-3, 3] \cap [0, 5]$



106. $(-\infty, 1) \cap (-4, \infty)$



Critical Thinking

107. $|x| < 4$

108. $|x| \geq 5$

109. $|x - 1| \leq 5$

110. $|x - 7| \geq 7$

111. $|x + 2| < 2$

112. $|x + 3| > 2$

113. $|x - a| > b$

114. $|x - a| < b$

Projects or Group Activities

115. $A \cup B = A$

116. $A \cup A = A$

117. $B \cap B = B$

118. $A \cup C = A$

119. $A \cap R = A$

120. $C \cap R = C$

121. $B \cup R = R$

122. $A \cup R = R$

123. $R \cup R = R$

124. $B \cap C = \{0\}$

Section 1.2**Concept Check**

1. a) To add two numbers with the same sign add the absolute values of the numbers. Then attach the sign of the addends.

b) To add two numbers with different signs find the absolute value of each number. Subtract the smaller of these two numbers from the larger. Then attach the sign of the larger absolute value.

2. To rewrite $8 - (-12)$ as addition of the opposite value, we change the -12 to its opposite, change the subtraction sign to addition, and rewrite the expression as follows: $8 - (-12) = 8 + 12$.

3. No. For example, $-5 + (-3) = -8$

4. They are both positive or both negative.

5. One number is positive and one number is negative.

6. Yes. For instance, $-3 - (-7) = 4$

7. At least one of the numbers is zero.

8. 3^7

9. $(-5)^6$

10. $8 - 2 \cdot 5 < (8 - 2) \cdot 5$

Objective A Exercises

11. Negative

12. Positive

13. Positive

14. Negative

15. $-15(-11) = 165$

16. $135 \div (-5) = -27$

$$\begin{aligned} 17. \quad & -39 - 5 \\ & = -39 + (-5) \\ & = -44 \end{aligned}$$

18. $-16 + (-6) = -22$

19. $87 \div (-3) = -29$

20. $400 \div (-16) = 25$

$$\begin{aligned} 21. \quad & 39 - (-6) \\ & = 39 + (6) \\ & = 45 \end{aligned}$$

22. $2 + (-18) = -16$

23. $(-12)(15) = -180$

$$\begin{aligned} 24. \quad & 9 - (-29) \\ & = 9 + (29) = 38 \end{aligned}$$

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25. $238 \div 17 = 14$

26. $19 - (-13)$
 $= 19 + (13)$
 $= 32$

27. $-5 + 19 = 14$

28. $-9 + (-11) = -20$

29. $44 \div (-4) = -11$

30. $20(-13) = -260$

31. $-14 - 8$
 $= -14 + (-8)$
 $= -22$

32. $-24(-4) = 96$

33. $9 - (-9)$
 $= 9 + (9)$
 $= 18$

34. $7(-13) = -91$

35. $27 + (-8) = 19$

36. $15 + (-16) = -1$

37. $2(-26) = -52$

38. $399 \div (-21) = -19$

39. The total of -6 and 24 is 18 .
 $-6 + 24 = 18$

40. 363 divided by 363 is 1 .
 $363 \div 363 = 1$

41. 80 divided by 80 is 1 .
 $80 \div 80 = 1$

42. 5 less -27 is 32
 $5 - (-27)$
 $= 5 + (27)$
 $= 32$

43. $-56 \div (-7) = 8$

44. $-39(-5) = 195$

45. $-18 + (-10) = -28$

46. $9 \div (-3) = -3$

47. $(-17)(-12) = 204$

48. $-720 \div -18 = 40$

49. $26 + (-6) = 20$

50. $(-3)(15) = -45$

51. $24 - (-2)$
 $= 24 + (2)$
 $= 26$

52. $-4 - 39$
 $= -4 + (-39)$
 $= -43$

53. $-21 - (-10)$
 $= -21 + (10)$
 $= -11$

54. $8 - (-9)$
 $= 8 + (9)$
 $= 17$

55. $107 - (-153)$
 $= 107 + (153)$
 260

The difference between the average temperatures during the day and at night on the moon is 260°C .

56. $-12 + 9 = -3$

The temperature at the weather station at 10:00am was -3°F .

57. $-282 + 223 = -59$

The new elevation of the hiker is -59 ft.

58. The difference in elevation between Salton Sea and Coachella is 148 ft.

Objective B Exercises

59. Negative

$$\begin{aligned}
 60. \quad & -4^5 = -1024 \\
 & (-4)^5 = -1024 \\
 & -4^6 = -4096 \\
 & (-4)^6 = 4096 \\
 & (-4)^6 \text{ is a positive number.}
 \end{aligned}$$

$$61. \quad -2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$$

$$62. \quad -3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$$

$$63. \quad (-5)^3 = (-5) \cdot (-5) \cdot (-5) = -125$$

$$64. \quad (-7)^4 = (-7) \cdot (-7) \cdot (-7) \cdot (-7) = 2401$$

$$\begin{aligned}
 65. \quad & (-2)^{10} = (-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2) \\
 & = 1024
 \end{aligned}$$

$$66. \quad (-4)^5 = (-4)(-4)(-4)(-4)(-4) = -1024$$

$$67. \quad (-1)^{50} = 1$$

$$68. \quad -1^{50} = -1$$

$$\begin{aligned}
 69. \quad & 5 - 8 - 12 + 3 \\
 & = 5 + (-8) + (-12) + 3 \\
 & = -3 + (-12) + 3 \\
 & = -15 + 3 \\
 & = -12
 \end{aligned}$$

$$70. \quad 48 \cdot 6 \div 3 = 288 \div 3 = 96$$

$$71. \quad 24 \div 2 \cdot 6 = 12 \cdot 6 = 72$$

$$72. \quad 14 - 4 \cdot 9 = 14 - 36 = 14 + (-36) = -22$$

$$73. \quad -28 - 12 \div 4 = -28 - 3 = -28 + (-3) = -31$$

$$74. \quad -30 \div (2 \cdot 5) = -30 \div 10 = -3$$

$$\begin{aligned}
 75. \quad & -9 + 4(8 - 15) = -9 + 4(-7) = -9 + (-28) \\
 & = -37
 \end{aligned}$$

$$\begin{aligned}
 76. \quad & 2(5 - 7) - 6(18 - 4) \\
 & = 2(-2) - 6(14) \\
 & = -4 - 84 \\
 & = -4 + (-84) \\
 & = -88
 \end{aligned}$$

$$\begin{aligned}
 77. \quad & 27 \div (-3) + 5(2 - 8)^2 \\
 & = 27 \div (-3) + 5(-6)^2 \\
 & = 27 \div (-3) + 5(36) \\
 & = 27 \div (-3) + 180 \\
 & = -9 + 180 \\
 & = 171
 \end{aligned}$$

$$\begin{aligned}
 78. \quad & \frac{9 - 35}{13} - \frac{(6 \cdot 9)^3}{-9} \\
 & = \frac{-26}{13} - \frac{54^3}{-9} = \frac{-26}{13} - \frac{157464}{-9} \\
 & = -2 - (-17496) \\
 & = 17494
 \end{aligned}$$

$$\begin{aligned}
 79. \quad & 3 \cdot 8 - 6(4 - 9) + \frac{3 - 15}{4} \\
 & = 3 \cdot 8 - 6(-5) + \frac{-12}{4} = 24 - (-30) + (-3) \\
 & = 24 + (30) + (-3) \\
 & = 51
 \end{aligned}$$

$$\begin{aligned}
 80. \quad & 63 \div (2 - 5)^2 \cdot 16 - 4 \cdot 8 - 15 \\
 & = 63 \div (-3)^2 \cdot 16 - 4 \cdot 8 - 15 \\
 & = 63 \div 9 \cdot 16 - 4 \cdot 8 - 15 \\
 & = 7 \cdot 16 - 32 - 15 \\
 & = 112 - 32 - 15 \\
 & = 65
 \end{aligned}$$

$$\begin{aligned}
 81. \quad & -3(5 - 8)^3 + (19 - 7) \div (1 - 3) \\
 & = -3(-3)^3 + (19 - 7) \div (1 - 3) \\
 & = -3(-27) + (12) \div (-2) \\
 & = 81 + (-6) \\
 & = 75
 \end{aligned}$$

$$\begin{aligned}
 82. \quad & 28 \div (7 - 9)^2 \cdot (1 - 3)^4 \div 14 \\
 & = 28 \div (-2)^2 \cdot (-2)^4 \div 14 \\
 & = 28 \div 4 \cdot 16 \div 14 \\
 & = 7 \cdot 16 \div 14 \\
 & = 112 \div 14 \\
 & = 8
 \end{aligned}$$

$$\begin{aligned}
 83. \quad & \frac{15-19}{2^2} \cdot \frac{3(2-7)^2}{3 \cdot 2 - 1} \div \frac{3 \cdot 2 \cdot 2^4}{10-4 \cdot 3} \\
 &= \frac{-4}{4} \cdot \frac{3(25)}{5} \div \frac{96}{-2} = -1 \cdot 15 \div (-48) \\
 &= -15 \div -48 \\
 &= \frac{5}{16}
 \end{aligned}$$

$$\begin{aligned}
 h. \quad & \frac{11!}{8!(11-8)!} = \frac{11!}{8!(3!)} \\
 &= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} = 165
 \end{aligned}$$

Critical Thinking

84. There is a 3 in the tens place for 11^{33} .

85. There is a 7 in the ones place for 7^{25} .

86. The last three digits of 5^{202} are 625.

87. $(2^3)^4 < 2^{(3^4)}$

Projects or Group Activities

88. a. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

b. $8! - 6!$
 $= (8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) - (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$
 $= 39,600$

c. $0! = 1$

d. $\frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 504$

e. $\frac{10!}{5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 30,240$

f. $\frac{7!}{2!5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = 21$

g. $\frac{12!}{4!(12-4)!} = \frac{12!}{4!(8!)}$
 $= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = 495$

Section 1.3

Concept Check

1. The least common multiple of two numbers is the smallest number that is a multiple of each of the two numbers. When adding fractions with different denominators we need to first rewrite the fractions as equivalent fractions with a common denominator. The lowest common denominator is the LCM of the denominators.

2. The greatest common factor of two numbers is the largest integer that divides evenly into both numbers.

3. All integers are rational numbers.

4. Answers will vary. An example of two rational numbers that are not integers is -2.34 and $\frac{2}{3}$.

5. There is a smallest possible positive integer. However, there is not a smallest positive rational number.

6. a. The LCM of the denominators $\frac{5}{8}, -\frac{1}{28}$ and $\frac{2}{7}$ is 56.

b. $\frac{5}{8} = \frac{35}{56}, -\frac{1}{28} = -\frac{2}{56}, \frac{2}{7} = \frac{16}{56}$

7. a. The reciprocal of $-\frac{8}{27}$ is $-\frac{27}{8}$.

b. $\frac{2}{3} \cdot \left(-\frac{27}{8}\right); -\frac{9}{4}$

8. $\frac{1}{4} \div \frac{3}{8}$

Objective A Exercises

9. Find the prime factorization of 83 and 2.

$$83 = 83 \cdot 1$$

$$2 = 2 \cdot 1$$

$$\text{The LCM} = 83 \cdot 2 = 166$$

$$\text{The GCF is } 1.$$

10. Find the prime factorization of 57 and 9.

$$57 = 19 \cdot 3$$

$$9 = 3 \cdot 3$$

$$\text{The LCM is } 19 \cdot 3 \cdot 3 = 171$$

$$\text{The GCF is } 3.$$

11. Find the prime factorization of 15 and 75.

$$15 = 5 \cdot 3$$

$$75 = 5 \cdot 5 \cdot 3$$

$$\text{The LCM is } 5 \cdot 5 \cdot 3.$$

$$\text{The GCF is } 5 \cdot 3$$

12. Find the prime factorization of 26 and 2.

$$26 = 13 \cdot 2$$

$$2 = 2 \cdot 1$$

$$\text{The LCM is } 26.$$

$$\text{The GCF is } 2.$$

13. Find the prime factorization of 6 and 2.

$$6 = 3 \cdot 2$$

$$2 = 2 \cdot 1$$

$$\text{The LCM is } 6.$$

$$\text{The GCF is } 2.$$

14. Find the prime factorization of 110 and 5.

$$110 = 11 \cdot 5 \cdot 2$$

$$5 = 5 \cdot 1$$

$$\text{The LCM is } 110.$$

$$\text{The GCF is } 5.$$

15. Find the prime factorization of 10 and 55.

$$10 = 5 \cdot 2$$

$$55 = 11 \cdot 5$$

$$\text{The LCM is } 110.$$

$$\text{The GCF is } 5.$$

16. Find the prime factorization of 11 and 10.

$$11 = 11 \cdot 1$$

$$10 = 5 \cdot 2$$

$$\text{The LCM is } 110.$$

$$\text{The GCF is } 1.$$

17. Find the prime factorization of 34 and 85.

$$34 = 17 \cdot 2$$

$$85 = 17 \cdot 5$$

$$\text{The LCM is } 170.$$

$$\text{The GCF is } 17.$$

18. Find the prime factorization of 96 and 8.

$$96 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$8 = 2 \cdot 2 \cdot 2$$

$$\text{The LCM is } 96.$$

$$\text{The GCF is } 2.$$

19. Find the prime factorization of 140 and 7.

$$140 = 7 \cdot 5 \cdot 2 \cdot 2$$

$$7 = 7 \cdot 1$$

$$\text{The LCM is } 140.$$

$$\text{The GCF is } 7.$$

20. Find the prime factorization of 18 and 6.

$$18 = 3 \cdot 3 \cdot 2$$

$$6 = 3 \cdot 2$$

$$\text{The LCM is } 18.$$

$$\text{The GCF is } 6.$$

21. Find the prime factorization of 14, 42 and 18.

$$14 = 7 \cdot 2$$

$$42 = 7 \cdot 3 \cdot 2$$

$$18 = 3 \cdot 3 \cdot 2$$

$$\text{The LCM is } 126.$$

$$\text{The GCF is } 2.$$

22. Find the prime factorization of 30, 5 and 6.

$$30 = 5 \cdot 3 \cdot 2$$

$$5 = 5 \cdot 1$$

$$6 = 3 \cdot 2$$

$$\text{The LCM is } 30.$$

$$\text{The GCF is } 1.$$

10 Chapter 1 Review of Real Numbers

23. Find the prime factorization of 30, 5 and 10.

$$30 = 5 \cdot 3 \cdot 2$$

$$5 = 5 \cdot 1$$

$$10 = 5 \cdot 2$$

The LCM is 30.

The GCF is 5.

24. Find the prime factorization of 4, 27 and 6.

$$4 = 2 \cdot 2$$

$$27 = 3 \cdot 3 \cdot 3$$

$$6 = 3 \cdot 2$$

The LCM is 108.

The GCF is 1.

25. Find the prime factorization of 15, 45 and 10.

$$15 = 5 \cdot 3$$

$$45 = 5 \cdot 3 \cdot 3$$

$$10 = 5 \cdot 2$$

The LCM is 90.

The GCF is 5.

26. Find the prime factorization of 16, 3 and 12.

$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

$$3 = 3 \cdot 1$$

$$12 = 3 \cdot 2 \cdot 2$$

The LCM is 48.

The GCF is 1.

27. Find the prime factorization of 12, 42 and 14.

$$12 = 3 \cdot 2 \cdot 2$$

$$42 = 7 \cdot 3 \cdot 2$$

$$14 = 7 \cdot 2$$

The LCM is 84.

The GCF is 2.

28. Find the prime factorization of 12, 2 and 14.

$$12 = 3 \cdot 2 \cdot 2$$

$$2 = 2 \cdot 1$$

$$14 = 7 \cdot 2$$

The LCM is 84.

The GCF is 2.

Objective B Exercises

$$\begin{aligned} 29. \left(-\frac{35}{12}\right)\left(\frac{4}{3}\right) &= -\frac{35 \cdot 4}{12 \cdot 3} \\ &= -\frac{7 \cdot 5 \cdot 2 \cdot 2}{3 \cdot 2 \cdot 2 \cdot 3} = -\frac{35}{9} \end{aligned}$$

$$30. \left(-\frac{7}{9}\right)\left(-\frac{1}{2}\right) = \frac{7 \cdot 1}{9 \cdot 2} = \frac{7}{18}$$

$$\begin{aligned} 31. \left(-\frac{1}{3}\right)\left(\frac{9}{29}\right) &= -\frac{1 \cdot 9}{3 \cdot 29} \\ &= -\frac{1 \cdot 3 \cdot 3}{3 \cdot 29} = -\frac{3}{29} \end{aligned}$$

$$\begin{aligned} 32. \left(-\frac{8}{13}\right)\left(-\frac{1}{2}\right) &= \frac{8 \cdot 1}{13 \cdot 2} \\ &= \frac{2 \cdot 2 \cdot 2 \cdot 1}{13 \cdot 2} = \frac{4}{13} \end{aligned}$$

$$\begin{aligned} 33. -\frac{3}{2} - \left(-\frac{1}{9}\right) &= -\frac{3}{2} + \frac{1}{9} = -\frac{27}{18} + \frac{2}{18} \\ &= \frac{-27 + 2}{18} = \frac{25}{18} \end{aligned}$$

$$\begin{aligned} 34. \left(\frac{3}{2}\right)\left(-\frac{5}{12}\right) &= -\frac{3 \cdot 5}{2 \cdot 12} \\ &= -\frac{3 \cdot 5}{2 \cdot 3 \cdot 2 \cdot 2} = -\frac{5}{8} \end{aligned}$$

$$\begin{aligned} 35. -\frac{5}{4} + \frac{19}{22} &= \frac{-55}{44} + \frac{38}{44} \\ &= \frac{-55 + 38}{44} = -\frac{17}{44} \end{aligned}$$

$$36. \left(\frac{19}{11}\right)\left(-\frac{7}{19}\right) = -\frac{19 \cdot 7}{11 \cdot 19} = -\frac{7}{11}$$

$$\begin{aligned} 37. -\frac{11}{4} \div \left(-\frac{1}{8}\right) &= -\frac{11}{4} \cdot \left(-\frac{8}{1}\right) = \frac{11 \cdot 8}{4 \cdot 1} \\ &= \frac{11 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = \frac{22}{1} = 22 \end{aligned}$$

$$38. \left(-\frac{3}{23}\right)\left(-\frac{11}{3}\right) = \frac{3 \cdot 11}{23 \cdot 3} = \frac{11}{23}$$

$$39. \frac{32}{39} + \left(-\frac{14}{3}\right) = \frac{32}{39} + \frac{-182}{39}$$

$$= \frac{32 + (-182)}{39} = -\frac{150}{39} = -\frac{50}{13}$$

$$40. \frac{3}{4} - \left(-\frac{7}{20}\right) = \frac{3}{4} + \frac{7}{20} = \frac{15}{20} + \frac{7}{20}$$

$$= \frac{15+7}{20} = \frac{22}{20} = \frac{11}{10}$$

$$41. \left(\frac{3}{4}\right)\left(-\frac{17}{27}\right) = -\frac{3 \cdot 17}{4 \cdot 27}$$

$$= -\frac{3 \cdot 17}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = -\frac{17}{36}$$

$$42. \frac{20}{7} - \left(-\frac{25}{28}\right) = \frac{80}{28} + \frac{25}{28}$$

$$= \frac{80+25}{28} = \frac{105}{28} = \frac{15}{4}$$

$$43. \frac{3}{4} \div \left(-\frac{39}{16}\right) = \frac{3}{4} \cdot \left(-\frac{16}{39}\right) = -\frac{3 \cdot 16}{4 \cdot 39}$$

$$= -\frac{3 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 3 \cdot 13} = -\frac{4}{13}$$

$$44. \frac{1}{2} - \left(-\frac{14}{13}\right) = \frac{13}{26} + \frac{28}{26}$$

$$= \frac{13+28}{26} = \frac{41}{26}$$

$$45. -\frac{10}{37} \div \left(-\frac{5}{2}\right) = -\frac{10}{37} \cdot \left(-\frac{2}{5}\right) = \frac{10 \cdot 2}{37 \cdot 5}$$

$$= \frac{5 \cdot 2 \cdot 2}{37 \cdot 5} = \frac{4}{37}$$

$$46. -\frac{7}{2} + \frac{39}{11} = \frac{-77}{22} + \frac{78}{22}$$

$$= \frac{-77+78}{22} = \frac{1}{22}$$

$$47. -\frac{11}{24} \left(\frac{32}{9}\right) = -\frac{11 \cdot 32}{24 \cdot 9}$$

$$= -\frac{11 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} = -\frac{44}{27}$$

$$48. \frac{1}{3} \div \left(-\frac{19}{3}\right) = \frac{1}{3} \cdot \left(-\frac{3}{19}\right) = -\frac{1 \cdot 3}{3 \cdot 19} = -\frac{1}{19}$$

$$49. \frac{12}{11} \div \left(-\frac{1}{3}\right) = \frac{12}{11} \cdot \left(-\frac{3}{1}\right) = -\frac{12 \cdot 3}{11 \cdot 1} = -\frac{36}{11}$$

$$50. -\frac{13}{15} - \left(\frac{6}{5}\right) = \frac{-13}{15} - \frac{18}{15}$$

$$= \frac{-13-18}{15} = \frac{-31}{15} = -\frac{31}{15}$$

$$51. \frac{5}{9} + \left(-\frac{34}{9}\right) = -\frac{29}{9}$$

$$52. \frac{3}{37} - \left(-\frac{29}{37}\right) = \frac{3}{37} + \frac{29}{37} = \frac{32}{37}$$

$$53. \frac{14}{9} + \left(-\frac{11}{18}\right) = \frac{28}{18} - \frac{11}{18} = \frac{17}{18}$$

$$54. \frac{5}{3} - \left(-\frac{8}{9}\right) = \frac{15}{9} + \frac{8}{9} = \frac{15+8}{9} = \frac{23}{9}$$

$$55. -\frac{2}{5} \left(-\frac{8}{7}\right) = \frac{2 \cdot 8}{5 \cdot 7} = \frac{16}{35}$$

$$56. \left(\frac{16}{9}\right)\left(-\frac{23}{8}\right) = -\frac{16 \cdot 23}{9 \cdot 8}$$

$$= -\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 23}{3 \cdot 3 \cdot 2 \cdot 2 \cdot 2} = -\frac{46}{9}$$

$$57. \frac{5}{39} \div \left(-\frac{40}{13}\right) = \frac{5}{39} \cdot \left(-\frac{13}{40}\right) = -\frac{5 \cdot 13}{39 \cdot 40}$$

$$= -\frac{5 \cdot 13}{3 \cdot 13 \cdot 2 \cdot 2 \cdot 2 \cdot 5} = -\frac{1}{24}$$

$$58. -\frac{9}{20} \left(\frac{20}{21}\right) = -\frac{9 \cdot 20}{20 \cdot 21}$$

$$= -\frac{3 \cdot 3 \cdot 5 \cdot 2 \cdot 2}{5 \cdot 2 \cdot 2 \cdot 7 \cdot 3} = -\frac{3}{7}$$

$$59. -\frac{11}{4} - \left(-\frac{16}{9}\right) = \frac{-99}{36} + \frac{64}{36} = \frac{-99 + 64}{36}$$

$$= -\frac{35}{36}$$

$$60. \frac{11}{15} \div \left(-\frac{33}{7}\right) = \frac{11}{15} \cdot \left(-\frac{7}{33}\right) = -\frac{11 \cdot 7}{15 \cdot 33}$$

$$= -\frac{11 \cdot 7}{5 \cdot 3 \cdot 11 \cdot 3} = -\frac{7}{45}$$

$$61. -\frac{3}{4} + \frac{5}{6} = \frac{-9}{12} + \frac{10}{12} = \frac{1}{12}$$

$$62. -\frac{11}{15} + \frac{2}{3} = \frac{-11}{15} + \frac{10}{15} = -\frac{1}{15}$$

$$63. -\frac{11}{12} - \left(\frac{13}{18}\right) = \frac{-33}{36} - \frac{26}{36} = \frac{-33 - 26}{36} = -\frac{59}{36}$$

$$64. -\frac{15}{16} \left(\frac{8}{3}\right) = -\frac{15 \cdot 8}{16 \cdot 3}$$

$$= -\frac{5 \cdot 3 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} = -\frac{5}{2}$$

$$65. -\frac{4}{5} - \left(\frac{2}{3}\right) = \frac{-12}{15} - \frac{10}{15} = \frac{-12 - 10}{15} = -\frac{22}{15}$$

$$66. \frac{8}{9} - \left(\frac{5}{6}\right) = \frac{16}{18} - \frac{15}{18} = \frac{16 - 15}{18} = \frac{1}{18}$$

$$67. \frac{5}{8} \div \left(-\frac{7}{5}\right) = \frac{5}{8} \cdot \left(-\frac{5}{7}\right) = -\frac{5 \cdot 5}{8 \cdot 7} = -\frac{25}{56}$$

$$68. -\frac{8}{3} + \left(-\frac{32}{11}\right) = \frac{-88}{33} + \frac{-96}{33}$$

$$= \frac{-88 - 96}{33} = \frac{-184}{33} = -\frac{184}{33}$$

$$69. \frac{5}{8} + \left(\frac{13}{12}\right) = \frac{15}{24} + \frac{26}{24} = \frac{15 + 26}{24} = \frac{41}{24}$$

$$70. -\frac{7}{9} - \left(-\frac{5}{6}\right) = \frac{-14}{18} + \frac{15}{18} = \frac{-14 + 15}{18} = \frac{1}{18}$$

71. No. For instance, there is no integer between 2 and 3.

72. Yes. Add the two numbers and divide by 2.

Objective C Exercises

$$73. \frac{17}{20} = 0.85$$

$$74. \frac{15}{16} = 0.9375$$

$$75. \frac{7}{22} = 0.3\overline{18}$$

$$76. \frac{17}{60} = 0.28\overline{3}$$

$$77. \frac{5}{12} = 0.41\overline{6}$$

$$78. \frac{48}{55} = 0.87\overline{2}$$

$$79. \frac{6}{13} = 0.4615384\overline{6}$$

$$80. \frac{29}{32} = 0.90625$$

$$81. 0.0986 \div 0.29 = 0.34$$

$$82. -0.252 \div (-2.1) = 0.12$$

$$83. 3.4 + (-0.09) = 3.31$$

$$84. 0.19 - (-20) = 0.19 + 20 = 20.19$$

$$85. 0.08 + (-3.6) = 0.08 - 3.6 = -3.52$$

$$86. 0.05(-1.6) = -0.08$$

$$87. -60 - (-1.3) = -60 + 1.3 = -58.7$$

$$88. -0.05 - (-0.027) = -0.05 + 0.027 = -0.023$$

$$89. -1.8 + (-3.4) = -5.2$$

$$90. -1.5(0.03) = -0.045$$

$$91. 0.2 + (-90) = -89.8$$

$$92. -0.000035 \div -0.001 = 0.035$$

$$93. -2.7(-0.28) = 0.756$$

$$94. -0.37 + (-0.03) = -0.4$$

$$95. -0.033 - (-3.6) = -0.033 + 3.6 = 3.567$$

$$96. -0.00033 \div -0.15 = 0.0022$$

$$97. 0.13 - (-0.017) = 0.13 + 0.017 = 0.147$$

$$98. 13.2 \div 3300 = 0.004$$

$$99. -\frac{3}{4} \div \frac{5}{8} \cdot \left(-\frac{10}{11}\right) = -\frac{3}{4} \cdot \frac{8}{5} \cdot \left(-\frac{10}{11}\right) \\ = -\frac{24}{20} \cdot \left(-\frac{10}{11}\right) = \frac{12}{11}$$

$$100. \frac{1}{2} \div \left(-\frac{4}{5}\right) \div \frac{8}{9} = \frac{1}{2} \cdot \left(-\frac{5}{4}\right) \cdot \frac{9}{8} = -\frac{45}{64}$$

$$101. \left(\frac{2}{3} - \frac{5}{6}\right)^2 - \frac{5}{12} = \left(-\frac{1}{6}\right)^2 - \frac{5}{12} \\ = \frac{1}{36} - \frac{5}{12} = -\frac{14}{36} = -\frac{7}{18}$$

$$102. \frac{3}{4} - \frac{5}{3} \left(-\frac{5}{4}\right) = \frac{3}{4} + \frac{25}{12} = \frac{34}{12} = \frac{17}{6}$$

$$103. -\frac{5}{16} \cdot \frac{8}{9} - \frac{5}{6} \div \frac{3}{4} = -\frac{5}{16} \cdot \frac{8}{9} - \frac{5}{6} \cdot \frac{4}{3} \\ = -\frac{5}{18} - \frac{20}{18} = -\frac{25}{18}$$

$$104. -\frac{9}{8} \left(\frac{5}{18} - \frac{3}{4}\right) + \frac{5}{8} = -\frac{9}{8} \left(-\frac{17}{36}\right) + \frac{5}{8} \\ = \frac{17}{32} + \frac{5}{8} = \frac{37}{32}$$

$$105. \frac{1}{2} - \left(\frac{2}{3} \div \frac{5}{9}\right) + \frac{5}{6} = \frac{1}{2} - \left(\frac{2}{3} \cdot \frac{9}{5}\right) + \frac{5}{6} \\ = \frac{1}{2} - \frac{18}{15} + \frac{5}{6} = \frac{15 - 36 + 25}{30} = \frac{4}{30} = \frac{2}{15}$$

$$106. \frac{3}{4} \div \left[\frac{5}{8} - \frac{5}{12}\right] + 2 = \frac{3}{4} \div \left(\frac{5}{24}\right) + 2 \\ = \frac{3}{4} \cdot \frac{24}{5} + 2 = \frac{18}{5} + 2 = \frac{18}{5} + \frac{10}{5} = \frac{28}{5}$$

$$107. \frac{1}{6} - \frac{5}{4} \left(-\frac{7}{12} + \frac{1}{24}\right) = \frac{1}{6} - \frac{5}{4} \left(-\frac{13}{24}\right) \\ = \frac{1}{6} + \frac{65}{96} = \frac{81}{96} = \frac{27}{32}$$

$$108. \frac{2}{3} - \left[\frac{3}{8} + \frac{5}{6}\right] \div \frac{3}{5} = \frac{2}{3} - \frac{29}{24} \div \frac{3}{5} \\ = \frac{2}{3} - \frac{29}{24} \cdot \frac{5}{3} = \frac{2}{3} - \frac{145}{72} = -\frac{97}{72}$$

$$\begin{aligned}
 109. \quad & \left(-\frac{1}{2}\right)^3 \div \left(-\frac{3}{2} - \frac{1}{4}\right) - \frac{2}{3} \\
 & = \left(-\frac{1}{2}\right)^3 \div \left(-\frac{7}{4}\right) - \frac{2}{3} = -\frac{1}{8} \div -\frac{7}{4} - \frac{2}{3} \\
 & = -\frac{1}{8} \cdot \frac{4}{7} - \frac{2}{3} = \frac{1}{14} - \frac{2}{3} = -\frac{25}{42}
 \end{aligned}$$

$$\begin{aligned}
 110. \quad & \left(\frac{7}{8} - \frac{11}{12}\right) \div \left(\frac{3}{4} - \frac{4}{5}\right) - \frac{1}{2} \left(\frac{5}{8} - \frac{2}{3}\right) \\
 & = -\frac{1}{24} \div \left(-\frac{1}{20}\right) - \frac{1}{2} \left(-\frac{1}{24}\right) \\
 & = -\frac{1}{24} \cdot \left(-\frac{20}{1}\right) - \frac{1}{2} \left(-\frac{1}{24}\right) \\
 & = \frac{20}{24} + \frac{1}{48} = \frac{41}{48}
 \end{aligned}$$

$$\begin{aligned}
 111. \quad & 0.4(1.2 - 2.3)^2 + 5.8 \\
 & = 0.4(-1.1)^2 + 5.8 \\
 & = 0.4(1.21) + 5.8 \\
 & = 6.284
 \end{aligned}$$

$$\begin{aligned}
 112. \quad & 5.4 - (0.3)^2 \div 0.09 \\
 & = 5.4 - 0.09 \div 0.09 \\
 & = 5.4 - 1 \\
 & = 4.4
 \end{aligned}$$

$$\begin{aligned}
 113. \quad & 1.75 \div 0.25 - (1.25)^2 \\
 & = 1.75 \div 0.25 - 1.5625 \\
 & = 7 - 1.5625 \\
 & = 5.4375
 \end{aligned}$$

$$\begin{aligned}
 114. \quad & (3.5 - 4.2)^2 - 3.50 \div 2.5 \\
 & = (-0.7)^2 - 3.50 \div 2.5 \\
 & = 0.49 - 1.4 \\
 & = -0.91
 \end{aligned}$$

$$\begin{aligned}
 115. \quad & 25.76 \div (6.54 \div 3.27)^2 \\
 & = 25.76 \div (2)^2 \\
 & = 25.76 \div 4 \\
 & = 6.44
 \end{aligned}$$

$$\begin{aligned}
 116. \quad & (3.09 - 4.77)^2 - 4.07 \cdot 3.66 \\
 & = (-1.68)^2 - 4.07 \cdot 3.66 \\
 & = 2.8224 - 14.8962 \\
 & = -12.0738
 \end{aligned}$$

$$117. \quad \frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \cdot \frac{5}{4} = \frac{5}{6}$$

$$118. \quad \frac{-\frac{5}{6}}{\frac{2}{3}} = -\frac{5}{6} \div \frac{2}{3} = -\frac{5}{6} \cdot \frac{3}{2} = -\frac{5}{4}$$

$$119. \quad \frac{\frac{2}{3} - \frac{5}{6}}{\frac{3}{4} - \frac{1}{2}} = \frac{-\frac{1}{6}}{\frac{1}{4}} = -\frac{1}{6} \div \frac{1}{4} = -\frac{1}{6} \cdot \frac{4}{1} = -\frac{2}{3}$$

$$120. \quad \frac{\frac{1}{8} - \frac{1}{12}}{\frac{2}{3} - \frac{3}{4}} = \frac{\frac{1}{24}}{-\frac{1}{12}} = \frac{1}{24} \div -\frac{1}{12} = \frac{1}{24} \cdot -\frac{12}{1} = -\frac{1}{2}$$

$$\begin{aligned}
 121. \quad & \frac{\frac{2}{3} - \left(\frac{1}{2}\right)^2}{\frac{5}{4} \left(\frac{1}{2} - \frac{3}{4}\right)} = \frac{\frac{2}{3} - \frac{1}{4}}{\frac{5}{4} \left(-\frac{1}{4}\right)} = \frac{\frac{5}{12}}{-\frac{5}{16}} \\
 & = \frac{5}{12} \div -\frac{5}{16} = \frac{5}{12} \cdot -\frac{16}{5} = -\frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 122. \quad & \frac{\left(\frac{2}{3} - \frac{3}{4}\right)^3}{\frac{3}{8} \left(\frac{5}{6} - \frac{5}{12}\right)} = \frac{\left(-\frac{1}{12}\right)^3}{\frac{3}{8} \left(\frac{5}{12}\right)} = \frac{-\frac{1}{1728}}{\frac{15}{96}} \\
 & = -\frac{1}{1728} \div \frac{15}{96} = -\frac{1}{1728} \cdot \frac{96}{15} = -\frac{1}{270}
 \end{aligned}$$

$$\begin{aligned}
 123. \quad & \frac{5}{8} - \frac{\frac{2}{3} + \frac{1}{4}}{\frac{5}{7}} = \frac{5}{8} - \frac{\frac{11}{12}}{\frac{5}{7}} = \frac{5}{8} - \frac{11}{12} \div -\frac{1}{24} \\
 & = \frac{5}{8} - \frac{11}{12} \cdot \frac{24}{1} = \frac{5}{8} + \frac{22}{1} = \frac{181}{8}
 \end{aligned}$$

$$\begin{aligned}
 124. \quad \frac{1}{2} + \frac{\frac{5}{2} - \frac{7}{3}}{\frac{18}{2} + \frac{9}{3}} &= \frac{1}{2} + \frac{-\frac{9}{6}}{\frac{18}{6}} = \frac{1}{2} + \frac{-9}{18} \div \frac{7}{6} \\
 &= \frac{1}{2} + \frac{-9}{18} \cdot \frac{6}{7} = \frac{1}{2} + \frac{-3}{7} = \frac{1}{14}
 \end{aligned}$$

$$\begin{aligned}
 125. \quad \frac{1}{2} - \frac{\frac{17}{4} - \frac{25}{5}}{\frac{3}{5}} + \frac{1}{5} &= \frac{1}{2} - \frac{\frac{17}{5} - \frac{25}{5}}{\frac{3}{5}} + \frac{1}{5} \\
 &= \frac{1}{2} - \frac{17}{25} \div \frac{3}{5} + \frac{1}{5} = \frac{1}{2} - \frac{17}{25} \cdot \frac{5}{3} + \frac{1}{5} \\
 &= \frac{1}{2} - \frac{1}{3} + \frac{1}{5} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 126. \quad \frac{3}{4} + \frac{3 - \frac{7}{9}}{\frac{5}{6}} \cdot \frac{2}{3} &= \frac{3}{4} + \frac{\frac{20}{9}}{\frac{5}{6}} \cdot \frac{2}{3} \\
 &= \frac{3}{4} + \frac{20}{9} \div \frac{5}{6} \cdot \frac{2}{3} = \frac{3}{4} + \frac{20}{9} \cdot \frac{6}{5} \cdot \frac{2}{3} \\
 &= \frac{3}{4} + \frac{16}{9} = \frac{91}{36}
 \end{aligned}$$

$$\begin{aligned}
 127. \quad \frac{1 - 2 \cdot 3}{4(5 - 4)} &= \frac{-5}{4} = \frac{-5}{4} \div \frac{-7}{14} \\
 \frac{3 \cdot 5 - 1}{3 \cdot 5 - 1} &= \frac{14}{14} \\
 &= \frac{-5}{4} \cdot \frac{14}{7} = \frac{5}{2}
 \end{aligned}$$

$$\begin{aligned}
 128. \quad \frac{3(2 - 5)}{-2(2 - 6)} &= \frac{-9}{8} = \frac{-9}{8} \div \frac{10}{5} \\
 \frac{1 - 3(2 - 5)}{2 \cdot 3 - 1} &= \frac{10}{5} \\
 &= \frac{-9}{8} \cdot \frac{5}{10} = -\frac{9}{16}
 \end{aligned}$$

Critical Thinking

129. The decimal representation of $\frac{5}{23}$ is a rational number since it is the ratio of two integers. Therefore it cannot be a non-terminating, non-repeating decimal.

130. No. By the Order of Operations Agreement, the correct expression to enter is $(2/3)(3/4)$

$$\begin{aligned}
 131. \quad 2 + \frac{2}{2 - \frac{2}{2+1}} &= 2 + \frac{2}{2 - \frac{2}{3}} = 2 + \frac{2}{\frac{4}{3}} \\
 &= 2 + 2 \div \frac{4}{3} = 2 + 2 \cdot \frac{3}{4} = 2 + \frac{3}{2} = \frac{7}{2}
 \end{aligned}$$

$$\begin{aligned}
 132. \quad 1 + \frac{2}{3 + \frac{2}{5 + \frac{6}{7+8}}} &= 1 + \frac{2}{3 + \frac{2}{5 + \frac{6}{15}}} \\
 &= 1 + \frac{2}{3 + \frac{2}{3 + 4 \div \frac{81}{15}}} \\
 &= 1 + \frac{2}{3 + \frac{2}{60}} = 1 + \frac{2}{\frac{303}{81}} = 1 + 2 \div \frac{303}{81} \\
 &= 1 + 2 \cdot \frac{81}{303} = 1 + \frac{162}{303} = \frac{155}{101}
 \end{aligned}$$

$$\begin{aligned}
 133. \quad 3 - \frac{1}{3 - \frac{1}{3 - \frac{1}{8}}} &= 3 - \frac{1}{3 - \frac{1}{\frac{21}{8}}} \\
 &= 3 - \frac{1}{3 - 1 \div \frac{8}{21}} = 3 - \frac{1}{\frac{21}{8}} = 3 - 1 \div \frac{21}{8} \\
 &= 3 - 1 \cdot \frac{8}{21} = 3 - \frac{8}{21} = \frac{55}{21}
 \end{aligned}$$

$$\begin{aligned}
 134. \quad \frac{1}{\frac{1}{2} - \frac{1}{\frac{1}{2} - \frac{1}{1 - \frac{1}{2}}}} &= \frac{1}{\frac{1}{2} - \frac{1}{\frac{1}{2} - \frac{1}{\frac{1}{2}}}} = \frac{1}{\frac{1}{2} - \frac{1}{\frac{1}{2} - 2}} \\
 &= \frac{1}{\frac{1}{2} - \frac{1}{-\frac{3}{2}}} = \frac{1}{\frac{1}{2} + \frac{2}{3}} = \frac{1}{\frac{7}{6}} = \frac{6}{7}
 \end{aligned}$$

Projects or Group Activities

135. The denominator is the product of a power of 2 and a power of 5.

Check Your Progress: Chapter 1

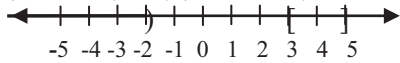
1. $\{x - 4 < x \leq 5\}$

2. $\{x | x > 2\}$

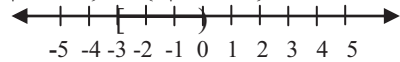
3. $[4, 8]$

4. $(-\infty, -3)$

5. $\{x | x \leq -2\} \cup \{x | 3 \leq x \leq 5\}$



6. $\{x | x < 0\} \cap \{x | x \geq -3\}$



7. $-495 \div (-33) = 15$

8. $-13 - 6 = -19$

9. $9(-15) = -135$

10. $-6 + (-30) = -36$

11. $-28 + (-10) = -38$

12. $31 + (-7) = 24$

13. $-13 - (-17) = -13 + 17 = 4$

14. $-18 + (-4) = -22$

15. $-\frac{3}{17} \left(-\frac{17}{39} \right) = \frac{3 \cdot 17}{17 \cdot 3 \cdot 13} = \frac{1}{13}$

16. $-\frac{7}{39} - \frac{8}{3} = -\frac{7}{39} + \frac{-104}{39} = -\frac{111}{39} = -\frac{37}{13}$

17. $\frac{5}{16} + \left(-\frac{5}{2} \right) = \frac{5}{16} + \frac{-40}{16} = -\frac{35}{16}$

18. $\frac{37}{18} + \left(-\frac{1}{18} \right) = \frac{36}{18} = 2$

19. $\frac{35}{4} + \left(-\frac{35}{12} \right) = \frac{105}{12} + \frac{-35}{12} = \frac{70}{12} = \frac{35}{6}$

20. $-\frac{13}{9} - \frac{10}{9} = -\frac{13}{9} + \frac{-10}{9} = -\frac{23}{9}$

21. $\frac{3}{8} - \left(-\frac{1}{6} \right) = \frac{9}{24} + \frac{4}{24} = \frac{13}{24}$

22. $\frac{15}{6} \div \left(-\frac{3}{2} \right) = \frac{15}{6} \cdot \left(-\frac{2}{3} \right) = -\frac{5 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 3} = -\frac{5}{3}$

23. $-5 - (-32) = -5 + 32 = 27$

24. $-2 - 3 = -2 + (-3) = -5$

25. $-4 + 28 = 24$

26. $-84 \div (-4) = 21$

27. $\frac{7}{8} \div \left(\frac{5}{16} \right) = \frac{7}{8} \cdot \left(\frac{16}{5} \right) = \frac{7 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 5} = \frac{14}{5}$

28. $\frac{7}{15} + \left(-\frac{11}{20} \right) = \frac{28}{60} - \frac{33}{60} = -\frac{5}{60} = -\frac{1}{12}$

$$29. \frac{11}{12} = 0.9\overline{16}$$

$$30. \frac{27}{32} = 0.84375$$

$$31. 48 - 36 \div 2^2 = 48 - 36 \div 4 = 48 - 9 = 39$$

$$\begin{aligned} 32. 3(4-7)^2 - 6 \div 2 \cdot 3 \\ &= 3(-3)^2 - 3 \cdot 3 \\ &= 3(9) - 9 \\ &= 27 - 9 \\ &= 18 \end{aligned}$$

$$\begin{aligned} 33. \frac{3}{4} - \left(\frac{1}{3} - \frac{1}{2}\right)^2 \div \frac{4}{9} &= \frac{3}{4} - \left(-\frac{1}{6}\right)^2 \div \frac{4}{9} \\ &= \frac{3}{4} - \left(\frac{1}{36}\right) \div \frac{4}{9} = \frac{3}{4} - \left(\frac{1}{36}\right) \cdot \frac{9}{4} = \frac{3}{4} - \frac{1}{16} \\ &= \frac{12}{16} - \frac{1}{16} = \frac{11}{16} \end{aligned}$$

$$\begin{aligned} 34. \frac{5}{6} \div \frac{2}{3} - \frac{1}{6} \left(\frac{4}{5} - \frac{7}{15}\right) &= \frac{5}{6} \div \frac{2}{3} - \frac{1}{6} \left(\frac{5}{15}\right) \\ &= \frac{5}{6} \cdot \frac{3}{2} - \frac{1}{6} \left(\frac{5}{15}\right) = \frac{5}{4} - \frac{1}{18} \\ &= \frac{45}{36} - \frac{2}{36} = \frac{43}{36} \end{aligned}$$

$$\begin{aligned} 35. \frac{1}{2} - \frac{\frac{3}{4} - \frac{5}{6}}{\left(\frac{1}{2} - \frac{3}{4}\right)^2} &= \frac{1}{2} - \frac{-\frac{1}{12}}{\frac{1}{16}} = \frac{1}{2} - \frac{-1}{12} \div \frac{1}{16} \\ &= \frac{1}{2} - \frac{-1}{12} \cdot \frac{16}{1} = \frac{1}{2} + \frac{4}{3} = \frac{11}{6} \end{aligned}$$

Section 1.4

Concept Check

1. Addition and multiplication have a commutative property.

2. Addition and multiplication have an associative property.

3. The inverse of $-a$ is a .

4. The multiplicative inverse of c is $\frac{1}{c}$.

5. Distributive Property

6. Expressions ii and iii are undefined.

7. No. The variable parts are not the same.

8. Yes. They have the same variable part.

Objective A Exercises

9. $3 \cdot 4 = 4 \cdot 3$

10. $7 + 15 = 15 + 7$

11. $(3 + 4) + 5 = 3 + (4 + 5)$

12. $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$

13. $\frac{5}{0}$ is undefined.

14. $5 \cdot 0 = 0$

15. $3(x + 2) = 3x + 6$

16. $5(y + 4) = 5y + 20$

17. $\frac{0}{-6} = 0$

18. $(x + y) + [-(x + y)] = 0$

19. $\frac{1}{mn} \cdot mn = 1$

20. $x \cdot 1 = x$

18 Chapter 1 Review of Real Numbers

21. $2(3x) = (2 \cdot 3)x$

22. $ab + bc = bc + ab$

23. The Division Property of Zero

24. The Inverse Property of Addition

25. The Inverse Property of Multiplication

26. The Commutative Property of Multiplication

27. The Addition Property of Zero

28. The Associative Property of Addition

29. The Division Property of Zero

30. The Distributive Property

31. The Distributive Property

32. The Multiplication Property of Zero

33. The Associative Property of Multiplication

34. The Commutative Property of Addition

35. When the sum of a positive number n and its additive inverse are multiplied by the reciprocal of the number n , the result is 0.

36. When the product of a negative number n and its reciprocal are multiplied by the number n , the result is n .

Objective B Exercises

37. $ab + dc$

$$2(3) + (-4)(-1) = 6 + 4 = 10$$

38. $2ab - 3dc$

$$2(2)(3) - 3(-4)(-1) = 4(3) - 3(4) \\ = 12 - 12 = 0$$

39. $4cd \div a^2$

$$4(-1)(-4) \div (2)^2 = 4(-1)(-4) \div 4 \\ = (-4)(-4) \div 4 = 16 \div 4 = 4$$

40. $b^2 - (d - c)^2$
 $3^2 - (-4 - (-1))^2 = 3^2 - (-4 + 1)^2$
 $= 3^2 - (-3)^2 = 9 - 9 = 0$

41. $(b - 2a)^2 + c$
 $(3 - 2(2))^2 + (-1)$
 $= (3 - 4)^2 + (-1)$
 $= (-1)^2 + (-1) = 1 + (-1) = 0$

42. $(b - c)^2 \div (b - d)$
 $(3 - (-4))^2 \div (3 - (-4)) = (3 + 4)^2 \div (3 + 4)$
 $= 7^2 \div 7 = 49 \div 7 = 7$

43. $(bc + a)^2 \div (d - b)$
 $(3(-1) + 2)^2 \div (-4 - 3) = (-3 + 2)^2 \div (-4 - 3)$
 $= (-1)^2 \div (-7) = 1 \div (-7) = -\frac{1}{7}$

44. $\frac{1}{3}b^3 - \frac{1}{4}d^3$
 $\frac{1}{3}(3)^3 - \frac{1}{4}(-4)^3 = \frac{1}{3}(27) - \frac{1}{4}(-64)$
 $= 9 - (-16) = 9 + 16 = 25$

45. $\frac{1}{4}a^4 - \frac{1}{6}bc$
 $\frac{1}{4}(2)^4 - \frac{1}{6}(3)(-1) = \frac{1}{4}(16) - \frac{1}{6}(-3)$
 $= 4 + \frac{1}{2} = \frac{9}{2}$

46. $2b^2 \div \frac{ad}{2}$
 $2(3)^2 \div \frac{(2)(-4)}{2} = 2(9) \div \frac{-8}{2}$
 $= 18 \div (-4) = -\frac{18}{4} = -\frac{9}{2}$

47. $\frac{3ac}{-4} - c^2$
 $\frac{3(2)(-1)}{-4} - (-1)^2 = \frac{3(2)(-1)}{-4} - 1$
 $= \frac{6(-1)}{-4} - 1 = \frac{-6}{-4} - 1$
 $= \frac{6}{4} - \frac{4}{4} = \frac{2}{4} = \frac{1}{2}$

$$48. \frac{2d-2a}{2bc}$$

$$\frac{2(-4)-2(2)}{2(3)(-1)} = \frac{-8-4}{6(-1)} = \frac{-12}{-6} = 2$$

$$49. \frac{3b-5c}{3a-c}$$

$$\frac{3(3)-5(-1)}{3(2)-(-1)} = \frac{9+5}{6-(-1)}$$

$$= \frac{14}{6+1} = \frac{14}{7} = 2$$

$$50. \frac{2d-a}{b-2c}$$

$$\frac{2(-4)-(-2)}{3-2(-1)} = \frac{-8-2}{3-(-2)}$$

$$= \frac{-10}{3+2} = \frac{-10}{5} = -2$$

$$51. \frac{a-d}{b+c}$$

$$\frac{2-(-4)}{3+(-1)} = \frac{2+4}{2} = \frac{6}{2} = 3$$

$$52. |a^2 + d|$$

$$|(2)^2 + (-4)| = |4 + (-4)| = |0| = 0$$

$$53. -a|a + 2d|$$

$$-2|2 + 2(-4)| = -2|2 + (-8)|$$

$$= -2|-6| = -2(6) = -12$$

$$54. d|b - 2d|$$

$$-4|3 - 2(-4)| = -4|3 - (-8)|$$

$$= -4|3 + 8| = -4|11| = -4(11) = -44$$

$$55. \frac{2a-4d}{3b-c}$$

$$\frac{2(2)-4(-4)}{3(3)-(-1)} = \frac{4-(-16)}{9+1}$$

$$= \frac{4+16}{10} = \frac{20}{10} = 2$$

$$56. \frac{3d-b}{b-2c}$$

$$\frac{3(-4)-3}{3-2(-1)} = \frac{-12-3}{3+2} = \frac{-15}{5} = -3$$

$$57. -3d \div \left| \frac{ab-4c}{2b+c} \right|$$

$$-3(-4) \div \left| \frac{2(3)-4(-1)}{2(3)+(-1)} \right| = 12 \div \left| \frac{6+4}{6-1} \right|$$

$$= 12 \div \left| \frac{10}{5} \right| = 12 \div |2| = 12 \div 2 = 6$$

$$58. -2bc + \left| \frac{bc+d}{ab-c} \right|$$

$$-2(3)(-1) + \left| \frac{3(-1)+(-4)}{2(3)-(-1)} \right| = 6 + \left| \frac{-3-4}{6+1} \right|$$

$$= 6 + \left| \frac{-7}{7} \right| = 6 + |-1| = 6 + 1 = 7$$

$$59. \quad 2(d-b) \div (3a-c)$$

$$2(-4-3) \div (3(2)-(-1))$$

$$= 2(-7) \div (6-(-1))$$

$$= 2(-7) \div (6+1) = 2(-7) \div 7$$

$$= -14 \div 7 = -2$$

$$60. (d-4a)^2 \div c^3$$

$$(-4-4(2))^2 \div (-1)^3 = (-4-8)^2 \div (-1)^3$$

$$= (-12)^2 \div (-1)^3 = 144 \div (-1) = -144$$

$$61. -d^2 - c^3a$$

$$-(-4)^2 - (-1)^3(2) = -16 - (-1)(2)$$

$$= -16 - (-2) = -16 + 2 = -14$$

$$62. a^2c - d^3$$

$$(2)^2(-1) - (-4)^3 = (4)(-1) - (-64)$$

$$= -4 + 64 = 60$$

$$63. -d^3 + 4ac$$

$$-(-4)^3 + 4(2)(-1) = -(-64) + 4(2)(-1)$$

$$= -(-64) + (8)(-1) = -(-64) + (-8)$$

$$= 64 + (-8) = 56$$

64. b^a
 $(3)^2 = 9$

65. $4^{(a^2)}$
 $4^{(2^2)} = 4^4 = 256$

66. a^b
 $(2)^3 = 8$

67. If $a = -38$, $b = -52$ and $c > 0$, the numerator is positive and the denominator is negative therefore, the expression is negative.

68. If $a > 20$, $b = 18$ and $c < 0$, the numerator is negative and the denominator is negative therefore, the expression is positive.

Objective C Exercises

69. $5x + 7x = 12x$

70. $3x + 10x = 13x$

71. $3x - 5x + 9x = 7x$

72. $-2x + 5x - 7x = -4x$

73. $5b - 8a - 12b = -8a - 7b$

74. $-2a + 7b + 9a = 7a + 7b$

75. $\frac{1}{3}(3y) = y$

76. $12\left(\frac{1}{12}x\right) = x$

77. $-\frac{2}{5}\left(-\frac{5}{2}z\right) = z$

78. $5(x + 2) = 5x + 10$

79. $3(a - 5) = 3a - 15$

80. $-3(x - 2) = -3x + 6$

81. $-5(x - 9) = -5x + 45$

82. $-(-x - y) = x + y$

83. $-(x + y) = -x - y$

84. $3(x - 2y) - 5 = 3x - 6y - 5$

85. $4x - 3(2y - 5) = 4x - 6y + 15$

86. $3x + 8(3x + 5) = 3x + 24x + 40$
 $= 27x + 40$

87. $25x + 10(9 - x) = 25x + 90 - 10x$
 $= 15x + 90$

88. $2x - 3(x - 2y) = 2x - 3x + 6y = -x + 6y$

89. $3[x - 2(x + 2y)] = 3[x - 2x - 4y]$
 $= 3[-x - 4y] = -3x - 12y$

90. $5 + 2(3x - 7) = 5 + 6x - 14 = 6x - 9$

91. $7 - 3(4a - 5) = 7 - 12a + 15 = -12a + 22$

92. $5v - 3(2 - 4v) = 5v - 6 + 12v = 17v - 6$

93. $-3m - 2(4m + 3) = -3m - 8m - 6$
 $= -11m - 6$

94. $-3 + 4(2z - 9) = -3 + 8z - 36 = 8z - 39$

95. $-5 - 6(2y - 3) = -5 - 12y + 18 = -12y + 13$

96. $5[-2 - 6(a - 5)] = 5[-2 - 6a + 30]$
 $= 5[28 - 6a] = 140 - 30a$

97. $3[a - 5(5 - 3a)] = 3[a - 25 + 15a]$
 $= 3[16a - 25] = 48a - 75$

98. $5[y - 3(y - 2x)] = 5[y - 3y + 6x]$
 $= 5[6x - 2y] = -10y + 30x$

99. $-2(x - 3y) + 2(3y - 5x) = -2x + 6y + 6y - 10x$
 $= -12x + 12y$

100. $4(-a - 2b) - 2(3a - 5b)$
 $= -4a - 8b - 6a + 10b$
 $= -10a + 2b$

$$\begin{aligned}
 101. \quad & 5(3a - 2b) - 3(-6a + 5b) \\
 & = 15a - 10b + 18a - 15b \\
 & = 33a - 25b
 \end{aligned}$$

$$\begin{aligned}
 102. \quad & -7(2a - b) + 2(-3b + a) \\
 & = -14a + 7b - 6b + 2a \\
 & = -12a + b
 \end{aligned}$$

$$\begin{aligned}
 103. \quad & 3x - 2[y - 2(x + 3[2x + 3y])] \\
 & = 3x - 2[y - 2(x + 6x + 9y)] \\
 & = 3x - 2[y - 2(7x + 9y)] \\
 & = 3x - 2[y - 14x - 18y] \\
 & = 3x - 2[-14x - 17y] = 3x + 28x + 34y \\
 & = 31x + 34y
 \end{aligned}$$

$$\begin{aligned}
 104. \quad & 2x - 4[x - 4(y - 2[5y + 3])] \\
 & = 2x - 4[x - 4(y - 10y - 6)] \\
 & = 2x - 4[x - 4(-9y - 6)] \\
 & = 2x - 4[x + 36y + 24] \\
 & = 2x - 4x - 144y - 96 \\
 & = -2x - 144y - 96
 \end{aligned}$$

$$\begin{aligned}
 105. \quad & 4 - 2(7x - 2y) - 3(-2x + 3y) \\
 & = 4 - 14x + 4y + 6x - 9y \\
 & = 4 - 8x - 5y
 \end{aligned}$$

$$\begin{aligned}
 106. \quad & 3x + 8(x - 4) - 3(2x - y) \\
 & = 3x + 8x - 32 - 6x + 3y = 5x + 3y - 32
 \end{aligned}$$

107. Simplify $31a - 102b + 73 - 88a + 256b - 73$ to $-57a + 154b$.
- The coefficient of a is negative.
 - The coefficient of b is positive.
 - The constant term is 0.

108. The correct answer is (b). Using the Distributive Property we have $3[5 - 2(y - 6)] = 15 - 6(y - 6)$

Critical Thinking

$$\begin{aligned}
 109. \quad & 0.052x + 0.072(x + 1000) \\
 & = 0.052x + 0.072x + 72 \\
 & = 0.124x + 72
 \end{aligned}$$

$$\begin{aligned}
 110. \quad & 0.07x + 0.08(10,000 - x) \\
 & = 0.07x + 800 - 0.08x \\
 & = -0.01x + 800
 \end{aligned}$$

$$111. \quad \frac{t}{20} + \frac{t}{30} = \frac{3t}{60} + \frac{2t}{60} = \frac{5t}{60} = \frac{t}{12}$$

$$112. \quad \frac{t}{4} + \frac{t}{5} = \frac{5t}{20} + \frac{4t}{20} = \frac{9t}{20}$$

Projects or Groups Activities

113. a. $4 \otimes 5 = 6$
 b. $6 \otimes 3 = 4$
 c. $3^2 = 3 \otimes 3 = 2$

114. Yes, the operation \otimes is commutative.

115. Yes, because $2 \otimes (3 \otimes 5) = 2$ and $(2 \otimes 3) \otimes 5 = 2$.

116. Yes, it appears that the operation \otimes is associative.

117. The multiplicative inverse of for the \otimes operation is 3 because $5 \otimes 3 = 1$.

Section 1.5

Concept Check

- “The sum of a and b ” means to add a and b .
 - “The product of a and b ” means to multiply a and b .
 - “The quotient of a and b ” means to divide a and b .
 - “The difference between a and b ” means to subtract a and b .

2. No. The first is $x - 2$; the second is $2 - x$.

3. No. The first is $m - 10$; the second is $10 - m$.

4. The second number would be expressed as $14 - x$.

Objective A Exercises

5. Let n represent the unknown number.
Eight less than a number: $n - 8$
6. Let n represent the unknown number.
The product of a negative six and a number:
 $-6n$
7. Let n represent the unknown number.
Four-fifths of a number: $\frac{4}{5}n$
8. Let n represent the unknown number.
The difference between a number and twenty:
 $n - 20$
9. Let n represent the unknown number.
The quotient of a number and fourteen: $\frac{n}{14}$
10. Let n represent the unknown number.
A number increased by two hundred: $n + 200$
11. Let n represent the unknown number.
Five subtracted from the product of the cube of eight and a number: $8^3n - 5$
No. This is not the same as the given expression $8n^3 - 5$.
12. Let n represent the unknown number.
Fifteen more than the sum of five times a number and two: $(5n + 2) + 15$.
Yes. This is the same as the given expression $(5n + 2) + 15$.
13. Let n represent the unknown number.
A number minus the sum of the number and two: $n - (n + 2)$
 $n - (n + 2) = n - n - 2 = -2$
14. Let n represent the unknown number.
A number decreased by the difference between five and a number: $n - (5 - n)$
 $n - (5 - n) = n - 5 + n = 2n - 5$
15. Let n represent the unknown number.
Five times the product of eight and a number:
 $5(8n) = 40n$
16. Let n represent the unknown number.
A number increased by two-thirds of the number: $n + \frac{2}{3}n = \frac{5}{3}n$
17. Let n represent the unknown number.
The difference between seventeen times a number and twice the number:
 $17n - 2n = 15n$
18. Let n represent the unknown number.
One-half of the total of six times a number and twenty-two: $\frac{1}{2}(6n + 22) = 3n + 11$
19. Let n represent the unknown number.
The difference between the square of a number and the total of twelve and the square of the number:
The square of the number: n^2
The total of twelve and the square of the number: $(12 + n^2)$
 $n^2 - (12 + n^2) = n^2 - 12 - n^2 = -12$
20. Let n represent the unknown number.
Eleven more than the square of a number added to the difference between the number and seventeen
Eleven more than the square of a number: $n^2 + 11$
The difference between the number and seventeen: $n - 17$
 $(n^2 + 11) + (n - 17) = n^2 + 11 + n - 17 = n^2 + n - 6$
21. Let n represent the unknown number.
The sum of five times a number and twelve is added to the product of fifteen and the number.
Five times the number and twelve: $5n + 12$
The product of fifteen and the number: $15n$
 $(5n + 12) + 15n = 20n + 12$

22. Let n represent the unknown number.
 Four less than twice the sum of a number and eleven.
 Twice the sum of a number and eleven:
 $2(n + 11)$
 $2(n + 11) - 4 = 2n + 22 - 4 = 2n + 18$
23. The sum of two numbers is fifteen. Let x represent the smaller of the two numbers.
 The larger number is $15 - x$.
 The sum of two more than the larger number and twice the smaller number:
 $(15 - x + 2) + 2x = 17 + x$
24. The sum of two numbers is twenty. Let x represent the smaller of the two numbers.
 The larger number is $20 - x$.
 The difference between two more than the larger number and twice the smaller number:
 $(20 - x + 2) - 2x = 22 - 3x$
25. The sum of two numbers is thirty-four. Let x represent the larger of the two numbers.
 The smaller number is $34 - x$.
 The quotient of five times the smaller number and the difference between the larger number and three:

$$\frac{5(34 - x)}{x - 3}$$
26. The sum of two numbers is thirty-three. Let x represent the larger of the two numbers.
 The smaller number is $33 - x$.
 The difference between six more than twice the larger number and the sum of the smaller number and three:
 $(2x + 6) - (33 - x + 3) = 2x + 6 - 33 + x - 3 = 3x - 30$
- Objective B Exercises**
27. a) Let j represent the number of jobs in November.
 December jobs number: $j + 200,000$
 b) Let r represent the unemployment rate in November.
 December unemployment rate: $r - 0.2$.
28. Let d represent the number of Internet users.
 Number of tablet users: $\frac{1}{3}t$
29. Let d represent the distance from Earth to the moon.
 Distance from the Earth to the sun is 390 times the distance from the Earth to the moon:
 $390d$
30. Let L represent the length of the longest road tunnel in Norway.
 The longest rail tunnel in Japan is 18.2 mi longer than the longest road tunnel in Norway: $L + 18.2$
31. Let x represent the amount in the first account. The total amount in both accounts is \$10,000.
 The amount in the second account: $10,000 - x$
32. Let L represent the length of the shorter piece. The total length of the board is 12 ft.
 The length of the longer piece: $12 - L$
33. Let x represent the measure of angle B .
 The measure of angle A is twice the measure of angle B : $2x$
 The measure of angle C is twice the measure of angle A : $2(2x) = 4x$
34. Let L represent the length of the longer piece.
 The total length of the fishing line is 3 ft.
 The length of the shorter piece: $3 - L$
35. In 2013, a house sold for \$30,000 less than the same house in sold for in 2010.
 s represents the selling price of the house in 2013.
36. In 2013, a house sold for \$30,000 less than the same house in sold for in 2010.
 p represents the selling price of the house in 2010.

Critical Thinking

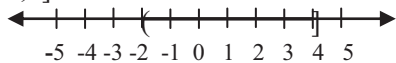
37. a. The product of mass m and acceleration a :
 ma
- b. The product of area A and the square of the velocity v : Av^2
- c. the sum of the principal P and the interest I : $P + I$
- d. the sum of twice the length L and twice the width W : $2L + 2W$
- e. the product of 16 and the time t squared:
 $16t^2$
- f. the product of four-thirds π and the radius r cubed: $\frac{4}{3}\pi r^3$.
- g. the quotient of the product of mass 1 m and mass 2 M and the distance r squared:
 $\frac{mM}{r^2}$

Projects or Group Activities

38. Answers will vary. For instance, the product of 5 and x ; 5 times x .
39. Answers will vary. For instance, the quotient of a and b ; the ratio of a to b .
40. The difference between 5 times y and 4.
41. The sum of twice x and 3.
42. 5 times the difference between y and 4.
43. Twice the sum of x and 3.

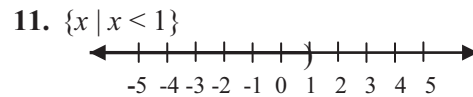
Chapter 1 Review Exercises

1. $\{-2, -1, 0, 1, 2, 3\}$
2. $A \cap B = \{2, 3\}$
3. $(-2, 4]$



4. The Associative Property of Multiplication

5. $-4.07 + 2.3 = -1.77$
6. $(a - 2b^2) \div (ab)$
 $(4 - 2(-3)^2) \div (4(-3))$
 $= (4 - 2(9)) \div (4(-3))$
 $= (4 - 18) \div (-12) = -14 \div (-12)$
 $= \frac{-14}{-12} = \frac{7}{6}$
7. $-2 \cdot (4^2) \cdot (-3)^2 = -2 \cdot 16 \cdot 9 = -288$
8. $4y - 3[x - 2(3 - 2x) - 4y]$
 $= 4y - 3[x - 6 + 4x - 4y]$
 $= 4y - 3[5x - 4y - 6]$
 $= 4y - 15x + 12y + 18 = 16y - 15x + 18$
9. The additive inverse of $-\frac{3}{4}$ is $\frac{3}{4}$.
10. $\{x \mid x < -3\}$



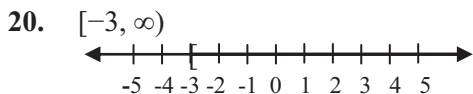
12. $-10 - (-3) = -10 + 3 = -7$
13. $-\frac{2}{3} + \frac{3}{5} = -\frac{20}{30} + \frac{18}{30}$
 $= \frac{-20 + 18}{30} = \frac{-2}{30} = -\frac{1}{15}$
14. $3 + (4 + y) = (3 + 4) + y$
15. $-\frac{3}{8} \div \frac{3}{5} = -\frac{3}{8} \cdot \frac{5}{3} = -\frac{3 \cdot 5}{8 \cdot 3} = -\frac{5}{8}$

16. Replace x with each element in the set to determine if the inequality is true or false.
- a) $-4 > -1$ False
- b) $-2 > -1$ False
- c) $0 > -1$ True
- d) $2 > -1$ True

17. $2a^2 - \frac{3b}{a}$
 $2(-3)^2 - \frac{3(2)}{-3} = 2(9) - \frac{6}{-3}$
 $= 18 - (-2) = 18 + 2 = 20$

18. $18 - |-12 + 8| = 18 - |-4| = 18 - 4 = 14$

19. $20 \div \frac{3^2 - 2^2}{3^2 + 2^2} = 20 \div \frac{9 - 4}{9 + 4} = 20 \div \frac{5}{13}$
 $= 20 \cdot \frac{13}{5} = 52$



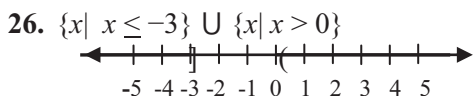
21. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

22. $-204 \div (-17) = 12$

23. $|x| - 2 \leq x \leq 3$

24. $\frac{\frac{2}{3} - \frac{5}{6}}{\frac{1}{2} - \frac{4}{4}} - \frac{2}{3} \div \frac{4}{9} = \frac{-\frac{1}{6}}{-\frac{1}{2}} - \frac{2}{3} \div \frac{4}{9}$
 $= -\frac{1}{6} \div \left(-\frac{1}{4}\right) - \frac{2}{3} \div \frac{4}{9}$
 $= -\frac{1}{6} \cdot \left(-\frac{4}{1}\right) - \frac{2}{3} \cdot \frac{9}{4}$
 $= \frac{2}{3} - \frac{3}{2} = -\frac{5}{6}$

25. $6x - 21y = 3(2x - 7y)$



27. $-2(x - 3) + 4(2 - x) = -2x + 6 + 8 - 4x$
 $= -6x + 14$

28. Replace p with each element in the set to determine the value of $-|p|$

a) $-|-4| = -4$

b) $-|0| = 0$

c) $-|7| = -7$

29. The Inverse Property of Addition

30. $-3.286 \div (-1.06) = 3.1$

31. $\frac{7}{12} = 0.58\bar{3}$

32. Replace y with each element in the set to determine if the inequality is true or false.

a) $-4 > -2$ False

b) $-1 > -2$ True

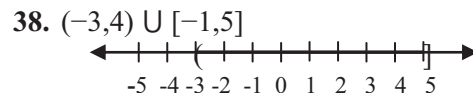
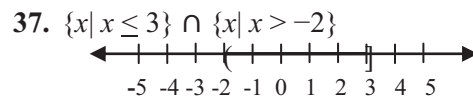
c) $4 > -2$ True

33. $\{-3, -2, -1, 0, 1\}$

34. $\{x | x < 7\}$

35. $A \cup B = \{-4, -2, 0, 2, 4, 5, 10\}$

36. $A \cap B = \emptyset$



39. $2 - (-3) = 2 + 3 = 5$

40. $-\frac{5}{6} + \left(\frac{1}{3}\right) = -\frac{5}{6} + \left(\frac{2}{6}\right) = -\frac{3}{6} = -\frac{1}{2}$

41. $(-3)^4 - 3(20-1) = (-3)^4 - 3(19)$
 $= 81 - 57 = 24$

42. $(-3)^3 - (2-6)^2 \cdot 5 = (-3)^3 - (-4)^2 \cdot 5$
 $= -27 - 16 \cdot 5 = -27 - 80 = -107$

43. $-8ac \div b^2$
 $= -8(-1)(-3) \div 2^2 = -8(-1)(-3) \div 4$
 $= -24 \div 4 = -6$

44. $-(3a + b) - 2(-4a - 5b) = -3a - b + 8a + 10b$
 $= 5a + 9b$

45. Let x represent the unknown number.

Four times the sum of a number and four:

$$4(x + 4) = 4x + 16$$

46. Let t represent the flying time between San Diego to New York.
Total flying time is 13 h
The flying time between New York and San Diego: $13 - t$

47. Let C represent the number of calories burned by walking at 4 mph for one hour.
The number of calories burned cross country skiing for one hour is 396 more than the number of calories burned walking: $C + 396$

48. Let x represent the unknown number.
Eight more than twice the difference between a number and two: $2(x - 2) + 8$
 $2(x - 2) + 8 = 2x - 4 + 8 = 2x + 4$

49. Let x represent the first integer.
The second integer is 5 more than four times the first integer.
The second integer: $4x + 5$

50. Let x represent the unknown number.
Twelve minus the quotient of three more than a number and four. $12 - \frac{x + 3}{4}$

$$12 - \frac{x + 3}{4} = \frac{48}{4} - \frac{x + 3}{4}$$

$$= \frac{48 - x + 3}{4} = \frac{45 - x}{4}$$

51. The sum of two numbers is forty.
Let x represent the smaller number.
Let $40 - x$ represent the larger number.
The sum of twice the smaller number and five more than the larger number: $2x + (40 - x) + 5$
 $2x + (40 - x) + 5 = 2x + 40 - x + 5 = x + 45$

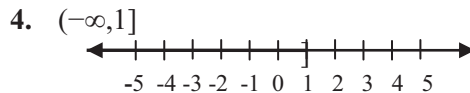
52. Let w represent the width of the rectangle.
The length is three feet less than three times the width: $3w - 3$

Chapter 1 Test

1. $-52(4) = -208$

2. $A \cap B = \{5, 7\}$

3. $(-2)^3(-3)^2 = (-8)(9) = -72$



5. $A \cap B = \{-1, 0, 1\}$

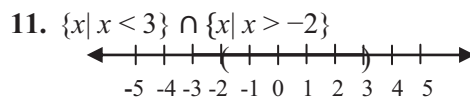
6. $(a - b)^2 \div (2b + 1)$
 $(2 - (-3))^2 \div (2(-3) + 1)$
 $= (2 + 3)^2 \div (-6 + 1)$
 $= (5)^2 \div (-5)$
 $= 25 \div (-5) = -5$

7. $-3 - (-5) = -3 + 5 = 2$

8. $2x - 4[2 - 3(x + 4y) - 2]$
 $= 2x - 4[2 - 3x - 12y - 2]$
 $= 2x - 4[-3x - 12y] = 2x + 12x + 48y$
 $= 14x + 48y$

9. The additive inverse of -12 is 12.

10. $-5^2 \cdot 4 = -25 \cdot 4 = -100$



12. $8 - 5(3 - 5)^3 \div 10 \cdot 2 = 8 - 5(-8) \div 10 \cdot 2$
 $= 8 + 40 \div 10 \cdot 2 = 8 + 4 \cdot 2 = 8 + 8$
 $= 16$

13. $\frac{3}{4} - \frac{\left(\frac{2}{3} - \frac{5}{6}\right)^2}{\frac{2}{3} - \frac{3}{4}} = \frac{3}{4} - \frac{\left(-\frac{1}{6}\right)^2}{-\frac{1}{12}} = \frac{3}{4} - \frac{\frac{1}{36}}{-\frac{1}{12}}$
 $= \frac{3}{4} - \frac{1}{36} \div \left(-\frac{1}{12}\right) = \frac{3}{4} - \frac{1}{36} \cdot \left(-\frac{12}{1}\right)$
 $= \frac{3}{4} + \frac{1}{3} = \frac{13}{12}$

14. $(3 + 4) + 2 = (4 + 3) + 2$

15. $\left(-\frac{2}{3}\right)\left(\frac{9}{16}\right) = -\frac{2 \cdot 3 \cdot 3}{3 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = -\frac{3}{8}$

16. Replace x with each element in the set to determine if the inequality is true or false.

$$\begin{aligned} -5 < -1 & \quad \text{True} \\ 3 < -1 & \quad \text{False} \\ 7 < -1 & \quad \text{False} \end{aligned}$$

17. $\frac{b^2 - c^2}{a - 2c}$

$$\frac{(3)^2 - (-1)^2}{2 - 2(-1)} = \frac{9 - 1}{2 + 2} = \frac{8}{4} = 2$$

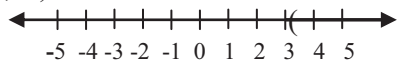
18. $-180 \div 12 = -15$

19. $12 - 4\left(\frac{5^2 - 1}{3}\right) \div 16 = 12 - 4\left(\frac{25 - 1}{3}\right) \div 16$

$$= 12 - 4\left(\frac{24}{3}\right) \div 16 = 12 - 4(8) \div 16$$

$$= 12 - 32 \div 16 = 12 - 2 = 10$$

20. $(3, \infty)$



21. $A \cup B = \{1, 2, 3, 4, 5, 7\}$

22. $3x - 2(x - y) - 3(y - 4x)$

$$= 3x - 2x + 2y - 3y + 12x$$

$$= 13x - y$$

23. $8 - 4(2 - 3)^2 \div 2 = 8 - 4(-1)^2 \div 2$

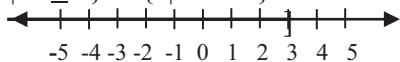
$$= 8 - 4(1) \div 2 = 8 - 4 \div 2$$

$$= 8 - 2 = 6$$

24. $\frac{3}{5}\left(-\frac{10}{21}\right) = \frac{-3 \cdot 2 \cdot 5}{5 \cdot 3 \cdot 7} = -\frac{2}{7}$

25. The Distributive Property

26. $\{x \mid x \leq 3\} \cup \{x \mid x < -2\}$



27. $\frac{7}{18} = 0.\overline{38}$

28. $A \cup B = \{-2, -1, 0, 1, 2, 3\}$

29. The sum of two numbers is nine.

Let x represent the larger of the two numbers.
Let $9 - x$ represent the smaller number.

The difference between one more than the larger number and twice the smaller number:

$$(x + 1) - 2(9 - x)$$

$$(x + 1) - 2(9 - x) = x + 1 - 18 + 2x = 3x - 17$$

30. Let x represent the amount of cocoa produced in Ghana.

The Ivory Coast produces three times the amount of cocoa produced in Ghana: $3x$

NOT FOR SALE

Chapter 10: Exponential and Logarithmic Functions

Prep Test

1. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

2. $\left(\frac{1}{2}\right)^{-4} = \left(\frac{2}{1}\right)^4 = 2^4 = 16$

3. $\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

4. $f(x) = x^4 + x^3$
 $f(-1) = (-1)^4 + (-1)^3 = 1 + (-1) = 0$
 $f(3) = (3)^4 + (3)^3 = 81 + 27 = 108$

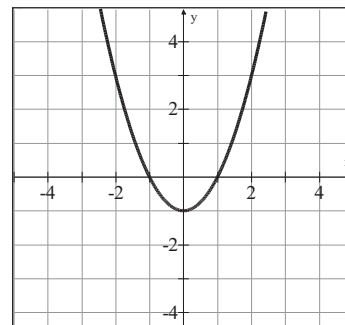
5. $3x + 7 = x - 5$
 $2x + 7 = -5$
 $2x = -12$
 $x = -6$
The solution is -6 .

6. $16 = x^2 - 6x$
 $0 = x^2 - 6x - 16$
 $0 = (x - 8)(x + 2)$
 $x - 8 = 0 \quad x + 2 = 0$
 $x = 8 \quad x = -2$
The solutions are -2 and 8 .

7. $A(1 + r)^n$
 $5000(1 + 0.04)^6 = 5000(1.04)^6$
 $= \$6326.60$

8. $f(x) = x^2 - 1$
 $x = -\frac{b}{2a} = \frac{0}{2(1)} = \frac{0}{2} = 0$
 $f(0) = (0)^2 - 1 = -1$

Vertex:
 $(0, -1)$.
Axis of
symmetry:
 $x = 0$.



Section 10.1

Concept Check

1. An exponential function with base b is defined by $f(x) = b^x$, $b > 0$, $b \neq 1$, and x is any real number.
2. The natural exponential function is the function defined by $f(x) = e^x$ where e is an irrational number approximately equal to 2.71828183.
3. $f(x) = b^x$, $b > 0$, $b \neq 1$
(iii) (iv) and (vi) cannot be the base.
4. a) No
b) Yes
c) Yes
d) Yes
e) No

Objective A Exercises

5. $f(x) = 3^x$
- a) $f(2) = 3^2 = 9$
 - b) $f(0) = 3^0 = 1$
 - c) $f(-2) = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

INSTRUCTOR USE ONLY

6. $H(x) = 2^x$

a) $H(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

b) $H(0) = 2^0 = 1$

c) $H(2) = 2^2 = 4$

7. $g(x) = 2^{x+1}$

a) $g(3) = 2^{3+1} = 2^4 = 16$

b) $g(1) = 2^{1+1} = 2^2 = 4$

c) $g(-3) = 2^{-3+1} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

8. $F(x) = 3^{x-2}$

a) $F(-4) = 3^{-4-2} = 3^{-6} = \frac{1}{3^6} = \frac{1}{729}$

b) $F(-1) = 3^{-1-2} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

c) $F(0) = 3^{0-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

9. $P(x) = \left(\frac{1}{2}\right)^{2x}$

a) $P(0) = \left(\frac{1}{2}\right)^{2(0)} = \left(\frac{1}{2}\right)^0 = 1$

b) $P\left(\frac{3}{2}\right) = \left(\frac{1}{2}\right)^{2(3/2)} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

c) $P(-2) = \left(\frac{1}{2}\right)^{2(-2)} = \left(\frac{1}{2}\right)^{-4} = 2^4 = 16$

10. $R(t) = \left(\frac{1}{3}\right)^{3t}$

a) $R\left(-\frac{1}{3}\right) = \left(\frac{1}{3}\right)^{3(-1/3)} = \left(\frac{1}{3}\right)^{-1} = 3^1 = 3$

b) $R(1) = \left(\frac{1}{3}\right)^{3(1)} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

c) $R(-2) = \left(\frac{1}{3}\right)^{3(-2)} = \left(\frac{1}{3}\right)^{-6} = 3^6 = 729$

11. $G(x) = e^{x/2}$

a) $G(4) = e^{4/2} = e^2 \approx 7.3891$

b) $G(-2) = e^{-2/2} = e^{-1} = \frac{1}{e^1} \approx 0.3679$

c) $G\left(\frac{1}{2}\right) = e^{(1/2)/2} = e^{1/4} \approx 1.2840$

12. $f(x) = e^{2x}$

a) $f(-2) = e^{2(-2)} = e^{-4} = \frac{1}{e^4} \approx 0.0183$

b) $f\left(-\frac{2}{3}\right) = e^{2(-2/3)} = e^{-4/3} = \frac{1}{e^{4/3}} \approx 0.2636$

c) $f(2) = e^{2(2)} = e^4 \approx 54.5982$

13. $H(r) = e^{-r+3}$

a) $H(-1) = e^{-(-1)+3} = e^4 \approx 54.5982$

b) $H(3) = e^{-3+3} = e^0 = 1$

c) $H(5) = e^{-5+3} = e^{-2} = \frac{1}{e^2} \approx 0.1353$

14. $P(t) = e^{-\frac{1}{2}t}$

a) $P(-3) = e^{-\frac{1}{2}(-3)} = e^{3/2} \approx 4.4817$

b) $P(4) = e^{-\frac{1}{2}(4)} = e^{-2} = \frac{1}{e^2} \approx 0.1353$

c) $P\left(\frac{1}{2}\right) = e^{-\frac{1}{2}(\frac{1}{2})} = e^{-1/4} = \frac{1}{e^{1/4}} \approx 0.7788$

15. $F(x) = 2^{x^2}$

a) $F(2) = 2^{2^2} = 2^4 = 16$

b) $F(-2) = 2^{(-2)^2} = 2^4 = 16$

c) $F\left(\frac{3}{4}\right) = 2^{(3/4)^2} = 2^{9/16} \approx 1.4768$

16. $Q(x) = 2^{-x^2}$

a) $Q(3) = 2^{-3^2} = 2^{-9} = \frac{1}{2^9} = \frac{1}{512}$

b) $Q(-1) = 2^{-(-1)^2} = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

c) $Q(-2) = 2^{-(-2)^2} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

17. $f(x) = e^{-x^2/2}$

a) $f(-2) = e^{-(-2)^2/2} = e^{-2} = \frac{1}{e^2} = 0.1353$

b) $f(2) = e^{-2^2/2} = e^{-2} = \frac{1}{e^2} = 0.1353$

c) $f(-3) = e^{-(-3)^2/2} = e^{-9/2} = \frac{1}{e^{9/2}} = 0.0111$

18. $f(x) = e^{-2x} + 1$

a) $f(-1) = e^{-2(-1)} + 1 = e^2 + 1 = 8.3891$

b) $f(3) = e^{-2(3)} + 1 = e^{-6} + 1 = \frac{1}{e^6} + 1 = 1.0025$

c) $f(-2) = e^{-2(-2)} + 1 = e^4 + 1 = 55.5982$

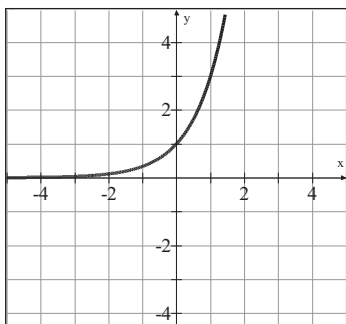
19. $f(a) > f(b)$

20. $b > 1$

Objective B Exercises

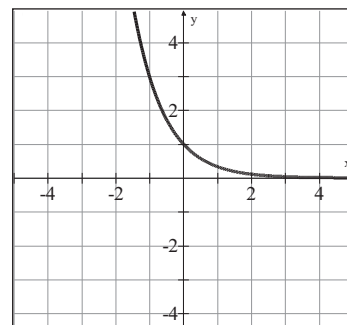
21. $f(x) = 3^x$

x	y
0	1
-1	$\frac{1}{3}$
1	3



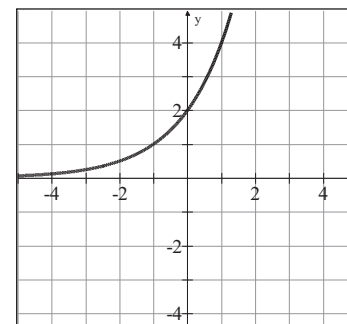
22. $f(x) = 3^{-x}$

x	y
0	1
-1	3
1	$\frac{1}{3}$



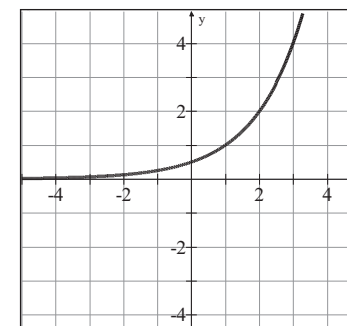
23. $f(x) = 2^{x+1}$

x	y
0	2
-1	1
1	4



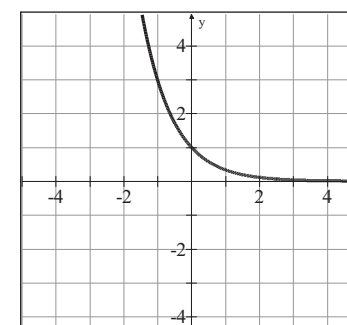
24. $f(x) = 2^{x-1}$

x	y
0	$\frac{1}{2}$
-1	$\frac{1}{4}$
1	1
2	2



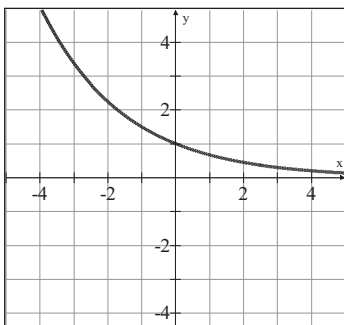
25. $f(x) = \left(\frac{1}{3}\right)^x$

x	y
0	1
-1	3
1	$\frac{1}{3}$



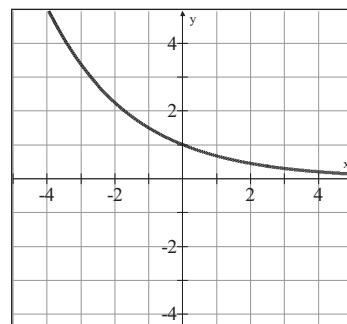
26. $f(x) = \left(\frac{2}{3}\right)^x$

x	y
0	1
-1	$\frac{3}{2}$
1	$\frac{2}{3}$



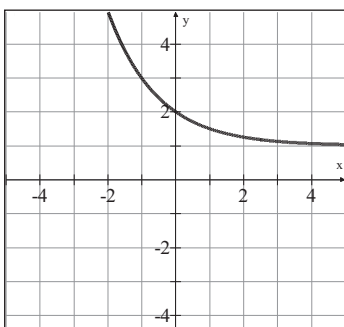
30. $f(x) = \left(\frac{3}{2}\right)^{-x}$

x	y
0	1
-1	$\frac{3}{2}$
1	$\frac{2}{3}$

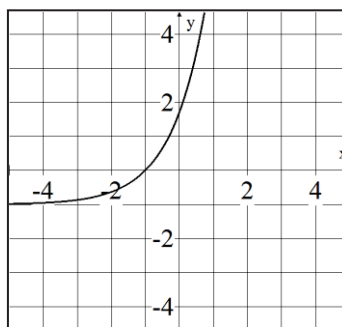


27. $f(x) = 2^{-x} + 1$

x	y
0	2
-1	3
1	$\frac{3}{2}$

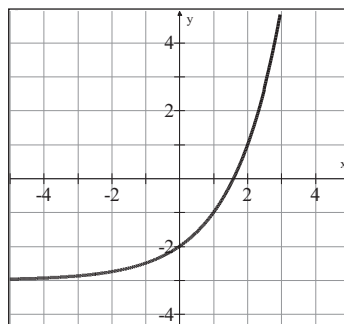


31. $f(x) = e^{x+1} - 1$

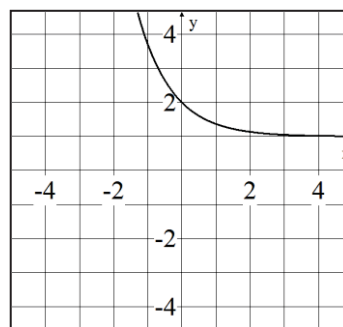


28. $f(x) = 2^x - 3$

x	y
0	-2
-1	$-\frac{5}{2}$
1	-1

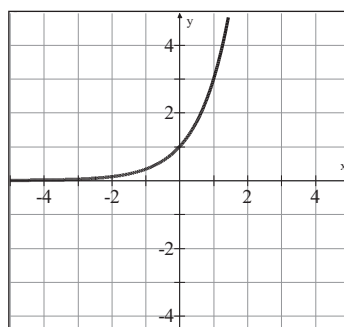


32. $f(x) = e^{-x} + 1$



29. $f(x) = \left(\frac{1}{3}\right)^{-x}$

x	y
0	1
-1	$\frac{1}{3}$
1	3



33. $v(t) = 32(1 - e^{-t})$

$v(4) = 32(1 - e^{-4}) \approx 31.41$

The speed of the object after 4 s is 31.41 ft/sec.

34. $F(n) = 500(1.00021918)^{365n}$

$F(9) = 500(1.00021918)^{365(9)} \approx 1027.14$

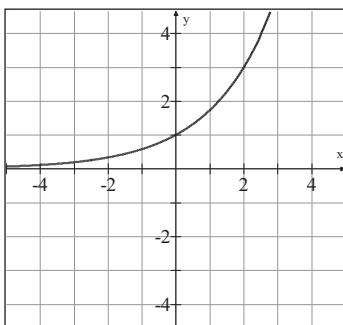
The value of the investment after 9 years is \$1027.14.

35. (i) and (iii) have the same graphs.
(ii) and (iv) have the same graphs.

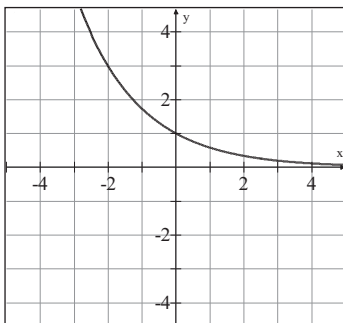
36. (i) and (iii) have the same graphs.
(ii) and (iv) have the same graphs.

Critical Thinking

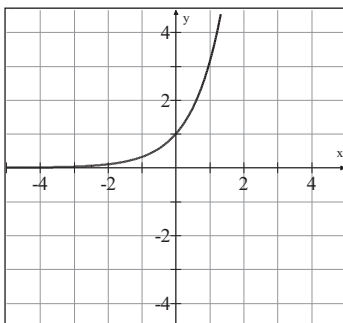
37. $P(x) = (\sqrt{3})^x$



38. $Q(x) = (\sqrt{3})^{-x}$



39. $f(x) = \pi^x$



Projects or Group Activities

40.

n	$\left(1 + \frac{1}{n}\right)^n$
100	2.704814
1000	2.716924
10,000	2.718146
100,000	2.718268

As n increases, $\left(1 + \frac{1}{n}\right)^n$ becomes closer to e .

41.

n	$(1+n)^{1/n}$
0.01	2.704814
0.001	2.716924
0.0001	2.718146
0.00001	2.718268

As n decreases, $(1+n)^{1/n}$ becomes closer to e .

Section 10.2

Concept Check

- A common logarithm is a logarithm with a base of 10.
- A natural logarithm is a logarithm with a base of e .
- $\log_5 25 = 2$
- $\log_{10} 1000 = 3$
- $\log_4 \frac{1}{16} = -2$
- $\ln u = v$
- $3^4 = 81$
- $10^{-3} = 0.001$
- $e^q = p$

10. $5^{-1} = \frac{1}{5}$

11. False

12. True

13. True

14. False

15. True

16. True

Objective A Exercises

17. $\log_3 81 = x$

$$3^x = 81$$

$$x = 4$$

$$\log_3 81 = 4$$

18. $\log_7 49 = x$

$$7^x = 49$$

$$x = 2$$

$$\log_7 49 = 2$$

19. $\log_2 128 = x$

$$2^x = 128$$

$$x = 7$$

$$\log_2 128 = 7$$

20. $\log_5 125 = x$

$$5^x = 125$$

$$x = 3$$

$$\log_5 125 = 3$$

21. $\log 100 = x$

$$10^x = 100$$

$$x = 2$$

$$\log 100 = 2$$

22. $\log 0.001 = x$

$$10^x = 0.001$$

$$x = -3$$

$$\log 0.001 = -3$$

23. $\ln e^3 = x$

$$3 \ln e = x$$

$$3(1) = x$$

$$x = 3$$

$$\ln e^3 = 3$$

24. $\ln e^2 = x$

$$2 \ln e = x$$

$$2(1) = x$$

$$x = 2$$

$$\ln e^2 = 2$$

25. $\log_8 1 = x$

$$8^x = 1$$

$$x = 0$$

$$\log_8 1 = 0$$

26. $\log_3 243 = x$

$$3^x = 243$$

$$x = 5$$

$$\log_3 243 = 5$$

27. $\log_5 625 = x$

$$5^x = 625$$

$$x = 4$$

$$\log_5 625 = 4$$

28. $\log_2 64 = x$

$$2^x = 64$$

$$x = 6$$

$$\log_2 64 = 6$$

INSTRUCTOR USE ONLY

29. $\log_3 x = 2$

$3^2 = x$

$x = 9$

30. $\log_5 x = 1$

$5^1 = x$

$x = 5$

31. $\log_4 x = 3$

$4^3 = x$

$x = 64$

32. $\log_2 x = 6$

$2^6 = x$

$x = 64$

33. $\log_7 x = -1$

$7^{-1} = x$

$x = \frac{1}{7}$

34. $\log_8 x = -2$

$8^{-2} = x$

$x = \frac{1}{64}$

35. $\log_6 x = 0$

$6^0 = x$

$x = 1$

36. $\log_4 x = 0$

$4^0 = x$

$x = 1$

37. $\log x = 2.5$

$10^{2.5} = x$

$x \approx 316.23$

38. $\log x = 3.2$

$10^{3.2} = x$

$x \approx 1584.89$

39. $\log x = -1.75$

$10^{-1.75} = x$

$x \approx 0.02$

40. $\log x = -2.1$

$10^{-2.1} = x$

$x \approx 0.01$

41. $\ln x = 2$

$e^2 = x$

$x \approx 7.39$

42. $\ln x = 1.4$

$e^{1.4} = x$

$x \approx 4.06$

43. $\ln x = -\frac{1}{2}$

$e^{-1/2} = x$

$x \approx 0.61$

44. $\ln x = -1.7$

$e^{-1.7} = x$

$x \approx 0.18$

45. $x > 1$

46. Suppose $\log_5(-2) = x$. Then $5^x = -2$.
However, $5^x > 0$ for all real numbers x .

Objective B Exercises

47. $\log_b(xy) = \log_b(x) + \log_b(y)$

48. $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

49. False

50. $x > 0$

51. $\log_{12} 1 = 0$

52. $\ln 1 = 0$

53. $\ln e = 1$

$$54. \log_{10} 10 = 1$$

$$55. \log_3 3^x = x$$

$$56. 8^{\log_8 p} = p$$

$$57. e^{\ln v} = v$$

$$58. \ln e^{3x} = 3x$$

$$59. 2^{\log_2(x^2+1)} = x^2 + 1$$

$$60. \log_4 4^{3x+1} = 3x + 1$$

$$61. \log_5 5^{x^2-x-1} = x^2 - x - 1$$

$$62. 8^{\log_8(3x-7)} = 3x - 7$$

$$63. \log_8(xz) = \log_8 x + \log_8 z$$

$$64. \log_7(rt) = \log_7 r + \log_7 t$$

$$65. \log_3 x^5 = 5 \log_3 x$$

$$66. \log_2 y^7 = 7 \log_2 y$$

$$67. \log_b \left(\frac{r}{s} \right) = \log_b r - \log_b s$$

$$68. \log_c \left(\frac{z}{4} \right) = \log_c z - \log_c 4$$

$$69. \log_3(x^2 y^6) = \log_3 x^2 + \log_3 y^6 \\ = 2 \log_3 x + 6 \log_3 y$$

$$70. \log_4(t^4 u^2) = \log_4 t^4 + \log_4 u^2 \\ = 4 \log_4 t + 2 \log_4 u$$

$$71. \log_7 \left(\frac{u^3}{v^4} \right) = \log_7 u^3 - \log_7 v^4 \\ = 3 \log_7 u - 4 \log_7 v$$

$$72. \log_{10} \left(\frac{s^5}{t^2} \right) = \log_{10} s^5 - \log_{10} t^2 \\ = 5 \log_{10} s - 2 \log_{10} t$$

$$73. \log_2(rs)^2 = 2 \log_2(rs) = 2[\log_2 r + \log_2 s]$$

$$74. \log_3(x^2 y)^3 = 3 \log_3(x^2 y) \\ = 3[\log_3 x^2 + \log_3 y] \\ = 3[2 \log_3 x + \log_3 y] \\ = 6 \log_3 x + 3 \log_3 y$$

$$75. \ln(x^2 yz) = \ln x^2 + \ln y + \ln z \\ = 2 \ln x + \ln y + \ln z$$

$$76. \ln(xy^2 z^3) = \ln x + \ln y^2 + \ln z^3 \\ = \ln x + 2 \ln y + 3 \ln z$$

$$77. \log_5 \left(\frac{xy^2}{z^4} \right) = \log_5 xy^2 - \log_5 z^4 \\ = \log_5 x + \log_5 y^2 - \log_5 z^4 \\ = \log_5 x + 2 \log_5 y - 4 \log_5 z$$

$$78. \log_b \left(\frac{r^2 s}{t^3} \right) = \log_b r^2 s - \log_b t^3 \\ = \log_b r^2 + \log_b s - \log_b t^3 \\ = 2 \log_b r + \log_b s - 3 \log_b t$$

$$79. \log_8 \left(\frac{x^2}{yz^2} \right) = \log_8 x^2 - \log_8 yz^2 \\ = \log_8 x^2 - (\log_8 y + \log_8 z^2) \\ = \log_8 x^2 - \log_8 y - \log_8 z^2 \\ = 2 \log_8 x - \log_8 y - 2 \log_8 z$$

INSTRUCTOR USE ONLY

$$\begin{aligned}
 80. \log_9 \left(\frac{x}{y^2 z^3} \right) &= \log_9 x - \log_9 y^2 z^3 \\
 &= \log_9 x - (\log_9 y^2 + \log_9 z^3) \\
 &= \log_9 x - \log_9 y^2 - \log_9 z^3 \\
 &= \log_9 x - 2 \log_9 y - 3 \log_9 z
 \end{aligned}$$

$$\begin{aligned}
 81. \log_4 \sqrt{x^3 y} &= \log_4 (x^3 y)^{1/2} = \frac{1}{2} \log_4 (x^3 y) \\
 &= \frac{1}{2} [\log_4 x^3 + \log_4 y] \\
 &= \frac{1}{2} [3 \log_4 x + \log_4 y] \\
 &= \frac{3}{2} \log_4 x + \frac{1}{2} \log_4 y
 \end{aligned}$$

$$\begin{aligned}
 82. \log_3 \sqrt{x^5 y^3} &= \log_3 (x^5 y^3)^{1/2} = \frac{1}{2} \log_3 (x^5 y^3) \\
 &= \frac{1}{2} [\log_3 x^5 + \log_3 y^3] \\
 &= \frac{1}{2} [5 \log_3 x + 3 \log_3 y] \\
 &= \frac{5}{2} \log_3 x + \frac{3}{2} \log_3 y
 \end{aligned}$$

$$\begin{aligned}
 83. \log_7 \sqrt{\frac{x^3}{y}} &= \log_7 \left(\frac{x^3}{y} \right)^{1/2} = \frac{1}{2} \log_7 \frac{x^3}{y} \\
 &= \frac{1}{2} [\log_7 x^3 - \log_7 y] \\
 &= \frac{1}{2} [3 \log_7 x - \log_7 y] \\
 &= \frac{3}{2} \log_7 x - \frac{1}{2} \log_7 y
 \end{aligned}$$

$$\begin{aligned}
 84. \log_b \sqrt[3]{\frac{r^2}{t}} &= \log_b \left(\frac{r^2}{t} \right)^{1/3} = \frac{1}{3} \log_b \frac{r^2}{t} \\
 &= \frac{1}{3} [\log_b r^2 - \log_b t] \\
 &= \frac{1}{3} [2 \log_b r - \log_b t] \\
 &= \frac{2}{3} \log_b r - \frac{1}{3} \log_b t
 \end{aligned}$$

$$\begin{aligned}
 85. \log_3 \left(\frac{t}{\sqrt{x}} \right) &= \log_3 \left(\frac{t}{x^{1/2}} \right) \\
 &= \log_3 t - \log_3 x^{1/2} \\
 &= \log_3 t - \frac{1}{2} \log_3 x
 \end{aligned}$$

$$\begin{aligned}
 86. \log_4 \left(\frac{x}{\sqrt{y^2 z}} \right) &= \log_4 \left(\frac{x}{(y^2 z)^{1/2}} \right) = \log_4 \left(\frac{x}{y z^{1/2}} \right) \\
 &= \log_4 x - \log_4 y z^{1/2} \\
 &= \log_4 x - [\log_4 y + \log_4 z^{1/2}] \\
 &= \log_4 x - \log_4 y - \log_4 z^{1/2} \\
 &= \log_4 x - \log_4 y - \frac{1}{2} \log_4 z
 \end{aligned}$$

$$87. \log_3 x^3 + \log_3 y^2 = \log_3 (x^3 y^2)$$

$$88. \log_7 x + \log_7 z^2 = \log_7 (xz^2)$$

$$89. \ln x^4 - \ln y^2 = \ln \left(\frac{x^4}{y^2} \right)$$

$$90. \ln x^2 - \ln y = \ln \left(\frac{x^2}{y} \right)$$

$$91. 3 \log_7 x = \log_7 x^3$$

$$92. 4 \log_8 y = \log_8 y^4$$

$$93. 3 \ln x + 4 \ln y = \ln x^3 + \ln y^4 = \ln(x^3 y^4)$$

$$94. 2 \ln x - 5 \ln y = \ln x^2 - \ln y^5 = \ln \left(\frac{x^2}{y^5} \right)$$

$$95. \begin{aligned} 2(\log_4 x + \log_4 y) &= 2 \log_4(xy) \\ &= \log_4(xy)^2 \\ &= \log_4(x^2 y^2) \end{aligned}$$

$$96. \begin{aligned} 3(\log_5 r + \log_5 t) &= 3 \log_5(rt) \\ &= \log_5(rt)^3 \\ &= \log_5(r^3 t^3) \end{aligned}$$

$$97. \begin{aligned} 2 \log_3 x - \log_3 y + 2 \log_3 z \\ &= \log_3 x^2 - \log_3 y + \log_3 z^2 \\ &= \log_3 \left(\frac{x^2}{y} \right) + \log_3 z^2 \\ &= \log_3 \left(\frac{x^2 z^2}{y} \right) \end{aligned}$$

$$98. \begin{aligned} 4 \log_5 r - 3 \log_5 s + \log_5 t \\ &= \log_5 r^4 - \log_5 s^3 + \log_5 t \\ &= \log_5 \left(\frac{r^4}{s^3} \right) + \log_5 t \\ &= \log_5 \left(\frac{r^4 t}{s^3} \right) \end{aligned}$$

$$99. \begin{aligned} \ln x - (2 \ln y + \ln z) &= \ln x - (\ln y^2 + \ln z) \\ &= \ln x - \ln(y^2 z) \\ &= \ln \left(\frac{x}{y^2 z} \right) \end{aligned}$$

$$100. \begin{aligned} 2 \log_b x - 3(\log_b y + \log_b z) \\ &= 2 \log_b x - 3 \log_b(yz) \\ &= \log_b x^2 - \log_b (yz)^3 \\ &= \log_b \left(\frac{x^2}{(yz)^3} \right) \\ &= \log_b \left(\frac{x^2}{y^3 z^3} \right) \end{aligned}$$

$$101. \begin{aligned} \frac{1}{2}(\log_6 x - \log_6 y) &= \frac{1}{2} \log_6 \left(\frac{x}{y} \right) \\ &= \log_6 \left(\frac{x}{y} \right)^{1/2} \\ &= \log_6 \sqrt{\frac{x}{y}} \end{aligned}$$

$$102. \begin{aligned} \frac{1}{3}(\log_8 x - \log_8 y) &= \frac{1}{3} \log_8 \left(\frac{x}{y} \right) \\ &= \log_8 \left(\frac{x}{y} \right)^{1/3} \\ &= \log_8 \sqrt[3]{\frac{x}{y}} \end{aligned}$$

$$103. \begin{aligned} 2(\log_4 s - 2 \log_4 t + \log_4 r) \\ &= 2(\log_4 s - \log_4 t^2 + \log_4 r) \\ &= 2(\log_4 \frac{s}{t^2} + \log_4 r) \\ &= 2 \log_4 \left(\frac{sr}{t^2} \right) \\ &= \log_4 \left(\frac{sr}{t^2} \right)^2 \\ &= \log_4 \frac{s^2 r^2}{t^4} \end{aligned}$$

INSTRUCTOR USE ONLY

$$\begin{aligned}
 104. \quad & 3(\log_9 x + 2\log_9 y - 2\log_9 z) \\
 &= 3(\log_9 x + \log_9 y^2 - \log_9 z^2) \\
 &= 3(\log_9 xy^2 - \log_9 z^2) \\
 &= 3\log_9 \left(\frac{xy^2}{z^2} \right) \\
 &= \log_9 \left(\frac{xy^2}{z^2} \right)^3 \\
 &= \log_9 \frac{x^3 y^6}{z^6}
 \end{aligned}$$

$$\begin{aligned}
 105. \quad & \ln x - 2(\ln y + \ln z) \\
 &= \ln x - 2\ln(yz) \\
 &= \ln x - \ln(yz)^2 \\
 &= \ln \left(\frac{x}{(yz)^2} \right) \\
 &= \ln \frac{x}{y^2 z^2}
 \end{aligned}$$

$$\begin{aligned}
 106. \quad & \ln t - 3(\ln u + \ln v) \\
 &= \ln t - 3\ln(uv) \\
 &= \ln t - \ln(uv)^3 \\
 &= \ln \left(\frac{t}{(uv)^3} \right) \\
 &= \ln \frac{t}{u^3 v^3}
 \end{aligned}$$

$$\begin{aligned}
 107. \quad & \frac{1}{2}(3\log_4 x - 2\log_4 y + \log_4 z) \\
 &= \frac{1}{2}(\log_4 x^3 - \log_4 y^2 + \log_4 z) \\
 &= \frac{1}{2}(\log_4 \left(\frac{x^3}{y^2} \right) + \log_4 z) \\
 &= \log_4 \left(\frac{x^3 z}{y^2} \right)^{1/2} \\
 &= \log_4 \sqrt{\frac{x^3 z}{y^2}}
 \end{aligned}$$

$$\begin{aligned}
 108. \quad & \frac{1}{3}(4\log_5 t - 5\log_5 u - 7\log_5 v) \\
 &= \frac{1}{3}(\log_5 t^4 - \log_5 u^5 - \log_5 v^7) \\
 &= \frac{1}{3}(\log_5 \left(\frac{t^4}{u^5 v^7} \right)) \\
 &= \frac{1}{3}\log_5 \left(\frac{t^4}{u^5 v^7} \right) \\
 &= \log_5 \left(\frac{t^4}{u^5 v^7} \right)^{1/3} \\
 &= \log_5 \sqrt[3]{\frac{t^4}{u^5 v^7}}
 \end{aligned}$$

$$\begin{aligned}
 109. \quad & \frac{1}{2}\log_2 x - \frac{2}{3}\log_2 y + \frac{1}{2}\log_2 z \\
 &= \log_2 x^{1/2} - \log_2 y^{2/3} + \log_2 z^{1/2} \\
 &= \log_2 \left(\frac{x^{1/2}}{y^{2/3}} \right) + \log_2 z^{1/2} \\
 &= \log_2 \left(\frac{x^{1/2} z^{1/2}}{y^{2/3}} \right) \\
 &= \log_2 \frac{\sqrt{xz}}{\sqrt[3]{y^2}}
 \end{aligned}$$

$$\begin{aligned}
 110. \quad & \frac{2}{3}\log_3 x + \frac{1}{3}\log_3 y - \frac{1}{2}\log_3 z \\
 &= \log_3 x^{2/3} + \log_3 y^{1/3} - \log_3 z^{1/2} \\
 &= \log_3 (x^{2/3} y^{1/3}) - \log_3 z^{1/2} \\
 &= \log_3 \left(\frac{x^{2/3} y^{1/3}}{z^{1/2}} \right) \\
 &= \log_3 \frac{\sqrt[3]{x^2 y}}{\sqrt{z}}
 \end{aligned}$$

Objective C Exercises

$$111. \log_8 6 = \frac{\log_{10} 6}{\log_{10} 8} \approx 0.8617$$

$$112. \log_4 8 = \frac{\log_{10} 8}{\log_{10} 4} \approx 1.5000$$

$$113. \log_5 30 = \frac{\log_{10} 30}{\log_{10} 5} \approx 2.1133$$

$$114. \log_6 28 = \frac{\log_{10} 28}{\log_{10} 6} \approx 1.8597$$

$$115. \log_3 0.5 = \frac{\log_{10} 0.5}{\log_{10} 3} \approx -0.6309$$

$$116. \log_5 0.6 = \frac{\log_{10} 0.6}{\log_{10} 5} \approx -0.3174$$

$$117. \log_7 1.7 = \frac{\log_{10} 1.7}{\log_{10} 7} \approx 0.2727$$

$$118. \log_6 3.2 = \frac{\log_{10} 3.2}{\log_{10} 6} \approx 0.6492$$

$$119. \log_5 15 = \frac{\log_{10} 15}{\log_{10} 5} \approx 1.6826$$

$$120. \log_3 25 = \frac{\log_{10} 25}{\log_{10} 3} \approx 2.9299$$

$$121. \log_{12} 120 = \frac{\log_{10} 120}{\log_{10} 12} \approx 1.9266$$

$$122. \log_9 90 = \frac{\log_{10} 90}{\log_{10} 9} \approx 2.0480$$

$$123. \log_4 2.55 = \frac{\log_{10} 2.55}{\log_{10} 4} \approx 0.6752$$

$$124. \log_8 6.42 = \frac{\log_{10} 6.42}{\log_{10} 8} \approx 0.8942$$

$$125. \log_5 67 = \frac{\log_{10} 67}{\log_{10} 5} \approx 2.6125$$

$$126. \log_8 35 = \frac{\log_{10} 35}{\log_{10} 8} \approx 1.7098$$

$$127. \log_5 x = \frac{\log_{10} x}{\log_{10} 5}$$

$$128. \log_5 x = \frac{\ln x}{\ln 5}$$

Critical Thinking

$$129. \log_3(\log_3 x) = 2$$

$$\text{Let } y = \log_3(x)$$

$$\log_3(\log_3(x)) = 2$$

$$\log_3(y) = 2$$

$$3^2 = y$$

$$y = 9$$

$$y = \log_3(x)$$

$$9 = \log_3(x)$$

$$3^9 = x$$

$$x = 19,683$$

$$130. \log_2(\log_2 16) = x$$

$$\text{Let } y = \log_2(16)$$

$$y = \log_2(16) = 4$$

$$\log_2(4) = x$$

$$x = 2$$

$$131. \log_2(\log_2 64) = x$$

$$\text{Let } y = \log_2(64)$$

$$y = \log_2(64) = 6$$

$$\log_2(6) = x$$

$$x \approx 2.58$$

INSTRUCTOR USE ONLY

$$132. \quad \log_2(\log_4 x) = 3$$

$$\text{Let } y = \log_4(x)$$

$$\log_2(\log_4(x)) = 3$$

$$\log_2(y) = 3$$

$$2^3 = y$$

$$y = 8$$

$$y = \log_4(x)$$

$$8 = \log_4(x)$$

$$4^8 = x$$

$$x = 65,536$$

133. Because $x = 4$, $x - 5 = -1$. The logarithm of a negative number is undefined.

$$134. \quad e^{(3 \ln 2)x}$$

Projects or Groups Activities

$$135. \quad \text{a) } D = -(p_1 \log_2 p_1 + p_2 \log_2 p_2 + p_3 \log_2 p_3 + p_4 \log_2 p_4 + p_5 \log_2 p_5)$$

$$D = -\left(\frac{1}{5} \log_2 \frac{1}{5} + \frac{1}{5} \log_2 \frac{1}{5} + \frac{1}{5} \log_2 \frac{1}{5} + \frac{1}{5} \log_2 \frac{1}{5} + \frac{1}{5} \log_2 \frac{1}{5}\right)$$

$$D = -5\left(\frac{1}{5} \log_2 5\right) = \log_2 5 = 2.3219281$$

$$\text{b) } D = -(p_1 \log_2 p_1 + p_2 \log_2 p_2 + p_3 \log_2 p_3 + p_4 \log_2 p_4 + p_5 \log_2 p_5)$$

$$D = -\left(\frac{1}{8} \log_2 \frac{1}{8} + \frac{3}{8} \log_2 \frac{3}{8} + \frac{1}{16} \log_2 \frac{1}{16} + \frac{1}{8} \log_2 \frac{1}{8} + \frac{5}{16} \log_2 \frac{5}{16}\right)$$

$$D = 2.055036$$

Less diversity

$$\text{c) } D = -(p_1 \log_2 p_1 + p_2 \log_2 p_2 + p_3 \log_2 p_3 + p_4 \log_2 p_4 + p_5 \log_2 p_5)$$

$$D = -\left(0 \log_2 0 + \frac{1}{4} \log_2 \frac{1}{4} + 0 \log_2 0 + 0 \log_2 0 + \frac{3}{3} \log_2 \frac{3}{4}\right)$$

$$D = -\left(0 + \frac{1}{4} \log_2 \frac{1}{4} + 0 + 0 + \frac{3}{3} \log_2 \frac{3}{4}\right) = 0.82600$$

Less diversity

$$\text{d) } D = -(p_1 \log_2 p_1 + p_2 \log_2 p_2 + p_3 \log_2 p_3 + p_4 \log_2 p_4 + p_5 \log_2 p_5)$$

$$D = -(0 \log_2 0 + 0 \log_2 0 + 0 \log_2 0 + 0 \log_2 0 + 1 \log_2 1)$$

$$D = -(0 + 0 + 0 + 0 + 0) = 0$$

Because this system has only one species, there is no diversity in the system.

Section 10.3

Concept Check

- They have the same graph.
- They are mirror images of each other with respect to the line $y = x$.

3. $x = 2^{\frac{y}{3}}$

4. $x = 5^{\frac{y}{4}} + 3$

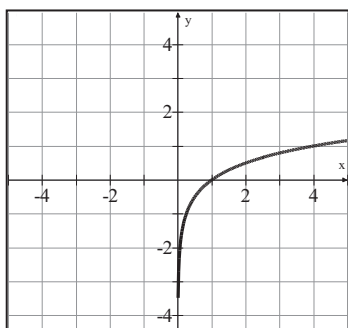
5. $x = 3^{\frac{y-2}{3}}$

6. $x = 10^{\frac{y+5}{2}}$

Objective A Exercises

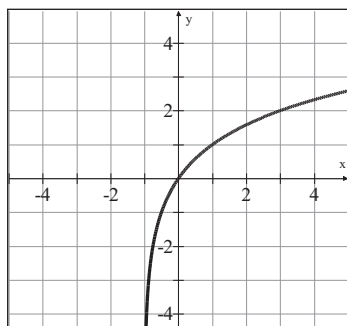
7. $f(x) = \log_4 x$
 $y = \log_4 x$ is equivalent to $x = 4^y$.

x	y
$\frac{1}{16}$	-2
$\frac{1}{4}$	-1
1	0
4	1



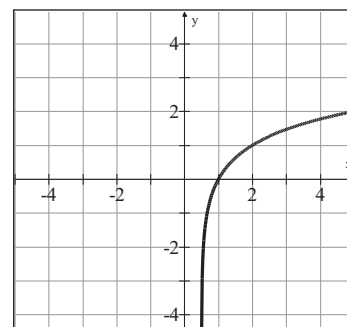
8. $f(x) = \log_2(x+1)$
 $y = \log_2(x+1)$ is equivalent to
 $x+1 = 2^y$ or $x = 2^y - 1$.

x	y
$-\frac{1}{2}$	-1
0	0
1	1
3	2



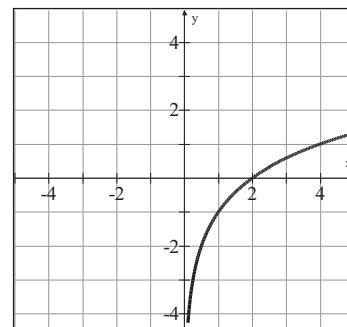
9. $f(x) = \log_3(2x-1)$
 $y = \log_3(2x-1)$ is equivalent to
 $2x-1 = 3^y$ or $x = \frac{1}{2}(3^y + 1)$.

x	y
$\frac{2}{3}$	-1
1	0
2	1
5	2



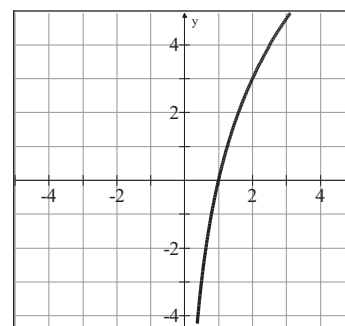
10. $f(x) = \log_2\left(\frac{1}{2}x\right)$
 $y = \log_2\left(\frac{1}{2}x\right)$ is equivalent to
 $\frac{1}{2}x = 2^y$ or $x = 2^{y+1}$.

x	y
1	-1
2	0
4	1
8	2



11. $f(x) = 3 \log_2 x$
 $y = 3 \log_2 x$ is equivalent to $x = 2^{y/3}$.

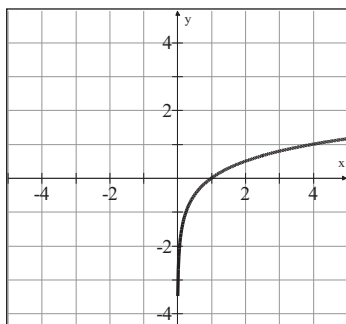
x	y
$\frac{1}{2}$	-3
1	0
2	3
4	6



12. $f(x) = \frac{1}{2} \log_2 x$

$y = \frac{1}{2} \log_2 x$ is equivalent to $x = 2^{2y}$.

x	y
$\frac{1}{4}$	-1
1	0
4	1
16	2

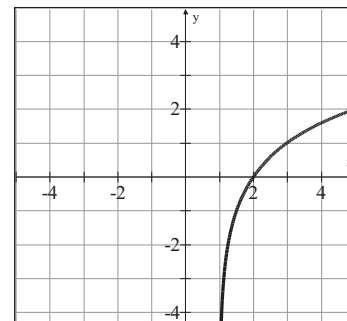


15. $f(x) = \log_2(x-1)$

$y = \log_2(x-1)$ is equivalent to $x-1 = 2^y$

or $x = 2^y + 1$.

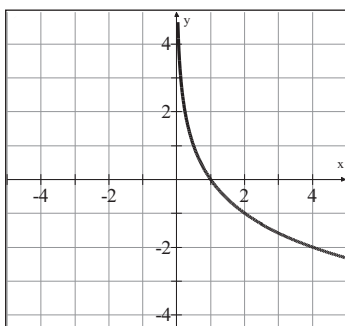
x	y
$\frac{3}{2}$	-1
2	0
3	1
5	2



13. $f(x) = -\log_2 x$

$y = -\log_2 x$ is equivalent to $x = 2^{-y}$.

x	y
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2

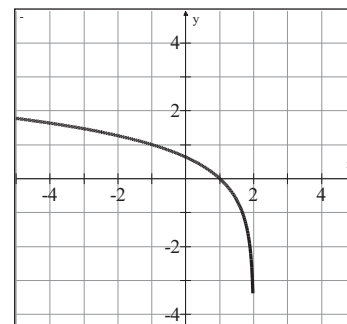


16. $f(x) = \log_3(2-x)$

$y = \log_3(2-x)$ is equivalent to $2-x = 3^y$

or $x = 2 - 3^y$.

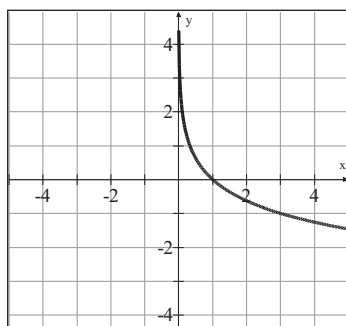
x	y
$\frac{5}{3}$	-1
1	0
-1	1
-7	2



14. $f(x) = -\log_3 x$

$y = -\log_3 x$ is equivalent to $x = 3^{-y}$.

x	y
3	-1
1	0
$\frac{1}{3}$	1
$\frac{1}{9}$	2

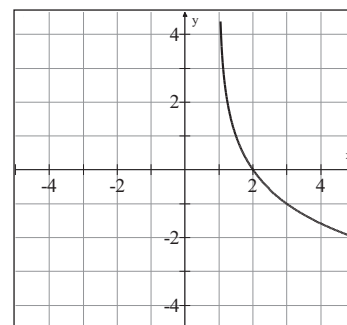


17. $f(x) = -\log_2(x-1)$

$y = -\log_2(x-1)$ is equivalent to

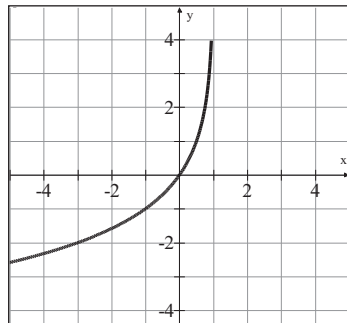
$x-1 = 2^{-y}$ or $x = 2^{-y} + 1$.

x	y
3	-1
2	0
$\frac{3}{2}$	1
$\frac{5}{4}$	2



18. $f(x) = -\log_2(1-x)$
 $y = -\log_2(1-x)$ is equivalent to
 $1-x = 2^{-y}$ or $x = 1 - 2^{-y}$.

x	y
-1	-1
0	0
$\frac{1}{2}$	1
$\frac{3}{4}$	2

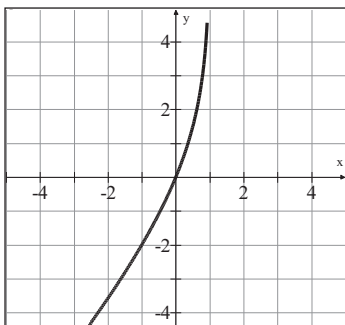


19. They intersect at the point (1, 0).

20. 0

Critical Thinking

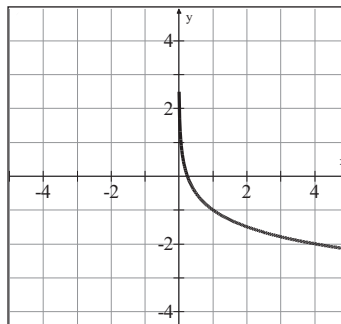
21. $f(x) = x - \log_2(1-x)$
 $y = x - \log_2(1-x)$
 $y = x - \frac{\log(1-x)}{\log 2}$



22. $f(x) = -\frac{1}{2}\log_2 x - 1$

$$y = -\frac{1}{2}\log_2 x - 1$$

$$y = -\frac{\log x}{2 \log 2} - 1$$

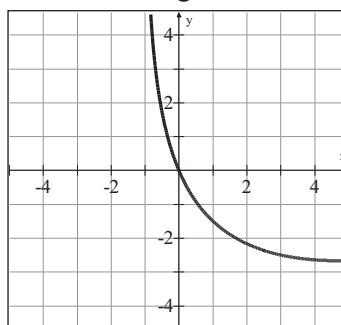


23. $f(x) = \frac{x}{2} - 2\log_2(x+1)$

$$y = \frac{x}{2} - 2\log_2(x+1)$$

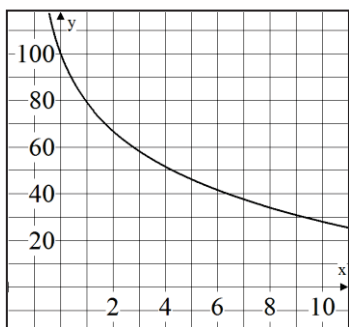
$$y = \frac{x}{2} - \log_2(x+1)^2$$

$$y = \frac{x}{2} - \frac{\log(x+1)^2}{\log 2}$$



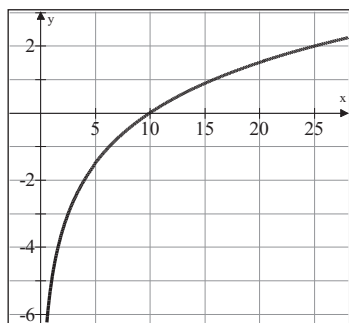
Projects or Group Activities

24. a) $P = 100 - 30 \ln(t + 1)$



- b) The ordered pair (6,42) means that after 6 weeks, a person will remember 42% of the words on the list.
c) 11 weeks

25. a) $M = 5 \log s - 5$



- b) The point (25.1, 2) means that a star that is 25.1 parsecs from Earth has a distance modulus of 2.
c) 6.3 parsecs

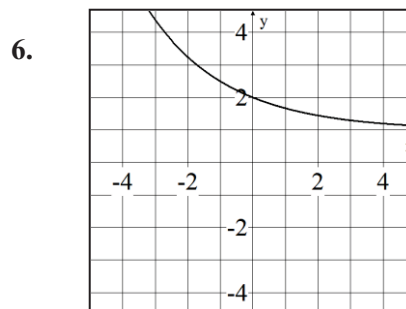
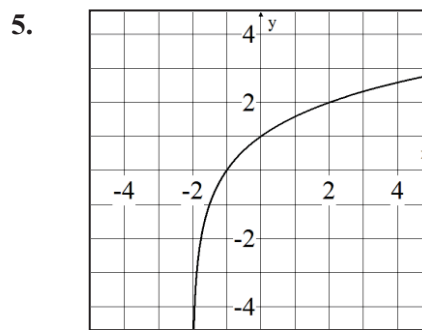
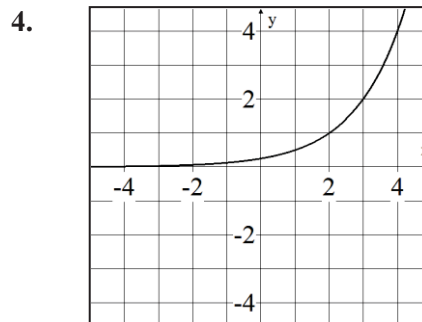
Check Your Progress: Chapter 10

1. $f(x) = 3^x$
 $f(4) = 3^4 = 81$

2. $f(x) = 2^{x-5}$
 $f(2) = 2^{2-5} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

3. $f(x) = 4^{2x+3}$

$$f(-2) = 4^{2(-2)+3} = 4^{-1} = \frac{1}{4^1} = \frac{1}{4}$$



7. $\log_3 81 = x$

$3^x = 81$

$x = 4$

$\log_3 81 = 4$

8. $\log_4\left(\frac{1}{64}\right) = x$

$$4^x = \frac{1}{64}$$

$$x = -3$$

$$\log_4\left(\frac{1}{64}\right) = -3$$

9. $\log_5\left(\frac{1}{5}\right) = x$

$$5^x = \frac{1}{5}$$

$$x = -1$$

$$\log_5\left(\frac{1}{5}\right) = -1$$

10. $\log_7 7^{33} = x$

$$7^x = 7^{33}$$

$$x = 33$$

$$\log_7 7^{33} = 33$$

11. $\log_5 x = 4$

$$5^4 = x$$

$$x = 625$$

12. $\log_3 x = -3$

$$3^{-3} = x$$

$$x = \frac{1}{27}$$

13. $\log_7 x = 1$

$$7^1 = x$$

$$x = 7$$

14. $\log x = -4$

$$10^{-4} = x$$

$$x = 0.0001$$

15. $\log_7(x^2 y^5) = \log_7 x^2 + \log_7 y^5$

$$= 2\log_7 x + 5\log_7 y$$

16. $\log_8\left(\frac{x}{y^3}\right) = \log_8 x - \log_8 y^3$

$$= \log_8 x - 3\log_8 y$$

17. $\log_3\left(\frac{x^2}{\sqrt{yz}}\right) = \log_3 x^2 - \log_3 (yz)^{1/2}$

$$= 2\log_3 x - \frac{1}{2}\log_3 (yz)$$

$$= 2\log_3 x - \frac{1}{2}\log_3 y - \frac{1}{2}\log_3 z$$

18. $3\log_3 x - 4\log_3 y = \log_3 x^3 - \log_3 y^4$

$$= \log_3\left(\frac{x^3}{y^4}\right)$$

19. $\ln x - (4\ln y - 5\ln z) = \ln x - (\ln y^4 - \ln z^5)$

$$= \ln x - \ln \frac{y^4}{z^5} = \ln \frac{x}{\frac{y^4}{z^5}} = \ln \frac{xz^5}{y^4}$$

20. $\frac{1}{2}(\log x + \log y) = \frac{1}{2}\log(xy) = \log(xy)^{1/2}$

$$= \log \sqrt{xy}$$

21. $\log_3 12 = \frac{\log_{10} 12}{\log_{10} 3} \approx 2.2619$

22. $\log_5 0.1 = \frac{\log_{10} 0.1}{\log_{10} 5} \approx -1.4307$

23. $\log_7 5 = \frac{\log_{10} 5}{\log_{10} 7} \approx 0.8271$

INSTRUCTOR USE ONLY

Section 10.4**Concept Check**

- The 1-1 Property of Exponential Functions states that for $b > 0$, $b \neq 1$, if $b^x = b^y$, then $x = y$.
- The 1-1 Property of Logarithms states that for $b > 0$, $b \neq 1$ and x and y positive numbers, if $\log_b x = \log_b y$, then $x = y$.
- $x < 0$
- (ii) and (iii) have no solution.

Objective A Exercises

$$5. \quad 5^{4x-1} = 5^{x-2}$$

$$4x - 1 = x - 2$$

$$3x - 1 = -2$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

The solution is $-\frac{1}{3}$.

$$6. \quad 7^{4x-3} = 7^{2x+1}$$

$$4x - 3 = 2x + 1$$

$$2x - 3 = 1$$

$$2x = 4$$

$$x = 2$$

The solution is 2.

$$7. \quad 8^{x-4} = 8^{5x+8}$$

$$x - 4 = 5x + 8$$

$$-4 = 4x + 8$$

$$4x = -12$$

$$x = -3$$

The solution is -3.

$$8. \quad 10^{4x-5} = 10^{x+4}$$

$$4x - 5 = x + 4$$

$$3x - 5 = 4$$

$$3x = 9$$

$$x = 3$$

The solution is 3.

$$9. \quad 9^x = 3^{x+1}$$

$$3^{2x} = 3^{x+1}$$

$$2x = x + 1$$

$$x = 1$$

The solution is 1.

$$10. \quad 2^{x-1} = 4^x$$

$$2^{x-1} = 2^{2x}$$

$$x - 1 = 2x$$

$$x = -1$$

The solution is -1.

$$11. \quad 8^{x+2} = 16^x$$

$$(2^3)^{x+2} = 2^{4x}$$

$$2^{3x+6} = 2^{4x}$$

$$3x + 6 = 4x$$

$$x = 6$$

The solution is 6.

$$12. \quad 9^{3x} = 81^{x-4}$$

$$(3^2)^{3x} = (3^4)^{x-4}$$

$$3^{6x} = 3^{4x-16}$$

$$6x = 4x - 16$$

$$2x = -16$$

$$x = -8$$

The solution is -8.

13. $16^{2-x} = 32^{2x}$
 $(2^4)^{2-x} = (2^5)^{2x}$
 $2^{8-4x} = 2^{10x}$
 $8 - 4x = 10x$
 $8 = 14x$
 $x = \frac{8}{14} = \frac{4}{7}$

The solution is $\frac{4}{7}$.

14. $27^{2x-3} = 81^{4-x}$
 $(3^3)^{2x-3} = (3^4)^{4-x}$
 $3^{6x-9} = 3^{16-4x}$
 $6x - 9 = 16 - 4x$
 $10x - 9 = 16$
 $10x = 25$
 $x = \frac{25}{10} = \frac{5}{2}$

The solution is $\frac{5}{2}$.

15. $25^{3-x} = 125^{2x-1}$
 $(5^2)^{3-x} = (5^3)^{2x-1}$
 $5^{6-2x} = 5^{6x-3}$
 $6 - 2x = 6x - 3$
 $6 = 8x - 3$
 $9 = 8x$
 $x = \frac{9}{8}$

The solution is $\frac{9}{8}$.

16. $8^{4x-7} = 64^{x-3}$
 $(2^3)^{4x-7} = (2^6)^{x-3}$
 $2^{12x-21} = 2^{6x-18}$
 $12x - 21 = 6x - 18$
 $6x - 21 = -18$
 $6x = 3$
 $x = \frac{3}{6} = \frac{1}{2}$

The solution is $\frac{1}{2}$.

17. $5^x = 6$
 $\log 5^x = \log 6$
 $x \log 5 = \log 6$
 $x = \frac{\log 6}{\log 5}$
 $x \approx 1.1133$
 The solution is 1.1133.

18. $7^x = 10$
 $\log 7^x = \log 10$
 $x \log 7 = \log 10$
 $x = \frac{\log 10}{\log 7}$
 $x \approx 1.1833$
 The solution is 1.1833.

19. $8^{x/4} = 0.4$
 $\log 8^{x/4} = \log 0.4$
 $\frac{x}{4} \log 8 = \log 0.4$
 $\frac{x}{4} = \frac{\log 0.4}{\log 8}$
 $x = 4 \cdot \frac{\log 0.4}{\log 8}$
 $x \approx -1.7626$
 The solution is -1.7626 .

INSTRUCTOR USE ONLY

20. $5^{x/2} = 0.5$

$$\log 5^{x/2} = \log 0.5$$

$$\frac{x}{2} \log 5 = \log 0.5$$

$$\frac{x}{2} = \frac{\log 0.5}{\log 5}$$

$$x = 2 \cdot \frac{\log 0.5}{\log 5}$$

$$x \approx -0.8614$$

The solution is -0.8614 .

21. $2^{3x} = 5$

$$\log 2^{3x} = \log 5$$

$$3x \log 2 = \log 5$$

$$3x = \frac{\log 5}{\log 2}$$

$$3x = 2.3219$$

$$x \approx 0.7740$$

The solution is 0.7740 .

22. $3^{6x} = 0.5$

$$\log 3^{6x} = \log 0.5$$

$$6x \log 3 = \log 0.5$$

$$6x = \frac{\log 0.5}{\log 3}$$

$$6x = -0.6309$$

$$x \approx -0.1052$$

The solution is -0.1052 .

23. $2^{-x} = 7$

$$\log 2^{-x} = \log 7$$

$$-x \log 2 = \log 7$$

$$-x = \frac{\log 7}{\log 2}$$

$$-x = 2.8074$$

$$x \approx -2.8074$$

The solution is -2.8074 .

24. $3^{-x} = 14$

$$\log 3^{-x} = \log 14$$

$$-x \log 3 = \log 14$$

$$-x = \frac{\log 14}{\log 3}$$

$$-x = 2.4022$$

$$x \approx -2.4022$$

The solution is -2.4022 .

25. $2^{x-1} = 6$

$$\log 2^{x-1} = \log 6$$

$$(x-1) \log 2 = \log 6$$

$$x-1 = \frac{\log 6}{\log 2}$$

$$x = \frac{\log 6}{\log 2} + 1$$

$$x \approx 3.5850$$

The solution is 3.5850 .

26. $4^{x+1} = 9$

$$\log 4^{x+1} = \log 9$$

$$(x+1) \log 4 = \log 9$$

$$x+1 = \frac{\log 9}{\log 4}$$

$$x = \frac{\log 9}{\log 4} - 1$$

$$x \approx 0.5850$$

The solution is 0.5850 .

27. $3^{2x-1} = 4$

$$\log 3^{2x-1} = \log 4$$

$$(2x-1) \log 3 = \log 4$$

$$2x-1 = \frac{\log 4}{\log 3}$$

$$2x-1 = 1.2619$$

$$2x = 2.2619$$

$$x \approx 1.1309$$

The solution is 1.1309 .

28. $4^{-x+2} = 12$
 $\log 4^{-x+2} = \log 12$
 $(-x + 2) \log 4 = \log 12$
 $-x + 2 = \frac{\log 12}{\log 4}$
 $-x + 2 = 1.7925$
 $-x = -0.2075$
 $x \approx 0.2075$
 The solution is 0.2075.

29. $\left(\frac{1}{2}\right)^{x+1} = 3$
 $\log\left(\frac{1}{2}\right)^{x+1} = \log 3$
 $(x + 1) \log\left(\frac{1}{2}\right) = \log 3$
 $x + 1 = \frac{\log 3}{\log \frac{1}{2}}$
 $x = \frac{\log 3}{\log \frac{1}{2}} - 1$
 $x \approx -2.5850$
 The solution is -2.5850.

30. $\left(\frac{3}{5}\right)^{-2x} = 2$
 $\log\left(\frac{3}{5}\right)^{-2x} = \log 2$
 $-2x \log\left(\frac{3}{5}\right) = \log 2$
 $-2x = \frac{\log 2}{\log \frac{3}{5}}$
 $-2x = -1.3569$
 $x \approx 0.6785$
 The solution is 0.6785.

31. $3 \cdot 2^x = 7$
 $\log(3 \cdot 2^x) = \log 7$
 $\log 3 + \log 2^x = \log 7$
 $\log 3 + x \log 2 = \log 7$
 $x \log 2 = \log 7 - \log 3$
 $x = \frac{\log 7 - \log 3}{\log 2}$
 $x \approx 1.2224$
 The solution is 1.2224.

32. $5 \cdot 3^{2-x} = 4$
 $\log(5 \cdot 3^{2-x}) = \log 4$
 $\log 5 + \log 3^{2-x} = \log 4$
 $\log 5 + (2 - x) \log 3 = \log 4$
 $(2 - x) \log 3 = \log 4 - \log 5$
 $2 - x = \frac{\log 4 - \log 5}{\log 3}$
 $2 - x = -0.2031$
 $-x = -2.2031$
 $x \approx 2.2031$
 The solution is 2.2031.

33. $7 = 10\left(\frac{1}{2}\right)^{x/8}$
 $\log 7 = \log 10\left(\frac{1}{2}\right)^{x/8}$
 $\log 7 = \log 10 + \log\left(\frac{1}{2}\right)^{x/8}$
 $\log 7 = \log 10 + \frac{x}{8} \log \frac{1}{2}$
 $\log 7 - \log 10 = \frac{x}{8} \log \frac{1}{2}$
 $\frac{\log 7 - \log 10}{\log \frac{1}{2}} = \frac{x}{8}$
 $0.5146 = \frac{x}{8}$
 $4.1166 \approx x$
 The solution is 4.1166.

INSTRUCTOR USE ONLY

$$34. 8 = 15\left(\frac{1}{2}\right)^{x/22}$$

$$\log 8 = \log 15\left(\frac{1}{2}\right)^{x/22}$$

$$\log 8 = \log 15 + \log\left(\frac{1}{2}\right)^{x/22}$$

$$\log 8 = \log 15 + \frac{x}{22}\log \frac{1}{2}$$

$$\log 8 - \log 15 = \frac{x}{22}\log \frac{1}{2}$$

$$\frac{\log 8 - \log 15}{\log \frac{1}{2}} = \frac{x}{22}$$

$$0.9069 = \frac{x}{22}$$

$$19.9516 \approx x$$

The solution is 19.9516.

$$35. 15 = 12(e)^{0.05x}$$

$$\ln 15 = \ln 12(e)^{0.05x}$$

$$\ln 15 = \ln 12 + \ln(e)^{0.05x}$$

$$\ln 15 = \ln 12 + 0.05x$$

$$\ln 15 - \ln 12 = 0.05x$$

$$0.2231 = 0.05x$$

$$4.4629 \approx x$$

The solution is 4.4629.

$$36. 7 = 42(e)^{-3x}$$

$$\ln 7 = \ln 42(e)^{-3x}$$

$$\ln 7 = \ln 42 + \ln(e)^{-3x}$$

$$\ln 7 = \ln 42 - 3x$$

$$\ln 7 - \ln 42 = -3x$$

$$-1.7918 = -3x$$

$$0.5973 \approx x$$

The solution is 0.5973.

Objective B Exercises

$$37. \log x = \log(1-x)$$

$$x = 1-x$$

$$2x = 1$$

$$x = \frac{1}{2}$$

The solution is $\frac{1}{2}$.

$$38. \ln(3x-2) = \ln(x+1)$$

$$3x-2 = x+1$$

$$2x-2 = 1$$

$$2x = 3$$

$$x = \frac{3}{2}$$

The solution is $\frac{3}{2}$.

$$39. \ln(3x+2) = \ln(5x+4)$$

$$3x+2 = 5x+4$$

$$-2x = 2$$

$$x = -1$$

When we substitute $x = -1$ in either side of the equation, we get a logarithm of a negative number.

Because the logarithm of a negative number is not a real number, there is no solution.

$$40. \log_3(x-2) = \log_3(2x)$$

$$x-2 = 2x$$

$$-2 = x$$

When we substitute $x = -2$ in either side of the equation, we get a logarithm of a negative number.

Because the logarithm of a negative number is not a real number, there is no solution.